

 ${\bf \acute{I}ndice}$

1. algorithm

#include <algorithm> #include <numeric>

Algo	Params	Funcion
sort, stable_sort	f, 1	ordena el intervalo
nth_element	f, nth, l	void ordena el n-esimo, y
		particiona el resto
fill, fill_n	f, l / n, elem	void llena [f, l) o [f,
		f+n) con elem
lower_bound, upper_bound	f, l, elem	it al primer / ultimo donde se
		puede insertar elem para que
		quede ordenada
binary_search	f, l, elem	bool esta elem en [f, l)
copy	f, l, resul	hace resul+ i =f+ $i \forall i$
find, find_if, find_first_of	f, l, elem	it encuentra i \in [f,l) tq. i=elem,
	/ pred / f2, l2	$\operatorname{pred}(i), i \in [f2,l2)$
count, count_if	f, l, elem/pred	cuenta elem, pred(i)
search	f, l, f2, l2	busca $[f2,l2) \in [f,l)$
replace, replace_if	f, l, old	cambia old / pred(i) por new
	/ pred, new	
reverse	f, 1	da vuelta
partition, stable_partition	f, l, pred	pred(i) ad, !pred(i) atras
min_element, max_element	f, l, [comp]	$it \min, \max de [f,l]$
lexicographical_compare	f1,l1,f2,l2	bool con [f1,l1]<[f2,l2]
next/prev_permutation	f,l	deja en [f,l) la perm sig, ant
set_intersection,	f1, l1, f2, l2, res	[res,) la op. de conj
set_difference, set_union,		
_set_symmetric_difference,		
push_heap, pop_heap,	f, l, e / e /	mete/saca e en heap [f,l),
make_heap		hace un heap de [f,l)
is_heap	f,l	bool es [f,l) un heap
accumulate	f,l,i,[op]	$T = \sum /\text{oper de [f,l)}$
inner_product	f1, l1, f2, i	$T = i + [f1, 11) \cdot [f2, \dots)$
partial_sum	f, l, r, [op]	$r+i = \sum /oper de [f,f+i] \forall i \in [f,l)$
builtin_ffs	unsigned int	Pos. del primer 1 desde la derecha
builtin_clz	unsigned int	Cant. de ceros desde la izquierda.
builtin_ctz	unsigned int	Cant. de ceros desde la derecha.
builtin_popcount	unsigned int	Cant. de 1's en x.
builtin_parity	unsigned int	1 si x es par, 0 si es impar.
builtin_XXXXXXII	unsigned ll	= pero para long long's.

2. Estructuras

2.1. RMQ (static)

Dado un arreglo y una operacion asociativa *idempotente*, get(i, j) opera sobre el rango [i, j). Restriccion: LVL ≥ ceil(logn); Usar [] para llenar arreglo y luego build().

```
1 struct RMQ{
     #define LVL 10
2
     tipo vec[LVL] [1<<(LVL+1)];
     tipo &operator[](int p){return vec[0][p];}
     tipo get(int i, int j) {//intervalo [i,j)
5
       int p = 31- builtin clz(j-i);
6
       return min(vec[p][i],vec[p][j-(1<<p)]);
7
8
     void build(int n) {//O(nlogn)
9
       int mp = 31-_builtin_clz(n);
10
       forn(p, mp) forn(x, n-(1<<p))
11
         vec[p+1][x] = min(vec[p][x], vec[p][x+(1<<p)]);
12
    }};
13
```

2.2. RMQ (dynamic)

```
1 //Dado un arreglo y una operacion asociativa con neutro, get(i, j) opera
       sobre el rango [i, j).
   #define MAXN 100000
   #define operacion(x, y) max(x, y)
   const int neutro=0;
   struct RMQ{
     int sz;
6
     tipo t[4*MAXN];
7
     tipo &operator[](int p){return t[sz+p];}
8
     void init(int n){//O(nlgn)
9
       sz = 1 << (32-_builtin_clz(n));</pre>
10
       forn(i, 2*sz) t[i]=neutro;
11
12
     void updall(){//0(n)}
13
       dforn(i, sz) t[i]=operacion(t[2*i], t[2*i+1]);}
14
     tipo get(int i, int j){return get(i,j,1,0,sz);} // [i,j) !
15
     tipo get(int i, int j, int n, int a, int b){\frac{1}{0}}
16
       if(j<=a || i>=b) return neutro;
17
       if(i<=a && b<=j) return t[n];</pre>
18
       int c=(a+b)/2;
19
```

```
return operacion(get(i, j, 2*n, a, c), get(i, j, 2*n+1, c, b));
20
     }
21
     void set(int p, tipo val){//O(lgn)
22
       for(p+=sz; p>0 && t[p]!=val;){
23
         t[p]=val;
24
         p/=2;
25
         val=operacion(t[p*2], t[p*2+1]);
26
27
     }
28
   }rmq;
   //Usage:
| cin >> n; rmq.init(n); forn(i, n) cin >> rmq[i]; rmq.updall();
                             2.3. RMQ (lazy)
1 //Dado un arreglo y una operacion asociativa con neutro, get(i, j) opera
       sobre el rango [i, j).
   typedef int Elem; //Elem de los elementos del arreglo
   typedef int Alt;//Elem de la alteracion
   #define operacion(x,y) x+y
   const Elem neutro=0; const Alt neutro2=0;
   #define MAXN 1024000
   struct RMQ{
     int sz:
     Elem t[4*MAXN]:
     Alt dirty[4*MAXN];//las alteraciones pueden ser de distinto Elem
10
     Elem &operator[](int p){return t[sz+p];}
11
     void init(int n){//O(nlgn)
12
       sz = 1 \ll (32-\_builtin\_clz(n));
13
       forn(i, 2*sz) t[i]=neutro;
14
       forn(i, 2*sz) dirty[i]=neutro2;
15
16
     void updall(){//0(n)}
17
       dforn(i, sz) t[i]=operacion(t[2*i], t[2*i+1]);}
18
     void opAltT(int n,int a,int b){//altera el valor del nodo n segun su
19
         dirty y el intervalo que le corresponde.
       t[n] += dirty[n]*(b-a);
20
     } //en este caso la alteracion seria sumarle a todos los elementos del
21
         intervalo [a,b) el valor dirty[n]
     void opAltD(int n ,Alt val){
22
       dirty[n] += val;
23
     }//actualiza el valor de Dirty "sumandole" val. podria cambiar el valor
24
```

de dirty dependiendo de la operacion que se quiera al actualizar un

```
t[p]=val:
          rango. Ej:11402.cpp
                                                                                       64
     void push(int n, int a, int b){//propaga el dirty a sus hijos
25
                                                                                       65
       if(dirty[n]!=neutro2){
                                                                                       66
26
         opAltT(n,a,b); //t[n]+=dirty[n]*(b-a);//altera el nodo
                                                                                       67
27
         if(n<sz){
28
           opAltD(2*n,dirty[n]);//dirty[2*n]+=dirty[n];
                                                                                       69
29
           opAltD(2*n+1,dirty[n]);//dirty[2*n+1]+=dirty[n];
                                                                                           }
                                                                                       70
30
                                                                                       71 | };
31
         dirty[n]=neutro2;
32
33
34
     Elem get(int i, int j, int n, int a, int b){\frac{1}{0}}
35
       if(i<=a || i>=b) return neutro:
                                                                                        typedef int tipo;
36
       push(n, a, b);//corrige el valor antes de usarlo
37
       if(i<=a && b<=j) return t[n];</pre>
                                                                                              return a+b;
38
       int c=(a+b)/2:
39
                                                                                        4
       return operacion(get(i, j, 2*n, a, c), get(i, j, 2*n+1, c, b));
40
                                                                                          struct node{
41
                                                                                            tipo v; node *1,*r;
     Elem get(int i, int j){return get(i,j,1,0,sz);}
42
     //altera los valores en [i, j) con una alteración de val
43
     void alterar(Alt val, int i, int j, int n, int a, int b){\frac{}{(0)}}
                                                                                                   if(!1) v=r->v;
44
                                                                                       9
       push(n, a, b);
                                                                                                   else if(!r) v=l->v;
45
                                                                                       10
       if(j<=a || i>=b) return;
46
       if(i<=a && b<=j){
                                                                                              }
47
                                                                                       12
         opAltD(n,val);//actualiza el valor de Dirty por val.
                                                                                          }:
48
                                                                                       13
         push(n,a,b);
49
         return;//este nodo esta totalmente contenido por el intervalo a
50
              alterar, no es necesario que se lo pases a los hijos.. por ahora
                                                                                            int tm=(tl + tr)>>1;
                                                                                       17
       }
51
                                                                                       18
       int c=(a+b)/2;
52
       alterar(val, i, j, 2*n, a, c), alterar(val, i, j, 2*n+1, c, b);
53
                                                                                       20
       t[n]=operacion(t[2*n], t[2*n+1]);//por esto es el push de arriba
                                                                                            int tm=(tl+tr)>>1;
54
                                                                                       21
55
                                                                                       22
     void alterar(Alt val, int i, int j){alterar(val,i,j,1,0,sz);}
56
                                                                                       23
57
                                                                                       24
     //setea de a un elemento. Esto lo "hace" dinámico.
58
                                                                                       25
     void set(int p, Elem val){//O(lgn)
59
                                                                                       26
       if(p<0) return; //OJO chequear que p no sea muy grande
                                                                                            int tm=(tl + tr)>>1:
60
                                                                                       27
       this->get(p,p+1); //para que acomode los dirty del camino de la raíz a
61
                                                                                       28
                                                                                       29
       int a=p, b=p+1, ancho=1, vecino;
62
                                                                                       30
       for(p+=sz; p>0 && t[p]!=val; ancho*=2){
63
                                                                                       31 }
```

```
if(p&1){ vecino=p-1; push(vecino,a,b); a-=ancho; }
else{ vecino=p+1; push(vecino,a,b); b+=ancho; }
val=operacion(t[p*2], t[p*2+1]);
```

2.4. RMQ (persistente)

```
tipo oper(const tipo &a, const tipo &b){
  node(tipo v):v(v), 1(NULL), r(NULL) {}
    node(node *1, node *r) : 1(1), r(r){
        else v=oper(1->v, r->v);
node *build (tipo *a, int tl, int tr) {//modificar para que tome tipo a
  if (tl+1==tr) return new node(a[tl]);
  return new node(build(a, tl, tm), build(a, tm, tr));
node *update(int pos, int new_val, node *t, int tl, int tr){
 if (tl+1==tr) return new node(new_val);
  if(pos < tm) return new node(update(pos, new_val, t->1, tl, tm), t->r);
  else return new node(t->1, update(pos, new val, t->r, tm, tr));
tipo get(int 1, int r, node *t, int tl, int tr){
    if(l==t1 && tr==r) return t->v:
    if(r<=tm) return get(l, r, t->l, tl, tm);
    else if(l>=tm) return get(l, r, t->r, tm, tr);
 return oper(get(1, tm, t->1, t1, tm), get(tm, r, t->r, tm, tr));
```

2.5. Fenwick Tree

```
1 //For 2D threat each column as a Fenwick tree, by adding a nested for in
        each operation
  struct Fenwick{
     int sz; //los elementos van de 1 a sz-1
     tipo t[MAXN] [MAXN];
     void init (int n){
5
       sz = n;
6
       forn(i,MAXN) forn(j,MAXN) t[i][j] = 0;
7
8
     //le suma v al valor de (p,q)
     void adjust(int p, int q, tipo v){//valid with p in [1, sz), q in [1,sz)
          --> 0(lgn*lgn)
       for(int i=p; i<sz; i+=(i&-i))</pre>
11
         for(int j=q; j<sz; j+=(j&-j))
12
           t[i][i]+=v: }
13
     tipo sum(int p, int q){//\text{cumulative sum in [(1,1), (p,q)], 0(lgn*lgn)} --
14
         OJO: los rangos son cerrados!
15
       for(int i=p; i; i-=(i&-i)) for(int j=q; j; j-=(j&-j)) s+=t[i][j];
16
       return s;
17
18
     tipo sum(int a1, int b1, int a2, int b2){return sum(a2,b2)-sum(a1-1,b2) -
19
           sum(a2,b1-1) + sum(a1-1,b1-1);
     //get largest value with cumulative sum less than or equal to x;
20
     //for smallest, pass x-1 and add 1 to result
21
     int getind(tipo x) {//O(lgn) -- VER!
22
         int idx = 0, mask = N;
23
         while(mask && idx < N) {
24
           int t = idx + mask:
25
         if(x >= tree[t])
26
             idx = t, x -= tree[t];
27
           mask >>= 1;
28
29
         return idx:
30
     }} f;
31
                              2.6. Union Find
```

```
struct UnionFind{
vector<int> f;//the array contains the parent of each node
void init(int n){f.clear(); f.insert(f.begin(), n, -1);}
```

```
int comp(int x){return (f[x]=-1?x:f[x]=comp(f[x]));}//0(1)
4
     bool join(int i, int j) {
5
       bool con=comp(i)==comp(j);
6
       if(!con) f[comp(i)] = comp(j);
       return con;
8
    }};
9
                          2.7. Disjoint Intervals
bool operator (const ii &a, const ii &b) {return a.fst <b.fst;}
   //Stores intervals as [first, second]
   //in case of a collision it joins them in a single interval
   struct disjoint_intervals {
     set<ii>> segs;
5
     void insert(ii v) {//O(lgn)
6
       if(v.snd-v.fst==0.) return://0J0
7
       set<ii>>::iterator it.at;
8
       at = it = segs.lower bound(v);
9
       if (at!=segs.begin() && (--at)->snd >= v.fst)
10
         v.fst = at->fst, --it;
11
       for(; it!=segs.end() && it->fst <= v.snd; segs.erase(it++))</pre>
12
         v.snd=max(v.snd, it->snd):
13
       segs.insert(v);
14
    }
15
16 };
                              2.8. RMQ (2D)
1 | struct RMQ2D{//n filas x m columnas
     int sz;
     RMQ t[4*MAXN];
     void init(int n, int m){\frac{1}{0}(n*m)}
       sz = 1 << (32-_builtin_clz(n));</pre>
5
       forn(i, 2*sz) t[i].init(m); }
6
     void set(int i, int j, tipo val){//0(lgm.lgn)
       for(i+=sz; i>0;){
         t[i].set(j, val);
9
10
         val=operacion(t[i*2][j], t[i*2+1][j]);
11
       } }
12
     tipo get(int i1, int j1, int i2, int j2){return get(i1, j1, i2, j2, 1, 0, sz);}
13
     //O(lgm.lgn), rangos cerrado abierto
14
     int get(int i1, int j1, int i2, int j2, int n, int a, int b){
15
       if(i2<=a || i1>=b) return neutro;
16
```

fill(n+l, n+LMAX, 0);

30

```
if(i1<=a && b<=i2) return t[n].get(j1, j2);</pre>
                                                                                             while(1>1 && !n[1-1]) 1--;
17
                                                                                      31
       int c=(a+b)/2;
                                                                                          }
                                                                                      32
18
       return operacion(get(i1, j1, i2, j2, 2*n, a, c),
                                                                                         };
                                                                                      33
19
            get(i1, j1, i2, j2, 2*n+1, c, b));
                                                                                         bint operator+(const bint&a, const bint&b){
20
                                                                                           bint c;
     }
                                                                                      35
21
   } rma;
                                                                                             c.1 = max(a.1, b.1);
22
   //Example to initialize a grid of M rows and N columns:
                                                                                             11 q = 0;
   RMQ2D rmq; rmq.init(n,m);
                                                                                             forn(i, c.l) q += a.n[i]+b.n[i], c.n[i]=q BASE, q/=BASE;
   forn(i, n) forn(j, m){
                                                                                             if(q) c.n[c.l++] = q;
                                                                                             c.invar();
     int v; cin >> v; rmq.set(i, j, v);}
                                                                                             return c;
                                                                                      41
                                2.9. Big Int
                                                                                      42
                                                                                         pair bint, bool lresta(const bint& a, const bint& b) // c = a - b
                                                                                         {
                                                                                      44
   #define BASEXP 6
                                                                                           bint c:
   #define BASE 1000000
                                                                                      45
                                                                                             c.1 = max(a.1, b.1);
   #define LMAX 1000
                                                                                             11 q = 0;
   struct bint{
                                                                                             forn(i, c.1) q += a.n[i]-b.n[i], c.n[i]=(q+BASE) %BASE, q=(q+BASE)/BASE
       int 1;
5
                                                                                                  -1:
       11 n[LMAX];
                                                                                             c.invar();
       bint(11 x=0){
                                                                                             return make_pair(c, !q);
           1=1:
                                                                                      50
8
           forn(i, LMAX){
                                                                                      51
9
                                                                                         bint& operator-= (bint& a, const bint& b){return a=lresta(a, b).first;}
               if (x) l=i+1;
10
                                                                                         bint operator- (const bint&a, const bint&b) {return lresta(a, b).first;}
               n[i]=x BASE:
11
                                                                                         bool operator (const bint&a, const bint&b) {return !lresta(a, b).second;}
               x/=BASE:
12
                                                                                         bool operator = (const bint&a, const bint&b){return lresta(b, a).second;}
13
                                                                                         bool operator==(const bint&a, const bint&b){return a <= b && b <= a;}
           }
14
                                                                                         bint operator*(const bint&a, ll b){
15
                                                                                             bint c;
       bint(string x){
                                                                                      58
16
                                                                                             11 q = 0;
       l=(x.size()-1)/BASEXP+1;
                                                                                      59
17
                                                                                             forn(i, a.1) q += a.n[i]*b, c.n[i] = q %BASE, q/=BASE;
           fill(n, n+LMAX, 0);
                                                                                      60
18
                                                                                             c.1 = a.1;
           ll r=1;
                                                                                      61
19
                                                                                             while(q) c.n[c.l++] = q \ BASE, q/=BASE;
           forn(i, sz(x)){
                                                                                      62
20
                                                                                             c.invar():
               n[i / BASEXP] += r * (x[x.size()-1-i]-'0');
                                                                                      63
21
                                                                                             return c:
                                                                                      64
               r*=10; if (r==BASE)r=1;
22
                                                                                      65
           }
23
                                                                                         bint operator*(const bint&a, const bint&b){
       }
                                                                                      66
24
                                                                                             bint c:
       void out(){
25
                                                                                             c.1 = a.1+b.1;
       cout << n[l-1]:
26
                                                                                             fill(c.n, c.n+b.1, 0);
       dforn(i, l-1) printf("%6.61lu", n[i]);//6=BASEXP!
27
                                                                                             forn(i, a.1){
                                                                                      70
28
                                                                                                 11 q = 0;
     void invar(){
                                                                                     71
29
                                                                                                 forn(j, b.1) q += a.n[i]*b.n[j]+c.n[i+j], c.n[i+j] = q BASE, q/=
```

72

```
c.invar():
                BASE:
            c.n[i+b.1] = q;
                                                                                               return make_pair(c, rm);
                                                                                      116
73
                                                                                          1
                                                                                      117
74
        c.invar();
                                                                                          bint operator/(const bint&a, const bint&b){return ldiv(a, b).first;}
75
                                                                                      bint operator %(const bint&a, const bint&b) {return ldiv(a, b).second;}
        return c;
76
77
                                                                                                                     2.10. HashTables
    pair bint, 11> ldiv(const bint& a, 11 b) \{// c = a / b : rm = a \% b \}
78
     bint c;
79
                                                                                        1 //Compilar: g++ --std=c++11
     11 \text{ rm} = 0;
80
                                                                                        2 struct Hash{
      dform(i, a.1){
81
                                                                                            size_t operator()(const ii &a)const{
                rm = rm * BASE + a.n[i];
82
                                                                                               size_t s=hash<int>()(a.fst);
                c.n[i] = rm / b;
83
                                                                                               return hash<int>()(a.snd)+0x9e3779b9+(s<<6)+(s>>2);
                                                                                        5
                rm %= b:
84
                                                                                        6
        }
85
                                                                                            size t operator()(const vector<int> &v)const{
                                                                                       7
        c.1 = a.1;
86
                                                                                               size t s=0:
        c.invar();
87
                                                                                              for(auto &e : v)
                                                                                       9
        return make pair(c, rm);
88
                                                                                                 s = hash(int)()(e)+0x9e3779b9+(s<<6)+(s>>2):
                                                                                       10
89
                                                                                       11
                                                                                              return s;
    bint operator/(const bint&a, ll b){return ldiv(a, b).first;}
                                                                                            }
                                                                                       12
    ll operator %(const bint&a, ll b){return ldiv(a, b).second;}
                                                                                          };
                                                                                       13
    pair<bint, bint> ldiv(const bint& a, const bint& b){
92
                                                                                          unordered set<ii, Hash> s;
     bint c;
93
                                                                                       unordered map<ii, int, Hash> m;//map<key, value, hasher>
        bint rm = 0;
94
        dform(i, a.1){
                                                                                                                      2.11. Modnum
95
            if (rm.l==1 && !rm.n[0])
96
                rm.n[0] = a.n[i];
                                                                                        1 //lindos valores para hash
97
            else{
                                                                                          #define MOD 100000000000000000001LL
98
                dforn(j, rm.l) rm.n[j+1] = rm.n[j];
                                                                                          #define PRIME 1009LL
99
                rm.n[0] = a.n[i];
100
                rm.l++;
                                                                                          mnum inv[MAXMOD];//inv[i]*i=1 mod MOD
101
            }
                                                                                          void calc(int p){//calcula inversos de 1 a p en O(p)
102
            ll q = rm.n[b.1] * BASE + rm.n[b.1-1];
103
                                                                                            inv[1]=1;
            ll u = q / (b.n[b.l-1] + 1);
                                                                                            forr(i, 2, p) inv[i] = p - (p/i)*inv[p\%];
104
            ll v = q / b.n[b.l-1] + 1;
105
                                                                                        9
            while (u < v-1)
106
                                                                                       10
                11 m = (u+v)/2:
                                                                                          ll mul(ll a, ll b, ll m) { //hace (a*b) /m
107
                                                                                       11
                if (b*m \le rm) u = m;
                                                                                            11 q = (11)((long double)a*b/m);
108
                else v = m:
                                                                                            ll r = a*b-m*a:
109
                                                                                       13
                                                                                            while(r<0) r += m:
110
            c.n[i]=u;
                                                                                            while(r \ge m) r -= m:
111
            rm-=b*u;
112
                                                                                            return r;
                                                                                       16
113
                                                                                       17 }
     c.l=a.l;
114
                                                                                       18
```

```
struct mnum{
                                                                                                Key key;
                                                                                         4
     static const tipo mod=MOD;
                                                                                                int prior, size;
                                                                                         5
20
                                                                                                pnode l,r;
                                                                                         6
21
     mnum(tipo v=0): v((v mod+mod) mod) {}
                                                                                                node(Key key=0): key(key), prior(rand()), size(1), 1(0), r(0) {}
^{22}
     mnum operator+(mnum b){return v+b.v;}
                                                                                            };
                                                                                         8
23
     mnum operator-(mnum b){return v-b.v;}
                                                                                            static int size(pnode p) { return p ? p->size : 0; }
24
     mnum operator*(mnum b){return v*b.v;} //Si mod<=1e9+9
                                                                                            void push(pnode p) {
25
     //~ mnum operator*(mnum b){return mul(v,b.v,mod);} //Si mod<=1e18+9</pre>
                                                                                              // modificar y propagar el dirty a los hijos aca(para lazy)
26
     mnum operator(11 n) \{ //0 (log n) \}
                                                                                        12
27
       if(!n) return 1;
                                                                                            // Update function and size from children's Value
28
       mnum q = (*this)^(n/2);
                                                                                            void pull(pnode p) {//recalcular valor del nodo aca (para rmq)
29
                                                                                              p->size = 1 + size(p->1) + size(p->r);
       return n 1/2 ? q*q*v : q*q;
30
                                                                                            }
                                                                                        16
31
     mnum operator/(mnum n){return ~n*v;} //O(log n) //OJO! mod tiene que ser
                                                                                            //junta dos arreglos
32
          primo! Sino no siempre existe inverso
                                                                                            pnode merge(pnode 1, pnode r) {
                                                                                              if (!1 || !r) return 1 ? 1 : r;
33
     mnum operator~(){ //inverso, O(log mod)
                                                                                             push(1), push(r);
34
                                                                                        20
       assert(v!=0);
                                                                                             pnode t;
35
                                                                                        21
       //return (*this)^(eulerphi(mod)-1); //si mod no es primo (sacar a mano)
                                                                                              if (1-\text{>prior} < r-\text{>prior}) 1-\text{>r=merge}(1-\text{>r}, r), t = 1;
36
             PROBAR! Ver si rta*x == 1 modulo mod
                                                                                              else r\rightarrow l=merge(1, r\rightarrow 1), t = r;
       return (*this)^(mod-2);//si mod es primo
                                                                                              pull(t);
                                                                                        24
37
                                                                                              return t;
38
                                                                                        25
                                                                                        26
39
                                                                                            //parte el arreglo en dos, l<key<=r
40
                                                                                            void split(pnode t, Key key, pnode &1, pnode &r) {
   DIVISIÓN MODULAR
   Para dividir hay que multiplicar por el inverso multiplicativo. x/y = x*(y)
                                                                                                if (!t) return void(1 = r = 0);
                                                                                        29
        ^-1).
                                                                                                push(t);
                                                                                        30
   El inverso multiplicativo de y módulo n es y^-1 tal que y*(y^-1) = 1 \mod n.
                                                                                                if (\text{key} \leftarrow \text{t->key}) split(t->1, key, 1, t->1), r = t;
                                                                                        31
   Por ejemplo, si n=7, y=2, o sea que quiero dividir por y,
                                                                                                else split(t->r, key, t->r, r), l = t;
                                                                                        32
   y^-1 = 4 porque y*(y^-1) = 8 = 1 \mod 7.
                                                                                                pull(t);
                                                                                        33
                                                                                            }
                                                                                        34
46
47
                                                                                        35
   #define MOD 1000000009LL
                                                                                            void erase(pnode &t, Key key) {
   #define PRIME 1009LL
                                                                                                if (!t) return:
                                                                                                push(t);
                                                                                        38
50
                                                                                                if (key == t->key) t=merge(t->1, t->r);
   #define MOD 100000000003LL
                                                                                                else if (key < t->key) erase(t->1, key);
52 #define PRIME 1009LL
                                                                                                else erase(t->r, key);
                                                                                        41
                            2.12. Treap para set
                                                                                                if(t) pull(t);
                                                                                        42
                                                                                            }
                                                                                        43
  typedef int Key;
                                                                                        44
                                                                                            ostream& operator<<(ostream &out, const pnode &t) {
  typedef struct node *pnode;
                                                                                             if(!t) return out;
3 struct node{
```

```
return out << t->l << t->key << ''' << t->r;
                                                                                           p->mini.fst+=p->dirty;
47
                                                                                           if(p->l) p->l->dirty+=p->dirty;
48
                                                                                     13
                                                                                           if(p->r) p->r->dirty+=p->dirty;
   pnode find(pnode t, Key key) {
                                                                                     14
49
       if (!t) return 0;
                                                                                           p->dirty=0;
                                                                                      15
50
       if (key == t->key) return t;
                                                                                         }
                                                                                      16
       if (key < t->key) return find(t->1, key);
                                                                                         static Value mini(pnode p) { return p ? push(p), p->mini : ii(1e9, -1); }
52
                                                                                         // Update function and size from children's Value
       return find(t->r, key);
53
                                                                                         void pull(pnode p) {//recalcular valor del nodo aca (para rmq)
54
   struct treap {
                                                                                           p->size = 1 + size(p->1) + size(p->r);
55
                                                                                           p->mini = min(min(p->val, mini(p->l)), mini(p->r));//operacion del rmg!
       pnode root;
56
       treap(pnode root=0): root(root) {}
                                                                                           p->parent=0;
57
                                                                                     22
       int size() { return ::size(root); }
                                                                                           if(p->1) p->1->parent=p;
58
                                                                                     23
       void insert(Key key) {
                                                                                           if(p->r) p->r->parent=p;
                                                                                     24
59
           pnode t1, t2; split(root, key, t1, t2);
                                                                                         }
                                                                                      25
60
           t1=::merge(t1,new node(key));
                                                                                         //junta dos arreglos
61
           root=::merge(t1,t2);
                                                                                         pnode merge(pnode 1, pnode r) {
62
       }
                                                                                           if (!1 || !r) return 1 ? 1 : r;
63
       void erase(Key key1, Key key2) {
                                                                                           push(1), push(r);
64
           pnode t1,t2,t3;
                                                                                           pnode t:
                                                                                     30
65
                                                                                           if (1->prior < r->prior) 1->r=merge(1->r, r), t = 1;
           split(root,key1,t1,t2);
66
                                                                                           else r\rightarrow l=merge(l, r\rightarrow l), t = r;
           split(t2,key2, t2, t3);
67
           root=merge(t1,t3);
                                                                                           pull(t);
68
                                                                                           return t;
                                                                                      34
69
       void erase(Key key) {::erase(root, key);}
                                                                                      35
70
       pnode find(Key key) { return ::find(root, key); }
                                                                                         //parte el arreglo en dos, sz(1)==tam
71
       Key &operator[](int pos){return find(pos)->key;}//ojito
                                                                                         void split(pnode t, int tam, pnode &1, pnode &r) {
72
                                                                                           if (!t) return void(l = r = 0);
73
                                                                                      38
  treap merge(treap a, treap b) {return treap(merge(a.root, b.root));}
                                                                                           push(t);
                                                                                      39
                                                                                           if (tam \le size(t->1)) split(t->1, tam, 1, t->1), r = t;
                                                                                     40
                        2.13. Treap para arreglo
                                                                                           else split(t->r, tam - 1 - size(t->l), t->r, r), l = t;
                                                                                           pull(t);
                                                                                      42
   typedef struct node *pnode;
                                                                                         pnode at(pnode t, int pos) {
   struct node{
                                                                                           if(!t) exit(1);
       Value val, mini;
3
                                                                                           push(t);
                                                                                      46
       int dirty;
4
                                                                                           if(pos == size(t->1)) return t;
       int prior, size;
5
                                                                                           if(pos < size(t->1)) return at(t->1, pos);
       pnode l,r,parent;
                                                                                      48
6
                                                                                           return at(t->r, pos - 1 - size(t->l));
       node(Value val): val(val), mini(val), dirty(0), prior(rand()), size(1),
                                                                                      49
            1(0), r(0), parent(0) {}
                                                                                      50
                                                                                         int getpos(pnode t){//inversa de at
8
                                                                                           if(!t->parent) return size(t->1);
   static int size(pnode p) { return p ? p->size : 0; }
                                                                                     52
                                                                                           if(t==t->parent->1) return getpos(t->parent)-size(t->r)-1;
   void push(pnode p) {//propagar dirty a los hijos(aca para lazy)
                                                                                      53
                                                                                           return getpos(t->parent)+size(t->l)+1;
     p->val.fst+=p->dirty;
                                                                                     54
```

```
55 }
   void split(pnode t, int i, int j, pnode &1, pnode &m, pnode &r) {
     split(t, i, l, t), split(t, j-i, m, r);}
   Value get(pnode &p, int i, int j){//like rmq
     pnode l,m,r;
59
       split(p, i, j, l, m, r);
60
       Value ret=mini(m);
61
       p=merge(1, merge(m, r));
62
       return ret;
63
64
   void print(const pnode &t) {//for debugging
65
     if(!t) return;
       push(t):
67
       print(t->1);
68
       cout << t->val.fst << '';</pre>
       print(t->r);
70
71 }
```

2.14. Convex Hull Trick

```
struct Line{tipo m,h;};
   tipo inter(Line a, Line b){
       tipo x=b.h-a.h, y=a.m-b.m;
3
       return x/y+(x\%?!((x>0)^(y>0)):0);//==ceil(x/y)
4
5
   struct CHT {
6
     vector<Line> c;
7
     bool mx;
8
     int pos;
9
     CHT(bool mx=0):mx(mx),pos(0){}//mx=1 si las query devuelven el max
10
     inline Line acc(int i){return c[c[0].m>c.back().m? i : sz(c)-1-i];}
11
     inline bool irre(Line x, Line y, Line z){
12
       return c[0].m>z.m? inter(y, z) <= inter(x, y)</pre>
13
                             : inter(y, z) >= inter(x, y);
14
15
     void add(tipo m, tipo h) {//O(1), los m tienen que entrar ordenados
16
           if(mx) m*=-1, h*=-1;
17
       Line l=(Line)\{m, h\}:
18
           if(sz(c) && m==c.back().m) { l.h=min(h, c.back().h), c.pop back();
19
                if(pos) pos--; }
           while(sz(c) \ge 2 \&\& irre(c[sz(c)-2], c[sz(c)-1], 1)) { c.pop back();
20
                if(pos) pos--; }
           c.pb(1);
21
```

```
22
     inline bool fbin(tipo x, int m) {return inter(acc(m), acc(m+1))>x;}
23
     tipo eval(tipo x){
24
       int n = sz(c);
25
       //query con x no ordenados O(lgn)
26
       int a=-1, b=n-1;
27
       while(b-a>1) { int m = (a+b)/2;
         if(fbin(x, m)) b=m;
         else a=m;
30
31
       return (acc(b).m*x+acc(b).h)*(mx?-1:1);
32
           //query 0(1)
33
       while(pos>0 && fbin(x, pos-1)) pos--;
34
       while(pos<n-1 && !fbin(x, pos)) pos++;</pre>
       return (acc(pos).m*x+acc(pos).h)*(mx?-1:1);
36
    }
37
38 } ch;
```

2.15. Convex Hull Trick (Dynamic)

```
const ll is query = -(1LL<<62);</pre>
2 struct Line {
        ll m, b;
        mutable multiset<Line>::iterator it:
        const Line *succ(multiset<Line>::iterator it) const:
5
        bool operator<(const Line& rhs) const {
6
            if (rhs.b != is query) return m < rhs.m;</pre>
7
            const Line *s=succ(it);
8
            if(!s) return 0;
            11 x = rhs.m;
10
            return b - s -> b < (s -> m - m) * x;
11
        }
12
   };
13
   struct HullDynamic : public multiset<Line>{ // will maintain upper hull for
         maximum
        bool bad(iterator y) {
15
            iterator z = next(y);
16
            if (v == begin()) {
17
                 if (z == end()) return 0:
18
                 return y->m == z->m && y->b <= z->b;
19
20
            iterator x = prev(y);
21
            if (z == end()) return y \rightarrow m == x \rightarrow m \&\& y \rightarrow b <= x \rightarrow b;
22
```

```
return (x-b - y-b)*(z-m - y-m) >= (y-b - z-b)*(y-m - x-m);
23
       }
^{24}
       iterator next(iterator y){return ++y;}
25
       iterator prev(iterator y){return --y;}
26
       void insert_line(ll m, ll b) {
27
           iterator y = insert((Line) { m, b });
28
           y->it=y;
29
           if (bad(y)) { erase(y); return; }
30
           while (next(y) != end() && bad(next(y))) erase(next(y));
31
           while (y != begin() && bad(prev(y))) erase(prev(y));
32
       }
33
       ll eval(ll x) {
34
           Line 1 = *lower_bound((Line) { x, is_query });
35
           return 1.m * x + 1.b;
36
       }
37
   }h;
38
   const Line *Line::succ(multiset<Line>::iterator it) const{
39
       return (++it==h.end()? NULL : &*it);}
40
                           2.16. Gain-Cost Set
```

```
//esta estructura mantiene pairs(beneficio, costo)
   //de tal manera que en el set quedan ordenados
   //por beneficio Y COSTO creciente. (va borrando los que no son optimos)
   struct V{
     int gain, cost;
     bool operator<(const V &b)const{return gain<b.gain;}</pre>
7
   set<V> s;
   void add(V x){
     set<V>::iterator p=s.lower_bound(x);//primer elemento mayor o igual
10
     if(p!=s.end() && p->cost <= x.cost) return;//ya hay uno mejor
11
     p=s.upper_bound(x);//primer elemento mayor
12
     if(p!=s.begin()){//borro todos los peores (<=beneficio y >=costo)
13
       --p;//ahora es ultimo elemento menor o igual
14
       while(p->cost >= x.cost){
15
         if(p==s.begin()){s.erase(p); break;}
16
         s.erase(p--);
17
18
19
     s.insert(x);
20
21
  int get(int gain){//minimo costo de obtener tal ganancia
```

```
set<V>::iterator p=s.lower_bound((V){gain, 0});
23
     return p==s.end()? INF : p->cost;}
24
                 2.17. Set con búsq. binaria (Treap)
 1 #include bits stdc++.h>
  #include<ext/pb_ds/assoc_container.hpp>
   #include<ext/pb ds/tree policy.hpp>
   using namespace gnu pbds;
   using namespace std;
   template <typename T>
   using ordered_set = tree<T, null_type, less<T>, rb_tree_tag,
       tree_order_statistics_node_update>;
   //o bien usar así:
   typedef tree<int,null_type,less<int>,//key, mapped type, comparator. Se
       puede usar como map<a,b> poniendo tree<a,b,...
       rb tree tag, tree order statistics node update> set t;
   int main(){
     ordered set<int> s;
     s.insert(1);
16
     s.insert(3):
17
     cout << s.order_of_key(3) << endl; // s.order_of_key(x): number of</pre>
18
         elements in s strictly less than x.
     cout << *s.find by order(0) << endl; // s.find by order(i): i-th smallest</pre>
19
          number in s. (empieza en 0)
     cout << *s.lower_bound(1) << endl;</pre>
20
21
   //order_of_key(k): devuelve la pos del lower bound de k
   //find_by_order(i) devuelve iterador al i-esimo elemento
   //Ej: 12, 100, 505, 1000, 10000.
   //order_of_key(10) == 0, order_of_key(100) == 1,
   //order of key(707) == 3, order of key(9999999) == 5
27
28
   Si son int se puede hacer con un rmq y busqueda binaria.
30
   rmq[i] = 1 si i esta
   rmq[i] = 0 si i no esta
33
```

rmq.get(i,j) = suma en el intervalo [i,j)

```
order_of_key(i) == rmq.get(0,i)
find_by_order(o) == busqueda binaria en i / rmq.get(0,i+1) == o
lower_bound(i) == find_by_order(order_of_key(i)-1)

*/
```

2.18. Árbol de costo n-ésimo

```
#include <ext/pb ds/assoc container.hpp>
   #include <ext/pb_ds/tree_policy.hpp>
   using namespace std;
   using namespace __gnu_pbds;
    template<class Node_CItr, class Node_Itr, class Cmp_Fn, class _Alloc>
   struct order_cost_update
8
9
     typedef struct { ll order, cost; } metadata_type;
10
11
     typedef typename Node CItr::value type const iter;
12
     typedef typename Node CItr::value type iter;
13
14
     virtual Node CItr node begin() const = 0;
15
     virtual Node CItr node end() const = 0;
16
17
     inline void operator()(Node_Itr it, Node_CItr end_it) const {
18
       auto &im = it.get metadata();
19
       auto &order = const_cast<ll&>(im.order);
20
       auto &cost = const_cast<ll&>(im.cost);
21
22
       order = (*it)->second;
23
       cost = (*it)->first * order;
24
25
       auto 1 = it.get_l_child();
26
       if(1 != end it) {
27
         auto &lm = l.get_metadata();
28
         order += lm.order;
29
         cost += lm.cost:
30
31
32
       auto r = it.get r child();
33
       if(r != end it) {
34
         auto &rm = r.get metadata();
35
```

```
order += rm.order;
36
37
         cost += rm.cost;
       }
38
     }
39
40
     // permite calcular costo de n comprar los n primeros items
41
     inline pair<const_iter,metadata_type> get_kth(ll x) {
42
       metadata_type d = {};
43
       auto it = node_begin();
44
       const iter last = *node end();
46
       while(it != node end())
47
48
         metadata type lm = {};
49
         auto 1 = it.get 1 child();
50
         if (1 != node end()) {
            auto &lm2 = 1.get metadata();
52
           lm.order = lm2.order;
           lm.cost = lm2.cost:
54
55
56
         if (!Cmp_Fn()(lm.order, x)) {
57
            it = 1; // contenido a la izq
58
         } else if (!Cmp_Fn()(lm.order + (*it)->second, x)) {
59
            d.order += x; // contenido en este
60
            d.cost += lm.cost + (x-lm.order) * (*it)->first;
61
            return make_pair(*it, d);
62
         } else { // contiene este y más
63
            d.order += lm.order + (*it)->second:
64
            d.cost += lm.cost + (*it)->first * (*it)->second;
65
66
            x = lm.order + (*it) -> second;
67
            last = *it:
68
           it = it.get_r_child();
69
70
       }
71
72
       return make pair(last,d);
73
74
   };
75
   // OJO! no actualizar elementos ni usar map[x]=y, siempre
78 // usar find() + erase() + insert()
```

5 | }qs[MAXN];

```
79 // map.insert({cost,qty})
80 typedef tree<11, 11, less<11>, rb_tree_tag, order_cost_update> rb_map;
                                    2.19. BIT
  struct bitrie
      static const int sz=1<<5;//5=ceil(log(n))
 2
      int V;//valor del nodo
      vector<br/>bitrie> ch;//childs
 4
     bitrie():V(0){}//NEUTRO
 5
      void set(int p, int v, int bit=sz>>1){\frac{1}{0}} \log sz
 6
        if(bit){
 7
          ch.resize(2):
 8
          ch[(p&bit)>0].set(p, v, bit>>1);
 9
          V=max(ch[0].V, ch[1].V);
10
11
        else V=v;
12
13
      int get(int i, int j, int a=0, int b=sz){\frac{1}{0}} (\log sz)
14
        if(j<=a || i>=b) return 0;//NEUTRO
15
        if(i<=a && b<=j) return V;</pre>
16
        if(!sz(ch)) return V;
17
        int c=(a+b)/2;
18
        return max(ch[0].get(i, j, a, c), ch[1].get(i, j, c, b));
19
20
<sub>21</sub> |};
```

3. Algos

3.1. Longest Increasing Subsecuence

```
//Para non-increasing, cambiar comparaciones y revisar busq binaria
//Given an array, paint it in the least number of colors so that each color
turns to a non-increasing subsequence.
//Solution:Min number of colors=Length of the longest increasing
subsequence
int N, a[MAXN];//secuencia y su longitud
ii d[MAXN+1];//d[i]=ultimo valor de la subsecuencia de tamanio i
int p[MAXN];//padres
vector<int> R;//respuesta
void rec(int i){
if(i=-1) return;
R.push_back(a[i]);
```

```
rec(p[i]);
11
   }
12
   int lis(){//O(nlogn)
13
     d[0] = ii(-INF, -1); forn(i, N) d[i+1]=ii(INF, -1);
     forn(i, N){
15
       int j = upper_bound(d, d+N+1, ii(a[i], INF))-d;
16
       if (d[j-1].first < a[i]&&a[i] < d[j].first){
17
         p[i]=d[j-1].second;
         d[j] = ii(a[i], i);
19
20
     }
21
     R.clear();
22
     dforn(i, N+1) if(d[i].first!=INF){
23
       rec(d[i].second);//reconstruir
24
       reverse(R.begin(), R.end());
25
       return i;//longitud
26
    }
27
     return 0;
29 }
                       3.2. Alpha-Beta prunning
1 | 11 alphabeta(State &s, bool player = true, int depth = 1e9, 11 alpha = -INF
        , ll beta = INF) { //player = true -> Maximiza
       if(s.isFinal()) return s.score;
     //~ if (!depth) return s.heuristic();
3
       vector<State> children;
4
       s.expand(player, children);
5
       int n = children.size();
6
       forn(i, n) {
           11 v = alphabeta(children[i], !player, depth-1, alpha, beta);
8
           if(!player) alpha = max(alpha, v);
9
           else beta = min(beta, v);
10
           if(beta <= alpha) break;</pre>
11
12
       return !player ? alpha : beta;}
13
                           3.3. Mo's algorithm
int n,sq;
struct Qu{//queries [1, r]
       //intervalos cerrado abiertos !!! importante!!
       int 1, r, id;
```

```
6 int ans[MAXN], curans;//ans[i]=ans to ith query
   bool bymos(const Qu &a, const Qu &b){
       if(a.l/sq!=b.l/sq) return a.l<b.1;</pre>
       return (a.1/sq)&1? a.r<b.r : a.r>b.r;
9
10
   void mos(){
       forn(i, t) qs[i].id=i;
12
       sort(qs, qs+t, bymos);
13
       int cl=0, cr=0;
14
       sq=sqrt(n);
15
       curans=0;
16
       forn(i, t){ //intervalos cerrado abiertos !!! importante!!
17
            Qu &q=qs[i];
18
            while(cl>q.1) add(--cl);
19
            while(cr<q.r) add(cr++);</pre>
20
            while(cl<q.1) remove(cl++);</pre>
21
            while(cr>q.r) remove(--cr);
22
            ans[q.id]=curans;
23
       }
24
25
```

3.4. huffman

```
#include <bits/stdc++.h>
   using namespace std;
   typedef long long 11;
   /* idea from following webpage
    * https://www.siggraph.org/education/materials/HyperGraph/video/mpeg/
        mpegfaq/huffman_tutorial.html
    */
9
10
   struct huff {
11
    ll v; /* value */
12
     huff *r, *l; /* right, left branches */
13
   };
14
15
   typedef pair<11, huff*> pih;
16
  huff *build huff(vector<11> &e)
  |{
19
```

```
priority_queue<pih, vector<pih>, greater<pih>> pq;
20
     for (auto &x: e)
21
       pq.push(make_pair(x, nullptr));
22
23
     while(pq.size() != 1) {
24
       /* Get 2 nodes with lower value */
25
       pih x = pq.top();
26
       pq.pop();
       pih y = pq.top();
       pq.pop();
30
       /* Combine them in a new node */
31
       huff *w = new huff:
32
       w->r = x.second;
       w->l= y.second;
       w->v = x.first+y.first;
36
       /* Push new one to the pg */
       pq.push(make_pair(w->v, w));
38
     }
39
40
     /* Only one node left => tree complete */
41
     return pq.top().second;
^{42}
43
44
   11 sum_nuke_huff(huff *x)
46
     /* Recursively sum all the values of the tree nodes while
47
      * destroying the tree */
48
     if (!x)
49
       return 0;
50
51
     ll tot = x->v + sum nuke huff(x->r) + sum nuke huff(x->l);
52
     delete x->r:
53
     delete x->1:
54
     return tot;
55
56
57
   int main()
   {
59
     11 n;
60
     cin >> n;
61
62
```

```
for (ll i = 0; i < n; i++) {
63
       11 t;
64
       cin >> t;
65
       vector<ll> a(t);
66
       for (11 j = 0; j < t; j++)
67
          cin >> a[i];
68
69
        huff *o = build_huff(a);
70
        cout << sum_nuke_huff(o) << endl;</pre>
71
        delete o;
72
     }
73
74 | }
```

3.5. Optimizaciones para DP

```
convex hull 1: dp[i] = min\{dp[j] + b[j] * a[i]\}, j < i. Si se cumple b[j]
       \Rightarrow b[i+1] y a[i] <= a[i+1] entonces pasa de O(n^2) a O(n) sino pasa a O
        (nlogn)
2
   convex hull 2: dp[i][j] = min\{dp[i-1][k] + b[k] * a[j]\}, k < j. Si se
        cumple b[k] >= b[k+1] y a[i] <= a[i+1] entonces pasa de <math>O(kn^2) a O(kn)
         sino pasa O(knlogn)
5
   divide and conquer: dp[i][j] = min\{dp[i-1][k] + C[k+1][j]\}, k < j. Se debe
        cumplir: A[i][j] <= A[i][j+1]. Pasa de O(kn^2) a O(knlogn)
   Donde A[i][j] es el minimo k tal que dp[i][j] = dp[i-1][k] + C[k][j]
    Tambien es aplicable si:
   C[a][c] + C[b][d] \leftarrow C[a][d] + C[b][c] \lor C[b][c] \leftarrow C[a][d], a \leftarrow b \leftarrow c \leftarrow d
11
   def ComputeDP(i, jleft, jright, kleft, kright):
12
     # Select the middle point
13
     jmid = (jleft + jright) / 2
14
     # Compute the value of dp[i][jmid] by definition of DP
15
     dp[i][imid] = +INFINITY
16
     bestk = -1
17
     for k in range[kleft, jmid):
18
     if dp[i-1][k] + C[k+1][jmid] < best:
19
       dp[i][jmid] = dp[i - 1][k] + C[k + 1][jmid]
20
       bestk = k
21
     # Divide and conquer
22
     if jleft < jmid:</pre>
```

```
ComputeDP(i, jleft, jmid, kleft, bestk)
     if jmid + 1 < jright:</pre>
25
     ComputeDP(i, jmid + 1, jright, bestk, kright)
26
27
   def ComputeFullDP:
28
     Initialize dp for i = 0 somehow
     for i in range(1, m):
30
     ComputeDP(i, 0, n, 0, n)
31
32
   knuth: dp[i][j] = min\{dp[i][k] + dp[k][j]\} + C[i][j], i < k < j. Se debe
        cumplir: A[i, j-1] \le A[i, j] \le A[i+1, j]. Pasa de O(n^3) a O(n^2)
35 | Donde A[i][j] es el minimo k tal que dp[i][j] = dp[i][k]+dp[k][j] + C[i][j
   Tambien es aplicable si:
   C[a][c] + C[b][d] \leftarrow C[a][d] + C[b][c] y C[b][c] \leftarrow C[a][d], a \leftarrow b \leftarrow c \leftarrow d
   for (int s = 0; s < = k; s + +)
     for (int 1 = 0; 1+s<=k; 1++) {
                                                   //l - left point
40
       int r = 1 + s;
                                                   //r - right point
       if (s < 2) {
42
       res[1][r] = 0;
                                                 //DP base - nothing to break
       A[1][r] = 1;
                                               //A is equal to left border
44
       continue;
45
       }
46
                                                 //Knuth's trick: getting bounds
       int aleft = A[l][r-1];
47
             on m
       int aright = A[l+1][r];
48
       res[1][r] = INF;
49
       for (int a = max(l+1,aleft); a<=min(r-1,aright); a++) {
                                                                   //iterating
50
             for a in the bounds only
       int act = res[l][a] + res[a][r] + (C[l][r]);
51
       if (res[l][r] > act) {
                                  //relax current solution
52
         res[1][r] = act;
53
         A[1][r] = a;
54
       }
55
       }
     }
```

4. Strings

4.1. Manacher

```
int d1[MAXN];//d1[i]=long del maximo palindromo impar con centro en i
int d2[MAXN];//d2[i]=analogo pero para longitud par
  //0 1 2 3 4
   //a a b c c <--d1[2]=3
   //a a b b <--d2[2]=2 (estan uno antes)
   void manacher(){
     int l=0, r=-1, n=sz(s);
     forn(i, n){
8
       int k=(i>r? 1 : min(d1[l+r-i], r-i));
9
       while(i+k< n \&\& i-k>=0 \&\& s[i+k]==s[i-k]) ++k;
10
       d1[i] = k--;
11
       if(i+k > r) l=i-k, r=i+k;
12
     }
13
     1=0, r=-1;
14
     forn(i, n){
15
       int k=(i>r? 0 : min(d2[l+r-i+1], r-i+1))+1;
16
       while(i+k-1<n && i-k>=0 && s[i+k-1]==s[i-k]) k++;
17
       d2[i] = --k;
18
       if(i+k-1 > r) l=i-k, r=i+k-1:
19
20
21 }
```

4.2. KMP

```
string T;//cadena donde buscar(where)
  string P;//cadena a buscar(what)
   int b[MAXLEN];//back table b[i] maximo borde de [0..i)
   void kmppre(){//by gabina with love
       int i =0, j=-1; b[0]=-1;
5
       while(i<sz(P)){</pre>
6
            while(j>=0 && P[i] != P[j]) j=b[j];
7
           i++, j++, b[i] = j;
8
       }
9
10
   void kmp(){
11
       int i=0, j=0;
12
       while(i<sz(T)){</pre>
13
            while(j>=0 && T[i]!=P[j]) j=b[j];
14
            i++, j++;
15
            if(j==sz(P)) printf("P<sub>i</sub> is found at index % in T\n", i-j), j=b[j];
16
       }
17
18 }
```

4.3. Booth

```
1 // Booth's lexicographically minimal string rotation algorithm
   template<class U, class T>
   int boothLCS(T &v) // O(n)
   {
4
     size_t len = 2 * v.size();
6
     // Duplicate original data to avoid modular arithmetic
7
     vector<U> S(len);
8
     for (size t i = 0, sz = v.size(); i < sz; i++)
       S[i] = S[v.size()+i] = v[i];
11
     // Failure function
     vector<int> f(len, -1);
14
     // Minimal rotation found so far
15
     int k = 0:
16
17
     for (size t j = 1; j < S.size(); j++) {</pre>
18
       int i = f[j-k-1];
19
       while (i != -1 \&\& S[i] != S[k+i+1]) {
20
         if (S[j] < S[k+i+1])
21
           k = j-i-1;
22
         i = f[i];
23
       }
24
       if (i == -1 \&\& S[i] != S[k+i+1]) {
25
         if (S[j] < S[k+i+1])
26
           k = j;
27
         f[j-k] = -1;
28
       } else {
         f[j-k] = i+1;
30
       }
31
     }
32
33
34
     return k;
35 }
                                   4.4. Trie
```

```
struct trie{
map<char, trie> m;
bool end=false;
```

11 needleman wunsnch(const char *A, const char *B) {

11 n = strlen(A), m = strlen(B);

forn(i, n+1) nwt[i][0] = i * INS;

forn(j, m+1) nwt[0][j] = j * DEL;

10

11

12

13

14

15

16

zero(f);

int sum=0:

forn(i,n)

10

11

13

14

15

forn(i, n) f[rBOUND(i+k)]++;

int t=f[i]; f[i]=sum; sum+=t;}

tmpsa[f[rBOUND(sa[i]+k)]++]=sa[i];

forn(i, max(255, n)){

```
void add(const string &s, int p=0){
                                                                                           forr(i, 1, n+1) forr(j, 1, m+1) {
4
                                                                                     17
                                                                                            nwt[i][j] = nwt[i-1][j-1] + (A[i-1] == B[j-1] ? MATCH : MISMATCH);
       if(s[p]) m[s[p]].add(s, p+1);
5
                                                                                     18
                                                                                            nwt[i][j] = max(nwt[i][j], nwt[i - 1][j] + DEL);
       else end=true;
6
                                                                                     19
                                                                                            nwt[i][j] = max(nwt[i][j], nwt[i][j-1] + INS);
    }
                                                                                     20
                                                                                          }
     void dfs(){
8
                                                                                     21
       //Do stuff
9
                                                                                     22
       forall(it, m)
                                                                                          return nwt[n][m];
10
                                                                                     23
         it->second.dfs();
11
                                                                                     24
    }
12
                                                                                     25
                                                                                         string lcs construct(const char *A, const char *B) {
13 };
                                                                                          11 len = needleman wunsnch(A, B), i = strlen(A), j = strlen(B);
                                                                                     27
                                 4.5. Regex
                                                                                           string s;
                                                                                     28
                                                                                           s.resize(len);
                                                                                     29
   string s = "hola mundo feliz.";
                                                                                     30
   regex r("^hola(\\smundo<sub>||</sub>(\\w+\\.))$");
                                                                                           while (i > 0 \&\& j > 0) {
                                                                                     31
  if (regex match(s, r))
                                                                                             if (nwt[i-1][j] == nwt[i][j]) --i;
                                                                                     32
     cout << "match" << endl:</pre>
                                                                                             else if (nwt[i][j-1] == nwt[i][j]) --j;
                                                                                     33
   smatch sm:
5
                                                                                             else {
   if (regex match(s, sm, r))
6
                                                                                               s[--len] = A[i-1];
    for (auto &m: sm)
7
                                                                                               --i, --j;
       cout << "[" << m << "]" << endl:
8
                                                                                            }
                                                                                     37
9
                                                                                          }
      [hola mundo feliz.]
10
                                                                                     39
      [ mundo feliz.]
11
                                                                                          return s;
                                                                                     40
      [feliz.] */
^{12}
                                                                                     41 }
                       4.6. Needleman Wunsnch
                                                                                                          4.7. Suffix Array (largo, nlogn)
   /* Longest common subsequence: DEL=INS=0, MATCH=1, MISMATCH=-INF
    * Hamming: DEL=INS=-INF, MATCH=0, MISMATCH=1
                                                                                      1 #define MAX_N 112345
    * String alignment: normalmente DEL=INS=-1, MATCH=+2, MISMATCH=-1 */
                                                                                        #define rBOUND(x) ((x) < n ? r[(x)] : 0)
                                                                                        //sa will hold the suffixes in order.
   #define DEL (0)
                                                                                        int sa[MAX_N], r[MAX_N], n;//OJO n = s.size()!
   #define INS (0)
   #define MATCH (1)
                                                                                        string s; //input string, n=s.size()
   #define MISMATCH (-10000000)
                                                                                        int f[MAX_N], tmpsa[MAX_N];
   #define MAXLEN 10000
   11 nwt[MAXLEN] [MAXLEN];
                                                                                         void countingSort(int k){
9
```

```
forn(i,n) sa[i] = tmpsa[i];
16
   }
17
   void constructsa(){\frac{}{0}}n log n)
18
     n = s.size();
19
     forn(i,n) sa[i]=i, r[i]=s[i];
20
     for(int k=1; k<n; k<<=1){</pre>
21
       countingSort(k), countingSort(0);
22
       int rank, tmpr[MAX_N];
23
       tmpr[sa[0]]=rank=0;
24
       forr(i, 1, n)
25
          tmpr[sa[i]] = (r[sa[i]] = r[sa[i-1]] \&\& r[sa[i]+k] = r[sa[i-1]+k]) ?
26
              rank: ++rank:
       forn(i,n) r[i]=tmpr[i];
27
       if(r[sa[n-1]]==n-1) break:
28
29
30
   void print(){//for debugging
31
     forn(i, n)
32
       cout << i << '..' <<
33
       s.substr(sa[i], s.find('$',sa[i])-sa[i]) << endl;}
34
```

4.8. String Matching With Suffix Array

```
1 //returns [lowerbound, upperbound] of the search -- los extremos estan
       incluidos!
pll stringMatching(string P){ //O(sz(P)lgn)
     int lo=0, hi=n-1, mid=lo;
3
     while(lo<hi){</pre>
4
       mid=(lo+hi)/2;
5
       int res=s.compare(sa[mid], sz(P), P);
6
       if(res>=0) hi=mid;
7
       else lo=mid+1;
8
9
     if(s.compare(sa[lo], sz(P), P)!=0) return {-1, -1};
10
     pll ans; ans.fst=lo;
11
     lo=0, hi=n-1, mid;
12
     while(lo<hi){</pre>
13
       mid=(lo+hi)/2:
14
       int res=s.compare(sa[mid], sz(P), P);
15
       if(res>0) hi=mid:
16
       else lo=mid+1:
17
18
     if(s.compare(sa[hi], sz(P), P)!=0) hi--;
19
```

```
ans.snd=hi;
20
     return ans;
21
22 }
                 4.9. LCP (Longest Common Prefix)
 1 //Calculates the LCP between consecutives suffixes in the Suffix Array.
   //LCP[i] is the length of the LCP between sa[i] and sa[i-1]
   int LCP[MAX_N], phi[MAX_N], PLCP[MAX_N];
   void computeLCP(){//0(n)}
     phi[sa[0]]=-1;
     forr(i, 1, n) phi[sa[i]]=sa[i-1];
     int L=0;
7
     forn(i, n){
8
       if(phi[i]==-1) {PLCP[i]=0; continue;}
9
       while(s[i+L]==s[phi[i]+L]) L++;
10
       PLCP[i]=L:
11
       L=max(L-1, 0):
12
    }
13
    forn(i, n) LCP[i]=PLCP[sa[i]];
14
15 }
                              4.10. Corasick
1 struct trie{
     map<char, trie> next;
     trie* tran[256];//transiciones del automata
     int idhoja, szhoja;//id de la hoja o 0 si no lo es
     //link lleva al sufijo mas largo, nxthoja lleva al mas largo pero que es
5
     trie *padre, *link, *nxthoja;
6
     char pch;//caracter que conecta con padre
     trie(): next(), tran(), idhoja(), szhoja(), padre(), link(),nxthoja(),
         pch() {}
     void insert(const string &s, int id=1, int p=0){//id>0!!!
9
       if(p \le z(s)){
10
         trie &ch=next[s[p]];
11
         tran[(int)s[p]]=&ch;
12
         ch.padre=this, ch.pch=s[p];
13
         ch.insert(s, id, p+1);
14
       }
15
       else idhoja=id, szhoja=sz(s);
16
17
```

trie* get link() {

```
if(!link){
19
          if(!padre) link=this;//es la raiz
20
          else if(!padre->padre) link=padre;//hijo de la raiz
21
         else link=padre->get_link()->get_tran(pch);
22
       }
23
       return link; }
24
      trie* get_tran(int c) {
25
       if(!tran[c]) tran[c] = !padre? this : this->get_link()->get_tran(c);
26
       return tran[c]; }
27
      trie *get nxthoja(){
28
       if(!nxthoja) nxthoja = get link()->idhoja? link : link->nxthoja;
29
       return nxthoja; }
30
     void print(int p){
       if(idhoja) cout << "found<sub>i</sub>" << idhoja << "<sub>i,i</sub> at position<sub>i</sub>" << p-szhoja
32
            << endl:
       if(get nxthoja()) get nxthoja()->print(p); }
33
     void matching(const string &s, int p=0){
       print(p); if(p<sz(s)) get tran(s[p])->matching(s, p+1); }
36 }tri;
```

4.11. Suffix Automaton

```
struct state {
     int len. link:
     map<char,int> next;
     state() { }
5
   const int MAXLEN = 10010;
   state st[MAXLEN*2];
   int sz, last;
   void sa_init() {
     forn(i,sz) st[i].next.clear();
10
     sz = last = 0;
11
     st[0].len = 0;
12
     st[0].link = -1;
13
     ++sz;
14
15
   // Es un DAG de una sola fuente y una sola hoja
  // cantidad de endpos = cantidad de apariciones = cantidad de caminos de la
        clase al nodo terminal
  // cantidad de miembros de la clase = st[v].len-st[st[v].link].len (v>0) =
       caminos del inicio a la clase
19 // El arbol de los suffix links es el suffix tree de la cadena invertida.
```

```
La string de la arista link(v)->v son los caracteres que difieren
   void sa_extend (char c) {
     int cur = sz++;
     st[cur].len = st[last].len + 1;
     // en cur agregamos la posicion que estamos extendiendo
23
     //podria agregar tambien un identificador de las cadenas a las cuales
24
         pertenece (si hay varias)
     int p;
25
     for (p=last; p!=-1 && !st[p].next.count(c); p=st[p].link) // modificar
26
         esta linea para hacer separadores unicos entre varias cadenas (c=='$
       st[p].next[c] = cur;
27
     if (p == -1)
28
       st[cur].link = 0;
29
     else {
30
       int q = st[p].next[c];
31
       if (st[p].len + 1 == st[q].len)
32
         st[cur].link = q;
33
       else {
34
         int clone = sz++;
         // no le ponemos la posicion actual a clone sino indirectamente por
36
              el link de cur
         st[clone].len = st[p].len + 1;
37
         st[clone].next = st[q].next;
         st[clone].link = st[q].link;
39
         for (; p!=-1 && st[p].next.count(c) && st[p].next[c]==q; p=st[p].link
           st[p].next[c] = clone;
41
         st[q].link = st[cur].link = clone;
42
       }
43
     }
44
     last = cur;
46 }
```

4.12. Z Function

```
char s[MAXN];
int z[MAXN]; // z[i] = i==0 ? 0 : max k tq s[0,k) match with s[i,i+k)
void z_function(char s[],int z[]) {
   int n = strlen(s);
   forn(i, n) z[i]=0;
   for (int i = 1, l = 0, r = 0; i < n; ++i) {
      if (i <= r) z[i] = min (r - i + 1, z[i - l]);
}</pre>
```

```
while (i + z[i] < n \&\& s[z[i]] == s[i + z[i]]) ++z[i];
           if (i + z[i] - 1 > r) l = i, r = i + z[i] - 1;
9
       }
10
  |}
11
```

Geometria

5.1. Punto

```
const double EPS=1e-9;
   struct pto{
     double x, v;
     pto(double x=0, double y=0):x(x),y(y){}
     pto operator+(pto a){return pto(x+a.x, y+a.y);}
     pto operator-(pto a){return pto(x-a.x, y-a.y);}
     pto operator+(double a){return pto(x+a, y+a);}
     pto operator*(double a){return pto(x*a, y*a);}
     pto operator/(double a){return pto(x/a, y/a);}
     //dot product, producto interno:
     //Significado: a*b = a.norm * b.norm * cos(ang).
     double operator*(pto a){return x*a.x+y*a.y;}
     //module of the cross product or vectorial product:
     //if a is less than 180 clockwise from b, a^b>0. Significado: abs(a^b) =
         area del paralelogramo.
     double operator^(pto a){return x*a.y-y*a.x;}
15
     //returns true if this is at the left side of line gr
16
     bool left(pto q, pto r){return ((q-*this)^(r-*this))>0;}
17
     bool operator<(const pto &a) const{return x<a.x-EPS || (abs(x-a.x)<EPS &&
18
          v<a.v-EPS);}
     bool operator==(pto a){return abs(x-a.x)<EPS && abs(y-a.y)<EPS;}
19
     double norm(){return sqrt(x*x+y*y);}
20
     double norm_sq(){return x*x+y*y;}
21
^{22}
   double dist(pto a, pto b){return (b-a).norm();}
   double dist_sq(pto a, pto b){return (b-a).norm_sq();}
   typedef pto vec;
^{25}
    //positivo si aob están en sentido antihorario con un ángulo <180º
   double angle(pto a, pto o, pto b){ //devuelve radianes! (-pi,pi)
     pto oa=a-o, ob=b-o;
29
     return atan2(oa^ob, oa*ob);}
30
31
   //rotate p by theta rads CCW w.r.t. origin (0,0)
```

```
pto rotate(pto p, double theta) {
     return pto(p.x*cos(theta)-p.y*sin(theta),
        p.x*sin(theta)+p.y*cos(theta));
35
36 }
                      5.2. Orden radial de puntos
struct Cmp{//orden total de puntos alrededor de un punto r
     pto r;
2
     Cmp(pto r):r(r) {}
3
     int cuad(const pto &a) const{
       if(a.x > 0 \&\& a.y >= 0)return 0;
5
       if(a.x \le 0 \&\& a.y > 0)return 1;
6
       if(a.x < 0 && a.v <= 0)return 2;
       if(a.x >= 0 && a.y < 0)return 3;
8
       assert(a.x ==0 && a.y==0);
9
       return -1:
10
11
     bool cmp(const pto&p1, const pto&p2)const{
12
       int c1 = cuad(p1), c2 = cuad(p2);
13
       if(c1==c2) return p1.y*p2.x<p1.x*p2.y;
14
           else return c1 < c2:
15
     }
16
       bool operator()(const ptokp1, const ptokp2) const{
17
       return cmp(pto(p1.x-r.x,p1.y-r.y),pto(p2.x-r.x,p2.y-r.y));
18
       }
19
20 };
                                  5.3. Line
int sgn(ll x){return x<0? -1 : !!x;}</pre>
   struct line{
     line() {}
```

```
double a,b,c;//Ax+By=C
4
   //pto MUST store float coordinates!
     line(double a, double b, double c):a(a),b(b),c(c){}
     line(pto p, pto q): a(q.y-p.y), b(p.x-q.x), c(a*p.x+b*p.y) {}
     int side(pto p){return sgn(ll(a) * p.x + ll(b) * p.y - c);}
8
9
   bool parallels(line 11, line 12){return abs(11.a*12.b-12.a*11.b)<EPS;}</pre>
   pto inter(line 11, line 12){//intersection
11
     double det=11.a*12.b-12.a*11.b;
12
     if(abs(det) < EPS) return pto(INF, INF);//parallels
13
     return pto(12.b*11.c-11.b*12.c, 11.a*12.c-12.a*11.c)/det;
```

```
15 }
                               5.4. Segment
  struct segm{
     pto s,f;
2
     segm(pto s, pto f):s(s), f(f) {}
     pto closest(pto p) {//use for dist to point
        double 12 = dist_sq(s, f);
        if(12==0.) return s;
        double t = ((p-s)*(f-s))/12;
        if (t<0.) return s;//not write if is a line
8
        else if(t>1.)return f;//not write if is a line
        return s+((f-s)*t);
10
11
     bool inside(pto p){return abs(dist(s, p)+dist(p, f)-dist(s, f))<EPS;}</pre>
12
13
14
    //NOTA: Si los segmentos son colineales sólo devuelve un punto de
       intersección
  pto inter(segm s1, segm s2){
       if(s1.inside(s2.s)) return s2.s; //Fix cuando son colineales
17
       if(s1.inside(s2.f)) return s2.f; //Fix cuando son colineales
18
       pto r=inter(line(s1.s, s1.f), line(s2.s, s2.f));
19
       if(s1.inside(r) && s2.inside(r)) return r;
20
       return pto(INF, INF);
21
22 }
                              5.5. Rectangle
  struct rect{
     //lower-left and upper-right corners
     pto lw, up;
3
  };
4
   //returns if there's an intersection and stores it in r
   bool inter(rect a, rect b, rect &r){
     r.lw=pto(max(a.lw.x, b.lw.x), max(a.lw.y, b.lw.y));
    r.up=pto(min(a.up.x, b.up.x), min(a.up.y, b.up.y));
   //check case when only a edge is common
    return r.lw.x<r.up.x && r.lw.y<r.up.y;
11 }
```

```
5.6. Polygon Area
```

```
double area(vector<pto> &p){//0(sz(p))
     double area=0;
     forn(i, sz(p)) area+=p[i]^p[(i+1) %z(p)];
     //if points are in clockwise order then area is negative
     return abs(area)/2;
   }
6
  //Area ellipse = M_PI*a*b where a and b are the semi axis lengths
   //Area triangle = sqrt(s*(s-a)(s-b)(s-c)) where s=(a+b+c)/2
_{9} //o mejor area triángulo = abs(x0 * (y1 - y2) + x1 * (y2 - y0) + x2 * (y0 -
        v1)) / 2;
                                 5.7. Circle
   vec perp(vec v){return vec(-v.y, v.x);}
   line bisector(pto x, pto y){
     line l=line(x, y); pto m=(x+y)/2;
     return line(-1.b, 1.a, -1.b*m.x+1.a*m.y);
5
   struct Circle{
     pto o;
     double r:
     Circle(pto x, pto y, pto z){
       o=inter(bisector(x, y), bisector(y, z));
10
       r=dist(o, x):
11
     }
12
     pair<pto, pto> ptosTang(pto p){
13
       pto m=(p+o)/2;
14
       tipo d=dist(o, m);
15
       tipo a=r*r/(2*d);
16
       tipo h=sqrt(r*r-a*a);
17
       pto m2=o+(m-o)*a/d;
18
       vec per=perp(m-o)/d;
19
       return make_pair(m2-per*h, m2+per*h);
20
21
   };
22
   //finds the center of the circle containing p1 and p2 with radius r
   //as there may be two solutions swap p1, p2 to get the other
   bool circle2PtsRad(pto p1, pto p2, double r, pto &c){
           double d2=(p1-p2).norm sq(), det=r*r/d2-0.25;
26
           if(det<0) return false:
27
           c=(p1+p2)/2+perp(p2-p1)*sqrt(det);
28
           return true:
29
30 }
```

```
#define sqr(a) ((a)*(a))
   #define feq(a,b) (fabs((a)-(b))<EPS)</pre>
   pair<tipo, tipo > ecCuad(tipo a, tipo b, tipo c){//a*x*x+b*x+c=0
     tipo dx = sqrt(b*b-4.0*a*c);
     return make_pair((-b + dx)/(2.0*a), (-b - dx)/(2.0*a));
36
   pair<pto, pto> interCL(Circle c, line 1){
     bool sw=false;
     if((sw=feq(0,1.b))){
39
     swap(1.a, 1.b);
     swap(c.o.x, c.o.y);
41
42
     pair<tipo, tipo> rc = ecCuad(
43
     sqr(1.a)+sqr(1.b),
     2.0*1.a*1.b*c.o.y-2.0*(sqr(1.b)*c.o.x+1.c*1.a),
     sqr(1.b)*(sqr(c.o.x)+sqr(c.o.y)-sqr(c.r))+sqr(1.c)-2.0*1.c*1.b*c.o.y
46
     );
47
     pair<pto, pto> p( pto(rc.first, (l.c - l.a * rc.first) / l.b),
48
                pto(rc.second, (1.c - 1.a * rc.second) / 1.b));
49
     if(sw){
50
     swap(p.first.x, p.first.y);
51
     swap(p.second.x, p.second.y);
52
53
     return p;
54
55
    pair<pto, pto> interCC(Circle c1, Circle c2){
     line 1:
57
     1.a = c1.o.x-c2.o.x;
58
     1.b = c1.o.y-c2.o.y;
59
     1.c = (sqr(c2.r) - sqr(c1.r) + sqr(c1.o.x) - sqr(c2.o.x) + sqr(c1.o.y)
60
     -sqr(c2.o.y))/2.0;
61
     return interCL(c1, 1);
62
63 }
                             5.8. Point in Poly
  //checks if v is inside of P, using ray casting
   //works with convex and concave.
   bool inPolygon(pto v, vector<pto>& P) {
```

```
//checks if v is inside of P, using ray casting
//works with convex and concave.
bool inPolygon(pto v, vector<pto>& P) {
   bool c = false;
   forn(i, sz(P)){
    int j=(i+1) %sz(P);
}
```

```
segm lado(P[i],P[j]);
8
       if(lado.inside(v)) return true; //OJO: return true: incluye lados.
9
            return false: excluye lados.
10
       if((P[i].v > v.v) != (P[i].v > v.v) &&
11
       (v.x < (P[i].x-P[j].x) * (v.y-P[j].y) / (P[i].y-P[j].y) + P[j].x))
12
         c = !c;
13
     }
14
     return c;
16 }
```

5.9. Point in Convex Poly log(n)

```
void normalize(vector<pto> &pt){//delete collinear points first!
     //this makes it clockwise:
       if(pt[2].left(pt[0], pt[1])) reverse(pt.begin(), pt.end());
     int n=sz(pt), pi=0;
4
     forn(i, n)
5
       if(pt[i].x<pt[pi].x || (pt[i].x==pt[pi].x && pt[i].y<pt[pi].y))</pre>
6
7
     vector<pto> shift(n);//puts pi as first point
8
       forn(i, n) shift[i]=pt[(pi+i) %n];
9
       pt.swap(shift);
10
   }
11
12
   /* left debe decir >0 para que considere los bordes. Ojo que Convex Hull
       necesita que left diga >= 0 para limpiar los colineales, hacer otro
14
            left
       si hace falta */
15
   bool inPolygon(pto p, const vector<pto> &pt){
     //call normalize first!
     if(p.left(pt[0], pt[1]) || p.left(pt[sz(pt)-1], pt[0])) return false;
     int a=1, b=sz(pt)-1;
     while(b-a>1){
20
       int c=(a+b)/2;
21
       if(!p.left(pt[0], pt[c])) a=c;
22
       else b=c;
23
24
     return !p.left(pt[a], pt[a+1]);
25
26 }
```

5.10. Convex Check CHECK

bool isConvex(vector<int> &p){//O(N), delete collinear points!

17

```
int N=sz(p);
2
     if(N<3) return false;
3
     bool isLeft=p[0].left(p[1], p[2]);
4
     forr(i, 1, N)
5
       if(p[i].left(p[(i+1) \mathbb{N}], p[(i+2) \mathbb{N}])!=isLeft)
         return false;
7
     return true; }
                            5.11. Convex Hull
  //stores convex hull of P in S, CCW order
    //left must return >=0 to delete collinear points!
   void CH(vector<pto>& P, vector<pto> &S){
     S.clear();
4
     sort(P.begin(), P.end());//first x, then y
5
     forn(i, sz(P)){//lower hull
6
       while(sz(S) \ge 2 \&\& S[sz(S)-1].left(S[sz(S)-2], P[i])) S.pop_back();
       S.pb(P[i]);
8
9
     S.pop_back();
10
     int k=sz(S):
11
     dforn(i, sz(P)){//upper hull
12
       while(sz(S) \ge k+2 \&\& S[sz(S)-1].left(S[sz(S)-2], P[i])) S.pop back();
13
       S.pb(P[i]);
14
     }
15
     S.pop_back();
16
17 }
                            5.12. Cut Polygon
1 //cuts polygon Q along the line ab
   //stores the left side (swap a, b for the right one) in P
   void cutPolygon(pto a, pto b, vector<pto> Q, vector<pto> &P){
     P.clear();
4
     forn(i, sz(Q)){
5
       double left1=(b-a)^(Q[i]-a), left2=(b-a)^(Q[(i+1)/sz(Q)]-a);
6
       if(left1>=0) P.pb(Q[i]);
       if(left1*left2<0)
8
         P.pb(inter(line(Q[i], Q[(i+1) \sl z(Q)]), line(a, b)));
9
     }
10
11 }
                             5.13. Bresenham
```

```
1 //plot a line approximation in a 2d map
   void bresenham(pto a, pto b){
     pto d=b-a; d.x=abs(d.x), d.y=abs(d.y);
     pto s(a.x<b.x? 1: -1, a.y<b.y? 1: -1);
     int err=d.x-d.v;
     while(1){
      m[a.x][a.y]=1;//plot
      if(a==b) break;
      int e2=err;
      if(e2 >= 0) err-=2*d.y, a.x+=s.x;
      if(e2 <= 0) err+= 2*d.x, a.y+= s.y;
11
12
   }
13 }
                          5.14. Rotate Matrix
 1 //rotates matrix t 90 degrees clockwise
  //using auxiliary matrix t2(faster)
   void rotate(){
     forn(x, n) forn(y, n)
       t2[n-y-1][x]=t[x][y];
5
     memcpy(t, t2, sizeof(t));
6
7 }
             5.15. Interseccion de Circulos en n3log(n)
struct event {
       double x; int t;
       event(double xx, int tt) : x(xx), t(tt) {}
       bool operator <(const event &o) const { return x < o.x; }</pre>
4
   };
5
   typedef vector<Circle> VC;
   typedef vector<event> VE;
   double cuenta(VE &v, double A,double B) {
       sort(v.begin(), v.end());
       double res = 0.0, lx = ((v.empty())?0.0:v[0].x);
11
       int contador = 0;
12
       forn(i,sz(v)) {
13
           //interseccion de todos (contador == n), union de todos (contador >
14
           //conjunto de puntos cubierto por exacta k Circulos (contador == k)
15
           if (contador == n) res += v[i].x - lx;
16
```

contador += v[i].t, lx = v[i].x;

```
}
18
       return res;
19
20
    // Primitiva de sqrt(r*r - x*x) como funcion double de una variable x.
21
   inline double primitiva(double x,double r) {
22
       if (x \ge r) return r*r*M PI/4.0;
23
       if (x \le -r) return -r*r*M PI/4.0;
24
       double raiz = sqrt(r*r-x*x);
25
       return 0.5 * (x * raiz + r*r*atan(x/raiz));
26
27
   double interCircle(VC &v) {
28
       vector<double> p; p.reserve(v.size() * (v.size() + 2));
29
       forn(i,sz(v)) p.push back(v[i].c.x + v[i].r), p.push back(v[i].c.x - v[i].r)
30
            [i].r);
       forn(i,sz(v)) forn(j,i) {
31
           Circle &a = v[i], b = v[j];
32
           double d = (a.c - b.c).norm();
33
           if (fabs(a.r - b.r) < d \&\& d < a.r + b.r) {
34
                double alfa = acos((sqr(a.r) + sqr(d) - sqr(b.r)) / (2.0 * d *
35
                    a.r));
                pto vec = (b.c - a.c) * (a.r / d);
36
                p.pb((a.c + rotate(vec, alfa)).x), p.pb((a.c + rotate(vec, -
37
                    alfa)).x);
           }
38
39
       sort(p.begin(), p.end());
40
       double res = 0.0:
41
       forn(i,sz(p)-1) {
42
            const double A = p[i], B = p[i+1];
43
           VE ve; ve.reserve(2 * v.size());
44
           forn(j,sz(v)) {
45
                const Circle &c = v[j];
46
                double arco = primitiva(B-c.c.x,c.r) - primitiva(A-c.c.x,c.r);
47
                double base = c.c.y * (B-A);
48
                ve.push back(event(base + arco,-1));
49
                ve.push back(event(base - arco, 1));
50
51
           res += cuenta(ve,A,B);
52
53
       return res;
54
55 }
```

6. Math

6.1. Identidades

$$\sum_{i=0}^{n} \binom{n}{i} = 2^{n}$$

$$\sum_{i=0}^{n} i \binom{n}{i} = n * 2^{n-1}$$

$$\sum_{i=m}^{n} i = \frac{n(n+1)}{2} - \frac{m(m-1)}{2} = \frac{(n+1-m)(n+m)}{2}$$

$$\sum_{i=0}^{n} i = \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=0}^{n} i^{2} = \frac{n(n+1)(2n+1)}{2} = \frac{n^{3}}{3} + \frac{n^{2}}{2} + \frac{n}{6}$$

$$\sum_{i=0}^{n} i(i-1) = \frac{8}{6} (\frac{n}{2})(\frac{n}{2} + 1)(n+1) \text{ (doubles)} \rightarrow \text{Sino ver caso impar y par}$$

$$\sum_{i=0}^{n} i^{3} = \left(\frac{n(n+1)}{2}\right)^{2} = \frac{n^{4}}{4} + \frac{n^{3}}{2} + \frac{n^{2}}{4} = \left[\sum_{i=1}^{n} i\right]^{2}$$

$$\sum_{i=0}^{n} i^{4} = \frac{n(n+1)(2n+1)(3n^{2}+3n-1)}{30} = \frac{n^{5}}{5} + \frac{n^{4}}{2} + \frac{n^{3}}{3} - \frac{n}{30}$$

$$\sum_{i=0}^{n} i^{p} = \frac{(n+1)^{p+1}}{p+1} + \sum_{k=1}^{p} \frac{B_{k}}{p-k+1} \binom{p}{k}(n+1)^{p-k+1}$$

$$\sum_{i=0}^{n} a^{i} = \frac{a^{n+1}-1}{a-1} \text{ sólo si } a \neq 1$$

$$r = e - v + k + 1$$

Teorema de Pick: (Area, puntos interiores y puntos en el borde) $A = I + \frac{B}{2} - 1$

6.2. Ec. Caracteristica

```
\begin{aligned} a_0T(n) + a_1T(n-1) + \ldots + a_kT(n-k) &= 0 \\ p(x) &= a_0x^k + a_1x^{k-1} + \ldots + a_k \\ \text{Sean } r_1, r_2, \ldots, r_q \text{ las raices distintas, de mult. } m_1, m_2, \ldots, m_q \\ T(n) &= \sum_{i=1}^q \sum_{j=0}^{m_i-1} c_{ij} n^j r_i^n \\ \text{Las constantes } c_{ij} \text{ se determinan por los casos base.} \end{aligned}
```

6.3. Combinatorio

```
forn(i, MAXN+1){//comb[i][k]=i tomados de a k
    comb[i][0]=comb[i][i]=1;
    forr(k, 1, i) comb[i][k]=(comb[i-1][k]+comb[i-1][k-1]) MOD;
}

ll lucas (ll n, ll k, int p){ //Calcula (n,k) % teniendo comb[p][p]
    precalculado.

ll aux = 1;
    while (n + k) aux = (aux * comb[n%][k%]) %p, n/=p, k/=p;
    return aux;
}
```

6.4. Log. Discreto

1 // !!! TESTEAR !!!

```
2 //Baby step - giant step.
                                                                                                  tal que a^x=b
   //Returns x such that a^x = b \mod MOD. O(sqrt(MOD)*log(sqrt(MOD))).
                                                                                     41
                                                                                             now = now*a;
                                                                                           }
                                                                                     ^{42}
   // IDEA: a^x=b mod MOD <=> x = i*sqrt(MOD)+j con i,j <= sqrt(MOD)=m
                                                                                     43
   // entonces guardo todos los a^j: T[a^j mod MOD]=j
                                                                                           mnum inv = inverso(now.v,MOD); // = a^-m
                                                                                     44
   // v después busco si vi T[b/(a^{(i*m)} \mod MOD] = T[b*a^{(i*m)} \mod MOD],
                                                                                           forn(i,m){
                                                                                     45
       return j+i*m
                                                                                             if(T.find(b.v) != T.end()) return i*m + T[b.v]; //found!
                                                                                     46
                                                                                             b = b*inv:
8
                                                                                     47
   #define mod(x) (((x) MOD+MOD) MOD)
                                                                                          }
                                                                                     48
10
   11 discrete log(ll a, ll b, ll MOD)
                                                                                          return -1; //not found
11
                                                                                     50
                                                                                     51 }
12
     a = mod(a); b = mod(b);
13
                                                                                                 6.5. Exp. de Matrices y Fibonacci en log(n)
     unordered map<11,11> T;
14
     11 m = min(MOD, (11)sqrt(MOD)+5); // m >= ceil(sqrt(MOD))
15
                                                                                      1 #define SIZE 350
16
                                                                                        int NN:
     11 \text{ now} = 1;
17
                                                                                         double tmp[SIZE] [SIZE];
     forn(j,m){
18
                                                                                         void mul(double a[SIZE] [SIZE], double b[SIZE] [SIZE]){ zero(tmp);
       if(T.find(now) == T.end()) T[now] = j; //con este if da el primer x, si
19
                                                                                             forn(i, NN) forn(j, NN) forn(k, NN) tmp[i][j]+=a[i][k]*b[k][j];
                                                                                      5
            se saca el if sigue andando pero puede no devolver el primer x tal
                                                                                             forn(i, NN) forn(j, NN) a[i][j]=tmp[i][j];
                                                                                      6
            que a^x=b
                                                                                        1
                                                                                      7
       now = mod(now*a);
20
                                                                                         void powmat(double a[SIZE] [SIZE], int n, double res[SIZE] [SIZE]){
     }
21
                                                                                             forn(i, NN) forn(j, NN) res[i][j]=(i==j);
22
                                                                                             while(n){
                                                                                     10
     11 inv = inverso(now,MOD); // = a^-m
23
                                                                                                 if(n&1) mul(res, a), n--;
                                                                                     11
     forn(i,m){
24
                                                                                                 else mul(a, a), n/=2;
                                                                                     12
       if(T.find(b) != T.end()) return i*m + T[b]; //found!
25
                                                                                            } }
                                                                                     13
       b = mod(b*inv);
26
     }
                                                                                                       6.6. Matrices y determinante O(n^3)
27
28
     return -1; //not found
29
                                                                                      1 struct Mat {
30
                                                                                             vector<vector<double> > vec;
                                                                                      2
                                                                                             Mat(int n): vec(n, vector<double>(n) ) {}
31
                                                                                      3
    //con mnum
                                                                                             Mat(int n, int m): vec(n, vector<double>(m) ) {}
                                                                                      4
   11 discrete_log(mnum a, mnum b)
                                                                                             vector<double> &operator[](int f){return vec[f];}
33
                                                                                      5
                                                                                             const vector<double> &operator[](int f) const {return vec[f];}
34
                                                                                      6
     unordered map<11,11> T;
                                                                                             int size() const {return sz(vec);}
35
     11 m = min(MOD, (11)sqrt(MOD)+5); // m >= ceil(sqrt(MOD))
                                                                                             Mat operator+(Mat &b) { ///this de n x m entonces b de n x m
36
                                                                                      8
                                                                                                 Mat m(sz(b), sz(b[0]));
37
                                                                                      9
     mnum now = 1;
                                                                                                 forn(i,sz(vec)) forn(j,sz(vec[0])) m[i][j] = vec[i][j] + b[i][j];
38
                                                                                     10
     forn(j,m){
39
                                                                                     11
       if(T.find(now.v) == T.end()) T[now.v] = j; //con este if da el primer x
                                                                                             Mat operator*(const Mat &b) { ///this de n x m entonces b de m x t
40
                                                                                     12
            , si se saca el if sigue andando pero puede no devolver el primer x
                                                                                                 int n = sz(vec), m = sz(vec[0]), t = sz(b[0]);
                                                                                     13
```

```
Mat mat(n,t);
14
            forn(i,n) forn(j,t) forn(k,m) mat[i][j] += vec[i][k] * b[k][j];
15
            return mat:
16
       double determinant(){//sacado de e maxx ru
17
            double det = 1;
18
            int n = sz(vec);
19
            Mat m(*this);
20
            forn(i, n){//para cada columna
21
                int k = i;
^{22}
                forr(j, i+1, n)//busco la fila con mayor val abs
23
                    if(abs(m[j][i])>abs(m[k][i])) k = j;
24
                if(abs(m[k][i])<1e-9) return 0;
25
                m[i].swap(m[k]);//la swapeo
26
                if(i!=k) det = -det;
27
                det *= m[i][i];
28
                forr(j, i+1, n) m[i][j] /= m[i][i];
29
                //hago 0 todas las otras filas
30
                forn(j, n) if (j!= i && abs(m[j][i])>1e-9)
31
                    forr(k, i+1, n) m[j][k]-=m[i][k]*m[j][i];
32
33
            return det;
34
35
36
37
   int n;
38
   int main() {
    //DETERMINANTE:
    //https://uva.onlinejudge.org/index.php?option=com_onlinejudge&Itemid=8&
41
        page=show problem&problem=625
     freopen("input.in", "r", stdin);
42
       ios::sync with stdio(0);
43
       while(cin >> n && n){
44
            Mat m(n):
45
            forn(i, n) forn(j, n) cin >> m[i][j];
46
            cout << (ll)round(m.determinant()) << endl;</pre>
47
48
       cout << "*" << endl;
49
     return 0:
50
51 }
```

6.7. Teorema Chino del Resto

$$y = \sum_{j=1}^{n} (x_j * (\prod_{i=1, i \neq j}^{n} m_i)_{m_j}^{-1} * \prod_{i=1, i \neq j}^{n} m_i)$$

6.8. Criba

```
#define MAXP 100000 //no necesariamente primo
   int criba[MAXP+1]:
   void crearcriba(){
     int w[] = \{4,2,4,2,4,6,2,6\};
     for(int p=25;p<=MAXP;p+=10) criba[p]=5;</pre>
     for(int p=9;p<=MAXP;p+=6) criba[p]=3;</pre>
     for(int p=4;p<=MAXP;p+=2) criba[p]=2;</pre>
     for(int p=7,cur=0;p*p<=MAXP;p+=w[cur++&7]) if (!criba[p])
       for(int j=p*p; j<=MAXP; j+=(p<<1)) if(!criba[j]) criba[j]=p;</pre>
10
   vector<int> primos;
   void buscarprimos(){
     crearcriba();
     forr (i,2,MAXP+1) if (!criba[i]) primos.push_back(i);
15
   //\sim Useful for bit trick: #define SET(i) ( criba[(i)>>5]|=1<<((i)&31) ), #
       define INDEX(i) ( (criba[i>>5]>>((i)&31))&1 ), unsigned int criba[MAXP
       /32+1];
```

6.9. Funciones de primos

Sea $n = \prod p_i^{k_i}$, fact(n) genera un map donde a cada p_i le asocia su k_i

```
//factoriza bien numeros hasta MAXP^2
map<ll,ll> fact(ll n){ //0 (cant primos)
map<ll,ll> ret;
forall(p, primos){
    while(!(n%p)){
        ret[*p]++;//divisor found
        n/=*p;
     }
    }
    if(n>1) ret[n]++;
    return ret;
}
```

```
//factoriza bien numeros hasta MAXP
                                                                                          return r;
   map<11,11> fact2(11 n){ //0 (lg n)
                                                                                     56 }
     map<11,11> ret;
15
     while (criba[n]){
16
       ret[criba[n]]++;
                                                                                                  6.10. Test de primalidad naive O(\operatorname{sqrt}(n)/6)
       n/=criba[n];
18
                                                                                      int __attribute__((const)) is_prime(long long n)
19
     if(n>1) ret[n]++;
20
                                                                                      2
     return ret;
^{21}
                                                                                          if (n <= 1)
                                                                                      3
22
                                                                                             return 0:
    //Usar asi: divisores(fac, divs, fac.begin()); NO ESTA ORDENADO
                                                                                           else if (n \le 3)
   void divisores(const map<11,11> &f, vector<11> &divs, map<11,11>::iterator
                                                                                            return 1;
                                                                                      6
       it. 11 n=1){
                                                                                           else if (!(n %2) || !(n %3))
                                                                                      7
       if(it==f.begin()) divs.clear();
25
                                                                                            return 0:
                                                                                      8
       if(it==f.end()) { divs.pb(n); return; }
26
                                                                                      9
       ll p=it->fst, k=it->snd; ++it;
27
                                                                                          long long cap = sqrt(n) + 1;
                                                                                     10
       forn(_, k+1) divisores(f, divs, it, n), n*=p;
28
                                                                                          for (long long int i = 5; i \le cap; i += 6)
                                                                                     11
29
                                                                                             if (!(n%i) || !(n%(i+2)))
                                                                                     12
   ll sumDiv (ll n){
30
                                                                                               return 0;
                                                                                     13
     ll rta = 1;
31
                                                                                     14
     map<ll,ll> f=fact(n);
32
                                                                                          return 1;
                                                                                     15
     forall(it, f) {
33
                                                                                     16 }
     11 \text{ pot} = 1, \text{ aux} = 0;
34
                                                                                                          6.11. Phollard's Rho (rolando)
     forn(i, it->snd+1) aux += pot, pot *= it->fst;
35
     rta*=aux;
36
     }
                                                                                      1 | ll gcd(ll a, ll b){return a?gcd(b %, a):b;}
37
     return rta;
38
                                                                                      2
                                                                                        11 mulmod (11 a, 11 b, 11 c) { //returns (a*b) %, and minimize overfloor
39
   ll eulerPhi (ll n){ // con criba: O(lg n)
                                                                                          11 x = 0, y = a\%;
40
     11 \text{ rta} = n:
                                                                                          while (b > 0){
41
     map<11,11> f=fact(n);
                                                                                          if (b \% 2 == 1) x = (x+y) \% c;
     forall(it, f) rta -= rta / it->first;
                                                                                            y = (y*2) \% c;
     return rta:
                                                                                             b /= 2;
44
                                                                                      8
45
                                                                                      9
   return x %c;
                                                                                     10
     11 r = n;
47
                                                                                     11
     forr (i,2,n+1){
48
                                                                                     12
       if ((11)i*i > n) break:
                                                                                        | 11 expmod (11 b, 11 e, 11 m){\frac{1}{0}} \log b
49
       if (n \% i == 0){
                                                                                          if(!e) return 1:
50
                                                                                     14
         while (n\% == 0) n/=i;
                                                                                          11 q= expmod(b,e/2,m); q=mulmod(q,q,m);
51
         r = r/i; }
                                                                                          return e %2? mulmod(b,q,m) : q;
52
                                                                                     16
                                                                                     17 }
53
     if (n != 1) r= r/n;
                                                                                     18
```

```
|bool es_primo_prob (ll n, int a)
                                                                                          if (rabin(n)){
                                                                                            prim[n]++;
                                                                                    63
20
     if (n == a) return true;
                                                                                            return;
21
                                                                                    64
     11 s = 0, d = n-1;
                                                                                         }
^{22}
                                                                                    65
     while (d \% 2 == 0) s++, d/=2;
                                                                                          11 factor = rho(n);
                                                                                    66
23
                                                                                          factRho(factor);
24
     11 x = expmod(a,d,n);
                                                                                          factRho(n/factor);
25
     if ((x == 1) \mid | (x+1 == n)) return true;
                                                                                    69 }
26
27
                                                                                                                      6.12. GCD
     forn (i, s-1){
28
       x = mulmod(x, x, n);
29
                                                                                     tipo gcd(tipo a, tipo b){return a?gcd(b %a, a):b;}
       if (x == 1) return false;
30
       if (x+1 == n) return true:
                                                                                                              6.13. Extended Euclid
31
32
     return false;
                                                                                     void extendedEuclid (ll a, ll b){ //a * x + b * y = d
33
34
                                                                                          if (!b) { x = 1; y = 0; d = a; return;}
35
                                                                                          extendedEuclid (b, a %);
   bool rabin (ll n){ //devuelve true si n es primo
                                                                                          11 x1 = y;
     if (n == 1) return false:
37
                                                                                          11 y1 = x - (a/b) * y;
     const int ar[] = \{2,3,5,7,11,13,17,19,23\};
38
                                                                                          x = x1; y = y1;
     forn (j,9)
39
                                                                                     7 | }
       if (!es_primo_prob(n,ar[j]))
40
                                                                                                                      6.14. LCM
         return false;
41
     return true;
42
                                                                                     1 | tipo lcm(tipo a, tipo b){return a / gcd(a,b) * b;}
43
44
                                                                                                                   6.15. Simpson
   11 rho(11 n){
45
       if( (n \& 1) == 0 ) return 2;
46
                                                                                     1 double integral(double a, double b, int n=10000) \{//0(n), n=cantdiv\}
       11 x = 2, y = 2, d = 1;
47
                                                                                          double area=0, h=(b-a)/n, fa=f(a), fb;
       ll c = rand() % n + 1;
48
                                                                                          forn(i, n){
       while(d == 1){
49
                                                                                            fb=f(a+h*(i+1));
                                                                                     4
           x = (mulmod(x, x, n) + c) n;
50
                                                                                            area+=fa+ 4*f(a+h*(i+0.5)) +fb, fa=fb;
                                                                                     5
           y = (mulmod(y, y, n) + c) n;
51
                                                                                          }
                                                                                     6
           y = (mulmod(y, y, n) + c) n;
52
                                                                                          return area*h/6.;}
           if(x - y >= 0) d = gcd(x - y, n);
53
           else d = gcd(y - x, n);
                                                                                                                    6.16. Fraction
54
       }
55
       return d==n? rho(n):d:
                                                                                     bool comp(tipo a, tipo b, tipo c, tipo d)\frac{1}{a*d} < b*c
56
                                                                                            int s1 = signo(a)*signo(d), s2 = signo(b)*signo(c);
57
                                                                                            if(s1 == 0) return s2 > 0;
58
   map<11,11> prim;
                                                                                            if(s2 == 0) return s1 < 0;
                                                                                     4
   void factRho (ll n){ //O (lg n)^3. un solo numero
                                                                                            if(s1 > 0 and s2 < 0) return false;
                                                                                     5
     if (n == 1) return;
                                                                                            if (s1 < 0 \text{ and } s2 > 0) return true;
                                                                                     6
```

```
if(a / b != c / d) return a/b < c/d; //asume que b y d son positivos
7
       a /= b, c /= d;
8
                                                                                        8
       /*O(1) pero con double:
                                                                                        9
9
       long double d1 = ((long double)(a))/(b), d2 = ((long double)(c))/(d);
                                                                                       10
10
       return d1 + EPS < d2;
                                                                                       11
11
       */
12
                                                                                       12
       return comp(d, c, b, a);
                                                                                       13
13
                                                                                       14
14
15
                                                                                       15
   tipo mcd(tipo a, tipo b) { return a ? mcd(b%,a) : b; }
                                                                                       16
   struct frac{
                                                                                       17
17
     tipo p,q;
18
                                                                                       18
     frac(tipo p=0, tipo q=1):p(p),q(q) {norm();}
                                                                                       19
     void norm(){
20
                                                                                       20
       tipo a = mcd(p,q);
21
                                                                                       21
       if(a) p/=a, q/=a;
                                                                                       22
22
       else q=1;
23
                                                                                       23
       if (q<0) q=-q, p=-p;}
24
     frac operator+(const frac% o){
                                                                                       25
25
       tipo a = mcd(q, o.q);
                                                                                       26
26
       return frac(p*(o.q/a)+o.p*(q/a), q*(o.q/a));}
                                                                                       27
27
     frac operator-(const frac& o){
28
                                                                                       28
       tipo a = mcd(q, o.q);
29
                                                                                       29
       return frac(p*(o.q/a)-o.p*(q/a), q*(o.q/a));}
                                                                                       30
30
     frac operator*(frac o){
                                                                                       31
31
       tipo a = mcd(q,o.p), b = mcd(o.q,p);
                                                                                       32
32
       return frac((p/b)*(o.p/a), (q/a)*(o.q/b));}
                                                                                       33
33
     frac operator/(frac o){
                                                                                       34
34
       tipo a = mcd(q,o.q), b = mcd(o.p,p);
                                                                                       35
35
       return frac((p/b)*(o.q/a),(q/a)*(o.p/b));}
36
     bool operator (const frac &o) const{return p*o.q < o.p*q;}//usar comp
37
                                                                                       37
          cuando el producto puede dar overflow
     bool operator==(frac o){return p==o.p&kq==o.q;}
38
                                                                                       39
39 | };
                                                                                       40
                                                                                       41
                              6.17. Polinomio
                                                                                       42
                                                                                       43
struct poly {
                                                                                       44
       vector<tipo> c;//guarda los coeficientes del polinomio
2
       poly(const vector<tipo> &c): c(c) {}
3
       poly() {}
                                                                                       47
4
       void simplify(){
                                                                                       48
5
       int i = 0;
```

```
/*tipo a0=0:
       while(a0 == 0 && i < sz(c)) a0 = c[i], i++;*/
       int j = sz(c)-1;
       tipo an=0;
       while(an == 0 && j >= i) an = c[j], j--;
       vector<tipo> d;
       forr(k,i,j) d.pb(c[k]);
       c=d;
     }
     bool isnull() { simplify(); return c.empty();}
       poly operator+(const poly &o) const {
           int m = sz(c), n = sz(o.c);
           vector<tipo> res(max(m.n)):
           forn(i, m) res[i] += c[i];
           forn(i, n) res[i] += o.c[i];
           return poly(res); }
       poly operator*(const tipo cons) const {
       vector<tipo> res(sz(c));
           forn(i, sz(c)) res[i]=c[i]*cons:
           return poly(res); }
       poly operator*(const poly &o) const {
           int m = sz(c), n = sz(o.c);
           vector<tipo> res(m+n-1);
           forn(i, m) forn(j, n) res[i+j]+=c[i]*o.c[j];
           return poly(res); }
     tipo eval(tipo v) {
       tipo sum = 0;
       dforn(i, sz(c)) sum=sum*v + c[i];
       return sum: }
       //poly contains only a vector<int> c (the coeficients)
     //the following function generates the roots of the polynomial
    //it can be easily modified to return float roots
     set<tipo> roots(){
       set<tipo> roots;
       simplify();
       if(c[0]) roots.insert(0);
       int i = 0;
       tipo a0=0:
       while(a0 == 0 && i < sz(c)) a0 = abs(c[i]), i++;
       tipo an = abs(c[sz(c)-1]);
       vector<tipo> ps,qs;
       forr(p,1,sqrt(a0)+1) if (a0%p==0) ps.pb(p),ps.pb(a0/p);
       forr(q,1,sqrt(an)+1) if (an)q==0) qs.pb(q),qs.pb(an/q);
49
```

```
forall(pt,ps)
50
         forall(qt,qs) if ( (*pt) % (*qt)==0 ) { //sacar esto para obtener
51
             todas las raices racionales
           tipo root = abs((*pt) / (*qt));
52
           if (eval(root)==0) roots.insert(root);
53
           if (eval((-1)*root)==0) roots.insert((-1)*root);// las raices
54
                tambien pueden ser negativas!
         }
55
       return roots; }
56
57
   pair <poly, tipo > ruffini(const poly p, tipo r) { //divive el polinomio p por
         (x-r)
     int n = sz(p.c) - 1;
     vector<tipo> b(n);
     b[n-1] = p.c[n];
61
     dforn(k,n-1) b[k] = p.c[k+1] + r*b[k+1];
     tipo resto = p.c[0] + r*b[0];
63
     poly result(b);
64
     return make pair(result, resto);
65
66
   poly interpolate(const vector<tipo>& x,const vector<tipo>& y) { //0(n^2)
67
       poly A; A.c.pb(1);
68
       forn(i,sz(x)) { poly aux; aux.c.pb(-x[i]), aux.c.pb(1), A = A * aux; }
69
           // A = (x-x0) * ... * (x-xn)
     poly S; S.c.pb(0);
70
     forn(i,sz(x)) { poly Li;
71
       Li = ruffini(A,x[i]).fst;
72
       Li = Li * (1.0 / Li.eval(x[i])); // here put a multiple of the
73
           coefficients instead of 1.0 to avoid using double -- si se usa mod
           usar el inverso!
       S = S + Li * y[i];
74
     return S;
75
76 }
```

6.18. Ec. Lineales

```
#define eps 1e-10
#define feq(a, b) (fabs((a)-(b))<eps)

bool resolver_ev(Mat a, Vec y, Vec &x, Mat &ev){
   int n = a.size(), m = n?a[0].size():0, rw = min(n, m);
   vector<int> p; forn(i,m) p.push_back(i);
   forn(i, rw) {
```

```
int uc=i. uf=i:
8
       forr(f, i, n) forr(c, i, m) if(fabs(a[f][c])>fabs(a[uf][uc])) {uf=f;uc=
9
       if (feq(a[uf][uc], 0)) { rw = i; break; }
10
       forn(j, n) swap(a[j][i], a[j][uc]);
11
       swap(a[i], a[uf]); swap(y[i], y[uf]); swap(p[i], p[uc]);
12
       tipo inv = 1 / a[i][i]; //aca divide
13
       forr(j, i+1, n) {
14
         tipo v = a[j][i] * inv;
15
         forr(k, i, m) a[j][k]-=v * a[i][k];
         y[j] = v*y[i];
17
18
     } // rw = rango(a), aca la matriz esta triangulada
19
     forr(i, rw, n) if (!feq(y[i],0)) return false; // checkeo de
20
         compatibilidad
     x = vector < tipo > (m, 0);
21
     dforn(i, rw){
       tipo s = y[i];
       forr(j, i+1, rw) s = a[i][j]*x[p[j]];
       x[p[i]] = s / a[i][i]; //aca divide
25
     }
26
     ev = Mat(m-rw, Vec(m, 0)); // Esta parte va SOLO si se necesita el ev
27
     forn(k, m-rw) {
28
       ev[k][p[k+rw]] = 1;
29
       dforn(i, rw){
30
         tipo s = -a[i][k+rw];
31
         forr(j, i+1, rw) s -= a[i][j]*ev[k][p[j]];
         ev[k][p[i]] = s / a[i][i]; //aca divide
33
       }
34
     }
35
     return true;
36
37 }
```

6.19. FFT

```
//~ typedef complex<double> base; //menos codigo, pero mas lento
//elegir si usar complejos de c (lento) o estos
struct base{
    double r,i;
    base(double r=0, double i=0):r(r), i(i){}
    double real()const{return r;}
    void operator/=(const int c){r/=c, i/=c;}
};
```

```
base operator*(const base &a, const base &b){
       return base(a.r*b.r-a.i*b.i, a.r*b.i+a.i*b.r);}
10
   base operator+(const base &a, const base &b){
11
       return base(a.r+b.r, a.i+b.i);}
12
   base operator-(const base &a, const base &b){
13
       return base(a.r-b.r, a.i-b.i);}
   vector<int> rev; vector<base> wlen pw;
   inline static void fft(base a[], int n, bool invert) {
       forn(i, n) if(i<rev[i]) swap(a[i], a[rev[i]]);</pre>
17
     for (int len=2; len<=n; len<<=1) {
18
       double ang = 2*M PI/len * (invert?-1:+1);
19
       int len2 = len >> 1;
20
       base wlen (cos(ang), sin(ang)):
21
       wlen pw[0] = base (1, 0);
22
           forr(i, 1, len2) wlen pw[i] = wlen pw[i-1] * wlen;
23
       for (int i=0; i<n; i+=len) {</pre>
24
         base t, *pu = a+i, *pv = a+i+len2, *pu end = a+i+len2, *pw = &
25
              wlen pw[0];
         for (; pu!=pu end; ++pu, ++pv, ++pw)
26
           t = *pv * *pw, *pv = *pu - t,*pu = *pu + t;
27
       }
28
29
     if (invert) forn(i, n) a[i]/= n;}
30
   inline static void calc rev(int n){//precalculo: llamar antes de fft!!
31
       wlen pw.resize(n), rev.resize(n);
32
       int lg=31-_builtin_clz(n);
33
       forn(i, n){
34
       rev[i] = 0;
35
           forn(k, lg) if(i&(1<<k)) rev[i] |=1<<(lg-1-k);
36
       }}
37
    //multiplica vectores en nlgn
   inline static void multiply(const vector<int> &a, const vector<int> &b,
       vector<int> &res) {
     vector<base> fa (a.begin(), a.end()), fb (b.begin(), b.end());
40
       int n=1; while(n < max(sz(a), sz(b))) n <<= 1; n <<= 1;
41
       calc rev(n);
42
     fa.resize (n), fb.resize (n);
43
     fft (&fa[0], n, false), fft (&fb[0], n, false);
     forn(i, n) fa[i] = fa[i] * fb[i];
45
     fft (&fa[0], n, true);
46
     res.resize(n):
47
       forn(i, n) res[i] = int (fa[i].real() + 0.5); }
   void toPoly(const string &s, vector<int> &P){//convierte un numero a
```

```
polinomio
    P.clear();
    dforn(i, sz(s)) P.pb(s[i]-'0');}
```

6.20. Tablas y cotas (Primos, Divisores, Factoriales, etc)

```
Factoriales
0! = 1
                  11! = 39.916.800
1! = 1
                  12! = 479.001.600 \ (\in int)
2! = 2
                  13! = 6.227.020.800
                  14! = 87.178.291.200
3! = 6
4! = 24
                  15! = 1.307.674.368.000
5! = 120
                  16! = 20.922.789.888.000
6! = 720
                  17! = 355.687.428.096.000
7! = 5.040
                  18! = 6.402.373.705.728.000
8! = 40.320
                  19! = 121.645.100.408.832.000
9! = 362.880
                  20! = 2.432.902.008.176.640.000 (\in tint)
10! = 3.628.800 \mid 21! = 51.090.942.171.709.400.000
       max signed tint = 9.223.372.036.854.775.807
     max unsigned tint = 18.446.744.073.709.551.615
```

Primos

 $2\ 3\ 5\ 7\ 11\ 13\ 17\ 19\ 23\ 29\ 31\ 37\ 41\ 43\ 47\ 53\ 59\ 61\ 67\ 71\ 73\ 79\ 83\ 89\ 97\ 101\ 103\ 107\ 109$ $113\ 127\ 131\ 137\ 139\ 149\ 151\ 157\ 163\ 167\ 173\ 179\ 181\ 191\ 193\ 197\ 199\ 211\ 223\ 227$ 229 233 239 241 251 257 263 269 271 277 281 283 293 307 311 313 317 331 337 347 349 353 359 367 373 379 383 389 397 401 409 419 421 431 433 439 443 449 457 461 $463\ 467\ 479\ 487\ 491\ 499\ 503\ 509\ 521\ 523\ 541\ 547\ 557\ 563\ 569\ 571\ 577\ 587\ 593\ 599$ $601\ 607\ 613\ 617\ 619\ 631\ 641\ 643\ 647\ 653\ 659\ 661\ 673\ 677\ 683\ 691\ 701\ 709\ 719\ 727$ $733\ 739\ 743\ 751\ 757\ 761\ 769\ 773\ 787\ 797\ 809\ 811\ 821\ 823\ 827\ 829\ 839\ 853\ 857\ 859$ 863 877 881 883 887 907 911 919 929 937 941 947 953 967 971 977 983 991 997 1009 1013 1019 1021 1031 1033 1039 1049 1051 1061 1063 1069 1087 1091 1093 1097 1103 $1109\ 1117\ 1123\ 1129\ 1151\ 1153\ 1163\ 1171\ 1181\ 1187\ 1193\ 1201\ 1213\ 1217\ 1223\ 1229$ 1231 1237 1249 1259 1277 1279 1283 1289 1291 1297 1301 1303 1307 1319 1321 1327 1361 1367 1373 1381 1399 1409 1423 1427 1429 1433 1439 1447 1451 1453 1459 1471 $1481\ 1483\ 1487\ 1489\ 1493\ 1499\ 1511\ 1523\ 1531\ 1543\ 1549\ 1553\ 1559\ 1567\ 1571\ 1579$ 1583 1597 1601 1607 1609 1613 1619 1621 1627 1637 1657 1663 1667 1669 1693 1697 $1699\ 1709\ 1721\ 1723\ 1733\ 1741\ 1747\ 1753\ 1759\ 1777\ 1783\ 1787\ 1789\ 1801\ 1811\ 1823$ $1831\ 1847\ 1861\ 1867\ 1871\ 1873\ 1877\ 1879\ 1889\ 1901\ 1907\ 1913\ 1931\ 1933\ 1949\ 1951$ 1973 1979 1987 1993 1997 1999 2003 2011 2017 2027 2029 2039 2053 2063 2069 2081

Primos cercanos a 10^n

9941 9949 9967 9973 10007 10009 10037 10039 10061 10067 10069 10079 99961 99971 99989 99991 100003 100019 100043 100049 100057 100069 999959 999961 999979 999983 1000003 1000033 1000037 1000039

 $9999943\ 9999971\ 9999973\ 9999991\ 10000019\ 10000079\ 10000103\ 10000121$ $99999941\ 99999959\ 99999971\ 99999989\ 100000007\ 1000000037\ 1000000039\ 1000000049$ $999999893\ 999999929\ 999999937\ 1000000007\ 1000000009\ 1000000021\ 1000000033$

Cantidad de primos menores que 10^n

```
\pi(10^1) = 4 \; ; \; \pi(10^2) = 25 \; ; \; \pi(10^3) = 168 \; ; \; \pi(10^4) = 1229 \; ; \; \pi(10^5) = 9592 \\ \pi(10^6) = 78.498 \; ; \; \pi(10^7) = 664.579 \; ; \; \pi(10^8) = 5.761.455 \; ; \; \pi(10^9) = 50.847.534 \\ \pi(10^{10}) = 455.052,511 \; ; \; \pi(10^{11}) = 4.118.054.813 \; ; \; \pi(10^{12}) = 37.607.912.018
```

Divisores

```
Cantidad de divisores (\sigma_0) para algunos n/\neg \exists n' < n, \sigma_0(n') \ge \sigma_0(n)
       \sigma_0(60) = 12; \sigma_0(120) = 16; \sigma_0(180) = 18; \sigma_0(240) = 20; \sigma_0(360) = 24
    \sigma_0(720) = 30; \sigma_0(840) = 32; \sigma_0(1260) = 36; \sigma_0(1680) = 40; \sigma_0(10080) = 72
        \sigma_0(15120) = 80; \sigma_0(50400) = 108; \sigma_0(83160) = 128; \sigma_0(110880) = 144
    \sigma_0(498960) = 200; \sigma_0(554400) = 216; \sigma_0(1081080) = 256; \sigma_0(1441440) = 288
                            \sigma_0(4324320) = 384; \sigma_0(8648640) = 448
             Suma de divisores (\sigma_1) para algunos n/\neg \exists n' < n, \sigma_1(n') \ge \sigma_1(n)
    \sigma_1(96) = 252 : \sigma_1(108) = 280 : \sigma_1(120) = 360 : \sigma_1(144) = 403 : \sigma_1(168) = 480
        \sigma_1(960) = 3048; \sigma_1(1008) = 3224; \sigma_1(1080) = 3600; \sigma_1(1200) = 3844
     \sigma_1(4620) = 16128; \sigma_1(4680) = 16380; \sigma_1(5040) = 19344; \sigma_1(5760) = 19890
    \sigma_1(8820) = 31122; \sigma_1(9240) = 34560; \sigma_1(10080) = 39312; \sigma_1(10920) = 40320
\sigma_1(32760) = 131040; \sigma_1(35280) = 137826; \sigma_1(36960) = 145152; \sigma_1(37800) = 148800
\sigma_1(60480) = 243840; \sigma_1(64680) = 246240; \sigma_1(65520) = 270816; \sigma_1(70560) = 280098
            \sigma_1(95760) = 386880; \sigma_1(98280) = 403200; \sigma_1(100800) = 409448
        \sigma_1(491400) = 2083200; \sigma_1(498960) = 2160576; \sigma_1(514080) = 2177280
        \sigma_1(982800) = 4305280; \sigma_1(997920) = 4390848; \sigma_1(1048320) = 4464096
    \sigma_1(4979520) = 22189440; \sigma_1(4989600) = 22686048; \sigma_1(5045040) = 23154768
    \sigma_1(9896040) = 44323200; \sigma_1(9959040) = 44553600; \sigma_1(9979200) = 45732192
```

7. Grafos

7.1. Dijkstra

```
#define INF 1e9
int N;
#define MAX_V 250001
vector<ii>> G[MAX_V];

//To add an edge use
#define add(a, b, w) G[a].pb(make_pair(w, b))

Il dijkstra(int s, int t){//0(|E| log |V|)}
priority_queue<ii, vector<ii>>, greater<ii>> Q;
vector<11> dist(N, INF); vector<int> dad(N, -1);
```

```
Q.push(make pair(0, s)); dist[s] = 0;
10
     while(sz(Q)){
11
       ii p = Q.top(); Q.pop();
12
       if(p.snd == t) break;
13
       forall(it, G[p.snd])
14
         if(dist[p.snd]+it->first < dist[it->snd]){
15
           dist[it->snd] = dist[p.snd] + it->fst;
16
           dad[it->snd] = p.snd;
17
           Q.push(make_pair(dist[it->snd], it->snd)); }
18
     }
19
     return dist[t];
20
     if(dist[t]<INF)//path generator</pre>
21
       for(int i=t; i!=-1; i=dad[i])
22
         printf("%1%", i, (i==s?'\n':','));}
23
                            7.2. Bellman-Ford
1 #define INF 1e9
   #define MAX N 1001
   vector<ii> G[MAX N];//ady. list with pairs (weight, dst)
   //To add an edge use
   #define add(a, b, w) G[a].pb(make pair(w, b))
   int dist[MAX N]:
  int N: //cantidad de vertices -- setear!!
   void bford(int src){//O(VE)
     memset(dist,INF,sizeof dist);
9
     dist[src]=0;
10
     forn(i, N-1) forn(j, N) if(dist[j]!=INF) forall(it, G[j])
11
       dist[it->snd]=min(dist[it->snd], dist[j]+it->fst);
12
   }
13
14
   bool hasNegCycle(){
     forn(j, N) if(dist[j]!=INF) forall(it, G[j])
       if(dist[it->snd]>dist[j]+it->fst) return true;
17
     //inside if: all points reachable from it->snd will have -INF distance(do
18
          bfs) ?
     return false;
20 }
                           7.3. Floyd-Warshall
```

```
//G[i][j] contains weight of edge (i, j) or INF
//G[i][i]=0
int G[MAX N][MAX N];
```

```
void floyd(){//0(N^3)}
  forn(k, N) forn(i, N) if(G[i][k]!=INF) forn(j, N) if(G[k][j]!=INF)
     G[i][j]=min(G[i][j], G[i][k]+G[k][j]);
7
   bool inNegCycle(int v){
     return G[v][v]<0;}
   //checks if there's a neg. cycle in path from a to b
   bool hasNegCycle(int a, int b){
     forn(i, N) if(G[a][i]!=INF && G[i][i]<0 && G[i][b]!=INF)</pre>
12
       return true;
13
     return false;
14
15 }
                                7.4. Kruskal
   const int MAXN=100000:
   vector<ii> G[MAXN];
  int n;
3
  struct Ar{int a,b,w;}; //w y cost deberian tener el mismo tipo
   bool operator (const Ar& a, const Ar &b) {return a.w <b.w;}
   vector<Ar> E:
  11 kruskal(){ //no hace falta agregar las aristas en las dos direcciones! (
       en prim si)
       11 cost=0;
9
       sort(E.begin(), E.end());//ordenar aristas de menor a mayor -- OJO
10
           cuando ordena algo no necesariamente las cosas del mismo valor
           quedan en el mismo orden!!
       uf.init(n);
11
       forall(it, E){
^{12}
           if(uf.comp(it->a)!=uf.comp(it->b)){//si no estan conectados
13
               uf.join(it->a, it->b);//conectar
14
               cost+=it->w;
15
           }
16
       }
17
       return cost;
18
19
                                  7.5. Prim
  vector<ii> G[MAXN];
  bool taken[MAXN];
```

priority queue<ii, vector<ii>, greater<ii>> pq;//min heap

void process(int v){

```
taken[v]=true;
5
       forall(e, G[v])
6
           if(!taken[e->second]) pq.push(*e);
   }
8
9
   11 prim(){
       zero(taken);
11
       process(0);
       11 cost=0;
13
       while(sz(pq)){
           ii e=pq.top(); pq.pop();
15
           if(!taken[e.second]) cost+=e.first, process(e.second);
16
       }
17
       return cost;
18
19 }
                       7.6. 2-SAT + Tarjan SCC
 1 //We have a vertex representing a var and other for his negation.
2 //Every edge stored in G represents an implication. To add an equation of
       the form a | |b, use addor(a, b)
   //MAX=max cant var, n=cant var
   #define addor(a, b) (G[neg(a)].pb(b), G[neg(b)].pb(a))
   vector<int> G[MAX*2]:
   //idx[i]=index assigned in the dfs
   //lw[i]=lowest index(closer from the root) reachable from i
   int lw[MAX*2], idx[MAX*2], qidx;
   stack<int> q;
   int qcmp, cmp[MAX*2];
   //verdad[cmp[i]]=valor de la variable i
   bool verdad[MAX*2+1];
13
   int neg(int x) { return x>=n? x-n : x+n;}
   void tin(int v){
     lw[v]=idx[v]=++qidx;
     q.push(v), cmp[v]=-2;
17
     forall(it, G[v]){
18
       if(!idx[*it] || cmp[*it]==-2){
19
         if(!idx[*it]) tjn(*it);
20
         lw[v]=min(lw[v], lw[*it]);
21
       }
22
     }
23
```

 $if(lw[v]==idx[v]){$

```
int x:
25
       do{x=q.top(); q.pop(); cmp[x]=qcmp;}while(x!=v);
26
       verdad[qcmp] = (cmp[neg(v)] < 0);</pre>
27
        qcmp++;
28
29
30
    //remember to CLEAR G!!!
31
    bool satisf(){//0(n)}
     memset(idx, 0, sizeof(idx)), qidx=0;
33
     memset(cmp, -1, sizeof(cmp)), qcmp=0;
34
     forn(i, n){
35
       if(!idx[i]) tjn(i);
36
       if(!idx[neg(i)]) tjn(neg(i));
37
38
     forn(i, n) if(cmp[i] == cmp[neg(i)]) return false;
39
     return true:
40
41 |}
```

7.7. Articulation Points

```
1 int N;
   vector<int> G[1000000];
   //V[i]=node number(if visited), L[i]= lowest V[i] reachable from i
   int qV, V[1000000], L[1000000], P[1000000];
   void dfs(int v, int f){
     L[v]=V[v]=++qV;
6
     forall(it, G[v])
7
       if(!V[*it]){
8
         dfs(*it, v);
9
         L[v] = min(L[v], L[*it]); //a todo lo que pueden llegar mis hijos yo
10
              tmb puede llegar
         P[v] += L[*it]>=V[v]; // no puede llegar a ningun vertice u / V[u] <</pre>
11
              V[v] \Rightarrow si saco v quedan desconectados \Rightarrow v punto de articulación
         //con > detecto puentes: P[v] += L[*it] > V[v] (ver pag 131-132 halim
12
13
       else if(*it!=f) //backedge
14
         L[v]=\min(L[v], V[*it]);
15
16
   int cantart(int N){ //O(n)
17
18
     zero(V), zero(P);
19
     dfs(0, -1);
20
```

```
P[0]--; //la raiz debe tener al menos dos hijos para ser punto de
21
            articulazion
     int q=0;
22
     forn(i, N) if(P[i]) q++;
  return q;
25 }
                   7.8. Comp. Biconexas y Puentes
   const int MAXN=1010;
   int n, m;
   vector<int> G[MAXN];
   struct edge {
     int u,v, comp;
     bool bridge;
7
   };
8
   vector<edge> e;
   void addEdge(int u, int v) {
     G[u].pb(sz(e)), G[v].pb(sz(e));
     e.pb((edge){u,v,-1,false});
   }
13
   //V[i]=id de la dfs
   //L[i]=lowest id reachable from i
   int V[MAXN], L[MAXN], qV;
   int nbc;//cant componentes
   int comp[MAXN];//comp[i]=cant comp biconexas a la cual pertenece i
   void initDfs(int n) {
     zero(G), zero(comp);
     e.clear();
21
     forn(i,n) V[i]=-1;
22
     nbc = qV = 0;
23
24
   stack<int> st;
   void dfs(int u, int pe) \{//0(n + m)\}
       L[u] = V[u] = qV++;
27
     comp[u] = (pe != -1);
28
       for(auto &me: G[u]) if (ne != pe){
29
       int v = e[ne].u \cdot e[ne].v \cdot u; // x \cdot v \cdot x = v!
30
       if (V[v] == -1) \{ // \text{ todavia no se lo visito} \}
31
         st.push(ne);
32
```

if (L[v] > V[u]){// bridge => no pertenece a ninguna comp biconexa

dfs(v,ne);

33

34

```
e[ne].bridge = true;
35
                                                                                        77
         }
                                                                                           int main() {
36
                                                                                             while(cin >> n >> m){}
         if (L[v] \ge V[u]) \{ // art \}
37
                                                                                        79
                                                                                               initDfs(n);
            int last;
38
                                                                                        80
            do { //todas las aristas que estan entre dos puntos de articulacion
                                                                                               forn(i, m){
39
                                                                                        81
                 pertenecen a la misma componente biconexa
                                                                                                 int a,b; cin >> a >> b;
                                                                                        82
              last = st.top(); st.pop();
                                                                                                 addEdge(a,b);
40
                                                                                        83
              e[last].comp = nbc;
                                                                                               }
41
                                                                                        84
           } while (last != ne);
                                                                                                    dfs(0,-1);
42
                                                                                        85
                                                                                                   forn(i, n) cout << "comp[" << i << "] = " << comp[i] << endl;
           nbc++;
43
                                                                                               for(auto &ne: e) cout << ne.u << "->" << ne.v << " en la comp. " << ne.
            comp[u]++;
44
                                                                                        87
                                                                                                    comp << endl;</pre>
45
                                                                                               cout << "Cant., de, componentes, biconexas, =, " << nbc << endl;
         L[u] = min(L[u], L[v]);
                                                                                        88
46
                                                                                             }
                                                                                        89
47
       else if (V[v] < V[u]) { // back edge
                                                                                               return 0;
                                                                                        90
48
         st.push(ne);
                                                                                       91 }
49
         L[u] = min(L[u], V[v]);
50
                                                                                                                     7.9. LCA + Climb
51
52
                                                                                        const int MAXN=100001;
53
                                                                                           const int LOGN=20;
54
    set<int> C[2*MAXN];
                                                                                           //f[v][k] holds the 2^k father of v
    int compnodo[MAXN];
                                                                                           //L[v] holds the level of v
56
    int ptoart;
                                                                                           int f[MAXN] [LOGN], L[MAXN];
57
    void blockcuttree(){
                                                                                           //call before build:
58
       ptoart = 0; zero(compnodo);
                                                                                           void dfs(int v, int fa=-1, int lvl=0){//generate required data
59
       forn(i,2*MAXN) C[i].clear();
                                                                                             f[v][0]=fa, L[v]=lvl;
60
       for(auto &it: e){
                                                                                             forall(it, G[v])if(*it!=fa)
61
           int u = it.u, v = it.v;
                                                                                               dfs(*it, v, lvl+1);
62
                                                                                        10
            if(comp[u] == 1) compnodo[u] = it.comp;
63
                                                                                        11
            else{
                                                                                           void build(int N){//f[i][0] must be filled previously, O(nlgn)
64
                if(compnodo[u] == 0){ compnodo[u] = nbc+ptoart; ptoart++;}
                                                                                             forn(k, LOGN-1) forn(i, N) f[i][k+1]=f[f[i][k]][k];}
65
                                                                                        13
                    C[it.comp].insert(compnodo[u]);
66
                                                                                        14
                    C[compnodo[u]].insert(it.comp);
                                                                                           #define lg(x) (31-_builtin_clz(x))//=floor(log2(x))
67
                                                                                        15
           }
68
                                                                                        16
            if(comp[v] == 1) compnodo[v] = it.comp;
                                                                                           int climb(int a, int d){\frac{1}{0}}
69
                                                                                        17
70
                                                                                             if(!d) return a;
                                                                                        18
                if(compnodo[v] == 0){ compnodo[v] = nbc+ptoart; ptoart++;}
                                                                                             dforn(i, lg(L[a])+1)
71
                                                                                       19
                    C[it.comp].insert(compnodo[v]);
                                                                                               if(1<<i<=d)
72
                                                                                       20
                    C[compnodo[v]].insert(it.comp);
                                                                                                 a=f[a][i], d-=1<<i;
73
                                                                                       21
           }
74
                                                                                               return a;
                                                                                       22
                                                                                           }
75
                                                                                       23
76 }
                                                                                       _{24} int lca(int a, int b){//0(lgn)
```

```
if(L[a]<L[b]) swap(a, b);</pre>
25
     a=climb(a, L[a]-L[b]);
26
     if(a==b) return a;
27
     dforn(i, lg(L[a])+1)
28
       if(f[a][i]!=f[b][i])
29
         a=f[a][i], b=f[b][i];
30
     return f[a][0];
31
32
   int dist(int a, int b) {//returns distance between nodes
     return L[a]+L[b]-2*L[lca(a, b)];}
```

7.10. Heavy Light Decomposition

```
int treesz[MAXN];//cantidad de nodos en el subarbol del nodo v
   int dad[MAXN];//dad[v]=padre del nodo v
   void dfs1(int v, int p=-1){//pre-dfs
     dad[v]=p;
4
     treesz[v]=1;
5
     forall(it, G[v]) if(*it!=p){
       dfs1(*it. v):
       treesz[v]+=treesz[*it];
8
9
10
    //PONER Q EN O !!!!!
   int pos[MAXN], q;//pos[v]=posicion del nodo v en el recorrido de la dfs
   //Las cadenas aparecen continuas en el recorrido!
   int cantcad:
   int homecad[MAXN];//dada una cadena devuelve su nodo inicial
   int cad[MAXN];//cad[v]=cadena a la que pertenece el nodo
   void heavylight(int v, int cur=-1){
17
     if(cur==-1) homecad[cur=cantcad++]=v;
18
     pos[v]=q++;
19
     cad[v]=cur;
20
     int mx=-1;
21
     forn(i, sz(G[v])) if(G[v][i]!=dad[v])
22
       if(mx==-1 || treesz[G[v][mx]]<treesz[G[v][i]]) mx=i;</pre>
23
     if(mx!=-1) heavylight(G[v][mx], cur);
24
     forn(i, sz(G[v])) if(i!=mx && G[v][i]!=dad[v])
25
       heavylight(G[v][i], -1);
26
27
    //ejemplo de obtener el maximo numero en el camino entre dos nodos
   //RTA: max(query(low, u), query(low, v)), con low=lca(u, v)
   //esta funcion va trepando por las cadenas
```

```
int query(int an, int v){\frac{1}{0000}}
     //si estan en la misma cadena:
     if(cad[an]==cad[v]) return rmq.get(pos[an], pos[v]+1);
     return max(query(an, dad[homecad[cad[v]]]),
            rmq.get(pos[homecad[cad[v]]], pos[v]+1));
35
36 }
                    7.11. Centroid Decomposition
1 int n;
   vector<int> G[MAXN];
   bool taken[MAXN];//poner todos en FALSE al principio!!
   int padre[MAXN];//padre de cada nodo en el centroid tree
   int szt[MAXN];
   void calcsz(int v. int p) {
     szt[v] = 1:
     forall(it,G[v]) if (*it!=p && !taken[*it])
       calcsz(*it,v), szt[v]+=szt[*it];
10
   }
11
   void centroid(int v=0, int f=-1, int lvl=0, int tam=-1) {//O(nlogn)
     if(tam==-1) calcsz(v, -1), tam=szt[v]:
13
     forall(it, G[v]) if(!taken[*it] && szt[*it]>=tam/2)
14
       {szt[v]=0; centroid(*it, f, lvl, tam); return;}
15
     taken[v]=true;
16
     padre[v]=f;
17
     /*Analizar todos los caminos que pasan por este nodo:
18
      * Agregar la informacion de cada subarbol
19
      * Para cada subarbol:
20
      * -sacar la informacion
21
      * -analizar
22
      * -agregar de nuevo la informacion
      */
24
     forall(it, G[v]) if(!taken[*it])
25
       centroid(*it, v, lvl+1, -1);
26
27 }
                            7.12. Euler Cycle
1 #define MAXN 1005
   #define MAXE 1005005
3
   int n,ars[MAXE], eq;
   vector<int> G[MAXN];//fill G,ars,eq
```

queue<int> q;

```
6 | list<int> path;
  int used[MAXN]; //used[v] = i => para todo j<=i la arista v-G[v][j] fue</pre>
       usada y la arista v-G[v][i+1] no se uso
   bool usede[MAXE];
    //encuentra el ciclo euleriano, el grafo debe ser conexo y todos los nodos
        tener grado par para que exista
    //para encontrar el camino euleriano conectar los dos vertices de grado
        impar y empezar de uno de ellos.
12
    queue<list<int>::iterator> q;
    int get(int v){
     while(used[v]<sz(G[v]) && usede[ G[v][used[v]] ]) used[v]++;</pre>
     return used[v]:
17
    void explore(int v, int r, list<int>::iterator it){
     int ar=G[v][get(v)]; int u=v^ars[ar];
19
     usede[ar]=true;
20
     list<int>::iterator it2=path.insert(it, u);
21
     if(u!=r) explore(u, r, it2);
22
     if(get(v)<sz(G[v])) q.push(it);</pre>
23
^{24}
    void euler(int a){
25
     zero(used), zero(usede);
26
     path.clear();
27
     q=queue<list<int>::iterator>();
28
     path.push_back(a); q.push(path.begin());
29
     while(sz(q)){
30
       list<int>::iterator it=q.front(); q.pop();
31
       if(used[*it]<sz(G[*it])) explore(*it, *it, it);</pre>
32
33
     reverse(path.begin(), path.end());
34
35
    void addEdge(int u, int v){
     G[u].pb(eq), G[v].pb(eq);
37
     ars[eq++]=u^v;
38
39 }
                           7.13. Diametro árbol
   vector<int> G[MAXN]; int n,m,p[MAXN],d[MAXN],d2[MAXN];
  int bfs(int r, int *d) {
```

```
d[r]=0; q.push(r);
     int v;
5
     while(sz(q)) { v=q.front(); q.pop();
6
       forall(it,G[v]) if (d[*it]==-1)
         d[*it]=d[v]+1, p[*it]=v, q.push(*it);
8
9
     return v://ultimo nodo visitado
10
11
   vector<int> diams; vector<ii> centros;
   void diametros(){
     memset(d,-1,sizeof(d));
14
     memset(d2,-1,sizeof(d2));
15
     diams.clear(), centros.clear();
16
     forn(i, n) if(d[i]==-1){
17
       int v,c;
18
       c=v=bfs(bfs(i, d2), d);
19
       forn(,d[v]/2) c=p[c];
20
       diams.pb(d[v]);
       if(d[v]&1) centros.pb(ii(c, p[c]));
       else centros.pb(ii(c, c));
    }
24
25 }
```

7.14. Chu-liu

```
void visit(graph &h, int v, int s, int r,
     vector<int> &no, vector< vector<int> > &comp,
     vector<int> &prev, vector< vector<int> > &mext, vector<weight> &mcost,
     vector<int> &mark, weight &cost, bool &found) {
     if (mark[v]) {
5
       vector<int> temp = no;
6
       found = true;
7
       do {
8
         cost += mcost[v]:
9
         v = prev[v];
         if (v != s) {
11
           while (comp[v].size() > 0) {
12
              no[comp[v].back()] = s:
13
              comp[s].push back(comp[v].back());
14
              comp[v].pop back();
15
16
         }
17
       } while (v != s);
18
```

```
forall(j,comp[s]) if (*j != r) forall(e,h[*j])
19
         if (no[e->src] != s) e->w -= mcost[ temp[*j] ];
20
     }
21
     mark[v] = true;
^{22}
     forall(i,next[v]) if (no[*i] != no[v] && prev[no[*i]] == v)
23
       if (!mark[no[*i]] || *i == s)
24
         visit(h, *i, s, r, no, comp, prev, next, mcost, mark, cost, found);
25
26
   weight minimumSpanningArborescence(const graph &g, int r) {
27
       const int n=sz(g);
28
     graph h(n);
29
                                                                                       11
     forn(u,n) forall(e,g[u]) h[e->dst].pb(*e);
30
                                                                                       12
     vector<int> no(n):
                                                                                          }
                                                                                       13
     vector<vector<int> > comp(n);
32
     forn(u, n) comp[u].pb(no[u] = u);
                                                                                       15
33
     for (weight cost = 0; ;) {
34
       vector<int> prev(n, -1);
35
       vector<weight> mcost(n, INF);
36
       forn(j,n) if (j != r) forall(e,h[j])
                                                                                       19
37
         if (no[e->src] != no[i])
38
           if (e->w < mcost[ no[i] ])</pre>
                                                                                       21
39
             mcost[no[j]] = e->w, prev[no[j]] = no[e->src];
40
                                                                                       22
       vector< vector<int> > next(n);
                                                                                       23
41
       forn(u,n) if (prev[u] >= 0)
42
                                                                                       24
         next[ prev[u] ].push_back(u);
                                                                                       25
43
       bool stop = true;
                                                                                       26
44
       vector<int> mark(n);
                                                                                       27
45
       forn(u,n) if (u != r && !mark[u] && !comp[u].empty()) {
                                                                                       28
46
         bool found = false:
                                                                                       29
47
         visit(h, u, u, r, no, comp, prev, next, mcost, mark, cost, found);
                                                                                       30
48
         if (found) stop = false;
                                                                                       31
49
       }
                                                                                       32
50
       if (stop) {
51
         forn(u,n) if (prev[u] >= 0) cost += mcost[u];
                                                                                       33
52
         return cost:
                                                                                       34
53
       }
                                                                                       35
54
     }
55
                                                                                       36
56 }
                              7.15. Hungarian
```

```
#define tipo double const tipo EPS = 1e-9;
```

```
3 const tipo INF = 1e14;
   #define N 502
5 //Dado un grafo bipartito completo con costos no negativos, encuentra el
       matching perfecto de minimo costo.
6 | tipo cost[N][N], lx[N], ly[N], slack[N]; //llenar: cost=matriz de
       advacencia
7 | int n, max_match, xy[N], yx[N], slackx[N], prev2[N]; //n=cantidad de nodos
   bool S[N], T[N]; //sets S and T in algorithm
   void add_to_tree(int x, int prevx) {
     S[x] = true, prev2[x] = prevx;
     forn(y, n) if (lx[x] + ly[y] - cost[x][y] < slack[y] - EPS)
       slack[y] = lx[x] + ly[y] - cost[x][y], slackx[y] = x;
   void update labels(){
     tipo delta = INF;
     forn (y, n) if (!T[y]) delta = min(delta, slack[y]);
     forn (x, n) if (S[x]) lx[x] -= delta;
     form (y, n) if (T[y]) ly[y] += delta; else slack[y] -= delta;
   void init labels(){
     zero(lx), zero(ly);
     form (x,n) form(y,n) lx[x] = max(lx[x], cost[x][y]);
   void augment() {
     if (max match == n) return;
     int x, y, root, q[N], wr = 0, rd = 0;
     memset(S, false, sizeof(S)), memset(T, false, sizeof(T));
     memset(prev2, -1, sizeof(prev2));
     forn (x, n) if (xy[x] == -1){
       q[wr++] = root = x, prev2[x] = -2;
       S[x] = true; break; }
     forn (y, n) slack[y] = lx[root] + ly[y] - cost[root][y], slackx[y] = root
     while (true){
       while (rd < wr){
         x = q[rd++];
         for (y = 0; y < n; y++) if (cost[x][y] == lx[x] + ly[y] && !T[y]){
           if (yx[y] == -1) break; T[y] = true;
           q[wr++] = yx[y], add_to_tree(yx[y], x); }
         if (y < n) break; }</pre>
       if (y < n) break;
40
       update_labels(), wr = rd = 0;
41
       for (y = 0; y < n; y++) if (!T[y] \&\& slack[y] == 0){
42
```

23 | struct DynCon {

```
if (yx[y] == -1)\{x = slackx[y]; break;\}
                                                                                              vector<Query> q;
43
                                                                                      24
         else{
                                                                                             UnionFind dsu;
44
                                                                                      25
           T[v] = true;
                                                                                              vector<int> match,res;
45
                                                                                      26
           if (!S[yx[y]]) q[wr++] = yx[y], add_to_tree(yx[y], slackx[y]);
                                                                                              map<ii,int> last;//se puede no usar cuando hay identificador para cada
                                                                                      27
46
         }}
                                                                                                  arista (mejora poco)
47
                                                                                             DynCon(int n=0):dsu(n){}
       if (v < n) break; }
48
                                                                                      28
                                                                                              void add(int u, int v) {
     if (y < n){
49
                                                                                      29
                                                                                                  if(u>v) swap(u,v);
       max match++;
50
                                                                                      30
       for (int cx = x, cy = y, ty; cx != -2; cx = prev2[cx], cy = ty)
                                                                                                  q.pb((Query){ADD, u, v}), match.pb(-1);
51
                                                                                      31
         ty = xy[cx], yx[cy] = cx, xy[cx] = cy;
                                                                                                 last[ii(u,v)] = sz(q)-1;
52
                                                                                      32
       augment(); }
                                                                                      33
53
                                                                                              void remove(int u, int v) {
54
                                                                                      34
   tipo hungarian(){
                                                                                                  if(u>v) swap(u,v);
                                                                                      35
     tipo ret = 0; max match = 0, memset(xy, -1, sizeof(xy));
                                                                                                  q.pb((Query){DEL, u, v});
                                                                                      36
     memset(yx, -1, sizeof(yx)), init labels(), augment(); //steps 1-3
                                                                                                  int prev = last[ii(u,v)];
                                                                                      37
     forn (x,n) ret += cost[x][xy[x]]; return ret;
                                                                                                 match[prev] = sz(q)-1;
                                                                                      38
59 }
                                                                                                 match.pb(prev);
                                                                                      39
                                                                                             }
                                                                                      40
                      7.16. Dynamic Connectivity
                                                                                              void query() {//podria pasarle un puntero donde guardar la respuesta
                                                                                      41
                                                                                                  q.pb((Query){QUERY, -1, -1}), match.pb(-1);}
                                                                                              void process() {
struct UnionFind {
                                                                                      43
                                                                                                 form(i,sz(q)) if (q[i].type == ADD \&\& match[i] == -1) match[i] = sz
       int n, comp;
                                                                                      44
2
                                                                                                      (q);
       vector<int> pre,si,c;
3
                                                                                                  go(0,sz(q));
       UnionFind(int n=0):n(n), comp(n), pre(n), si(n, 1) {
                                                                                      45
4
                                                                                             }
           forn(i,n) pre[i] = i; }
                                                                                      46
5
                                                                                             void go(int 1, int r) {
       int find(int u){return u==pre[u]?u:find(pre[u]);}
                                                                                      47
6
                                                                                                 if(l+1==r){
       bool merge(int u, int v) {
                                                                                      48
7
                                                                                                      if (q[1].type == QUERY)//Aqui responder la query usando el dsu!
           if((u=find(u))==(v=find(v))) return false;
                                                                                      49
8
                                                                                                          res.pb(dsu.comp);//aqui query=cantidad de componentes
           if(si[u]<si[v]) swap(u, v);</pre>
                                                                                      50
9
           si[u]+=si[v], pre[v]=u, comp--, c.pb(v);
                                                                                                              conexas
10
                                                                                                      return;
           return true;
                                                                                      51
11
                                                                                      52
12
                                                                                                  int s=dsu.snap(), m = (l+r) / 2;
       int snap(){return sz(c);}
                                                                                      53
13
                                                                                                 forr(i,m,r) if(match[i]!=-1 && match[i]<1) dsu.merge(q[i].u, q[i].v</pre>
       void rollback(int snap){
                                                                                      54
14
                                                                                                      );
           while(sz(c)>snap){
15
                                                                                                  go(1,m);
               int v = c.back(); c.pop_back();
                                                                                      55
16
                                                                                                  dsu.rollback(s);
               si[pre[v]] = si[v], pre[v] = v, comp++;
17
                                                                                                  s = dsu.snap();
           }
                                                                                      57
18
                                                                                                 forr(i,1,m) if(match[i]!=-1 && match[i]>=r) dsu.merge(q[i].u, q[i].
       }
                                                                                      58
19
                                                                                                      v);
20
                                                                                                  go(m,r);
   enum {ADD,DEL,QUERY};
                                                                                      59
                                                                                                  dsu.rollback(s);
   struct Query {int type,u,v;};
                                                                                      60
                                                                                             }
```

61

62 }dc;

7.17. DFS Paralelo

```
S[1][szS[1]++] = b;//.push(b);
1
       it[b] = G[b].begin();
2
     }
3
     int act = 0;
     vis[a] = vis[b] = true;
6
     while(szS[act]){ //recorre las dos componentes en paralelo
7
       int v = S[act][szS[act]-1];//.top();
8
       int u = *it[v];
9
       it[v]++;
10
       if(it[v] == G[v].end()) szS[act]--;//.pop();
11
       if(vis[u]){act = 1 - act; continue;}
12
       szC[act]++;
13
       if(sz(G[u])>1 \text{ or } *G[u].begin() != v){
14
         S[act][szS[act]++] = u;//.push(u);
15
         vis[u] = true;
16
         it[u] = G[u].begin();
17
       }
18
       act = 1 - act;
19
20
     act = 1 - act; //ya recorrio la toda la componente de act
21
22
     //sigue recorriendo la otra componente hasta que ve un elemento más o no
23
          tiene más elementos.
     while(szC[act] < szC[1-act]+1 and szS[act]){</pre>
24
       int v = S[act] [szS[act]-1];//.top();
25
       int u = *it[v];
26
       it[v]++;
27
       if(it[v] == G[v].end()) szS[act]--;//.pop();
28
       if(vis[u]) continue;
29
       szC[act]++:
30
       if(sz(G[u])>1 \text{ or } *G[u].begin() != v){
31
         S[act][szS[act]++] = u;//.push(u);
32
         vis[u] = true;
33
         it[u] = G[u].begin();
34
  35
```

8. Network Flow

8.1. Dinic

```
1
2 const int MAX = 300;
3 // Corte minimo: vertices con dist[v]>=0 (del lado de src) VS. dist[v]==-1
         (del lado del dst)
4 // Para el caso de la red de Bipartite Matching (Sean V1 y V2 los conjuntos
        mas proximos a src y dst respectivamente):
5 // Reconstruir matching: para todo v1 en V1 ver las aristas a vertices de
       V2 con it->f>0, es arista del Matching
6 // Min Vertex Cover: vertices de V1 con dist[v] == -1 + vertices de V2 con
       dist[v]>0
7 // Max Independent Set: tomar los vertices NO tomados por el Min Vertex
8 // Max Clique: construir la red de G complemento (debe ser bipartito!) y
       encontrar un Max Independet Set
9 // Min Edge Cover: tomar las aristas del matching + para todo vertices no
       cubierto hasta el momento, tomar cualquier arista de el
10 //Complejidad:
   //Peor caso: O(V^2E)
   //Si todas las capacidades son 1: O(min(E^1/2,V^2/3)E)
   //Para matching bipartito es: O(sqrt(V)E)
   int nodes, src, dst;
   int dist[MAX], q[MAX], work[MAX];
   struct Edge {
       int to, rev;
18
       ll f, cap;
19
       Edge(int to, int rev, ll f, ll cap) : to(to), rev(rev), f(f), cap(cap)
20
           {}
   };
21
   vector<Edge> G[MAX];
   void addEdge(int s, int t, ll cap){
       G[s].pb(Edge(t, sz(G[t]), 0, cap)), G[t].pb(Edge(s, sz(G[s])-1, 0, 0))
24
25 bool dinic bfs(){
       fill(dist, dist+nodes, -1), dist[src]=0;
       int qt=0; q[qt++]=src;
       for(int qh=0; qh<qt; qh++){</pre>
28
           int u =q[qh];
29
           forall(e, G[u]){
30
```

```
7 | void koning() {//O(n)
                int v=e->to:
31
               if(dist[v]<0 && e->f < e->cap)
32
                    dist[v]=dist[u]+1, q[qt++]=v;
33
           }
34
       }
                                                                                       11
35
       return dist[dst]>=0;
36
                                                                                       12
37
                                                                                       13
   11 dinic_dfs(int u, 11 f){
38
                                                                                       14
       if(u==dst) return f;
39
                                                                                       15
       for(int &i=work[u]; i<sz(G[u]); i++){</pre>
40
           Edge &e = G[u][i];
                                                                                               } else {
41
                                                                                       17
           if(e.cap<=e.f) continue;</pre>
42
                                                                                       18
           int v=e.to:
43
                                                                                       19
           if(dist[v]==dist[u]+1){
                                                                                       20
                    11 df=dinic dfs(v, min(f, e.cap-e.f));
                                                                                       21
45
                    if(df>0){
                                                                                       22
46
                            e.f+=df, G[v][e.rev].f-= df;
                                                                                               }
47
                                                                                       23
                            return df; }
                                                                                             }
                                                                                        24
48
                                                                                       25 }
           }
49
50
       return 0;
51
52
   ll maxFlow(int _src, int _dst){
                                                                                        1 #define MAX V 1000
53
       src=_src, dst=_dst;
                                                                                          #define INF 1e9
54
       11 result=0;
                                                                                           //special nodes
55
       while(dinic_bfs()){
                                                                                           #define SRC 0
56
           fill(work, work+nodes, 0);
                                                                                           #define SNK 1
57
           while(ll delta=dinic_dfs(src,INF))
58
               result+=delta;
59
60
       // todos los nodos con dist[v]!=-1 vs los que tienen dist[v]==-1 forman
61
                                                                                           int f, p[MAX_V];
             el min-cut
       return result: }
62
                                                                                       11
                                                                                       12
                                  8.2. Konig
                                                                                       13
                                                                                       14
                                                                                             }
  // asume que el dinic YA ESTA tirado
                                                                                        15
   // asume que nodes-1 y nodes-2 son la fuente y destino
                                                                                           }
                                                                                        16
  int match[maxnodes]; // match[v]=u si u-v esta en el matching, -1 si v no
       esta matcheado
                                                                                             11 Mf=0;
                                                                                       18
  int s[maxnodes]; // numero de la bfs del koning
                                                                                             dof
                                                                                       19
   queue<int> kq;
                                                                                       20
6 // s[e] %2==1 o si e esta en V1 y s[e] ==-1-> lo agarras
```

```
r void koning() {//U(n)
    forn(v,nodes-2) s[v] = match[v] = -1;
    forn(v,nodes-2) forall(it,g[v]) if (it->to < nodes-2 && it->f>0)
    { match[v]=it->to; match[it->to]=v;}
    forn(v,nodes-2) if (match[v]==-1) {s[v]=0;kq.push(v);}

while(!kq.empty()) {
    int e = kq.front(); kq.pop();
    if (s[e] %2=1) {
        s[match[e]] = s[e]+1;
        kq.push(match[e]);
    } else {

        forall(it,g[e]) if (it->to < nodes-2 && s[it->to]==-1) {
            s[it->to] = s[e]+1;
            kq.push(it->to);
        }
     }
}
}
}
```

8.3. Edmonds Karp's

```
#define MAX_V 1000
#define INF 1e9

//special nodes
#define SRC 0

#define SNK 1

map<int, int> G[MAX_V];//limpiar esto

//To add an edge use
#define add(a, b, w) G[a][b]=w
int f, p[MAX_V];

void augment(int v, int minE){
    if(v==SRC) f=minE;
    else if(p[v]!=-1){
        augment(p[v], min(minE, G[p[v]][v]));
        G[p[v]][v]-=f, G[v][p[v]]+=f;
    }
}

11 maxflow(){//O(VE^2)
    ll Mf=0;
    do{
        f=0;
        char used[MAX_V]; queue<int> q; q.push(SRC);
```

27

```
zero(used), memset(p, -1, sizeof(p));
                                                                                               enqueue(v);
22
                                                                                       28
       while(sz(q)){
                                                                                            }
                                                                                       29
23
         int u=q.front(); q.pop();
                                                                                       30
^{24}
         if(u==SNK) break;
                                                                                           void relabel(int v) {
25
                                                                                       31
         forall(it, G[u])
                                                                                             count[height[v]]--;
26
                                                                                       32
           if(it->snd>0 && !used[it->fst])
                                                                                            height[v] = 2*N;
27
                                                                                       33
              used[it->fst]=true, q.push(it->fst), p[it->fst]=u;
                                                                                            forall(it, G[v])
28
                                                                                       34
       }
                                                                                              if(it->snd)
29
                                                                                       35
       augment(SNK, INF);
                                                                                                 height[v] = min(height[v], height[it->fst] + 1);
30
                                                                                       36
                                                                                             count[height[v]]++;
       Mf+=f;
31
                                                                                       37
     }while(f);
                                                                                             enqueue(v);
32
                                                                                       38
     return Mf;
33
                                                                                       39
                                                                                          11 \max flow() {\frac{}{\sqrt{0(V^3)}}}
34 | }
                                                                                       40
                                                                                             zero(height), zero(active), zero(count), zero(excess);
                        8.4. Push-Relabel O(N3)
                                                                                             count[0] = N-1:
                                                                                             count[N] = 1;
                                                                                            height[SRC] = N;
   #define MAX V 1000
                                                                                       44
                                                                                            active[SRC] = active[SNK] = true;
   int N://valid nodes are [0...N-1]
   #define INF 1e9
                                                                                            forall(it, G[SRC]){
                                                                                       46
                                                                                              excess[SRC] += it->snd;
   //special nodes
                                                                                               push(SRC, it->fst);
   #define SRC 0
                                                                                       48
                                                                                            }
   #define SNK 1
                                                                                       49
                                                                                            while(sz(Q)) {
                                                                                       50
   map<int, int> G[MAX V];
                                                                                              int v = Q.front(); Q.pop();
   //To add an edge use
                                                                                       51
                                                                                               active[v]=false;
   #define add(a, b, w) G[a][b]=w
                                                                                       52
                                                                                            forall(it, G[v]) push(v, it->fst);
   11 excess[MAX V]:
                                                                                       53
                                                                                             if(excess[v] > 0)
   int height[MAX V], active[MAX V], count[2*MAX V+1];
                                                                                       54
                                                                                               count[height[v]] == 1? gap(height[v]):relabel(v);
   queue<int> Q;
                                                                                       55
12
                                                                                            }
   void enqueue(int v) {
                                                                                       56
                                                                                            ll mf=0:
     if (!active[v] && excess[v] > 0) active[v]=true, Q.push(v); }
                                                                                       57
14
                                                                                            forall(it, G[SRC]) mf+=G[it->fst][SRC];
                                                                                       58
    void push(int a, int b) {
15
                                                                                            return mf:
                                                                                       59
     int amt = min(excess[a], 11(G[a][b]));
16
                                                                                       60 }
     if(height[a] <= height[b] || amt == 0) return;</pre>
17
     G[a][b]-=amt, G[b][a]+=amt;
18
                                                                                                                 8.5. Min-cost Max-flow
     excess[b] += amt, excess[a] -= amt;
19
     enqueue(b);
20
                                                                                        const int MAXN=10000;
^{21}
   void gap(int k) {
                                                                                        2 typedef ll tf;
22
     forn(v, N){
                                                                                          typedef ll tc;
23
       if (height[v] < k) continue;</pre>
                                                                                          const tf INFFLUJO = 1e14;
24
       count[height[v]]--;
                                                                                          const tc INFCOSTO = 1e14;
25
       height[v] = max(height[v], N+1);
                                                                                        6 struct edge {
26
       count[height[v]]++;
                                                                                            int u, v;
```

51 }

```
tf cap, flow;
8
     tc cost;
9
     tf rem() { return cap - flow; }
10
11
   int nodes; //numero de nodos
   vector<int> G[MAXN]; // limpiar!
    vector<edge> e; // limpiar!
    void addEdge(int u, int v, tf cap, tc cost) {
     G[u].pb(sz(e)); e.pb((edge){u,v,cap,0,cost});
16
     G[v].pb(sz(e)); e.pb((edge)\{v,u,0,0,-cost\});
17
18
   tc dist[MAXN], mnCost;
   int pre[MAXN];
   tf cap[MAXN], mxFlow;
   bool in queue[MAXN];
    void flow(int s, int t) {
     zero(in queue);
24
     mxFlow=mnCost=0;
25
     while(1){
26
       fill(dist, dist+nodes, INFCOSTO); dist[s] = 0;
27
       memset(pre, -1, sizeof(pre)); pre[s]=0;
28
       zero(cap); cap[s] = INFFLUJO;
29
       queue<int> q; q.push(s); in_queue[s]=1;
30
       while(sz(q)){
31
         int u=q.front(); q.pop(); in_queue[u]=0;
32
         for(auto it:G[u]) {
33
           edge &E = e[it];
34
           if(E.rem() && dist[E.v] > dist[u] + E.cost + 1e-9){ // ojo EPS
35
              dist[E.v] = dist[u] + E.cost;
36
             pre[E.v] = it;
37
             cap[E.v] = min(cap[u], E.rem());
38
             if(!in queue[E.v]) q.push(E.v), in queue[E.v]=1;
39
40
         }
41
42
       if (pre[t] == -1) break;
43
       mxFlow +=cap[t];
44
       mnCost +=cap[t]*dist[t];
45
       for (int v = t; v != s; v = e[pre[v]].u) {
46
         e[pre[v]].flow += cap[t];
47
         e[pre[v]^1].flow -= cap[t];
48
49
50
```

```
9. Template
```

```
#include <bits/stdc++.h>
using namespace std;
   #define forr(i,a,b) for(int i=(a); i<(b); i++)</pre>
   #define forn(i,n) forr(i,0,n)
   #define zero(v) memset(v, 0, sizeof(v))
   #define forall(it,v) for(auto it=v.begin();it!=v.end();++it)
   #define pb push back
   #define fst first
   #define snd second
   typedef long long 11;
   typedef pair<ll, ll> pll;
   #define dforn(i,n) for(int i=n-1; i>=0; i--)
   int main() {
       ios::sync_with_stdio(0); cin.tie(0);
15
       return 0;
16
17 }
```

10. Ayudamemoria

Cant. decimales

```
#include <iomanip>
cout << setprecision(2) << fixed;</pre>
```

Rellenar con espacios(para justificar)

```
#include <iomanip>
cout << setfill('u') << setw(3) << 2 << endl;</pre>
```

Leer hasta fin de linea

```
#include <sstream>
//hacer cin.ignore() antes de getline()
while(getline(cin, line)){
    istringstream is(line);
    while(is >> X)
        cout << X << """;
    cout << endl;
}</pre>
```

Aleatorios

```
1 #define RAND(a, b) (rand() %(b-a+1)+a)
srand(time(NULL));
                             Doubles Comp.
 const double EPS = 1e-9;
 #define feq(a, b) (fabs((a)-(b)) < EPS)
 x == y \iff fabs(x-y) \iff EPS
_4 | x > y <=> x > y + EPS
5 | x >= y <=> x > y - EPS
                                  Limites
  #include <limits>
  numeric limits<T>
    ::max()
    ::min()
    ::epsilon()
                                Muahaha
  #include <signal.h>
  void divzero(int p){
    while(true);}
  void segm(int p){
    exit(0);}
  //in main
  signal(SIGFPE, divzero);
8 | signal(SIGSEGV, segm);
                           Mejorar velocidad
ios::sync_with_stdio(false);
cin.tie(NULL); // OJO! no mezclar scanf con este tip
                          Mejorar velocidad 2
  //Solo para enteros positivos
  inline void Scanf(int& a){
    char c = 0;
    while(c<33) c = getc(stdin);</pre>
    while(c>33) a = a*10 + c - '0', c = getc(stdin);
```

7 |}

Expandir pila