

Probability and Statistics – Homework 1

Instructions: For each problem, remember you must briefly explain/justify how you obtained your answer, as correct answers without an explanation will receive **no credit**. Moreover, in the event of an incorrect answer, we can still try to give you partial credit based on the explanation you provide. It is fine for your answers to include summations, products, factorials, exponentials, or combinations; you don't need to calculate those all out to get a single numeric answer.

Submission: You must upload your working in PDF format to Canvas MATH2010

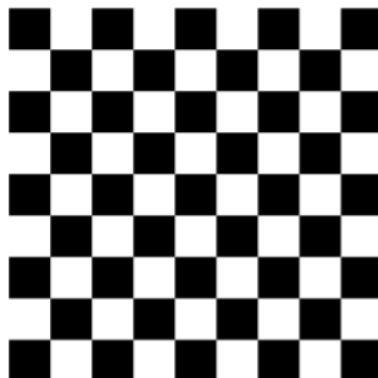
Question 1 (4 points):

How many ways can 16 people be seated in a row if ...

- a) ...there are no restrictions on the seating arrangement?
- b) ...two of the people, persons A and B, cannot sit (immediately) next to each other?
- c) ...there are 8 adults and 8 children, and no two adults nor two children can sit next to each other?
- d) ...there are 8 married couples, and each couple must sit together?

Question 2 (4 points):

Each square of a 9x9 checkerboard (see below) is initially slept on by one of 81 students. At noon, each student will wake up and randomly sleepwalk to a valid adjacent square horizontally or vertically (but not diagonally). What is the probability that two or more students end up on the same square?



Question 3 (4 points):

A home security system may detect movement using its two different sensors. If motion is detected by any of the sensors, the system will alert the police. If there is movement outside, sensor V (video camera) will detect it with probability 0.95, and sensor L (laser) will detect it with probability 0.8. If there is no movement outside, sensor L will detect motion anyway with probability 0.05, and sensor V will detect motion anyway with probability 0.1. Based on history, the probability that there is movement at a given time is 0.7. Assume these sensors

have proprietary algorithms, so that **conditioned** on there being movement (or not), the events of detecting motion (or not) for each sensor is **independent**.

- a) Given that there is movement outside and that sensor V does not detect motion, what is the probability that sensor L detects motion?
- b) Given that there is a moving object, what is the probability that the home security system alerts the police?
- c) What is the probability of a false alarm? That is, that there is no movement, but the police are alerted anyway.
- d) What is the probability that there is a moving object given that both sensors detect motion?

Question 4 (4 points):

A group of 50 people are comparing their birthdays (as usual, assume their birthdays are independent, are not February 29, etc.). Find ...

- (a) ... the expected number of pairs of people with the same birthday
- (b) ... the expected number of days in the year on which at least two of these people were born.

Question 5 (4 points):

Let $X \sim \text{Bin}\left(n, \frac{1}{2}\right)$ and $Y \sim \text{Bin}\left(n + 1, \frac{1}{2}\right)$, independently.

Let $V = \min(X, Y)$ be the smaller of X and Y , and let $W = \max(X, Y)$ be the larger of X and Y .

- a) Find $E(V) + E(W)$.
- b) Show that $E|X - Y| = E(W) - E(V)$