Mock Mid-term Exam

In this mock exam, there are:

- 10 multiple choice questions
- 5 written questions. You need to submit your working in the format of a file (pdf, docx, png, jpg files only).

You are given 75 minutes to work on the questions.

Note: For each written question, remember you must briefly explain/justify how you obtained your answer, as correct answers without an explanation will receive **no credit**. Moreover, in the event of an incorrect answer, we can still try to give you partial credit based on the explanation you provide. It is fine for your answers to include summations, products, factorials, exponentials, or combinations; you don't need to calculate those all out to get a single numeric answer.

1 Multiple Choice 0.5 points

Sam and Joan are playing a tennis match. If the probability of Sam's win is 0.59, then find the probability of Joan's win

- ○ 0.47
- ○ 0.36
- ◉ 0.41
- ○ 0.25

2 Multiple Choice 0.5 points

Let A and B be events on the same sample space, with $P(A) = 0.6$ and $P(B) = 0.7$. Can these two events be mutually exclusive?

- ○ Yes
- ◉ No
- ○ None of the above
- ○ All of the above

3 Multiple Choice 0.5 points

A family has two children. Find the probability that both the children are girls given that at least one of them is a girl?

- ○ $\frac{1}{4}$
- ○ $\frac{2}{3}$
- ◉ $\frac{1}{3}$
- ○ $\frac{1}{2}$

4 Multiple Choice 0.5 points

Tickets numbered 1 to 20 are mixed up and then a ticket is drawn at random. What is the probability that the ticket drawn has a number which is a multiple of 3 or 5?

- ○ $\frac{1}{2}$
- ○ $\frac{2}{5}$
- ○ $\frac{8}{15}$
- ◉ $\frac{9}{20}$

5 Multiple Choice 0.5 points

Three fair coins are tossed. What is the probability of getting at most two heads?

- ○ $\frac{3}{4}$
- ○ $\frac{1}{4}$
- ○ $\frac{3}{8}$
- ◉ $\frac{7}{8}$

6 Multiple Choice 0.5 points

A study has shown that there is a 40% chance that people who are 25 will still be alive when they are 85 years old. The researcher had randomly selected 3 college students aged 25. By using the binomial probability formula, what is the probability that the number alive at age 85 will be exactly two?

- (●) 28.8%
- ( ) 17.28%
- ( ) 40%
- ( ) 48%

7 Multiple Choice 0.5 points

A large collection of tires has 3% of defective tires. Suppose one chooses tires from this collection until he/she obtains 4 non-defective tires. Then the total number of defective tires drawn in this process has a Negative Binomial distribution with

- ( ) $r = 3, p = 0.04$
- (●) $r = 4, p = 0.97$
- ( ) $r = 3, p = 0.97$
- ( ) $r = 4, p = 0.03$

8 Multiple Choice 0.5 points

A stockbroker estimates that at the end of the year, there is a 40% chance a stock will be worth $50, a 35% chance it will be worth $60 and a 25% chance it will be worth $70. What expected value does this broker assign to this stock's end-of-the-year price?

- (●) $58.50
- ( ) $60.00
- ( ) $62.50
- ( ) $65.00

9 Multiple Choice 0.5 points

If Dzung's score is 87%, the class average is 82%, and the standard deviation is 4, what is Dzung's standardized score?

- ( ) - 1.25
- (●) 1.25
- ( ) 1.20
- ( ) 1.02

10 Multiple Choice 0.5 points

The mean and standard deviation of a population are 400 and 40, respectively. The sample size is 25. What is the standard deviation of the sampling distribution

- ○ 400
- ○ 25
- ○ 40
- ◉ 8

11 File Upload 3 points Written Question

This question type cannot be printed

(a) How many ways can you arrange the letters in the word STATISTICS? (e.g. SSSTTTIIAC counts as one arrangement.)
(b) If all arrangements are equally likely, what is the probability the two 'I's are next to each other.

Drag n' Drop here or Browse

pdf, png, jpg, docx files only!

12 File Upload 3 points Written Question

This question type cannot be printed

The random variable X takes in the value -1, 0, 1 with probabilities $\frac{1}{8}, \frac{2}{8}, \frac{5}{8}$ respectively.

1. Compute $E[X]$
2. Compute the PMF of $Y = X^2$ and use it compute $E[Y]$
3. Compute $Var(X)$

Drag n' Drop here or Browse

pdf, png, jpg, docx files only!

13 File Upload 3 points Written Question

This question type cannot be printed

Suppose you roll a fair 6-sided die 100 times (independently), and you get $3 every time you roll a 6.

Let X_1 be the number of dollars you win on rolls 1 through 25.
Let X_2 be the number of dollars you win on rolls 26 through 50.
Let X_3 be the number of dollars you win on rolls 51 through 75.
Let X_4 be the number of dollars you win on rolls 76 through 100.

Let $X = X_1 + X_2 + X_3 + X_4$ be the total number of dollars you will win over all 100 rolls.

1. What is the probability mass function of X?
2. What is expectation and variance of X?
3. Let $Y = 4X_1$. So instead of rolling 100 times, you just roll 25 times and multiply your winnings by 4. What are the expectation and variance of Y ?

Drag n' Drop here or Browse

pdf, docx, png, jpg files only!

14 File Upload 3 points Written Question

This question type cannot be printed

Suppose X and Y are continuous random variables with joint density function $f_{X,Y}(x, y) = x + y$ on the unit square [0, 1] × [0, 1].

1. Compute the joint CDF $F_{X,Y}(x, y)$.
2. Compute the marginal PDFs $f_X(x)$, $f_Y(y)$
3. Are X, Y independent?

Drag n' Drop here or Browse

pdf, docx, png, jpg files only!

15 File Upload 3 points Written Question

This question type cannot be printed

Suppose $X_1, X_2, ..., X_{100}$ are independently identically distributed random variable with mean $\frac{1}{5}$ and variance $\frac{1}{9}$. Use the central limit theorem to estimate $P\left(\sum_{i=1}^{100} X_i < 30\right)$.

Drag n' Drop here or Browse

pdf, jpg, png, docx files only!