

Probability and Statistics – Problem Set 2

Question 1. A spam filter is designed by looking at commonly occurring phrases in spam. Suppose that 80% of email is spam. In 10% of the spam emails, the phrase “free money” is used, whereas this phrase is only used in 1% of non-spam emails.

A new email has just arrived, which includes the phrase “free money”. What is the probability that it is spam?

Question 2. A bag contains one marble which is either green or blue, with equal probabilities. A green marble is put in the bag (so there are two marbles now), and then a random marble is taken out. The marble taken out is green. What is the probability that the remaining marble is also green?

Question 3. A crime is committed by one of two suspects, A and B. Initially, there is equal evidence against both of them. In further investigation at the crime scene, it is found that the guilty party had a blood type found in 10% of the population. Suspect A does match this blood type, whereas the blood type of Suspect B is unknown.

- a) Given this new information, what is the probability that A is the guilty party?
- b) Given this new information, what is the probability that B’s blood type matches that found at the crime scene?

Question 4. Is it always true that if A and B are independent events, then A^c and B^c are independent events? Show that it is, or give a counter example.

Question 5. A parallel system functions whenever at least one of its components works. Consider a parallel system of n components and suppose that each component works with probability p independently.

- a) What is the probability the system is functioning?
- b) If the system is functioning, what is the probability that component 1 is working?
- c) If the system is functioning and component 2 is working, what is the probability that component 1 is working?

Question 6. There are 3 people in Alex’s family; his mom, dad, and sister. Each family member decides whether or not they want to come to lunch in his social-distancing home restaurant, independently of the others.

- Mom wants to come with probability 0.8.
- Dad wants to come with probability 0.6.
- Sister wants to come with probability 0.1.

Unfortunately, if all 3 of them want to come, he must turn one of them away since the restaurant capacity is 2 guests. Otherwise, he will take everyone that comes. Let X be the number of customers that Alex serves at lunch. What is the range Ω_X , the PMF $p_X(k)$, and the expectation $E(X)$.

Question 7. There are 8 cards in a hat: $\{1\heartsuit, 1\spadesuit, 1\diamondsuit, 1\clubsuit, 2\heartsuit, 2\spadesuit, 2\diamondsuit, 2\clubsuit\}$. You draw one card at random. If its rank is 1 you draw one more card; if its rank is two you draw two more cards. Let X be the sum of the ranks on the 2 or 3 cards drawn. Find $E(X)$.