# Integer Linear Programming

- f(x1, x2, x3, x4, x5) = x1 + x2 → max
  s.t. 2x1 + x2 + x3 = 8
  3x1 + 4x2 + x4 = 24
  x1 - x2 + x5 = 2
  x1, x2, x3, x4, x5 $\geq$ 0 and integer

# Balanced Class Assignment

- *N* classes 0, 1, 2, …, *N*-1 need to be assigned to *M* teachers 0, 1, 2, …, *M*-1 (each class is taught by only one teacher).
- Class *i* has credit *c*(*i*), *i* = 0,1,…, *N*-1
- Each teacher can only teach some certain classes based on the expertise of the teacher: tc(i,j) = 1 indicates that teacher i can teach class j and tc(i,j) = 0 means that teacher *i* cannot be assigned to teach class j, forall i = 0,1,…, N-1, j = 0,1,…, M-1
- N classes have been scheduled to a timetable beforehand: it means that two classes *i* and j scheduled in the same time slot cannot be assigned to a teacher: f(i,j) = 1 indicates that classes i and j have been scheduled in a same time slot, for i, j = 0,1,..., N-1
- Load of a teacher is defined to be the sum of credits of classes assigned to the teacher
- Goal: assign N classes to M teachers such that the maximum load of teachers is minimal

# Balanced Class Assignment

| Class | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Credit | 3 | 3 | 4 | 3 | 4 | 3 | 3 | 3 | 4 | 3 | 3 | 4 | 4 |

| Teacher | List of classes the teacher can teach |
|---|---|
| 0 | 0, 2, 3, 4, 8, 10 |
| 1 | 0, 1, 3, 5, 6, 7, 8 |
| 2 | 1, 2, 3, 7, 9, 11, 12 |

**Classes scheduled to a same time slot**

| | |
|---|---|
| 0 | 2 |
| 0 | 4 |
| 0 | 8 |
| 1 | 4 |
| 1 | 10 |
| 3 | 7 |
| 3 | 9 |
| 5 | 11 |
| 5 | 12 |
| 6 | 8 |
| 6 | 12 |

ILP model { y[i]: load of teacher i
z: describe objective function.

Variables: x[i,j] = 1, means that class i is assigned to teacher j (x[i,j]=0, otherwise)

$\forall i = 0, \dots, N-1, \; j = 0, \dots, M-1$

Constraints: → $\sum_{j=0}^{M-1} x[i,j] = 1, \quad \forall i = 0, \dots, N-1$

→ x[i,j] = 0, for all i = 0, .. N-1, j = 0, .. M-1 s.t. tc[j,i] = 0

→ for i = 0, .., N-1, j = 0, .. M-1 with y[i,j] = 1:

x[i,k] + x[j,k] ≤ 1 $\quad \forall k = 0, 1, \dots M-1$ → x[i,k] = 1 and x[j,k] = 1

→ $y[j] = \sum_{i=0}^{N-1} x[i,j] * c[i], \quad \forall j = 0, 1, \dots M-1$ → $\sum_{i=0}^{N-1} x[i,j] * c[i] - y[j] = 0$

→ $z \geq y[j], \quad \forall j = 0, \dots, M-1$ → $0 \leq z - y[j] \leq INF$

Objective: ~~f(x,y,z)~~ $f(x,y,z) = z \rightarrow \min$

x

| | 0 | 1 | 2 |
|---|---|---|---|
| 0 | 1 | 0 | 0 |
| 1 | 0 | 1 | 0 |
| 2 | 0 | 0 | 1 |
| 3 | 1 | 0 | 0 |
| 4 | 1 | 0 | 0 |
| 5 | 0 | 0 | 1 |