HANOI UNIVERSITY OF SCIENCE AND TECHNOLOGY
SCHOOL OF APPLIED MATHEMATICS AND INFORMATICS

—————o0o—————

PROBLEMS

PROBABILITY AND STATISTICS

NGUYEN THI THU THUY

HANOI – 2021

HANOI UNIVERSITY OF SCIENCE AND TECHNOLOGY
SCHOOL OF APPLIED MATHEMATICS AND INFORMATICS

PROBLEMS

PROBABILITY AND STATISTICS

MI2026

NGUYEN THI THU THUY
DEPARTMENT OF APPLIED MATHEMATICS

HA NOI – 2021

Contents

INTRODUCTION

GENERAL INFORMATION

1. Course name: Probability and Statistics
2. Course ID: MI2026
3. Course units: 5(3-2-0-10)
   - Lectures: 45 hours
   - Tutorial: 30 hours
4. Expected participants: Third-year students
5. Requisites (Corequisites):
   - Calculus 1
   - Calculus 2

OBJECTIVE

The course provides students with the knowledge of probability such as concepts and inference rules of probability as well as random variables and common probability distributions (one-dimensional and two-dimensional); basic concepts of mathematical statistics which help students in dealing with statistical problems in estimation, hypothesis testing. Through the acquired knowledge, students are given a methodology for approaching practical models and finding out an appropriate solution.

CONTENTS

Random events and probability calculation, random variables, probability distributions, random vectors, statistical estimation theory, statistical decision theory.

Chapter 1

Probability

1.1 Experiments

Problem 1.1 A fax transmission can take place at any of three speeds depending on the condition of the phone connection between the two fax machines. The speeds are high (h) at 14400 b/s, medium (m) at 9600 b/s, and low (l) at 4800 b/s. In response to requests for information, a company sends either short faxes of two (t) pages, or long faxes of four (f) pages. Consider the experiment of monitoring a fax transmission and observing the transmission speed and length. An observation is a two-letter word, for example, a high-speed, two-page fax is ht.

(a) What is the sample space of the experiment?

(b) Let A_1 be the event "medium-speed fax." What are the outcomes in A_1?

(c) Let A_2 be the event "short (two-page) fax." What are the outcomes in A_2?

(d) Let A_3 be the event "high-speedfax or low-speed fax." What are the outcomes in A_3?

(e) Are A_1, A_2, and A_3 mutually exclusive?

(f) Are A_1, A_2, and A_3 collectively exhaustive?

Problem 1.2 An integrated circuit factory has three machines X, Y, and Z. Test one integrated circuit produced by each machine. Either a circuit is acceptable (a) or it fails (f). An observation is a sequence of three test results corresponding to the circuits from machines X, Y, and Z, respectively. For example, aaf is the observation that the circuits from X and Y pass the test and the circuit from Z fails the test.

(a) What are the elements of the sample space of this experiment?

(b) What are the elements of the sets

$$\begin{aligned} Z_F &= \{\text{circuit from } Z \text{ fails}\}, \\ X_A &= \{\text{circuit from } X \text{ is acceptable}\}. \end{aligned}$$

(c) Are Z_F and X_A mutually exclusive?

(d) Are Z_F and X_A collectively exhaustive?

(e) What are the elements of the sets

$$C = \{\text{more than one circuit acceptable}\},$$
$$D = \{\text{at least two circuits fail}\}.$$

(f) Are C and D mutually exclusive?

(g) Are C and D collectively exhaustive?

Problem 1.3 Find out the birthday (month and day but not year) of a randomly chosen person. What is the sample space of the experiment. How many outcomes are in the event that the person is born in July?

Problem 1.4 Let the sample space of the experiment consist of the measured resistances of two resistors. Give four examples of event spaces.

1.2 Counting Methods

Problem 1.5 Consider a binary code with 5 bits (0 or 1) in each code word. An example of a code word is 01010. How many different code words are there? How many code words have exactly three 0's?

Problem 1.6 Consider a language containing four letters: A, B, C, D. How many three-letter words can you form in this language? How many four-letter words can you form if each letter appears only once in each word?

Problem 1.7 On an American League baseball team with 15 field players and 10 pitchers, the manager must select for the starting lineup, 8 field players, 1 pitcher, and 1 designated hitter. A starting lineup specifies the players for these positions and the positions in a batting order for the 8 field players and designated hitter. If the designated hittermust be chosen among all the field players, howmany possible starting lineups are there?

Problem 1.8 A basketball team has three pure centers, four pure forwards, four pure guards, and one swingman who can play either guard or forward. A pure position player can play only the designated position. If the coach must start a lineup with one center, two forwards, and two guards, how many possible lineups can the coach choose?

1.3 Probability

Problem 1.9 In a certain city, three newspapers A, B, and C are published. Suppose that 60 percent of the families in the city subscribe to newspaper A, 40 percent of the families subscribe to newspaper B, and 30 percent of the families subscribe to newspaper C. Suppose also that 20 percent of the families subscribe to both A and B, 10 percent subscribe to both A and C, 20 percent subscribe to both B and C, and 5 percent subscribe to all three newspaper A, B, and C. What percentage of the families in the city subscribe to at least one of the three newspapers?

Problem 1.10 From a group of 3 freshmen, 4 sophomores, 4 juniors and 3 seniors a committee of size 4 is randomly selected. Find the probability that the committee will consist of

(a) 1 from each class;

(b) 2 sophomores and 2 juniors;

(c) Only sophomores and juniors.

Problem 1.11 A box contains 24 light bulbs of which four are defective. If one person selects 10 bulbs from the box in a random manner, and a second person then takes the remaining 14 bulbs, what is the probability that all 4 defective bulbs will be obtained by the same person?

Problem 1.12 Suppose that three runners from team A and three runners from team B participate in a race. If all six runners have equal ability and there are no ties, what is the probability that three runners from team A will finish first, second, and third, and three runners from team B will finish fourth, fifth, and sixth?

Problem 1.13 Suppose that a school band contains 10 students from the freshman class, 20 students from the sophomore class, 30 students from the junior class, and 40 students from the senior class. If 15 students are selected at random from the band, what is the probability that at least one students from each of the four classes?

Problem 1.14 Suppose that 10 cards, of which 5 are red and 5 are green, are placed at random in 10 envelopes, of which 5 are red and 5 are green. Determine the probability that exactly x envelopes will contain a card with a matching color ($x = 0, 1, 2, \ldots, 10$).

Problem 1.15 Consider two events A and B with $P(A) = 0.4$ and $P(B) = 0.7$. Determine the maximum and minimum possible values of $P(A \cap B)$ and the conditions under which each of these values is attained.

Problem 1.16 Suppose that four guests check their hats when they arrive at a restaurant, and that these hats are returned to them in a random order when they leave. Determine the probability that no guest will receive the proper hat.

Problem 1.17 Suppose that four guests check their hats when they arrive at a restaurant, and that these hats are returned to them in a random order when they leave. Determine the probability that at least 2 guests will receive the proper hat.

Problem 1.18 Suppose that A, B and C are three independent events such that $P(A) = 1/4$, $P(B) = 1/3$ and $P(C) = 1/2$.

(a) What is the probability that none of these three events will occur?

(b) Determine the probability that exactly one of these three events will occur.

Problem 1.19 Three players A, B and C take turns tossing a fair coin. Suppose that A tosses the coin first, B tosses the second and C tosses third and cycle is repeated indefinitely until someone wins by being the first player to obtain a head. Determine the probability that each of the three players will win.

Problem 1.20 Computer programs are classified by the length of the source code and by the execution time. Programs with more than 150 lines in the source code are big (B). Programs with $\leq$ 150 lines are little (L). Fast programs (F) run in less than 0.1 seconds. Slow programs (W) require at least 0.1 seconds. Monitor a program executed by a computer. Observe the length of the source code and the run time. The probability model for this experiment contains the following information: $P[LF] = 0.5$, $P[BF] = 0.2$, and $P[BW] = 0.2$. What is the sample space of the experiment? Calculate the following probabilities:

(a) $P[W]$;

(b) $P[B]$;

(c) $P[W \cup B]$.

Problem 1.21 You have a six-sided die that you roll once and observe the number of dots facing upwards. What is the sample space? What is the probability of each sample outcome? What is the probability of E, the event that the roll is even?

Problem 1.22 A student's score on a 10-point quiz is equally likely to be any integer between 0 and 10. What is the probability of an A, which requires the student to get a score of 9 or more? What is the probability the student gets an F by getting less than 4?

Problem 1.23 Mobile telephones perform handoffs as they move from cell to cell. During a call, a telephone either performs zero handoffs (H_0), one handoff (H_1), or more than one handoff (H_2). In addition, each call is either long (L), if it lasts more than three minutes, or brief (B). The following table describes the probabilities of the possible types of calls.

	H_0	H_1	H_2
L	0.1	0.1	0.2
B	0.4	0.1	0.1

What is the probability $P[H_0]$ that a phone makes no handoffs? What is the probability a call is brief? What is the probability a call is long or there are at least two handoffs?

Problem 1.24 Proving the following facts: (a) $P[A \cup B] \geq P[A]$; (b) $P[A \cup B] \geq P[B]$; (c) $P[A \cap B] \leq P[A]$; (d) $P[A \cap B] \leq P[B]$.

Problem 1.25 Proving by induction the union bound: For any collection of events $A_1, \ldots, A_n$,

$$P[A_1 \cup A_2 \cup \ldots \cup A_n] \leq \sum_{i=1}^{n} P[A_i].$$

Problem 1.26 Proving $P[\emptyset] = 0$.

1.4 Law of Total Probability

Problem 1.27 Given the model of handoffs and call lengths in Problem 1.23,

(a) What is the probability that a brief call will have no handoffs?

(b) What is the probability that a call with one handoff will be long?

(c) What is the probability that a long call will have one or more handoffs?

Problem 1.28 You have a six-sided die that you roll once. Let R_i denote the event that the roll is i. Let G_j denote the event that the roll is greater than j. Let E denote the event that the roll of the die is even-numbered.

(a) What is $P[R_3|G_1]$, the conditional probability that 3 is rolled given that the roll is greater than 1?

(b) What is the conditional probability that 6 is rolled given that the roll is greater than 3?

(c) What is $P[G3|E]$, the conditional probability that the roll is greater than 3 given that the roll is even?

(d) Given that the roll is greater than 3, what is the conditional probability that the roll is even?

Problem 1.29 You have a shuffled deck of three cards: 2, 3, and 4. You draw one card. Let C_i denote the event that card i is picked. Let E denote the event that card chosen is a even-numbered card.

(a) What is $P[C_2|E]$, the probability that the 2 is picked given that an even-numbered card is chosen?

(b) What is the conditional probability that an evennumbered card is picked given that the 2 is picked?

Problem 1.30 Two different suppliers, A and B, provide a manufacturer with the same part. All suppliers of this part are kept in a large bin. In the past, 5 percent of the parts supplied by A and 9 percent of the parts supplied by B have been defective. A supplies four times as many parts as B. Suppose you reach into the bin and select a part and find it is non-defective. What is the probability that it was supplied by A?

Problem 1.31 Suppose that 30 percent of the bottles produced in a certain plant are defective. If a bottle is defective, the probability is 0.9 that an inspector will notice it and remove it from the filling line. If a bottle is not defective, the probability is 0.2 that the inspector will think that it is defective and remove it from the filling line.

(a) If a bottle is removed from the filling line, what is the probability that it is defective?

(b) If a customer buys a bottle that has not been removed from the filling line, what is the probability that it is defective.

Problem 1.32 Suppose that traffic engineers have coordinated the timing of two traffic lights to encourage a run of green lights. In particular, the timing was designed so that with probability 0.75 a driver will find the second light to have the same color as the first. Assuming the first light is equally likely to be red or green.

(a) What is the probability that the second light is green?

(b) What is the probability that you wait for at least one light?

Problem 1.33 A factory has three machines A, B, and C. Past records show that the machine A produced 40% of the items of output, the machine B produced 35% of the items of output, and machine B produced 25% of the items. Further 2% of the items produced by machine A were defective, 1.5% produced by machine B were defective, and 1% produced by machine B were defective.

(a) If an item is drawn at random, what is the probability that it is defective?

(b) An item is acceptable if it is not defective. What is the probability that an acceptable item comes from machine A?

1.5 Independent

Problem 1.34 Is it possible for A and B to be independent events yet satisfy $A = B$?

Problem 1.35 Use a Venn diagram in which the event areas are proportional to their probabilities to illustrate two events A and B that are independent.

Problem 1.36 In an experiment, A, B, C, and D are events with probabilities $P[A] = 1/4$, $P[B] = 1/8$, $P[C] = 5/8$, and $P[D] = 3/8$. Furthermore, A and B are disjoint, while C and D are independent.

(a) Find $P[A \cap B]$, $P[A \cup B]$, $P[A \cap B^c]$, and $P[A \cup B^c]$.

(b) Are A and B independent?

(c) Find $P[C \cap D]$, $P[C \cap D^c]$, and $P[C^c \cap D^c]$.

(d) Are C^c and D^c independent?

Problem 1.37 In an experiment, A, B, C, and D are events with probabilities $P[A \cup B] = 5/8$, $P[A] = 3/8$, $P[C \cap D] = 1/3$, and $P[C] = 1/2$. Furthermore, A and B are disjoint, while C and D are independent.

(a) Find $P[A \cap B]$, $P[B]$, $P[A \cap B^c]$, and $P[A \cup B^c]$.

(b) Are A and B independent?

(c) Find $P[D]$, $P[C \cap D^c]$, $P[C^c \cap D^c]$, and $P[C|D]$.

(d) Find $P[C \cup D]$ and $P[C \cup D^c]$.

(e) Are C and D^c independent?

1.6 Bernoulli Trials

Problem 1.38 Consider a binary code with 5 bits (0 or 1) in each code word. An example of a code word is 01010. In each code word, a bit is a zero with probability 0.8, independent of any other bit.

Problem 1.39 Suppose each day that you drive to work a traffic light that you encounter is either green with probability 7/16, red with probability 7/16, or yellow with probability 1/8, independent of the status of the light on any other day. If over the course of five days, G, Y, and R denote the number of times the light is found to be green, yellow, or red, respectively, what is the probability that $P[G = 2, Y = 1, R = 2]$? Also, what is the probability $P[G = R]$?

Problem 1.40 We wish to modify the cellular telephone coding system in Example 1.29 in order to reduce the number of errors. In particular, if there are two or three zeroes in the received sequence of 5 bits, we will say that a deletion (event D) occurs. Otherwise, if at least 4 zeroes are received, then the receiver decides a zero was sent. Similarly, if at least 4 ones are received, then the receiver decides a one was sent. We say that an error occurs if either a one was sent and the receiver decides zero was sent or if a zero was sent and the receiver decides a one was sent. For this modified protocol, what is the probability $P[E]$ of an error? What is the probability $P[D]$ of a deletion?

Example 1.29 To communicate one bit of information reliably, cellular phones transmit the same binary symbol five times. Thus the information "zero" is transmitted as 00000 and "one" is 11111. The receiver detects the correct information if three or more binary symbols are received correctly. What is the information error probability $P[E]$, if the binary symbol error probability is $q = 0.1$?

Problem 1.41 An airline sells 200 tickets for a certain flight on an airplane that has only 198 seats because, on the average, 1 percent of purchasers of airline tickets do not appear for the departure of their flight. Determine the probability that everyone who appears for the departure of this flight will have a seat.

Chapter 2

Random Variables and Probability Distributions

2.1 Discrete Random Variables

Problem 2.1 A civil engineer is studying a left-turn lane that is long enough to hold 7 cars. Let X be the number of cars in the lane at the end of a randomly chosen red light. The engineer believes that the prob. that $X = x$ is proportional to $(x+1)(8-x)$ for $x = 0, 1, \ldots, 7$.

(a) Find the probability mass function of X.

(b) Find the probability that X will be at least 5.

Problem 2.2 A midterm test has 4 multiple choice questions with four choices with one correct answer each. If you just randomly guess on each of the 4 questions, what is the probability that you get exactly 2 questions correct? Assume that you answer all and you will get $(+5)$ points for 1 question correct, (-2) points for 1 question wrong. Let X is number of points that you get. Find the probability mass function of X and the expected value of X.

Problem 2.3 The random variable N has PMF

$$P_N(n) = \begin{cases} c(1/2)^n, & n = 0, 1, 2, \\ 0, & \text{otherwise.} \end{cases}$$

(a) What is the value of the constant c?

(b) What is $P[N \leq 1]$?

(c) What is the PD of X?

Problem 2.4 The random variable V has PMF

$$P_V(v) = \begin{cases} cv^2, & v = 1, 2, 3, 4, \\ 0, & \text{otherwise.} \end{cases}$$

(a) Find the value of the constant c.

(b) Find $P[V \in \ u^2|u = 1, 2, 3, \dots]$.

(c) Find the probability that V is an even number.

(d) Find $P[V > 2]$.

Problem 2.5 Suppose when a baseball player gets a hit, a single is twice as likely as a double which is twice as likely as a triple which is twice as likely as a home run. Also, the player's batting average, i.e., the probability the player gets a hit, is 0.300. Let B denote the number of bases touched safely during an at-bat. For example, $B = 0$ when the player makes an out, $B = 1$ on a single, and so on. What is the PMF of B?

Problem 2.6 In a package of M&Ms, Y, the number of yellow M&Ms, is uniformly distributed between 5 and 15.

(a) What is the PMF of Y?

(b) What is $P[Y < 10]$?

(c) What is $P[Y > 12]$?

(d) What is $P[8 \le Y \le 12]$?

Problem 2.7 When a conventional paging system transmits a message, the probability that the message will be received by the pager it is sent to is p. To be confident that a message is received at least once, a system transmits the message n times.

(a) Assuming all transmissions are independent, what is the PMF of K, the number of times the pager receives the same message?

(b) Assume $p = 0.8$. What is the minimum value of n that produces a probability of 0.95 of receiving the message at least once?

Problem 2.8 When a two-way paging system transmits a message, the probability that the message will be received by the pager it is sent to is p. When the pager receives the message, it transmits an acknowledgment signal (ACK) to the paging system. If the paging system does not receive the ACK, it sends the message again.

(a) What is the PMF of N, the number of times the system sends the same message?

(b) The paging company wants to limit the number of times it has to send the same message. It has a goal of $P[N \le 3] \ge 0.95$. What is the minimum value of p necessary to achieve the goal?

Problem 2.9 The number of bits B in a fax transmission is a geometric $(p = 2.5 \times 10^{-5})$ random variable. What is the probability $P[B > 500,000]$ that a fax has over 500,000 bits?

Problem 2.10 The random variable X has CDF

$$F_X(x) = \begin{cases} 0, & x < -1, \\ 0.2, & -1 \le x < 0, \\ 0.7, & 0 \le x < 1, \\ 1, & x \ge 1. \end{cases}$$

(a) Draw a graph of the CDF.

(b) Write $P_X(x)$, the PMF of X. Be sure to write the value of $P_X(x)$ for all x from $-\infty$ to ∞.

(c) Write the PD of X.

Problem 2.11 The random variable X has CDF

$$F_X(x) = \begin{cases} 0, & x < -3, \\ 0.4, & -3 \le x < 5, \\ 0.8, & 5 \le x < 7, \\ 1, & x \ge 7. \end{cases}$$

(a) Draw a graph of the CDF.

(b) Write $P_X(x)$, the PMF of X.

(c) Write the PDT of X.

Problem 2.12 In Problem 2.5, find and sketch the CDF of B, the number of bases touched safely during an at-bat.

Problem 2.13 Let X have the uniform PMF

$$P_X(x) = \begin{cases} 0.01, & x = 1, 2, \dots, 100, \\ 0, & \text{otherwise.} \end{cases}$$

(a) Find a mode x_{mod} of X. If the mode is not unique, find the set X_{mod} of all modes of X.

(b) Find a median x_{med} of X. If the median is not unique, find the set X_{med} of all numbers x that are medians of X.

Problem 2.14 Voice calls cost 20 cents each and data calls cost 30 cents each. C is the cost of one telephone call. The probability that a call is a voice call is $P[V] = 0.6$. The probability of a data call is $P[D] = 0.4$.

(a) Find $P_C(c)$, the PMF of C.

(b) Find the PD of C.

(c) What is $E[C]$, the expected value of C?

Problem 2.15 Find the expected value of the random variable X in Problem 2.10.

Problem 2.16 Find the expected value of the random variable X in Problem 2.11.

Problem 2.17 Find the expected value of a binomial $(n = 4, p = 1/2)$ random variable X.

Problem 2.18 Give examples of practical applications of probability theory that can be modeled by the following PMFs. In each case, state an experiment, the sample space, the range of the random variable, the PMF of the random variable, and the expected value: (a) Bernoulli; (b) Binomial; (c) Poisson. Make up your own examples.

Problem 2.19 Given the random variable X in Problem 2.10, let $V = g(X) = |X|$. (a) Find $P_V(v)$. (b) Find $F_V(v)$. (c) Find $E[V]$.

Problem 2.20 In a certain lottery game, the chance of getting a winning ticket is exactly one in a thousand. Suppose a person buys one ticket each day (except on the leap year day February 29) over a period of fifty years. What is the expected number $E[T]$ of winning tickets in fifty years? If each winning ticket is worth \$1000, what is the expected amount $E[R]$ collected on these winning tickets? Lastly, if each ticket costs \$2, what is your expected net profit $E[Q]$?

Problem 2.21 In an experiment to monitor two calls, the PMF of N, the number of voice calls, is

$$P_N(n) = \begin{cases} 0.2, & n = 0, \\ 0.7, & n = 1, \\ 0.1, & n = 2, \\ 0, & \text{otherwise.} \end{cases}$$

(a) Find $E[N]$, the expected number of voice calls.

(b) Find $E[N^2]$, the second moment of N.

(c) Find $Var[N]$, the variance of N.

(d) Find σ_N, the standard deviation of N.

Problem 2.22 Find the variance of the random variable X in Problem 2.10.

Problem 2.23 Let X have the binomial PMF $P_X(x) = C_4^x(1/2)^4$.

(a) Find the standard deviation of the random variable X.

(b) What is $P[\mu_X - \sigma_X \le X \le \mu_X + \sigma_X]$, the probability that X is within one standard deviation of the expected value?

Problem 2.24 Show that the variance of $Y = aX + b$ is $Var[Y] = a^2 Var[X]$.

Problem 2.25 Given a random variable X with mean μ_X and variance σ_X^2, find the mean and variance of the standardized random variable

$$Y = \frac{(X - \mu_X)}{\sigma_X}.$$

Problem 2.26 In Problem 2.10, find $P_{X|B}(x)$, where the condition $B = \{|X| > 0\}$. What are $E[X|B]$ and $Var[X|B]$?

Problem 2.27 In Problem 2.23, find $P_{X|B}(x)$, where the condition $B = \{X \neq 0\}$. What are $E[X|B]$ and $Var[X|B]$?

2.2 Continuous Random Variables

Problem 2.28 The cumulative distribution function of random variable X is

$$F_X(x) = \begin{cases} 0, & x \leq -1, \\ (x+1)/2, & -1 < x \leq 1, \\ 1, & x > 1. \end{cases}$$

(a) What is $P[X \geq 1/2]$?

(b) What is $P[-1/2 \leq X < 3/4]$?

(c) What is $P[|X| > 1/2]$?

(d) What is the value of a such that $P[X < a] = 0.8$?

Problem 2.29 The cumulative distribution function of the continuous random variable V is

$$F_V(v) = \begin{cases} 0, & v \leq -5, \\ c(v+5)^2, & -5 < v \leq 7, \\ 1, & v > 7. \end{cases}$$

(a) What is c?

(b) What is $P[V \geq 4]$?

(c) $P[-3 \leq V < 0]$?

(d) What is the value of a such that $P[V \geq a] = 2/3$?

Problem 2.30 The random variable X has probability density function

$$f_X(x) = \begin{cases} cx, & 0 \leq x \leq 2, \\ 0, & \text{otherwise.} \end{cases}$$

Use the PDF to find

(a) the constant c,

(b) $P[0 \leq X \leq 1]$,

(c) $P[-1/2 \leq X \leq 1/2]$,

(d) the CDF $F_X(x)$.

Problem 2.31 The cumulative distribution function of random variable X is

$$F_X(x) = \begin{cases} 0, & x \leq -1, \\ (x+1)/2, & -1 < x \leq 1, \\ 1, & x > 1. \end{cases}$$

Find the PDF $f_X(x)$ of X.

Problem 2.32 Continuous random variable X has PDF

$$f_X(x) = \begin{cases} 1/4, & -1 \leq x \leq 3, \\ 0, & \text{otherwise.} \end{cases}$$

Define the random variable Y by $Y = h(X) = X^2$.

(a) Find $E[X]$ and $Var[X]$.

(b) Find $h(E[X])$ and $E[h(X)]$.

(c) Find $E[Y]$ and $Var[Y]$.

Problem 2.33 Random variable X has CDF

$$F_X(x) = \begin{cases} 0, & x \leq 0, \\ x/2, & 0 < x \leq 2, \\ 1, & x > 2. \end{cases}$$

(a) What is $E[X]$?

(b) What is $Var[X]$?

Problem 2.34 Y is an exponential random variable with variance $Var[Y] = 25$.

(a) What is the PDF of Y?

(b) What is $E[Y^2]$?

(c) What is $P[Y > 5]$?

Problem 2.35 X is a continuous uniform $(-5, 5)$ random variable.

(a) What is the PDF $f_X(x)$?

(b) What is the CDF $F_X(x)$?

(c) What is $E[X]$?

(d) What is $E[X^5]$?

(e) What is $E[e^X]$?

Problem 2.36 X is a uniform random variable with expected value $\mu_X = 7$ and variance $Var[X] = 3$. What is the PDF of X?

Problem 2.37 The peak temperature T, as measured in degrees Fahrenheit, on a July day in New Jersey is the Gaussian $(85, 10)$ random variable. What is $P[T > 100]$, $P[T < 60]$, and $P[70 \le T \le 100]$?

Problem 2.38 What is the PDF of Z, the standard normal random variable?

Problem 2.39 X is a Gaussian random variable with $E[X] = 0$ and $P[|X| \le 10] = 0.1$. What is the standard deviation σ_X?

Problem 2.40 The peak temperature T, in degrees Fahrenheit, on a July day in Antarctica is a Gaussian random variable with a variance of 225. With probability $1/2$, the temperature T exceeds 10 degrees. What is $P[T > 32]$, the probability the temperature is above freezing? What is $P[T < 0]$? What is $P[T > 60]$?

Problem 2.41 The voltage X across a 1 Ω resistor is a uniform random variable with parameters 0 and 1. The instantaneous power is $Y = X^2$. Find the CDF $F_Y(y)$ and the PDF $f_Y(y)$ of Y.

Problem 2.42 X is uniform random variable with parameters 0 and 1. Find a function $g(x)$ such that the PDF of $Y = g(X)$ is

$$f_Y(y) = \begin{cases} 3y^2, & 0 \le y \le 1, \\ 0, & \text{otherwise.} \end{cases}$$

Problem 2.43 X is a uniform random variable with parameters -5 and 5. Given the event $B = \{|X| \le 3\}$,

(a) Find the conditional PDF, $f_{X|B}(x)$.

(b) Find the conditional expected value, $E[X|B]$.

(c) What is the conditional variance, $Var[X|B]$?

Problem 2.44 Y is an exponential random variable with parameter $\lambda = 0.2$. Given the event $A = \{Y < 2\}$,

(a) What is the conditional PDF, $f_{Y|A}(y)$?

(b) Find the conditional expected value, $E[Y|A]$.

Problem 2.45 The cumulative distribution function of the continuous random variable X is

$$F_X(x) = \begin{cases} 0, & x \le 0 \\ \dfrac{1}{2} - k\cos x, & 0 < x \le \pi \\ 1, & x > \pi. \end{cases}$$

(a) What is k?

(b) What is $P[0 < X < \dfrac{\pi}{2}]$?

(c) What is $E[X]$?

Problem 2.46 The cumulative distribution function of the continuous random variable X is

$$F(x) = \begin{cases} 0, & x \le -a \\ A + B\arcsin\dfrac{x}{a}, & x \in (-a, a) \\ 1, & x \ge a. \end{cases}$$

(a) What are A and B?

(b) What is the PDF $f_X(x)$?

Problem 2.47 The cumulative distribution function of the continuous random variable X is $F(x) = a + b\arctan x$, $(-\infty < x < +\infty)$

(a) What are a and b?

(b) What is the PDF $f_X(x)$?

(c) What is $P[-1 < X < 1]$?

Problem 2.48 The cumulative distribution function of the continuous random variable X is $F(x) = 1/2 + 1/\pi\arctan x/2$. What is the value of x_1 such that $P(X > x_1) = 1/4$?

Problem 2.49 The continuous random variable X has probability density function

$$f(x) = \begin{cases} k\sin 3x, & x \in \left(0, \dfrac{\pi}{3}\right) \\ 0, & x \notin \left(0, \dfrac{\pi}{3}\right). \end{cases}$$

Use the PDF to find

(a) the constant k,

(b) $P[\pi/6 \le X < \pi/3]$,

(c) the CDF $F_X(x)$.

Problem 2.50 The continuous random variable X has PDF

$$f(x) = \frac{c}{e^x + e^{-x}}.$$

What is $E[X]$?

Problem 2.51 The continuous random variable X has PDF $f(x) = ae^{-|x|}$, $(-\infty < x < +\infty)$. Define the random variable Y by $Y = X^2$.

(a) What is a?

(b) What is the CDF $F_Y(x)$?

(c) What is $E[X]$? What is $Var[X]$?

Problem 2.52 Y is an exponential random variable with the PDF $f_X(x)$ is

$$f(x) = \begin{cases} 5e^{-5x}, & x > 0 \\ 0, & x \leq 0 \end{cases}$$

(a) What is $E[X]$?

(b) What is $P[0,4 < X < 1]$?

Problem 2.53 X is a Gaussian random variable with $E[X] = 0$ and $\sigma_X = 0,4$.

(a) What is $P[X > 3]$?

(b) What is the value of c such that $P[3 - c < X < 3 + c] = 0,9$?

Chapter 3

Important Probability Distributions

3.1 Some Discrete Probability Distributions

Problem 3.1 (see Problem 2.6) In a package of M&Ms, Y, the number of yellow M&Ms, is uniformly distributed between 5 and 15.

(a) What is the PMF of Y?

(b) What is $P[Y < 10]$?

(c) What is $P[Y > 12]$?

(d) What is $P[8 \leq Y \leq 12]$?

Problem 3.2 (see Problem 2.7) When a conventional paging system transmits a message, the probability that the message will be received by the pager it is sent to is p. To be confident that a message is received at least once, a system transmits the message n times.

(a) Assuming all transmissions are independent, what is the PMF of K, the number of times the pager receives the same message?

(b) Assume $p = 0.8$. What is the minimum value of n that produces a probability of 0.95 of receiving the message at least once?

Problem 3.3 (see Problem 2.8) When a two-way paging system transmits a message, the probability that the message will be received by the pager it is sent to is p. When the pager receives the message, it transmits an acknowledgment signal (ACK) to the paging system. If the paging system does not receive the ACK, it sends the message again.

(a) What is the PMF of N, the number of times the system sends the same message?

(b) The paging company wants to limit the number of times it has to send the same message. It has a goal of $P[N \leq 3] \geq 0.95$. What is the minimum value of p necessary to achieve the goal?

Problem 3.4 Four microchips are to be placed in a computer. Two of the four chips are randomly selected for inspection before assembly of the computer. Let X denote the number of defective chips found among the two chips inspected. Find the probability mass and distribution function of X if

(a) Two of the microchips were defective.

(b) One of the microchips was defective.

(c) None of the microchips was defective.

Problem 3.5 A four engine plane can fly if at least two engines work.

(a) If the engines operate independently and each malfunctions with probability q, what is the probability that the plane will fly safely?

(b) A two engine plane can fly if at least one engine works and if an engine malfunctions with probability q, what is the probability that plane will fly safely?

(c) Which plane is the safest?

Problem 3.6 A rat maze consists of a straight corridor, at the end of which is a branch; at the branching point the rat must either turn right or left. Assume 10 rats are placed in the maze, one at a time.

(a) If each is choosing one of the two branches at random, what is the distribution of the number that turn right?

(b) What is the probability at least 9 will turn the same way?

Problem 3.7 A student who is trying to write a paper for a course has a choice of two topics, A and B. If topic A is chosen, the student will order 2 books through interlibrary loan, while if topic B is chosen, the student will order 4 books. The student feels that a good paper necessitates receiving and using at least half the books ordered for either topic chosen.

(a) If the probability that a book ordered through interlibrary loan actually arrives on time is 0.9 and books arrive independently of one another, which 2 topics should the student choose to maximize the probability of writing a good paper?

(b) What if, the arrival probability is only 0.5 instead of 0.9?

Problem 3.8 The number of phone calls at a post office in any time interval is a Poisson random variable. A particular post office has on average 2 calls per minute.

(a) What is the probability that there are 5 calls in an interval of 2 minutes?

(b) What is the probability that there are no calls in an interval of 30 seconds?

(c) What is the probability that there are no less than one call in an interval of 10 seconds?

Problem 3.9 (see Problem 1.41) An airline sells 200 tickets for a certain flight on an airplane that has only 198 seats because, on the average, 1 percent of purchasers of airline tickets do not appear for the departure of their flight. Determine the probability that everyone who appears for the departure of this flight will have a seat.

3.2 Some Continuous Probability Distributions

Problem 3.10 X is an exponential random variable with the PDF $f_X(x)$ is

$$f_X(x) = \begin{cases} 5e^{-5x}, & x \geq 0 \\ 0, & x < 0 \end{cases}$$

(a) What is $E[X]$?

(b) What is $P[0.4 < X < 1]$?

Problem 3.11 X is a Gaussian random variable with $E[X] = 0$ and $\sigma_X = 0.4$.

(a) What is $P[X > 3]$?

(b) What is the value of c such that $P[3 - c < X < 3 + c] = 0,9$?

Problem 3.12 Let X be an exponential random variable with parameter and define $Y = [X]$, the largest integer in X, (ie. $[x] = 0$ for $0 \leq x < 1$, $[x] = 1$ for $1 \leq x < 2$ etc.)

(a) Find the probability function for Y.

(b) Find $E(Y)$.

(c) Find the distribution function of Y.

(d) Let Y represent the number of periods that a machine is in use before failure. What is the probability that the machine is still working at the end of 10th period given that it does not fail before 6th period?

Problem 3.13 Starting at 5:00 am, every half hour there is a flight from San Francisco airport to Los Angeles International Airport. Suppose that none of these planes sold out and that they alwayshave room for passengers. A person who wants to fly LA arrives at the airport at a random time between 8:45–9:45 am. Find the probability that she waits at most 10 minutes and at least 15 minutes.

Problem 3.14 X is a Gaussian random variable with $E[X] = 0$ and $P[|X| \leq 10] = 0.1$. What is the standard deviation σ_X?

Chapter 4

Pairs of Random Variables

Problem 4.1 Random variables X and Y have the joint CDF

$$F_{X,Y}(x,y) = \begin{cases} (1-e^{-x})(1-e^{-y}), & x \geq 0,\ y \geq 0, \\ 0, & \text{otherwise.} \end{cases}$$

(a) What is $P[X < 2, Y < 3]$?

(b) What is the marginal CDF, $F_X(x)$?

(c) What is the marginal CDF, $F_Y(y)$?

Problem 4.2 Express the following extreme values of $F_{X,Y}(x,y)$ in terms of the marginal cumulative distribution functions $F_X(x)$ and $F_Y(y)$.

(a) $F_{X,Y}(x,-\infty)$.

(b) $F_{X,Y}(x,\infty)$.

(c) $F_{X,Y}(-\infty,\infty)$.

(d) $F_{X,Y}(-\infty,y)$.

(e) $F_{X,Y}(\infty,y)$.

Problem 4.3 Random variables X and Y have CDF $F_X(x)$ and $F_Y(y)$. Is $F_{X,Y}(x,y) = F_X(x)F_Y(y)$ a valid CDF? Explain your answer.

Problem 4.4 Random variables X and Y have the joint PMF

$$P_{X,Y}(x,y) = \begin{cases} cxy, & x = 1,2,4,\ y = 1,3, \\ 0, & \text{otherwise.} \end{cases}$$

(a) What is the value of the constant c?

(a) What is $P[Y < X]$?

(b) What is $P[Y > X]$?

(c) What is $P[Y = X]$?

(d) What is $P[Y = 3]$?

Problem 4.5 Random variables X and Y have the joint PMF

$$P_{X,Y}(x,y) = \begin{cases} c|x+y|, & x = -2,0,2,\ y = -1,0,1, \\ 0, & \text{otherwise.} \end{cases}$$

(a) What is the value of the constant c?

(b) What is $P[Y < X]$?

(c) What is $P[Y > X]$?

(d) What is $P[Y = X]$?

(e) What is $P[X < 1]$?

Problem 4.6 Given the random variables X and Y in Problem 4.4, find

(a) The marginal PMFs $P_X(x)$ and $P_Y(y)$,

(b) The expected values $E[X]$ and $E[Y]$,

(c) The standard deviations σ_X and σ_Y.

Problem 4.7 Given the random variables X and Y in Problem 4.5, find

(a) The marginal PMFs $P_X(x)$ and $P_Y(y)$,

(b) The expected values $E[X]$ and $E[Y]$,

(c) The standard deviations σ_X and σ_Y.

Problem 4.8 Random variables X and Y have the joint PDT

X \ Y	1	2	3
1	0.12	0.15	0.03
2	0.28	0.35	0.07

(a) Are X and Y independent?.

(b) Find the marginal PDTs of X and Y.

(c) Find the PDT of Z, where $Z = XY$.

(d) Find $E(Z)$. Proof $E(Z) = E(X).E(Y)$.

Problem 4.9 Random variables X and Y have the joint PDT

$X \backslash Y$	-1	0	1
-1	$4/15$	$1/15$	$4/15$
0	$1/15$	$2/15$	$1/15$
1	0	$2/15$	0

(a) Find $E(X)$, $E(Y)$, and $Cov(X,Y)$.

(b) Are X and Y independent?

(c) Find the marginal PDTs of X and Y.

Problem 4.10 Random variables X and Y have the joint PDT

$X \backslash Y$	1	2	3
1	0.17	0.13	0.25
2	0.10	0.30	0.05

(a) Find the marginal PDTs of X and Y.

(b) Find the covariance matrix of X and Y.

(c) Find the correlation coefficient of two random variables X and Y.

(d) Are X and Y independent?

Problem 4.11 Random variables X and Y have the joint PDF

$$f_{X,Y}(x,y) = \begin{cases} c, & x+y \leq 1,\ x \geq 0,\ y \geq 0, \\ 0, & \text{otherwise.} \end{cases}$$

(a) What is the value of the constant c?

(b) What is $P[X \leq Y]$?

(c) What is $P[X+Y \leq 1/2]$?

Problem 4.12 Random variables X and Y have joint PDF

$$f_{X,Y}(x,y) = \begin{cases} cxy^2, & 0 \leq x \leq 1,\ 0 \leq y \leq 1, \\ 0, & \text{otherwise} \end{cases}$$

(a) Find the constant c.

(b) Find $P[X > Y]$ and $P[Y < X^2]$.

(c) Find $P[\min(X,Y) \leq 1/2]$.

(d) Find $P[\max(X,Y) \leq 3/4]$.

Problem 4.13 Random variables X and Y have the joint PDF

$$f_{X,Y}(x,y) = \begin{cases} 1/2, & -1 \le x \le y \le 1, \\ 0, & \text{otherwise.} \end{cases}$$

(a) Sketch the region of nonzero probability.

(b) What is $P[X > 0]$?

(c) What is $f_X(x)$?

(d) What is $E[X]$?

Problem 4.14 X and Y are random variables with the joint PDF

$$f_{X,Y}(x,y) = \begin{cases} 2, & x+y \le 1,\ x \ge 0,\ y \ge 0, \\ 0, & \text{otherwise.} \end{cases}$$

(a) What is the marginal PDF $f_X(x)$?

(b) What is the marginal PDF $f_Y(y)$?

Problem 4.15 Random variables X and Y have the joint PDF

$$f_{X,Y}(x,y) = \begin{cases} cy, & 0 \le y \le x \le 1, \\ 0, & \text{otherwise.} \end{cases}$$

(a) Draw the region of nonzero probability.

(b) What is the value of the constant c?

(c) What is $F_X(x)$?

(d) What is $F_Y(y)$?

(e) What is $P[Y \le X/2]$?

Problem 4.16 Given random variables X and Y in Problem 4.5 and the function $W = X + 2Y$, find

(a) The probability mass function $P_W(w)$,

(b) The expected value $E[W]$,

(c) $P[W > 0]$.

Problem 4.17 Random variables X and Y have joint PDF

$$f_{X,Y}(x,y) = \begin{cases} x+y, & 0 \le x \le 1,\ 0 \le y \le 1, \\ 0, & \text{otherwise.} \end{cases}$$

Let $W = \max(X, Y)$.

(a) What is S_W, the range of W?

(b) Find $F_W(w)$ and $f_W(w)$.

Problem 4.18 Random variables X and Y have joint PDF

$$f_{X,Y}(x,y) = \begin{cases} 2, & 0 \le y \le x \le 1, \\ 0, & \text{otherwise.} \end{cases}$$

Let $W = Y/X$.

(a) What is S_W, the range of W?

(b) Find $F_W(w)$, $f_W(w)$, and $E[W]$.

Problem 4.19 For the random variables X and Y in Problem 4.4, find

(a) The expected value of $W = Y/X$,

(b) The correlation, $E[XY]$,

(c) The covariance, $Cov[X,Y]$,

(d) The correlation coefficient, $\rho_{X,Y}$,

(e) The variance of $X+Y$, $Var[X+Y]$.

Problem 4.20 Random variables X and Y have joint PMF

$$P_{X,Y}(x,y) = \begin{cases} 1/21, & x = 0,1,2,3,4,5,\ y = 0,1,\ldots,x, \\ 0, & \text{otherwise.} \end{cases}$$

Find the marginal PMFs $PX_(x)$ and $P_Y(y)$. Also find the covariance $Cov[X,Y]$.

Problem 4.21 Random variables X and Y have joint PDF

$$f_{X,Y}(x,y) = \begin{cases} (x+y)/3, & 0 \le x \le 1,\ 0 \le y \le 2, \\ 0, & \text{otherwise.} \end{cases}$$

(a) What are $E[X]$ and $Var[X]$?

(b) What are $E[Y]$ and $Var[Y]$?

(c) What is $Cov[X,Y]$?

(d) What is $E[X+Y]$?

(e) What is $Var[X+Y]$?

Problem 4.22 Random variables X and Y have joint PDF

$$f_{X,Y}(x,y) = \begin{cases} (x+y)/3, & 0 \le x \le 1,\ 0 \le y \le 2, \\ 0, & \text{otherwise.} \end{cases}$$

Let $A = \{Y \le 1\}$.

(a) What is $P[A]$?

(b) Find $f_{X,Y|A}(x,y)$, $f_{X|A}(x)$, and $f_{Y|A}(y)$.

Problem 4.23 Random variables X and Y have joint PDF

$$f_{X,Y}(x,y) = \begin{cases} 5x^2/2, & -1 \le x \le 1,\ 0 \le y \le x^2, \\ 0, & \text{otherwise.} \end{cases}$$

Let $A = \{Y \le 1/4\}$.

(a) What is the conditional PDF $f_{X,Y|A}(x,y)$?

(b) What is $f_{Y|A}(y)$?

(c) What is $E[Y|A]$?

(d) What is $f_{X|A}(x)$?

(e) What is $E[X|A]$?

Problem 4.24 X and Y have joint PDF

$$f_{X,Y}(x,y) = \begin{cases} (4x+2y)/3, & 0 \le x \le 1,\ 0 \le y \le 1, \\ 0, & \text{otherwise.} \end{cases}$$

(a) For which values of y is $f_{X|Y}(x|y)$ defined? What is $f_{X|Y}(x|y)$?

(b) For which values of x is $f_{Y|X}(y|x)$ defined? What is $f_{Y|X}(y|x)$?

Problem 4.25 The joint PDF of two random variables X and Y is

$$f_{X,Y}(x,y) = \begin{cases} kx, & \text{if } 0 < y < x < 1, \\ 0, & \text{otherwise.} \end{cases}$$

(a) Find the constant k.

(b) Find the PDFs of X and Y.

(c) Are X and Y independent?

Chapter 5

Random Sample

Problem 5.1 The lengths of time, in minutes, that 10 patients waited in a doctor's office before receiving treatment were recorded as follows: 5, 11, 9, 5, 10, 15, 6, 10, 5, and 10. Treating the data as a random sample, find (a) the mean; (b) the median; (c) the mode.

Problem 5.2 The numbers of incorrect answers on a true-false competency test for a random sample of 15 students were recorded as follows: 2, 1, 3, 0, 1, 3, 6, 0, 3, 3, 5, 2, 1, 4, and 2. Find (a) the mean; (b) the median; (c) the mode.

Problem 5.3 The grade-point averages of 20 college seniors selected at random from a graduating class are as follows: 3.2, 1.9, 2.7, 2.4, 2.8, 2.9, 3.8, 3.0, 2.5, 3.3, 1.8, 2.5, 3.7, 2.8, 2.0, 3.2, 2.3, 2.1, 2.5, 1.9. Calculate the standard deviation.

Problem 5.4 (a) Find $t_{0.025}$ when $\nu = 14$. (b) Find $-t_{0.10}$ when $\nu = 10$. (c) Find $t_{0.995}$ when $\nu = 7$.

Problem 5.5 (a) Find $P(T < 2.365)$ when $\nu = 7$. (b) Find $P(T > 1.318)$ when $\nu = 24$. (c) Find $P(-1.356 < T < 2.179)$ when $\nu = 12$. (d) Find $P(T > -2.567)$ when $\nu = 17$.

Problem 5.6 Given a random sample of size 24 from a normal distribution, find k such that

(a) $P(-2.069 < T < k) = 0.965$;

(b) $P(k < T < 2.807) = 0.095$;

(c) $P(-k < T < k) = 0.90$.

Problem 5.7 A manufacturing firm claims that the batteries used in their electronic games will last an average of 30 hours. To maintain this average, 16 batteries are tested each month. If the computed t-value falls between $-t_{0.025}$ and $t_{0.025}$, the firm is satisfied with its claim. What conclusion should the firm draw from a sample that has a mean of $\overline{x} = 27.5$ hours and a standard deviation of $s = 5$ hours? Assume the distribution of battery lives to be approximately normal.

Chapter 6

One-Sample Estimation Problems

Problem 6.1 The average zinc concentration recovered from a sample of measurements taken in 36 different locations in a river is found to be 2.6 grams per milliliter.

(a) Find the 95% and 99% confidence intervals for the mean zinc concentration in the river. Assume that the population standard deviation is 0.3 gram per milliliter.

(b) What are the errors?

(c) How large a sample is required if we want to be 95% confident that our estimate of μ is off by less than 0.05?

Problem 6.2 An electrical firm manufactures light bulbs that have a length of life that is approximately normally distributed with a standard deviation of 40 hours. If a sample of 30 bulbs has an average life of 780 hours, find a 96% confidence interval for the population mean of all bulbs produced by this firm.

Problem 6.3 In a psychological testing experiment, 25 subjects are selected randomly and their reaction time, in seconds, to a particular stimulus is measured. Past experience suggests that the variance in reaction times to these types of stimuli is 4 sec^2 and that the distribution of reaction times is approximately normal. The average time for the subjects is 6.2 seconds. Give an upper 95% bound for the mean reaction time.

Problem 6.4 The contents of seven similar containers of sulfuric acid are 9.8, 10.2, 10.4, 9.8, 10.0, 10.2, and 9.6 liters. Find a 95% confidence interval for the mean contents of all such containers, assuming an approximately normal distribution.

Problem 6.5 A machine produces metal pieces that are cylindrical in shape. A sample of pieces is taken, and the diameters are found to be 1.01, 0.97, 1.03, 1.04, 0.99, 0.98, 0.99, 1.01, and 1.03 centimeters. Find a 99% confidence interval for the mean diameter of pieces from this machine, assuming an approximately normal distribution.

Problem 6.6 The following measurements were recorded for the drying time, in hours, of a certain brand of latex paint:

3.4 2.5 4.8 2.9 3.6 2.8 3.3 5.6 3.7 2.8 4.4 4.0 5.2 3.0 4.8

Assuming that the measurements represent a random sample from a normal population, find a 95% prediction interval for the drying time for the next trial of the paint.

Problem 6.7 Scholastic Aptitude Test (SAT) mathematics scores of a random sample of 500 high school seniors in the state of Texas are collected, and the sample mean and standard deviation are found to be 501 and 112, respectively. Find a 99% confidence interval on the mean SAT mathematics score for seniors in the state of Texas.

Problem 6.8 The heights of a random sample of 50 college students showed a mean of 174.5 centimeters and a standard deviation of 6.9 centimeters.

(a) Construct a 98% confidence interval for the mean height of all college students.

(b) What can we assert with 98% confidence about the possible size of our error if we estimate the mean height of all college students to be 174.5 centimeters?

Problem 6.9 In a random sample of $n = 500$ families owning television sets in the city of Hamilton, Canada, it is found that $m = 340$ subscribe to HBO.

(a) Find a 95% confidence interval for the actual proportion of families with television sets in this city that subscribe to HBO.

(b) What is error?

(c) How large a sample is required if we want to be 95% confident that our estimate of p is within 0.02 of the true value?

Problem 6.10 In a random sample of 1000 homes in a certain city, it is found that 228 are heated by oil. Find 99% confidence intervals for the proportion of homes in this city that are heated by oil.

Problem 6.11 (a) A random sample of 200 voters in a town is selected, and 114 are found to support an annexation suit. Find the 96% confidence interval for the fraction of the voting population favoring the suit. (b) What can we assert with 96% confidence about the possible size of our error if we estimate the fraction of voters favoring the annexation suit to be 0.57?

Problem 6.12 A geneticist is interested in the proportion of African males who have a certain minor blood disorder. In a random sample of 100 African males, 24 are found to be afflicted.

(a) Compute a 99% confidence interval for the proportion of African males who have this blood disorder.

(b) What can we assert with 99% confidence about the possible size of our error if we estimate the proportion of African males with this blood disorder to be 0.24?

Chapter 7

One- and Two-Sample Test of Hypotheses

Problem 7.1 The average weekly earnings for female social workers is \$670. Do men in the same positions have average weekly earnings that are higher than those for women? A random sample of $n = 40$ male social workers showed $\overline{x} = \$725$. Assuming a population standard deviation of \$102, test the appropriate hypothesis using $\alpha = 0.01$.

Problem 7.2 A random sample of 64 bags of white cheddar popcorn weighed, on average, 5.23 ounces with a standard deviation of 0.24 ounce. Test the hypothesis that $\mu = 5.5$ ounces against the alternative hypothesis, $\mu < 5.5$ ounces, at the 0.05 level of significance.

Problem 7.3 A local telephone company claims that the average length of a phone call is 8 minutes. In a random sample of 18 phone calls, the sample mean was 7.8 minutes and the standard deviation was 0.5 minutes. Is there enough evidence to support this claim at $\alpha = 0.05$?

Problem 7.4 Test the hypothesis that the average content of containers of a particular lubricant is 10 liters if the contents of a random sample of 10 containers are 10.2, 9.7, 10.1, 10.3, 10.1, 9.8, 9.9, 10.4, 10.3, and 9.8 liters. Use a 0.01 level of significance and assume that the distribution of contents is normal.

Problem 7.5 According to a dietary study, high sodium intake may be related to ulcers, stomach cancer, and migraine headaches. The human requirement for salt is only 220 milligrams per day, which is surpassed in most single servings of ready-to-eat cereals. If a random sample of 20 similar servings of a certain cereal has a mean sodium content of 244 milligrams and a sample standard deviation of 24.5 milligrams, does this suggest at the 0.05 level of significance that the average sodium content for a single serving of such cereal is greater than 220 milligrams? Assume the distribution of sodium content to be normal.

Problem 7.6 The daily yield for a local chemical plant has averaged 880 tons for the last several years. The quality control manager would like to know whether this average has changed in recent months. She randomly selects 50 days from the computer database and computes the average and sample standard deviation of the $n = 50$ yields as $\overline{x} = 871$ tons and $s = 21$ tons, respectively. Test the appropriate hypothesis using $\alpha = 0.05$.

Problem 7.7 A college claims that more than 94% of their graduates find employment within 6 months of graduation. In a sample of 500 randomly selected graduates, 475 of them were employed. Is there enough evidence to support the college's claim at a 1% level of significance?

Problem 7.8 A cigarette manufacturer claims that 1/8 of the US adult population smokes cigarettes. In a random sample of 100 adults, 5 are cigarette smokers. Test the claim at $\alpha = 0.05$.

Problem 7.9 A marketing expert for a pasta-making company believes that 40% of pasta lovers prefer lasagna. If 9 out of 20 pasta lovers choose lasagna over other pastas, what can be concluded about the expert's claim? Use a 0.05 level of significance.

Problem 7.10 It is believed that at least 60% of the residents in a certain area favor an annexation suit by a neighboring city. What conclusion would you draw if only 110 in a sample of 200 voters favored the suit? Use a 0.05 level of significance.

Problem 7.11 A high school math teacher claims that students in her class will score higher on the math portion of the ACT then students in a colleague's math class. The mean ACT math score for 49 students in her class is 22.1 and the sample standard deviation is 4.8. The mean ACT math score for 44 of the colleague's students is 19.8 and the sample standard deviation is 5.4. At $\alpha = 0.10$, can the teacher's claim be supported?

Problem 7.12 To determine whether car ownership affects a student's academic achievement, two random samples of 100 male students were each drawn from the student body. The grade point average for the $n_1 = 100$ non-owners of cars had an average and variance equal to $\overline{x}_1 = 2.70$ and $s_1^2 = 0.36$, while $\overline{x}_2 = 2.54$ and $s_2^2 = 0.40$ for the $n_2 = 100$ car owners. Do the data present sufficient evidence to indicate a difference in the mean achievements between car owners and nonowners of cars? Test using $\alpha = 0.05$.

Problem 7.13 A manufacturer claims that the average tensile strength of thread A exceeds the average tensile strength of thread B by at least 12 kilograms. To test this claim, 50 pieces of each type of thread were tested under similar conditions. Type A thread had an average tensile strength of 86.7 kilograms with a standard deviation of 6.28 kilograms, while type B thread had an average tensile strength of 77.8 kilograms with a standard deviation of 5.61 kilograms. Test the manufacturer's claim using a 0.05 level of significance.

Problem 7.14 Engineers at a large automobile manufacturing company are trying to decide whether to purchase brand A or brand B tires for the company's new models. To help them arrive at a decision, an experiment is conducted using 12 of each brand. The tires are run until they wear out. The results are as follows:

Brand A: $\overline{x}_A = 37,900$ kilometers, $s_A = 5100$ kilometers.

Brand B: $\overline{x}_B = 39,800$ kilometers, $s_B = 5900$ kilometers.

Test the hypothesis that there is no difference in the average wear of the two brands of tires. Assume the populations to be approximately normally distributed with equal variances. Use a 0.01 level of significance.

Problem 7.15 A recent survey stated that male college students smoke less than female college students. In a survey of 1245 male students, 361 said they smoke at least one pack of cigarettes a day. In a survey of 1065 female students, 341 said they smoke at least one pack a day. At $\alpha = 0.01$, can you support the claim that the proportion of male college students who smoke at least one pack of cigarettes a day is lower then the proportion of female college students who smoke at least one pack a day?

Problem 7.16 In a study to estimate the proportion of residents in a certain city and its suburbs who favor the construction of a nuclear power plant, it is found that 63 of 100 urban residents favor the construction while only 59 of 125 suburban residents are in favor. Is there a significant difference between the proportions of urban and suburban residents who favor the construction of the nuclear plant? Use a 0.01 level of significance.