FREQUENCY MODULATION
SIGNAL GENERATION

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We have message signal:

$$V_m(t) = V_m \cos(2\pi f_m t) \tag{1}$$

Which V_m is amplitude of message signal and f_m is frequency of message signal. And then, we have carrier signal:

$$V_c(t) = V_c \cos(2\pi f_c t) \tag{2}$$

Which V_c is amplitude of carrier signal and f_c is frequency of carrier signal. Then, the FM modulated signal is:

$$V_{FM}(t) = V_c\ cos(2\pi f_c t + \beta \sin(2\pi f_m t)) \tag{3}$$

Which β is modulation index:

$$\beta = \frac{\Delta_{fc}}{f_m} \tag{4}$$

Δ_{fc} is frequency deviation:

$$\Delta_{fc} = k_f \,.\, V_m \tag{5}$$

And k_f is frequency sensitivity constant.

For example purpose, let's assume we have $V_m = 2\ volt$, $f_m = 10\ Hz$, $V_c = 2\ volt$, $f_c = 100\ Hz$, and $k_f = 10\ Hz/volt$. And then, we got:

Message signal:

$$V_m(t) = 2\cos(2\pi 10 t)$$

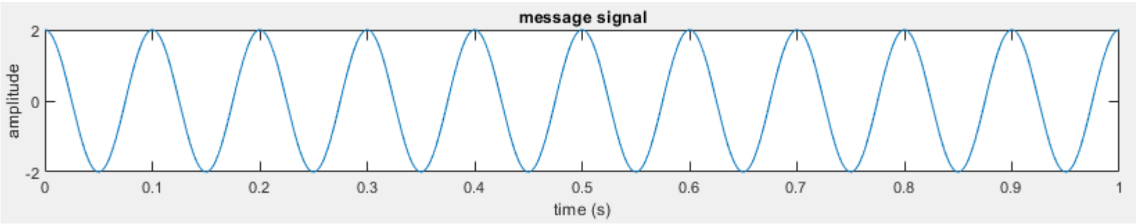

Figure 1. Message signal

Carrier signal:

$$V_c(t) = 2\cos(2\pi 100t)$$

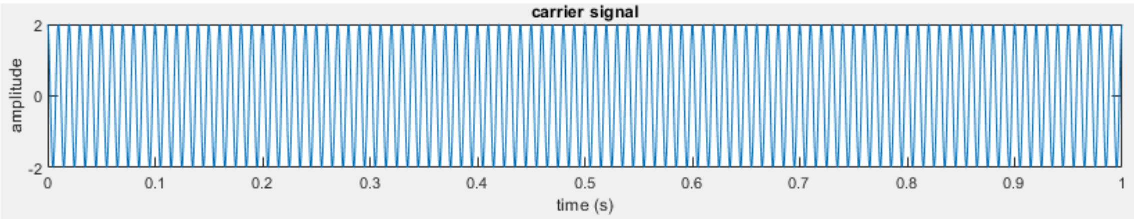

Figure 2. Carrier signal

Frequency deviation:

$$\Delta_{fc} = 10\frac{Hz}{volt}\ .\ 2\ volt = 20\ Hz$$

Bandwidth (using Carson rule):

$$B \cong 2(\Delta_{fc} + f_m) \tag{6}$$

$$B \cong 2(20\ Hz + 10\ Hz) \cong 60\ Hz$$

Modulation index:

$$\beta = \frac{20\ Hz}{10\ Hz} = 2$$

FM modulated signal:

$$V_{FM}(t) = 2\ cos(2\pi 100t + 2\sin(2\pi 10t))$$

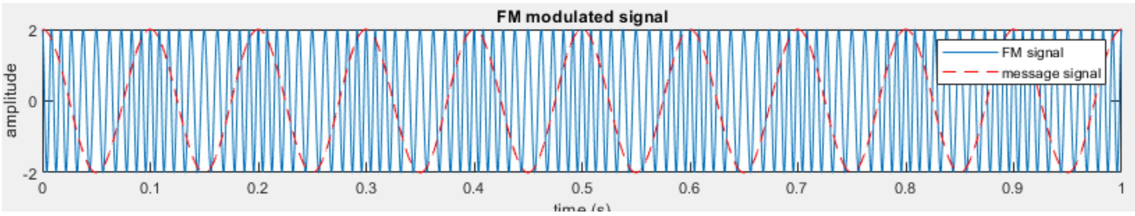

Figure 3. FM Modulated signal

If we represent it in Bessel function, we have:

$$V_{FM}(t) = V_c \sum_{n=-\infty}^{\infty} J_n(\beta)\cos((\omega_c + n.\omega_m)\,t) \tag{7}$$

$$V_{FM}(t) = 2 \sum_{n=-\infty}^{\infty} J_n(2)\cos(\,2\pi(100 + 10n)\,t)$$

By looking at the Bessel function table for $\beta = 2$,

Table 1. Bessel functions

Modulation index	Sideband amplitude																
	Carrier	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
0.00	1.00																
0.25	0.98	0.12															
0.5	0.94	0.24	0.03														
1.0	0.77	0.44	0.11	0.02													
1.5	0.51	0.56	0.23	0.06	0.01												
2.0	0.22	0.58	0.35	0.13	0.03												
2.41	0.00	0.52	0.43	0.20	0.06	0.02											
2.5	−0.05	0.50	0.45	0.22	0.07	0.02	0.01										
3.0	−0.26	0.34	0.49	0.31	0.13	0.04	0.01										
4.0	−0.40	−0.07	0.36	0.43	0.28	0.13	0.05	0.02									
5.0	−0.18	−0.33	0.05	0.36	0.39	0.26	0.13	0.05	0.02								
5.53	0.00	−0.34	−0.13	0.25	0.40	0.32	0.19	0.09	0.03	0.01							
6.0	0.15	−0.28	−0.24	0.11	0.36	0.36	0.25	0.13	0.06	0.02							
7.0	0.30	0.00	−0.30	−0.17	0.16	0.35	0.34	0.23	0.13	0.06	0.02						
8.0	0.17	0.23	−0.11	−0.29	−0.10	0.19	0.34	0.32	0.22	0.13	0.06	0.03					
8.65	0.00	0.27	0.06	−0.24	−0.23	0.03	0.26	0.34	0.28	0.18	0.10	0.05	0.02				
9.0	−0.09	0.25	0.14	−0.18	−0.27	−0.06	0.20	0.33	0.31	0.21	0.12	0.06	0.03	0.01			
10.0	−0.25	0.04	0.25	0.06	−0.22	−0.23	−0.01	0.22	0.32	0.29	0.21	0.12	0.06	0.03	0.01		
12.0	0.05	−0.22	−0.08	0.20	0.18	−0.07	−0.24	−0.17	0.05	0.23	0.30	0.27	0.20	0.12	0.07	0.03	0.01

We can represent the spectrum of FM modulated signal with normalized amplitude as below:

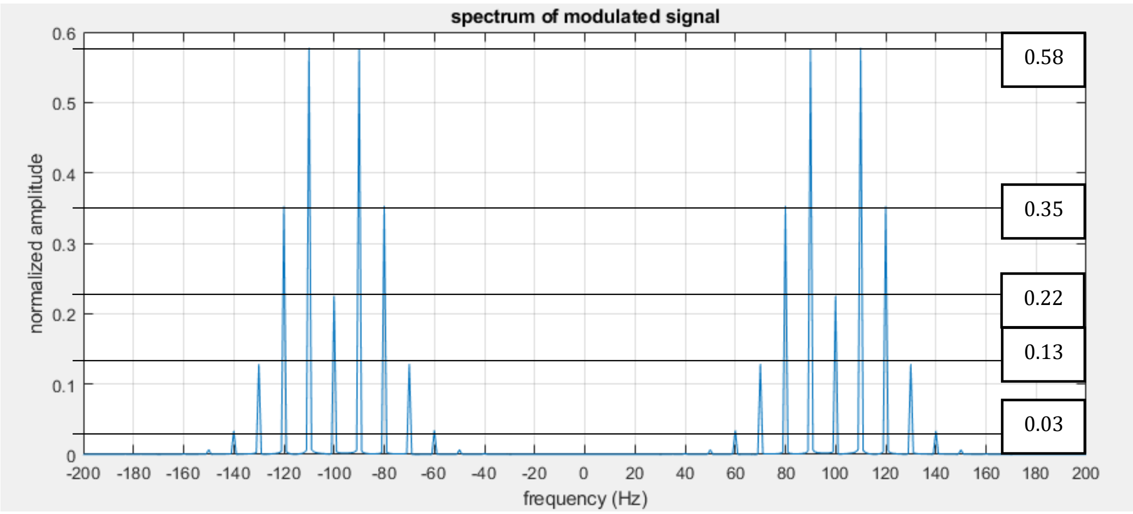

Figure 4. Spectrum of FM modulated signal

And we can see the bandwidth approximation using Carson rule represented in the graph below:

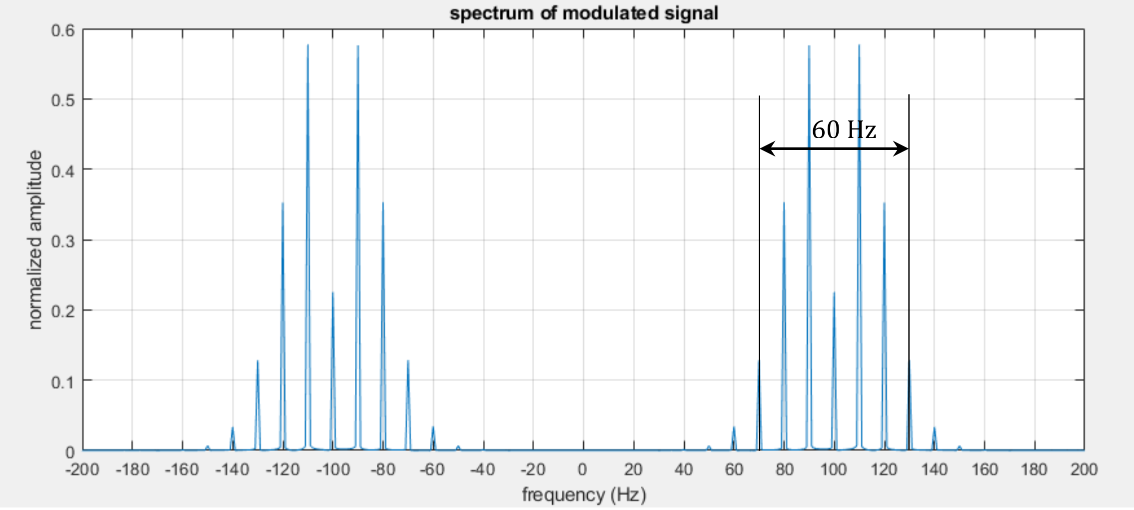

Figure 5. Bandwidth approximation using Carson rule

For MATLAB code, you can access here:

https://github.com/elyaserben/frequency-modulation