techniques of Integration

ARITHMATIC SIMPLIFICATION

- USING ALG TO SIMPLIFY EXPRESSION TO HELP SOLVE IT EASIER
- CHAIN RULE → OUTSIDE TO INSIDE
- MULTIPLY → $f(x) \times g(x) = f'(x)g(x) + f(x)g'(x)$
- DIVISION → $f(x) \div g(x) = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$

U-SUBSTITUTION

- FIND A FUNCTION HARD TO INTEGRATE & SET TO u
- ADJUST BOUNDS BASED ON u(x)
  - IE $\int_2^4 x+2\,dx,\ u = x+2 \rightarrow \int_4^6 u\,du$
- INTEGRATE

INTEGRATION BY PARTS

- IDENTIFY HARD TO INTEGRATE FUNCTION & SET TO u
- REST IS SET TO dv
- SOLVE FOR $uv - \int v\,du$
- REPEAT (FOR)
  - COS/SIN:
    - UNTIL IT LOOKS SIMILAR TO BASE FUNCTION (USUALLY 2X)
  - MULTI POWER FUNCTIONS:
    - UNTIL $x^{\square}$ → POWER = 1

— REMEMBER u + dv STAY SAME DISPITE CHANGE —

- FOR TAN + SEC
  - $\int \tan^{\#} x \sec^2 dx$
    $u = \tan x \quad du = \sec^2 dx$
    $\int u^{\#}\,du\,du$
  - $\sec^{\#} x \sec x \tan x\,dx$
    $u = \sec^{\#} x \quad du = \sec x \tan x\,dx$
- RULES FOR ln(x)
  - $\ln(x^{\frac{1}{2}}) = \frac{1}{2}\ln(x)$
  - $\ln(a+b)$: NULL
  - $\ln(ab) = \ln(a) + \ln(b)$
  - $\ln(\frac{a}{b}) = \ln(a) - \ln(b)$

TRIG SUB

— GOAL IS TO GET TO SUBSTITUTION —

1. IDENTIFY FUNCTIONS TO HAVE SAME ARGUMENT
   - a & b NEED TO BE THE SAME

IF NOT
- INTEGRATE BY PARTS 2X

IF m OR n IS ODD

2a. FIGURE OUT WHICH IS ODD:
- TAKE OUT 1 SO ITS EVEN
  → ($\sin^3 \rightarrow \sin^2 \sin$)
- IF m IS ODD
  - CHANGE $\sin^2$ TO $1 - \cos^2$
- IF n IS ODD
  - CHANGE $\cos^2$ TO $1 - \sin^2$

IF m OR n IS EVEN

2b. - $\sin^2 = \frac{1 - \cos 2x}{2}$
- $\cos^2 = \frac{1 + \cos 2x}{2}$

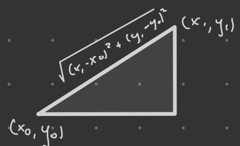
FOR TAN + SEC

$\int \tan^{\#} x \sec^2 dx$
$u = \tan \quad du = \sec^2$
$\int u^{\#}\,du$

$\int \sec^{\#} x \sec x \tan x\,dx$
$u = \sec^{\#} x \quad du = \sec \tan\,dx$

$\int \sec^{2n+1} x\,dx$ → INTEGRATE BY PARTS
- RARELY SEE

TRIG SUBSTITUTION

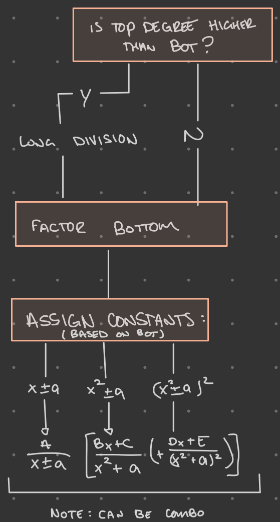


IDEA: FIND PERFECT SQUARE

$1 - \sin^2 = \cos^2$
$1 + \tan^2 = \sec^2$
$\sec^2 - 1 = \tan^2$

IF WE HAVE	SUB	SO
$\sqrt{a^2 - u^2}$	$u = a\sin\theta$	$= a\cos\theta$
$\sqrt{a^2 + u^2}$	$u = a\tan\theta$	$= a\sec^2\theta$
$\sqrt{u^2 - a^2}$	$u = a\sec\theta$	$= a\sec\tan$

PARTIAL FRACTIONS



NUMERICAL INTEGRATION + ERROR

$\Delta x = \frac{b-a}{n}$

TRAPEZOIDAL $= \frac{\Delta x}{2}(y_0 + 2y_1 + 2y_2 \cdots + 2y_{n-1} + y_n)$

$|\text{ERROR}| \le \frac{m(b-a)^3}{12n^2} \qquad m = |f''(x)| \le K \quad a \le x \le b$

SIMPSONS $= \frac{\Delta x}{3}(y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 \cdots + 4y_{n-1} + y_n)$

$|\text{ERROR}| \le \frac{m(b-a)^5}{180n^4} \qquad m = |f^{4}(x)| \le K \quad a \le x \le b$

IMPROPER INTEGRATION

$\int_1^{\infty} \frac{1}{x^p}$
- IF $p > 1$ → CONVERGES
- OTHERWISE DIVERGES