Algebra Final Exam - Duration: 90 Minutes

No materials or electronic devices shall be used or viewed during the examination.

Q1. Let $f(x) = -x^2 - 2x + 3$.
Find a such that $f : \mathbb{R} \to (-\infty, a]$ is surjective.

Q2. Solve the equation $\overline{z^7} = \frac{1}{z^3}$, where $z \in \mathbb{C}$.

Q3. Find the change-of-basis matrix from S_2 to S_1 of $\mathbb{R}^2$, where $\begin{cases} S_1 = \{u_1 = (1,-2),\ u_2 = (3,-4)\} \\ S_2 = \{v_1 = (1,3),\ v_2 = (3,8)\} \end{cases}$.

Q4. Let $A = \begin{bmatrix} 1 & 3 & -1 \\ 2 & 0 & 5 \\ 6 & -2 & 4 \end{bmatrix}$ be the matrix of the linear transformation $f : P_2[x] \to P_2[x]$ with respect to the basis

$$v_1 = 3x + 3x^2,\ v_2 = -1 + 3x + 2x^2,\ v_3 = 3 + 7x + 2x^2.$$

Find $f(1 + x^2)$.

Q5. Let $p \cdot q = \int\limits_0^1 p(x)q(x)dx$ be the inner product in $P_2[X]$.

a) Apply the Gram-Schmidt process to $\{1, x, x^2\}$.

b) Find the projection of $1 - x$ on to $1 + x$.

Q6. Orthogonal diagonalization the quadratic form

$$q(x,y,z) = 4xy + 4yz + 4xz.$$

Q7. Solve the system of linear equations

$$\begin{cases} ax_1 + x_2 + x_3 + x_4 = 1 \\ x_1 + ax_2 + x_3 + x_4 = a \\ x_1 + x_2 + ax_3 + x_4 = a^2. \end{cases}$$

Q8. Let u, v and w be linearly independent vectors. Show that $u + v, u - v$ and $u - 2v + w$ are linearly independent.

Q9. Compute the determinant $\begin{vmatrix} 1+x & 1 & 1 & 1 \\ 1 & 1-x & 1 & 1 \\ 1 & 1 & 1+z & 1 \\ 1 & 1 & 1 & 1-z \end{vmatrix}$

Algebra Final Exam - Duration: 90 Minutes

No materials or electronic devices shall be used or viewed during the examination.

Q1. Let $f(x) = -x^2 - 2x + 3$.
Find b such that $f : [b, +\infty) \to (-\infty, 3]$ is injective.

Q2. Solve the equation $\overline{z^3} = \frac{1}{z^7}$, where $z \in \mathbb{C}$.

Q3. Find the change-of-basis matrix from S_1 to S_2 of $\mathbb{R}^2$, where $\begin{cases} S_1 = \{u_1 = (1,-2),\ u_2 = (3,-4)\} \\ S_2 = \{v_1 = (1,3),\ v_2 = (3,8)\} \end{cases}$.

Q4. Let $A = \begin{bmatrix} 1 & 3 & -1 \\ 2 & 0 & 5 \\ 6 & -2 & 4 \end{bmatrix}$ be the matrix of the linear transformation $f : P_2[x] \to P_2[x]$ with respect to the basis

$$v_1 = 3x + 3x^2,\ v_2 = -1 + 3x + 2x^2,\ v_3 = 3 + 7x + 2x^2.$$

Find $f(4 + 4x)$.

Q5. Let $p \cdot q = \int\limits_0^1 p(x)q(x)dx$ be the inner product in $P_2[X]$.

a) Apply the Gram-Schmidt process to $\{1, x, x^2\}$.

b) Find the projection of $1 + x$ on to $1 - x$.

Q6. Orthogonal diagonalization the quadratic form

$$q(x,y,z) = 2xy + 2yz + 2xz.$$

Q7. Solve the system of linear equations

$$\begin{cases} (2-a)x_1 + x_2 + x_3 = 0 \\ x_1 + (2-a)x_2 + x_3 = 0 \\ x_1 + x_2 + (2-a)x_3 = 0. \end{cases}$$

Q8. Let u, v and w be linearly independent vectors. Show that $u + w, u - w$ and $u + v - 2w$ are linearly independent.

Q9. Compute the determinant $\begin{vmatrix} 1 & 1 & 2 & 3 \\ 1 & 2-x^2 & 2 & 3 \\ 2 & 3 & 1 & 5 \\ 2 & 3 & 1 & 9-x^2 \end{vmatrix}$