

# GENERAL PHYSICS PH1110

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## 2. THERMODYNAMICS

### 2.4 REAL GASES

1 VAN DER WAALS EQUATION OF STATE

2 EXPERIMENTAL ISOTHERMS

## 1. Van der Waals equation of state

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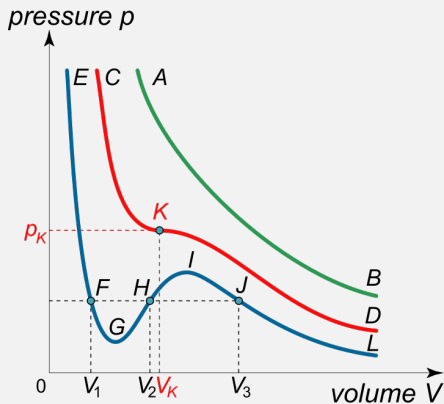
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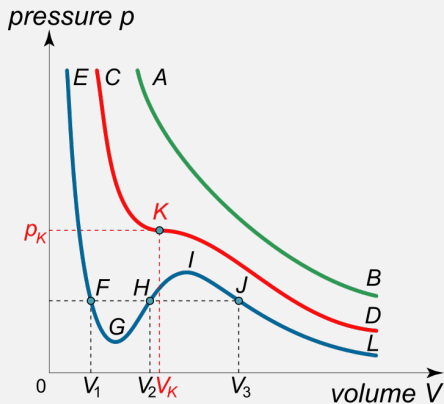
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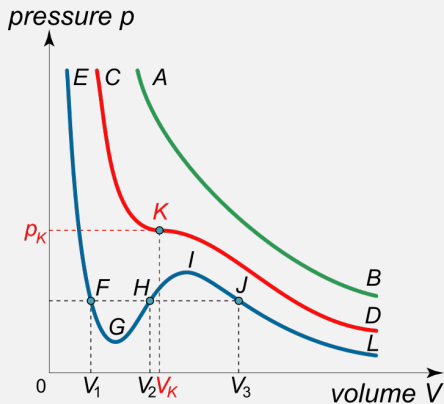
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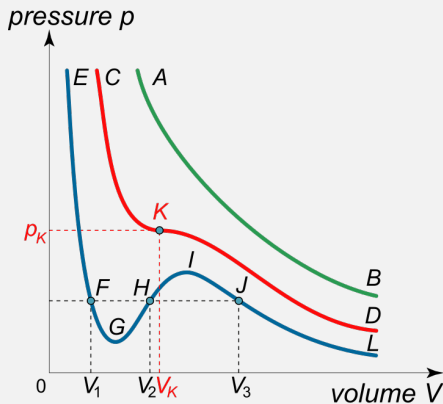
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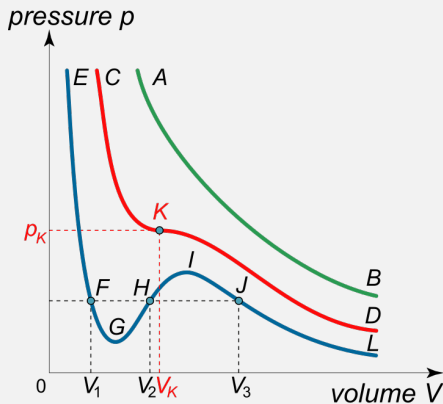
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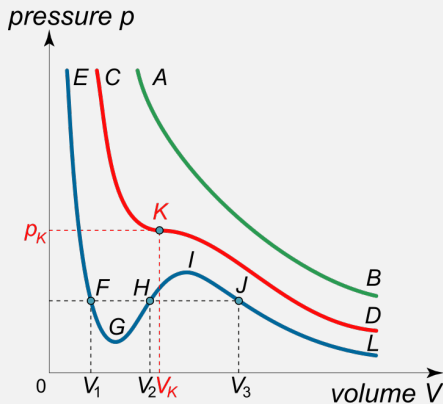




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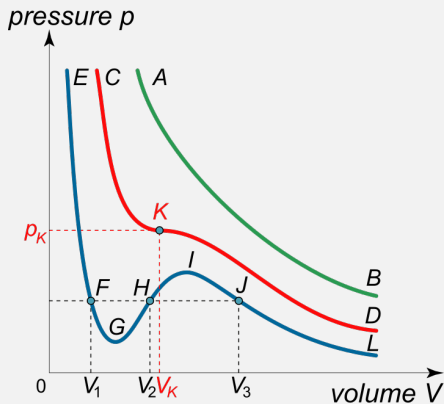
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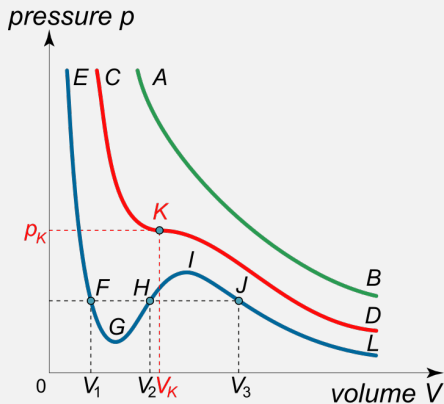
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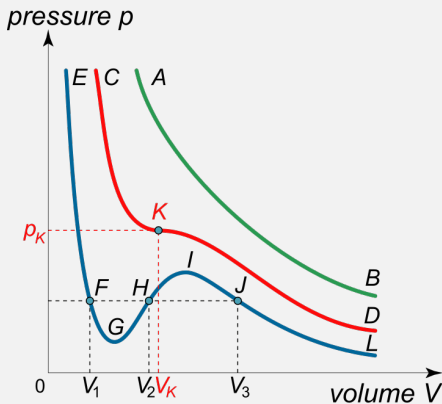
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## ♣ Exercise

- For one mole of a real gas at critical point, find its volume, temperature and pressure in terms of  $a$ ,  $b$  and  $R$ .

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- Solutions:

$$V_c = 3b$$

$$T_c = \frac{8a}{27bR}$$

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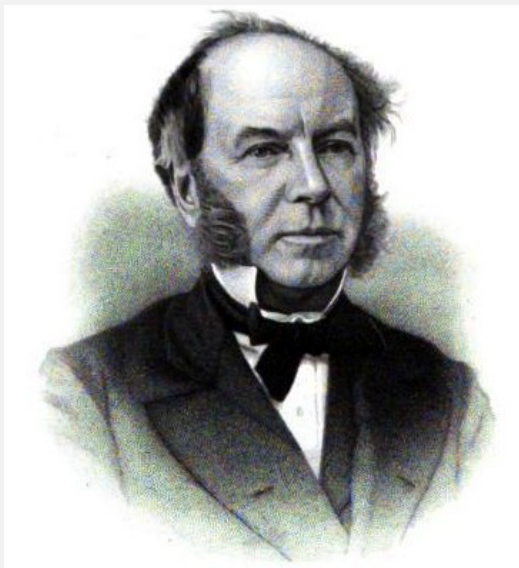
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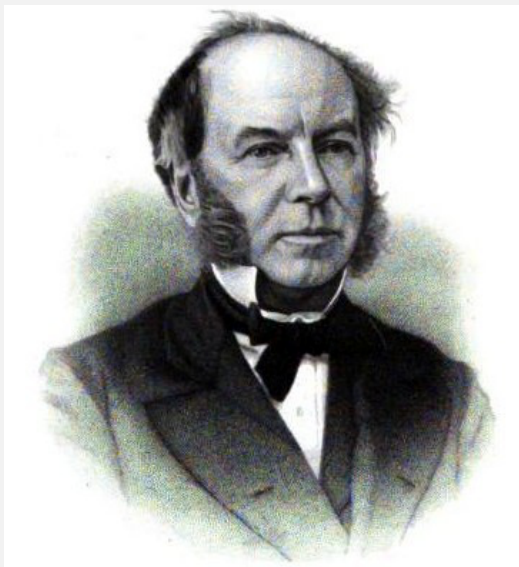
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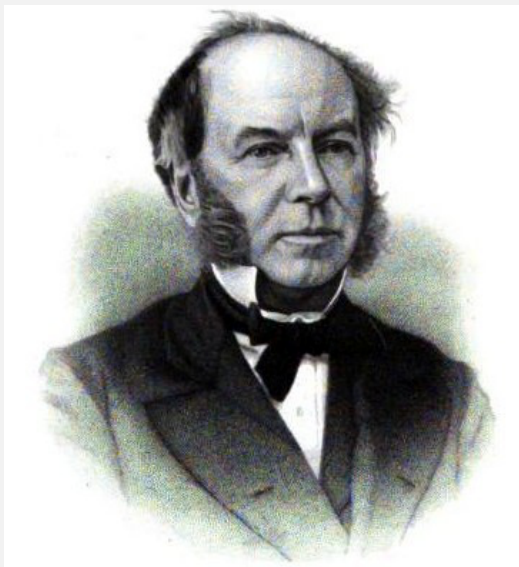


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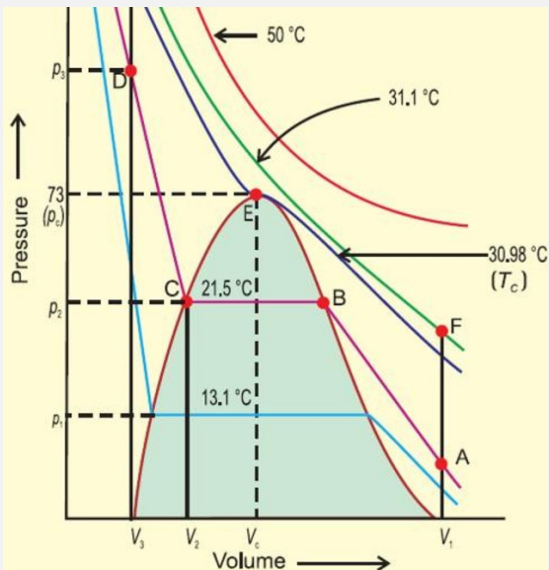


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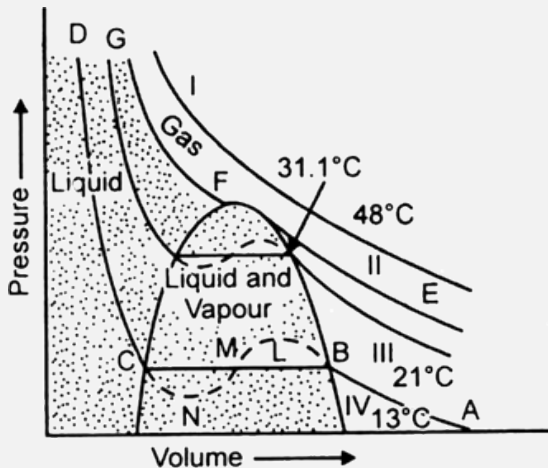


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