Probability and Statistics – Homework 2

Instructions: For each problem, remember you must briefly explain/justify how you obtained your answer, as correct answers without an explanation will receive **no credit**. Moreover, in the event of an incorrect answer, we can still try to give you partial credit based on the explanation you provide. It is fine for your answers to include summations, products, factorials, exponentials, or combinations; you don't need to calculate those all out to get a single numeric answer.

Submission: You must upload your working in PDF format to Canvas MATH2010

Question 1 (4 points):

You're playing minigolf for the very first time. You play a total of 18 holes. Your score on each hole is the number of strokes you need to get the ball in the hole. Since you're a novice, the success of each of your strokes is independent. For the first 9 holes, each stroke has 20% chance of getting the ball in the hole. For the last 9 holes, you improve slightly and each stroke has 25% chance of success.

a) What is the probability that you take more than 4 strokes to complete the first hole?
b) A hole-in-one happens when your score for a hole is 1; that is, you get the ball in the hole on your first attempt. What is the probability that you get exactly one hole-in-one as you play all 18 holes?
c) Given that your cumulative score after the first 9 holes is 50 strokes, what is the probability that your total score after all 18 holes will be exactly 80 strokes?

Question 2 (4 points):

Buses arrive at a specified stop at 15-minute intervals staring at 7 a.m. That is, they arrive at 7 a.m., 7:15 a.m., 7:30 a.m. and so on. If a passenger arrives at the top at a time that is uniformly distributed between 7 a.m. and 7:30 a.m., find the probability that he/she waits

a) … less than 5 minutes for a bus.
b) … more than 10 minutes for a bus.

Question 3 (4 points):

1. Let $X \sim \mathcal{N}(-1, 4)$. What is $P(|X| < 3)$.
2. Let $Y = |Z|$ with $Z \sim \mathcal{N}(0, 1)$. The distribution of Y is called a Folded Normal with parameters $\mu = 0$ and $\sigma^2 = 1$
   a. Find $E[Y]$.
   b. Find $Var(Y)$.
   c. Find the CDF and PDF of Y.

Question 4 (4 points):

a) An instructor has 50 papers that will be graded in sequence. The time required to grade the 50 papers are independent, with a common distribution that has mean 20 minutes and standard deviation 4 minutes. Approximate the probability that the instructor will grade at least 25 of the papers in the first 450 minutes of work using central limit theorem.

b) There are 300 students registered in the MATH2010 class. On the day of homework is due, each student comes to the instructor's office hour with probability $\frac{1}{6}$ independently of the other students. What is the approximate probability that between 37 and 41 people come to MATH2010 office hours?

Question 5 (4 points):
A miner is trapped in a mine containing 3 doors. The first door leads to a tunnel that will take him to safety 3 hours of travel. The second door leads to a tunnel that will return him to the mine after 5 hours of travel. The third door leads to a tunnel that will return him to the mine after 7 hours. If we assume that the miner is at all times equally likely to choose any one of the doors, what is the expected length of time until he reaches safety?