Probability and Statistics – Problem Set 4

Question 1. Suppose X is continuous with density

$$f_X(x) = \begin{cases} cx^2 & 0 \le x < 9 \\ 0 & otherwise \end{cases}$$

Write an expression for the value of c that makes X a valid PDF, and set up expressions (integrals) for its mean and variance. Also, find the CDF of X, F_X.

Question 2.

a) Suppose the current (in Amperes) flowing through a 1-ohm resistor is a $Uniform(a, b)$ random variable I for $a, b > 0$. The power dissipated by this resistor is $X = I^2$. What is the expected power dissipated by the resistor?

b) Continuing with the previous example, suppose that the current I instead follows an $Exponential(\lambda)$ distribution. What is the expected power dissipated by the resistor?

Question 3: A flea of negligible size is trapped in a large, spherical, inflated beach ball with radius r. At this moment, it is equally likely to be at any point within the ball. Let X be the distance of the flea from the center of the ball.

a. Find the range of X, Ω_X.
b. Find the cumulative distribution function $F_X(x) = P(X \le x)$.
c. Find the probability density function $f_X(x)$.
d. Find an integral for $E[X]$.

Question 4: Suppose that you are s minutes early for an appointment, then you incur the cost cs, and if you are s minutes late, then you incur the cost ks. Suppose also the travel time from where you presently are to the location of your appointment is a continuous random variable having probability density function f. Determine the time at which you should depart if you want to minimize your expected cost.

Question 5:

You are waiting for a bus to take home from VinUni. You can either take the A-line, B-line, and C-line. The distribution of the waiting time in minutes for each is the following:

- A-line: $A \sim Exp(\lambda = 0.1)$
- B-line: $B \sim Unif(0, 20)$
- C-line: has range $(1, \infty)$ and density function $f_C(x) = \frac{1}{x^2}$

Assume the three bus arrival times are independent. You take the first bus that arrives.

- Find the CDFs of A, B, and C.
- What is the probability you wait more than 5 minutes for a bus?
- What is the probability you wait more than 30 minutes for a bus?

- What is the expected amount of time you will wait for a bus?