1 Path Integral

1.

$$\int_{-\infty}^{\infty} \exp\left(\sum_i -r_i x_i^2 + c_i x_i\right) \prod_{i=1}^{N} dx_i$$

$$= \prod_i \int_{-\infty}^{\infty} \exp\left(-r_i x_i^2 + c_i x_i\right) dx_i$$

$$= \int_{-\infty}^{\infty} \prod_i \left(\exp\left(-r_i(x_i - \frac{c_i}{2r_i})^2\right) \exp\left(\frac{c_i^2}{4r_i}\right) dx_i\right)$$

$$= \prod_{i=1}^{N} \sqrt{\frac{\pi}{r_i}} \exp\left(\frac{1}{4}\sum_i \frac{c_i^2}{r_i}\right)$$

2. The matrix form of

$$\int_{-\infty}^{\infty} \exp\left(\sum_i (-i a_i x_i^2 + i b_i x_i) \prod_i dx_i\right)$$

$$= \prod_{i=1}^{N} \sqrt{\frac{\pi}{i a_i}} \exp\left(\frac{i}{4}\sum_i \frac{b_i^2}{a_i}\right)$$

is

$$\int_{-\infty}^{\infty} \exp\left(-i X^T A X + i B X\right) \prod_{i=1}^{N} dx_i = \sqrt{\frac{\pi^N}{\det(iA)}} \exp\left(\frac{i}{4} B^T A^{-1} B\right)$$

We know that $A = O^T S O$,

$$\int_{-\infty}^{\infty} \exp\left(-i X^T O^T S O X + i B O^T O X\right) \prod_{i=1}^{N} dx_i = \sqrt{\frac{\pi^N}{\det(S)}} \exp\left(\frac{i}{4} B^T O^T O A^{-1} O^T O B\right)$$

Since $Y = OX$, $D = OB$,

$$\int_{-\infty}^{\infty} \exp\left(-i Y^T S Y + i D^T Y\right) \prod_{i=1}^{N} dy_i = \sqrt{\frac{\pi^N}{\det(S)}} \exp\left(\frac{i}{4} D^T S^{-1} D\right)$$

3. Denote

$$I = \int_{-\infty}^{\infty} \exp\left(-i Y^T S Y + i D^T Y\right) \prod_i^{N} dy_i$$

Variation of I

$$\delta I = \int_{-\infty}^{\infty} \left[\exp\left(-i(Y + \delta Y)^T S(Y + \delta Y) + iD^T(Y + \delta Y)\right) - \exp\left(-iY^T SY + iD^T Y\right) \right] \prod_i^N dy_i$$

$$= \int_{-\infty}^{\infty} \exp\left(-iY^T SY + iD^T Y\right) \left[\exp\left(-i(2Y^T S - D^T)\delta Y\right) - 1 \right] \prod_i^N dy_i$$

To make I stationary, $\delta I = 0$, that is

$$2\bar{Y}^T D - D^T = 0$$

Then we have

$$\bar{Y} = \frac{1}{2} S^{-1} D$$

At its stationary point, the integrand of I becomes

$$\exp\left[-i \left(\frac{1}{2}S^{-1}D\right)^T S \left(\frac{1}{2}S^{-1}D\right) + iD \left(\frac{1}{2}S^{-1}D\right) \right]$$

$$= \exp\left(\frac{1}{4} iD^T S^{-1} D\right)$$

which is different from the integral result of I with a prefactor $\sqrt{\pi^N / \det(iS)}$.

4. Denote

$$I = \int_{-\infty}^{\infty} \exp\left(-Y^T SY + D^T Y\right) \prod_i^N dy_i$$

Then

$$\delta I = \int_{-\infty}^{\infty} \left[\exp\left(-(Y + \delta Y)^T S(Y + \delta Y) + D^T(Y + \delta Y)\right) - \exp\left(-Y^T SY + D^T Y\right) \right] \prod_i^N dy_i$$

$$= \int_{-\infty}^{\infty} \exp\left(-Y^T SY + D^T Y\right) \left[\exp\left(-(2Y^T S - D^T)\delta Y\right) - 1 \right] \prod_i^N dy_i$$

To make I stationary,

$$2\bar{Y}^T S - D^T = 0.$$

i.e.,

$$\bar{Y} = \frac{1}{2} S^{-1} D$$

Put $\bar{Y} = \frac{1}{2}S^{-1}D$ into the integrand of I, we get

$$\exp\left[-\left(\frac{1}{2}S^{-1}D\right)^T S\frac{1}{2}S^{-1}D + D^T\frac{1}{2}S^{-1}D\right]$$

$$= \exp\left(-\frac{1}{4}D^T S^{-1}SS^{-1}D + \frac{1}{2}D^T S^{-1}D\right)$$

$$= \exp\left(\frac{1}{4}D^T S^{-1}D\right)$$

which is the same as the integral result of I apart from a prefactor.

5.

$$\langle q|e^{-itH}\rangle = \int\int dp'\,dp''\langle q|p'\rangle\langle p'|e^{-ip^2/(2m\hbar)t}|p''\rangle\langle p''|0\rangle$$

$$= \int dp'\frac{1}{(2\hbar\pi)^3}e^{-\frac{-it}{2m\hbar}p'^2}e^{iqp/\hbar}$$

$$= \frac{1}{(2\pi\hbar)^3}\sqrt{\frac{\pi^3}{(it/(2m\hbar))}}e^{2mq^2/(2\hbar t)}$$

$$= \left(\frac{m}{2\pi i\hbar t}\right)^{3/2}e^{imq^2/(2\hbar t)}$$

6.

7.

$$S[q] = \int_0^t \left(\frac{1}{2}m\dot{q}^2 - \frac{1}{2}m\omega^2 q^2\right)dt'$$

$$= \int_0^t \frac{1}{2}m\left((-\omega q'\sin\omega t' + \dot{q}_0\cos\omega t')^2 - \omega^2(q'\cos\omega t' + \frac{\dot{q}^2}{\omega}\sin\omega t')^2\right)dt'$$

$$= \frac{m\omega}{2\sin(\omega t)}\left((q'^2 + q''^2)\cos\omega t - 2q'q''\right)$$

8.

$$S[\delta q] = \int_0^t dt'\left(\frac{1}{2}m\left(\sum_{n=1}a_n n\pi/t\cos\frac{n\pi t'}{t}\right)^2 - \frac{1}{2}m\omega^2\left(\sum_{n=1}a_n\sin\frac{n\pi t'}{t}\right)^2\right)$$

$$= \int_0^t dt'\left(\frac{1}{2}m\sum_{n=1}a_n^2\frac{n^2\pi^2}{t^2}\cos^2\frac{n\pi t'}{t} - \frac{1}{2}m\omega^2\sum_{n=0}a_n^2\sin^2\frac{n\pi t'}{t}\right)$$

$$= \sum_{n=1}\frac{1}{2}ma_n^2\int_0^t dt'\left(\frac{n^2\pi^2}{t^2}\cos^2\frac{n\pi t'}{t} - \omega^2\sin^2\frac{n\pi t'}{t}\right)$$

$$= \sum_{n=1}\frac{mt}{4}a_n^2\left(n^2\pi^2/t^2 - \omega^2\right)$$

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9. When $q' = 0$ and $q'' = q$, it becomes

$$\langle q|e^{-itH/\hbar}|0\rangle = \sqrt{\frac{m\omega}{2\pi i\hbar \sin \omega t}} \exp\left[i\frac{m\omega[q^2 \cos \omega t]}{2\hbar \sin \omega t}\right]$$

In the limit of $t \to 0$, the trigonometric functions used in our calculation becomes $\sin \omega t \to \omega t$ and $\cos \omega t \to 1$.

$$\lim_{t\to 0}\langle q|e^{-itH/\hbar}|0\rangle = \sqrt{\frac{m}{2\pi i\hbar t}} \exp\left(\frac{imq^2}{2\hbar t}\right)$$

10.

11.

$$
\begin{aligned}
S_e[q] &= \int_0^\beta \left[\frac{1}{2}m\dot{q}^2 + \frac{1}{2}m\omega^2 q^2\right] dt \\
&= \int_0^\beta \frac{1}{2}m\left[(A\omega e^{\omega t} - B\omega e^{-\omega t})^2 + \omega(Ae^{\omega t} + Be^{-\omega t})^2\right] \\
&= \frac{1}{2}m\omega^2 \int_0^\beta 2(A^2 e^{2\omega t} + B^2 e^{-2\omega t}) dt \\
&= m\omega^2 \left[A^2(e^{2\omega t} - 1) - B^2(e^{-2\omega t} - 1)\right]
\end{aligned}
$$

12.

$$
\begin{aligned}
S_0[\phi] &= \int \frac{1}{2}[-\partial_a\phi(x)\partial^a\phi(x) - m^2\phi(x)]d^4x \\
&= \int \frac{1}{2}\left[-\int ip_a e^{ip'x}\tilde{\phi}(p')\frac{1}{(2\pi)^4}d^4p' \int \left(-ip_a e^{-ip''x}\tilde{\phi}(-p'')\right)\frac{d^4p''}{(2\pi)^4}\right. \\
&\qquad \left. -m^2 \int\int \frac{dp'}{(2\pi)^4}\frac{dp''}{(2\pi)^4}e^{ip'-p''}x\tilde{\phi}(p')\tilde{\phi}(-p'')\right]d^4x \\
&= -\int d^4x e^{i(p'-p'')x} \int \frac{1}{2}(p^2 + m^2)\tilde{\phi}(p')\tilde{\phi}(-p'')\frac{d^4p'}{(2\pi)^4}\frac{d^4p''}{(2\pi)^4} \\
&= -\delta(p' - p'') \int \frac{1}{2}(p^2 + m^2)\tilde{\phi}(p')\tilde{\phi}(-p'')\frac{d^4p'}{(2\pi)^4}\frac{d^4p''}{(2\pi)^4} \\
&= -\frac{1}{2}\int |\tilde{\phi}(p)|^2 (p^2 + m^2)\frac{d^4p}{(2\pi)^4}
\end{aligned}
$$

4

13.

$$\lim_{\epsilon \to 0+} \epsilon \int_{-\infty}^{\infty} e^{-\epsilon|t|} dt$$

$$= \lim_{\epsilon \to 0+} \left(\epsilon \int_{-\infty}^{0} f(t)e^{\epsilon t} dt + \epsilon \int_{0}^{\infty} f(t)e^{-\epsilon t} dt \right)$$

$$= \lim_{\epsilon \to 0+} \left(\int_{-\infty}^{0} f(t) de^{\epsilon t} - \int_{0}^{\infty} f(t) de^{-\epsilon t} \right)$$

$$= \lim_{\epsilon \to 0+} \left(f(t)e^{\epsilon t}|_{-\infty}^{0} - \int_{-\infty}^{0} e^{\epsilon t} df(t) - f(t)e^{-\epsilon t}|_{0}^{\infty} + \int_{0}^{\infty} e^{-\epsilon t} df(t) \right)$$

$$= \lim_{\epsilon \to 0} \left(2f(0) + \int_{0}^{\infty} e^{-\epsilon t} df(t) - \int_{-\infty}^{0} e^{\epsilon t} df(t) \right)$$

$$= 2f(0) + f(\infty) - f(0) - f(0) + f(-\infty)$$

$$= f(\infty) + f(-\infty)$$

14. Check out it after the other problems are solved.

Fourier transform of $\phi(\vec{x}, t)$ and $\phi(p)$ are

$$\tilde{\phi}(\vec{p}, t) = \int e^{-i\vec{p}\cdot\vec{x}} \phi(\vec{x}, t) d^3 x$$

$$\phi(\vec{x}, t) = \int e^{i\vec{p}\cdot\vec{x}} e^{-ip_0 t} \phi(p') \frac{d^4 p'}{(2\pi)^4}$$

Then

$$\tilde{\phi}(\vec{p}, t) = \iint e^{-i\vec{p}\cdot\vec{x}} e^{-i\vec{p}'\cdot\vec{x}} e^{-ip_0 t} \phi(p') \frac{d^4}{(2\pi)^4} d^3 x$$

$$= \iint e^{-i(\vec{p}-\vec{p}')\cdot\vec{x}} e^{-ip_0 t} \phi(p') \frac{d^4 p'}{(2\pi)^4} d^3 x$$

15.

$$S_0[\phi, \epsilon, j] = -\frac{1}{2} \int \left[|\tilde{\phi}(p)|^2 (p^2 + m^2 - i\epsilon) - \tilde{j}^*(p)\tilde{\phi}(p) - \tilde{\phi}^*(p)\tilde{j}(p) \right] \frac{d^4 p}{(2\pi)^4}$$

$$= -\frac{1}{2} \int \left[\left(\tilde{\psi}(p) + \frac{\tilde{j}(p)}{p^2 + m^2 - i\epsilon} \right)^* \left(\tilde{\psi}(p) + \frac{\tilde{j}(p)}{p^2 + m^2 - i\epsilon} \right) (p^2 + m^2 - i\epsilon) \right.$$

$$\left. - \tilde{j}^*(p) \left(\tilde{\psi}(p) + \frac{\tilde{j}(p)}{p^2 + m^2 - i\epsilon} \right) - \left(\tilde{\psi}^*(p) + \frac{\tilde{j}^*(p)}{p^2 + m^2 + i\epsilon} \right) \tilde{j}(p) \right] \frac{d^4}{(2\pi)^4}$$

$$= -\frac{1}{2} \int \left[|\tilde{\psi}(p)|^2 (p^2 + m^2 - i\epsilon) - \frac{\tilde{j}^*(p)\tilde{j}(p)}{p^2 + m^2 - i\epsilon} \right] \frac{d^4 p}{(2\pi)^4}$$

$$= S_0[\psi, \epsilon] + \frac{1}{2} \int \frac{\tilde{j}^*(p)\tilde{j}(p)}{p^2 + m^2 - i\epsilon} \frac{d^4 p}{(2\pi)^4}$$

16.

$$
\begin{aligned}
Z_0[j] &= \frac{\int \exp\left[i \int j(x)\phi(x)d^4x\right] e^{iS_0[\phi,\epsilon]} D\phi}{\int e^{iS_0[\phi,\epsilon]} D\phi} \\
&= \frac{\int e^{iS_0[\phi,\epsilon,j]} D\phi}{\int e^{iS_0[\phi,\epsilon]} D\phi} \\
&= \frac{\int e^{iS_0[\psi,\epsilon]} D\psi \cdot e^{\frac{i}{2} \int \frac{\tilde{j}^*(p)\tilde{j}(p)}{p^2+m^2-i\epsilon} \frac{d^4p}{(2\pi)^4}}}{\int e^{iS_0[\psi,\epsilon]} D\psi} \\
&= \exp\left[\frac{i}{2} \int \frac{|\tilde{j}(p)|^2}{p^2+m^2-i\epsilon} \frac{d^4p}{(2\pi)^4}\right]
\end{aligned}
$$

17. Applying

$$
\begin{aligned}
\tilde{j}(p) &= \int e^{-ipx} j(x) d^4x \\
\tilde{j}^*(p) &= \int e^{ipx'} j(x') d^4x'
\end{aligned}
$$

to $Z_0[j]$, we get

$$
\begin{aligned}
Z_0[j] &= \exp\left[\frac{i}{2} \int \frac{\iint d^4x d^4x' e^{ip(x-x')} j(x)j(x')}{p^2+m^2-i\epsilon} \frac{d^4p}{(2\pi)^4}\right] \\
&= \exp\left[\frac{i}{2} \int j(x)\Delta(x-x')j(x')d^4x d^4x'\right],
\end{aligned}
$$

in which $\Delta(x-x')$ is the Feynmann's propagrator.

18.

$$\frac{1}{i^4}\frac{\delta^4 Z_0[j]}{\delta_j(x_1)\delta_j(x_2)\delta_j(x_3)\delta_j(x_4)}\bigg|_{j=0}$$

$$= \left(Z_0[j] \int d^4x' d^4x'' d^4x''' d^4x'''' \Delta(x_4 - x')\Delta(x_3 - x'')\Delta(x_2 - x''')\Delta(x_1 - x'''') \right.$$

$$j(x')j(x'')j(x''')j(x'''')$$

$$+ \frac{1}{i} Z_0[j]\Delta(x_3 - x_4) \int d^4x' d^4x'' \Delta(x_1 - x')\Delta(x_1 - x'')j(x')j(x'')$$

$$+ \frac{1}{i} Z_0[j]\Delta(x_2 - x_4) \int d^4x' d^4x'' \Delta(x_3 - x')\Delta(x_1 - x'')j(x')j(x'')$$

$$+ \frac{1}{i} Z_0[j]\Delta(x_1 - x_4) \int d^4x' d^4x'' \Delta(x_3 - x')\Delta(x_2 - x'')j(x')j(x'')$$

$$+ \frac{1}{i} Z_0[j]\Delta(x_2 - x_3) \int d^4x' d^4x'' \Delta(x_3 - x')\Delta(x_2 - x'')j(x')j(x'')$$

$$+ \frac{1}{i} Z_0[j]\Delta(x_1 - x_3) \int d^4x' d^4x'' \Delta(x_2 - x')\Delta(x_4 - x'')j(x')j(x'')$$

$$+ i^2 Z_0[j]\Delta(x_2 - x_3)\Delta(x_1 - x_4) + i^2 Z_0[j]\Delta(x_1 - x_3)\Delta(x_2 - x_4)$$

$$+ \frac{1}{i} Z_0[j]\Delta(x_1 - x_2) \int d^4x' d^4x'' \Delta(x_3 - x')\Delta(x_4 - x'')j(x')j(x'')$$

$$\left. \frac{1}{i^2} Z_0[j]\Delta(x_3 - x_4)\Delta(x_1 - x_2) \right)\bigg|_{j=0}$$

$$= -\Delta(x_1 - x_4)\Delta(x_2 - x_3) - \Delta(x_2 - x_4)\Delta(x_1 - x_3) - \Delta(x_1 - x_2)\Delta(x_3 - x_4)$$

19.

$$\frac{1}{2}\int \left(\boldsymbol{\nabla}\Delta^{-1}j^0 \right)^2 d^4x$$

$$= \frac{1}{2}\int \left[\boldsymbol{\nabla}\left(-\int \frac{1}{4\pi}\frac{j^0(x')}{|x - x'|}d^3x' \right) \right]^2 d^4x$$

$$= \frac{1}{2}\iiint \frac{1}{4\pi}\frac{j(x')}{|x - x'|}\nabla^2 \frac{j(x'')}{|x' - x''|}d^3x' d^3x'' d^4x$$

$$= \frac{1}{2}\iiint \frac{1}{4\pi}\frac{j^){(x)}j^0(x)}{|x - x'|}\delta(x' - x'')d^3x d^3x'' d^4x$$

$$= \frac{1}{2}\iiint \frac{j^0(x)j^0(x')}{4\pi|x - x'|}d^3x' d^3x dt$$

$$= \int V_c dt$$

That is

$$V_c = \frac{1}{2}\int \frac{j^0(\boldsymbol{x}, t)j^0(\boldsymbol{y}, t)}{4\pi|\boldsymbol{x} - \boldsymbol{y}|}d^3x d^3y$$

20.

$$S_0 = \int d^4x \left(-\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + J^\mu A_\mu \right)$$

$$= \frac{1}{2} \int \frac{d^4k}{(2\pi)^4} \left[-\tilde{A}_\mu(k)(k^2 g^{\mu\nu} - k^\mu k^\nu)\tilde{A}_\nu(-k) + \tilde{J}^\mu(k)\tilde{A}_\mu(-k) + \tilde{J}^\mu(-k)\tilde{A}_\mu(k) \right]$$

in which $k^2 g^{\mu\nu - k^\mu k^\nu} = k^2 g^{\mu\nu}(k)$j.

Then we have

$$Z_0 = \int DA e^{iS_0}$$

$$= \exp\left[\frac{i}{2} \int \frac{d^4k}{(2\pi)^4} \tilde{J}_\mu(k) \frac{j^{\mu\nu}(k)}{k^2 - i\epsilon} \tilde{J}_\nu(-k) \right]$$

Finally

$$\langle 0|\mathcal{T}[A_\mu(x)A_\nu(y)]|0\rangle$$

$$= \int \frac{y_{\mu\nu}}{k^2 - i\epsilon} e^{ik(x-y)} \frac{d^4k}{(2\pi)^4}$$

21. Using

$$|\theta\rangle = \exp\left(\psi^\dagger \theta - \frac{1}{2}\theta^*\theta \right) |0\rangle$$

$$= \left(1 + \psi^\dagger \theta - \frac{1}{2}\theta^*\theta \right) |0\rangle$$

and

$$\langle\xi| = \langle 0| \left(1 + \xi^*\psi - \frac{1}{2}\xi^*\xi \right),$$

we get

$$\langle\xi|\theta\rangle = \langle 0| \left(1 + \xi^*\psi - \frac{1}{2}\xi^*\xi \right) \left(1 + \psi^\dagger \theta - \frac{1}{2}\theta^*\theta \right) |0\rangle$$

$$= \langle 0|1 + \xi^*\psi\psi^\dagger\theta - \frac{1}{2}\xi^*\xi - \frac{1}{2}\theta^*\theta + \frac{1}{4}\xi^*\xi\theta^*\theta|0\rangle$$

$$= \langle 0|1 + \xi^*\theta - \frac{1}{2}\xi^*\xi - \frac{1}{2}\theta^*\theta + \frac{1}{4}\xi^*\xi\theta^*\theta|0\rangle$$

$$= \exp\left(\xi^*\theta - \frac{1}{2}(\xi^*\xi + \theta^*\theta) \right)$$

22.

$$\int |\theta\rangle\langle\theta| d\theta^* d\theta$$

$$= \left(1 + \psi^\dagger\theta - \frac{1}{2}\theta^*\theta\right)|0\rangle\langle0|\left(1 + \theta^*\psi - \frac{1}{2}\theta^*\theta\right)d\theta^* d\theta$$

$$= \int\left(|0\rangle\langle0|\theta|1\rangle\langle0| - \frac{1}{2}\theta^*\theta|0\rangle\langle0| + \theta\theta^*|1\rangle\langle1| - \frac{1}{2}\theta^*\theta|0\rangle\langle0|\right)d\theta^* d\theta$$

$$= |0\rangle\langle0| + |1\rangle\langle1|$$

$$= I$$

23. We can expand the state

$$|\theta\rangle = \exp\left(\sum_{k=1}^{n}\psi_k^\dagger\theta_k - \frac{1}{2}\theta_k^*\theta_k\right)|0\rangle$$

$$= \left[\prod_{k=1}^{n}\left(1 + \psi_k^\dagger\theta_k - \frac{1}{2}\theta_k^*\theta_k\right)\right]|0\rangle$$

The ground state is

$$|0\rangle = \left(\prod_{k=1}^{n}\psi_k\right)|s\rangle$$

Then

$$\psi_k|\theta\rangle = \prod_{i\neq k}^{n}\left(1 + \psi_i^\dagger\theta_i - \frac{1}{2}\theta_i^*\theta_i\right)\psi_k\left(1 + \psi_k^\dagger\theta_k - \frac{1}{2}\theta_k^*\theta_k\right)|0\rangle$$

$$= \prod_{i\neq k}^{n}\left(1 + \psi_i^\dagger\theta_i - \frac{1}{2}\theta_i^*\theta_i\right)\psi_k\psi_k^\dagger\theta_k|0\rangle$$

$$= \prod_{i\neq k}^{n}\left(1 + \psi_i^\dagger\theta_i - \frac{1}{2}\theta_i^*\theta_i\right)\left(1 - \psi_k^\dagger\psi_k\right)\theta_k|0\rangle$$

$$= \theta_k\prod_{k=1}^{n}\left(1 + \psi_k^\dagger\theta_k - \frac{1}{2}\theta_k^*\theta_k\right)|0\rangle$$

$$= \theta_k|\theta\rangle$$

24.

$$|0\rangle = \left(\prod_{m}\psi_m(\boldsymbol{x}, 0)\right)|s\rangle$$

$$|\chi\rangle = \exp\left[\int \sum_m \psi_m^\dagger(\boldsymbol{x},0)\chi_m(\boldsymbol{x}) - \frac{1}{2}\chi_m^*(\boldsymbol{x})\chi_m(\boldsymbol{x})d^3x\right]|0\rangle$$

$$= \exp\left[\int\left(\psi^\dagger\chi - \frac{1}{2}\chi^\dagger\chi\right)d^3x\right]|0\rangle$$

$$\psi_m(\boldsymbol{x},0)|\chi\rangle = \exp\left[\int\sum_{i\neq m}\left(\psi_i^\dagger(\boldsymbol{x},0)\chi_m(\boldsymbol{x}) - \frac{1}{2}\chi_m^*(\boldsymbol{x})\chi_m(\boldsymbol{x})\right)d^3x\right]|0\rangle$$

$$\psi_m(\boldsymbol{x},0)\left[\int\left(1 + \psi_m^\dagger(\boldsymbol{x},0)\chi_m(\boldsymbol{x}) - \frac{1}{2}\chi_m^*(\boldsymbol{x})\chi_m(\boldsymbol{x})\right)d^3x\right]|0\rangle$$

$$= \exp\left[\int\sum_{i\neq m}\left(\psi_i^\dagger(\boldsymbol{x},0)\chi_i(\boldsymbol{x}) - \frac{1}{2}\chi_i^*(\boldsymbol{x})\chi_i(\boldsymbol{x})\right)d^3x\right]$$

$$(1 - \psi_m\psi_m^\dagger)\chi_m(\boldsymbol{x})|0\rangle$$

$$= \chi_m(\boldsymbol{x})\exp\left[\int\sum_m\left(\psi_m^\dagger(\boldsymbol{x},0)\chi_m(\boldsymbol{x}) - \frac{1}{2}\chi_m^*(\boldsymbol{x})\chi_m\right)d^3x\right]|0\rangle$$

$$= \chi_m(\boldsymbol{x})|\chi\rangle$$

25.

$$|\chi\rangle = \exp\left[\int\left(\psi^\dagger\chi - \frac{1}{2}\chi^\dagger\chi\right)d^3x\right]|0\rangle$$

$$\langle\chi| = \langle0|\left[\exp\int\left(\chi^\dagger\psi - \frac{1}{2}\chi^\dagger\chi\right)d^3x\right]$$

$$\langle\chi|\chi\rangle = \langle0|\exp\left(\int(\psi^\dagger\chi + \chi'^\dagger\psi - 1/2\chi^\dagger\chi - 1/2\chi'^\dagger\chi')d^3x\right)|0\rangle$$

$$= \langle0|\int\prod_m\left(1 + \psi_m^\dagger\chi_m + \chi_m'^\dagger\psi_m - \frac{1}{2}\chi_m^\dagger\chi_m - \frac{1}{2}\chi_m'^\dagger\chi_m' + \psi_m^\dagger\chi_m\chi_m'^\dagger\psi_m\right)d^3x|0\rangle$$

$$= \langle0|\int\prod_m\left(1 + \chi_m'^\dagger\chi_m - \frac{1}{2}\chi_m^\dagger\chi_m - \frac{1}{2}\chi_m'^\dagger\chi_m'\right)d^3x|0\rangle$$

$$= \exp\left[\int\left(\chi'^\dagger\chi - \frac{1}{2}\chi'^\dagger\chi' - \frac{1}{2}\chi^\dagger\chi\right)d^3x\right]$$