FINAL EXAM CALCULUS 2
(90 minutes)

1. Find the volume of the parallelepiped determined by the vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ as follows:

$$\vec{a} = (0,0,2), \quad \vec{b} = (0,2,-3), \quad \vec{c} = (1,3,4).$$

2. Find the equation of plane that goes through the point $(2,1,3)$ and perpendicular to the line $x = 1+t$, $y = 2t$, $z = 2-t$.

3. Sketch the region and change the order of the integration

$$\int_0^1 dx \int_0^{\sqrt{x}} f(x,y)dy + \int_1^2 dx \int_0^{2-x} f(x,y)dy.$$

4. Evaluate $\iint_D \sqrt{1+x^2+y^2}dxdy$, where $D = \{(x,y) | 1 \le x^2+y^2 \le 4, x \ge 0, x \ge y \ge 0\}$.

5. Sketch and find the volume of the solid V enclosed by the cone $z = \sqrt{x^2+y^2}$ and the paraboloid $z = 2 - x^2 - y^2$.

6. Evaluate $\int_C zds$, where C is the curve given by $x = t^2$, $y = t$, $z = -2t$, $0 \le t \le 1$.

7. Using Green's Theorem to evaluate $\int_C (x^3 + \sin e^x + y)dx + (\cos y - x)dy$, where C is the boundary of the region between the circles $1 \le x^2 + y^2 \le 2x$.

8. Determine a function f such that $\vec{F} = \nabla f$, where

$$\vec{F}(x,y,z) = (2xz + y^2)\vec{i} + 2xy\vec{j} + (x^2 + 3z^2)\vec{k}.$$

Then, evaluate $\int_C \vec{F} \cdot d\vec{r}$ along the curve $(C) : x = t, y = t^2, z = t^3, t : 0 \to 1$.

9. Evaluate $\iint_S zdS$, where S is the part of the plane $x + y + z = 1$ that lies in the first octant.

10. Using Divergence Theorem to calculate the surface integral $\iint_S \vec{F} \cdot d\vec{S}$, where

$$\vec{F}(x,y,z) = (e^z + xy^2)\vec{i} + (2zy + xe^{-z})\vec{j} + (\sin(xy) + x^2 z)\vec{k},$$

and S is the surface of the solid bounded by paraboloid $z = 1 - x^2 - y^2$ and plane $z = 0$.