

# Probability and Statistics – Problem Set 1

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**Question 1.** How many ways are there to permute the 8 letters A, B, C, D, E, F, G, H so that A is not at the beginning and H is not at the end.

**Question 2.**

- (i) Count the number of ways to get 3 heads in a sequence of 10 flips of a coin.
- (ii) If the coin is fair, what is the probability of exactly 3 heads in 10 flips.

**Question 3.** There are 40 seats and 40 students in a classroom. Suppose that the front row contains 10 seats, and there are 5 students who must sit in the front row in order to see the board clearly. How many seating arrangements are possible with this restriction?

**Question 4.** Suppose you have five semesters left and you want to take exactly two classes per semester. You want to take CECS1, CECS2, ..., CECS10, but CECS1 and CECS2 must both be taken before CECS3, which must be taken before CECS4. CECS1 and CECS2 can be taken in any order, or together. The other classes can be taken any semester, in any order, and have no prerequisites. How many different schedules can be formed (assume the two classes in a semester are unordered)?

**Question 5.** Find the number of ways to rearrange the word “INGREDIENT”, such that no two identical letters are adjacent to each other (there are two E’s, two N’s, and two I’s). For example, “INGREEDINT” is invalid because the two E’s are adjacent. Repeat the question for the letters “AAAAABBB”.

**Question 6.** A deck of 52 cards has 13 ranks (2, 3, . . . , 9, 10, J, Q, K, A) and 4 suits (♥, ♠, ♦, ♣). A poker hand consists of 5 cards. A one-pair hand consists of two cards having one rank and three cards having three other ranks, e.g., {2♥, 2♠, 5♥, 8♣, K♦}

The probability of a one-pair hand is (pick one answer):

- (a) less than 5%
- (b) between 5% and 10%
- (c) between 10% and 20%
- (d) between 20% and 40%
- (e) greater than 40%

**Question 7.** After one-pair, the next most common hands are two-pair and three-of-a-kind: Two-pair: Two cards have one rank, two cards have another rank, and the remaining card has a third rank. e.g. {2♥, 2♠, 5♥, 5♣, K♦} Three-of-a-kind: Three cards have one rank and the remaining two cards have two other ranks. e.g. {2♥, 2♠, 2♣, 5♣, K♦} Calculate the probability of each type of hand. Which is more likely?

**Question 8.** Suppose you went trick-or-treating (as an adult) and were able to nab 50 total candies, 13 of which are kit-kats. Your responsible parent says you can only eat 6 of them tonight. Let  $X$  be the number of kit-kats you grabbed out of 6. What is  $P(X = k)$  for valid values of  $k$  ( $k \in \{0, 1, 2, \dots, 6\}$ )?

**Question 9.** Suppose we have 13 chairs (in a row) with 8 TA's, and 5 professors to be seated. Suppose all seatings are equally likely. What is the probability that every professor has a TA to their immediate left and right?