

Chapter 1

VECTORS AND THE GEOMETRY OF SPACE

Reference: James Stewart. *Calculus*, sixth edition. Thomson, USA 2008.

1.1 Three-dimensional coordinate systems

1. Find the lengths of the sides of the triangle PQR . Is it a right triangle?
Is it an isosceles triangle?

a) $P(3; -2; -3)$, $Q(7; 0; 1)$, $R(1; 2; 1)$.

b) $P(2; -1; 0)$, $Q(4; 1; 1)$, $R(4; -5; 4)$.

2. Find an equation of the sphere with center $(1; -4; 3)$ and radius 5.
Describe its intersection with each of the coordinate planes.

3. Find an equation of the sphere that passes through the origin and whose center is $(1; 2; 3)$.

4. Find an equation of a sphere if one of its diameters has end points $(2; 1; 4)$ and $(4; 3; 10)$.

5. Find an equation of the largest sphere with center $(5, 4, 9)$ that is contained in the first octant.

6. Write inequalities to describe the following regions

a) The region consisting of all points between (but not on) the spheres of radius r and R centered at the origin, where $r < R$.

b) The solid upper hemisphere of the sphere of radius 2 centered at the origin.

7. Consider the points P such that the distance from P to $A(-1; 5; 3)$ is twice the distance from P to $B(6; 2; -2)$. Show that the set of all such points is a sphere, and find its center and radius.

8. Find an equation of the set of all points equidistant from the points $A(-1; 5; 3)$ and $B(6; 2; -2)$. Describe the set.

1.2 Vectors

9. Find the unit vectors that are parallel to the tangent line to the parabola $y = x^2$ at the point $(2; 4)$.

10. Find the unit vectors that are parallel to the tangent line to the curve $y = 2 \sin x$ at the point $(\pi/6; 1)$.

11. Find the unit vectors that are perpendicular to the tangent line to the curve $y = 2 \sin x$ at the point $(\pi/6; 1)$.

12. Let C be the point on the line segment AB that is twice as far from B as it is from A . If $\mathbf{a} = \overrightarrow{OA}$, $\mathbf{b} = \overrightarrow{OB}$, and $\mathbf{c} = \overrightarrow{OC}$, show that $\mathbf{c} = \frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{b}$.

1.3 The dot product

13. Determine whether the given vectors are orthogonal, parallel, or neither

a) $a = (-5; 3; 7)$, $b = (6; -8; 2)$

b) $a = (4; 6)$, $b = (-3; 2)$

c) $a = -i + 2j + 5k$, $b = 3i + 4j - k$

d) $u = (a, b, c)$, $v = (-b; a; 0)$

14. For what values of b are the vectors $(-6; b; 2)$ and $(b; b^2; b)$ orthogonal?

15. Find two unit vectors that make an angle of 60° with $v = (3; 4)$.

16. If a vector has direction angles $\alpha = \pi/4$ and $\beta = \pi/3$, find the third direction angle γ .

17. Find the angle between a diagonal of a cube and one of its edges.

18. Find the angle between a diagonal of a cube and a diagonal of one of its faces.

1.4 The cross product

19. Find the area of the parallelogram with vertices $A(-2; 1)$, $B(0; 4)$, $C(4; 2)$, and $D(2; -1)$.

20. Find the area of the parallelogram with vertices $K(1; 2; 3)$, $L(1; 3; 6)$, $M(3; 8; 6)$ and $N(3; 7; 3)$.

21. Find the volume of the parallelepiped determined by the vectors a , b , and c .

a) $a = (6; 3; -1)$, $b = (0; 1; 2)$, $c = (4; -2; 5)$.

b) $a = i + j - k$, $b = i - j + k$, $c = -i + j + k$.

22. Let $v = 5j$ and let u be a vector with length 3 that starts at the origin and rotates in the xy -plane. Find the maximum and minimum values of the length of the vector $u \times v$. In what direction does $u \times v$ point?

1.5 Equations of lines and planes

23. Determine whether each statement is true or false.

- a) Two lines parallel to a third line are parallel.
- b) Two lines perpendicular to a third line are parallel.
- c) Two planes parallel to a third plane are parallel.
- d) Two planes perpendicular to a third plane are parallel.
- e) Two lines parallel to a plane are parallel.
- f) Two lines perpendicular to a plane are parallel.
- g) Two planes parallel to a line are parallel.
- h) Two planes perpendicular to a line are parallel.
- i) Two planes either intersect or are parallel.
- j) Two lines either intersect or are parallel.
- k) A plane and a line either intersect or are parallel.

24. Find a vector equation and parametric equations for the line.

- a) The line through the point $(6; -5; 2)$ and parallel to the vector $(1; 3; -2/3)$.
- b) The line through the point $(0; 14; -10)$ and parallel to the line $x = -1 + 2t; y = 6 - 3t; z = 3 + 9t$.
- c) The line through the point $(1, 0, 6)$ and perpendicular to the plane $x + 3y + z = 5$.

25. Find parametric equations and symmetric equations for the line of intersection of the plane $x + y + z = 1$ and $x + z = 0$.

26. Find a vector equation for the line segment from $(2; -1; 4)$ to $(4; 6; 1)$.

27. Determine whether the lines L_1 and L_2 are parallel, skew, or intersecting. If they intersect, find the point of intersection.

- a) $L_1 : x = -6t, y = 1 + 9t, z = -3t; \quad L_2 : x = 1 + 2s, y = 4 - 3s, z = s$.
- b) $L_1 : \frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}; \quad L_2 : \frac{x-3}{-4} = \frac{y-2}{-3} = \frac{z-1}{2}$.

28. Find an equation of the plane.

- a) The plane through the point $(6; 3; 2)$ and perpendicular to the vector $(-2; 1; 5)$
- b) The plane through the point $(-2; 8; 10)$ and perpendicular to the line $x = 1 + t, y = 2t, z = 4 - 3t$.
- c) The plane that contains the line $x = 3 + 2t, y = t, z = 8 - t$ and is parallel to the plane $2x + 4y + 8z = 17$.

29. Find the cosine of the angle between the planes $x + y + z = 0$ and $x + 2y + 3z = 1$.

30. Find parametric equations for the line through the point $(0; 1; 2)$ that is perpendicular to the line $x = 1 + t, y = 1 - t, z = 2t$, and intersects this line.

31. Find the distance between the skew lines with parametric equations $x = 1 + t, y = 1 + 6t, z = 2t$ and $x = 1 + 2s, y = 5 + 15s, z = -2 + 6s$.

1.6 Quadric surfaces

32. Find an equation for the surface obtained by rotating the parabola $y = x^2$ about the y -axis.

33. Find an equation for the surface consisting of all points that are equidistant from the point $(-1; 0; 0)$ and the plane $x = 1$. Identify the surface.

Chapter 2

VECTOR FUNCTIONS

Reference: James Stewart. *Calculus*, sixth edition. Thomson, USA 2008.

2.1 Vector functions

34. Find the domain of the vector function.

a) $r(t) = (\sqrt{4-t^2}, e^{-3t}, \ln(t+1))$

b) $r(t) = \frac{t-2}{t+2}i + \sin tj + \ln(9-t^2)k$

35. Find the limit

a) $\lim_{t \rightarrow 0} \left(\frac{e^t-1}{t}, \frac{\sqrt{1+t}-1}{t}, \frac{3}{t+1} \right)$

b) $\lim_{t \rightarrow \infty} \left(\arctan t, e^{-2t}, \frac{\ln t}{t+1} \right)$

36. Find a vector function that represents the curve of intersection of the two surfaces.

a) The cylinder $x^2 + y^2 = 4$ and the surface $z = xy$.

b) The paraboloid $z = 4x^2 + y^2$ and the parabolic cylinder $y = x^2$.

37. Suppose u and v are vector functions that possess limits as $t \rightarrow a$ and let c be a constant. Prove the following properties of limits.

a) $\lim_{t \rightarrow a} [u(t) + v(t)] = \lim_{t \rightarrow a} u(t) + \lim_{t \rightarrow a} v(t)$

b) $\lim_{t \rightarrow a} cu(t) = c \lim_{t \rightarrow a} u(t)$

c) $\lim_{t \rightarrow a} [u(t) \cdot v(t)] = \lim_{t \rightarrow a} u(t) \cdot \lim_{t \rightarrow a} v(t)$

d) $\lim_{t \rightarrow a} [u(t) \times v(t)] = \lim_{t \rightarrow a} u(t) \times \lim_{t \rightarrow a} v(t)$

38. Find the derivative of the vector function.

a) $r(t) = (t \sin t, t^3, t \cos 2t).$

b) $r(t) = \arcsin ti + \sqrt{1 - t^2}j + k$

c) $r(t) = e^{t^2}i - \sin^2 tj + \ln(1 + 3t)$

39. Find parametric equations for the tangent line to the curve with the given parametric equations at the specified point. Illustrate by graphing both the curve and the tangent line on a common screen.

a) $x = t, y = e^{-t}, z = 2t - t^2; (0; 1; 0)$

b) $x = 2 \cos t, y = 2 \sin t, z = 4 \cos 2t; (\sqrt{3}, 1, 2)$

c) $x = t \cos t, y = t, z = t \sin t; (-\pi, \pi, 0)$

40. Find the point of intersection of the tangent lines to the curve $r(t) = (\sin \pi t, 2 \sin \pi t, \cos \pi t)$ at the points where $t = 0$ and $t = 0.5$

41. Evaluate the integral

a) $\int_0^{\pi/2} (3 \sin^2 t \cos t i + 3 \sin t \cos^2 t j + 2 \sin t \cos t k) dt$

b) $\int_1^2 (t^2 i + t\sqrt{t-1} j + t \sin \pi t k) dt$

c) $\int (e^t i + 2t j + \ln t k) dt$

d) $\int (\cos \pi t i + \sin \pi t j + t^2 k) dt$

42. If a curve has the property that the position vector $r(t)$ is always perpendicular to the tangent vector $r'(t)$, show that the curve lies on a sphere with center the origin.

2.2 Arc length and curvature

43. Find the length of the curve.

a) $r(t) = (2 \sin t, 5t, 2 \cos t), \quad -10 \leq t \leq 10$

b) $r(t) = (2t, t^2, \frac{1}{3}t^3), \quad 0 \leq t \leq 1$

c) $r(t) = \cos t \, i + \sin t \, j + \ln \cos t \, k, \quad 0 \leq t \leq \pi/4$

44. Let C be the curve of intersection of the parabolic cylinder $x^2 = 2y$ and the surface $3z = xy$. Find the exact length of C from the origin to the point $(6; 18; 36)$.

45. Suppose you start at the point $(0; 0; 3)$ and move 5 units along the curve $x = 3 \sin t, y = 4t, z = 3 \cos t$ in the positive direction. Where are you now?

46. Reparametrize the curve

$$r(t) = \left(\frac{2}{t^2 + 1} - 1 \right) i + \frac{2t}{t^2 + 1} j$$

with respect to arc length measured from the point $(1; 0)$ in the direction of increasing t . Express the reparametrization in its simplest form. What can you conclude about the curve?

47. Find the curvature

a) $r(t) = t^2 i + t k$

b) $r(t) = t i + t j + (1 + t^2) k$

c) $r(t) = 3t i + 4 \sin t j + 4 \cos t k$

d) $x = e^t \cos t, y = e^t \sin t$

e) $x = t^3 + 1, y = t^2 + 1$

48. Find the curvature of $r(t) = (e^t \cos t, e^t \sin t, t)$ at the point $(1, 0, 0)$.

49. Find the curvature of $r(t) = (t, t^2, t^3)$ at the point $(1, 1, 1)$.

50. Find the curvature

a) $y = 2x - x^2,$

b) $y = \cos x,$

c) $y = 4x^{5/2}.$

51. At what point does the curve have maximum curvature? What happens to the curvature as $x \rightarrow \infty$?

a) $y = \ln x,$

b) $y = e^x.$

52. Find an equation of a parabola that has curvature 4 at the origin.

Chapter 3

DOUBLE INTEGRALS

Reference: James Stewart. *Calculus*, sixth edition. Thomson, USA 2008.

3.1 Double integrals

53. Calculate the iterated integral

$$\begin{array}{lll} \text{a)} \int_1^3 \int_0^1 (1 + 4xy) dx dy & \text{b)} \int_0^2 \int_0^1 (2x + y)^8 dx dy & \text{c)} \int_0^1 \int_1^2 \frac{xe^x}{y} dy dx \\ \text{d)} \int_1^4 \int_1^2 \left(\frac{x}{y} + \frac{y}{x} \right) dx dy & \text{e)} \int_0^1 \int_0^1 xy \sqrt{x^2 + y^2} dx dy & \text{f)} \int_0^2 \int_0^\pi r \sin^2 \varphi d\varphi dr. \end{array}$$

54. Calculate the double integral

$$\begin{array}{ll} \text{a)} \iint_D \frac{1+x^2}{1+y^2} dx dy, & D = \{(x, y) | 0 \leq x \leq 1, 0 \leq y \leq 1\} \\ \text{b)} \iint_D \frac{x}{1+xy} dx dy, & D = \{(x, y) | 0 \leq x \leq 1, 0 \leq y \leq 1\} \\ \text{c)} \iint_D \frac{x}{x^2+y^2} dx dy, & D = [1, 2] \times [0, 1] \\ \text{d)} \iint_D xy e^{x^2 y} dx dy, & D = [0, 1] \times [0, 2] \end{array}$$

55. Find the volume of the solid that lies under the hyperbolic paraboloid $z = 4 + x^2 - y^2$ and above the square $D = [-1; 1] \times [0; 2]$

56. Find the volume of the solid enclosed by the surface $z = 1 + e^x \sin y$ and the planes $x = \pm 1, y = 0, y = \pi$ and $z = 0$.

57. Find the volume of the solid in the first octant bounded by the cylinder $z = 16 - x^2$ and the plane $y = 5$.

58. Evaluate the iterated integral

$$\text{a)} \int_0^4 \int_0^{\sqrt{y}} xy^2 dx dy, \quad \text{b)} \int_0^2 \int_y^{2y} xy dx dy, \quad \text{c)} \int_0^1 \int_0^v \sqrt{1-v^2} du dv.$$

59. Evaluate the double integral

- a) $\iint_D \frac{y}{1+x^5} dx dy, \quad D = \{(x, y) | 0 \leq x \leq 1, 0 \leq y \leq x^2\}$
- b) $\iint_D y^2 e^{xy} dx dy, \quad D = \{(x, y) | 0 \leq y \leq 4, 0 \leq x \leq y\}$
- c) $\iint_D x \sqrt{y^2 - x^2} dx dy, \quad D = \{(x, y) | 0 \leq y \leq 1, 0 \leq x \leq y\}$
- d) $\iint_D (x + y) dx dy, \quad D$ is bounded by $y = \sqrt{x}$ and $y = x^2$
- e) $\iint_D y^3 dx dy, \quad D$ is the triangle region with vertices $(0; 2)$, $(1; 1)$ and $(3; 2)$
- f) $\iint_D xy^2 dx dy, \quad D$ is enclosed by $x = 0$ and $x = \sqrt{1 - y^2}$

60. Find the volume of the given solid

- a) Under the surface $z = 2x + y^2$ and above the region bounded by $x = y^2$ and $x = y^3$.
- b) Enclosed by the paraboloid $z = x^2 + 3y^2$ and the planes $x = 0$, $y = 1$, $y = x$, $z = 0$
- c) Enclosed by the cylinders $z = x^2$, $y = x^2$ and the planes $z = 0$, $y = 4$.
- d) Bounded by the cylinder $y^2 + z^2 = 4$ and the planes $x = 2y$, $x = 0$, $z = 0$ in the first octant
- e) Bounded by the cylinders $x^2 + y^2 = r^2$ and $y^2 + z^2 = r^2$.
- f) The solid enclosed by the parabolic cylinder $y = x^2$ and the planes $z = 3y$, $z = 2 + y$.

61. Sketch the region of integration and change the order of integration.

- a) $\int_0^4 \int_0^{\sqrt{x}} f(x, y) dy dx,$
- b) $\int_0^1 \int_{4x}^4 f(x, y) dy dx,$
- c) $\int_0^3 \int_{-\sqrt{9-y^2}}^{\sqrt{9-y^2}} f(x, y) dx dy.$
- d) $\int_0^3 \int_0^{\sqrt{9-y}} f(x, y) dx dy,$
- e) $\int_1^2 \int_0^{\ln x} f(x, y) dy dx,$
- f) $\int_0^1 \int_{\arctan x}^{\pi/4} f(x, y) dy dx.$

62. Evaluate the integral by reversing the order of integration

- a) $\int_0^1 \int_{3y}^3 e^{x^2} dx dy$
- b) $\int_0^{\sqrt{\pi}} \int_y^{\sqrt{\pi}} \cos(x^2) dx dy$
- c) $\int_0^4 \int_{\sqrt{x}}^2 \frac{1}{y^3 + 1} dy dx$
- d) $\int_0^1 \int_x^1 e^{x/y} dy dx$
- e) $\int_0^1 \int_{\arcsin y}^{\pi/2} \cos x \sqrt{1 + \cos^2 x} dx dy$
- f) $\int_0^8 \int_{\sqrt[3]{y}}^2 e^{x^4} dx dy.$

3.2 Double integrals in polar coordinates

63. Evaluate the given integral by changing to polar coordinates.

- $\iint_D (x+y) dx dy$ where D is the region that lies to the left of the y -axis, between the circles $x^2 + y^2 = 1$, and $x^2 + y^2 = 4$.
- $\iint_D \cos(x^2 + y^2) dx dy$ where D is the region that lies above the x -axis within the circle $x^2 + y^2 = 9$.
- $\iint_D \sqrt{4 - x^2 - y^2} dx dy$ where $D = \{(x, y) | x^2 + y^2 \leq 4, x \geq 0\}$.
- $\iint_D ye^x dx dy$ where D is the region in the first quadrant enclosed by the circle $x^2 + y^2 = 25$.
- $\iint_D \arctan(y/x) dx dy$ where $D = \{(x, y) | 1 \leq x^2 + y^2 \leq 4, 0 \leq y \leq x\}$.
- $\iint_D x dx dy$ where D is the region in the first quadrant that lies between the circles $x^2 + y^2 = 4$ and $x^2 + y^2 = 2x$.

64. Use a double integral to find the area of the region.

- The region enclosed by the curve $r = 4 + 3 \cos \varphi$
- The region inside the cardioid $r = 1 + \cos \varphi$ and outside the circle $r = 3 \cos \varphi$.

65. Use polar coordinates to find the volume of the given solid.

- Below the paraboloid $z = 18 - 2x^2 - 2y^2$ and above the xy -plane
- Bounded by the paraboloid $z = 1 + 2x^2 + 2y^2$ and the plane $z = 7$ in the first octant.
- Above the cone $z = \sqrt{x^2 + y^2}$ and below the sphere $x^2 + y^2 + z^2 = 1$
- Bounded by the paraboloids $z = 3x^2 + 3y^2$ and $z = 4 - x^2 - y^2$.

66. Evaluate the iterated integral by converting to polar coordinates

$$\text{a) } \int_0^a \int_{-\sqrt{a^2-y^2}}^0 x^2 y dx dy, \quad \text{b) } \int_0^1 \int_y^{\sqrt{2y-y^2}} (x+y) dx dy, \quad \text{c) } \int_0^2 \int_0^{\sqrt{2x-x^2}} \sqrt{x^2+y^2} dy dx.$$

Chapter 4

TRIPLE INTEGRALS

Reference: James Stewart. *Calculus*, sixth edition. Thomson, USA 2008.

69. Evaluate the iterated integral.

$$\begin{aligned} \text{a) } & \int_0^1 \int_x^{2x} \int_0^y 2xyz \, dz \, dy \, dx, & \text{b) } & \int_0^3 \int_0^1 \int_0^{\sqrt{1-z^2}} ze^y \, dx \, dz \, dy, & \text{c) } & \int_0^1 \int_0^z \int_0^y ze^{-y^2} \, dx \, dy \, dz. \\ \text{d) } & \int_0^{\pi/2} \int_0^y \int_0^x \cos(x+y+z) \, dz \, dx \, dy, & \text{e) } & \int_0^{\sqrt{\pi}} \int_0^x \int_0^{xz} x^2 \sin y \, dy \, dz \, dx. \end{aligned}$$

70. Evaluate the triple integral

- a) $\iiint_E y \, dV$, where E is bounded by the planes $x = 0$, $y = 0$, $z = 0$, and $2x + 2y + z = 4$
- b) $\iiint_E x^2 e^y \, dV$, where E is bounded by the parabolic cylinder $z = 1 - y^2$ and the planes, $z = 0$, $x = 1$, and $x = -1$.
- c) $\iiint_E xy \, dV$, where E is bounded by the parabolic cylinder $y = x^2$ and $x = y^2$ and the planes, $z = 0$ and $z = x + y$.
- d) $\iiint_E xyz \, dV$, where E is the solid tetrahedron with vertices $(0, 0, 0)$, $(1, 0, 0)$, $(0, 1, 0)$ and $(0, 0, 1)$.
- e) $\iiint_E x \, dV$, where E is the bounded by the paraboloid $x = 4y^2 + 4z^2$ and the plane $x = 4$.
- f) $\iiint_E z \, dV$, where E is the bounded by the cylinder $y^2 + z^2 = 9$ and the planes $x = 0$, $y = 3x$, and $z = 0$ in the first octant.

3.3 Applications of double integrals

67. Find the mass and center of mass of the lamina that occupies the region D and has the given density function ρ .

- a) D is the triangular region enclosed by the lines $x = 0$, $y = x$ and $2x + y = 6$, $\rho(x, y) = x^2$.
- b) D is bounded by $y = e^x$, $y = 0$, $x = 0$, and $x = 1$, $\rho(x, y) = y$.
- c) D is bounded by $y = \sqrt{x}$, $y = 0$, and $x = 1$, $\rho(x, y) = x$.
- d) D is bounded by the parabolas $y = x^2$, and $x = y^2$, $\rho(x, y) = x$.

68. A lamina occupies the region inside the circle $x^2 + y^2 = 2y$ but outside the circle $x^2 + y^2 = 1$. Find the center of mass if the density at any point is inversely proportional to its distance from the origin.

71. Find the volume of the given solid

- a) The solid bounded by the cylinder $y = x^2$ and the planes $z = 0$, $z = 4$, and $y = 9$.
- b) The solid enclosed by the cylinder $x^2 + y^2 = 9$ and the planes $y + z = 5$ and $z = 1$.
- c) The solid enclosed by the paraboloid $x = y^2 + z^2$ and the plane $x = 16$.

72. Evaluate $\iiint_E (x^3 + xy^2) dV$, where E is the solid in the first octant that lies beneath the paraboloid $z = 1 - x^2 - y^2$.

73. Evaluate $\iiint_E e^z dV$, where E is enclosed by the paraboloid $z = 1 + x^2 + y^2$, the cylinder $x^2 + y^2 = 5$, and the xy -plane.

74. Evaluate $\iiint_E x dV$, where E is enclosed by the planes $z = 0$ and $z = x + y + 5$ and by the cylinders $x^2 + y^2 = 4$ and $x^2 + y^2 = 9$.

75. Find the volume of the solid that lies within both the cylinder $x^2 + y^2 = 1$ and the sphere $x^2 + y^2 + z^2 = 4$.

76. Find the volume of the region E bounded by the paraboloids $z = x^2 + y^2$ and $z = 36 - 3x^2 - 3y^2$.

77. Evaluate the integral by changing to cylindrical coordinates

- a) $\int_{-2}^2 \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} \int_{\sqrt{x^2+y^2}}^2 xz dz dx dy.$
- b) $\int_{-3}^3 \int_0^{\sqrt{9-x^2}} \int_0^{9-x^2-y^2} \sqrt{x^2+y^2} dz dy dx.$

78. A solid lies above the cone $z = \sqrt{x^2 + y^2}$ and below the sphere $x^2 + y^2 + z^2 = z$. Write a description of the solid in terms of inequalities involving spherical coordinates.

79. Use spherical coordinates

- a) Evaluate $\iiint_H (9 - x^2 - y^2) dV$, where H is the solid hemisphere $x^2 + y^2 + z^2 \leq 9$, $z \geq 0$.
- b) Evaluate $\iiint_E z dV$, where E lies between the spheres $x^2 + y^2 + z^2 = 1$ and $x^2 + y^2 + z^2 = 4$ in the first octant.
- c) Evaluate $\iiint_E e^{\sqrt{x^2+y^2+z^2}} dV$, where E is enclosed by the sphere $x^2 + y^2 + z^2 = 9$ in the first octant.

- d) Evaluate $\iiint_E x^2 dV$, where E is bounded by the xz -plane and the hemispheres $y = \sqrt{9 - x^2 - z^2}$ and $y = \sqrt{16 - x^2 - z^2}$.

80. Evaluate the integral by changing to spherical coordinates.

- a) $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_{x^2+y^2}^{\sqrt{2-x^2-y^2}} xy dz dy dx.$
 b) $\int_{-a}^a \int_{-\sqrt{a^2-y^2}}^{\sqrt{a^2-y^2}} \int_{-\sqrt{a^2-x^2-y^2}}^{\sqrt{a^2-x^2-y^2}} (x^2 z + y^2 z + z^3) dz dx dy..$

81. Calculate $\iiint_E y^2 z^2 dV$, where E is bounded by the paraboloid $x = 1 - y^2 - z^2$ and the plane $x = 0$.

82. Evaluate the triple integral $\iiint_V y dx dy dz$, where V is bounded by the cone $y = \sqrt{x^2 + z^2}$ and the plane $y = h$, ($h > 0$).

83. Evaluate the triple integral

$$\iiint_V \left(\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} \right) dx dy dz, \quad \text{where } V : \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} \leq 1, (a, b, c > 0).$$

84. Evaluate $\iiint_V \sqrt{x^2 + y^2 + z^2} dx dy dz$, where V is defined by $x^2 + y^2 + z^2 \leq z$.

85. Evaluate $\iiint_V \sqrt{(6x - x^2 - y^2 - z^2)^3} dx dy dz$, where V is the sphere defined by $x^2 + y^2 + z^2 \leq 6x$.

86. Evaluate $\iiint_V \frac{z}{1 + x^2 + y^2} dx dy dz$, where V is bounded by $z = 6 - \sqrt{x^2 + y^2}$, $z = 5$.

Chapter 5

Line integrals

87. Evaluate the line integral, where C is the given curve

- a) $\int_C x \sin y ds$, C is the line segment from $(0, 3)$ to $(4, 6)$.
- b) $\int_C (x^2 y^3 - \sqrt{x}) dy$, C is the arc of the curve $y = \sqrt{x}$ from $(1, 1)$ to $(4, 2)$.
- c) $\int_C x e^y dx$, C is the arc of the curve $x = e^y$ from $(1, 0)$ to $(e, 1)$.
- d) $\int_C \sin x dx + \cos y dy$, C consists of the top half of the circle $x^2 + y^2 = 1$ from $(1, 0)$ to $(-1, 0)$ and the line segment from $(-1, 0)$ to $(-2, 3)$.
- e) $\int_C xyz ds$, $C : x = 2 \sin t, y = t, z = -2 \cos t, 0 \leq t \leq \pi$.
- f) $\int_C xyz^2 ds$, C is the line segment from $(-1, 5, 0)$ to $(1, 6, 4)$.
- g) $\int_C x^2 y \sqrt{z} dz$, $C : x = t^3, y = t, z = t^2, 0 \leq t \leq 1$.
- h) $\int_C z dx + x dy + y dz$, $C : x = t^2, y = t^3, z = t^2, 0 \leq t \leq 1$.
- k) $\int_C (x + yz) dx + 2x dy + xyz dz$, C consists of line segments from $(1, 0, 1)$ to $(2, 3, 1)$ and from $(2, 3, 1)$ to $(2, 5, 2)$.
- l) $\int_C x^2 dx + y^2 dy + z^2 dz$, C consists of line segments from $(0, 0, 0)$ to $(1, 2, -1)$ and from $(1, 2, -1)$ to $(3, 2, 0)$.

88. Evaluate the following line integrals

- a) $\int_C (x - y) ds$, where C is the circle $x^2 + y^2 = 2x$.



- b) $\int_C (x^2 + y^2 + z^2) ds$, where C is the helix $x = a \cos t$, $y = a \sin t$, $z = bt$,
 $(0 \leq t \leq 2\pi)$.

89. Evaluate the line integral $\int_C F \cdot dr$, where $F(x, y, z) = xi - zj + yk$ and C is given by $r(t) = 2ti + 3tj - t^2k$, $-1 \leq t \leq 1$. **-2**

90. Find the work done by the force field $F(x, y, z) = (y + z, x + z, x + y)$ on a particle that moves along the line segment from $(1; 0; 0)$ to $(3; 4; 2)$.

91. Evaluate the line integral by two methods: (a) directly and using Green's Theorem

- a) $\oint_C (x - y)dx + (x + y)dy$, C is the circle with center the origin and radius 2.
- b) $\oint_C xydx + x^2dy$, C is the rectangle with vertices $(0; 0)$, $(3; 0)$, $(3; 1)$, and $(0; 1)$.
- c) $\oint_C ydx + xdy$, C consists of the line segments from $(0; 1)$ to $(0; 0)$ and from $(0; 0)$ to $(1; 0)$ and the parabola $y = 1 - x^2$ from $(1; 0)$ to $(0; 1)$.

92. Use Green's Theorem to evaluate the line integral along given positively oriented curve

- a) $\int_C (y + e^{\sqrt{x}})dx + (2x + \cos y)dy$, C is the boundary of the region enclosed by the parabolas $y = x^2$ and $x = y^2$.
- b) $\int_C xe^{-2x}dx + (x^4 + 2x^2y^2)dy$, C is the boundary of the region between the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$.
- c) $\int_C (e^x + x^2y)dx + (e^y - xy^2)dy$, C is the circle $x^2 + y^2 = 25$.
- d) $\int_C (2x - x^3y^5)dx + x^3y^8dy$, C is the ellipse $4x^2 + y^2 = 4$.

93. Show that the line integral is independent of path and evaluate the integral

- a) $\int_C (1 - ye^{-x})dx + e^{-x}dy$, C is any path from $(0, 1)$ to $(1, 2)$.
- b) $\int_C 2y^{3/2}dx + 3x\sqrt{y}dy$, C is any path from $(1, 1)$ to $(2, 4)$.

Curl and Divergence

94. Determine whether or not F is a conservative vector field. If it is, find a function f such that $F = \nabla f$.

- a) $F(x, y) = (2x - 3y)i + (-3x + 4y - 8)j$
- b) $F(x, y) = e^x \cos y i + e^x \sin y j$
- c) $F(x, y) = (xy \cos xy + \sin xy)i + (x^2 \cos xy)j$
- d) $F(x, y) = (\ln y + 2xy^3)i + (3x^2y^2 + x/y)j$
- e) $F(x, y) = (ye^x + \sin y)i + (e^x + x \cos y)j$

95. Find a function f such that $F = \nabla f$ and then evaluate $\int_C F \cdot dr$ along the given curve C .

- a) $F(x, y) = xy^2i + x^2yj, C : r(t) = (t + \sin \frac{1}{2}\pi t, t + \cos \frac{1}{2}\pi t), 0 \leq t \leq 1.$
- b) $F(x, y) = \frac{y^2}{1+x^2}i + 2y \arctan x j, C : r(t) = t^2i + 2tj, 0 \leq t \leq 1.$
- c) $F(x, y) = (2xz + y^2)i + 2xyj + (x^2 + 3z^2)k, C : x = t^2, y = t + 1, z = 2t - 1, 0 \leq t \leq 1.$
- d) $F(x, y) = e^y i + x e^y j + (z + 1)e^z k, C : x = t, y = t^2, z = t^3, 0 \leq t \leq 1.$

Chapter 6

Surface Integrals

96. Evaluate the surface integral

- a) $\iint_S xy dS$, S is the triangular region with vertices $(1, 0, 0)$, $(0, 2, 0)$, and $(0, 0, 2)$.
- b) $\iint_S yz dS$, S is the part of the plane $x + y + z = 1$ that lies in the first octant.
- c) $\iint_S yz dS$, S is the surface with parametric equations $x = u^2$, $y = u \sin v$, $z = u \cos v$, $0 \leq u \leq 1$, $0 \leq v \leq \pi/2$.
- d) $\iint_S z dS$, S is the surface $x = y + 2z^2$, $0 \leq y \leq 1$, $0 \leq z \leq 1$.
- e) $\iint_S y^2 dS$, S is the part of the sphere $x^2 + y^2 + z^2 = 4$ that lies inside the cylinder $x^2 + y^2 = 1$ and above the xy -plane.

97. Evaluate the surface integral $\iint_S F \cdot dS$ for the given vector field F and the oriented surface S . In other words, find the flux of F across S . For closed surfaces, use the positive (outward) orientation.

- a) $F(x, y, z) = xze^y i - xze^y j + zk$, S is the part of the plane $x + y + z = 1$ in the first octant and has downward orientation.
- b) $F(x, y, z) = xi + yj + z^4 k$, S is the part of the cone $z = \sqrt{x^2 + y^2}$ beneath the plane $z = 1$ with downward orientation.
- c) $F(x, y, z) = xzi + xj + yk$, S is the hemisphere $x^2 + y^2 + z^2 = 25$, $y \geq 0$, oriented in the direction of the positive y -axis.

- d) $F(x, y, z) = xyi + 4x^2j + yzk$, S is the surface $z = xe^y$, $0 \leq x \leq 1$, $0 \leq y \leq 1$, with upward orientation.
- e) $F(x, y, z) = x^2i + y^2j + z^2k$, S is the boundary of the solid half-cylinder $0 \leq z \leq \sqrt{1-y^2}$, $0 \leq x \leq 2$.

98. a) Find the center of mass of the hemisphere $x^2 + y^2 + z^2 = a^2$, $z \geq 0$, if it has constant density.

b) Find the mass of a thin funnel in the shape of a cone $z = \sqrt{x^2 + y^2}$, $1 \leq z \leq 4$, if its density function is $\rho(x, y, z) = 10 - z$.

Stokes Theorem

99. Use Stokes Theorem to evaluate $\iint_S \text{curl} F \cdot dS$

- a) $F(x, y, z) = 2y \cos z i + e^x \sin z j + xe^y k$, S is the hemisphere $x^2 + y^2 + z^2 = 9$, $z \geq 0$, oriented upward.
- b) $F(x, y, z) = x^2 z^2 i + y^2 z^2 j + xyz k$, S is the part of the paraboloid $z = x^2 + y^2$ that lies inside the cylinder $x^2 + y^2 = 4$, oriented upward.

The Divergence Theorem

100. Use the Divergence Theorem to calculate the surface integral $\iint_S F \cdot dS$; that is, calculate the flux of F across S

- a) $F(x, y, z) = x^3 y i - x^2 y^2 j - x^2 y z k$, S is the surface of the solid bounded by the hyperboloid $x^2 + y^2 - z^2 = 1$ and the planes $z = -2$ and $z = 2$.
- b) $F(x, y, z) = (\cos z + xy^2)i + xe^{-z}j + (\sin y + x^2 z)k$, S is the surface of the solid bounded by the paraboloid $z = x^2 + y^2$ and the plane $z = 4$.
- c) $F(x, y, z) = 4x^3 z i + 4y^3 z j + 3z^4 k$, S is the sphere with radius R and center the origin.