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IT3160E
Introduction to Artificial Intelligence

Chapter 3 – Problem solving

Part 3: problem-solving by searching using more advanced search strategies

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Content of the course

- Chapter 1: Introduction
- Chapter 2: Intelligent agents
- Chapter 3: Problem Solving
  - Search algorithms, adversarial search
  - Constraint Satisfaction Problems
- Chapter 4: Knowledge and Inference
  - Knowledge representation
  - Propositional and first-order logic
- Chapter 5: Uncertain knowledge and reasoning
- Chapter 6: Advanced topics
  - Machine learning
  - Computer Vision

Outline

- Chapter 3 – part 1: un-informed (basic) algorithms
- Chapter3 - part 2: informed search strategies in graphs
- Chapter 3 – part 3: advanced search strategies
  - Memory-bounded heursitic search
    - Introduction
    - IDA*
    - RBFS
    - SMA*
  - Local search algorithms
    - Hill-climbing search
    - Simulated annealing search
  - Exercise
  - Summary

Goal of this Lecture

Goal	Description of the goal or output requirement	Output division/ Level (I/T/U)
M1	Understand basic concepts and techniques of AI	1.2

Memory-bounded heuristic search

Introduction

Memory-bounded heuristic search

- Recall from the last lecture:
  - A* is a very good algorithm, but its main drawback is the space cost (exponential in the path length)
    - In large search spaces, might be impossible to use on regular compters
- Memory-bounded heuristic search are solutions to this problem, maintaining completeness and optimality
  - Iterative-deepening A* (IDA*)
    - Close to IDS, but with cutoff based on the the *f*-cost (*g*+*h*) instead of depth
  - Recursive best-first search (RBFS)
    - Recursive algorithm that attempts to mimic standard best-first search with linear space
  - (simple) Memory-bounded A* ((S)MA*)
    - Drop the worst-leaf node when memory is full

Memory-bounded heuristic search

IDA*

Iterative Deeping A* (IDA*)

- IDA* is a **tree-search** algorithm
  - Can be applied to graphs only after they're converted to trees
    - An example of that is given in an exercise, at the end of this lecture

- It is a variant of Iterative Deepening (depth-first) Search (IDS) that borrows the idea of A* to use an evaluation function in the form $f(n)=g(n)+h(n)$
  - $g(n)$: true cost from the root to node n
  - $h(n)$: heuristic cost from the node n to the goal

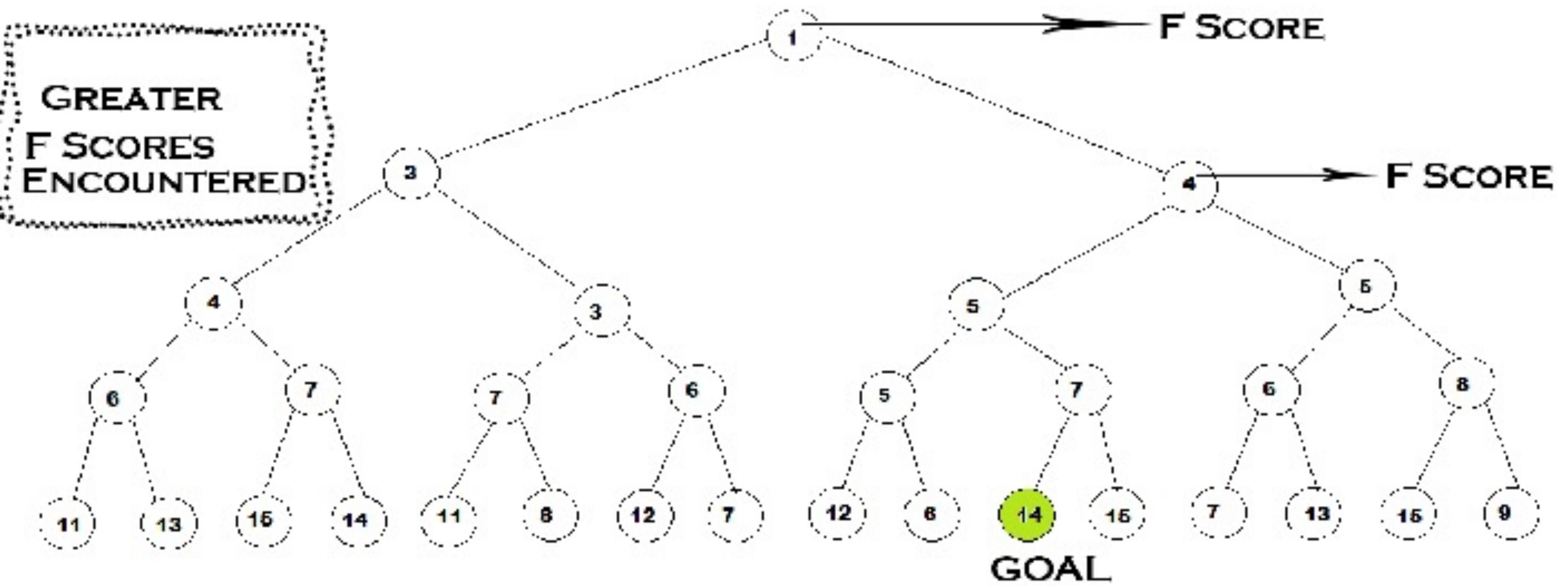
- Combines advantages of IDS and A*:
  - It's a depth-first search algorithm=> space cost lower than A*
  - Concentrates on exploring the most promising nodes => doesn't go to the same depth everywhere in the search tree => more time efficient than IDS

Iterative Deeping A* (IDA*)

- Iterative Deeping version of A* (general idea)
  - Define a threshold, θ_1 = *f(root_node)*
  - *t<-1*
  - Then, at each iteration *t*
    - Expand the non-goal nodes *n* in a depth-first way, with stopping criterion *f()* > θ_t
      - *I.e.* If *f(n)* > θ_t then **prune** the node *n* (do not expand it), and put this node in the fringe
      - If any goal node in the fringe, return the goal node with lowest value of *f()*
        - Simple strategy in case of ties: pick randomly; other strategies are possible
      - Set θ_{t+1} = minimum *f()* of all nodes in the fringe
      - t <- t+1

Iterative Deeping A* (IDA*)

- **To save space**, between iterations, IDA* retains only a single number: θ
  - ⇒ Suffers from too many node re-generations in general
  - ⇒ Costly, in terms of **time complexity**
- Example:
  - N.B. in this example, the f() values have been pre-computed from *f(n)=g(n)+h(n)*
  - N.B. Initially, threshold=1



Iterative Deeping A* (IDA*)

- **Recall**:
  - *b:* branching factor; *d:* depth of the **shallowest** goal node
- IDA* is complete and optimal under the same conditions as A* **tree** search:
  - *h* is admissible
  - All arc costs > epsilon > 0
  - Finite branching factor
- Time complexity: same as IDS: $O(b^d)$ – in the worst case
  - In practice, it strongly depends on the number of different values that the *f()* value can take on
- Space complexity is decreased to O(bd) – in the worst case
  - More precisely, If δ is the smallest cost, and f^* is the optimal solution cost, then IDA* will require bf^*/δ nodes.

Memory-bounded heuristic search

Recursive best-first search

Recursive best first search example

- Algorithm: similar to recursive depth-first search, but rather than continuing indefinitely down the current path, it uses the attribute *f-limit*
  - *f-limit* keeps track of the best alternative path, **starting from any ancestor** of the current node
    - So that the algorithm can backtrack there, if needed
    - The whole alternative paths **are not kept in memory**, only their *f_limit*

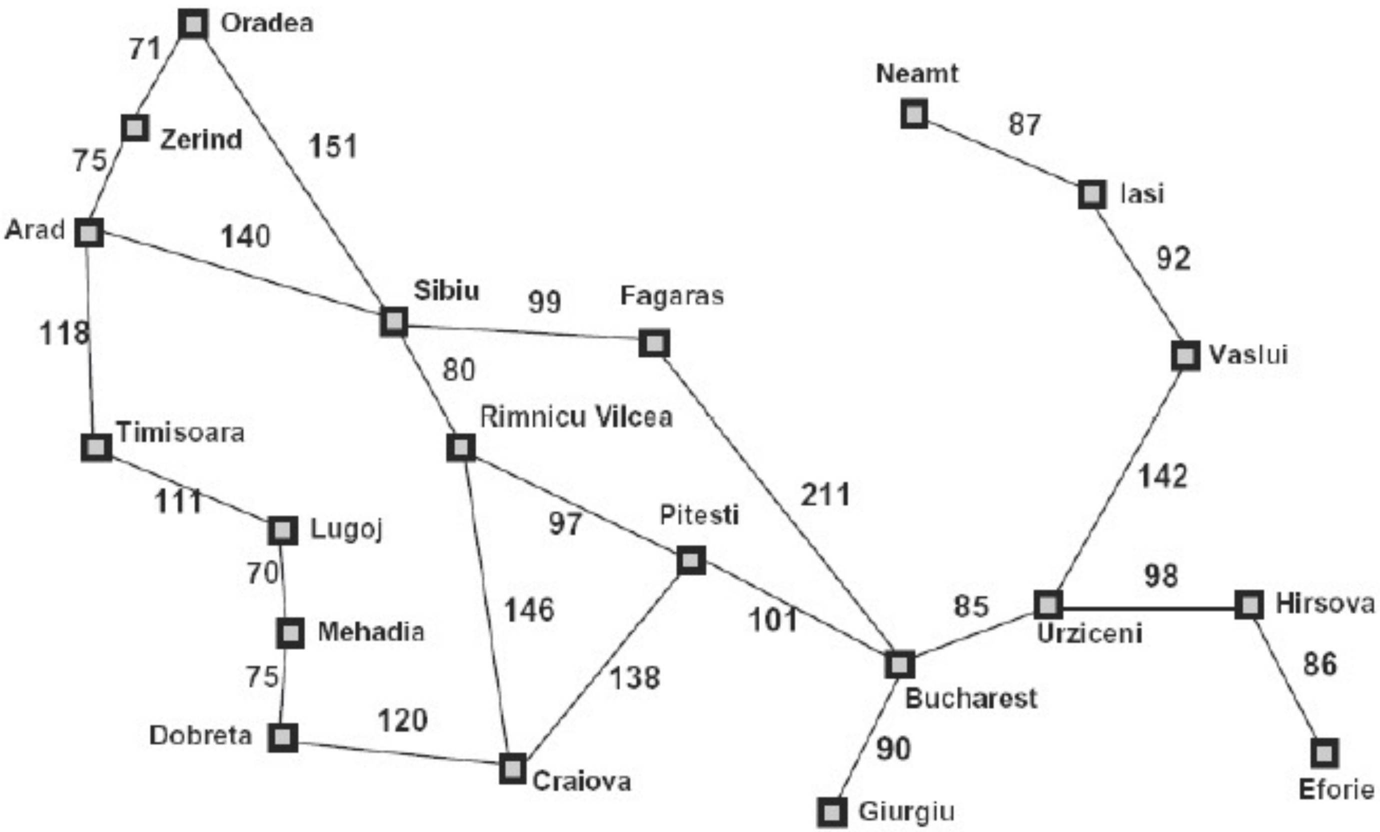
    => When the algorithm backtracks there, then it has to **regenerate** the alternative path
      - Costly, in terms of time complexity
  - For the root, the value of *f-limit* is initially set to +infinity

Recursive best-first search algorithm

function RECURSIVE-BEST-FIRST-SEARCH(*problem*) **return** a solution or failure
    return RFBS(*problem*,MAKE-NODE(INITIAL-STATE[*problem*]),∞)

function RFBS(*problem, node, f_limit*) **return** a solution or failure and a new *f-cost* limit
    if GOAL-TEST[*problem*](STATE[*node*]) **then return** *node*
    successors ← EXPAND(*node, problem*)
    if *successors* is empty then return failure, ∞
    for each *s* **in** *successors* **do**
        f [*s*] ← max(*g(s)* + *h(s)*, *f* [*node*])
    repeat
        best ← the lowest *f*-value node in *successors*
        if *f* [*best*] > *f_limit* **then return** failure, *f* [*best*]
        alternative ← the second lowest *f*-value among *successors*
        result, *f* [*best*] ← RBFS*(problem, best,* min(*f_limit, alternative*))
        if *result* ≠ failure **then return** *result*

Recursive best first search example (with A*)



Straight−line distance to Bucharest

Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	~~178~~ 176
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
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Oradea	380
Pitesti	~~98~~ 100
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Recursive best first search example (with A*)

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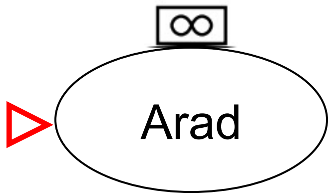
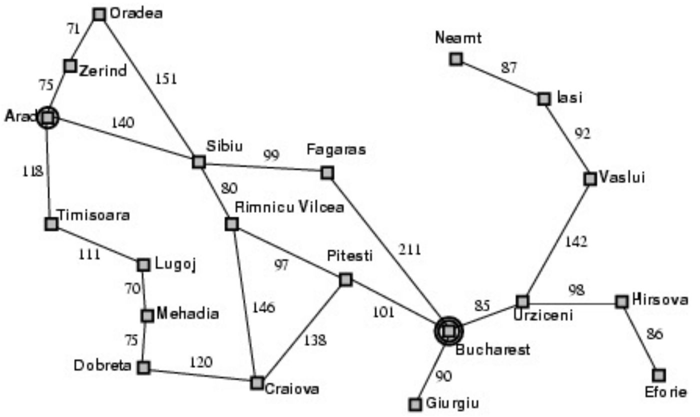
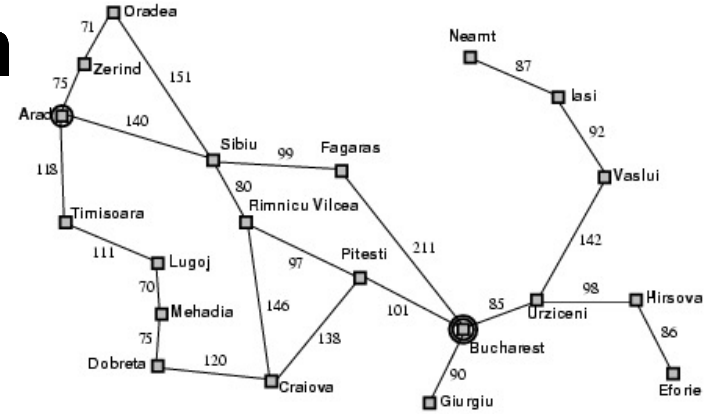


366 = 0 + 366



Recursive best first search



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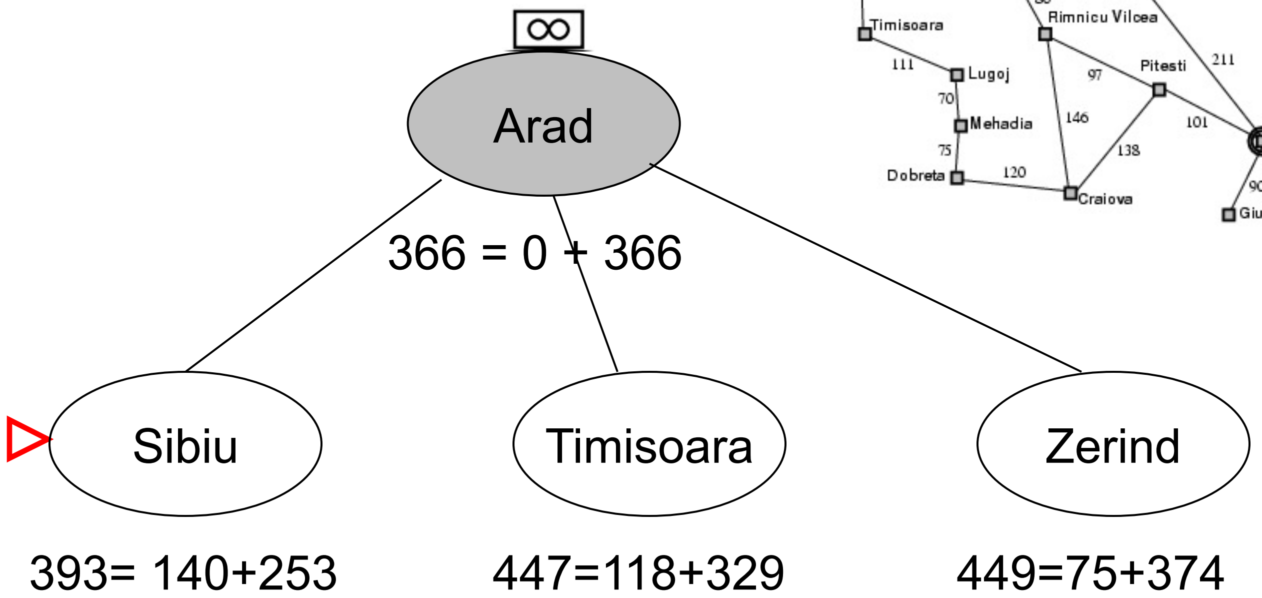

F_limit=+infinity
Sibiu is now best, with f(best)=393
Alternative is Timisoara, with value=447
-> f_limit becomes 447

(*result*, *f* [*best*] ← RBFS*(problem, best,* min(*f_limit, alternative*)))

Recursive best first search

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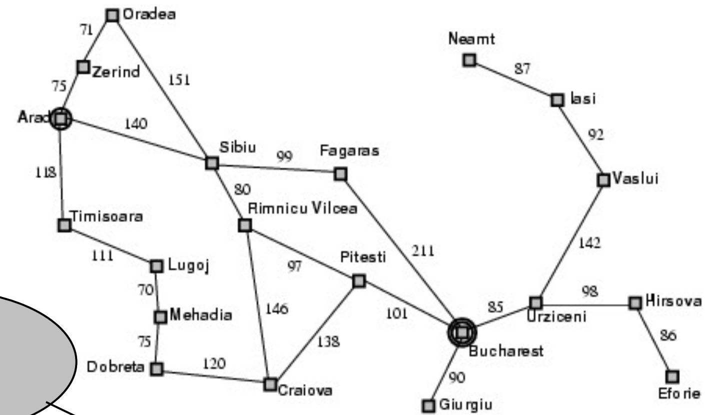
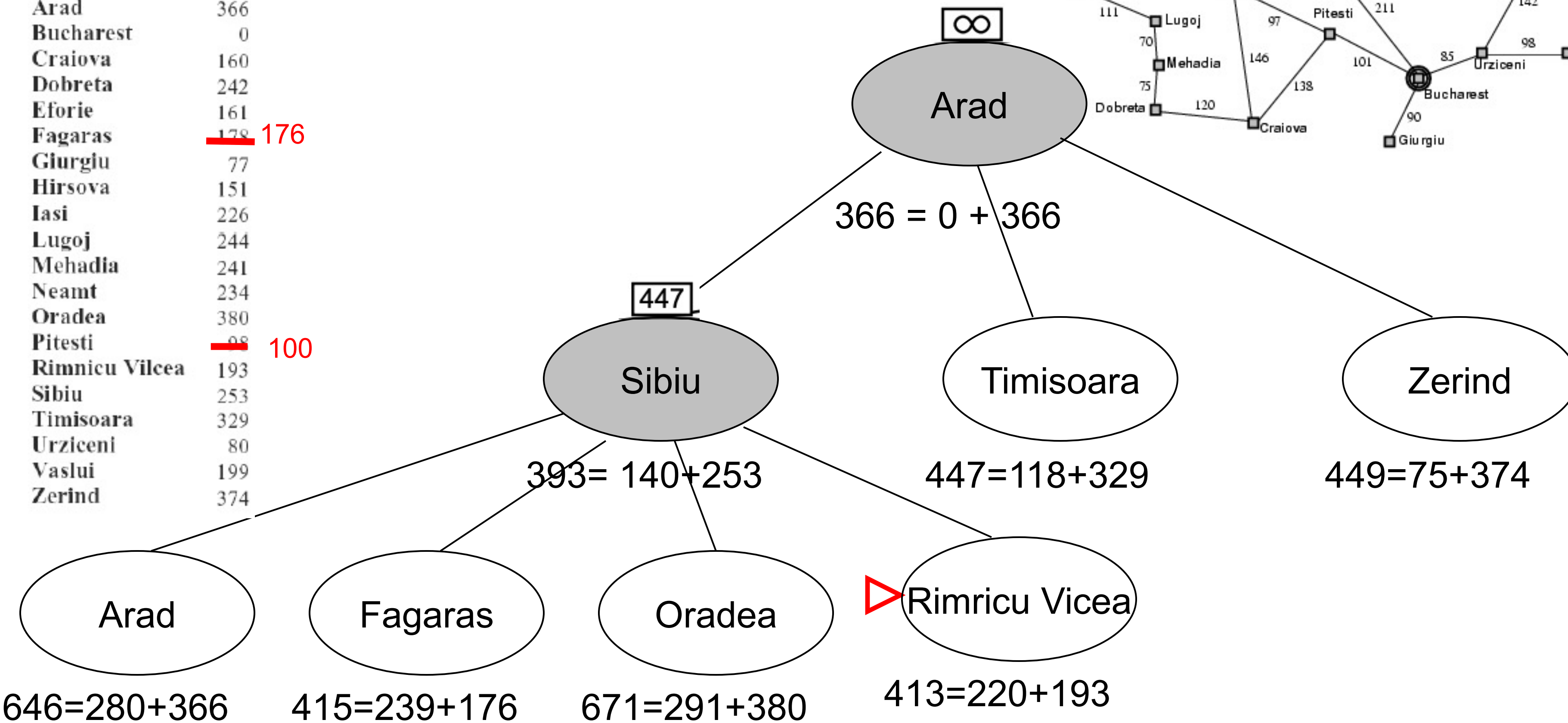
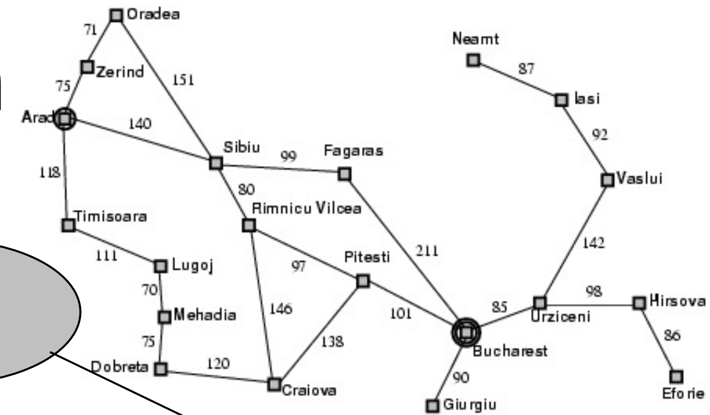




F_limit= 447
Rimnicu-Vicea is now best, with f(best)=413
Alternative is Fagaras, with value=415
-> f_limit becomes 415

(*result*, *f* [*best*] ← RBFS(*problem, best,* min(*f_limit, alternative*)))

Recursive best first search



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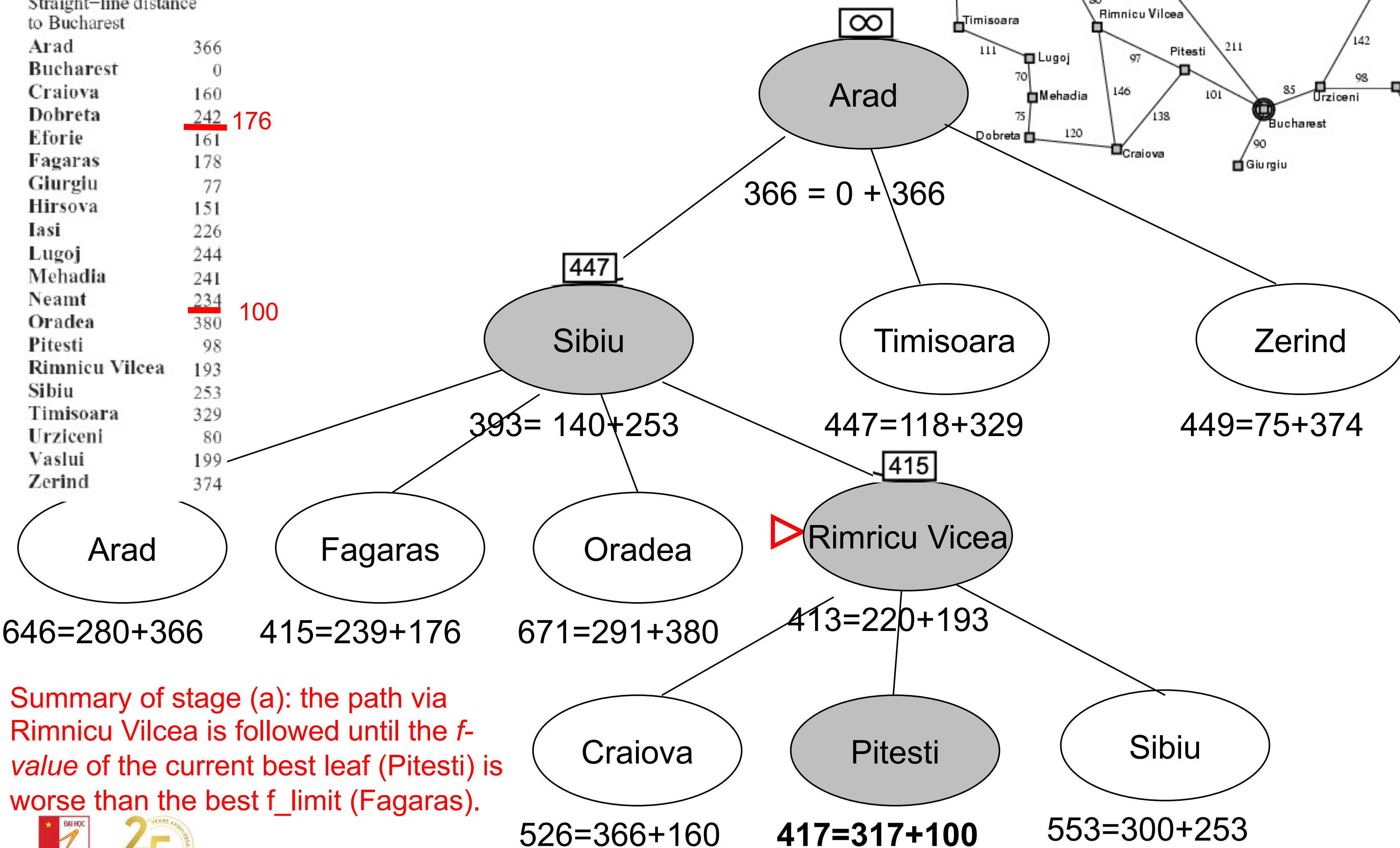

Summary of stage (a): the path via Rimnicu Vilcea is followed until the *f-value* of the current best leaf (Pitesti) is worse than the best f_limit (Fagaras).

Recursive best first search

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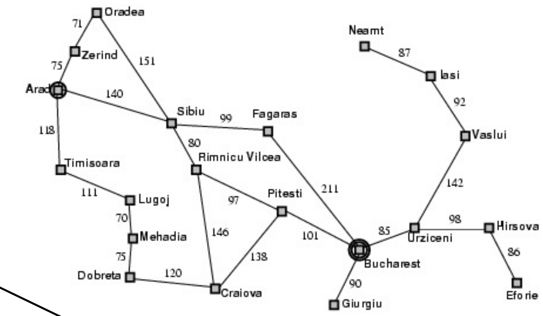
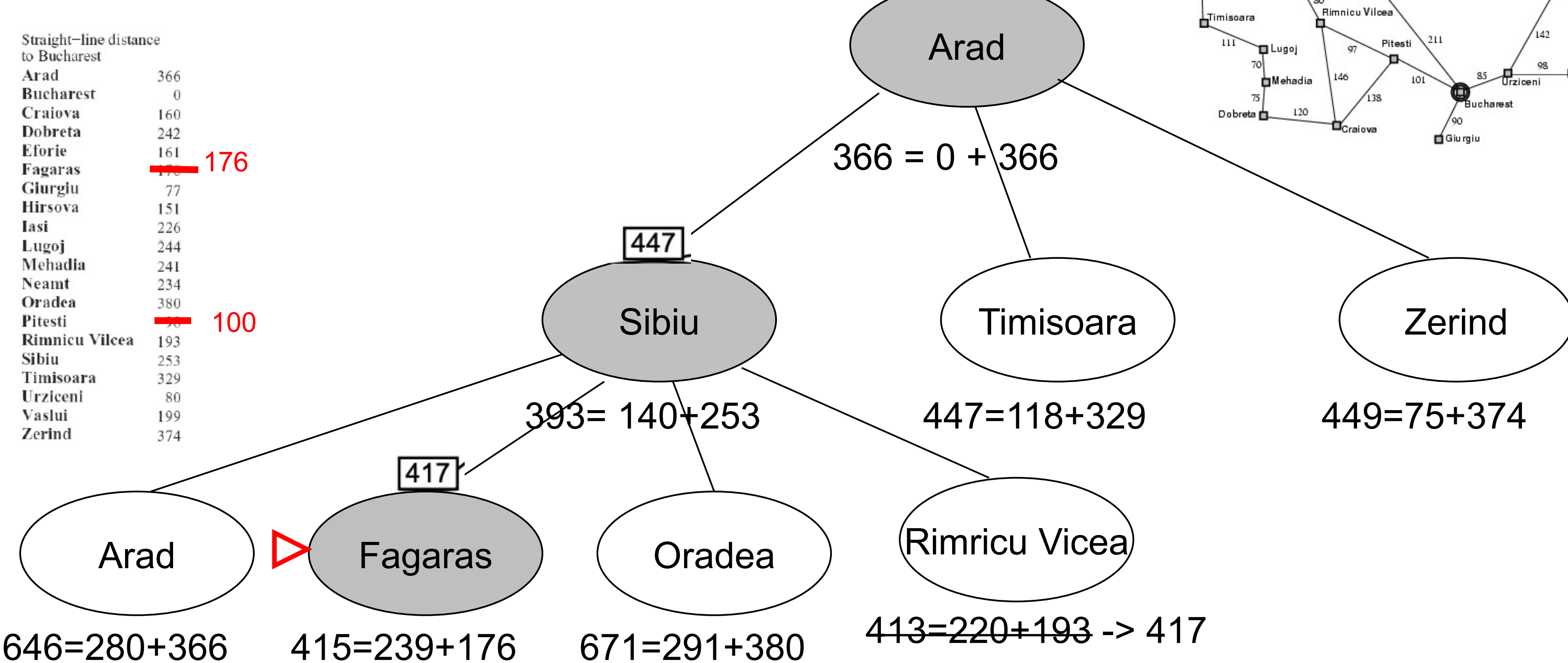


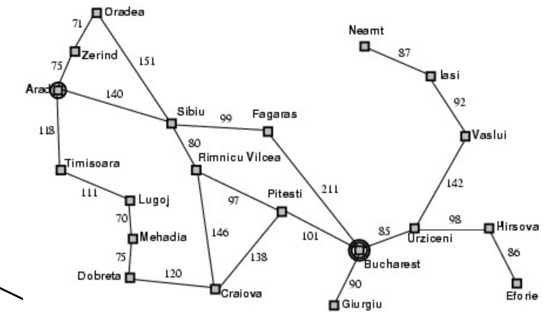


if *f* [*best*] > *f_limit* **then return** failure, *f* [*best*]

Unwind recursion and store best *f*-value for current best leaf Pitesti
best is now Fagaras, with f_limit 415, and alternative is Rimricu Vicea, with f_limit 417

result, *f* [*best*] ← RBFS(*problem, best,* min(*f_limit, alternative*))

Recursive best first search



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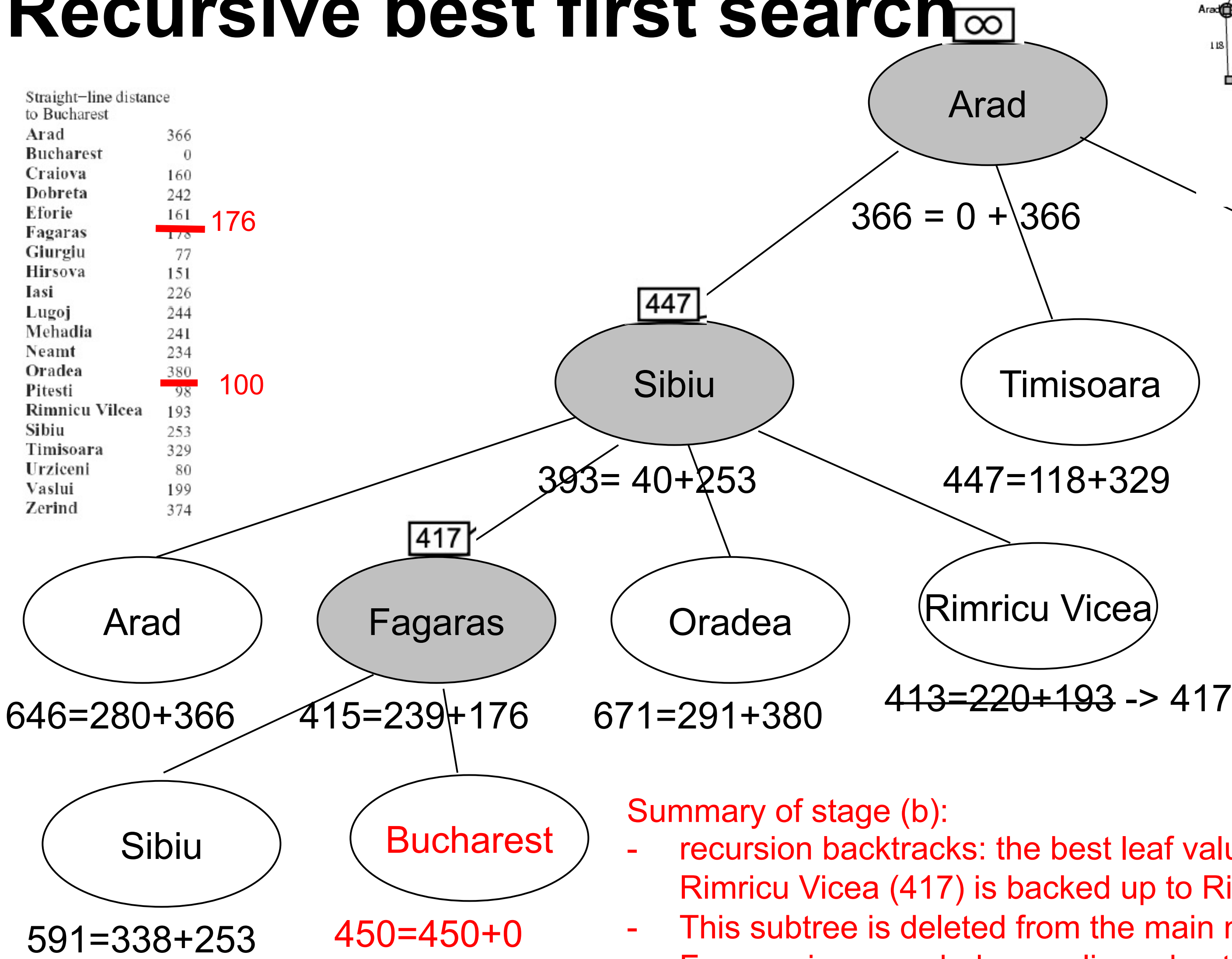

Summary of stage (b):

- recursion backtracks: the best leaf value of the subtree starting at Rimricu Vicea (417) is backed up to Rimnicu Vicea
- This subtree is deleted from the main memory
- Fagaras is expanded, revealing a best value of 450, for a goal node
- But the f_limit (417) is < 450 so the algorithm continues

Recursive best first search

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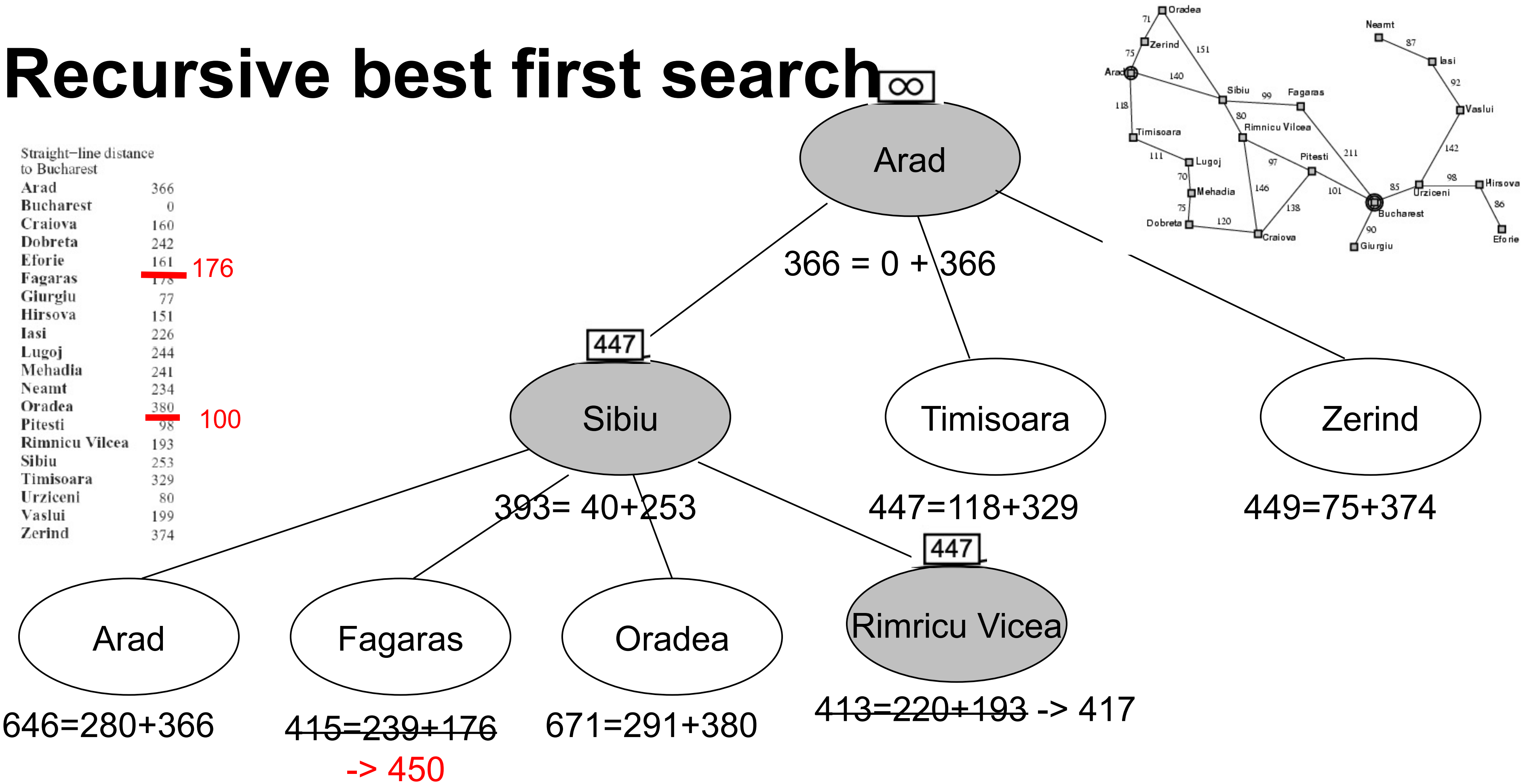

if *f* [*best*] > *f_limit* **then return** failure, *f* [*best*]

Recursive best first search

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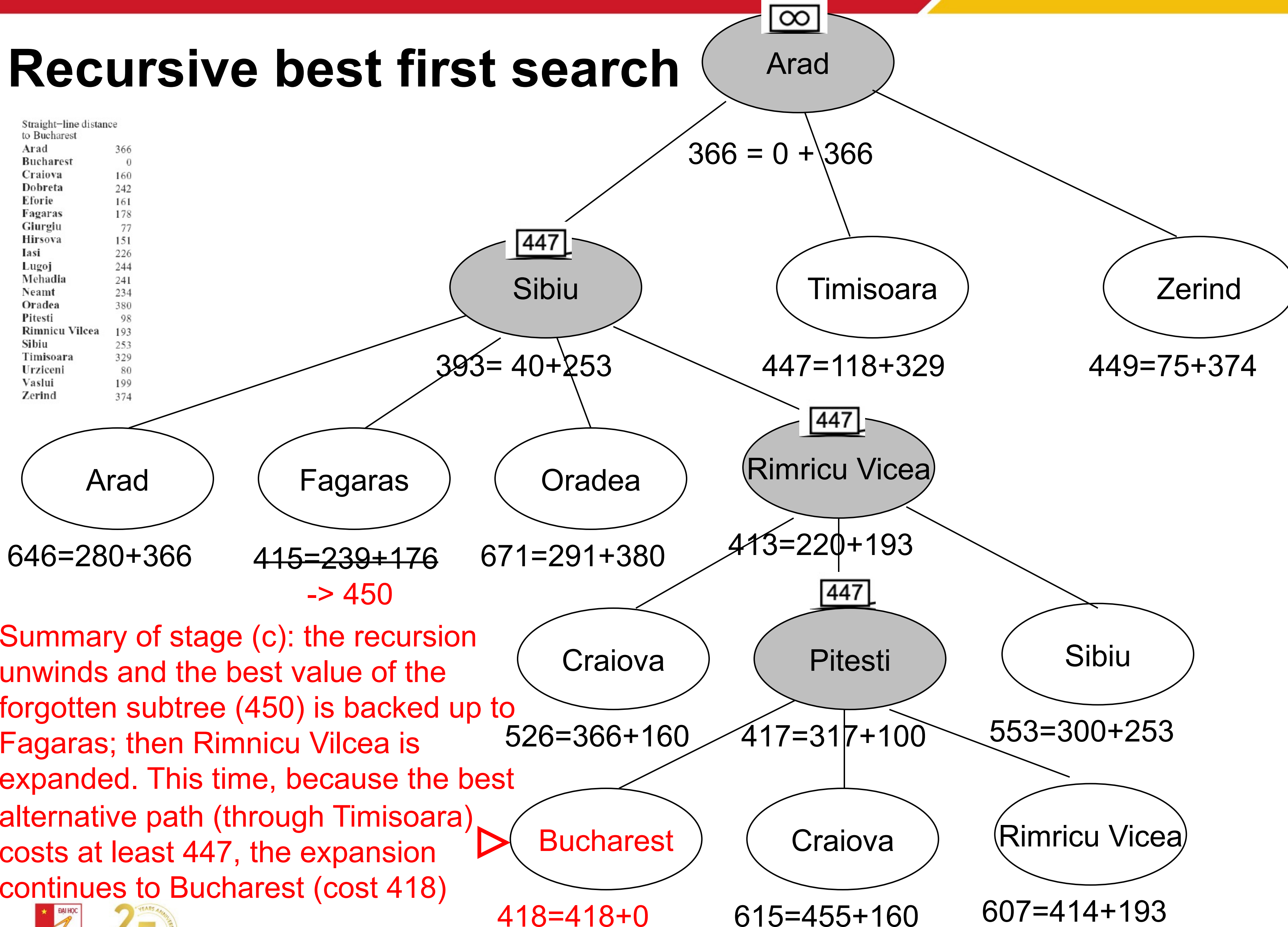


Summary of stage (c): the recursion unwinds and the best value of the forgotten subtree (450) is backed up to Fagaras; then Rimnicu Vilcea is expanded. This time, because the best alternative path (through Timisoara) costs at least 447, the expansion continues to Bucharest (cost 418)

We found a goal node, less costly than the alternative path -> **this node is returned as solution**

Recursive best first search evaluation

- RBFS is a bit more efficient than IDA*
  - But, still has excessive node re-generation (backtrackings)
- Like A*, optimal if *h(n)* is admissible
- Space complexity is *O(bd)*
- Time complexity difficult to characterize
  - Depends on accuracy of *h(n)* and how often best path changes
  - In practice:
    - If the costs grow monotically, then RBFS expands less nodes (in total, considering also re-expansions) than IDA*
    - Else, RBFS and IDA* are difficult to compare, as they traverse entirely different search trees
      - Need to compare both in practice
  - Time complexity is $O(b^d)$ in the worst case

Recursive best first search evaluation

=> IDA* and RBFS can suffer from ***too little*** memory, leading to re-expanding many nodes (thus, higher time complexity)

- However, they are great in the sense that the space complexity is linear instead of exponential!
  - Good in the presence of big search spaces
- Could be more efficient if they could use a bit more memory, instead of having to re-expand so many nodes

Memory-bounded heuristic search

(Simplified) memory-bounded A*

(Simplified) memory-bounded A* (SMA*)

- Use all available memory.
  - *i.e.* expand best leaves until available memory is full
  - When full, SMA* drops the worst leaf node (with highest *f*-value)
  - Like RBFS, we remember the value for the best descendant in the branch we prune (f_limit in RBFS)

- What if all leaves have the same *f*-value?
  - Problem: the same node could be selected for expansion <u>and</u> deletion
  - SMA* solves this by expanding *newest* best leaf and pruning *oldest* worst leaf
    - In short, 2 strategies for ties: “LIFO for the Fringe, FIFO for deletion”

- The pruned branch is regenerated when all other candidates look worse than its best value

(Simplified) memory-bounded A* (SMA*)

- SMA* is complete if solution is reachable, optimal if optimal solution is reachable
  - *i.e.* if the depth of the shallowest goal node is less than the memory size (expressed in nodes)
- But, time complexity is still $O(b^d)$ in the worst case
- SMA* is a good choice for finding solutions, particularly when:
  - the state space is a graph
  - step costs are **not** uniform
  - node generation is expensive compared to the overhead of keeping the fringe nodes, and the explored nodes, in memory
    - Tradeoff RAM / CPU…

Local search algorithms

Introduction

Local search algorithms

- In many optimization problems, the path to the goal is irrelevant; the goal state itself is the solution
  - Example: the 8-puzzle

start

1	2	5
	7	4
8	6	3

goal

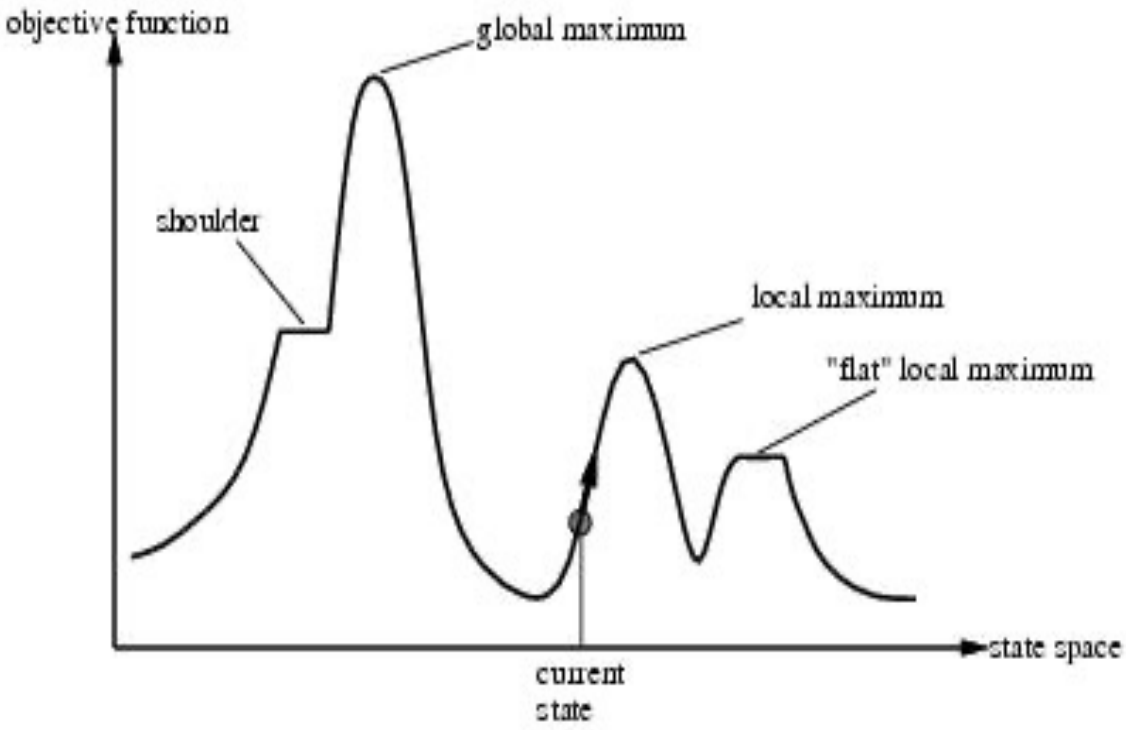
1	2	3
8		4
7	6	5

- In such cases, we can use local search algorithms
  - Standard mathematical **optimization techniques**, belong to numerical analysis
  - Can be used for AI (but **do not belong to AI** algorithms)

- Local search= use single current state and move to neighboring states

Local search and optimization

- In local search algorithms (general optimization algorithms)
  - Sometimes, it can happen that the objective function has to be maximized
  - Example: hill-climbing algorithm
  - So, different from the previous search algorithms, negative cost values (rewards) can be considered
- But, **do NOT** consider negative costs with the search algorithms previously introduced (uniform cost search, greedy best-first search, A* search, IDA*, RBFS, SMA*)!!!
  - Otherwise, you would need to modify the algorithms (out of the scope of this lecture)



Local search algorithms

Hill-climbing search

Hill-climbing search

- Also called “greedy local search” as it only looks to its best **immediate** neighbor state and <u>not beyond that</u>
  - In case of ties, selects the neighbor randomly

- Simple, general idea:
  - Start wherever
  - Always choose the best neighbor
  - If no neighbors have better scores than current, quit

- Types of Hill Climbing Algorithm: https://www.javatpoint.com/hill-climbing-algorithm-in-ai
  - Simple hill Climbing: examines only children nodes
  - Steepest-Ascent hill-climbing: examines also neighbor nodes
  - Stochastic hill Climbing: select randomly some neighbor nodes to consider
  - Many other variants…

Simple hill-climbing search

function HILL-CLIMBING(*problem*) **return** a state that is a local maximum

  input: *problem*, a problem

  local variables: *current*, **a node.**

   neighbor, **a node.**

  current $\leftarrow$ MAKE-NODE(INITIAL-STATE[*problem*])

  loop do

    neighbor $\leftarrow$ a highest valued successor of *current*

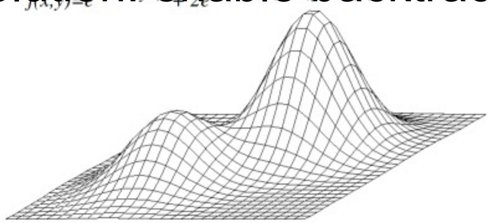
    if VALUE [*neighbor*] ≤ VALUE[*current*] **then return** STATE[*current*]

    current $\leftarrow$ *neighbor*

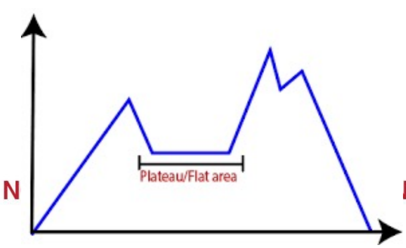
Hill-climbing search evaluation

- Advantages:
  - Use very little memory
  - Find often reasonable solutions in large or infinite state spaces
- But, 3 main problems:

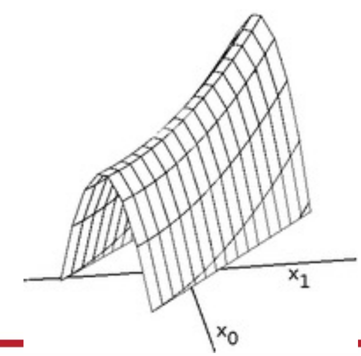
Local maximum:
Hill climbing might be stuck in a local maximum
Possible solution: enable backtracking



Plateau:
Hill climbing might not find the best direction to move to in a plateau (random walk)
Possible solution: enable bigger steps



Ridge:
Flat like a plateau, but with dropoffs to the sides. *E.g.* steps to the North, East, South and West may go down, but a combination of two steps (e.g. N, W) may go up
https://en.wikipedia.org/wiki/Hill_climbing
Possible solution: use a gradient method instead if the function is differentiable



Hill-climbing search

- Some problem spaces are great for hill climbing and others are terrible
- For example, hill climbing can be applied to the travelling salesman problem
  - Initialization: solution visiting all the cities (even if very long)
  - The algorithm makes small improvements to it, such as switching the order in which two cities are visited
  - Eventually, the algorithm will likely return:
    - a much shorter path than the original path
    - a longer path than the optimal solution (local optimum)

Local search algorithms

Simulated annealing search

Simulated Annealing

- **What is annealing? -> in metallurgy**
  - Process of heating, then slowly cooling down a compound or a substance (often, metals)
  - Heating makes the substance "unstable"
  - Then, slow cooling let the substance "flow around" → thermodynamic equilibrium
  - Molecules get optimum conformation
  - Example: annealing silver makes it softer

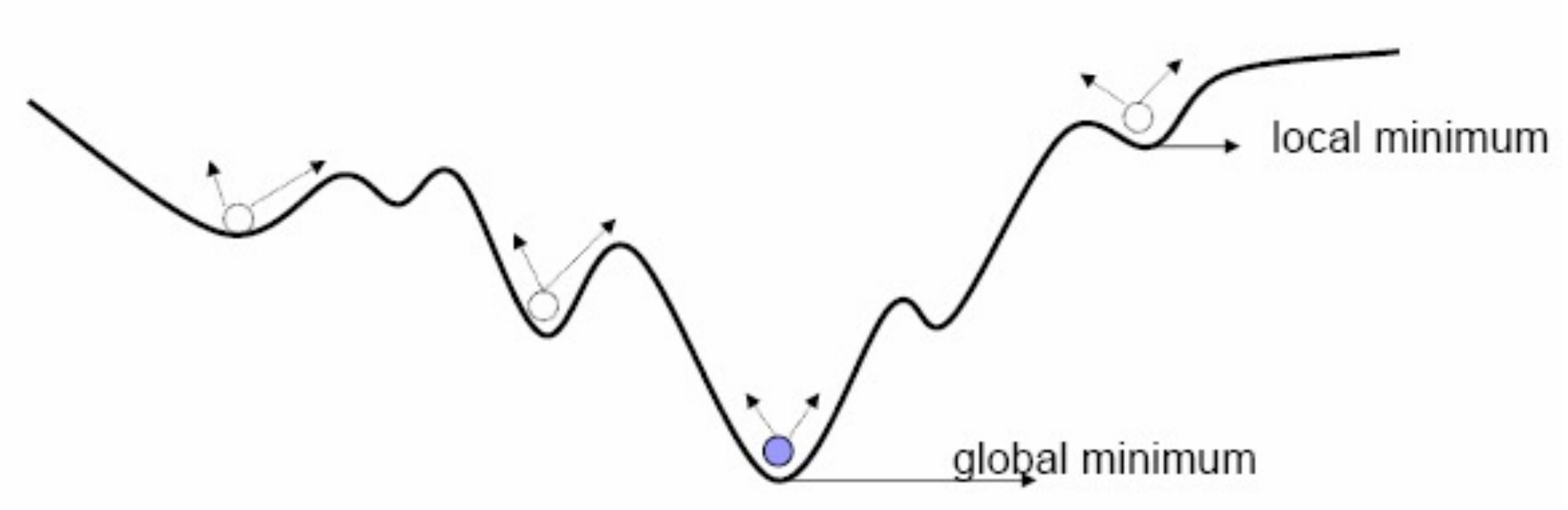
Simulated Annealing

- Simulates the slow cooling of the annealing process

- Applied for combinatorial optimization problem for the first time by S. Kirkpatric (‘83)

- N.B. In this part, we’ll use (as usual except for hill climbing) a cost function to minimize ☺

Simulated annealing



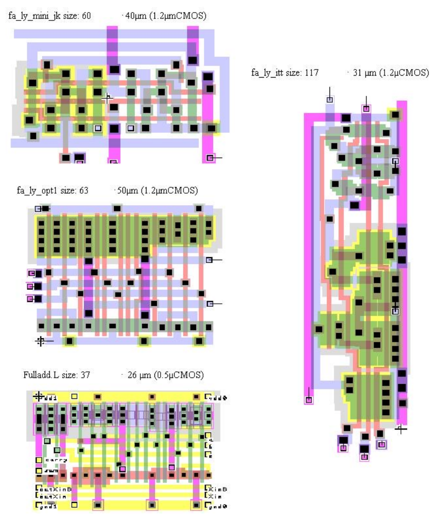
gradually decrease “shaking” to make sure the ball escape from local minima and fall into the global minimum

Simulated annealing

- Idea: Escape local minima by allowing "bad" moves….
  - … but gradually decrease their loss amplitude, and frequency

- Implement:
  - Not always select the best move
  - Accept a "worse" move with probability less than 1 (p<1)
    - The probability of accepting a move decreases inversely to how bad the move is
  - p decreases with time, as the (temperature) parameter T decreases

- If T decreases slowly enough and with a good "schedule", best state is reached
- Applied for travelling salesman problem, VLSI layout, airline scheduling, etc



Simulated annealing

function SIMULATED-ANNEALING(*problem, schedule*) **return** a solution state

input: *problem*, a problem

schedule, a mapping from time to temperature

local variables: *current***,** a node; *next***,** a node.

*T***,** a "temperature" controlling the probability of downward steps

current ← MAKE-NODE(INITIAL-STATE[*problem*])

for t ← 1 to ∞ do

T ← *schedule*[*t*]

if *T = 0* **then return** *current*

next ← a randomly selected successor of *current*

ΔE ← VALUE[*next*] - VALUE[*current*]

if $\Delta E > 0$ **then** *current* ← *next*

else *current* ← *next* only with probability $e^{\Delta E / T}$

Similar to hill climbing, but a **random** move instead of best move

case of improvement, make the move

Otherwise, choose the move with probability that decreases exponentially with the "badness" of the move.

What's the probability when: T → +∞?

What's the probability when: T → 0?

What's the probability when: Δ=0?

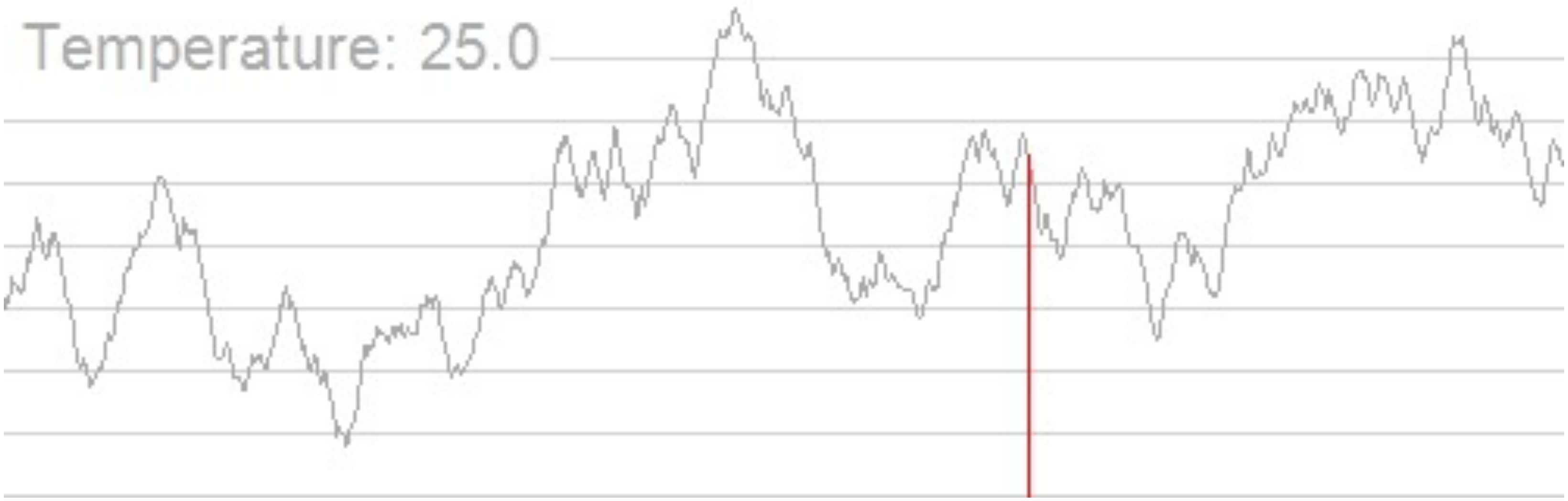
What's the probability when: Δ→-∞?

Simulated Annealing parameters

- Temperature T
  - Used to determine the probability
  - High T : large changes
  - Low T : small changes
- Cooling Schedule
  - Determines rate at which the temperature T is lowered
  - Lowers T "slowly enough", the algorithm will find a global optimum
- In the beginning, aggressive for searching alternatives, become conservative when time goes by

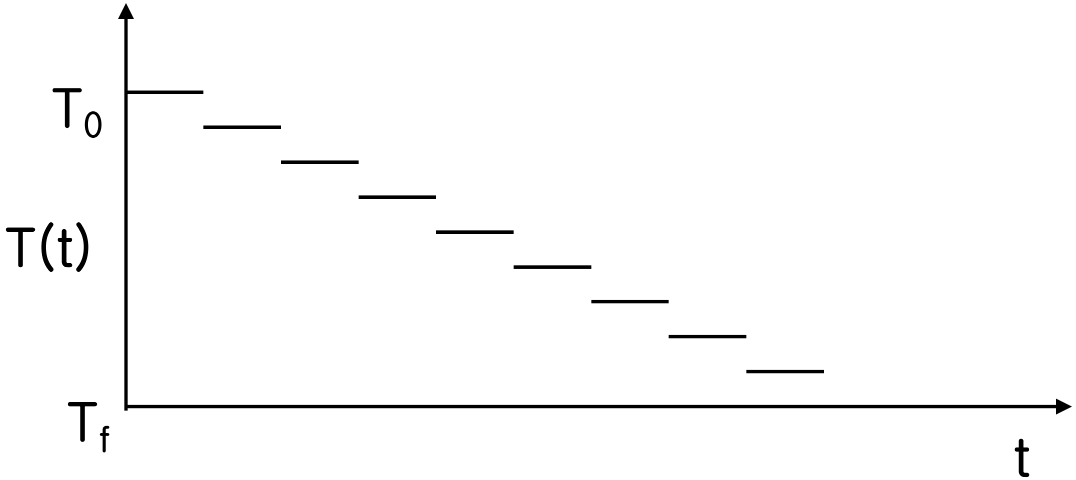
Simulated Annealing parameters

- Visual illustration (from Wikipedia)



Simulated Annealing Cooling Schedule

- The manner in which the temperature T is decreased over the iterations called the cooling **schedule**
  - if Ti is reduced too fast, poor quality
- Many schedules have been proposed, *e.g.*:
  - if Tt >= T(0) / log(1+t) - Geman
    - System will converge to minimun configuration
  - Tt = k/1+t - Szu
  - Tt = a T(t-1) where a is in between 0.8 and 0.99
  - The simple **geometric schedule**, where the temperature is reduced by a constant factor at each chain of moves, and the chain lengths are equal, works quite well on various problems:



Tips for Simulated Annealing

- To avoid getting stuck in local minima
  - Choosing the annealing schedule by trial and error
    - *E.g.*, for the geometric schedule
    - Choice of initial temperature
    - How many iterations are performed at each temperature
    - How much the temperature is decremented at each step as cooling proceeds
- Difficulties
  - Determination of parameters
  - If cooling is too slow → Too much time to get solution
  - If cooling is too rapid → Solution may not be the global optimum

Properties of simulated annealing

- Theoretical guarantee:
  - If T decreases slowly enough (infinite number of plateaux in the schedule, each plateau being “long enough”), it will converge to optimal state!
  - Impossible to reach in practice, but simulated annealing with slow cooling usually produce a good solution

- Advantages of simulated annealing:
  - Easy to implement
  - Can get **un-stuck** from a local optimum
  - Can provide satisfactory results with a relatively low number of iterations => efficient

Summary

Summary

- In this lecture, you have learnt about:

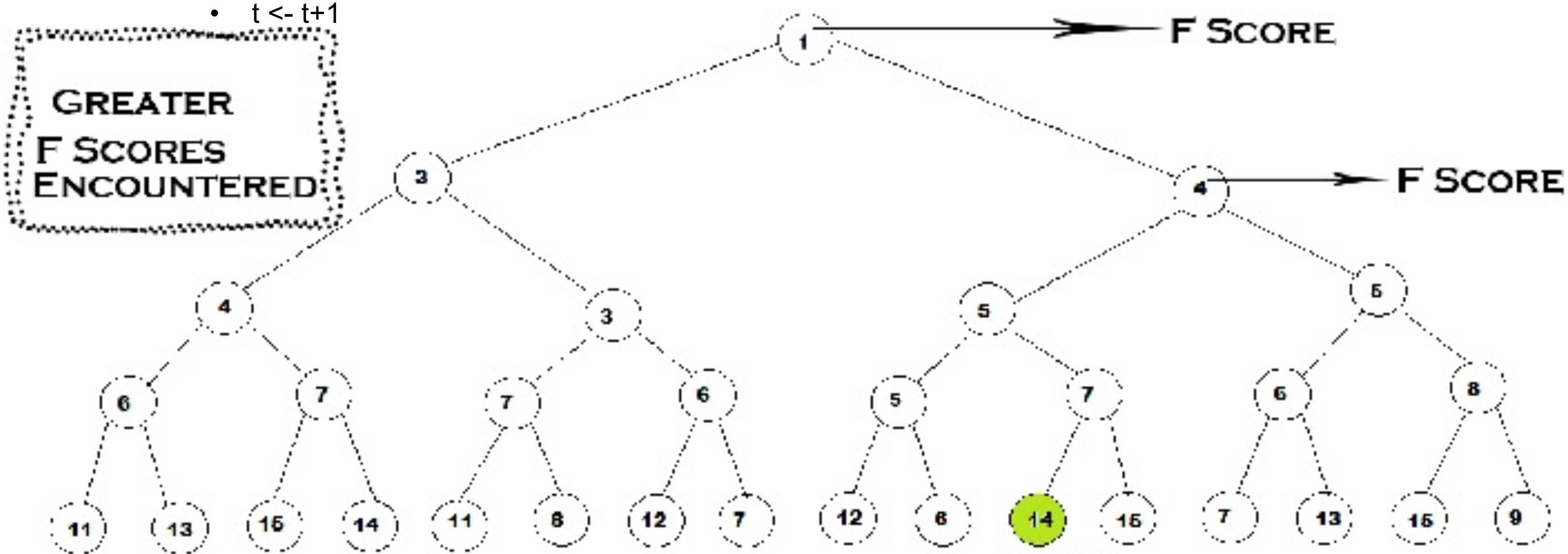
  - Variants of A* search, less memory-consuming
    - Memory-bounded heuristic search: IDA*, RBFS, SMA*

  - Local search algorithms: Hill-climbing search, Simulated annealing search
    - Standard mathematical **optimization techniques**, belong to numerical analysis
    - Can be used for AI (but do not strictly belong to AI)
      - The vocabulary is sometimes a bit different…

Exercise

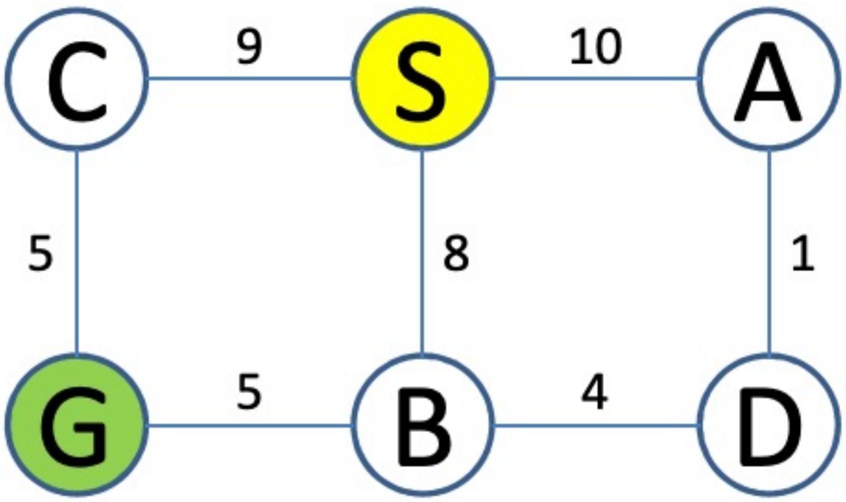
Recall: iterative Deeping A* (IDA*)

- Iterative Deeping version of A* (general idea)
  - If necessary, transform the graph into a tree; remove loopy paths; compute *f* from *g* and *h*
  - Define a threshold, $\theta_1 = f(root\_node)$
  - *t<-1*
  - Then, at each iteration *t*
    - Expand the non-goal nodes *n* in a depth-first way, with stopping criterion $f() > \theta_t$
      - *I.e.* If $f(n) > \theta_t$ then **prune** the node *n* (do not expand it), and put this node in the fringe
      - If any goal node in the fringe, return the goal node with lowest value of *f()*
        - Simple strategy in case of ties: pick randomly; other strategies are possible
      - Set θ_{t+1} = minimum *f()* of all nodes in the fringe
      - t <- t+1



Exercise: IDA* on a graph

- Apply IDA* on this graph search



	S	A	B	C	D	G
heuristic	0	0	4	3	0	0

Chapter 3 – part 3

Questions

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Thank you
for your
attention!