Experiment 3

MOMENT OF INERTIA OF THE SYMMETRIC RIGID BODIES

I. THEORETICAL BACKGROUND

It is known that the moment of inertia of the body about the axis of rotation is determined by

$$I = \int r^2 dm \tag{1}$$

Where dm is the mass element and r is the distance from the mass element to the axis of rotation. In the m.k.s. system of units, the units of I are kgm^2/s.

If the axis of rotation is chosen to be through the center of mass of the object, then the moment of inertia about the center of mass axis is call I_{cm}. In case of the typical symmetric and homogenous rigid bodies, I_{cm}.is calculated as follows

- For a long bar: $I_{cm} = \frac{1}{12} ml^2$ (2)

- For a thin disk or a solid cylinder: $I_{cm} = \frac{1}{2} mR^2$ (3)

- For a hollow cylinder having very thin wall: $I_{cm} = mR^2$ (4)

- For a solid sphere: $I_{cm} = \frac{2}{5} mR^2$ (5)

The parallel-axis theorem relates the moment of inertia Icm about an axis through the center of mass to the moment of inertia I about a parallel axis through some other point. The theorem states that,

$$I = I_{cm} + Md^2 \tag{6}$$

This implies I_{cm} is always less than I about any other axis.

In this experiment, the moment of inertia of a rigid body will be determined by using an apparatus which consists of a spiral spring (made of brass). The object whose moment of inertia is to be measured can be mounted on the axis of this torsion spring which tends to restrict the rotary motion of the object and provides a restoring torque. If the object is rotated by an angle ϕ, the torque acting on it will be

$$\tau_z = D_z..\phi \tag{7}$$

where D_z. is a elastic constant of spring.

This torque will make the object oscillation. Using the theorem of angular momentum of a rigide body in rotary motion.

$$\tau = \frac{dL}{dt} = I\frac{d\omega}{dt} = I\frac{d^2\phi}{dt^2} \tag{8}$$

We get the typical equation of oscillation as

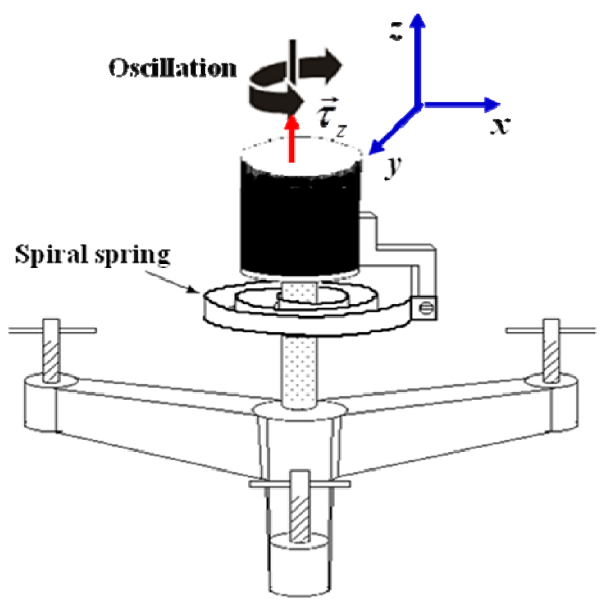

Fig. 1. *Experimental model to determine the moment of inertia of the rigid body*

$$\frac{d^2\phi}{dt^2} + \frac{D_z}{I}\phi = 0 \tag{9}$$

The oscillation is corresponds to a period

$$T = 2\pi\sqrt{\frac{I}{D_z}} \tag{10}$$

According to (10), for a known D_z, the unknown moment of inertia of an object can be found if the period T is measured.

II. EQUIPMENT

1. Rotation axle with spiral spring having the elastic constant, D_z = 0,044 *Nm/Rad*;
2. Light barrier (or photogate) with counter;
3. Rod with length of 620mm and mass of 240g;
4. Solid sphere with mass of 2290g and diameter of 146mm;
5. Solid disk with mass of 795g and diameter of 220mm;
6. Hollow cylinder with mass of 780g and diameter of 89mm;
7. Supported thin disk;
8. A set of screws for mounting the objects;
9. Tripod base.

Fig. 2. *Equipments for measurment*

III. EXPERIMENTAL PROCEDURE

3. 1. Measurement of the rod

- ***Step 1:*** Equipment is setup corresponding to Fig.3. A mask (width ~ 3 mm) is stuck on the rod to ensure the rod went through the photogate.
- ***Step 2:*** Press the button "Start" to turn on the counter. Then, you can see the light of LED on the photogate.
- ***Step 3:*** Push the rod to rotate with an angle of 180^0, then let it to oscillate freely. The time of a vibration period of the rod will be measured. In this case, the result you got is averaged after several periods. Make 5 trials and record the measurement result of period T in a data sheet.

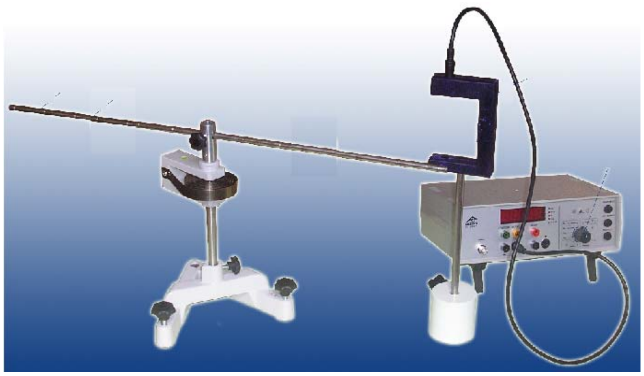

Fig. 3. *Experimental setup for measurement of the rod*

- ***Step 4:*** Press the button "Reset" to turn the display of the counter being 0. Uninstall the rod for next measurement.

3.2 Measurement of the solid disk

- Using the suitable screws to mount the solid disk on the rotation axle of the spiral spring as shown in Fig.4. A piece of note paper is stuck on the disk to ensure it passing through the photogate.
- Perform the measurement procedure similar to that of the rod. Record the measurement result of period *T* in a data sheet.
- Press the button "Reset" to turn the display of the counter being 0. Uninstall the disk for next measurement.

Fig. 4. *Experimental setup for measurement of the solid disk*

3.3 Measurement of the hollow cylinder

- Using the suitable screws to mount the hollow cylinder coupled with a supported disk below on the rotation axle of the spiral spring as shown in Fig.5. A piece of note paper is also stuck on the disk to ensure the system passing through the photogate.
- Perform the measurement procedure similar to that of the disk. Record the measurement result of period *T* (5 trials) in a data sheet.
- Push the button "Reset" to turn the display of the counter being 0. Uninstall the hollow cylinder and repeat the measurment to get its rotary period *T* (5 trials) ..

Fig. 5. *Experimental setup for measurement of the hollow cylinder*

- Press the button "Reset" to turn the display of the counter being 0. Uninstall the supported disk for next measurement.

3.4 Measurement of the Solid Sphere

- Mount the solid sphere on the rotation axle of the spiral spring as shown in Fig.6. A piece of note paper is also stuck on the sphere to ensure its passing through the photogate.
- Push the sphere to rotate with an angle of 270^0, then let it to oscillate freely. The obtained vibration period of the sphere will be recorded (5 trials) in the data sheet.
- Uninstall the solid sphere and switch off the

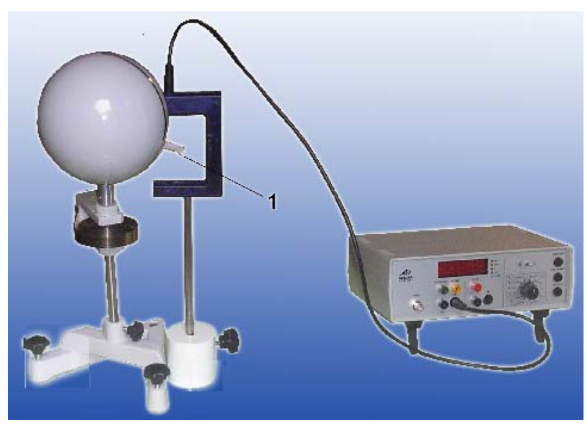

Fig. 6. *Experimental setup for measurement of the solid sphere*

counter to finish the measurements.

III. LAB REPORT

Your lab report should include:
1. A data sheet of the vibration periods of the measured rigid bodies.
2. Determine the average value of the vibration periods of corresponding bodies and then calculate the moment of inertia of the rod, solid disk, and solid sphere using equation (10).
3. The moment of inertia of the hollow cylinder is calculated by subtracting that of alone supported disk from the coupled object (consisting of the cylinder and supported disk).
4. Calculate the uncertainty of the moment of inertia obtained by experiment.
5. Calculate the value of moment of inertia of the rigid bodies based on the theoretical formula (2 to 5) and compare them to the measured values. Note that you use the relatively difference as an estimate of the errors.
6. ***Note***: Please read the instruction of "***Significant Figures***" on page 6 of the document "***Theory of Uncertainty***" to know the way for reporting the last result.