



# Distributed Roubust Learning

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2021-12-15

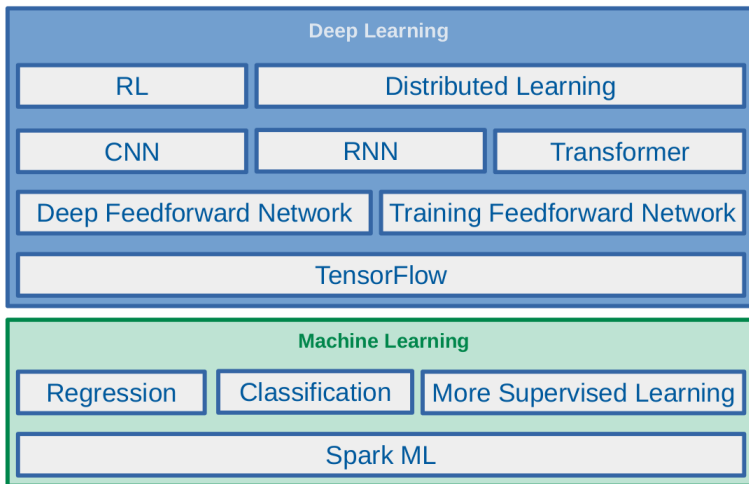




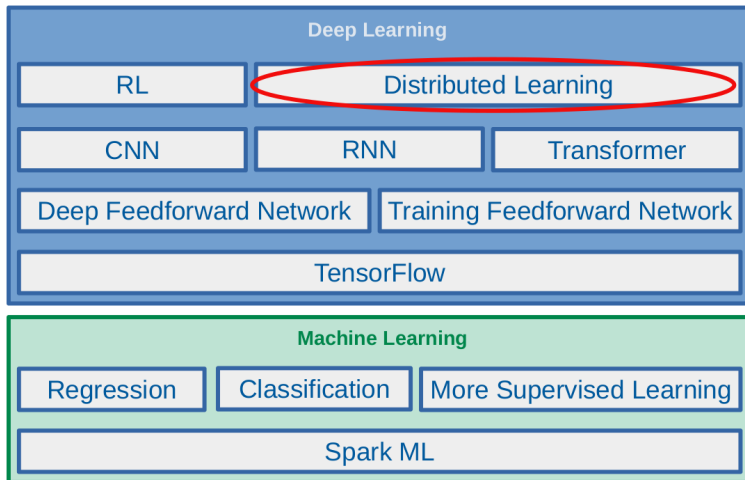
## The Course Web Page

<https://id2223kth.github.io>  
<https://tinyurl.com/6s5jy46a>

# Where Are We?



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# Adversarial Goals



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- ▶ Confidentiality and privacy
  - Confidentiality of the model or the data.



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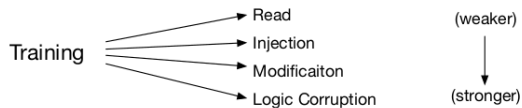
- ▶ Confidentiality and privacy
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- ▶ Integrity
  - Integrity of the **predictions**
- ▶ Availability
  - Availability of the **system** deploying machine learning





# Adversarial Capabilities for Integrity Attacks

## ► Training phase



[Papernot et al., SoK: Security and Privacy in Machine Learning, 2018]

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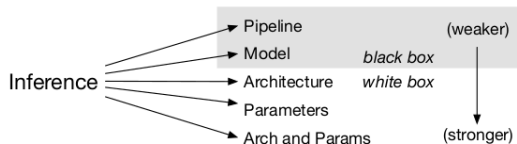
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## ► Inference phase

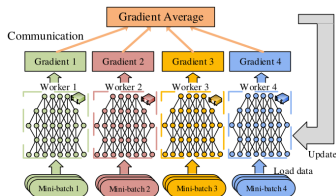
- White box
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[Papernot et al., SoK: Security and Privacy in Machine Learning, 2018]

# Our Focus and Goal

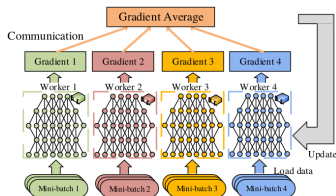
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[Tang et al., Communication-Efficient Distributed Deep Learning: A Comprehensive Survey, 2020]

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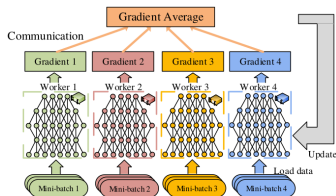
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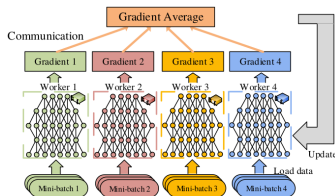
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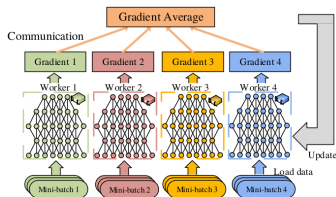
- ▶ Data parallelization
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- ▶ Our goal: integrity of the model in the training phase



[Tang et al., Communication-Efficient Distributed Deep Learning: A Comprehensive Survey, 2020]

# Distributed Stochastic Gradient Descent (1/3)

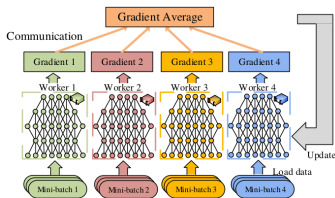
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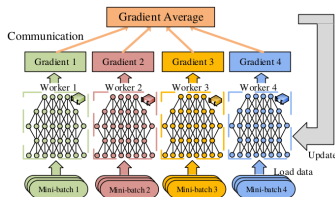


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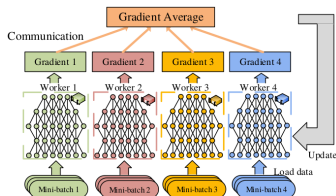
- ▶ One **parameter server**, and **n** **workers**.
- ▶ Computation is divided into **synchronous rounds**.
- ▶ During round **t**, the **parameter server** broadcasts its parameter vector  $w \in \mathbb{R}^d$  to all the **workers**.



[Tang et al., Communication-Efficient Distributed Deep Learning: A Comprehensive Survey, 2020]

# Distributed Stochastic Gradient Descent (2/3)

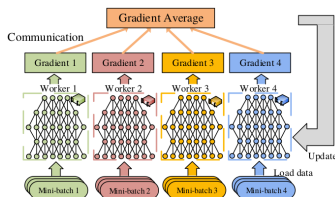
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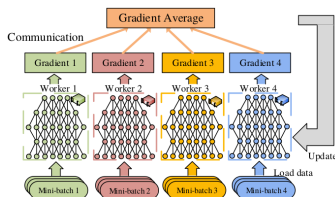
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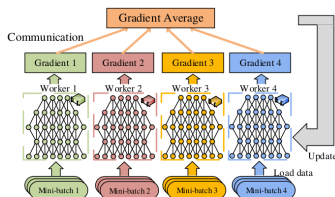
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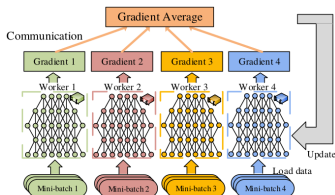
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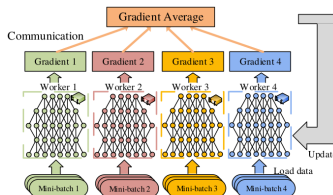
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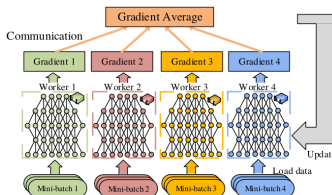
- ▶ The parameter server computes  $F(G_1, G_2, \dots, G_n)$
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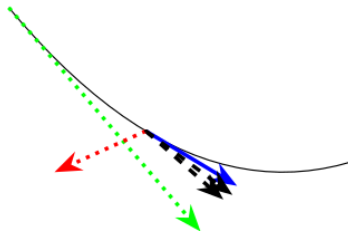
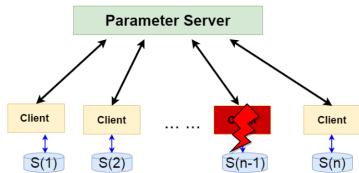


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# Distributed SGD with Byzantine Workers

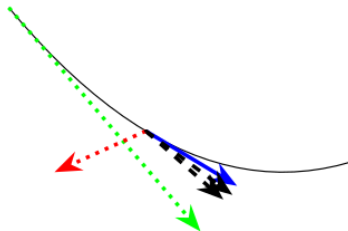
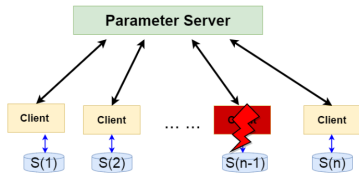
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[El-Mhamdi et al., Fast and Secure Distributed Learning in High Dimension, 2019]

# Distributed SGD with Byzantine Workers

- ▶ Among the  $n$  workers,  $f$  of them are possibly Byzantine (behaving arbitrarily).
- ▶ A Byzantine worker  $b$  proposes a vector  $G_b$  that can deviate arbitrarily from the vector it is supposed.



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# Averaging GAR and Byzantine Workers

- ▶ Averaging GAR:  $F(G_1, G_2, \dots, G_n) = \frac{1}{n} \sum_{i=1}^n G_i$
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- ▶ Even a **single Byzantine** worker can **prevent convergence**.
- ▶ **Proof:** if the Byzantine worker proposes  $G_n = nU - \sum_{i=1}^{n-1} G_i$ , then  $F = U$ .



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- ▶  $(B_1, \dots, B_f) \in (\mathbb{R}^d)^f$  are random vectors, possibly dependent between them and the vectors  $(G_1, \dots, G_{n-f})$



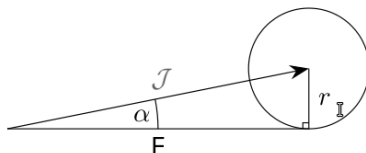
## $(\alpha, f)$ -Byzantine-Resilience (2/2)

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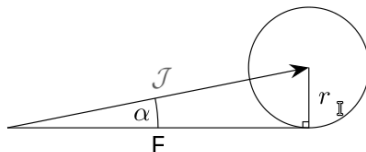
1. Vector  $\mathbf{F}$  that is not too far from the real gradient  $\mathcal{J}$ , i.e.,  $\|\mathbb{E}[\mathbf{F}] - \mathcal{J}\| \leq r$ .



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  2. Moments of  $\mathbf{F}$  should be controlled by the moments of the (correct) gradient estimator  $\mathbf{g}$ , where  $\mathbb{E}[\mathbf{g}] = \mathcal{J}$ .



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# Byzantine-Resilience GAR

- ▶ Median
- ▶ Krum
- ▶ Multi-Krum
- ▶ Brute



# Median

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►  $\text{median}(x_1, \dots, x_n) = \arg \min_{x \in \mathbb{R}} \sum_{i=1}^n |x_i - x|$





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- ▶  $d$ : the gradient vectors **dimension**.

- ▶ **Geometric** median

$$F = \text{GeoMed}(G_1, \dots, G_n) = \arg \min_{G \in \mathbb{R}^d} \sum_{i=1}^n \|G_i - G\|$$



- Geometric median

► Marginal median

1)



Krum

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- ▶  $G_{i_*}$  refers to the worker minimizing the score,  $s(i_*) \leq s(i)$  for all  $i$ .





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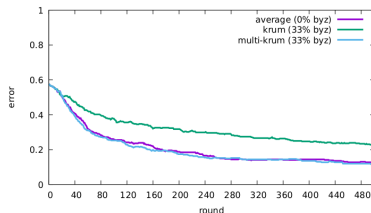
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- ▶ The cases  $m = 1$  and  $m = n$  correspond to **Krum** and **averaging**, respectively.



[Blanchard et al., Machine Learning with Adversaries: Byzantine Tolerant Gradient Descent, 2017]



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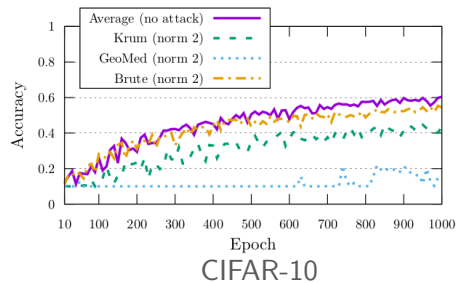
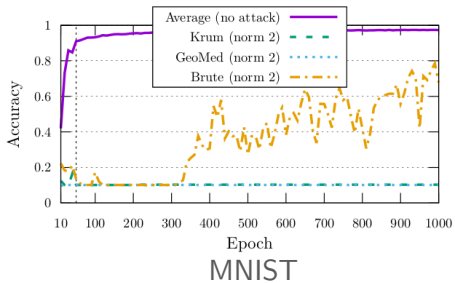
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- ▶  $F(G_1, \dots, G_n) = \frac{1}{n-f} \sum_{G \in \mathcal{S}} G$



[El Mhamdi et al., The Hidden Vulnerability of Distributed Learning in Byzantium, 2018]



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  - ▶ 2. Does  $X$  and  $Y$  **disagree** a **lot** on **only one**?



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- ▶ **A** can be Brute, Krum, Median, etc.
- ▶ **Bulyan** is a strong Byzantine-resilience algorithm.

# The Hidden Vulnerability of Distributed Learning in Byzantium



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  3. Loop back to step 1 if  $|S| < \theta$ .



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- ▶  $\theta = n - 2f \geq 2f + 3$ , thus  $S = (S_1, \dots, S_\theta)$  contains a majority of non-Byzantine gradients.
- ▶ For each  $i \in [1..d]$ , the median of the  $\theta$  coordinates  $i$  of the selected gradients is always bounded by coordinates from non-Byzantine submissions.



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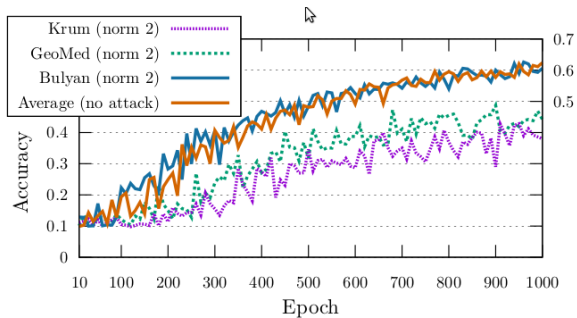
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- ▶  $\text{median}[i] = \arg \min_{m=Y[i], Y \in S} (\sum_{Z \in S} |Z[i] - m|)$
- ▶ Each  $i$ th coordinate of  $\mathbf{F}$  is equal to the average of the  $\beta$  closest  $i$ th coordinates to the median  $i$ th coordinate of the  $\theta$  selected gradients.



[El Mhamdi et al., The Hidden Vulnerability of Distributed Learning in Byzantium, 2018]



What if parameter servers are Byzantine?

# SGD: Decentralized Byzantine Resilience

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- ▶ GuanYu uses a GAR for aggregating workers' gradients and Median for aggregating models received from servers.





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- ▶  $d$  the dimension of the parameter space  $\mathbb{R}^d$ .



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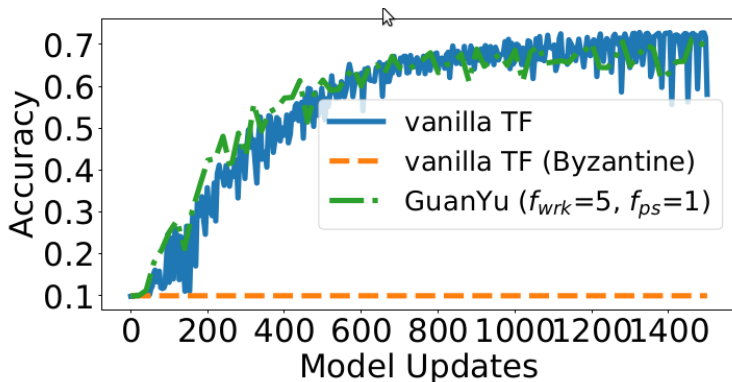
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- ▶ This aggregated parameter vector is  $\bar{w}_i^{t+1}$ .



[El Mhamdi et al., SGD: Decentralized Byzantine Resilience, 2019]

# Summary



# Summary

- ▶ Integrity in data-parallel learning
- ▶ Weak Byzantine resilience
- ▶ Strong Byzantine resilience
- ▶ Byzantine parameter servers

- ▶ Xie et al., Generalized Byzantine-tolerant SGD, 2018
- ▶ Blanchard et al., Machine Learning with Adversaries: Byzantine Tolerant Gradient Descent, 2017
- ▶ El Mhamdi et al., The Hidden Vulnerability of Distributed Learning in Byzantium, 2018
- ▶ Damaskinos et al., AGGREGATHOR: Byzantine Machine Learning via Robust Gradient Aggregation, 2019
- ▶ El Mhamdi et al., SGD: Decentralized Byzantine Resilience, 2019
- ▶ El Mhamdi et al., Fast Machine Learning with Byzantine Workers and Servers, 2019



Questions?