

## Estructuras

### RMQ (static)

Dado un arreglo y una operacion asociativa *idempotente*,  $get(i, j)$  opera sobre el rango  $[i, j]$ .  
 Restriccion:  $LVL \geq \text{ceil}(\log n)$ ; Usar  $[]$  para llenar arreglo y luego  $build()$ .

```

1 struct RMQ{
2     #define LVL 10
3     tipo vec[LVL] [1<<(LVL+1)];
4     tipo &operator[] (int p){return vec[0] [p];}
5     tipo get(int i, int j) { //intervalo [i,j]
6         int p = 31-__builtin_clz(j-i);
7         return min(vec[p] [i], vec[p] [j-(1<<p)]);
8     }
9     void build(int n) { //O(nlogn)
10         int mp = 31-__builtin_clz(n);
11         forn(p, mp) forn(x, n-(1<<p))
12             vec[p+1] [x] = min(vec[p] [x], vec[p] [x+(1<<p)]);
13     };

```

### RMQ (dynamic)

```

1 //Dado un arreglo y una operacion asociativa con neutro, get(i, j) opera sobre el
  rango [i, j].
2 #define MAXN 100000
3 #define operacion(x, y) max(x, y)
4 const int neutro=0;
5 struct RMQ{
6     int sz;
7     tipo t[4*MAXN];
8     tipo &operator[] (int p){return t[sz+p];}
9     void init(int n){ //O(nlgn)
10         sz = 1 << (32-__builtin_clz(n));
11         forn(i, 2*sz) t[i]=neutro;
12     }
13     void updall(){ //O(n)
14         dforn(i, sz) t[i]=operacion(t[2*i], t[2*i+1]);}
15     tipo get(int i, int j){return get(i,j,1,0,sz);} // [i,j] !
16     tipo get(int i, int j, int n, int a, int b){ //O(lgn)
17         if(j<=a || i>=b) return neutro;
18         if(i<=a && b<=j) return t[n];
19         int c=(a+b)/2;
20         return operacion(get(i, j, 2*n, a, c), get(i, j, 2*n+1, c, b));
21     }
22     void set(int p, tipo val){ //O(lgn)
23         for(p+=sz; p>0 && t[p]!=val;){
24             t[p]=val;
25             p/=2;

```

```

26         val=operacion(t[p*2], t[p*2+1]);
27     }
28 }
29 }rmq;
30 //Usage:
31 cin >> n; rmq.init(n); forn(i, n) cin >> rmq[i]; rmq.updall();

```

### RMQ (lazy)

```

1 //Dado un arreglo y una operacion asociativa con neutro, get(i, j) opera sobre el
  rango [i, j].
2 typedef int Elem; //Elem de los elementos del arreglo
3 typedef int Alt; //Elem de la alteracion
4 #define operacion(x,y) x+y
5 const Elem neutro=0; const Alt neutro2=0;
6 #define MAXN 1024000
7 struct RMQ{
8     int sz;
9     Elem t[4*MAXN];
10    Alt dirty[4*MAXN]; //las alteraciones pueden ser de distinto Elem
11    Elem &operator[] (int p){return t[sz+p];}
12    void init(int n){ //O(nlgn)
13        sz = 1 << (32-__builtin_clz(n));
14        forn(i, 2*sz) t[i]=neutro;
15        forn(i, 2*sz) dirty[i]=neutro2;
16    }
17    void updall(){ //O(n)
18        dforn(i, sz) t[i]=operacion(t[2*i], t[2*i+1]);}
19    void opAltT(int n,int a,int b){ //altera el valor del nodo n segun su dirty y el
      intervalo que le corresponde.
20        t[n] += dirty[n]*(b-a);
21    } //en este caso la alteracion seria sumarle a todos los elementos del intervalo
      [a,b) el valor dirty[n]
22    void opAltD(int n ,Alt val){
23        dirty[n] += val;
24    } //actualiza el valor de Dirty "sumandole" val. podria cambiar el valor de dirty
      dependiendo de la operacion que se quiera al actualizar un rango. Ej:11402.
      cpp
25    void push(int n, int a, int b){ //propaga el dirty a sus hijos
26        if(dirty[n] !=neutro2){
27            opAltT(n,a,b); //t[n]+=dirty[n]*(b-a); //altera el nodo
28            if(n<sz){
29                opAltD(2*n,dirty[n]); //dirty[2*n]+=dirty[n];
30                opAltD(2*n+1,dirty[n]); //dirty[2*n+1]+=dirty[n];
31            }
32            dirty[n]=neutro2;
33        }
34    }

```

```

35 Elem get(int i, int j, int n, int a, int b){//O(lgn)
36     if(j<=a || i>=b) return neutro;
37     push(n, a, b);//corrige el valor antes de usarlo
38     if(i<=a && b<=j) return t[n];
39     int c=(a+b)/2;
40     return operacion(get(i, j, 2*n, a, c), get(i, j, 2*n+1, c, b));
41 }
42 Elem get(int i, int j){return get(i,j,1,0,sz);}
43 //altera los valores en [i, j] con una alteracion de val
44 void alterar(Alt val, int i, int j, int n, int a, int b){//O(lgn)
45     push(n, a, b);
46     if(j<=a || i>=b) return;
47     if(i<=a && b<=j){
48         opAltD(n,val);//actualiza el valor de Dirty por val.
49         push(n,a,b);
50         return;//este nodo esta totalmente contenido por el intervalo a alterar, no
                    es necesario que se lo pases a los hijos.. por ahora..
51     }
52     int c=(a+b)/2;
53     alterar(val, i, j, 2*n, a, c), alterar(val, i, j, 2*n+1, c, b);
54     t[n]=operacion(t[2*n], t[2*n+1]);//por esto es el push de arriba
55 }
56 void alterar(Alt val, int i, int j){alterar(val,i,j,1,0,sz);}
57
58 //setea de a un elemento. Esto lo "hace" dinámico.
59 void set(int p, Elem val){//O(lgn)
60     if(p<0) return; //OJO chequear que p no sea muy grande
61     this->get(p,p+1); //para que acomode los dirty del camino de la raíz a p
62     int a=p, b=p+1, ancho=1, vecino;
63     for(p+=sz; p>0 && t[p]!=val; ancho*=2){
64         t[p]=val;
65         if(p&1){ vecino=p-1; push(vecino,a,b); a-=ancho; }
66         else{ vecino=p+1; push(vecino,a,b); b+=ancho; }
67         p/=2;
68         val=operacion(t[p*2], t[p*2+1]);
69     }
70 }
71 };

```

## RMQ (persistente)

```

1 typedef int tipo;
2 tipo oper(const tipo &a, const tipo &b){
3     return a+b;
4 }
5 struct node{
6     tipo v; node *l,*r;
7     node(tipo v):v(v), l(NULL), r(NULL) {}

```

```

8     node(node *l, node *r) : l(l), r(r){
9         if(!l) v=r->v;
10        else if(!r) v=l->v;
11        else v=oper(l->v, r->v);
12    }
13 };
14 node *build (tipo *a, int tl, int tr) {//modificar para que tome tipo a
15     if (tl+1==tr) return new node(a[tl]);
16     int tm=(tl + tr)>>1;
17     return new node(build(a, tl, tm), build(a, tm, tr));
18 }
19 node *update(int pos, int new_val, node *t, int tl, int tr){
20     if (tl+1==tr) return new node(new_val);
21     int tm=(tl+tr)>>1;
22     if(pos < tm) return new node(update(pos, new_val, t->l, tl, tm), t->r);
23     else return new node(t->l, update(pos, new_val, t->r, tm, tr));
24 }
25 tipo get(int l, int r, node *t, int tl, int tr){
26     if(l==tl && tr==r) return t->v;
27     int tm=(tl + tr)>>1;
28     if(r<=tm) return get(l, r, t->l, tl, tm);
29     else if(l>=tm) return get(l, r, t->r, tm, tr);
30     return oper(get(l, tm, t->l, tl, tm), get(tm, r, t->r, tm, tr));
31 }

```

## Union Find

```

1 struct UnionFind{
2     vector<int> f;//the array contains the parent of each node
3     void init(int n){f.clear(); f.insert(f.begin(), n, -1);}
4     int comp(int x){return (f[x]==-1?x:f[x]=comp(f[x]));};//O(1)
5     bool join(int i, int j) {
6         bool con=comp(i)==comp(j);
7         if(!con) f[comp(i)] = comp(j);
8         return con;
9     };

```

## Disjoint Intervals

```

1 bool operator< (const ii &a, const ii &b) {return a.fst<b.fst;}
2 //Stores intervals as [first, second]
3 //in case of a collision it joins them in a single interval
4 struct disjoint_intervals {
5     set<ii> segs;
6     void insert(ii v) {//O(lgn)
7         if(v.snd-v.fst==0.) return;//OJO
8         set<ii>::iterator it,at;
9         at = it = segs.lower_bound(v);

```

```

10     if (at!=segs.begin() && (--at)->snd >= v.fst)
11         v.fst = at->fst, --it;
12     for(; it!=segs.end() && it->fst <= v.snd; segs.erase(it++))
13         v.snd=max(v.snd, it->snd);
14     segs.insert(v);
15 }
16 };

```

## RMQ (2D)

```

1 struct RMQ2D{//n filas x m columnas
2     int sz;
3     RMQ t[4*MAXN];
4     void init(int n, int m){//O(n*m)
5         sz = 1 << (32-__builtin_clz(n));
6         forn(i, 2*sz) t[i].init(m); }
7     void set(int i, int j, tipo val){//O(lgm.lgn)
8         for(i+=sz; i>0;){
9             t[i].set(j, val);
10            i/=2;
11            val=operacion(t[i*2][j], t[i*2+1][j]);
12        } }
13     tipo get(int i1, int j1, int i2, int j2){return get(i1,j1,i2,j2,1,0,sz);}
14     //O(lgm.lgn), rangos cerrado abierto
15     int get(int i1, int j1, int i2, int j2, int n, int a, int b){
16         if(i2<=a || i1>=b) return neutro;
17         if(i1<=a && b<=i2) return t[n].get(j1, j2);
18         int c=(a+b)/2;
19         return operacion(get(i1, j1, i2, j2, 2*n, a, c),
20             get(i1, j1, i2, j2, 2*n+1, c, b));
21     }
22 } rmq;
23 //Example to initialize a grid of M rows and N columns:
24 RMQ2D rmq; rmq.init(n,m);
25 forn(i, n) forn(j, m){
26     int v; cin >> v; rmq.set(i, j, v);}

```

## Big Int

```

1 #define BASEXP 6
2 #define BASE 1000000
3 #define LMAX 1000
4 struct bint{int l;ll n[LMAX];bint(ll x=0){l=1;forn(i,LMAX){if(x)l=i+1;n[i]=x%BASE;
5     x/=BASE;}}bint(string x){l=(x.size()-1)/BASEXP+1;fill(n,n+LMAX,0);ll r=1;forn(
6     i,sz(x)){n[i/BASEXP]+=r*(x[x.size()-1-i]-'0');r*=10;if(r==BASE)r=1;}}void out
7     (){cout<<n[l-1];dforn(i,l-1)printf("%6.6llu",n[i]);}void invar(){fill(n,n+
8     LMAX,0);while(l>1&&!n[l-1])l--;}};bint operator+(const bint&a,const bint&b){
9     bint c;c.l=max(a.l,b.l);ll q=0;forn(i,c.l)q+=a.n[i]+b.n[i],c.n[i]=q%BASE,q/=

```

```

BASE;if(q)c.n[c.l++]=q;c.invar();return c;}pair<bint,bool>lresta(const bint&a,
const bint&b){bint c;c.l=max(a.l,b.l);ll q=0;forn(i,c.l)q+=a.n[i]-b.n[i],c.n[i
]=(q+BASE)%BASE,q=(q+BASE)/BASE-1;c.invar();return make_pair(c,!q);}bint&
operator-=(bint&a,const bint&b){return a=lresta(a,b).first;}bint operator-(
const bint&a,const bint&b){return lresta(a,b).first;}bool operator<(const bint
&a,const bint&b){return!lresta(a,b).second;}bool operator<=(const bint&a,const
bint&b){return lresta(b,a).second;}bool operator==(const bint&a,const bint&b)
{return a<=b&&b<=a;}bint operator*(const bint&a,ll b){bint c;ll q=0;forn(i,a.l
)q+=a.n[i]*b,c.n[i]=q%BASE,q/=BASE;c.l=a.l;while(q)c.n[c.l++]=q%BASE,q/=BASE;c
.invar();return c;}bint operator*(const bint&a,const bint&b){bint c;c.l=a.l+b.
l;fill(c.n,c.n+b.l,0);forn(i,a.l){ll q=0;forn(j,b.l)q+=a.n[i]*b.n[j]+c.n[i+j],
c.n[i+j]=q%BASE,q/=BASE;c.n[i+b.l]=q;}c.invar();return c;}pair<bint,ll>ldiv(
const bint&a,ll b){bint c;ll rm=0;dforn(i,a.l){rm=rm*BASE+a.n[i];c.n[i]=rm/b;
rm%=b;}c.l=a.l;c.invar();return make_pair(c,rm);}bint operator/(const bint&a,
ll b){return ldiv(a,b).first;}ll operator%(const bint&a,ll b){return ldiv(a,b)
.second;}pair<bint,bint>ldiv(const bint&a,const bint&b){bint c;bint rm=0;dforn
(i,a.l){if(rm.l==1&&!rm.n[0])rm.n[0]=a.n[i];else{dforn(j,rm.l)rm.n[j+1]=rm.n[j
];rm.n[0]=a.n[i];rm.l++;}ll q=rm.n[b.l]*BASE+rm.n[b.l-1];ll u=q/(b.n[b.l-1]+1)
;ll v=q/b.n[b.l-1]+1;while(u<v-1){ll m=(u+v)/2;if(b*m<=rm)u=m;else v=m;}c.n[i
]=u;rm-=b*u;}c.l=a.l;c.invar();return make_pair(c,rm);}bint operator/(const
bint&a,const bint&b){return ldiv(a,b).first;}bint operator%(const bint&a,const
bint&b){return ldiv(a,b).second;}

```

## Modnum

```

1 //lindos valores para hash
2 #define MOD 1000000000000000009LL
3 #define PRIME 1009LL
4
5 mnum inv[MOD]; //inv[i]*i=1 mod MOD
6 void calc(int p){//calcula inversos de 1 a p en O(p)
7     inv[1]=1;
8     forr(i, 2, p) inv[i] = p - (p/i)*inv[p/i];
9 }
10
11 ll mul(ll a, ll b, ll m) { //hace (a*b)%m
12     ll q = (ll)((long double)a*b/m);
13     ll r = a*b-m*q;
14     while(r<0) r += m;
15     while(r>=m) r -= m;
16     return r;
17 }
18
19 struct mnum{
20     static const tipo mod=MOD;
21     tipo v;
22     mnum(tipo v=0): v((v%mod+mod)%mod) {}
23     mnum operator+(mnum b){return v+b.v;}

```

```

24 mnum operator-(mnum b){return v-b.v;}
25 mnum operator*(mnum b){return v*b.v;} //Si mod<=1e9+9
26 //~ mnum operator*(mnum b){return mul(v,b.v,mod);} //Si mod<=1e18+9
27 mnum operator^(ll n){ //O(log n)
28     if(!n) return 1;
29     mnum q = (*this)^(n/2);
30     return n%2 ? q*q*v : q*q;
31 }
32 mnum operator/(mnum n){return ~n*v;} //O(log n) //OJ0! mod tiene que ser primo!
    Sino no siempre existe inverso
33
34 mnum operator~(){ //inverso, O(log mod)
35     assert(v!=0);
36     //return (*this)^(eulerphi(mod)-1); //si mod no es primo (sacar a mano) PROBAR
        ! Ver si rta*x == 1 modulo mod
37     return (*this)^(mod-2); //si mod es primo
38 }
39 };

```

## Convex Hull Trick

```

1 struct Line{tipo m,h};
2 tipo inter(Line a, Line b){
3     tipo x=b.h-a.h, y=a.m-b.m;
4     return x/y+(x/y?((x>0)^(y>0)):0); //==ceil(x/y)
5 }
6 struct CHT {
7     vector<Line> c;
8     bool mx;
9     int pos;
10    CHT(bool mx=0):mx(mx),pos(0){ //mx=1 si las query devuelven el max
11    inline Line acc(int i){return c[c[0].m>c.back().m? i : sz(c)-1-i];}
12    inline bool irre(Line x, Line y, Line z){
13        return c[0].m>z.m? inter(y, z) <= inter(x, y)
14            : inter(y, z) >= inter(x, y);
15    }
16    void add(tipo m, tipo h) { //O(1), los m tienen que entrar ordenados
17        if(mx) m*=-1, h*=-1;
18        Line l=(Line){m, h};
19        if(sz(c) && m==c.back().m) { l.h=min(h, c.back().h), c.pop_back(); if(pos)
20            pos--; }
21        while(sz(c)>=2 && irre(c[sz(c)-2], c[sz(c)-1], l)) { c.pop_back(); if(pos)
22            pos--; }
23        c.pb(l);
24    }
25    inline bool fbin(tipo x, int m) {return inter(acc(m), acc(m+1))>x;}
26    tipo eval(tipo x){
27        int n = sz(c);

```

```

26 //query con x no ordenados O(lgn)
27 int a=-1, b=n-1;
28 while(b-a>1) { int m = (a+b)/2;
29     if(fbin(x, m)) b=m;
30     else a=m;
31 }
32 return (acc(b).m*x+acc(b).h)*(mx?-1:1);
33 //query O(1)
34 while(pos>0 && fbin(x, pos-1)) pos--;
35 while(pos<n-1 && !fbin(x, pos)) pos++;
36 return (acc(pos).m*x+acc(pos).h)*(mx?-1:1);
37 }
38 } ch;

```

## Convex Hull Trick (Dynamic)

```

1 const ll is_query = -(1LL<<62);
2 struct Line {
3     ll m, b;
4     mutable multiset<Line>::iterator it;
5     const Line *succ(multiset<Line>::iterator it) const;
6     bool operator<(const Line& rhs) const {
7         if (rhs.b != is_query) return m < rhs.m;
8         const Line *s=succ(it);
9         if(!s) return 0;
10        ll x = rhs.m;
11        return b - s->b < (s->m - m) * x;
12    }
13 };
14 struct HullDynamic : public multiset<Line>{ // will maintain upper hull for
    maximum
15    bool bad(iterator y) {
16        iterator z = next(y);
17        if (y == begin()) {
18            if (z == end()) return 0;
19            return y->m == z->m && y->b <= z->b;
20        }
21        iterator x = prev(y);
22        if (z == end()) return y->m == x->m && y->b <= x->b;
23        return (x->b - y->b)*(z->m - y->m) >= (y->b - z->b)*(y->m - x->m);
24    }
25    iterator next(iterator y){return ++y;}
26    iterator prev(iterator y){return --y;}
27    void insert_line(ll m, ll b) {
28        iterator y = insert((Line) { m, b });
29        y->it=y;
30        if (bad(y)) { erase(y); return; }
31        while (next(y) != end() && bad(next(y))) erase(next(y));

```

```

32     while (y != begin() && bad(prev(y))) erase(prev(y));
33 }
34 ll eval(ll x) {
35     Line l = *lower_bound((Line) { x, is_query });
36     return l.m * x + l.b;
37 }
38 }h;
39 const Line *Line::succ(multiset<Line>::iterator it) const{
40     return (++it==h.end())? NULL : &*it;}
```

## Set con búsq. binaria (Treap)

```

1 #include<bits/stdc++.h>
2 #include<ext/pb_ds/assoc_container.hpp>
3 #include<ext/pb_ds/tree_policy.hpp>
4 using namespace __gnu_pbds;
5 using namespace std;
6
7 template <typename T>
8 using ordered_set = tree<T, null_type, less<T>, rb_tree_tag,
9     tree_order_statistics_node_update>;
10
11 //o bien usar así:
12 typedef tree<int,null_type,less<int>,//key, mapped type, comparator. Se puede usar
13     como map<a,b> poniendo tree<a,b,...
14     rb_tree_tag,tree_order_statistics_node_update> set_t;
15
16 int main(){
17     ordered_set<int> s;
18     s.insert(1);
19     s.insert(3);
20     cout << s.order_of_key(3) << endl; // s.order_of_key(x): number of elements in s
21     strictly less than x.
22     cout << *s.find_by_order(0) << endl; // s.find_by_order(i): i-th smallest number
23     in s. (empieza en 0)
24     cout << *s.lower_bound(1) << endl;
25 }
```

## Algoritmos

### Longest Increasing Subsequence

```

1 //Para non-increasing, cambiar comparaciones y revisar busq binaria
2 //Given an array, paint it in the least number of colors so that each color turns
3 //to a non-increasing subsequence.
4 //Solution:Min number of colors=Length of the longest increasing subsequence
5 int N, a[MAXN]; //secuencia y su longitud
6 ii d[MAXN+1]; //d[i]=ultimo valor de la subsecuencia de tamaño i
```

```

6 int p[MAXN]; //padres
7 vector<int> R; //respuesta
8 void rec(int i){
9     if(i==1) return;
10    R.push_back(a[i]);
11    rec(p[i]);
12 }
13 int lis(){ //O(nlogn)
14    d[0] = ii(-INF, -1); forn(i, N) d[i+1]=ii(INF, -1);
15    forn(i, N){
16        int j = upper_bound(d, d+N+1, ii(a[i], INF))-d;
17        if (d[j-1].first < a[i] && a[i] < d[j].first){
18            p[i]=d[j-1].second;
19            d[j] = ii(a[i], i);
20        }
21    }
22    R.clear();
23    dforn(i, N+1) if(d[i].first!=INF){
24        rec(d[i].second); //reconstruir
25        reverse(R.begin(), R.end());
26        return i; //longitud
27    }
28    return 0;
29 }
```

## Optimizaciones para DP

```

1 convex hull 1: dp[i] = min{dp[j] + b[j] * a[i]}, j < i. Si se cumple b[j] >= b[j
2     +1] y a[i] <= a[i+1] entonces pasa de O(n^2) a O(n) sino pasa a O(nlogn)
3
4 convex hull 2: dp[i][j] = min{dp[i-1][k] + b[k] * a[j]}, k < j. Si se cumple b[k]
5     >= b[k+1] y a[j] <= a[j+1] entonces pasa de O(kn^2) a O(kn) sino pasa O(knlogn)
6
7 divide and conquer: dp[i][j] = min{dp[i-1][k] + C[k+1][j]}, k < j. Se debe cumplir
8     : A[i][j] <= A[i][j+1]. Pasa de O(kn^2) a O(knlogn)
9     Donde A[i][j] es el minimo k tal que dp[i][j] = dp[i-1][k] + C[k][j]
10    Tambien es aplicable si:
11    C[a][c] + C[b][d] <= C[a][d] + C[b][c] y C[b][c] <= C[a][d], a<=b<=c<=d
12
13 def ComputeDP(i, jleft, jright, kleft, kright):
14     # Select the middle point
15     jmid = (jleft + jright) / 2
16     # Compute the value of dp[i][jmid] by definition of DP
17     dp[i][jmid] = +INFINITY
18     bestk = -1
```

```

18 for k in range[kleft, jmid]:
19     if dp[i - 1][k] + C[k + 1][jmid] < best:
20         dp[i][jmid] = dp[i - 1][k] + C[k + 1][jmid]
21         bestk = k
22 # Divide and conquer
23 if jleft < jmid:
24     ComputeDP(i, jleft, jmid, kleft, bestk)
25 if jmid + 1 < jright:
26     ComputeDP(i, jmid + 1, jright, bestk, kright)
27
28 def ComputeFullDP:
29     Initialize dp for i = 0 somehow
30     for i in range(1, m):
31         ComputeDP(i, 0, n, 0, n)
32
33 knuth: dp[i][j] = min{dp[i][k] + dp[k][j] + C[i][j], i < k < j. Se debe cumplir: A[i
    , j - 1] <= A[i, j] <= A[i + 1, j]. Pasa de O(n^3) a O(n^2)
35 Donde A[i][j] es el minimo k tal que dp[i][j] = dp[i][k] + dp[k][j] + C[i][j]
36 Tambien es aplicable si:
37 C[a][c] + C[b][d] <= C[a][d] + C[b][c] y C[b][c] <= C[a][d], a <= b <= c <= d
38
39 for (int s = 0; s <= k; s++)
40     for (int l = 0; l + s <= k; l++) {
41         int r = l + s;
42         if (s < 2) {
43             res[l][r] = 0;
44             A[l][r] = 1;
45             continue;
46         }
47         int aleft = A[l][r - 1];
48         int aright = A[l + 1][r];
49         res[l][r] = INF;
50         for (int a = max(l + 1, aleft); a <= min(r - 1, aright); a++) {
51             in the bounds only
52             int act = res[l][a] + res[a][r] + (C[l][r]);
53             if (res[l][r] > act) {
54                 res[l][r] = act;
55                 A[l][r] = a;
56             }
57         }

```

## Strings

### KMP

```

1 string T; //cadena donde buscar(what)
2 string P; //cadena a buscar(what)
3 int b[MAXLEN]; //back table b[i] maximo borde de [0..i)
4 void kmppre(){ //by gabina with love
5     int i = 0, j = -1; b[0] = -1;
6     while(i < sz(P)){
7         while(j >= 0 && P[i] != P[j]) j = b[j];
8         i++, j++, b[i] = j;
9     }
10 }
11 void kmp(){
12     int i = 0, j = 0;
13     while(i < sz(T)){
14         while(j >= 0 && T[i] != P[j]) j = b[j];
15         i++, j++;
16         if(j == sz(P)) printf("P is found at index %d in T\n", i - j), j = b[j];
17     }
18 }

```

### Trie

```

1 struct trie{
2     map<char, trie> m;
3     bool end = false;
4     void add(const string &s, int p = 0){
5         if(s[p]) m[s[p]].add(s, p + 1);
6         else end = true;
7     }
8     void dfs(){
9         //Do stuff
10        forall(it, m)
11            it->second.dfs();
12    }
13 };

```

### Suffix Array (largo, nlogn)

```

1 #define MAX_N 112345
2 #define rBOUND(x) ((x) < n ? r[(x)] : 0)
3 //sa will hold the suffixes in order.
4 int sa[MAX_N], r[MAX_N], n; // OJO n = s.size()!
5 string s; //input string, n=s.size()
6
7 int f[MAX_N], tmpsa[MAX_N];
8 void countingSort(int k){
9     zero(f);
10    forn(i, n) f[rBOUND(i + k)]++;
11    int sum = 0;

```



```

12   forn(i, max(255, n)){
13       int t=f[i]; f[i]=sum; sum+=t;}
14   forn(i,n)
15       tmpsa[f[rBOUND(sa[i]+k)]++] = sa[i];
16   forn(i,n) sa[i] = tmpsa[i];
17   }
18   void constructsa(){//O(n log n)
19       n = s.size();
20       forn(i,n) sa[i]=i, r[i]=s[i];
21       for(int k=1; k<n; k<=<=1){
22           countingSort(k), countingSort(0);
23           int rank, tmpr[MAX_N];
24           tmpr[sa[0]]=rank=0;
25           forr(i, 1, n)
26               tmpr[sa[i]] = (r[sa[i]]==r[sa[i-1]] && r[sa[i]+k]==r[sa[i-1]+k]) ? rank : ++
                           rank;
27           forn(i,n) r[i]=tmpr[i];
28           if(r[sa[n-1]]==n-1) break;
29       }
30   }
31   void print(){//for debugging
32       forn(i, n)
33           cout << i << '␣' <<
34           s.substr(sa[i], s.find('$',sa[i])-sa[i]) << endl;}

```

## LCP (Longest Common Prefix)

```

1 //Calculates the LCP between consecutives suffixes in the Suffix Array.
2 //LCP[i] is the length of the LCP between sa[i] and sa[i-1]
3 int LCP[MAX_N], phi[MAX_N], PLCP[MAX_N];
4 void computeLCP(){//O(n)
5     phi[sa[0]]=-1;
6     forr(i, 1, n) phi[sa[i]]=sa[i-1];
7     int L=0;
8     forn(i, n){
9         if(phi[i]==-1) {PLCP[i]=0; continue;}
10        while(s[i+L]==s[phi[i]+L]) L++;
11        PLCP[i]=L;
12        L=max(L-1, 0);
13    }
14    forn(i, n) LCP[i]=PLCP[sa[i]];
15 }

```

## Corasick

```

1 struct trie{
2     map<char, trie> next;
3     trie* tran[256]; //transiciones del automata

```

```

4     int idhoja, szhoja; //id de la hoja o 0 si no lo es
5     //link lleva al sufijo mas largo, nxthoja lleva al mas largo pero que es hoja
6     trie *padre, *link, *nxthoja;
7     char pch; //caracter que conecta con padre
8     trie(): next(), tran(), idhoja(), szhoja(), padre(), link(), nxthoja(), pch() {}
9     void insert(const string &s, int id=1, int p=0){//id>0!!!
10         if(p<sz(s)){
11             trie &ch=next[s[p]];
12             tran[(int)s[p]]=&ch;
13             ch.padre=this, ch.pch=s[p];
14             ch.insert(s, id, p+1);
15         }
16         else idhoja=id, szhoja=sz(s);
17     }
18     trie* get_link() {
19         if(!link){
20             if(!padre) link=this; //es la raiz
21             else if(!padre->padre) link=padre; //hijo de la raiz
22             else link=padre->get_link()->get_tran(pch);
23         }
24         return link; }
25     trie* get_tran(int c) {
26         if(!tran[c]) tran[c] = !padre? this : this->get_link()->get_tran(c);
27         return tran[c]; }
28     trie *get_nxthoja(){
29         if(!nxthoja) nxthoja = get_link()->idhoja? link : link->nxthoja;
30         return nxthoja; }
31     void print(int p){
32         if(idhoja) cout << "found␣" << idhoja << "␣at␣position␣" << p-szhoja << endl;
33         if(get_nxthoja()) get_nxthoja()->print(p); }
34     void matching(const string &s, int p=0){
35         print(p); if(p<sz(s)) get_tran(s[p])->matching(s, p+1); }
36 }tri;

```

## Suffix Automaton

```

1 struct state {
2     int len, link;
3     map<char,int> next;
4     state() { }
5 };
6 const int MAXLEN = 10010;
7 state st[MAXLEN*2];
8 int sz, last;
9 void sa_init() {
10     forn(i,sz) st[i].next.clear();
11     sz = last = 0;
12     st[0].len = 0;

```

```

13   st[0].link = -1;
14   ++sz;
15 }
16 // Es un DAG de una sola fuente y una sola hoja
17 // cantidad de endpos = cantidad de apariciones = cantidad de caminos de la clase
   al nodo terminal
18 // cantidad de miembros de la clase = st[v].len-st[st[v].link].len (v>0) = caminos
   del inicio a la clase
19 // El arbol de los suffix links es el suffix tree de la cadena invertida. La
   string de la arista link(v)->v son los caracteres que difieren
20 void sa_extend (char c) {
21     int cur = sz++;
22     st[cur].len = st[last].len + 1;
23     // en cur agregamos la posicion que estamos extendiendo
24     //podria agregar tambien un identificador de las cadenas a las cuales pertenece
   (si hay varias)
25     int p;
26     for (p=last; p!=-1 && !st[p].next.count(c); p=st[p].link) // modificar esta
   linea para hacer separadores unicos entre varias cadenas (c=='$')
27         st[p].next[c] = cur;
28     if (p == -1)
29         st[cur].link = 0;
30     else {
31         int q = st[p].next[c];
32         if (st[p].len + 1 == st[q].len)
33             st[cur].link = q;
34         else {
35             int clone = sz++;
36             // no le ponemos la posicion actual a clone sino indirectamente por el link
   de cur
37             st[clone].len = st[p].len + 1;
38             st[clone].next = st[q].next;
39             st[clone].link = st[q].link;
40             for (; p!=-1 && st[p].next.count(c) && st[p].next[c]==q; p=st[p].link)
41                 st[p].next[c] = clone;
42             st[q].link = st[cur].link = clone;
43         }
44     }
45     last = cur;
46 }

```

## Z Function

```

1 char s[MAXN];
2 int z[MAXN]; // z[i] = i==0 ? 0 : max k tq s[0,k) match with s[i,i+k)
3 void z_function(char s[],int z[]) {
4     int n = strlen(s);
5     forn(i, n) z[i]=0;

```

```

6     for (int i = 1, l = 0, r = 0; i < n; ++i) {
7         if (i <= r) z[i] = min (r - i + 1, z[i - l]);
8         while (i + z[i] < n && s[z[i]] == s[i + z[i]]) ++z[i];
9         if (i + z[i] - 1 > r) l = i, r = i + z[i] - 1;
10    }
11 }

```

## Geometría

### Punto

```

1 const double EPS=1e-9;
2 struct pto{
3     double x, y;
4     pto(double x=0, double y=0):x(x),y(y){}
5     pto operator+(pto a){return pto(x+a.x, y+a.y);}
6     pto operator-(pto a){return pto(x-a.x, y-a.y);}
7     pto operator+(double a){return pto(x+a, y+a);}
8     pto operator*(double a){return pto(x*a, y*a);}
9     pto operator/(double a){return pto(x/a, y/a);}
10    //dot product, producto interno:
11    //Significado: a*b = a.norm * b.norm * cos(ang).
12    double operator*(pto a){return x*a.x+y*a.y;}
13    //module of the cross product or vectorial product:
14    //if a is less than 180 clockwise from b, a~b>0. Significado: abs(a~b) = area
   del paralelogramo.
15    double operator^(pto a){return x*a.y-y*a.x;}
16    //returns true if this is at the left side of line qr
17    bool left(pto q, pto r){return ((q-*this)^(r-*this))>0;}
18    bool operator<(const pto &a) const{return x<a.x-EPS || (abs(x-a.x)<EPS && y<a.y-
   EPS);}
19    bool operator==(pto a){return abs(x-a.x)<EPS && abs(y-a.y)<EPS;}
20    double norm(){return sqrt(x*x+y*y);}
21    double norm_sq(){return x*x+y*y;}
22 };
23 double dist(pto a, pto b){return (b-a).norm();}
24 double dist_sq(pto a, pto b){return (b-a).norm_sq();}
25 typedef pto vec;
26
27 //positivo si aob están en sentido antihorario con un ángulo <180º
28 double angle(pto a, pto o, pto b){ //devuelve radianes! (-pi,pi)
29     pto oa=a-o, ob=b-o;
30     return atan2(oa^ob, oa*ob);}
31
32 //rotate p by theta rads CCW w.r.t. origin (0,0)
33 pto rotate(pto p, double theta){
34     return pto(p.x*cos(theta)-p.y*sin(theta),

```



```

35     p.x*sin(theta)+p.y*cos(theta));
36 }

```

## Orden radial de puntos

```

1 struct Cmp{//orden total de puntos alrededor de un punto r
2     pto r;
3     Cmp(pto r):r(r) {}
4     int cuad(const pto &a) const{
5         if(a.x > 0 && a.y >= 0)return 0;
6         if(a.x <= 0 && a.y > 0)return 1;
7         if(a.x < 0 && a.y <= 0)return 2;
8         if(a.x >= 0 && a.y < 0)return 3;
9         assert(a.x ==0 && a.y==0);
10        return -1;
11    }
12    bool cmp(const pto&p1, const pto&p2)const{
13        int c1 = cuad(p1), c2 = cuad(p2);
14        if(c1==c2) return p1.y*p2.x<p1.x*p2.y;
15        else return c1 < c2;
16    }
17    bool operator()(const pto&p1, const pto&p2) const{
18        return cmp(pto(p1.x-r.x,p1.y-r.y),pto(p2.x-r.x,p2.y-r.y));
19    }
20 };

```

## Line

```

1 int sgn(ll x){return x<0? -1 : !!x;}
2 struct line{
3     line() {}
4     double a,b,c;//Ax+By=C
5     //pto MUST store float coordinates!
6     line(double a, double b, double c):a(a),b(b),c(c){}
7     line(pto p, pto q): a(q.y-p.y), b(p.x-q.x), c(a*p.x+b*p.y) {}
8     int side(pto p){return sgn(ll(a) * p.x + ll(b) * p.y - c);}
9 };
10 bool parallels(line l1, line l2){return abs(l1.a*l2.b-l2.a*l1.b)<EPS;}
11 pto inter(line l1, line l2){//intersection
12     double det=l1.a*l2.b-l2.a*l1.b;
13     if(abs(det)<EPS) return pto(INF, INF);//parallels
14     return pto(l2.b*l1.c-l1.b*l2.c, l1.a*l2.c-l2.a*l1.c)/det;
15 }

```

## Segment

```

1 struct segm{
2     pto s,f;
3     segm(pto s, pto f):s(s), f(f) {}

```

```

4     pto closest(pto p) {//use for dist to point
5         double l2 = dist_sq(s, f);
6         if(l2==0.) return s;
7         double t=((p-s)*(f-s))/l2;
8         if (t<0.) return s;//not write if is a line
9         else if(t>1.)return f;//not write if is a line
10        return s+((f-s)*t);
11    }
12    bool inside(pto p){return abs(dist(s, p)+dist(p, f)-dist(s, f))<EPS;}
13 };
14
15 //NOTA: Si los segmentos son colineales sólo devuelve un punto de intersección
16 pto inter(segm s1, segm s2){
17     if(s1.inside(s2.s)) return s2.s; //Fix cuando son colineales
18     if(s1.inside(s2.f)) return s2.f; //Fix cuando son colineales
19     pto r=inter(line(s1.s, s1.f), line(s2.s, s2.f));
20     if(s1.inside(r) && s2.inside(r)) return r;
21     return pto(INF, INF);
22 }

```

## Rectangle

```

1 struct rect{
2     //lower-left and upper-right corners
3     pto lw, up;
4 };
5 //returns if there's an intersection and stores it in r
6 bool inter(rect a, rect b, rect &r){
7     r.lw=pto(max(a.lw.x, b.lw.x), max(a.lw.y, b.lw.y));
8     r.up=pto(min(a.up.x, b.up.x), min(a.up.y, b.up.y));
9     //check case when only a edge is common
10    return r.lw.x<r.up.x && r.lw.y<r.up.y;
11 }

```

## Polygon Area

```

1 double area(vector<pto> &p){//0(sz(p))
2     double area=0;
3     forn(i, sz(p)) area+=p[i]^p[(i+1)%sz(p)];
4     //if points are in clockwise order then area is negative
5     return abs(area)/2;
6 }
7 //Area ellipse = M_PI*a*b where a and b are the semi axis lengths
8 //Area triangle = sqrt(s*(s-a)(s-b)(s-c)) where s=(a+b+c)/2
9 //o mejor area triángulo = abs(x0 * (y1 - y2) + x1 * (y2 - y0) + x2 * (y0 - y1)) /
10    2;

```

## Circle

```

1 vec perp(vec v){return vec(-v.y, v.x);}
2 line bisector(pto x, pto y){
3     line l=line(x, y); pto m=(x+y)/2;
4     return line(-l.b, l.a, -l.b*m.x+l.a*m.y);
5 }
6 struct Circle{
7     pto o;
8     double r;
9     Circle(pto x, pto y, pto z){
10         o=inter(bisector(x, y), bisector(y, z));
11         r=dist(o, x);
12     }
13     pair<pto, pto> ptosTang(pto p){
14         pto m=(p+o)/2;
15         tipo d=dist(o, m);
16         tipo a=r*r/(2*d);
17         tipo h=sqrt(r*r-a*a);
18         pto m2=o+(m-o)*a/d;
19         vec per=perp(m-o)/d;
20         return make_pair(m2-per*h, m2+per*h);
21     }
22 };
23 //finds the center of the circle containing p1 and p2 with radius r
24 //as there may be two solutions swap p1, p2 to get the other
25 bool circle2PtsRad(pto p1, pto p2, double r, pto &c){
26     double d2=(p1-p2).norm_sq(), det=r*r/d2-0.25;
27     if(det<0) return false;
28     c=(p1+p2)/2+perp(p2-p1)*sqrt(det);
29     return true;
30 }
31 #define sqr(a) ((a)*(a))
32 #define feq(a,b) (fabs((a)-(b))<EPS)
33 pair<tipo, tipo> ecCuad(tipo a, tipo b, tipo c){//a*x*x+b*x+c=0
34     tipo dx = sqrt(b*b-4.0*a*c);
35     return make_pair((-b + dx)/(2.0*a), (-b - dx)/(2.0*a));
36 }
37 pair<pto, pto> interCL(Circle c, line l){
38     bool sw=false;
39     if((sw=feq(0,l.b))){
40         swap(l.a, l.b);
41         swap(c.o.x, c.o.y);
42     }
43     pair<tipo, tipo> rc = ecCuad(
44         sqr(l.a)+sqr(l.b),
45         2.0*l.a*l.b*c.o.y-2.0*(sqr(l.b)*c.o.x+l.c*l.a),
46         sqr(l.b)*(sqr(c.o.x)+sqr(c.o.y)-sqr(c.r))+sqr(l.c)-2.0*l.c*l.b*c.o.y
47     );

```

```

48     pair<pto, pto> p( pto(rc.first, (l.c - l.a * rc.first) / l.b),
49         pto(rc.second, (l.c - l.a * rc.second) / l.b) );
50     if(sw){
51         swap(p.first.x, p.first.y);
52         swap(p.second.x, p.second.y);
53     }
54     return p;
55 }
56 pair<pto, pto> interCC(Circle c1, Circle c2){
57     line l;
58     l.a = c1.o.x-c2.o.x;
59     l.b = c1.o.y-c2.o.y;
60     l.c = (sqr(c2.r)-sqr(c1.r)+sqr(c1.o.x)-sqr(c2.o.x)+sqr(c1.o.y)
61         -sqr(c2.o.y))/2.0;
62     return interCL(c1, l);
63 }

```

## Point in Poly

```

1 //checks if v is inside of P, using ray casting
2 //works with convex and concave.
3 bool inPolygon(pto v, vector<pto>& P) {
4     bool c = false;
5     forn(i, sz(P)){
6         int j=(i+1)%sz(P);
7
8         segm lado(P[i],P[j]);
9         if(lado.inside(v)) return true; //OJO: return true: incluye lados. return
            false: excluye lados.
10
11         if((P[j].y > v.y) != (P[i].y > v.y) &&
12             (v.x < (P[i].x-P[j].x) * (v.y-P[j].y) / (P[i].y-P[j].y) + P[j].x))
13             c = !c;
14     }
15     return c;
16 }

```

## Point in Convex Poly log(n)

```

1 void normalize(vector<pto> &pt){//delete collinear points first!
2     //this makes it clockwise:
3     if(pt[2].left(pt[0], pt[1])) reverse(pt.begin(), pt.end());
4     int n=sz(pt), pi=0;
5     forn(i, n)
6         if(pt[i].x<pt[pi].x || (pt[i].x==pt[pi].x && pt[i].y<pt[pi].y))
7             pi=i;
8     vector<pto> shift(n);//puts pi as first point
9     forn(i, n) shift[i]=pt[(pi+i)%n];

```

```

10 pt.swap(shift);
11 }
12
13 /* left debe decir >0 para que considere los bordes. Ojo que Convex Hull
14 necesita que left diga >= 0 para limpiar los colineales, hacer otro left
15 si hace falta */
16 bool inPolygon(pto p, const vector<pto> &pt){
17     //call normalize first!
18     if(p.left(pt[0], pt[1]) || p.left(pt[sz(pt)-1], pt[0])) return false;
19     int a=1, b=sz(pt)-1;
20     while(b-a>1){
21         int c=(a+b)/2;
22         if(!p.left(pt[0], pt[c])) a=c;
23         else b=c;
24     }
25     return !p.left(pt[a], pt[a+1]);
26 }

```

### Convex Check CHECK

```

1 bool isConvex(vector<int> &p){//O(N), delete collinear points!
2     int N=sz(p);
3     if(N<3) return false;
4     bool isLeft=p[0].left(p[1], p[2]);
5     forr(i, 1, N)
6         if(p[i].left(p[(i+1)%N], p[(i+2)%N])!=isLeft)
7             return false;
8     return true; }

```

### Convex Hull

```

1 //stores convex hull of P in S, CCW order
2 //left must return >=0 to delete collinear points!
3 void CH(vector<pto>& P, vector<pto> &S){
4     S.clear();
5     sort(P.begin(), P.end());//first x, then y
6     forn(i, sz(P)){//lower hull
7         while(sz(S)>= 2 && S[sz(S)-1].left(S[sz(S)-2], P[i])) S.pop_back();
8         S.pb(P[i]);
9     }
10    S.pop_back();
11    int k=sz(S);
12    dforn(i, sz(P)){//upper hull
13        while(sz(S) >= k+2 && S[sz(S)-1].left(S[sz(S)-2], P[i])) S.pop_back();
14        S.pb(P[i]);
15    }
16    S.pop_back();
17 }

```

### Cut Polygon

```

1 //cuts polygon Q along the line ab
2 //stores the left side (swap a, b for the right one) in P
3 void cutPolygon(pto a, pto b, vector<pto> Q, vector<pto> &P){
4     P.clear();
5     forn(i, sz(Q)){
6         double left1=(b-a)^(Q[i]-a), left2=(b-a)^(Q[(i+1)%sz(Q)]-a);
7         if(left1>=0) P.pb(Q[i]);
8         if(left1*left2<0)
9             P.pb(inter(line(Q[i], Q[(i+1)%sz(Q)]), line(a, b)));
10    }
11 }

```

### Bresenham

```

1 //plot a line approximation in a 2d map
2 void bresenham(pto a, pto b){
3     pto d=b-a; d.x=abs(d.x), d.y=abs(d.y);
4     pto s(a.x<b.x? 1: -1, a.y<b.y? 1: -1);
5     int err=d.x-d.y;
6     while(1){
7         m[a.x][a.y]=1;//plot
8         if(a==b) break;
9         int e2=err;
10        if(e2 >= 0) err-=2*d.y, a.x+=s.x;
11        if(e2 <= 0) err+= 2*d.x, a.y+= s.y;
12    }
13 }

```

### Rotate Matrix

```

1 //rotates matrix t 90 degrees clockwise
2 //using auxiliary matrix t2(faster)
3 void rotate(){
4     forn(x, n) forn(y, n)
5         t2[n-y-1][x]=t[x][y];
6     memcpy(t, t2, sizeof(t));
7 }

```

### Interseccion de Circulos en $n^3\log(n)$

```

1 struct event {
2     double x; int t;
3     event(double xx, int tt) : x(xx), t(tt) {}
4     bool operator <(const event &o) const { return x < o.x; }
5 };
6 typedef vector<Circle> VC;
7 typedef vector<event> VE;
8 int n;

```

# Math

## Identidades

$$\sum_{i=0}^n \binom{n}{i} = 2^n$$

$$\sum_{i=0}^n i \binom{n}{i} = n * 2^{n-1}$$

$$\sum_{i=m}^n i = \frac{n(n+1)}{2} - \frac{m(m-1)}{2} = \frac{(n+1-m)(n+m)}{2}$$

$$\sum_{i=0}^n i = \sum_{i=1}^n i = \frac{n(n+1)}{2}$$

$$\sum_{i=0}^n i^2 = \frac{n(n+1)(2n+1)}{6} = \frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6}$$

$$\sum_{i=0}^n i(i-1) = \frac{6}{6} \left(\frac{n}{2}\right) \left(\frac{n}{2} + 1\right) (n+1) \text{ (doubles)} \rightarrow \text{Sino ver caso impar y par}$$

$$\sum_{i=0}^n i^3 = \left(\frac{n(n+1)}{2}\right)^2 = \frac{n^4}{4} + \frac{n^3}{2} + \frac{n^2}{4} = \left[\sum_{i=1}^n i\right]^2$$

$$\sum_{i=0}^n i^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30} = \frac{n^5}{5} + \frac{n^4}{2} + \frac{n^3}{3} - \frac{n}{30}$$

$$\sum_{i=0}^n i^p = \frac{(n+1)^{p+1}}{p+1} + \sum_{k=1}^p \frac{B_k}{p-k+1} \binom{p}{k} (n+1)^{p-k+1}$$

$$\sum_{i=0}^n a^i = \frac{a^{n+1}-1}{a-1} \text{ sólo si } a \neq 1$$

$$r = e - v + k + 1$$

Teorema de Pick: (Area, puntos interiores y puntos en el borde)  $A = I + \frac{B}{2} - 1$

## Ec. Característica

$$a_0 T(n) + a_1 T(n-1) + \dots + a_k T(n-k) = 0$$

$$p(x) = a_0 x^k + a_1 x^{k-1} + \dots + a_k$$

Sean  $r_1, r_2, \dots, r_q$  las raíces distintas, de mult.  $m_1, m_2, \dots, m_q$

$$T(n) = \sum_{i=1}^q \sum_{j=0}^{m_i-1} c_{ij} n^j r_i^n$$

Las constantes  $c_{ij}$  se determinan por los casos base.

## Combinatorio

```
1 forn(i, MAXN+1){ //comb[i][k]=i tomados de a k
2   comb[i][0]=comb[i][i]=1;
3   forr(k, 1, i) comb[i][k]=(comb[i-1][k]+comb[i-1][k-1])%MOD;
4 }
5 ll lucas (ll n, ll k, int p){ //Calcula (n,k)%p teniendo comb[p][p] precalculado.
6   ll aux = 1;
7   while (n + k) aux = (aux * comb[n%p][k%p]) %p, n/=p, k/=p;
8   return aux;
9 }
```

## Log. Discreto

```
1 // IDEA: a^x=b mod MOD <=> x = i*sqrt(MOD)+j con i,j <= sqrt(MOD)=m
2 // entonces guardo todos los a^j: T[a^j mod MOD]=j
3 // y después busco si vi T[b/(a^(i*m) mod MOD)] = T[b*a^-(i*m) mod MOD], return j+i*m
```

## Exp. de Matrices y Fibonacci en log(n)

```
1 #define SIZE 350
2 int NN;
3 double tmp[SIZE][SIZE];
```

```
9 double cuenta(VE &v, double A,double B) {
10   sort(v.begin(), v.end());
11   double res = 0.0, lx = ((v.empty())?0.0:v[0].x);
12   int contador = 0;
13   forn(i,sz(v)) {
14     //interseccion de todos (contador == n), union de todos (contador > 0)
15     //conjunto de puntos cubierto por exacta k Circulos (contador == k)
16     if (contador == n) res += v[i].x - lx;
17     contador += v[i].t, lx = v[i].x;
18   }
19   return res;
20 }
21 // Primitiva de sqrt(r*r - x*x) como funcion double de una variable x.
22 inline double primitiva(double x,double r) {
23   if (x >= r) return r*r*M_PI/4.0;
24   if (x <= -r) return -r*r*M_PI/4.0;
25   double raiz = sqrt(r*r-x*x);
26   return 0.5 * (x * raiz + r*r*atan(x/raiz));
27 }
28 double interCircle(VC &v) {
29   vector<double> p; p.reserve(v.size() * (v.size() + 2));
30   forn(i,sz(v)) p.push_back(v[i].c.x + v[i].r), p.push_back(v[i].c.x - v[i].r);
31   forn(i,sz(v)) forn(j,i) {
32     Circle &a = v[i], b = v[j];
33     double d = (a.c - b.c).norm();
34     if (fabs(a.r - b.r) < d && d < a.r + b.r) {
35       double alfa = acos((sqr(a.r) + sqr(d) - sqr(b.r)) / (2.0 * d * a.r));
36       pto vec = (b.c - a.c) * (a.r / d);
37       p.pb((a.c + rotate(vec, alfa)).x), p.pb((a.c + rotate(vec, -alfa)).x);
38     }
39   }
40   sort(p.begin(), p.end());
41   double res = 0.0;
42   forn(i,sz(p)-1) {
43     const double A = p[i], B = p[i+1];
44     VE ve; ve.reserve(2 * v.size());
45     forn(j,sz(v)) {
46       const Circle &c = v[j];
47       double arco = primitiva(B-c.c.x,c.r) - primitiva(A-c.c.x,c.r);
48       double base = c.c.y * (B-A);
49       ve.push_back(event(base + arco,-1));
50       ve.push_back(event(base - arco, 1));
51     }
52     res += cuenta(ve,A,B);
53   }
54   return res;
55 }
```

```

4 void mul(double a[SIZE][SIZE], double b[SIZE][SIZE]){ zero(tmp);
5     forn(i, NN) forn(j, NN) forn(k, NN) tmp[i][j] += a[i][k] * b[k][j];
6     forn(i, NN) forn(j, NN) a[i][j] = tmp[i][j];
7 }
8 void powmat(double a[SIZE][SIZE], int n, double res[SIZE][SIZE]){
9     forn(i, NN) forn(j, NN) res[i][j] = (i==j);
10    while(n){
11        if(n&1) mul(res, a), n--;
12        else mul(a, a), n/=2;
13    } }

```

### Matrices y determinante $O(n^3)$

```

1 struct Mat {
2     vector<vector<double>> > vec;
3     Mat(int n): vec(n, vector<double>(n) ) {}
4     Mat(int n, int m): vec(n, vector<double>(m) ) {}
5     vector<double> &operator[] (int f){return vec[f];}
6     const vector<double> &operator[] (int f) const {return vec[f];}
7     int size() const {return sz(vec);}
8     Mat operator+(Mat &b) { ///this de n x m entonces b de n x m
9         Mat m(sz(b),sz(b[0]));
10        forn(i,sz(vec)) forn(j,sz(vec[0])) m[i][j] = vec[i][j] + b[i][j];
11        return m;    }
12    Mat operator*(const Mat &b) { ///this de n x m entonces b de m x t
13        int n = sz(vec), m = sz(vec[0]), t = sz(b[0]);
14        Mat mat(n,t);
15        forn(i,n) forn(j,t) forn(k,m) mat[i][j] += vec[i][k] * b[k][j];
16        return mat;    }
17    double determinant(){//sacado de e maxx ru
18        double det = 1;
19        int n = sz(vec);
20        Mat m(*this);
21        forn(i, n){//para cada columna
22            int k = i;
23            forr(j, i+1, n)//busco la fila con mayor val abs
24                if(abs(m[j][i])>abs(m[k][i])) k = j;
25            if(abs(m[k][i])<1e-9) return 0;
26            m[i].swap(m[k]);//la swapeo
27            if(i!=k) det = -det;
28            det *= m[i][i];
29            forr(j, i+1, n) m[i][j] /= m[i][i];
30            //hago 0 todas las otras filas
31            forn(j, n) if (j!= i && abs(m[j][i])>1e-9)
32                forr(k, i+1, n) m[j][k] -= m[i][k] * m[j][i];
33        }
34        return det;
35    }

```

```

36 }

```

### Teorema Chino del Resto

$$y = \sum_{j=1}^n (x_j * (\prod_{i=1, i \neq j}^n m_i)^{-1}_{m_j} * \prod_{i=1, i \neq j}^n m_i)$$

### Criba

```

1 #define MAXP 100000 //no necesariamente primo
2 int criba[MAXP+1];
3 void crearcriba(){
4     int w[] = {4,2,4,2,4,6,2,6};
5     for(int p=25;p<=MAXP;p+=10) criba[p]=5;
6     for(int p=9;p<=MAXP;p+=6) criba[p]=3;
7     for(int p=4;p<=MAXP;p+=2) criba[p]=2;
8     for(int p=7,cur=0;p*p<=MAXP;p+=w[cur++&7]) if (!criba[p])
9         for(int j=p*p;j<=MAXP;j+=(p<<1)) if(!criba[j]) criba[j]=p;
10 }
11 vector<int> primos;
12 void buscarprimos(){
13     crearcriba();
14     forr (i,2,MAXP+1) if (!criba[i]) primos.push_back(i);
15 }
16 //~ Useful for bit trick: #define SET(i) ( criba[(i)>>5] |= 1<<((i)&31) ), #define
    INDEX(i) ( (criba[i]>>5)>>((i)&31))&1 ), unsigned int criba[MAXP/32+1];

```

### Funciones de primos

Sea  $n = \prod p_i^{k_i}$ , fact(n) genera un map donde a cada  $p_i$  le asocia su  $k_i$

```

1 //factoriza bien numeros hasta MAXP^2
2 map<ll,ll> fact(ll n){ //0 (cant primos)
3     map<ll,ll> ret;
4     forall(p, primos){
5         while(!(n%p)){
6             ret[*p]++;//divisor found
7             n/=p;
8         }
9     }
10    if(n>1) ret[n]++;
11    return ret;
12 }
13 //factoriza bien numeros hasta MAXP
14 map<ll,ll> fact2(ll n){ //0 (lg n)
15     map<ll,ll> ret;
16     while (criba[n]){
17         ret[criba[n]]++;
18         n/=criba[n];

```

```

19     }
20     if(n>1) ret[n]++;
21     return ret;
22 }
23 //Usar asi:  divisores(fac, divs, fac.begin()); NO ESTA ORDENADO
24 void divisores(const map<ll,ll> &f, vector<ll> &divs, map<ll,ll>::iterator it, ll
        n=1){
25     if(it==f.begin()) divs.clear();
26     if(it==f.end()) { divs.pb(n); return; }
27     ll p=it->fst, k=it->snd; ++it;
28     forn(_, k+1) divisores(f, divs, it, n), n*=p;
29 }
30 ll sumDiv (ll n){
31     ll rta = 1;
32     map<ll,ll> f=fact(n);
33     forall(it, f) {
34         ll pot = 1, aux = 0;
35         forn(i, it->snd+1) aux += pot, pot *= it->fst;
36         rta*=aux;
37     }
38     return rta;
39 }
40 ll eulerPhi (ll n){ // con criba: O(lg n)
41     ll rta = n;
42     map<ll,ll> f=fact(n);
43     forall(it, f) rta -= rta / it->first;
44     return rta;
45 }
46 ll eulerPhi2 (ll n){ // O (sqrt n)
47     ll r = n;
48     forr (i,2,n+1){
49         if ((ll)i*i > n) break;
50         if (n % i == 0){
51             while (n%i == 0) n/=i;
52             r -= r/i; }
53     }
54     if (n != 1) r-= r/n;
55     return r;
56 }

```

Test de primalidad naive  $O(\sqrt{n}/6)$

```

1 int __attribute__((const)) is_prime(long long n)
2 {
3     if (n <= 1)
4         return 0;
5     else if (n <= 3)
6         return 1;

```

```

7     else if (!(n % 2) || !(n % 3))
8         return 0;
9
10    long long cap = sqrt(n) + 1;
11    for (long long int i = 5; i <= cap; i += 6)
12        if (!(n%i) || !(n%(i+2)))
13            return 0;
14
15    return 1;
16 }

```

Phollard's Rho (rolando)

```

1 ll gcd(ll a, ll b){return a?gcd(b %a, a):b;}
2
3 ll mulmod (ll a, ll b, ll c) { //returns (a*b)%c, and minimize overflow
4     ll x = 0, y = a%c;
5     while (b > 0){
6         if (b % 2 == 1) x = (x+y) % c;
7         y = (y*2) % c;
8         b /= 2;
9     }
10    return x % c;
11 }
12
13 ll expmod (ll b, ll e, ll m){//O(log b)
14     if(!e) return 1;
15     ll q= expmod(b,e/2,m); q=mulmod(q,q,m);
16     return e%2? mulmod(b,q,m) : q;
17 }
18
19 bool es_primo_prob (ll n, int a)
20 {
21     if (n == a) return true;
22     ll s = 0,d = n-1;
23     while (d % 2 == 0) s++,d/=2;
24
25     ll x = expmod(a,d,n);
26     if ((x == 1) || (x+1 == n)) return true;
27
28     forn (i, s-1){
29         x = mulmod(x, x, n);
30         if (x == 1) return false;
31         if (x+1 == n) return true;
32     }
33     return false;
34 }
35

```



```

36 bool rabin (ll n){ //devuelve true si n es primo
37     if (n == 1) return false;
38     const int ar[] = {2,3,5,7,11,13,17,19,23};
39     forn (j,9)
40         if (!es_primo_prob(n,ar[j]))
41             return false;
42     return true;
43 }
44
45 ll rho(ll n){
46     if( (n & 1) == 0 ) return 2;
47     ll x = 2 , y = 2 , d = 1;
48     ll c = rand() %n + 1;
49     while( d == 1 ){
50         x = (mulmod( x , x , n ) + c)%n;
51         y = (mulmod( y , y , n ) + c)%n;
52         y = (mulmod( y , y , n ) + c)%n;
53         if( x - y >= 0 ) d = gcd( x - y , n );
54         else d = gcd( y - x , n );
55     }
56     return d==n? rho(n):d;
57 }
58
59 map<ll,ll> prim;
60 void factRho (ll n){ //0 (lg n)^3. un solo numero
61     if (n == 1) return;
62     if (rabin(n)){
63         prim[n]++;
64         return;
65     }
66     ll factor = rho(n);
67     factRho(factor);
68     factRho(n/factor);
69 }

```

## GCD

```

1 | tipo gcd(tipo a, tipo b){return a?gcd(b %a, a):b;}

```

## Extended Euclid

```

1 void extendedEuclid (ll a, ll b){ //a * x + b * y = d
2     if (!b) { x = 1; y = 0; d = a; return;}
3     extendedEuclid (b, a%b);
4     ll x1 = y;
5     ll y1 = x - (a/b) * y;
6     x = x1; y = y1;
7 }

```

## LCM

```

1 | tipo lcm(tipo a, tipo b){return a / gcd(a,b) * b;}

```

## Simpson

```

1 double integral(double a, double b, int n=10000) { //0(n), n=cantdiv
2     double area=0, h=(b-a)/n, fa=f(a), fb;
3     forn(i, n){
4         fb=f(a+h*(i+1));
5         area+=fa+ 4*f(a+h*(i+0.5)) +fb, fa=fb;
6     }
7     return area*h/6.;}

```

## Fraction

```

1 bool comp(tipo a, tipo b, tipo c, tipo d){ //a*d < b*c
2     int s1 = signo(a)*signo(d), s2 = signo(b)*signo(c);
3     if(s1 == 0) return s2 > 0;
4     if(s2 == 0) return s1 < 0;
5     if(s1 > 0 and s2 < 0) return false;
6     if(s1 < 0 and s2 > 0) return true;
7     if(a / b != c / d) return a/b < c/d; //asume que b y d son positivos
8     a %= b, c %= d;
9     /*0(1) pero con double:
10    long double d1 = ((long double)(a))/(b), d2 = ((long double)(c))/(d);
11    return d1 + EPS < d2;
12    */
13    return comp(d, c, b, a);
14 }
15
16 tipo mcd(tipo a, tipo b){ return a ? mcd(b%a,a) : b; }
17 struct frac{
18     tipo p,q;
19     frac(tipo p=0, tipo q=1):p(p),q(q) {norm();}
20     void norm(){
21         tipo a = mcd(p,q);
22         if(a) p/=a, q/=a;
23         else q=1;
24         if (q<0) q=-q, p=-p;}
25     frac operator+(const frac& o){
26         tipo a = mcd(q,o.q);
27         return frac(p*(o.q/a)+o.p*(q/a), q*(o.q/a));}
28     frac operator-(const frac& o){
29         tipo a = mcd(q,o.q);
30         return frac(p*(o.q/a)-o.p*(q/a), q*(o.q/a));}
31     frac operator*(frac o){
32         tipo a = mcd(q,o.p), b = mcd(o.q,p);
33         return frac((p/b)*(o.p/a), (q/a)*(o.q/b));}

```

```

34  frac operator/(frac o){
35      tipo a = mcd(q,o.q), b = mcd(o.p,p);
36      return frac((p/b)*(o.q/a), (q/a)*(o.p/b));}
37  bool operator<(const frac &o) const{return p*o.q < o.p*q;}//usar comp cuando el
    producto puede dar overflow
38  bool operator==(frac o){return p==o.p&&q==o.q;}
39  };

```

## Polinomio

```

1  struct poly {
2      vector<tipo> c;//guarda los coeficientes del polinomio
3      poly(const vector<tipo> &c): c(c) {}
4      poly() {}
5      void simplify(){
6          int i = 0;
7          /*tipo a0=0;
8          while(a0 == 0 && i < sz(c)) a0 = c[i], i++;*/
9          int j = sz(c)-1;
10         tipo an=0;
11         while(an == 0 && j >=i) an = c[j], j--;
12         vector<tipo> d;
13         forr(k,i,j) d.pb(c[k]);
14         c=d;
15     }
16     bool isnull() { simplify(); return c.empty();}
17     poly operator+(const poly &o) const {
18         int m = sz(c), n = sz(o.c);
19         vector<tipo> res(max(m,n));
20         forn(i, m) res[i] += c[i];
21         forn(i, n) res[i] += o.c[i];
22         return poly(res);    }
23     poly operator*(const tipo cons) const {
24         vector<tipo> res(sz(c));
25         forn(i, sz(c)) res[i]=c[i]*cons;
26         return poly(res);    }
27     poly operator*(const poly &o) const {
28         int m = sz(c), n = sz(o.c);
29         vector<tipo> res(m+n-1);
30         forn(i, m) forn(j, n) res[i+j]+=c[i]*o.c[j];
31         return poly(res);    }
32     tipo eval(tipo v) {
33         tipo sum = 0;
34         dform(i, sz(c)) sum=sum*v + c[i];
35         return sum; }
36     //poly contains only a vector<int> c (the coeficients)
37     //the following function generates the roots of the polynomial
38     //it can be easily modified to return float roots

```

```

39  set<tipo> roots(){
40      set<tipo> roots;
41      simplify();
42      if(c[0]) roots.insert(0);
43      int i = 0;
44      tipo a0=0;
45      while(a0 == 0 && i < sz(c)) a0 = abs(c[i]), i++;
46      tipo an = abs(c[sz(c)-1]);
47      vector<tipo> ps,qs;
48      forr(p,1,sqrt(a0)+1) if (a0%p==0) ps.pb(p),ps.pb(a0/p);
49      forr(q,1,sqrt(an)+1) if (an%q==0) qs.pb(q),qs.pb(an/q);
50      forall(pt,ps)
51          forall(qt,qs) if ( (*pt) % (*qt)==0 ) { //sacar esto para obtener todas las
                raices racionales
52              tipo root = abs((*pt) / (*qt));
53              if (eval(root)==0) roots.insert(root);
54              if (eval((-1)*root)==0) roots.insert((-1)*root);// las raices tambien
                pueden ser negativas!
55          }
56      return roots; }
57 };
58 pair<poly,tipo> ruffini(const poly p, tipo r) { //divide el polinomio p por (x-r)
59     int n = sz(p.c) - 1 ;
60     vector<tipo> b(n);
61     b[n-1] = p.c[n];
62     dforn(k,n-1) b[k] = p.c[k+1] + r*b[k+1];
63     tipo resto = p.c[0] + r*b[0];
64     poly result(b);
65     return make_pair(result,resto);
66 }
67 poly interpolate(const vector<tipo>& x,const vector<tipo>& y) { //O(n^2)
68     poly A; A.c.pb(1);
69     forn(i,sz(x)) { poly aux; aux.c.pb(-x[i]), aux.c.pb(1), A = A * aux; } // A =
        (x-x0) * ... * (x-xn)
70     poly S; S.c.pb(0);
71     forn(i,sz(x)) { poly Li;
72         Li = ruffini(A,x[i]).fst;
73         Li = Li * (1.0 / Li.eval(x[i])); // here put a multiple of the coefficients
            instead of 1.0 to avoid using double -- si se usa mod usar el inverso!
74         S = S + Li * y[i]; }
75     return S;
76 }

```

## Ec. Lineales

```

1  bool resolver_ev(Mat a, Vec y, Vec &x, Mat &ev){
2      int n = a.size(), m = n?a[0].size():0, rw = min(n, m);
3      vector<int> p; forn(i,m) p.push_back(i);

```

```

4   forn(i, rw) {
5       int uc=i, uf=i;
6       forr(f, i, n) forr(c, i, m) if(fabs(a[f][c])>fabs(a[uf][uc])) {uf=f;uc=c;}
7       if (feq(a[uf][uc], 0)) { rw = i; break; }
8       forn(j, n) swap(a[j][i], a[j][uc]);
9       swap(a[i], a[uf]); swap(y[i], y[uf]); swap(p[i], p[uc]);
10      tipo inv = 1 / a[i][i]; //aca divide
11      forr(j, i+1, n) {
12          tipo v = a[j][i] * inv;
13          forr(k, i, m) a[j][k]-=v * a[i][k];
14          y[j] -= v*y[i];
15      }
16  } // rw = rango(a), aca la matriz esta triangulada
17  forr(i, rw, n) if (!feq(y[i],0)) return false; // chequeo de compatibilidad
18  x = vector<tipo>(m, 0);
19  dforn(i, rw){
20      tipo s = y[i];
21      forr(j, i+1, rw) s -= a[i][j]*x[p[j]];
22      x[p[i]] = s / a[i][i]; //aca divide
23  }
24  ev = Mat(m-rw, Vec(m, 0)); // Esta parte va SOLO si se necesita el ev
25  forn(k, m-rw) {
26      ev[k][p[k+rw]] = 1;
27      dforn(i, rw){
28          tipo s = -a[i][k+rw];
29          forr(j, i+1, rw) s -= a[i][j]*ev[k][p[j]];
30          ev[k][p[i]] = s / a[i][i]; //aca divide
31      }
32  }
33  return true;
34  }

```

## FFT

```

1  //~ typedef complex<double> base; //menos codigo, pero mas lento
2  //elegir si usar complejos de c (lento) o estos
3  struct base{
4      double r,i;
5      base(double r=0, double i=0):r(r), i(i){}
6      double real()const{return r;}
7      void operator/=(const int c){r/=c, i/=c;}
8  };
9  base operator*(const base &a, const base &b){
10     return base(a.r*b.r-a.i*b.i, a.r*b.i+a.i*b.r);}
11  base operator+(const base &a, const base &b){
12     return base(a.r+b.r, a.i+b.i);}
13  base operator-(const base &a, const base &b){
14     return base(a.r-b.r,a.i-b.i);}

```

```

15 vector<int> rev; vector<base> wlen_pw;
16 inline static void fft(base a[], int n, bool invert) {
17     forn(i, n) if(i<rev[i]) swap(a[i], a[rev[i]]);
18     for (int len=2; len<=n; len<=1) {
19         double ang = 2*M_PI/len * (invert?-1:+1);
20         int len2 = len>>1;
21         base wlen (cos(ang), sin(ang));
22         wlen_pw[0] = base (1, 0);
23         forr(i, 1, len2) wlen_pw[i] = wlen_pw[i-1] * wlen;
24         for (int i=0; i<n; i+=len) {
25             base t, *pu = a+i, *pv = a+i+len2, *pu_end = a+i+len2, *pw = &wlen_pw[0];
26             for (; pu!=pu_end; ++pu, ++pv, ++pw)
27                 t = *pv * *pw, *pv = *pu - t,*pu = *pu + t;
28         }
29     }
30     if (invert) forn(i, n) a[i]/= n;}
31 inline static void calc_rev(int n){//precalculo: llamar antes de fft!!
32     wlen_pw.resize(n), rev.resize(n);
33     int lg=31-__builtin_clz(n);
34     forn(i, n){
35         rev[i] = 0;
36         forn(k, lg) if(i&(1<<k)) rev[i]|=1<<(lg-1-k);
37     }
38     //multiplica vectores en nlgn
39 inline static void multiply(const vector<int> &a, const vector<int> &b, vector<int>
40     > &res) {
41     vector<base> fa (a.begin(), a.end()), fb (b.begin(), b.end());
42     int n=1; while(n < max(sz(a), sz(b))) n <= 1; n <= 1;
43     calc_rev(n);
44     fa.resize (n), fb.resize (n);
45     fft (&fa[0], n, false),  fft (&fb[0], n, false);
46     forn(i, n) fa[i] = fa[i] * fb[i];
47     fft (&fa[0], n, true);
48     res.resize(n);
49     forn(i, n) res[i] = int (fa[i].real() + 0.5); }
50 void toPoly(const string &s, vector<int> &P){//convierte un numero a polinomio
51     P.clear();
52     dforn(i, sz(s)) P.pb(s[i]-'0');}

```

## Cantidad de primos menores que $10^n$

$\pi(10^6) = 78.498$  ;  $\pi(10^7) = 664.579$  ;  $\pi(10^8) = 5.761.455$  ;  $\pi(10^9) = 50.847.534$   
 $\pi(10^{10}) = 455.052,511$  ;  $\pi(10^{11}) = 4.118.054.813$  ;  $\pi(10^{12}) = 37.607.912.018$

## Divisores

Cantidad de divisores ( $\sigma_0$ ) para *algunos*  $n/\neg\exists n' < n, \sigma_0(n') \geq \sigma_0(n)$

$\sigma_0(60) = 12$  ;  $\sigma_0(120) = 16$  ;  $\sigma_0(180) = 18$  ;  $\sigma_0(240) = 20$  ;  $\sigma_0(360) = 24$

$\sigma_0(720) = 30$  ;  $\sigma_0(840) = 32$  ;  $\sigma_0(1260) = 36$  ;  $\sigma_0(1680) = 40$  ;  $\sigma_0(10080) = 72$

$\sigma_0(15120) = 80$  ;  $\sigma_0(50400) = 108$  ;  $\sigma_0(83160) = 128$  ;  $\sigma_0(110880) = 144$   
 $\sigma_0(498960) = 200$  ;  $\sigma_0(554400) = 216$  ;  $\sigma_0(1081080) = 256$  ;  $\sigma_0(1441440) = 288$   $\sigma_0(4324320)$   
 $= 384$  ;  $\sigma_0(8648640) = 448$   
 Suma de divisores ( $\sigma_1$ ) para *algunos*  $n/\neg\exists n' < n, \sigma_1(n') \geq \sigma_1(n)$   
 $\sigma_1(96) = 252$  ;  $\sigma_1(108) = 280$  ;  $\sigma_1(120) = 360$  ;  $\sigma_1(144) = 403$  ;  $\sigma_1(168) = 480$   
 $\sigma_1(960) = 3048$  ;  $\sigma_1(1008) = 3224$  ;  $\sigma_1(1080) = 3600$  ;  $\sigma_1(1200) = 3844$   
 $\sigma_1(4620) = 16128$  ;  $\sigma_1(4680) = 16380$  ;  $\sigma_1(5040) = 19344$  ;  $\sigma_1(5760) = 19890$   
 $\sigma_1(8820) = 31122$  ;  $\sigma_1(9240) = 34560$  ;  $\sigma_1(10080) = 39312$  ;  $\sigma_1(10920) = 40320$   
 $\sigma_1(32760) = 131040$  ;  $\sigma_1(35280) = 137826$  ;  $\sigma_1(36960) = 145152$  ;  $\sigma_1(37800) = 148800$   
 $\sigma_1(60480) = 243840$  ;  $\sigma_1(64680) = 246240$  ;  $\sigma_1(65520) = 270816$  ;  $\sigma_1(70560) = 280098$   
 $\sigma_1(95760) = 386880$  ;  $\sigma_1(98280) = 403200$  ;  $\sigma_1(100800) = 409448$   
 $\sigma_1(491400) = 2083200$  ;  $\sigma_1(498960) = 2160576$  ;  $\sigma_1(514080) = 2177280$   
 $\sigma_1(982800) = 4305280$  ;  $\sigma_1(997920) = 4390848$  ;  $\sigma_1(1048320) = 4464096$   
 $\sigma_1(4979520) = 22189440$  ;  $\sigma_1(4989600) = 22686048$  ;  $\sigma_1(5045040) = 23154768$   
 $\sigma_1(9896040) = 44323200$  ;  $\sigma_1(9959040) = 44553600$  ;  $\sigma_1(9979200) = 45732192$

## Grafos

### Dijkstra

```

1  #define INF 1e9
2  int N;
3  #define MAX_V 250001
4  vector<ii> G[MAX_V];
5  //To add an edge use
6  #define add(a, b, w) G[a].pb(make_pair(w, b))
7  ll dijkstra(int s, int t){//O(|E| log |V|)
8      priority_queue<ii, vector<ii>, greater<ii> > Q;
9      vector<ll> dist(N, INF); vector<int> dad(N, -1);
10     Q.push(make_pair(0, s)); dist[s] = 0;
11     while(sz(Q)){
12         ii p = Q.top(); Q.pop();
13         if(p.snd == t) break;
14         forall(it, G[p.snd])
15             if(dist[p.snd]+it->first < dist[it->snd]){
16                 dist[it->snd] = dist[p.snd] + it->fst;
17                 dad[it->snd] = p.snd;
18                 Q.push(make_pair(dist[it->snd], it->snd)); }
19     }
20     return dist[t];
21     if(dist[t]<INF)//path generator
22         for(int i=t; i!=-1; i=dad[i])
23             printf("%d%c", i, (i==s?' \n': ' '));}

```

### Bellman-Ford

```

1  #define INF 1e9

```

```

2  #define MAX_N 1001
3  vector<ii> G[MAX_N]; //ady. list with pairs (weight, dst)
4  //To add an edge use
5  #define add(a, b, w) G[a].pb(make_pair(w, b))
6  int dist[MAX_N];
7  int N; //cantidad de vertices -- setear!!
8  void bford(int src){//O(VE)
9      memset(dist, INF, sizeof dist);
10     dist[src]=0;
11     forn(i, N-1) forn(j, N) if(dist[j]!=INF) forall(it, G[j])
12         dist[it->snd]=min(dist[it->snd], dist[j]+it->fst);
13 }
14
15 bool hasNegCycle(){
16     forn(j, N) if(dist[j]!=INF) forall(it, G[j])
17         if(dist[it->snd]>dist[j]+it->fst) return true;
18     //inside if: all points reachable from it->snd will have -INF distance(do bfs) ?
19     return false;
20 }

```

### Floyd-Warshall

```

1  //G[i][j] contains weight of edge (i, j) or INF
2  //G[i][i]=0
3  int G[MAX_N][MAX_N];
4  void floyd(){//O(N^3)
5      forn(k, N) forn(i, N) if(G[i][k]!=INF) forn(j, N) if(G[k][j]!=INF)
6          G[i][j]=min(G[i][j], G[i][k]+G[k][j]);
7  }
8  bool inNegCycle(int v){
9      return G[v][v]<0;}
10 //checks if there's a neg. cycle in path from a to b
11 bool hasNegCycle(int a, int b){
12     forn(i, N) if(G[a][i]!=INF && G[i][i]<0 && G[i][b]!=INF)
13         return true;
14     return false;
15 }

```

### Kruskal

```

1  const int MAXN=100000;
2  vector<ii> G[MAXN];
3  int n;
4
5  struct Ar{int a,b,w;}; //w y cost deberian tener el mismo tipo
6  bool operator<(const Ar& a, const Ar &b){return a.w<b.w;}
7  vector<Ar> E;
8  ll kruskal(){ //no hace falta agregar las aristas en las dos direcciones! (en prim
9      si)

```

```

9   ll cost=0;
10  sort(E.begin(), E.end()); //ordenar aristas de menor a mayor -- OJO cuando
    ordena algo no necesariamente las cosas del mismo valor quedan en el mismo
    orden!!
11  uf.init(n);
12  forall(it, E){
13      if(uf.comp(it->a)!=uf.comp(it->b)){//si no estan conectados
14          uf.join(it->a, it->b); //conectar
15          cost+=it->w;
16      }
17  }
18  return cost;
19 }

```

## 2-SAT + Tarjan SCC

```

1  //We have a vertex representing a var and other for his negation.
2  //Every edge stored in G represents an implication. To add an equation of the form
    a||b, use addor(a, b)
3  //MAX=max cant var, n=cant var
4  #define addor(a, b) (G[neg(a)].pb(b), G[neg(b)].pb(a))
5  vector<int> G[MAX*2];
6  //idx[i]=index assigned in the dfs
7  //lw[i]=lowest index(closer from the root) reachable from i
8  int lw[MAX*2], idx[MAX*2], qidx;
9  stack<int> q;
10 int qcmp, cmp[MAX*2];
11 //verdad[cmp[i]]=valor de la variable i
12 bool verdad[MAX*2+1];
13
14 int neg(int x) { return x>=n? x-n : x+n;}
15 void tjn(int v){
16     lw[v]=idx[v]=++qidx;
17     q.push(v), cmp[v]=-2;
18     forall(it, G[v]){
19         if(!idx[*it] || cmp[*it]==-2){
20             if(!idx[*it]) tjn(*it);
21             lw[v]=min(lw[v], lw[*it]);
22         }
23     }
24     if(lw[v]==idx[v]){
25         int x;
26         do{x=q.top(); q.pop(); cmp[x]=qcmp;}while(x!=v);
27         verdad[qcmp]=(cmp[neg(v)]<0);
28         qcmp++;
29     }
30 }
31 //remember to CLEAR G!!!

```

```

32 bool satisf(){//O(n)
33     memset(idx, 0, sizeof(idx)), qidx=0;
34     memset(cmp, -1, sizeof(cmp)), qcmp=0;
35     forn(i, n){
36         if(!idx[i]) tjn(i);
37         if(!idx[neg(i)]) tjn(neg(i));
38     }
39     forn(i, n) if(cmp[i]==cmp[neg(i)]) return false;
40     return true;
41 }

```

## Comp. Biconexas y Puentes

```

1  const int MAXN=1010;
2  int n, m;
3  vector<int> G[MAXN];
4
5  struct edge {
6      int u,v, comp;
7      bool bridge;
8  };
9  vector<edge> e;
10 void addEdge(int u, int v) {
11     G[u].pb(sz(e)), G[v].pb(sz(e));
12     e.pb((edge){u,v,-1,false});
13 }
14 //V[i]=id de la dfs
15 //L[i]=lowest id reachable from i
16 int V[MAXN], L[MAXN], qV;
17 int nbc;//cant componentes
18 int comp[MAXN]; //comp[i]=cant comp biconexas a la cual pertenece i
19 void initDfs(int n) {
20     zero(G), zero(comp);
21     e.clear();
22     forn(i,n) V[i]=-1;
23     nbc = qV = 0;
24 }
25 stack<int> st;
26 void dfs(int u, int pe) { //O(n + m)
27     L[u] = V[u] = qV++;
28     comp[u] = (pe != -1);
29     for(auto &ne: G[u]) if (ne != pe){
30         int v = e[ne].u ^ e[ne].v ^ u; // x ^ y ^ x = y!
31         if (V[v] == -1) { // todavia no se lo visito
32             st.push(ne);
33             dfs(v,ne);
34             if (L[v] > V[u]){ // bridge => no pertenece a ninguna comp biconexa
35                 e[ne].bridge = true;

```

```

36     }
37     if (L[v] >= V[u]){ // art
38         int last;
39         do { //todas las aristas que estan entre dos puntos de articulacion
                pertenecen a la misma componente biconexa
40             last = st.top(); st.pop();
41             e[last].comp = nbc;
42         } while (last != ne);
43         nbc++;
44         comp[u]++;
45     }
46     L[u] = min(L[u], L[v]);
47 }
48 else if (V[v] < V[u]) { // back edge
49     st.push(ne);
50     L[u] = min(L[u], V[v]);
51 }
52 }
53 }
54
55 set<int> C[2*MAXN];
56 int compnodo[MAXN];
57 int ptoart;
58 void blockcuttree(){
59     ptoart = 0; zero(compnodo);
60     forn(i,2*MAXN) C[i].clear();
61     for(auto &it: e){
62         int u = it.u, v = it.v;
63         if(comp[u] == 1) compnodo[u] = it.comp;
64         else{
65             if(compnodo[u] == 0){ compnodo[u] = nbc+ptoart; ptoart++;}
66             C[it.comp].insert(compnodo[u]);
67             C[compnodo[u]].insert(it.comp);
68         }
69         if(comp[v] == 1) compnodo[v] = it.comp;
70         else{
71             if(compnodo[v] == 0){ compnodo[v] = nbc+ptoart; ptoart++;}
72             C[it.comp].insert(compnodo[v]);
73             C[compnodo[v]].insert(it.comp);
74         }
75     }
76 }
77
78 int main() {
79     while(cin >> n >> m){
80         initDfs(n);
81         forn(i, m){

```

```

82         int a,b; cin >> a >> b;
83         addEdge(a,b);
84     }
85     dfs(0,-1);
86     forn(i, n) cout << "comp[" << i << "]_=" << comp[i] << endl;
87     for(auto &ne: e) cout << ne.u << "->" << ne.v << "_en_la_comp_" << ne.comp <<
            endl;
88     cout << "Cant._de_componentes_biconexas_=" << nbc << endl;
89 }
90     return 0;
91 }

```

## LCA + Climb

```

1  const int MAXN=100001;
2  const int LOGN=20;
3  //f[v][k] holds the 2^k father of v
4  //L[v] holds the level of v
5  int f[MAXN][LOGN], L[MAXN];
6  //call before build:
7  void dfs(int v, int fa=-1, int lvl=0){//generate required data
8      f[v][0]=fa, L[v]=lvl;
9      forall(it, G[v])if(*it!=fa)
10         dfs(*it, v, lvl+1);
11 }
12 void build(int N){//f[i][0] must be filled previously, O(nlgn)
13     forn(k, LOGN-1) forn(i, N) f[i][k+1]=f[f[i][k]][k];}
14
15 #define lg(x) (31-__builtin_clz(x))//=floor(log2(x))
16
17 int climb(int a, int d){//O(lgn)
18     if(!d) return a;
19     dforn(i, lg(L[a])+1)
20         if(1<=i<=d)
21             a=f[a][i], d-=1<=i;
22     return a;
23 }
24 int lca(int a, int b){//O(lgn)
25     if(L[a]<L[b]) swap(a, b);
26     a=climb(a, L[a]-L[b]);
27     if(a==b) return a;
28     dforn(i, lg(L[a])+1)
29         if(f[a][i]!=f[b][i])
30             a=f[a][i], b=f[b][i];
31     return f[a][0];
32 }
33 int dist(int a, int b) { //returns distance between nodes
34     return L[a]+L[b]-2*L[lca(a, b)];}

```



## Heavy Light Decomposition

```

1 int treesz[MAXN]; //cantidad de nodos en el subarbol del nodo v
2 int dad[MAXN]; //dad[v]=padre del nodo v
3 void dfs1(int v, int p=-1){ //pre-dfs
4     dad[v]=p;
5     treesz[v]=1;
6     forall(it, G[v]) if(*it!=p){
7         dfs1(*it, v);
8         treesz[v]+=treesz[*it];
9     }
10 }
11 //PONER Q EN 0 !!!!!
12 int pos[MAXN], q; //pos[v]=posicion del nodo v en el recorrido de la dfs
13 //Las cadenas aparecen continuas en el recorrido!
14 int cantcad;
15 int homecad[MAXN]; //dada una cadena devuelve su nodo inicial
16 int cad[MAXN]; //cad[v]=cadena a la que pertenece el nodo
17 void heavylight(int v, int cur=-1){
18     if(cur==-1) homecad[cur=cantcad++]=v;
19     pos[v]=q++;
20     cad[v]=cur;
21     int mx=-1;
22     forn(i, sz(G[v])) if(G[v][i]!=dad[v])
23         if(mx==-1 || treesz[G[v][mx]]<treesz[G[v][i]]) mx=i;
24     if(mx!=-1) heavylight(G[v][mx], cur);
25     forn(i, sz(G[v])) if(i!=mx && G[v][i]!=dad[v])
26         heavylight(G[v][i], -1);
27 }
28 //ejemplo de obtener el maximo numero en el camino entre dos nodos
29 //RTA: max(query(low, u), query(low, v)), con low=lca(u, v)
30 //esta funcion va trepando por las cadenas
31 int query(int an, int v){ //O(logn)
32     //si estan en la misma cadena:
33     if(cad[an]==cad[v]) return rmq.get(pos[an], pos[v]+1);
34     return max(query(an, dad[homecad[cad[v]]]),
35               rmq.get(pos[homecad[cad[v]]], pos[v]+1));
36 }

```

## Centroid Decomposition

```

1 int n;
2 vector<int> G[MAXN];
3 bool taken[MAXN]; //poner todos en FALSE al principio!!
4 int padre[MAXN]; //padre de cada nodo en el centroid tree
5
6 int szt[MAXN];
7 void calcsz(int v, int p) {

```

```

8     szt[v] = 1;
9     forall(it, G[v]) if (*it!=p && !taken[*it])
10         calcsz(*it, v), szt[v]+=szt[*it];
11 }
12 void centroid(int v=0, int f=-1, int lvl=0, int tam=-1) { //O(nlogn)
13     if(tam==-1) calcsz(v, -1), tam=szt[v];
14     forall(it, G[v]) if(!taken[*it] && szt[*it]>=tam/2)
15         {szt[v]=0; centroid(*it, f, lvl, tam); return;}
16     taken[v]=true;
17     padre[v]=f;
18     /*Analizar todos los caminos que pasan por este nodo:
19     * Agregar la informacion de cada subarbol
20     * Para cada subarbol:
21     * -sacar la informacion
22     * -analizar
23     * -agregar de nuevo la informacion
24     */
25     forall(it, G[v]) if(!taken[*it])
26         centroid(*it, v, lvl+1, -1);
27 }

```

## Euler Cycle

```

1 #define MAXN 1005
2 #define MAXE 1005005
3
4 int n, ars[MAXE], eq;
5 vector<int> G[MAXN]; //fill G, ars, eq
6 list<int> path;
7 int used[MAXN]; //used[v] = i => para todo j<=i la arista v-G[v][j] fue usada y la
8   arista v-G[v][i+1] no se uso
9 bool usede[MAXE];
10
11 //encuentra el ciclo euleriano, el grafo debe ser conexo y todos los nodos tener
12   grado par para que exista
13 //para encontrar el camino euleriano conectar los dos vertices de grado impar y
14   empezar de uno de ellos.
15
16 queue<list<int>::iterator> q;
17 int get(int v){
18     while(used[v]<sz(G[v]) && usede[ G[v][used[v]] ]) used[v]++;
19     return used[v];
20 }
21 void explore(int v, int r, list<int>::iterator it){
22     int ar=G[v][get(v)]; int u=v^ars[ar];
23     usede[ar]=true;
24     list<int>::iterator it2=path.insert(it, u);
25     if(u!=r) explore(u, r, it2);

```

```

23     if(get(v)<sz(G[v])) q.push(it);
24 }
25 void euler(int a){
26     zero(used), zero(usede);
27     path.clear();
28     q=queue<list<int>::iterator>();
29     path.push_back(a); q.push(path.begin());
30     while(sz(q)){
31         list<int>::iterator it=q.front(); q.pop();
32         if(used[*it]<sz(G[*it])) explore(*it, *it, it);
33     }
34     reverse(path.begin(), path.end());
35 }
36 void addEdge(int u, int v){
37     G[u].pb(eq), G[v].pb(eq);
38     ars[eq++]=u^v;
39 }

```

## Diametro árbol

```

1 vector<int> G[MAXN]; int n,m,p[MAXN],d[MAXN],d2[MAXN];
2 int bfs(int r, int *d) {
3     queue<int> q;
4     d[r]=0; q.push(r);
5     int v;
6     while(sz(q)) { v=q.front(); q.pop();
7         forall(it,G[v]) if (d[*it]==-1)
8             d[*it]=d[v]+1, p[*it]=v, q.push(*it);
9     }
10    return v;//ultimo nodo visitado
11 }
12 vector<int> diams; vector<ii> centros;
13 void diametros(){
14     memset(d,-1,sizeof(d));
15     memset(d2,-1,sizeof(d2));
16     diams.clear(), centros.clear();
17     forn(i, n) if(d[i]==-1){
18         int v,c;
19         c=v=bfs(bfs(i, d2), d);
20         forn(_,d[v]/2) c=p[c];
21         diams.pb(d[v]);
22         if(d[v]&1) centros.pb(ii(c, p[c]));
23         else centros.pb(ii(c, c));
24     }
25 }

```

## Chu-liu

```

1 void visit(graph&h,int v,int s,int r,vector<int>&no,vector<vector<int>>&comp,
    vector<int>&prev,vector<vector<int>>&next,vector<weight>&mcost,vector<int>&
    mark,weight&cost,bool&found){if(mark[v]){vector<int>temp=no;found=true;do{cost
    +=mcost[v];v=prev[v];if(v!=s){while(comp[v].size()>0){no[comp[v].back()]=s;
    comp[s].push_back(comp[v].back());comp[v].pop_back();}}while(v!=s);forall(j,
    comp[s])if(*j!=r)forall(e,h[*j])if(no[e->src]!=s)e->w-=mcost[ temp[*j] ];mark
    [v]=true;forall(i,next[v])if(no[*i]!=no[v]&&prev[no[*i]]==v)if(!mark[no[*i]
    ])|!*i==s)visit(h,*i,s,r,no,comp,prev,next,mcost,mark,cost,found);}weight
    minimumSpanningArborescence(const graph&g,int r){const int n=sz(g);graph h(n);
    forn(u,n)forall(e,g[u])h[e->dst].pb(*e);vector<int>no(n);vector<vector<int>>
    comp(n);forn(u,n)comp[u].pb(no[u]=u);for(weight cost=0;;){vector<int>prev(n
    ,-1);vector<weight>mcost(n,INF);forn(j,n)if(j!=r)forall(e,h[j])if(no[e->src]!=
    no[j])if(e->w<mcost[ no[j] ])mcost[ no[j] ]=e->w,prev[ no[j] ]=no[e->src];
    vector<vector<int>>next(n);forn(u,n)if(prev[u]>=0)next[ prev[u] ].push_back(u)
    ;bool stop=true;vector<int>mark(n);forn(u,n)if(u!=r&&!mark[u]&&!comp[u].empty
    ()){bool found=false;visit(h,u,u,r,no,comp,prev,next,mcost,mark,cost,found);if
    (found)stop=false;if(stop){forn(u,n)if(prev[u]>=0)cost+=mcost[u];return cost
    ;}}}

```

## Hungarian

```

1 //Dado un grafo bipartito completo con costos no negativos, encuentra el matching
  perfecto de minimo costo.
2 const tipo EPS=1e-9;const tipo INF=1e14;
3 #define N 502
4 tipo cost[N][N],lx[N],ly[N],slack[N];int n,max_match,xy[N],yx[N],slackx[N],prev2[N
  ];bool S[N],T[N];void add_to_tree(int x,int prevx){S[x]=true,prev2[x]=prevx;
  forn(y,n)if(lx[x]+ly[y]-cost[x][y]<slack[y]-EPS)slack[y]=lx[x]+ly[y]-cost[x][y
  ],slackx[y]=x;void update_labels(){tipo delta=INF;forn(y,n)if(!T[y])delta=min
  (delta,slack[y]);forn(x,n)if(S[x])lx[x]-=delta;forn(y,n)if(T[y])ly[y]+=delta;
  else slack[y]-=delta;void init_labels(){zero(lx),zero(ly);forn(x,n)forn(y,n)
  lx[x]=max(lx[x],cost[x][y]);void augment(){if(max_match==n)return;int x,y,
  root,q[N],wr=0,rd=0;memset(S,false,sizeof(S)),memset(T,false,sizeof(T));memset
  (prev2,-1,sizeof(prev2));forn(x,n)if(xy[x]==-1){q[wr++]=root=x,prev2[x]=-2;S[x
  ]=true;break;}forn(y,n)slack[y]=lx[root]+ly[y]-cost[root][y],slackx[y]=root;
  while(true){while(rd<wr){x=q[rd++];for(y=0;y<n;y++)if(cost[x][y]==lx[x]+ly[y]
  )&&!T[y]){if(yx[y]==-1)break;T[y]=true;q[wr++]=yx[y],add_to_tree(yx[y],x);}if(
  y<n)break;}if(y<n)break;update_labels(),wr=rd=0;for(y=0;y<n;y++)if(!T[y]&&
  slack[y]==0){if(yx[y]==-1){x=slackx[y];break;}else{T[y]=true;if(!S[yx[y]])q[wr
  ++]=yx[y],add_to_tree(yx[y],slackx[y]);}}if(y<n)break;if(y<n){max_match++;for
  (int cx=x,cy=y,ty;cx!=-2;cx=prev2[cx],cy=ty)ty=xy[cx],yx[cy]=cx,xy[cx]=cy;
  augment();}}tipo hungarian(){tipo ret=0;max_match=0,memset(xy,-1,sizeof(xy));
  memset(yx,-1,sizeof(yx)),init_labels(),augment();forn(x,n)ret+=cost[x][xy[x]];
  return ret;}

```

## Dynamic Connectivity

```

1 struct UnionFind {

```

```

2   int n, comp;
3   vector<int> pre, si, c;
4   UnionFind(int n=0):n(n), comp(n), pre(n), si(n, 1) {
5       forn(i,n) pre[i] = i; }
6   int find(int u){return u==pre[u]?u:find(pre[u]);}
7   bool merge(int u, int v) {
8       if((u=find(u))== (v=find(v))) return false;
9       if(si[u]<si[v]) swap(u, v);
10      si[u]+=si[v], pre[v]=u, comp--, c.pb(v);
11      return true;
12  }
13  int snap(){return sz(c);}
14  void rollback(int snap){
15      while(sz(c)>snap){
16          int v = c.back(); c.pop_back();
17          si[pre[v]] -= si[v], pre[v] = v, comp++;
18      }
19  }
20 };
21 enum {ADD,DEL,QUERY};
22 struct Query {int type,u,v;};
23 struct DynCon {
24     vector<Query> q;
25     UnionFind dsu;
26     vector<int> match,res;
27     map<ii,int> last; //se puede no usar cuando hay identificador para cada arista
                        (mejora poco)
28     DynCon(int n=0):dsu(n){}
29     void add(int u, int v) {
30         if(u>v) swap(u,v);
31         q.pb((Query){ADD, u, v}), match.pb(-1);
32         last[ii(u,v)] = sz(q)-1;
33     }
34     void remove(int u, int v) {
35         if(u>v) swap(u,v);
36         q.pb((Query){DEL, u, v});
37         int prev = last[ii(u,v)];
38         match[prev] = sz(q)-1;
39         match.pb(prev);
40     }
41     void query() { //podria pasarle un puntero donde guardar la respuesta
42         q.pb((Query){QUERY, -1, -1}), match.pb(-1);}
43     void process() {
44         forn(i,sz(q)) if (q[i].type == ADD && match[i] == -1) match[i] = sz(q);
45         go(0,sz(q));
46     }
47     void go(int l, int r) {

```

```

48         if(l+1==r){
49             if (q[l].type == QUERY) //Aqui responder la query usando el dsu!
50                 res.pb(dsu.comp); //aqui query=cantidad de componentes conexas
51             return;
52         }
53         int s=dsu.snap(), m = (l+r) / 2;
54         forr(i,m,r) if(match[i]!=-1 && match[i]<l) dsu.merge(q[i].u, q[i].v);
55         go(l,m);
56         dsu.rollback(s);
57         s = dsu.snap();
58         forr(i,l,m) if(match[i]!=-1 && match[i]>=r) dsu.merge(q[i].u, q[i].v);
59         go(m,r);
60         dsu.rollback(s);
61     }
62 }dc;

```

## DFS Paralelo

```

1  #define MAXN 212345
2  set<int> G[MAXN];
3  set<int>::iterator it[MAXN];
4  int S[2][MAXN]; //pila
5  int szS[2]; //tamaño de la pila
6  int szC[2]; //tamaño de la componente
7  bool vis[MAXN];
8  void dfsparalelo(int a, int b){ //0(componente mas chica)
9      zero(vis);
10     szS[0] = szS[1] = 0;
11     szC[0] = szC[1] = 1;
12     if(sz(G[a])){
13         S[0][szS[0]++] = a; //push(a);
14         it[a] = G[a].begin();
15     }
16     if(sz(G[b])){
17         S[1][szS[1]++] = b; //push(b);
18         it[b] = G[b].begin();
19     }
20     int act = 0;
21     vis[a] = vis[b] = true;
22
23     while(szS[act]){ //recorre las dos componentes en paralelo
24         int v = S[act][szS[act]-1]; //top();
25         int u = *it[v];
26         it[v]++;
27         if(it[v] == G[v].end()) szS[act]--; //pop();
28         if(vis[u]){act = 1 - act; continue;}
29         szC[act]++;
30         if(sz(G[u])>1 or *G[u].begin() != v){

```

```

31     S[act][szS[act]++] = u; //.push(u);
32     vis[u] = true;
33     it[u] = G[u].begin();
34 }
35 act = 1 - act;
36 }
37 act = 1 - act; //ya recorrio la toda la componente de act
38
39 //sigue recorriendo la otra componente hasta que ve un elemento más o no tiene
    más elementos.
40 while(szC[act] < szC[1-act]+1 and szS[act]){
41     int v = S[act][szS[act]-1]; //.top();
42     int u = *it[v];
43     it[v]++;
44     if(it[v] == G[v].end()) szS[act]--; //.pop();
45     if(vis[u]) continue;
46     szC[act]++;
47     if(sz(G[u])>1 or *G[u].begin() != v){
48         S[act][szS[act]++] = u; //.push(u);
49         vis[u] = true;
50         it[u] = G[u].begin();
51 } } }

```

## Network Flow

### Dinic

```

1
2 const int MAX = 300;
3 // Corte minimo: vertices con dist[v]>=0 (del lado de src) VS. dist[v]==-1 (del
    lado del dst)
4 // Para el caso de la red de Bipartite Matching (Sean V1 y V2 los conjuntos mas
    proximos a src y dst respectivamente):
5 // Reconstruir matching: para todo v1 en V1 ver las aristas a vertices de V2 con
    it->f>0, es arista del Matching
6 // Min Vertex Cover: vertices de V1 con dist[v]==-1 + vertices de V2 con dist[v]>0
7 // Max Independent Set: tomar los vertices NO tomados por el Min Vertex Cover
8 // Max Clique: construir la red de G complemento (debe ser bipartito!) y encontrar
    un Max Independet Set
9 // Min Edge Cover: tomar las aristas del matching + para todo vertices no cubierto
    hasta el momento, tomar cualquier arista de el
10 //Complejidad:
11 //Peor caso:  $O(V^2E)$ 
12 //Si todas las capacidades son 1:  $O(\min(E^{1/2}, V^{2/3})E)$ 
13 //Para matching bipartito es:  $O(\sqrt{V}E)$ 
14
15 int nodes, src, dst;

```

```

16 int dist[MAX], q[MAX], work[MAX];
17 struct Edge {
18     int to, rev;
19     ll f, cap;
20     Edge(int to, int rev, ll f, ll cap) : to(to), rev(rev), f(f), cap(cap) {}
21 };
22 vector<Edge> G[MAX];
23 void addEdge(int s, int t, ll cap){
24     G[s].pb(Edge(t, sz(G[t]), 0, cap)), G[t].pb(Edge(s, sz(G[s])-1, 0, 0));}
25 bool dinic_bfs(){
26     fill(dist, dist+nodes, -1), dist[src]=0;
27     int qt=0; q[qt++] = src;
28     for(int qh=0; qh<qt; qh++){
29         int u = q[qh];
30         forall(e, G[u]){
31             int v = e->to;
32             if(dist[v]<0 && e->f < e->cap)
33                 dist[v] = dist[u] + 1, q[qt++] = v;
34         }
35     }
36     return dist[dst] >= 0;
37 }
38 ll dinic_dfs(int u, ll f){
39     if(u == dst) return f;
40     for(int &i = work[u]; i < sz(G[u]); i++){
41         Edge &e = G[u][i];
42         if(e.cap <= e.f) continue;
43         int v = e.to;
44         if(dist[v] == dist[u] + 1){
45             ll df = dinic_dfs(v, min(f, e.cap - e.f));
46             if(df > 0){
47                 e.f += df, G[v][e.rev].f -= df;
48                 return df; }
49         }
50     }
51     return 0;
52 }
53 ll maxFlow(int _src, int _dst){
54     src = _src, dst = _dst;
55     ll result = 0;
56     while(dinic_bfs()){
57         fill(work, work+nodes, 0);
58         while(ll delta = dinic_dfs(src, INF))
59             result += delta;
60     }
61     // todos los nodos con dist[v] != -1 vs los que tienen dist[v] == -1 forman el min
    -cut

```

62 |     return result; }

## Min-cost Max-flow

```

1  const int MAXN=10000;
2  typedef ll tf;
3  typedef ll tc;
4  const tf INFFLUJO = 1e14;
5  const tc INFCOSTO = 1e14;
6  struct edge {
7      int u, v;
8      tf cap, flow;
9      tc cost;
10     tf rem() { return cap - flow; }
11 };
12 int nodes; //numero de nodos
13 vector<int> G[MAXN]; // limpiar!
14 vector<edge> e; // limpiar!
15 void addEdge(int u, int v, tf cap, tc cost) {
16     G[u].pb(sz(e)); e.pb((edge){u,v,cap,0,cost});
17     G[v].pb(sz(e)); e.pb((edge){v,u,0,0,-cost});
18 }
19 tc dist[MAXN], mnCost;
20 int pre[MAXN];
21 tf cap[MAXN], mxFlow;
22 bool in_queue[MAXN];
23 void flow(int s, int t) {
24     zero(in_queue);
25     mxFlow=mnCost=0;
26     while(1){
27         fill(dist, dist+nodes, INFCOSTO); dist[s] = 0;
28         memset(pre, -1, sizeof(pre)); pre[s]=0;
29         zero(cap); cap[s] = INFFLUJO;
30         queue<int> q; q.push(s); in_queue[s]=1;
31         while(sz(q)){
32             int u=q.front(); q.pop(); in_queue[u]=0;
33             for(auto it:G[u]) {
34                 edge &E = e[it];
35                 if(E.rem() && dist[E.v] > dist[u] + E.cost + 1e-9){ // ojo EPS
36                     dist[E.v]=dist[u]+E.cost;
37                     pre[E.v] = it;
38                     cap[E.v] = min(cap[u], E.rem());
39                     if(!in_queue[E.v]) q.push(E.v), in_queue[E.v]=1;
40                 }
41             }
42         }
43         if (pre[t] == -1) break;
44         mxFlow +=cap[t];

```

```

45     mnCost +=cap[t]*dist[t];
46     for (int v = t; v != s; v = e[pre[v]].u) {
47         e[pre[v]].flow += cap[t];
48         e[pre[v]^1].flow -= cap[t];
49     }
50 }
51 }

```

## Template

```

1  #include <bits/stdc++.h>
2  using namespace std;
3  #define forr(i,a,b) for(int i=(a); i<(b); i++)
4  #define forn(i,n) forr(i,0,n)
5  #define zero(v) memset(v, 0, sizeof(v))
6  #define forall(it,v) for(auto it=v.begin();it!=v.end();++it)
7  #define pb push_back
8  #define fst first
9  #define snd second
10 typedef long long ll;
11 typedef pair<ll,ll> pll;
12 #define dforn(i,n) for(int i=n-1; i>=0; i--)
13 ios::sync_with_stdio(0); cin.tie(0);

```

## Ayudamemoria

### Doubles Comp.

```

1  const double EPS = 1e-9;
2  #define feq(a, b) (fabs((a)-(b))<EPS)
3  x == y  <=> fabs(x-y) < EPS
4  x > y   <=> x > y + EPS
5  x >= y  <=> x > y - EPS

```

### Expandir pila

```

1  #include <sys/resource.h>
2  rlimit rl;
3  getrlimit(RLIMIT_STACK, &rl);
4  rl.rlim_cur=1024L*1024L*256L;//256mb
5  setrlimit(RLIMIT_STACK, &rl);

```

### Iterar subconjunto

```

1  for(int sbm=bm; sbm; sbm=(sbm-1)&bm)

```