

Probability and Statistics – Homework 4

Instructions: For each problem, remember you must briefly explain/justify how you obtained your answer, as correct answers without an explanation will receive **no credit**. Moreover, in the event of an incorrect answer, we can still try to give you partial credit based on the explanation you provide. It is fine for your answers to include summations, products, factorials, exponentials, or combinations; you don't need to calculate those all out to get a single numeric answer.

Submission: You must upload your working in PDF format to Canvas MATH2010

Question 1 (5 points)

Let X_1, X_2, \dots, X_n be random samples from a $Uniform(0, \theta)$ distribution, where θ is unknown. Define the estimator for θ :

$$\hat{\theta}_n = \max\{X_1, X_2, \dots, X_n\}$$

- a) Find the bias of $\hat{\theta}_n$.
- b) Find the mean squared error (MSE) of $\hat{\theta}_n$.
- c) Is $\hat{\theta}_n$ a consistent estimator of θ ?

Question 2 (5 points)

Let X be the amount of orange juice (in grams per day) consumed by an adult. Suppose it is known that the standard deviation of X is $\sigma = 96$. To estimate the mean μ of X , an orange growers' association took a random sample of $n = 576$ adults and found that they consumed on the average, $\bar{x} = 133$ grams of orange juice per day. **Please construct an approximate 90% confidence interval for μ .**

Question 3 (5 points)

Two detergents were tested for their ability to remove stains of certain type. An inspector judged the first one to be successful on 63 out of 91 independent trials and the second one to be successful on 42 out of 79 independent trials. Suppose that the probabilities of success for the first one and second one are p_1 and p_2 respectively. **Give an approximate 90% confidence interval for the difference $p_1 - p_2$ of the two detergents.**

Question 4 (5 points)

The mean birth weight of infants in the United States is 3315 grams. Let X be the birth weight (in grams) of a randomly selected infant in Vietnam. Assume the distribution of X is $\mathcal{N}(\mu, \sigma^2)$, where μ and σ^2 are unknown. We want to test the null hypothesis:

$$H_0: \mu = 3315$$

against the alternative hypothesis

$$H_0: \mu < 3315$$

using $n = 30$ randomly selected Vietnamese infants.

- a) Define a critical region that has a significance level of $\alpha = 0.05$.

- b) If the random sample of $n = 30$ yields the sample mean $\bar{x} = 3189$ and the sample standard deviation $s = 488$, what would be your conclusion?
- c) What is the approximate $p - value$ of your test.