Experiment 2

VERIFICATION OF CONSERVATION OF MOMENTUM AND KINETIC ENERGY USING AIR TRACK

Instruments:
1. Air track;
2. Blower;
3. Two photogates;
4. Two digital timers;
5. Two gliders attached with flags (devices set on top of gliders to facilitate timing);
6. Glider brackets and steel spring.

I. THEORETICAL BACKGROUND

1. Momentum and conservation of momentum

Newton's Second Law of motion states that force acting on a mass causes it to accelerate in the direction of the force as follows,

$$\Sigma \vec{F} = m\vec{a} \tag{1}$$

Equation (1) is also called the fundamental equation of dynamics of materials point. It also can be applied for translational motion of the rigid body.
Based on the definition of acceleration, we have:

$$\Sigma \vec{F} = m\vec{a} = m\frac{d\vec{v}}{dt} = \frac{d(m\vec{v})}{dt} \tag{2}$$

A product of the particle's mass and velocity is called the momentum or linear momentum of the particle ($\vec{p} = m\vec{v}$), then we get:

$$\Sigma \vec{F} = \frac{d\vec{p}}{dt} \tag{3}$$

Keep in mind that momentum is a vector quantity with the same direction as the particle's velocity.

The concept of momentum is particularly important in situations in which we have two or more interacting bodies. For any system, the forces that the particles of the system exert on each other are called internal forces. Forces exerted on any part of the system by some object outside it are called external forces. For the system, the internal forces are cancelled due to the Newton's third law. Then, if the vector sum of the external forces is zero, the time rate of change of the total momentum is zero. Hence, the total momentum of the system is constant:

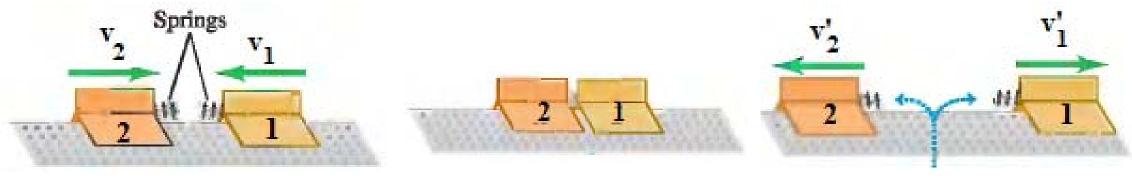
$$\Sigma \vec{F} = 0 = \frac{d\vec{p}}{dt} \Rightarrow \vec{p} = const \tag{3}$$

This result is called the principle of conservation of momentum.

2. Elastic and inelastic collision

2.1 Elastic collision

If the forces between the bodies are much larger than any external forces, as is the case in most collisions, we can neglect the external forces entirely and treat the bodies as an isolated system. The momentum of an individual object may change, but the total for the system does not. Then momentum is conserved and the total momentum of the system has the same value before and after the collision. If the forces between the bodies are also conservative, so that no mechanical energy is lost or gained in the collision, the total kinetic energy of the system is the same after the collision as before. *Such a collision is called an elastic collision*. This case can be illustrated by an example in which two gliders undergoing a collision on a frictionless surface and each glider has a steel spring bumper that exerts a conservative force on the other glider as shown in Fig.1.



(a) (b) (c)

Fig.1. *Before collision (a), elastic collision (b) and after collision (c)*

Remember this rule:

- In any collision in which external forces can be neglected, momentum is conserved and the total momentum before equals the total momentum after:

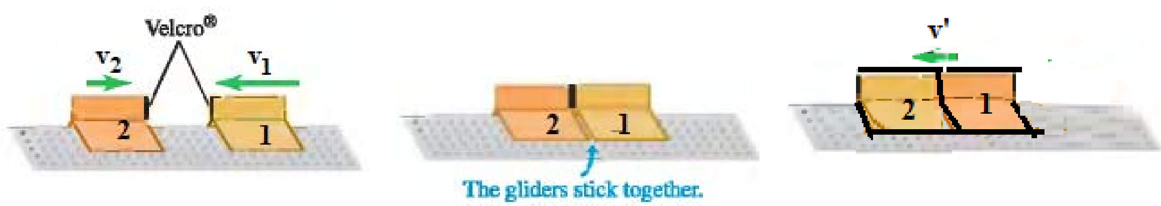
$$m_1\vec{v}_1 + m_2\vec{v}_2 = m_1\vec{v}_1' + m_2\vec{v}_2' \tag{4}$$

- In elastic collisions only, the total kinetic energy before equals the total kinetic energy after:

$$\frac{1}{2}m_1\vec{v}_1^2 + \frac{1}{2}m_1\vec{v}_2^2 = \frac{1}{2}m_1\vec{v}'^2_1 + \frac{1}{2}m_1\vec{v}'^2_2 \tag{5}$$

2.2 Inelastic collision

A collision in which the total kinetic energy after the collision is less than before the collision is called an inelastic collision. An inelastic collision in which the colliding bodies stick together and move as one body after the collision is often called a completely inelastic collision. The phenomenon is represented in Fig.2.



(a) (b) (c)

Fig.2. *Before collision (a), completely inelastic collision (b) and after collision (c)*

Conservation of momentum gives the relationship:

$$m_1\vec{v}_1 + m_1\vec{v}_2 = (m_1 + m_2)\vec{v}' \tag{6}$$

Suppose, for example, that a body with mass m_A and initial x-component of velocity v_{A1} collides inelastically with a body with mass m_B and initially at rest ($v_2 = 0$). Velocity of both bodies after the collision is:

$$v' = \frac{m_1}{m_1 + m_2} v_1 \tag{7}$$

Let's verify that the total kinetic energy after this completely inelastic collision is less than before the collision. The motion is purely along the x-axis, so the kinetic energies K_l and K_2 before and after the collision, respectively, are:

$$K = \frac{1}{2} m_1 v_1^2 \tag{8}$$

$$K' = \frac{1}{2}(m_1 + m_2)v'^2 = \frac{1}{2}(m_1 + m_2)\left(\frac{m_1}{m_1 + m_2}\right)^2 v_1^2 \tag{9}$$

Then, the ratio of final to initial kinetic energy is

$$\frac{K'}{K} = \frac{m_1}{m_1 + m_2} \tag{10}$$

It is obviously that the kinetic energy after a completely inelastic collision is always less than before.

II. EXPERIMENTAL PROCEDURE

2.1. Preparation

In this lab, you will use the air tracks to perform several collisions between two gliders attached with "flag". The moving time before and after a collision through the respective gates is measured allowing you to calculate the corresponding velocities if the length of the "flag" for that gate is known. In this case, the equipments are setup so that the glide 2 will be stationary in the center of the track between the gates, that is, its initial velocity $v_2 = 0$ and the glide 1 is placed in the right as shown in Fig.3. Make several trial runs of the collision before doing any measurements.

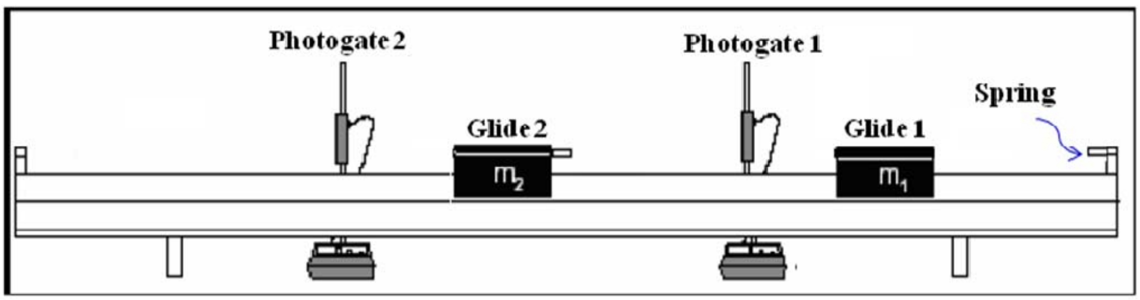


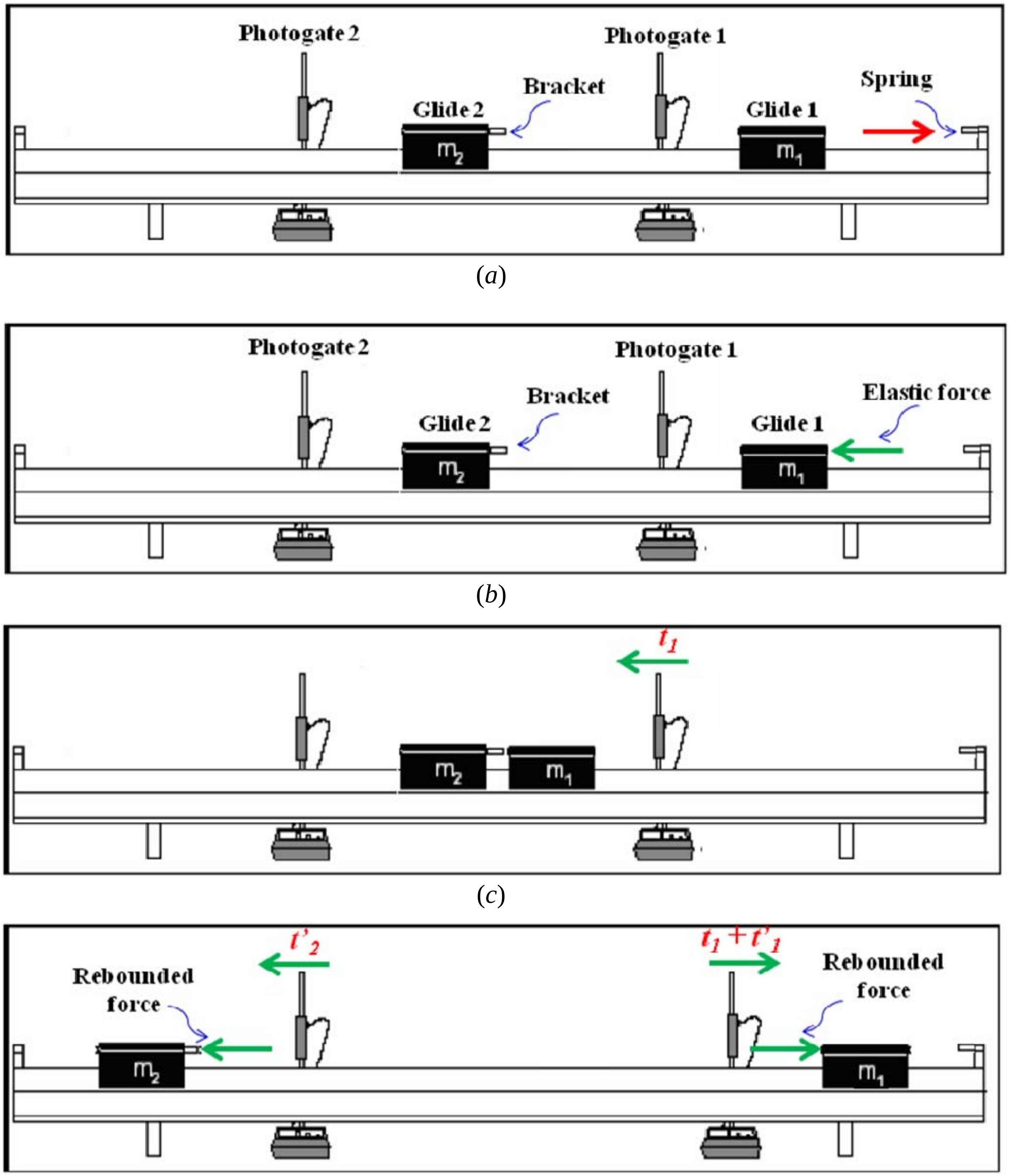
Fig.3. *Experimental setup for examination of elastic and completely inelastic collision*

2.2. Elastic collision

- ***Step 1***: Gently push the glide 1, from one end to make it moving to the right (direction of the arrow) toward the steel spring fixed onto the air track (Fig. 4a). Glide 1 will move to the left due to the elastic force exerted by the spring (Fig 4b) and pass through the photogate 1 (Fig.4c). Quickly record the moving time t_1 displayed on the first digital timer. The glide 1 will collide with the glide 2 in the middle. Two glides bounce apart, and go through the photogates (Fig. 4d) once again before you catch them to ensure that they do not go through the photogates a third time. Then, record both the time t'_2 displayed on the second timer and the total time $t_1 + t'_1$ on the first timer. The moving time of the glide 1 after collision (t'_1) is determined by subtract t_1 from the total time $t_1 + t'_1$.
- ***Step2***: Repeat the measurement procedure for more 9 times and record all the measurement results in a data sheet.

2.3. Inelastic collision

- ***Step 1***: Attach a piece of clay on one end of glide 2 facing to glide 1 to make them stick together after collision.
- ***Step 2***: Perform measurement procedure and record the moving time of two glides before and after collision as illustrated in Fig5.
- ***Step3***: Repeat the measurement procedure for more 9 times and record all the measurement results in a data sheet.



(*a*) (*b*) (*c*) (*d*)

Fig.4. *Experimental procedure to investigate the elastic collision*

III. LAB REPORT

Your lab report should include the following:

1. Data sheet of time and velocity measured for the system before and after the collision (should be 10 trials) in case of elastic and inelastic collision.
2. Calculate the average momentum and kinetic energy for the system before and after the collision in case of elastic and inelastic collision by using the equations from (4) to (10).
3. Calculations for percent changes in momentum and kinetic energy (KE) through the collision for the two sets of data specified above (change should be less than 5%). Describe what you changed to achieve changes less than 5%.

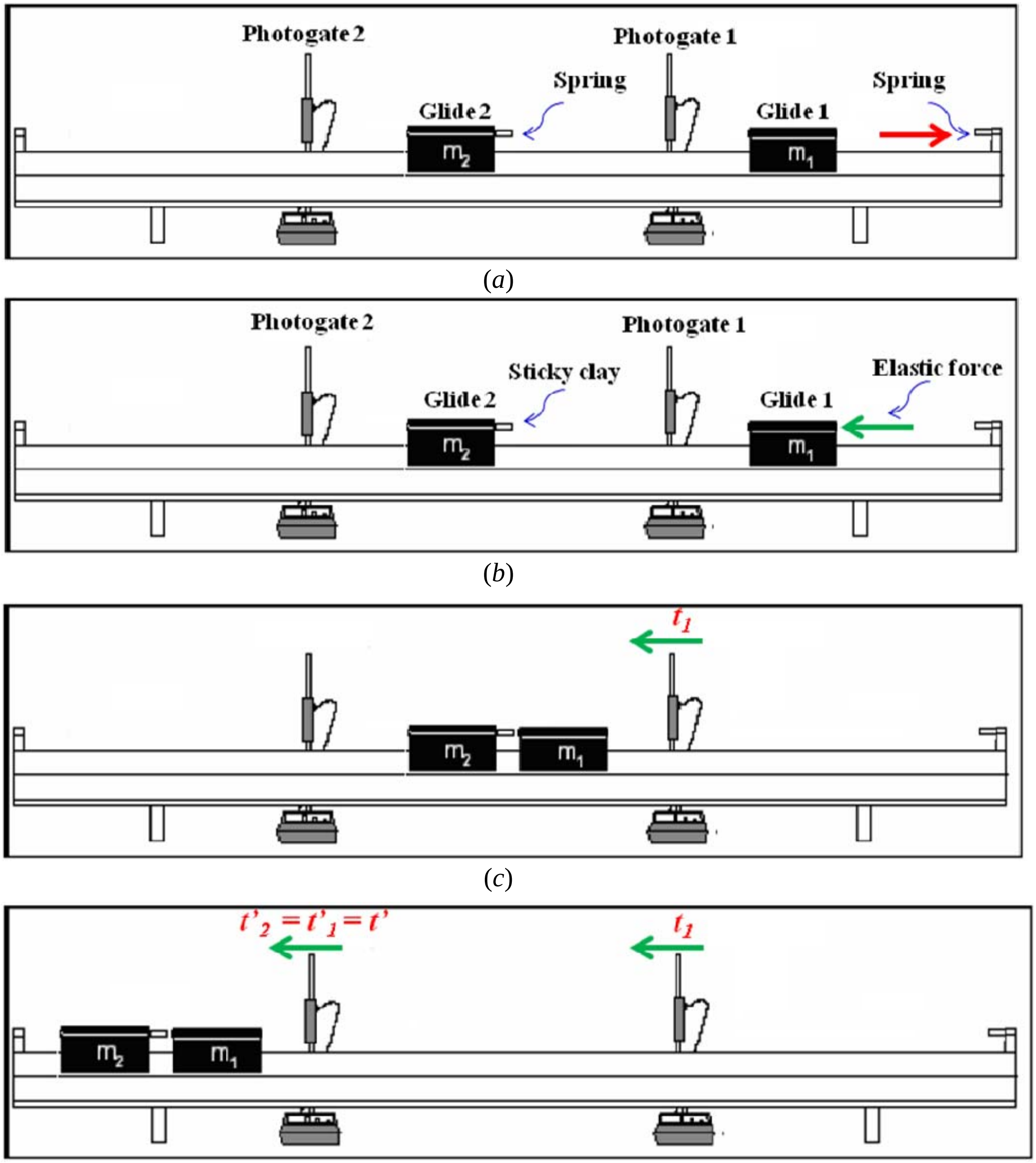


Fig.5. *Experimental procedure to investigate the inelastic collision*

4. Conclusions and possible explanations for any change in KE.and momentum.
5. Comment on the experimental uncertainties in the momentum and kinetic energy changes.
6. Comparison of the results of the elastic collision and inelastic collision experiments.

Remember: If the percentage change in momentum or kinetic energy before and after the collision is greater than 5%, repeat the measurement more carefully (collide slower/faster, etc.). Since the spreadsheet is set up it is easy to see whether momentum/energy is better conserved with every trial you do.

7. ***Note***: Please read the instruction of "***Significant Figures***" on page 6 of the document "***Theory of Uncertainty***" to know the way for reporting the last result.