Proofs and logic	
Direct proof- $p \rightarrow q$	Proof by contrapositive - $\sim q \rightarrow \sim p$
Division into cases $(p \lor q) \rightarrow r \equiv (p \rightarrow r) \land (q \rightarrow r)$	Transitivity - $p \rightarrow q, q \rightarrow r, \therefore p \rightarrow r$
Elimination - $(p \lor q)$, $\sim q$, $\therefore p$	Specialisation - $(p \land q)$, $\therefore p$, $\therefore q$
Inverse error - $\sim p \rightarrow \sim q$	Converse error - $q \rightarrow p$
Uniqueness	
$\exists! x, P(x) \equiv \exists x, P(x) \land \forall a \forall b, (P(a) \land P(b)) \rightarrow a = b$	
Number theory	
Direct proof/Contrapositive	Pigeonhole principle
Constructive	Example/Counterexample – for existential statements
Contradiction (assume $p \rightarrow q$ and get a contradiction)	Division into cases (modulo, even/odd, +/-/0)
Mathematical Induction	
Strong PMI – use of every base case	PMI – 1 base case and 1 inductive step
Multiple base cases	PMI –inductive steps in both ways

Logical axioms	
Commutative: $p \land q \equiv q \land p$	Associative: $(p \land q) \land r \equiv p \land (q \land r)$,
	$(p \lor q) \lor r \equiv p \lor (q \lor r)$
Distributive: $p \land (q \lor r) \equiv (p \land q) \lor (p \land r)$	Identity: $p \land true \equiv p, \ p \lor false \equiv p$
Negation: $p \lor \sim p \equiv true, p \land \sim p \equiv false$	$Idempotent: p \lor p \equiv p \land p \equiv p$
De Morgan: $\sim (p \lor q) \equiv \sim p \land \sim q, \sim (p \land q) \equiv \sim p \lor \sim q$	Absorption: $p \lor (p \land q) \equiv p \land (p \lor q) \equiv p$
Universal bound: $p \lor true \equiv true, p \land false \equiv false$	Cases $(p \lor q) \to r \equiv (p \to r) \land (q \to r)$
Conditional: $p \rightarrow q \equiv \sim p \lor q$	Biconditional: $p \leftrightarrow q \equiv p \rightarrow q \land q \rightarrow p$
Number system - \mathbb{R} , \mathbb{Q} , \mathbb{Z}	
Identity: $x + 0 = x$, $x \cdot 1 = x$	Inverse: $x + (-x) = 0$, $x \cdot \left(\frac{1}{x}\right) = 1$ if $x \neq 0$
Commutative: $x + y = y + x$, $x \cdot y = y \cdot x$	Associative: $x + (y + z) = (x + y) + z$, $x \cdot (y \cdot z) =$
	$(x \cdot y) \cdot z$
Distributive: $x \cdot (y + z) = x \cdot y + x \cdot z$	
Closure properties	
Integers: closed under addition and multiplication	Rational numbers: addition, multiplication, division
Even integers: closed under addition and multiplication	Odd integers: closed under multiplication

Number Theory	
Tut3, q1: n is even if and only if n^3 is even	Tut3, q8: If a is even, and $a^2 = b^3$, then $4 a$ and $4 b$
extension: n^k is even/odd if and only if n is even/odd	
$a b \wedge b a \Longrightarrow a = \pm b$	4.1.1: If $n \in \mathbb{Z}$ then $n^2 + n$ is even
4.1.2: If $n \in \mathbb{Z}$, then $3 n^3 - n$ (proven by mod cases)	4.1.4: Pigeonhole principle – if m pigeons go into r
	pigeonholes, at least one hole has more than one
Tut4, q5: There are no integers a and n with $n \ge 2$ and	4.3.6: Standard factored form of $\forall n > 1, n \in \mathbb{Z}$ is $n =$
$a^2 + 1 = 2^n$	$p_1^{e_1}p_2^{e_2}p_k^{e_k}$ where p_1p_k are primes, e_1e_k are
	positive integers, and $p_1 < p_2 < \cdots < p_k$
5.2.1: Every positive integer can be written as the sum	Bernoulli inequality: $\forall x \in \mathbb{R}, x > -1, n \in \mathbb{Z}, n \geq 2$,
of distinct powers of any integer	$1 + nx < (1+x)^n$
Rational numbers	
3.3.5: \forall positive $x, y \in \mathbb{R}, x \neq y, \frac{x}{y} + \frac{y}{x} > 2$	3.3.6: A rational number in its lowest term $\frac{m}{n}$
Congruence/Modulo	
Symmetric: $a \equiv b \mod n \leftrightarrow b \equiv a \mod n$	Transitive: $a \equiv b \mod n \land b \equiv c \mod n \rightarrow a \equiv c \mod n$
$\forall \ a \in \mathbb{Z} \text{ and } n \in \mathbb{Z}^+, a \equiv r \mod n \text{ for exactly one}$	$a \equiv b \mod n$ and $c \equiv d \mod n \Rightarrow a+c \equiv b+d \mod n$
integer r such that $0 \le r \le n-1$	
$a \equiv b \mod n \text{ and } c \equiv d \mod n \Rightarrow ac \equiv bd \mod n$	$a \equiv b \mod n \Rightarrow a^k \equiv b^k \mod n \text{ for all } k \in \mathbb{Z}^+$
Absolute value	
Triangle inequality: $\forall x, y \in \mathbb{R}, x + y \le x + y $	Tut4, q1a: $\forall x, y \in \mathbb{R}, xy = x y $
Primes: No factors except 1 and itself	Composites: Not a prime

Tut4, q4: The set of primes is infinite 4.2-3-derived Left p, p_0 , p_k be a sequence of primes. For any prime p_k , $p_{k+1} \le p_k$, p_k , p_k be a sequence of primes. For any prime p_k , $p_{k+1} \le p_k$, p_k , p_k and p		
For any prime $p_n, p_{n+1} \le p_1p_2 \dots p_n + 1$ Irrational Numbers (2) Definition: Not rational Sum of a rational and irrational number is irrational Trut3, q_0 : if x is irrational and irrational number is irrational extension: any roto of x is irrational \sqrt{p} is an irrational and irrational number is irrational \sqrt{p} is an irrational and irrational number is irrational vectors of x is irrational Sequences AP: $\sum_{k=m}^n k = \frac{(n+m)(n-m+1)}{2}$ Sum of squares: $\sum_{k=1}^n k^2 = \frac{n(n+1)(2n+1)}{2}$ Sum of squares: $\sum_{k=1}^n k^2 = \frac{n(n+1)(2n+1)}{2}$ Product of a rational and irrational number is irrational \sqrt{p} is an irrational number \sqrt{p} is an irration	Tut4, q4: The set of primes is infinite	
Irrational Numbers($\sqrt{2}$) Definition: Not rational Sum of a rational and irrational number is irrational Tuts, q4a: if x is irrational then \sqrt{x} is irrational extension: any root of x is irrational extension: any extension: any extension is any root of x is irrational extension: any extension: x is irrational extension: x is irrational extension: x is irrational extension: x is irrational extension: x is an irrational number x in irrational number x is irrational extension: x is an irrational number x in irrational number x is irrational extension: x is an irrational number x in irrational number x is irrational extension: x is an irrational number x in irrational number x is irrational extension: x in x	4.2.3-derived: Let $p_1, p_2, \dots p_n$ be a sequence of primes.	Assn1, q5b: $n , \forall n \in \mathbb{Z}, n > 2$
Sum of a rational and irrational number is irrational ruts, q4a; if x is irrational then \sqrt{x} is irrational \sqrt{p} is an irrational number \sqrt{x} is irrational \sqrt{p} is an irrational number \sqrt{x} is irrational \sqrt{p} is an irrational number \sqrt{x} is irrational \sqrt{x} in irr	For any prime $p_n, p_{n+1} \le p_1 p_2 p_n + 1$	
$ \begin{array}{ll} \operatorname{Tut3}, \operatorname{q4a} : f \ x \ \text{ is irrational } \\ \operatorname{extension: any root of } x \ \text{ is irrational} \\ \operatorname{extension: any root of } x \ \text{ is irrational} \\ \operatorname{Sequences} \\ \operatorname{AP:} \sum_{k=m}^{\infty} k = \frac{(n+m)(n-m+1)}{2} \\ \operatorname{Sum of squares:} \sum_{k=1}^{\infty} k^2 = \frac{n(n+1)(2n+1)}{2} \\ \operatorname{Sum of squares:} \sum_{k$	Irrational Numbers($\sqrt{2}$) Definition: Not rational	
	Sum of a rational and irrational number is irrational	Product of a rational and irrational number is irrational
	Tut3, q4a: if x is irrational then \sqrt{x} is irrational	\sqrt{p} is an irrational number
$\begin{array}{lll} \textbf{Sequences} & \textbf{AP}: \sum_{k=m}^{\infty} k = \frac{(n+m)(n-m+1)}{2} & \textbf{GP}: \text{Given a series } a + ar + ar^2 + ar^3 + \cdots + ar^{n-1}, \\ \text{where } r \neq 1, \sum_{k=1}^{\infty} ar^{k-1} = \frac{a(1-r^n)}{1-r} & \textbf{where } r \neq 1, \sum_{k=1}^{\infty} ar^{k-1} = \frac{a(1-r^n)}{1-r} & \textbf{where } r \neq 1, \sum_{k=1}^{\infty} ar^{k-1} = \frac{a(1-r^n)}{1-r} & \textbf{where } r \neq 1, \sum_{k=1}^{\infty} ar^{k-1} = \frac{a(1-r^n)}{1-r} & \textbf{where } r \neq 1, \sum_{k=1}^{\infty} ar^{k-1} = \frac{a(1-r^n)}{1-r} & \textbf{where } r \neq 1, \sum_{k=1}^{\infty} ar^{k-1} = \frac{a(1-r^n)}{1-r} & \textbf{where } r \neq 1, \sum_{k=1}^{\infty} ar^{k-1} = \frac{a(1-r^n)}{1-r} & \textbf{where } r \neq 1, \sum_{k=1}^{\infty} ar^{k-1} = \frac{a(1-r^n)}{1-r} & \textbf{where } r \neq 1, \sum_{k=1}^{\infty} ar^{k-1} = \frac{a(1-r^n)}{1-r} & \textbf{where } r \neq 1, \sum_{k=1}^{\infty} ar^{k-1} = \frac{a(1-r^n)}{1-r} & \textbf{where } r \neq 1, \sum_{k=1}^{\infty} ar^{k-1} = \frac{a(1-r^n)}{1-r} & \textbf{where } r \neq 1, \sum_{k=1}^{\infty} ar^{k-1} = \frac{a(1-r^n)}{1-r} & \textbf{where } r \neq 1, \sum_{k=1}^{\infty} ar^{k-1} = \frac{a(1-r^n)}{1-r} & \textbf{where } r \neq 1, \sum_{k=1}^{\infty} ar^{k-1} = \frac{a(1-r^n)}{1-r} & \textbf{where } r \neq 1, \sum_{k=1}^{\infty} ar^{k-1} = \frac{a(1-r^n)}{1-r} & \textbf{where } r \neq 1, \sum_{k=1}^{\infty} ar^{k-1} = \frac{a(1-r^n)}{1-r} & \textbf{where } r \neq 1, \sum_{k=1}^{\infty} ar^{k-1} = \frac{a(1-r^n)}{1-r} & \textbf{where } r \neq 1, \sum_{k=1}^{\infty} ar^{k-1} = \frac{a(1-r^n)}{1-r} & \textbf{where } r \neq 1, \sum_{k=1}^{\infty} ar^{k-1} = \frac{a(1-r^n)}{1-r} & \textbf{where } r \neq 1, \sum_{k=1}^{\infty} ar^{k-1} = \frac{a(1-r^n)}{1-r} & \textbf{where } r \neq 1, \sum_{k=1}^{\infty} ar^{k-1} = \frac{a(1-r^n)}{1-r} & \textbf{where } r \neq 1, \sum_{k=1}^{\infty} ar^{k-1} = \frac{a(1-r^n)}{1-r} & \textbf{where } r \neq 1, \sum_{k=1}^{\infty} ar^{k-1} = \frac{a(1-r^n)}{1-r} & \textbf{where } r \neq 1, \sum_{k=1}^{\infty} ar^{k-1} = \frac{a(1-r^n)}{1-r} & \textbf{where } r \neq 1, \sum_{k=1}^{\infty} ar^{k-1} = \frac{a(1-r^n)}{1-r} & \textbf{where } r \neq 1, \sum_{k=1}^{\infty} ar^{k-1} = \frac{a(1-r^n)}{1-r} & \textbf{where } r \neq 1, \sum_{k=1}^{\infty} ar^{k-1} = \frac{a(1-r^n)}{1-r} & \textbf{where } r \neq 1, \sum_{k=1}^{\infty} ar^{k-1} = \frac{a(1-r^n)}{1-r} & \textbf{where } r \neq 1, \sum_{k=1}^{\infty} ar^{k-1} = \frac{a(1-r^n)}{1-r} & \textbf{where } r \neq 1, \sum_{k=1}^{\infty} ar^{k-1} = \frac{a(1-r^n)}{1-r} & \textbf{where } r \neq 1, \sum_{k=1}^{\infty} ar^{k-1} = \frac{a(1-r^n)}{1-r} & \textbf{where } r \neq 1, \sum_{k=1}^{\infty} ar^{k-1} = \frac{a(1-r^n)}{1-r} & \textbf{where } r \neq 1, \sum_{k=1}^$		V .
$\begin{array}{lll} \textbf{Sequences} & \textbf{AP}: \sum_{k=m}^{\infty} k = \frac{(n+m)(n-m+1)}{2} & \textbf{GP}: \text{Given a series } a + ar + ar^2 + ar^3 + \cdots + ar^{n-1}, \\ \text{where } r \neq 1, \sum_{k=1}^{\infty} ar^{k-1} = \frac{a(1-r^n)}{1-r} & \textbf{where } r \neq 1, \sum_{k=1}^{\infty} ar^{k-1} = \frac{a(1-r^n)}{1-r} & \textbf{where } r \neq 1, \sum_{k=1}^{\infty} ar^{k-1} = \frac{a(1-r^n)}{1-r} & \textbf{where } r \neq 1, \sum_{k=1}^{\infty} ar^{k-1} = \frac{a(1-r^n)}{1-r} & \textbf{where } r \neq 1, \sum_{k=1}^{\infty} ar^{k-1} = \frac{a(1-r^n)}{1-r} & \textbf{where } r \neq 1, \sum_{k=1}^{\infty} ar^{k-1} = \frac{a(1-r^n)}{1-r} & \textbf{where } r \neq 1, \sum_{k=1}^{\infty} ar^{k-1} = \frac{a(1-r^n)}{1-r} & \textbf{where } r \neq 1, \sum_{k=1}^{\infty} ar^{k-1} = \frac{a(1-r^n)}{1-r} & \textbf{where } r \neq 1, \sum_{k=1}^{\infty} ar^{k-1} = \frac{a(1-r^n)}{1-r} & \textbf{where } r \neq 1, \sum_{k=1}^{\infty} ar^{k-1} = \frac{a(1-r^n)}{1-r} & \textbf{where } r \neq 1, \sum_{k=1}^{\infty} ar^{k-1} = \frac{a(1-r^n)}{1-r} & \textbf{where } r \neq 1, \sum_{k=1}^{\infty} ar^{k-1} = \frac{a(1-r^n)}{1-r} & \textbf{where } r \neq 1, \sum_{k=1}^{\infty} ar^{k-1} = \frac{a(1-r^n)}{1-r} & \textbf{where } r \neq 1, \sum_{k=1}^{\infty} ar^{k-1} = \frac{a(1-r^n)}{1-r} & \textbf{where } r \neq 1, \sum_{k=1}^{\infty} ar^{k-1} = \frac{a(1-r^n)}{1-r} & \textbf{where } r \neq 1, \sum_{k=1}^{\infty} ar^{k-1} = \frac{a(1-r^n)}{1-r} & \textbf{where } r \neq 1, \sum_{k=1}^{\infty} ar^{k-1} = \frac{a(1-r^n)}{1-r} & \textbf{where } r \neq 1, \sum_{k=1}^{\infty} ar^{k-1} = \frac{a(1-r^n)}{1-r} & \textbf{where } r \neq 1, \sum_{k=1}^{\infty} ar^{k-1} = \frac{a(1-r^n)}{1-r} & \textbf{where } r \neq 1, \sum_{k=1}^{\infty} ar^{k-1} = \frac{a(1-r^n)}{1-r} & \textbf{where } r \neq 1, \sum_{k=1}^{\infty} ar^{k-1} = \frac{a(1-r^n)}{1-r} & \textbf{where } r \neq 1, \sum_{k=1}^{\infty} ar^{k-1} = \frac{a(1-r^n)}{1-r} & \textbf{where } r \neq 1, \sum_{k=1}^{\infty} ar^{k-1} = \frac{a(1-r^n)}{1-r} & \textbf{where } r \neq 1, \sum_{k=1}^{\infty} ar^{k-1} = \frac{a(1-r^n)}{1-r} & \textbf{where } r \neq 1, \sum_{k=1}^{\infty} ar^{k-1} = \frac{a(1-r^n)}{1-r} & \textbf{where } r \neq 1, \sum_{k=1}^{\infty} ar^{k-1} = a(1-r^n)} & \textbf{where } r \neq 1, \sum_{k=1}^{\infty} ar^{k-1} = a(1-r^n)} & \textbf{where } r \neq 1, \sum_{k=1}^{\infty} ar^{k-1} = a(1-r^n)} & \textbf{where } r \neq 1, \sum_{k=1}^{\infty} ar^{k-1} = a(1-r^n)} & \textbf{where } r \neq 1, \sum_{k=1}^{\infty} ar^{k-1} = a(1-r^n)} & \textbf{where } r \neq 1, \sum_{k=1}^{\infty} ar^{k-1}} & \textbf{where } r \neq $	$\sqrt{2} + \sqrt{3}$ is irrational	Assn1, $q4b\sqrt{2} + \sqrt{3} + \sqrt{5}$ is irrational
$\begin{array}{lll} \mathbf{AP}: \Sigma_{k=m}^m k = \frac{(n+m)(n-m+1)}{2} & \mathbf{GP}: \text{Given a series } a + ar + ar^2 + ar^3 + \cdots + ar^{m-1}, \\ & \text{where } r \neq 1, \Sigma_{k=1}^m ar^{k-1} = \frac{a(1-r^n)}{1-r} \\ & \text{Sum of squares}: \sum_{n=1}^n k^2 = \frac{n(n+1)(2n+1)}{6} \\ & \text{Product}: \prod_{k=m}^n a_k = a_m \cdot a_{m+1} \cdot a_{m+2} \cdot \cdots \cdot a_n \\ & \text{Znots } r \text{ and } s: a_k = Aa_{k-1} + Ba_{k-2} \\ & \text{Znots } r \text{ and } s: a_k = Cr^k + Ds^k \\ & \text{Stet} \\ & \text{Notations}: \text{Listing } \{1,2,\dots\} \text{ or Set builder } \{x \in U \mid p(x)\} \\ & \text{Laws: idempotent, Commutative, Associative,} \\ & \text{Distributive, power Morgan's} \\ & \text{A} \times (B \text{ op } C) = (A \times B) \text{ op}(A \times C) \\ & \text{Functions} \\ & \text{RD implies } (a,b) \in R = \{(a,b) \in A \times B \mid p(x,y)\} \\ & \text{a } \in \text{dom}(R) \Rightarrow \exists b \text{ such that } (a,b) \in R \\ & \text{Equivalence relation when R is reflexive, symmetric and transitive} \\ & \text{Counting} \\ & \text{repetition not allowed, order matters: } \frac{n!}{(n-r)!} (r) \\ & \text{repetition not allowed, order matters: } \frac{n!}{(n-r)!} (r) \\ & \text{repetition not allowed, order matters: } \frac{n!}{n! n! n$	Sequences	
$ \begin{aligned} & \text{where } r \neq 1, \sum_{k=1}^n ar^{k-1} = \frac{n(n+1)(2(n+1))}{1-r} \\ & \text{Sum of squaries: } \sum_{k=1}^n k^2 = \frac{n(n+1)(2(n+1))}{6} \\ & \text{Product: } \prod_{k=m}^n a_k = a_m \cdot a_{m+1} \cdot a_{m+2} \cdot \dots \cdot a_n \\ & \text{Dark of the Inleas Homogeneous Recurrence Relation:} \\ & \text{Expression: } a_k = Aa_{k-1} + Ba_{k-2} \\ & \text{Z roots } r \text{ and } s : a_k = Cr^k + Ds^k \\ & \text{Sets} \\ & \text{Notations: Listing } \{1,2,\} \text{ or Set builder } \{x \in U p(x)\} \\ & \text{Laws: Idempotent, Commutative, Associative, Distributive, De Morgan's} \\ & \text{Functions} \\ & f : A(domain) \to B(codomain) \\ & f : A(b) \in R \Rightarrow 1 \text{ and } x = \text{preimage of } y \\ & \text{under } f, y \in range(f) \\ & \text{Bijective: } Ny \in B, \exists x \in A \text{ such that } y = f(x) \\ & \text{Bijective: } Ny \in B, \exists x \in A \text{ such that } y = f(x) \\ & \text{Bijective: Injective and Surjective} \\ & \text{Relations} \\ & \text{ARB implies } (a,b) \in R = \{(a,b) \in A \times B \mid p(x,y)\} \\ & a \in dom(R) \Leftrightarrow \exists b \text{ such that } (a,b) \in R \\ & \text{Equivalence relation when } R \text{ is reflexive, symmetric} \\ & \text{and transitive} \\ & \text{Counting} \\ & \text{repetition allowed, order matters: } n^k \text{ (multiplication rule)} \\ & \text{repetition allowed, order matters: } n^k \text{ (multiplication rule)} \\ & \text{repetition not allowed, order matters: } n^k \text{ (multiplication rule)} \\ & \text{repetition not allowed, order matters: } n^k \text{ (multiplication rule)} \\ & \text{repetition not allowed, order matters: } n^k \text{ (multiplication rule)} \\ & \text{repetition allowed, order does not matter: } (n-1) \\ & \text{repetition not allowed, order matters: } n^k \text{ (multiplication rule)} \\ & \text{repetition allowed, order does not matter: } (n-1) \\ & \text{repetition not allowed, order of does not matter: } (n-1) \\ & \text{repetition not allowed, order does not matter: } (n-1) \\ & \text{repetition not allowed, order does not matter: } (n-1) \\ & \text{repetition not allowed, order does not matter: } (n-1) \\ & repetitio$		GP: Given a series $a + ar + ar^2 + ar^3 + \cdots + ar^{n-1}$,
$\begin{array}{lll} \text{Sum of squares: } \sum_{k=1}^n k^2 = \frac{n(n+1)(2n+1)}{6} & \text{Product: } \prod_{k=m}^n a_k = a_m \cdot a_{m+1} \cdot a_{m+2} \cdot \dots \cdot a_n \\ \hline \textbf{Znd-Order Linear Homogeneous Recurrence Relation:} \\ \hline \textbf{Expression: } a_k = Aa_{k-1} + Ba_{k-2} & \text{Characteristic eqn: } t^2 - At - B = 0 \\ \hline \textbf{Zroots } r \text{ and } s: a_{k-1} + Ba_{k-2} & \text{Iroot } r: a_k = Cr^k + Dkr^k \\ \hline \textbf{Sets} & \text{Iroot } r: a_k = Cr^k + Dkr^k \\ \hline \textbf{Sets} & \text{Notations: Listing } \{1,2,\} \text{ or Set builder } \{x \in U p(x)\} \\ \hline \textbf{Laws: Idempotent, Commutative, Associative, Distributive, De Morgan's } & A \times (B \text{ op } C) = (A \times B) \text{ op}(A \times C) \\ \hline \textbf{Functions} & \text{Injective: } \forall x, y \in A, x \neq y \rightarrow f(x) \neq f(y) \\ \hline \textbf{Surjective: } \forall y \in B, \exists x \in A \text{ such that } y = f(x) \\ \hline \textbf{Surjective: } \forall y \in B, \exists x \in A \text{ such that } y = f(x) \\ \hline \textbf{Surjective: } \forall y \in B, \exists x \in A \text{ such that } y = f(x) \\ \hline \textbf{Surjective: } \forall y \in B, \exists x \in A \text{ such that } y = f(x) \\ \hline \textbf{Surjective: } \forall y \in B, \exists x \in A \text{ such that } y = f(x) \\ \hline \textbf{Surjective: } \forall y \in B, \exists x \in A \text{ such that } y = f(x) \\ \hline \textbf{Surjective: } \forall y \in B, \exists x \in A \text{ such that } y = f(x) \\ \hline \textbf{Surjective: } \forall y \in B, \exists x \in A \text{ such that } y = f(x) \\ \hline \textbf{Surjective: } \forall y \in B, \exists x \in A \text{ such that } y = f(x) \\ \hline \textbf{Surjective: } \forall y \in B, \exists x \in A \text{ such that } y = f(x) \\ \hline \textbf{Surjective: } \forall y \in B, \exists x \in A \text{ such that } y = f(x) \\ \hline \textbf{Surjective: } \forall y \in B, \exists x \in A \text{ such that } y = f(x) \\ \hline \textbf{Surjective: } \forall y \in B, \exists x \in A \text{ such that } y = f(x) \\ \hline \textbf{Surjective: } \forall y \in B, \exists x \in A \text{ such that } y = f(x) \\ \hline \textbf{Surjective: } \forall y \in B, \exists x \in A \text{ such that } y = f(x) \\ \hline \textbf{Surjective: } \forall y \in B, \exists x \in A \text{ such that } y = f(x) \\ \hline \textbf{Surjective: } \forall y \in B, \exists x \in A \text{ such that } y = f(x) \\ \hline \textbf{Surjective: } \forall y \in B, \exists x \in A \text{ such that } y = f(x) \\ \hline \textbf{Surjective: } \forall y \in B, \exists x \in A \text{ such that } y = f(x) \\ \hline \textbf{Surjective: } \forall y \in B, \exists x \in A \text{ such that } y = f(x) \\ \hline \textbf{Surjective: } \forall y \in B, \exists x \in A \text{ such that } y = f(x) \\ \hline \textbf{Surjective: } \forall y \in B, \exists x \in A \text{ such that } y = f(x) \\ \hline \textbf{Surjective: } \forall y \in B, \exists x $	$\sum_{k=m}^{n} k = 2$	where $r \neq 1$ $\sum_{n=1}^{n} ar^{k-1} - \frac{a(1-r^n)}{n}$
2nd-Order Linear Homogeneous Recurrence Relation:Expression: $a_k = Aa_{k-1} + Ba_{k-2}$ Characteristic eqn: $t^2 - At - B = 0$ 2 roots r and s : $a_k = Cr^k + Ds^k$ 1 root r : $a_k = Cr^k + Dkr^k$ SetsNotations: Listing $\{1,2,\}$ or Set builder $\{x \in U \mid p(x)\}$ Operators: $U, \cap, -$, complementLaws: Idempotent, Commutative, Associative, Distributive, be Morgan'sDistributive, be Morgan'sAx $(B \circ pC) = (A \times B)\circ p(A \times C)$ FunctionsInjective: Vary $e A, x \neq y \rightarrow f(x) \neq f(y)$ FunctionsInjective: $\forall x \in B, \exists x \in A \text{ such that } y = f(x)$ $f(x) = y \rightarrow y = image of x \text{ and } x = \text{preimage of } y$ Surjective: $\forall y \in B, \exists x \in A \text{ such that } y = f(x)$ $f(x) = y \rightarrow y = image of x \text{ and } x = \text{preimage of } y$ Surjective: $\forall y \in B, \exists x \in A \text{ such that } y = f(x)$ $f(x) = y \rightarrow y = image of x \text{ and } x = \text{preimage of } y$ Surjective: $\forall x \in A, (x, x) \notin B$ $f(x) = y \rightarrow y = image of x \text{ and } x = \text{preimage of } y$ Surjective: $\forall x \in A, (x, x) \notin A \text{ such that } y = f(x)$ $f(x) = x = x = x = x = x = x = x = x = x =$	n(n+1)(2n+1)	Product Π^n $\alpha = \alpha$ α
Expression: $a_k = Aa_{k-1} + Ba_{k-2}$ 2 roots r and s : $a_k = Cr^k + Ds^k$ 1 root r : $a_k = Cr^k + Dkr^k$ Sets Notations: Listing $\{1,2,\}$ or Set builder $\{x \in U p(x)\}$ Laws: Idempotent, Commutative, Associative, Distributive, De Morgan's $A \times (B \ ap \ C) = (A \times B) \ ap (A \times C)$ Functions $f: A(domain) \to B(codomain)$ $f(x) = y \Rightarrow y = image of x \ and x = preimage of y \ under f, y \in range(f) Relations aRb \ implies \ (a,b) \in R = \{(a,b) \in A \times B p(x,y)\} a \in dom(R) \Leftrightarrow \exists b \ such \ that \ (a,b) \in R b \in range(R) \Leftrightarrow \exists a \ such \ that \ (a,b) \in R Equivalence relation when R is reflexive, symmetric and transitive Counting repetition allowed, order matters: n^k (multiplication rule) repetition allowed, order matters: n^k (multiplication rule) repetition allowed, order matters: n^k (multiplication rule) Fepermutation) permutations of n objects with indistinguishable elements: n^k \ (n^k) = n^k \$	Sum of squares: $\sum_{k=1}^{n} k^2 = \frac{k(k+1)(2k+1)}{6}$	Product: $\prod_{k=m} a_k = a_m \cdot a_{m+1} \cdot a_{m+2} \cdot \dots \cdot a_n$
$ \begin{array}{lll} 2 \operatorname{roots} r \operatorname{and} s: a_k = Cr^k + Ds^k \\ \mathbf{Sets} \\ \mathbf{Notations}: \operatorname{Listing} \{1,2,\ldots\} \operatorname{or} \operatorname{Set} \operatorname{builder} \{x \in U p(x)\} \\ \mathbf{Notations}: \operatorname{Listing} \{1,2,\ldots\} \operatorname{or} \operatorname{Set} \operatorname{builder} \{x \in U p(x)\} \\ \mathbf{Notations}: \operatorname{Listing} \{1,2,\ldots\} \operatorname{or} \operatorname{Set} \operatorname{builder} \{x \in U p(x)\} \\ \mathbf{Notations}: \operatorname{Listing} \{1,2,\ldots\} \operatorname{or} \operatorname{Set} \operatorname{builder} \{x \in U p(x)\} \\ \mathbf{Notations}: \operatorname{Listing} \{1,2,\ldots\} \operatorname{or} \operatorname{Set} \operatorname{builder} \{x \in U p(x)\} \\ \mathbf{Notations}: \operatorname{Listing} \{1,2,\ldots\} \operatorname{or} \operatorname{Set} \operatorname{builder} \{x \in U p(x)\} \\ \mathbf{Notations}: \operatorname{Listing} \{1,2,\ldots\} \operatorname{or} \operatorname{Set} \operatorname{builder} \{x \in U p(x)\} \\ \mathbf{Notations}: \operatorname{Listing} \{1,2,\ldots\} \operatorname{or} \operatorname{Set} \operatorname{builder} \{x \in U p(x)\} \\ \mathbf{Notations}: \operatorname{Listing} \{1,2,\ldots\} \operatorname{or} \operatorname{Set} \operatorname{builder} \{x \in U p(x)\} \\ \mathbf{Notations}: \operatorname{Listing} \{1,2,\ldots\} \operatorname{or} \operatorname{Set} \operatorname{builder} \{x \in U p(x)\} \\ \mathbf{Notations}: \operatorname{Listing} \{1,2,\ldots\} \operatorname{or} \operatorname{Set} \operatorname{builder} \{x \in U p(x)\} \\ \mathbf{Notations}: \operatorname{Listing} \{1,2,\ldots\} \operatorname{or} \operatorname{Set} \operatorname{builder} \{x \in U p(x)\} \\ \mathbf{Notations}: \operatorname{Listing} \{1,2,\ldots\} \operatorname{or} \operatorname{Set} \operatorname{builder} \{x \in U p(x)\} \\ \mathbf{Notations}: \operatorname{Listing} \{1,2,\ldots\} \operatorname{or} \operatorname{Set} \operatorname{builder} \{x \in U p(x)\} \\ \mathbf{Notations}: \operatorname{Listing} \{1,2,\ldots\} \operatorname{or} \operatorname{Set} \operatorname{builder} \{x \in U p(x)\} \\ \mathbf{Notations}: \operatorname{Listing} \{1,2,\ldots\} \operatorname{or} \operatorname{Set} \operatorname{builder} \{x \in U p(x)\} \\ \mathbf{Notations}: \operatorname{Listing} \{1,2,\ldots\} \operatorname{or} \operatorname{Set} \operatorname{builder} \{x \in U p(x)\} \\ \mathbf{Notations}: \operatorname{Inting} \operatorname{Ling} \operatorname{under} (x) un$	2nd-Order Linear Homogeneous Recurrence Relation:	
SetsNotations: Listing $\{1,2,\}$ or Set builder $\{x \in U p(x)\}$ Operators: $U, \cap, -$, complementLaws: Idempotent, Commutative, Associative, Distributive, De Morgan'sDistributive law on Cartesian products: $A \times (B \ op \ C) = (A \times B) \ op (A \times C)$ FunctionsInjective: $V, y \in A, x \neq y \to f(x) \neq f(y)$ $f(x) = y \to y = image of x \ and x = preimage of y under f, y \in range(f)Injective: V, y \in A, x \neq y \to f(x) \neq f(y)B implies (a, b) \in R = \{(a, b) \in A \times B p(x, y)\}a \in dom(R) \Leftrightarrow \exists b \ such \ that \ (a, b) \in RReflexive: \forall x \in A, (x, x) \in RSymmetric: if \ (x, y) \in R \ then \ (y, x) \in RB Equivalence relation when R is reflexive, symmetric and transitiveReflexive: \forall x \in A, (x, x) \in RSymmetric: if \ (x, y) \ and \ (y, z) \in R \ then \ (x, z) \in RCountingEquivalence classes: a _R = \{E \in (x, a) \in R\}(a, b) \in R \Leftrightarrow a _R = b _R(a, b) \in R \Leftrightarrow a _R = b$	Expression: $a_k = Aa_{k-1} + Ba_{k-2}$	Characteristic eqn: $t^2 - At - B = 0$
SetsNotations: Listing $\{1,2,\}$ or Set builder $\{x \in U p(x)\}$ Operators: $U, \cap, -$, complementLaws: Idempotent, Commutative, Associative, Distributive, De Morgan'sDistributive law on Cartesian products: $A \times (B \ op \ C) = (A \times B) \ op (A \times C)$ FunctionsInjective: $V, y \in A, x \neq y \to f(x) \neq f(y)$ $f(x) = y \to y = image of x \ and x = preimage of y under f, y \in range(f)Injective: V, y \in A, x \neq y \to f(x) \neq f(y)B implies (a, b) \in R = \{(a, b) \in A \times B p(x, y)\}a \in dom(R) \Leftrightarrow \exists b \ such \ that \ (a, b) \in RReflexive: \forall x \in A, (x, x) \in RSymmetric: if \ (x, y) \in R \ then \ (y, x) \in RB Equivalence relation when R is reflexive, symmetric and transitiveReflexive: \forall x \in A, (x, x) \in RSymmetric: if \ (x, y) \ and \ (y, z) \in R \ then \ (x, z) \in RCountingEquivalence classes: a _R = \{E \in (x, a) \in R\}(a, b) \in R \Leftrightarrow a _R = b _R(a, b) \in R \Leftrightarrow a _R = b$	2 roots r and s : $a_k = Cr^k + Ds^k$	$1 \operatorname{root} r: a_k = Cr^k + Dkr^k$
Laws: Idempotent, Commutative, Associative, Distributive, De Morgan's $A \times (B \text{ op } C) = (A \times B) \text{ op}(A \times C)$ Functions $f: A(domain) \rightarrow B(codomain)$ $f(x) = y \Rightarrow y = \text{image of } x \text{ and } x = \text{preimage of } y$ $ARB \text{ implies } (a,b) \in R = \{(a,b) \in A \times B p(x,y)\}$ $a \in dom(R) \Leftrightarrow \exists b \text{ such that } (a,b) \in R$ $b \in range(R) \Leftrightarrow \exists a \text{ such that } (a,b) \in R$ $Equivalence relation when R is reflexive, symmetric and transitive (a,b) \in R \Leftrightarrow [a]_R = [b]_R (a,b) \in R \Leftrightarrow [a]_R $		
Laws: Idempotent, Commutative, Associative, Distributive, De Morgan's $A \times (B \text{ op } C) = (A \times B) \text{ op}(A \times C)$ Functions $f: A(domain) \rightarrow B(codomain)$ $f(x) = y \Rightarrow y = \text{image of } x \text{ and } x = \text{preimage of } y$ $ARB \text{ implies } (a,b) \in R = \{(a,b) \in A \times B p(x,y)\}$ $a \in dom(R) \Leftrightarrow \exists b \text{ such that } (a,b) \in R$ $b \in range(R) \Leftrightarrow \exists a \text{ such that } (a,b) \in R$ $Equivalence relation when R is reflexive, symmetric and transitive (a,b) \in R \Leftrightarrow [a]_R = [b]_R (a,b) \in R \Leftrightarrow [a]_R $	Notations : Listing $\{1,2,\}$ or Set builder $\{x \in U p(x)\}$	Operators: ∪,∩, –, complement
Functions $f: A(domain) \rightarrow B(codomain) \\ f: A(domain) \rightarrow B(codomain) \\ f(x) = y \Rightarrow y = image of x and x = preimage of y \\ under f, y \in range(f) \\ \textbf{Relations} \\ aRb implies (a,b) \in R = \{(a,b) \in A \times B p(x,y)\} \\ a \in dom(R) \Leftrightarrow \exists b \ such \ that \ (a,b) \in R \\ b \in range(R) \Leftrightarrow \exists a \ such \ that \ (a,b) \in R \\ b \in range(R) \Leftrightarrow \exists a \ such \ that \ (a,b) \in R \\ \textbf{Equivalence relation} \ \text{when R is reflexive, symmetric} \\ \text{and transitive} \ \ \ \ \ \ \ \ \ \ \ \ \ $	12 1 1	Distributive law on Cartesian products:
$ f: A(domain) \rightarrow B(codomain) \\ f(x) = y \Rightarrow y = \text{image of } x \text{ and } x = \text{preimage of } y \\ \text{under } f, y \in range(f) \\ \text{Relations} \\ aRb \text{ implies } (a,b) \in R = \{(a,b) \in A \times B \mid p(x,y)\} \\ a \in dom(R) \Leftrightarrow \exists b \text{ such } t \text{ hat } (a,b) \in R \\ b \in range(R) \Leftrightarrow \exists a \text{ such } t \text{ hat } (a,b) \in R \\ b \in range(R) \Leftrightarrow \exists a \text{ such } t \text{ hat } (a,b) \in R \\ b \in range(R) \Leftrightarrow \exists a \text{ such } t \text{ hat } (a,b) \in R \\ b \in range(R) \Leftrightarrow \exists a \text{ such } t \text{ hat } (a,b) \in R \\ equivalence relation \text{ when } R \text{ is reflexive, symmetric} \\ \text{and transitive} \\ \text{Counting} \\ \text{repetition allowed, order matters: } n^k \text{ (multiplication } \\ \text{rule}) \\ \text{repetition not allowed, order matters: } n^k \text{ (multiplication } \\ \text{rule}) \\ \text{repetition not allowed, order matters: } n^k \text{ (multiplication } \\ \text{rule}) \\ \text{repetition not allowed, order matters: } n^k \text{ (multiplication } \\ \text{rule}) \\ \text{repetition of } n \text{ objects with indistinguishable} \\ \text{elements: } n^1 \\ \text{lu}_{n_1!n_2!n_3!\dots n_k!} \\ \text{Graphs} \\ \text{Graphs} \\ \text{Graphs} \\ \text{Graphs} \text{ Agraph } G = \{V, E\} \text{ Consists of a } nonempty \text{ set of vertices } V(G) \text{ and set of edges } E(G) \\ \text{Bijective: } \forall y \in B, \exists x \in A \text{ such } t \text{ that } x \neq y \neq f(x) \neq f(x) \\ \text{Surjective: } \exists \text{ injective: } \exists \text{ such } t \text{ that } x \neq y \neq f(x) \neq f(x) \\ \text{Surjective: } \exists \text{ lojective: } loject$	Distributive, De Morgan's	$A \times (B \ op \ C) = (A \times B)op(A \times C)$
$f(x) = y \Rightarrow y = \text{image of } x \text{ and } x = \text{preimage of } y \text{ under } f, y \in range(f)$ $Relations$ $Relations$ $RR b implies (a, b) \in R = \{(a, b) \in A \times B \mid p(x, y)\}$ $a \in dom(R) \Leftrightarrow \exists b \text{ such that } (a, b) \in R$ $b \in range(R) \Leftrightarrow \exists a \text{ such that } (a, b) \in R$ $Equivalence relation when R is reflexive, symmetric and transitive$ $(a, b) \in R \Leftrightarrow [a]_R = \{x \in (x, a) \in R\}$ $(a, b) \notin R \Leftrightarrow [a]_R = \{b\}_R$ $(a, b) \in R \Leftrightarrow [a]_R = \{b\}_R$ $(a, b) \in R \Leftrightarrow [a]_R = \{b\}_R$ $(a, b) \notin R \Leftrightarrow [a]_R = \{b\}_R$ $(a, b) \in R \Leftrightarrow [a]_R = $	Functions	
under $f, y \in range(f)$ Bijective: Injective and SurjectiveRelations aRb implies $(a,b) \in R = \{(a,b) \in A \times B p(x,y)\}$ $a \in dom(R) \Leftrightarrow \exists b \ such that (a,b) \in RReflexive: \forall x \in A, (x,x) \in Rb \in range(R) \Leftrightarrow \exists a \ such that (a,b) \in RSymmetric: if(x,y) \in R then (y,x) \in REquivalence relation when R is reflexive, symmetricand transitiveEquivalence classes: [a]_R = \{x \in (x,a) \in R\}(a,b) \in R \Leftrightarrow [a]_R \cap [b]_R = \emptysetCountingrepetition allowed, order matters: n^k (multiplication rule)repetition allowed, order does not matter: \binom{r+n-1}{r} (r-combination with repetition)repetition not allowed, order matters: \frac{n!}{(n-r)!} (r-combination)permutations of n objects with indistinguishableelements: \frac{n!}{n_1!n_2!n_3!n_k!}repetition not allowed, order does not matter: \binom{r}{k} = \frac{n!}{r!(n-r)!} (r-combination)GraphsGraph: A graph G = \{V, E\} Consists of a nonempty set of vertices V(G) and set of edges E(G)Simple graph: No loops or parallel edgesGraph: graph with distinct vertices v and w such that there are an even number of vertices with an odd degreeSimple graph: No loops or parallel edgesClosed walk: Starts and ends at same vertexCircuit: Closed walk with no repeat edgesSimple circuit: Circuit with no repeat verticesEuler circuits, connected gegree count, possible circuits, connectedness$	$f: A(domain) \rightarrow B(codomain)$	Injective: $\forall x, y \in A, x \neq y \rightarrow f(x) \neq f(y)$
Relations aRb implies $(a,b) \in R = \{(a,b) \in A \times B p(x,y)\}$ $a \in dom(R) \Leftrightarrow \exists b \text{ such that } (a,b) \in R$ $b \in range(R) \Leftrightarrow \exists a \text{ such that } (a,b) \in R$ Symmetric: if $(x,y) = R$ then $(y,x) \in R$ Transitive: if (x,y) and $(y,z) \in R$ then $(x,z) \in R$ Equivalence relation when R is reflexive, symmetric and transitive	$f(x) = y \Rightarrow y = \text{image of } x \text{ and } x = \text{preimage of } y$	Surjective: $\forall y \in B, \exists x \in A \text{ such that } y = f(x)$
$ \begin{array}{ll} \textit{aRb} \text{ implies } (a,b) \in R = \{(a,b) \in A \times B p(x,y)\} \\ \textit{a} \in \textit{dom}(R) \Leftrightarrow \exists \textit{b} \textit{ such that } (a,b) \in R \\ \textit{b} \in \textit{range}(R) \Leftrightarrow \exists \textit{a} \textit{ such that } (a,b) \in R \\ \textit{Equivalence relation} \text{ when R is reflexive, symmetric} \\ \text{and transitive} \\ \text{and transitive} \\ \\ \hline \textbf{Counting} \\ \text{repetition allowed, order matters: } n^k \text{ (multiplication rule)} \\ \text{repetition not allowed, order matters: } n^k \text{ (multiplication rule)} \\ \text{repetition not allowed, order matters: } n^k \text{ (multiplication rule)} \\ \text{repetition not allowed, order matters: } \frac{n!}{(n-r)!} \text{ (r-combination with repetition)} \\ \text{repermutation)} \\ \text{permutations of } n \text{ objects with indistinguishable elements: } \frac{n!}{n_1!n_2!n_3!n_k!} \\ \text{Graphs} \\ \text{Graphs} \\ \text{Graphs} \\ \text{Graphs} \\ \text{Graphs a graph } G = \{V, E\} \text{ Consists of a nonempty set of vertices } V(G) \text{ and set of edges } E(G) \\ \text{Bipartite graph: graph with distinct vertices } v \text{ and } w \\ \text{such that there are no edges between any } v's \text{ or } w's \\ \text{such that there are no edges between any } v's \text{ or } w's \\ \text{such that there are no edges between any } v's \text{ or } w's \\ \text{suffice the number of edges, and is always even} \\ \text{Trails with no repeat edges} \\ \text{Closed walk: Starts and ends at same vertex} \\ \text{Clicouit: Closed walk with no repeat edges} \\ \text{Isomorphism: } G \text{ and } G'\text{ are isomorphic iff 3 bijective} \\ \text{functions } g: V(G) \rightarrow V(G') \text{ and } h: E(G) \rightarrow E(G') \\ \end{array}$	under $f, y \in range(f)$	Bijective: Injective and Surjective
$\begin{array}{ll} \mathbf{a} \in dom(R) \Leftrightarrow \exists b \ such \ that \ (a,b) \in R \\ b \in range(R) \Leftrightarrow \exists a \ such \ that \ (a,b) \in R \\ \mathbf{b} \in range(R) \Leftrightarrow \exists a \ such \ that \ (a,b) \in R \\ \mathbf{Equivalence \ relation} \ \text{ when R is reflexive, symmetric} \\ \text{and transitive} \\ & (a,b) \in R \Leftrightarrow [a]_R = [b]_R \\ (a,b) \notin R \Leftrightarrow [a]_R \cap [b]_R = \emptyset \\ \\ \mathbf{Counting} \\ \mathbf{repetition \ allowed, order \ matters: } \frac{n!}{(n-r)!} (\mathbf{r} \\ \mathbf{repetition \ not \ allowed, order \ matters: } \frac{n!}{(n-r)!} (\mathbf{r} \\ \mathbf{repetition \ not \ allowed, order \ matters: } \frac{n!}{(n-r)!} (\mathbf{r} \\ \mathbf{repetition \ not \ allowed, order \ does \ not \ matter: } (\frac{r+n-1}{r}) \\ \mathbf{repetition \ not \ allowed, order \ does \ not \ matter: } (\frac{r}{r}) \\ \mathbf{repetition \ not \ allowed, order \ does \ not \ matter: } (\frac{r}{r}) \\ \mathbf{repetition \ not \ allowed, order \ does \ not \ matter: } (\frac{n}{r}) \\ \mathbf{repetition \ not \ allowed, order \ does \ not \ matter: } (\frac{n}{r}) \\ \mathbf{repetition \ not \ allowed, order \ does \ not \ matter: } (\frac{n}{r}) \\ \mathbf{repetition \ not \ allowed, order \ does \ not \ matter: } (\frac{n}{r}) \\ \mathbf{repetition \ not \ allowed, order \ does \ not \ matter: } (\frac{n}{r}) \\ \mathbf{repetition \ not \ allowed, order \ does \ not \ matter: } (\frac{n}{r}) \\ \mathbf{repetition \ not \ allowed, order \ does \ not \ matter: } (\frac{n}{r}) \\ \mathbf{repetition \ not \ allowed, order \ does \ not \ matter: } (\frac{n}{r}) \\ \mathbf{repetition \ not \ allowed, order \ does \ not \ matter: } (\frac{n}{r}) \\ \mathbf{repetition \ not \ allowed, order \ does \ not \ matter: } (\frac{n}{r}) \\ \mathbf{repetition \ not \ allowed, order \ does \ not \ matter: } (\frac{n}{r}) \\ \mathbf{repetition \ not \ allowed, order \ does \ not \ matter: } (\frac{n}{r}) \\ \mathbf{repetition \ not \ allowed, order \ does \ not \ matter: } (\frac{n}{r}) \\ \mathbf{repetition \ not \ allowed, order \ does \ not \ matter: } (\frac{n}{r}) \\ \mathbf{repetition \ not \ allowed, order \ does \ not \ matter: } (\frac{n}{r}) \\ \mathbf{repetition \ not \ allowed, order \ does \ not \ matter: } (\frac{n}{r}) \\ \mathbf{repetition \ not \ allowed, order \ does \ not \ matter: } (\frac{n}{r}) \\ repetition \ not \ allowed, order \ does \ not \ matter: $	Relations	
$\begin{array}{ll} \textbf{b} \in range(R) \Leftrightarrow \exists a \ such \ that \ (a,b) \in R \\ \textbf{Equivalence relation} \ \text{when R is reflexive, symmetric} \\ \text{and transitive} \\ & (a,b) \in R \Leftrightarrow [a]_R = [b]_R \\ (a,b) \notin R \Leftrightarrow [a]_R \cap [b]_R = \emptyset \\ \\ \textbf{Counting} \\ \\ \text{repetition allowed, order matters: } n^k \ (\text{multiplication rule}) \\ \text{repetition not allowed, order matters: } n^k \ (\text{multiplication rule}) \\ \text{repetition not allowed, order matters: } n^k \ (\text{multiplication rule}) \\ \text{repetition not allowed, order matters: } n^k \ (\text{multiplication rule}) \\ \text{repetition not allowed, order does not matter: } (r+n-1) \ (r\text{-combination with repetition}) \\ \text{repetition not allowed, order does not matter: } (n-1) \ (r\text{-combination}) \\ \text{permutations of } n \text{ objects with indistinguishable} \\ \text{elements: } \frac{n!}{n_1!n_2!n_3!n_k!} \\ \text{Graphs} \\ \text{Graphs} \\ \text{Graph: A graph } G = \{V,E\} \text{ Consists of a } nonempty \text{ set of vertices } V(G) \text{ and set of edges } E(G) \\ \text{Bipartite graph: graph with distinct vertices } v \text{ and } w \text{ such that there are no edges between any } v'\text{s or } w'\text{s} \\ \text{10.1.9: In any graph, there are an even number of vertices with an odd degree} \\ \text{Closed walk: Starts and ends at same vertex} \\ \text{Closed walk: Starts and ends at same vertex} \\ \text{Circuit: Closed walk with no repeat deges} \\ \text{Isomorphism: } G \text{ and } G \text{ are isomorphic iff } \exists \text{ bijective} \\ \text{Isomorphism: } G \text{ and } G \text{ are isomorphic iff } \exists \text{ bijective} \\ \text{Isomorphic invariants: vertex/edge/degree count,} \\ \text{possible circuits, connected anes} \\ \text{Sumorphic invariants: vertex/edge/degree count,} \\ \text{possible circuits, connected enes} \\ \text{Sumorphic invariants: vertex/edge/degree count,} \\ \text{possible circuits, connected enes} \\ \text{Sumorphic invariants: vertex/edge/degree count,} \\ \text{possible circuits, connected enes} \\ \text{Transitive: } f (x, y) \text{ and } h (x, z) \in R \text{ then } (x, z) \in R t$	aRb implies $(a,b) \in R = \{(a,b) \in A \times B p(x,y)\}$	Reflexive : $\forall x \in A, (x, x) \in R$
Equivalence relation when R is reflexive, symmetric and transitiveEquivalence classes: $[a]_R = \{x \in (x, a) \in R\}$ $(a, b) \in R \Leftrightarrow [a]_R = [b]_R$ $(a, b) \notin R \Leftrightarrow [a]_R = [b]_R$ repetition allowed, order matters: n^k (multiplication rule)repetition allowed, order does not matter: $\binom{r+n-1}{r}$ $(r$ -combination with repetition)repetition not allowed, order matters: $\frac{n!}{(n-r)!}$ (r-combination)repetition not allowed, order does not matter: $\binom{r}{k} = \frac{n!}{r!(n-r)!}$ (r-combination)permutations of n objects with indistinguishable elements: $\frac{n!}{n_1!n_2!n_3!n_k!}$ repetition not allowed, order does not matter: $\binom{n}{k} = \frac{n!}{r!(n-r)!}$ (r-combination)Graphsgeneralized inclusion/exclusion rule: $N(A_1 \cup \cup A_n) = \sum_{1 \le i \le n} N(A_i) - \sum_{1 \le i \le j < n} N(A_i \cap A_j) + \cdots + (-1)^{n+1} N(A_1 \cap A_2 \cap \cap A_n)$ GraphsSimple graph: No loops or parallel edgesGraph: A graph $G = \{V, E\}$ Consists of a nonempty set of vertices $V(G)$ and set of edges $E(G)$ Simple graph: No loops or parallel edgesBipartite graph: graph with distinct vertices v and v such that there are no edges between any v 's or v 'sHandshake theorem: In any graph, the total degree of a graph is twice the number of edges, and is always even10.1.9: In any graph, there are an even number of vertices with an odd degreeTrail: No repeat edgesClosed walk: Starts and ends at same vertexEuler circuit: Visits every edge of G . G must be connected and all vertices with positive even degreesSimple circuit: Circuit with no repeat verticesIsomorphic invariants: vertex/edge/degree count, possible circuits, connectedness		
$(a,b) \notin R \Leftrightarrow [a]_R \cap [b]_R = \emptyset$ $(a,b) \notin R \Leftrightarrow [a]_R \cap [b]_R = \emptyset$ repetition allowed, order matters: n^k (multiplication rule) repetition not allowed, order matters: $\frac{n!}{(n-r)!}$ (r-combination with repetition) repetition not allowed, order does not matter: $\binom{r+n-1}{r}$ (r-combination with repetition) repetition not allowed, order does not matter: $\binom{n}{k} = \frac{n!}{r!(n-r)!}$ (r-combination) permutations of n objects with indistinguishable elements: $\frac{n!}{n_1!n_2!n_3!\dots n_k!}$ generalized inclusion/exclusion rule: $N(A_1 \cup \dots \cup A_n) = \sum_{1 \le i \le n} N(A_i) - \sum_{1 \le i < j < n} N(A_i \cap A_j) + \dots + (-1)^{n+1}N(A_1 \cap A_2 \cap \dots \cap A_n)$ Graphs Graphs Graph: A graph $G = \{V, E\}$ Consists of a nonempty set of vertices $V(G)$ and set of edges $E(G)$ Bipartite graph: graph with distinct vertices v and w such that there are no edges between any v 's or w 's graph is twice the number of edges, and is always even 10.1.9: In any graph, there are an even number of vertices with an odd degree Closed walk: Starts and ends at same vertex circuit: Closed walk with no repeat edges Simple circuit: Closed walk with no repeat edges Simple circuit: Closed walk with no repeat vertices Isomorphism: G and G are isomorphic iff B bijective functions $g: V(G) \to V(G')$ and $h: E(G) \to E(G')$ Isomorphic invariants: vertex/edge/degree count, possible circuits, connectedness	· ·	
Countingrepetition allowed, order matters: n^k (multiplication rule)repetition allowed, order does not matter: $\binom{r+n-1}{r}$ (r-combination with repetition)repetition not allowed, order matters: $\frac{n!}{(n-r)!}$ (r-permutation)repetition not allowed, order does not matter: $\binom{n}{k}$ = $\frac{n!}{r!(n-r)!}$ (r-combination)permutations of n objects with indistinguishable elements: $\frac{n!}{n_1!n_2!n_3!n_k!}$ generalized inclusion/exclusion rule: $N(A_1 \cup \cup A_n) = \sum_{1 \le i \le n} N(A_i) - \sum_{1 \le i < j < n} N(A_i \cap A_j) + \cdots + (-1)^{n+1} N(A_1 \cap A_2 \cap \cap A_n)$ GraphsSimple graph: No loops or parallel edgesGraphs A graph $G = \{V, E\}$ Consists of a nonempty set of vertices $V(G)$ and set of edges $E(G)$ Simple graph: No loops or parallel edgesBipartite graph: graph with distinct vertices v and w such that there are no edges between any v 's or w 's such that there are no edges between any v 's or w 's vertices with an odd degreeHandshake theorem: In any graph, the total degree of a graph is twice the number of edges, and is always even10.1.9: In any graph, there are an even number of vertices with an odd degreeTrail: No repeat edgesClosed walk: Starts and ends at same vertexEuler circuit: Visits every edge of G . G must be connected and all vertices with positive even degreesSimple circuit: Closed walk with no repeat edgesEuler circuit: Visits every edge of G . G must be connected and all vertices with positive even degreesIsomorphism: G and G are isomorphic iff \exists bijective functions $g: V(G) \to V(G')$ and $h: E(G) \to E(G')$ Isomorphic invariants: vertex/edge/degree count, possible circuits, connectedness	and transitive	1 1 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2
repetition allowed, order matters: n^k (multiplication rule) repetition not allowed, order matters: $\frac{n!}{(n-r)!}$ (r-combination with repetition) repetition not allowed, order matters: $\frac{n!}{(n-r)!}$ (r-combination with repetition) permutations of n objects with indistinguishable elements: $\frac{n!}{n_1!n_2!n_3!n_k!}$ and $\frac{n!}{n_1!n_2!n_3!n_k!}$ are permutations of n objects with indistinguishable elements: $\frac{n!}{n_1!n_2!n_3!n_k!}$ and n of n objects with indistinguishable elements: $\frac{n!}{n_1!n_2!n_3!n_k!}$ and n of n objects with indistinguishable elements: $\frac{n!}{n_1!n_2!n_3!n_k!}$ are permutation on allowed, order does not matter: $\binom{n}{k}$ = $\frac{n!}{n!(n-r)!}$ (r-combination) generalized inclusion/exclusion rule: $N(A_1 \cup \cup A_n) \cap A_n$ and n objects with indistinguishable elements: n objects with n oloops or parallel edges Simple graph: No loops or parallel edges Handshake theorem: In any graph, the total degree of a graph is twice the number of edges, and is always even Trail: No repeat edges Path: Trail with no repeat vertices Euler circuit: Visits every edge of n of n on n objects with positive even degrees Simple circuit: Circuit with no repeat vertices Isomorphism: n and n or n objects with n or n objects n or n objects n or n objects n or n		$(a,b) \notin R \Leftrightarrow [a]_R \cap [b]_R = \emptyset$
rule) $ (r\text{-combination with repetition}) $ repetition not allowed, order matters: $\frac{n!}{(n-r)!}$ (r-permutation) $ \frac{n!}{r!(n-r)!}$ (r-combination with repetition) $ \frac{n!}{r!(n-r)!}$ (r-combination) generalized inclusion/exclusion rule: $N(A_1 \cup \cup A_n) = \sum_{1 \le i \le n} N(A_i) - \sum_{1 \le i < j < n} N(A_i \cap A_j) + \cdots + (-1)^{n+1}N(A_1 \cap A_2 \cap \cap A_n) $ Simple graph: No loops or parallel edges of vertices $V(G)$ and set of edges $E(G)$ Bipartite graph with distinct vertices $V(G)$ and set of edges between any V' s or W' s graph is twice the number of edges, and is always even 10.1.9: In any graph, there are an even number of vertices with an odd degree Path: Trail with no repeat vertices Elosed walk: Starts and ends at same vertex Circuit: Closed walk with no repeat edges Simple circuit: Circuit with no repeat vertices Isomorphism: $V(G) \to V(G')$ and $V(G) \to E(G')$ Isomorphic invariants: vertex/edge/degree count, possible circuits, connectedness		(m m 1)
repetition not allowed, order matters: $\frac{n!}{(n-r)!}$ (r-permutation) repetition not allowed, order matters: $\frac{n!}{(n-r)!}$ (r-permutations of n objects with indistinguishable elements: $\frac{n!}{n_1!n_2!n_3!n_k!}$ generalized inclusion/exclusion rule: $N(A_1 \cup \cup A_n) = \sum_{1 \le i \le n} N(A_i) - \sum_{1 \le i < j < n} N(A_i \cap A_j) + \cdots + (-1)^{n+1} N(A_1 \cap A_2 \cap \cap A_n)$ Graphs Graphs: Graph: A graph $G = \{V, E\}$ Consists of a nonempty set of vertices $V(G)$ and set of edges $E(G)$ Bipartite graph: graph with distinct vertices v and w such that there are no edges between any v 's or w 's graph is twice the number of edges, and is always even 10.1.9: In any graph, there are an even number of vertices with an odd degree Closed walk: Starts and ends at same vertex Circuit: Closed walk with no repeat edges Simple circuit: Circuit with no repeat vertices Isomorphism: G and G 'are isomorphic iff B bijective functions $g: V(G) \to V(G')$ and $h: E(G) \to E(G')$ isomorphic invariants: vertex/edge/degree count, possible circuits, connectedness		repetition allowed, order does not matter: $\binom{r+n-1}{r}$
repetition not allowed, order matters: $\frac{n!}{(n-r)!}$ (r-permutation) $\frac{n!}{r!(n-r)!}$ (r-combination) $\frac{n!}{r!(n-r)!}$ (r-combination) generalized inclusion/exclusion rule: $N(A_1 \cup \cup A_n) = \sum_{1 \le i \le n} N(A_i) - \sum_{1 \le i < j < n} N(A_i) - \sum_{1 \le i < j < n} N(A_i \cap A_j) + \cdots + (-1)^{n+1} N(A_1 \cap A_2 \cap \cap A_n)$ Graphs Graphs Simple graph: A graph $G = \{V, E\}$ Consists of a nonempty set of vertices $V(G)$ and set of edges $E(G)$ Bipartite graph: graph with distinct vertices v and v such that there are no edges between any v 's or v 's in any graph, there are an even number of vertices with an odd degree Closed walk: Starts and ends at same vertex Circuit: Closed walk with no repeat edges Simple circuit: Circuit with no repeat vertices Simple circuit. Circuit with no repeat vertices	rule)	
permutations of n objects with indistinguishable elements: $\frac{n!}{n_1!n_2!n_3!n_k!}$ generalized inclusion/exclusion rule: $N(A_1 \cup \cup A_n) = \sum_{1 \le i \le n} N(A_i) - \sum_{1 \le i < j < n} N(A_i \cap A_j) + \cdots + (-1)^{n+1} N(A_1 \cap A_2 \cap \cap A_n)$ Graphs Graphs: A graph $G = \{V, E\}$ Consists of a nonempty set of vertices $V(G)$ and set of edges $E(G)$ Bipartite graph: graph with distinct vertices v and w such that there are no edges between any v 's or w 's graph is twice the number of edges, and is always even 10.1.9: In any graph, there are an even number of vertices with an odd degree Closed walk: Starts and ends at same vertex Circuit: Closed walk with no repeat edges Simple circuit: Circuit with no repeat vertices Isomorphism: G and G are isomorphic iff \exists bijective functions $g:V(G) \to V(G)$ and $h: E(G) \to E(G)$	repetition not allowed, order matters: $\frac{n!}{n!}$ (r-	(21)
permutations of n objects with indistinguishable elements: $\frac{n!}{n_1!n_2!n_3!n_k!}$ $\frac{n!}{n_1!n_2!n_3!n_k!}$ generalized inclusion/exclusion rule: $N(A_1 \cup \cup A_n) = \sum_{1 \le i \le n} N(A_i) - \sum_{1 \le i < j < n} N(A_i \cap A_j) + \cdots + (-1)^{n+1} N(A_1 \cap A_2 \cap \cap A_n)$ Graphs Graphs Graph: A graph $G = \{V, E\}$ Consists of a nonempty set of vertices $V(G)$ and set of edges $E(G)$ Bipartite graph: graph with distinct vertices v and w such that there are no edges between any v 's or w 's such that there are no edges between any v 's or w 's graph is twice the number of edges, and is always even 10.1.9: In any graph, there are an even number of vertices with an odd degree Path: Trail with no repeat vertices Closed walk: Starts and ends at same vertex Circuit: Closed walk with no repeat edges Simple circuit: Circuit with no repeat vertices Isomorphism: G and G are isomorphic iff \exists bijective functions $g:V(G) \to V(G')$ and $h: E(G) \to E(G')$ Isomorphic invariants: vertex/edge/degree count, possible circuits, connectedness	repetition for allowed, order matters: $(n-r)!$	107
elements: $\frac{n!}{n_1!n_2!n_3!n_k!}$ $A_n) = \sum_{1 \le i \le n} N(A_i) - \sum_{1 \le i < j < n} N\left(A_i \cap A_j\right) + \cdots + \left(-1\right)^{n+1} N(A_1 \cap A_2 \cap \ldots \cap A_n)$ Graphs Graphs Simple graph: No loops or parallel edges of vertices $V(G)$ and set of edges $E(G)$ Bipartite graph: graph with distinct vertices v and w such that there are no edges between any v 's or w 's graph is twice the number of edges, and is always even 10.1.9: In any graph, there are an even number of vertices with an odd degree Closed walk: Starts and ends at same vertex Circuit: Closed walk with no repeat edges Simple circuit: Circuit with no repeat vertices Simple circuit: Circuit with no repeat vertices Isomorphism: G and G are isomorphic iff \exists bijective functions $g: V(G) \to V(G')$ and $h: E(G) \to E(G')$ Isomorphic invariants: vertex/edge/degree count, possible circuits, connectedness	permutation)	$\frac{1}{r!(n-r)!}$ (r-combination)
Graphs Graph: A graph $G = \{V, E\}$ Consists of a nonempty set of vertices $V(G)$ and set of edges $E(G)$ Bipartite graph: graph with distinct vertices v and w such that there are no edges between any v 's or w 's 10.1.9: In any graph, there are an even number of vertices with an odd degree Closed walk: Starts and ends at same vertex Circuit: Closed walk with no repeat edges Simple circuit: Circuit with no repeat vertices Isomorphism: G and G are isomorphic iff \exists bijective functions $g: V(G) \rightarrow V(G)$ and $h: E(G) \rightarrow E(G)$ Simple graph: No loops or parallel edges Handshake theorem: In any graph, the total degree of a graph is twice the number of edges, and is always even Trail: No repeat edges Path: Trail with no repeat vertices Euler circuit: Visits every edge of G . G must be connected and all vertices with positive even degrees Isomorphic invariants: vertex/edge/degree count, possible circuits, connectedness		generalized inclusion/exclusion rule: $N(A_1 \cup \cup$
Graphs Graph: A graph $G = \{V, E\}$ Consists of a nonempty set of vertices $V(G)$ and set of edges $E(G)$ Bipartite graph: graph with distinct vertices v and w such that there are no edges between any v 's or w 's 10.1.9: In any graph, there are an even number of vertices with an odd degree Closed walk: Starts and ends at same vertex Circuit: Closed walk with no repeat edges Simple circuit: Circuit with no repeat vertices Isomorphism: G and G are isomorphic iff \exists bijective functions $g: V(G) \rightarrow V(G)$ and $h: E(G) \rightarrow E(G)$ Simple graph: No loops or parallel edges Handshake theorem: In any graph, the total degree of a graph is twice the number of edges, and is always even Trail: No repeat edges Path: Trail with no repeat vertices Euler circuit: Visits every edge of G . G must be connected and all vertices with positive even degrees Isomorphic invariants: vertex/edge/degree count, possible circuits, connectedness	elements: $\frac{n!}{n!}$	$A_n) = \sum_{1 \le i \le n} N(A_i) - \sum_{1 \le i < j < n} N(A_i \cap A_j) + \dots +$
Graph: A graph $G = \{V, E\}$ Consists of a <i>nonempty</i> set of vertices $V(G)$ and set of edges $E(G)$ Bipartite graph: graph with distinct vertices v and w such that there are no edges between any v 's or w 's 10.1.9: In any graph, there are an even number of vertices with an odd degree Path: Trail with no repeat vertices Closed walk: Starts and ends at same vertex Circuit: Closed walk with no repeat edges Simple circuit: Circuit with no repeat vertices Isomorphism: G and G are isomorphic iff \exists bijective functions $g: V(G) \rightarrow V(G)$ and $h: E(G) \rightarrow E(G)$ Simple graph: No loops or parallel edges Handshake theorem: In any graph, the total degree of a graph is twice the number of edges, and is always even Trail: No repeat edges Euler circuit: Visits every edge of G . G must be connected and all vertices with positive even degrees Isomorphic invariants: vertex/edge/degree count, possible circuits, connectedness	$n_1!n_2!n_3!n_k!$	$(-1)^{n+1}N(A_1\cap A_2\cap\cap A_n)$
Graph: A graph $G = \{V, E\}$ Consists of a <i>nonempty</i> set of vertices $V(G)$ and set of edges $E(G)$ Bipartite graph: graph with distinct vertices v and w such that there are no edges between any v 's or w 's 10.1.9: In any graph, there are an even number of vertices with an odd degree Path: Trail with no repeat vertices Closed walk: Starts and ends at same vertex Circuit: Closed walk with no repeat edges Simple circuit: Circuit with no repeat vertices Isomorphism: G and G are isomorphic iff \exists bijective functions $g: V(G) \rightarrow V(G)$ and $h: E(G) \rightarrow E(G)$ Simple graph: No loops or parallel edges Handshake theorem: In any graph, the total degree of a graph is twice the number of edges, and is always even Trail: No repeat edges Euler circuit: Visits every edge of G . G must be connected and all vertices with positive even degrees Isomorphic invariants: vertex/edge/degree count, possible circuits, connectedness		
of vertices $V(G)$ and set of edges $E(G)$ Bipartite graph: graph with distinct vertices v and w such that there are no edges between any v 's or w 's 10.1.9: In any graph, there are an even number of vertices with an odd degree Closed walk: Starts and ends at same vertex Circuit: Closed walk with no repeat edges Simple circuit: Circuit with no repeat vertices Isomorphism: G and G are isomorphic iff \exists bijective functions $g:V(G) \rightarrow V(G)$ and $h:E(G) \rightarrow E(G)$ Handshake theorem: In any graph, the total degree of a graph is twice the number of edges, and is always even Trail: No repeat edges Path: Trail with no repeat vertices Euler circuit: Visits every edge of G . G must be connected and all vertices with positive even degrees Isomorphic invariants: vertex/edge/degree count, possible circuits, connectedness	Graphs	
Bipartite graph: graph with distinct vertices v and w such that there are no edges between any v 's or w 's $10.1.9$: In any graph, there are an even number of vertices with an odd degree v and v is a graph is twice the number of edges, and is always even v is v is v in	Graph: A graph $G = \{V, E\}$ Consists of a <i>nonempty</i> set	Simple graph: No loops or parallel edges
such that there are no edges between any v 's or w 's graph is twice the number of edges, and is always even 10.1.9: In any graph, there are an even number of vertices with an odd degree Path: Trail with no repeat vertices Closed walk: Starts and ends at same vertex Circuit: Closed walk with no repeat edges connected and all vertices with positive even degrees Simple circuit: Circuit with no repeat vertices Isomorphism: G and G are isomorphic iff \exists bijective functions $g:V(G) \rightarrow V(G)$ and $h:E(G) \rightarrow E(G)$ Isomorphic invariants: vertex/edge/degree count, possible circuits, connectedness	, ,	
10.1.9: In any graph, there are an even number of vertices with an odd degree Path: Trail with no repeat vertices Closed walk: Starts and ends at same vertex Circuit: Closed walk with no repeat edges Simple circuit: Circuit with no repeat vertices Isomorphism: G and G are isomorphic iff \exists bijective functions $g:V(G) \rightarrow V(G)$ and $h:E(G) \rightarrow E(G)$ Trail: No repeat edges Euler circuit: Visits every edge of G . G must be connected and all vertices with positive even degrees Isomorphic invariants: vertex/edge/degree count, possible circuits, connectedness		
vertices with an odd degree Path: Trail with no repeat vertices Closed walk: Starts and ends at same vertex Circuit: Closed walk with no repeat edges Simple circuit: Circuit with no repeat vertices Isomorphism: G and G are isomorphic iff \exists bijective functions $g:V(G) \rightarrow V(G')$ and $h:E(G) \rightarrow E(G')$ Path: Trail with no repeat vertices Euler circuit: Visits every edge of G . G must be connected and all vertices with positive even degrees Isomorphic invariants: vertex/edge/degree count, possible circuits, connectedness		
Closed walk: Starts and ends at same vertex Circuit: Closed walk with no repeat edges Simple circuit: Circuit with no repeat vertices Isomorphism: G and G are isomorphic iff \exists bijective functions $g:V(G) \to V(G)$ and $h:E(G) \to E(G)$ Euler circuit: Visits every edge of G . G must be connected and all vertices with positive even degrees Isomorphic invariants: vertex/edge/degree count, possible circuits, connectedness		
Circuit: Closed walk with no repeat edges connected and all vertices with positive even degrees simple circuit: Circuit with no repeat vertices Isomorphism: G and G are isomorphic iff \exists bijective functions $g:V(G) \to V(G)$ and $h:E(G) \to E(G)$ Isomorphic invariants: vertex/edge/degree count, possible circuits, connectedness		
Simple circuit: Circuit with no repeat vertices Isomorphism: G and G are isomorphic iff \exists bijective functions $g:V(G) \to V(G)$ and $h:E(G) \to E(G)$ Isomorphic invariants: vertex/edge/degree count, possible circuits, connectedness		
Isomorphism: G and G are isomorphic iff \exists bijective functions $g:V(G) \to V(G')$ and $h:E(G) \to E(G')$ Isomorphic invariants: vertex/edge/degree count, possible circuits, connectedness		connected and all vertices with positive even degrees
functions $g:V(G) \to V(G')$ and $h:E(G) \to E(G')$ possible circuits, connectedness		
Graph isomorphism is an equivalence relation		possible circuits, connectedness
	Graph isomorphism is an equivalence relation	