Assignment Problems

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16 March 2018

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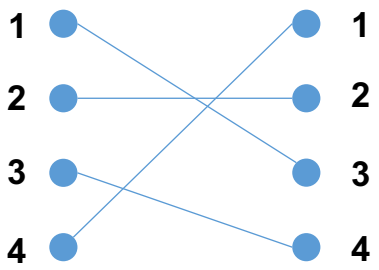
Assignment Problems

- How to assign *n* jobs to *n* workers in the best possible way (i.e. minimising cost)?
- Two components:
  - the assignment as underlying combinatorial structure
  - an objective function modeling the “best way”.

$$\phi = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 2 & 4 & 1 \end{pmatrix} \qquad X_\phi = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$



Different styles of representing the assignment of 4 tasks to 4 workers

Assignment.mod (Model File)

```
set STUD; # Set of Students


set PROJ; # Set of Project


param cost{STUD, PROJ} >= 0; # Preference costs (assigning a lesser preferred project
to a student is costlier)


var assign{STUD,PROJ} binary; # Decision variable; 1 if Project j is assigned to
Student i; 0 otherwise


minimize TotalCost:sum {i in STUD, j in PROJ} cost[i,j] *  assign[i,j]; # Objective
function to minimise the cost of assignment


subject to Max_Num_Projects {i in STUD}:sum {j in PROJ} assign[i,j] >= 1; # Each
student should be allocated a project


subject to Max_Num_Students {j in PROJ}:sum {i in STUD} assign[i,j] <= 1; # There may
be projects which are not allocated, but a project cannot be assigned to more than one
student
```

Assignment.dat
(Data File to Test the Model)

```
set STUD := S1 S2 S3; # Set of Students

set PROJ := P1 P2 P3 P4; # Set of Projects

param cost: P1 P2 P3 P4 := # Hypothetical Preference Structure
          S1 10  1 100 1000
          S2 1  100 10 1000
          S3 1  10 1000 100;
```

Solution to Test Data

```
CPLEX 12.7.0.0: optimal solution; objective 12
4 dual simplex iterations (0 in phase I)
ampl: display _varname, _var;
:          _varname          _var    :=
1     "assign['S1','P1']"     0
2     "assign['S1','P2']"     1
3     "assign['S1','P3']"     0
4     "assign['S1','P4']"     0
5     "assign['S2','P1']"     0
6     "assign['S2','P2']"     0
7     "assign['S2','P3']"     1
8     "assign['S2','P4']"     0
9     "assign['S3','P1']"     1
10    "assign['S3','P2']"     0
11    "assign['S3','P3']"     0
12    "assign['S3','P4']"     0
```