Probability and Statistics – Problem Set 5

Question 1. Suppose in MATH2010:

The time it takes for you to finish Problem set i $(i = 1, .., 5)$ is approximately

$$X_i \sim \mathcal{N}(\mu = 10,\ \sigma^2 = 9)$$

and the time it takes for you to finish the Homework i $(1, \ldots, 5)$

$$Y_i \sim \mathcal{N}(\mu = 20,\ \sigma^2 = 10)$$

Let $W = X_1 + X_2 + \cdots + X_5 + Y_1 + Y_2 + \cdots + Y_5$ be the time it takes to complete all the problem sets and the homework.

a) What is the mean and variance of W?
b) What is the distribution of W and what are its parameter(s)?
c) What is the probability that you complete all the problem sets and the homework in under 100 hours?

Question 2. Suppose $X \sim Exp\left(\lambda = \frac{1}{2}\right)$ is the waiting time in hours until your pizza delivery arrives, and suppose we decide to tip $Y = g(X) = \frac{24}{X+1}$ dollars.

a. What is the range, PDF, and CDF of X?
b. What is the range Ω_Y?
c. Find $F_Y(y)$ using the CDF method, then find $f_Y(y)$ afterwards.
e. Set up integrals for $E[Y]$ in two ways: one with LOTUS and $f_X(x)$, and one with $f_Y(y)$.

Question 3: Suppose $X \sim Unif(-1, 1)$ (continuous), then find the PDF of $Y = X^2$.

Question 4:

a. Let $X \sim Geo(p)$. Give a formula for $M_X(t)$, and specify for which values of t the formula converges.

b. Let $Y \sim NegBin(r, p)$. Give a formula for $M_Y(t)$.

c. Set up formulas to compute $E[Y]$ and $Var(Y)$ using the MGF M_Y (but don't compute anything).

Question 5: You have invited 64 guests to a party. You need to make sandwiches for the guests. You believe that a guest might need 0, 1 or 2 sandwiches with probabilities $\frac{1}{4}, \frac{1}{2}$, and $\frac{1}{4}$ respectively. You assume that the number of sandwiches each guest needs is independent from other guests. How many sandwiches should you make so that you are 95% sure that there is no shortage?

Question 6: Suppose that a market research analyst for a cell phone company conducts a study of their customers who exceed the time allowance included on their basic cell phone contract; the analyst finds that for those people who exceed the time included in their basic contract, the excess time used follows an exponential distribution with a mean of 22 minutes.

Consider a random sample of 80 customers who exceed the time allowance included in their basic cell phone contract.

Let X = the excess time used by one INDIVIDUAL cell phone customer who exceeds his contracted time allowance and $X \sim Exp\left(\lambda = \frac{1}{22}\right)$.

a) Find the probability that the average excess time used by the 80 customers in the sample is longer than 20 minutes. Draw the graph.
b) Suppose that one customer who exceeds the time limit for his cell phone contract is randomly selected. Find the probability that this individual customer's excess time is longer than 20 minutes. This is asking us to find P (X > 20)
c) Explain why the probabilities in (a) and (b) are different