Problems for Chapter 17 (Path Integral).

1. Derive the multiple Gaussian Integral (17.8) from (5.184).

$$\int_{-\infty}^{\infty} \exp\left(\sum_i -r_i x_i^2 + c_i x_i\right) \prod_{i=1}^{N} dx_i = \prod_i \int_{-\infty}^{\infty} \exp(-r_i x_i^2 + c_i x_i)\, dx_i$$

$$= \int_{-\infty}^{\infty} \prod_i \left\{ \exp\left[-r_i\left(x_i - \frac{c_i}{2r_i}\right)^2\right] \cdot \exp\left(\frac{c_i^2}{4r_i}\right) dx_i \right\}$$

$$= \prod_{i=1}^{N} \sqrt{\frac{\pi}{r_i}} \exp\left(\frac{1}{4} \sum_i \frac{c_i^2}{r_i}\right)$$

2. Derive the multiple Gaussian Integral (17.17) from (5.183)

According to (5.183) we have

$$\int_{-\infty}^{\infty} \exp\left(\sum_i (-i a_i x_i^2 + i b_i x_i)\right) \prod_i dx_i = \prod_{i=1} \sqrt{\frac{\pi}{i a_i}} \exp\left(\frac{i}{4} \sum_i \frac{b_i^2}{a_i}\right)$$

If A is a $N \times N$ diagonal matrix with positive entries $\{a_1, \dots a_N\}$ and X and B are N-vectors with real $\{x_i\}$ and real $\{b_i\}$ entries, then the above formula can be written in matrix forms

$$\int_{-\infty}^{\infty} \exp(-i X^T A X + i B X) \prod_{i=1}^{N} dx_i = \sqrt{\frac{\pi^N}{i \det(A)}} \exp\left(\frac{i}{4} B^T A^{-1} B\right)$$

Now we can write A as $A = O^T S O$ where S

is a positive symmetric matrix. insert it into the formula we have

$$\int_{-\infty}^{\infty} \exp(-iX^T O^T S O X + iB O^T O X) \prod_{i=1}^{N} dX_i$$

$$= \sqrt{\frac{\pi^N}{i\det(S)}} \exp\left(\frac{i}{4} B^T O^T \cdot O A^{-1} O^T O B\right)$$

The Jacobian of the orthogonal transformation is unity so ($Y = OX$ $D = OB$

$$\int_{-\infty}^{\infty} \exp(-iY^T S Y + iD^T Y) \prod_{i=1}^{N} dy_i = \sqrt{\frac{\pi^N}{i\det(S)}} \exp\left(\frac{i}{4} D^T S^{-1} D\right)$$

3. Show that the vector $\bar{Y}$ that makes the argument for the multiple gaussian integral (17.12) stationary is given by (17.13), and that the multiple gaussian integral (17.12) is equal to its exponential evaluated at its stationary point $\bar{Y}$ apart from a prefactor involving the determinant $\det(iS)$

If S is a diagonal matrix R. its easy to verify that the ~~sati~~ stationary point of $(-iX^T R X + iC^T X)$ is $\bar{X} = \frac{1}{2} R^{-1} C$, ~~the~~ under the transformation $Y = OX$ we have $\bar{Y} = O\bar{X} = \frac{1}{2} O R^{-1} C = \frac{1}{2} S^{-1} D$

$\exp(-i\bar{Y}^T S \bar{Y} + iD^T \bar{Y}) = \exp\left(\frac{i}{4} D^T S^{-1} D\right)$ so it is equal to the integral apart from a prefactor involving the determinant $\det(iS)$

4. Repeat the previous problem for the multiple gaussian integral (17.11).

it's easy to get the stationary point of the argument of the exponential $(-Y^T S Y + D^T Y)$

$$\bar{Y} = \frac{1}{2} S^{-1} D$$

So the integral is just the exponential evaluated at its stationary point $\exp(\frac{1}{4} D^T S^{-1} D)$ apart from a prefactor involving determinant $\det(S)$ which is $\sqrt{\frac{\pi^N}{\det(S)}} \exp(\frac{1}{4} D^T S^{-1} D)$

5. Insert a complete set of momentum dyadics $|p\rangle\langle p|$, use the inner product $\langle q|p\rangle = \exp(iqp)/\sqrt{2\pi}$, do the resulting Fourier transform, and so verify the free-particle path integral.

$$\langle \vec{q} | e^{-itH} | \vec{0} \rangle = \iint d\vec{p}' d\vec{p}'' \langle \vec{q} | \vec{p}' \rangle \langle \vec{p}' | e^{-it \cdot \frac{\vec{p}^2}{2m\hbar}} | \vec{p}'' \rangle \langle \vec{p}'' | \vec{0} \rangle$$

$$= \int \frac{d\vec{p}'}{(2\pi\hbar)^3} e^{-\frac{it}{2m\hbar}\vec{p}'^2} \cdot e^{i\vec{q}\cdot\vec{p}'/\hbar}$$

$$= \frac{1}{(2\pi\hbar)^3} \sqrt{\frac{\pi^3}{(it/2m\hbar)^3}} \cdot e^{\frac{1}{4} \cdot \frac{q^2}{\hbar^2} \cdot \frac{2m\hbar}{t}} = \left(\frac{m}{2\pi i\hbar t}\right)^{3/2} e^{imq^2/2\hbar t}$$

6. Show that for the Hamiltonian (17.58) of the simple harmonic oscillator the action $S[q_c]$ of the classical path is (17.65)

$$S[q] = \int_0^t \left[\frac{1}{2} m\dot{q}^2(t') - \frac{1}{2} m\omega^2 q^2(t')\right] dt'$$

and the classical path is $q_c(t) = q' \cos\omega t + \frac{\dot{q}_0}{\omega} \sin\omega t$

where $q' = q_c(0)$ $\dot{q}_0 = \dot{q}_c(t=0)$

insert it into the above integral we can easily get

$$S[q_c] = \int_0^t \frac{1}{2} m \Big\{ (-\omega q' \sin\omega t' + \dot{q}_0 \cos\omega t')^2$$

$$- \frac{1}{2} m\omega^2 \left(q' \cos\omega t' + \frac{\dot{q}_0^2}{\omega} \sin\omega t'\right)^2 \Big\} dt'$$

$$= \frac{m\omega}{2\sin(\omega t)} \left[(q'^2 + q''^2)\cos(\omega t) - 2q'q''\right]$$

7 Show that the harmonic-oscillator action of the loop (17.66) is (17.67).

$$S[\delta q] = \int_0^t dt' \left[\frac{1}{2} m \left(\sum_{n=1}^{\infty} a_n n\pi/t \cos\frac{n\pi t'}{t}\right)^2 - \frac{1}{2} m\omega^2 \left(\sum_{n=1}^{\infty} a_n \sin\frac{n\pi t'}{t}\right)^2\right]$$

Because of the orthoganality of cosines and sines we have

$$S[(\delta q)] = \int_0^t dt' \left[\frac{1}{2} m \sum_{n=1}^{\infty} a_n^2 \frac{n^2\pi^2}{t^2} \cos^2\frac{n\pi t'}{t} - \frac{1}{2} m\omega^2 \sum_{n=0}^{\infty} a_n^2 \sin^2\frac{n\pi t'}{t}\right]$$

$$= \sum_{n=1}^{\infty} \frac{1}{2} m a_n^2 \int_0^t \left(\frac{n^2\pi^2}{t^2} \cos^2\frac{n\pi t'}{t} - \omega^2 \sin^2\frac{n\pi t'}{t}\right) dt'$$

$$= \sum_{n=1}^{\infty} \frac{mt}{4} a_n^2 \left[\left(\frac{n\pi}{t}\right)^2 - \omega^2\right]$$

8. Show that the harmonic-oscillator amplitude (17.70) for $q'=0$ and $q''=q$ reduces as $t\to 0$ to the one-dim version of the free-particle amplitude (17.54)

$$\langle q|e^{-itH/\hbar}|0\rangle = \sqrt{\frac{m\omega}{2\pi i\hbar\sin(\omega t)}}\exp\left[i\frac{m\omega[q^2\cos(\omega t)\cancel{-}0]}{2\hbar\sin\omega t}\right]$$

when $t\to 0$ $\quad \sin\omega t\to\omega t \quad \cos\omega t\to 1$

$$\Rightarrow t\to 0 \quad \langle q|e^{-itH/\hbar}|0\rangle \to \sqrt{\frac{m}{2\pi i\hbar t}}\exp\left(\frac{imq^2}{2\hbar t}\right)$$

which is just 1-d version of the free particle amplitude.

9. Show that the action (17.72) of the stationary solution (17.75) is (17.77)

$$S_e[q] = \int_0^\beta [\tfrac{1}{2}m\dot{q}^2(t)+\tfrac{1}{2}m\omega^2q^2(t)]dt$$

$$= \int_0^\beta \frac{1}{2}m\left[(A\omega e^{\omega t}-B\omega e^{-\omega t})^2+\omega^2(Ae^{\omega t}+Be^{-\omega t})^2\right]dt$$

$$= \frac{1}{2}m\omega^2\int_0^\beta 2(A^2e^{2\omega t}+B^2e^{-2\omega t})dt$$

$$= m\omega^2\left(A^2\cdot\frac{e^{2\omega t}}{2\omega}\Big|_0^\beta - B^2\cdot\frac{e^{-2\omega t}}{2\omega}\Big|_0^\beta\right)$$

$$= \frac{1}{2}m\omega[A^2(e^{2\omega\beta}-1)-B^2(e^{-2\omega\beta}-1)]$$

10. Derive formula (17.130) for the action $S_0[\phi]$ from (17.128 & 17.129).

$$S_0[\phi] = \int \frac{1}{2}[-\partial_a\phi(x)\,\partial^a\phi(x) - m^2\phi(x)]\,d^4x$$

$$= \int \frac{1}{2}\Big[-\int ip_a e^{ip'x}\tilde{\phi}(p')\frac{d^4p'}{(2\pi)^4}\int -ip_a e^{-ip''x}\tilde{\phi}(-p'')\frac{d^4p''}{(2\pi)^4}$$

$$-m^2\iint \frac{d^4p'}{(2\pi)^4}\frac{d^4p''}{(2\pi)^4} e^{i(p'-p'')x}\tilde{\phi}(p')\tilde{\phi}(-p'')\Big]d^4x$$

$$= \int d^4x\, e^{i(p'-p'')x}\int -\frac{1}{2}(p^2+m^2)\tilde{\phi}(p')\tilde{\phi}(-p'')\frac{d^4p'}{(2\pi)^4}\frac{d^4p''}{(2\pi)^4}$$

$$= \delta(p'-p'')\int -\frac{1}{2}(p'+m^2)\tilde{\phi}(p')\tilde{\phi}(-p'')\frac{d^4p'}{(2\pi)^4}\frac{d^4p''}{(2\pi)^4}$$

$$= -\frac{1}{2}\int |\tilde{\phi}(p)|^2(p^2+m^2)\frac{d^4p}{(2\pi)^4}$$

11. Derive identity (17.134). Split the time integral at $t=0$ into two halves, use

$$\varepsilon e^{\pm\varepsilon t} = \pm\frac{d}{dt}e^{\pm\varepsilon t}$$

and then integrate each half by parts.

$$\lim_{\varepsilon\to 0^+}\varepsilon\int_{-\infty}^{\infty} f(t)e^{-\varepsilon|t|}dt$$

$$= \lim_{\varepsilon\to 0^+}\left(\varepsilon\int_{-\infty}^{0} f(t)e^{\varepsilon t}dt + \varepsilon\int_0^{\infty} f(t)e^{-\varepsilon t}dt\right)$$

$$= \lim_{\varepsilon\to 0^+}\left(\int_{-\infty}^{0} f(t)\,de^{\varepsilon t} - \int_{0}^{\infty} f(t)\,de^{-\varepsilon t}\right)$$

$$= \lim_{\varepsilon\to 0^+}\Big(f(t)e^{\varepsilon t}\Big|_{-\infty}^{0} - \int_{-\infty}^{0} e^{\varepsilon t}\,df(t)$$

$$- f(t)e^{-\varepsilon t}\Big|_{0}^{\infty} + \int_{0}^{\infty} e^{-\varepsilon t}\,df(t)\Big)$$

$$= f(\infty) + f(-\infty)$$

Not clear!

12. Derive the third term in eq. (17.136) from its 2nd term.
Donot understand this Problem!

13. Derive eq (17.145) from eqs. (17.142, 17.143 & 17.144)

$$S_0[\phi, \varepsilon, j] = -\frac{1}{2}\int\Big[|\tilde{\phi}(p)|^2(p^2+m^2-i\varepsilon) - \tilde{j}^*(p)\tilde{\phi}(p) - \tilde{\phi}^*(p)\,\tilde{j}(p)\Big]\frac{d^4p}{(2\pi)^4}$$

$$= -\frac{1}{2}\int\Big[\Big(\tilde{\Psi}(p) + \frac{\tilde{j}(p)}{p^2+m^2-i\varepsilon}\Big)^*\Big(\tilde{\Psi}(p) + \frac{\tilde{j}(p)}{p^2+m^2-i\varepsilon}\Big)(p^2+m^2-i\varepsilon)$$

$$- \tilde{j}^*(p)\Big(\tilde{\Psi}(p) + \frac{\tilde{j}(p)}{p^2+m^2-i\varepsilon}\Big) - \Big(\tilde{\Psi}^*(p) + \frac{\tilde{j}^*(p)}{p^2+m^2+i\varepsilon}\Big)$$

$$\Theta \cdot \tilde{j}(p)\Big]\frac{d^4p}{(2\pi)^4}$$

$$= -\frac{1}{2}\int \left[|\tilde{\psi}(p)|^2 (p^2+m^2-i\varepsilon) - \frac{\tilde{j}^*(p)\tilde{j}(p)}{p^2+m^2-i\varepsilon} \right] \frac{d^4p}{(2\pi)^4}$$

$$= S_0[\psi, \varepsilon] + \frac{1}{2}\int \frac{\tilde{j}^*(p)\tilde{j}(p)}{p^2+m^2-i\varepsilon} \frac{d^4p}{(2\pi)^4}$$

14. Derive the formula (17.146) for $Z_0[j]$ from the expression (17.145) for $S_0[\phi, \varepsilon, j]$.

$$Z_0[j] = \frac{\int \exp[i\int j(x)\phi(x)\, d^4x]\, e^{iS_0[\phi,\varepsilon]}\, D\phi}{\int e^{iS_0[\phi,\varepsilon]}\, D\phi}$$

$$= \frac{\int e^{iS_0[\phi,\varepsilon,j]}\, D\phi}{\int e^{iS_0[\phi,\varepsilon]}\, D\phi}$$

$$= \frac{\int e^{iS_0[\psi,\varepsilon]}\, D\psi \cdot e^{\frac{i}{2}\int \frac{\tilde{j}^*(p)\tilde{j}(p)}{p^2+m^2-i\varepsilon}\frac{d^4p}{(2\pi)^4}}}{\int e^{iS_0[\psi,\varepsilon]}\, D\psi}$$

$$= \exp\left[\frac{i}{2}\int |\tilde{j}(p)|^2 \cdot \frac{1}{p^2+m^2-i\varepsilon} \frac{d^4p}{(2\pi)^4}\right.$$

15. Derive equations (17.147 & 17.148) from formula (17.146)

$$\tilde{j}(p) = \int e^{-ipx} j(x)\, d^4x \qquad \tilde{j}^*(p) = \tilde{j}(-p) = \int e^{ipx'} j(x')\, d^4x'$$

put it into equation (17.146) we can get

$$Z_0[j] = \exp\left[\frac{i}{2}\int \frac{\iint d^4x\, d^4x'\, e^{ip(x-x')} j(x) j(x')}{p^2+m^2-i\varepsilon} \frac{d^4p}{(2\pi)^4}\right.$$

$$= \exp[\frac{i}{2}\int j(x)\,\Delta(x-x')\,j(x')\,d^4x\,d^4x'$$

Propogator

where $\Delta(x-x') = \int \frac{e^{ip(x-x')}}{p^2+m^2-i\varepsilon}\frac{d^4p}{(2\pi)^4}$ is Feynman

16. Derive equation (~~17.179~~ 17.152) from the formula (17.147) for $Z_0[j]$

$$Z_0[j] = \exp[\frac{i}{2}\int j(x)\,\Delta(x-x')\,j(x')\,d^4(x)\,d^4x'$$

so we have

$$\frac{1}{i^4}\frac{\delta^4 Z_0[j]}{\delta j(x_1)\,\delta j(x_2)\,\delta j(x_3)\,\delta j(x_4)}\Big|_{j=0}$$

$$= \Big\{ Z_0[j]\int \Delta(x_4-x')\,\Delta(x_3-x'')\,\Delta(x_2-x''')\,\Delta(x_1-x'''')\,j(x')\,j(x'')\,j(x'''),j(x'''')\,d^4x'\,d^4x''\,d^4x'''\,d^4x''''$$

$$+\frac{1}{i}Z_0(j)\,\Delta(x_3-x_4)\int\Delta(x_2-x')\,\Delta(x_1-x'')\,j(x')\,j(x'')\,d^4x'\,d^4x''$$

$$+\frac{1}{i}Z_0(j)\,\Delta(x_2-x_4)\int\Delta(x_3-x')\,\Delta(x_1-x'')\,j(x')\,j(x'')\,d^4x'\,d^4x''$$

$$+\frac{1}{i}Z_0(j)\,\Delta(x_1-x_4)\int\Delta(x_3-x')\,\Delta(x_2-x'')\,j(x')\,j(x'')\,d^4x'\,d^4x''$$

$$+\frac{1}{i}Z_0[j]\,\Delta(x_2-x_3)\int\Delta(x_4-x')\,\Delta(x_1-x'')\,j(x')\,j(x'')\,d^4x'\,d^4x''$$

$$+ i^2 Z_0[j]\,\Delta(x_2-x_3)\,\Delta(x_1-x_4) + \frac{1}{i}Z_0[j]\,\Delta(x_1-x_3)\int\Delta(x_2-x')\,\Delta(x_4-x'')\,j(x')\,j(x'')\,d^4x'\,d^4x'' + i^2 Z_0[j]\,\Delta(x_1-x_3)\,\Delta(x_2-x_4))$$

$$+ \frac{1}{i^2} Z_0[j]\, \Delta(x_3-x_4)\Delta(x_1-x_2) + \frac{1}{i} Z_0[j]\, \Delta(x_1-x_2) \int \Delta(x_3-x')\Delta(x_4-x'')\, j(x')\, j(x'')\, d^4x'\, d^4x'' \Big\} \Big|_{j=0}$$

$$= -\Delta(x_1-x_4)\Delta(x_2-x_3) - \Delta(x_2-x_4)\Delta(x_1-x_3) - \Delta(x_1-x_2)\Delta(x_3-x_4)$$

17. Show that the ~~limit~~ time integral of the coulomb term (17.157) is the negative of the term that is quadratic in j^0 in the number F defined by (17.162).

18. By following steps analogous to those that lead to (17.148), derive the formula (17.175) for the photon propagator in Feynman's Gauge.

19. Derive expression (17.190) for the inner product $\langle\zeta|\theta\rangle$

$$\langle\zeta|\theta\rangle = \langle 0|(1+\zeta^*\psi - \tfrac{1}{2}\zeta^*\zeta)(1+\psi^\dagger\theta - \tfrac{1}{2}\theta^*\theta)|0\rangle$$

$$= \langle 0|(1+\psi^\dagger\theta - \tfrac{1}{2}\theta^*\theta + \cancel{\zeta^*\zeta}\,\zeta^*\psi + \zeta^*\psi\psi^\dagger\theta - \tfrac{1}{2}\zeta^*\psi\theta^*\theta$$
$$- \tfrac{1}{2}\zeta^*\zeta - \tfrac{1}{2}\zeta^*\zeta\psi^\dagger\theta + \tfrac{1}{4}\zeta^*\zeta\theta^*\theta|0\rangle$$
$$= \langle 0|(1+\zeta^*\psi\psi^\dagger\theta - \tfrac{1}{2}\zeta^*\zeta - \tfrac{1}{2}\theta^*\theta + \tfrac{1}{4}\zeta^*\zeta\theta^*\theta)|0\rangle$$
$$= \langle 0|(1+\zeta^*\theta - \zeta^*\psi^\dagger\psi\theta - \tfrac{1}{2}\zeta^*\zeta - \tfrac{1}{2}\theta^*\theta + \tfrac{1}{4}\zeta^*\zeta\theta^*\theta)|0\rangle$$
$$= \langle 0|(1+\zeta^*\theta - \tfrac{1}{2}\zeta^*\zeta - \tfrac{1}{2}\theta^*\theta + \tfrac{1}{4}\zeta^*\zeta\theta^*\theta)|0\rangle$$
$$= \exp[\zeta^*\theta - \tfrac{1}{2}(\zeta^*\zeta + \theta^*\theta)]$$

20. Derive the representation (17.193) of the identity operator I for a single fermionic degree of freedom from the rules (17.180 & 17.183) for Grassmann integration and the anti-commutation relations (17.176 & 17.182)

$$|\theta\rangle\langle\theta| = (1+\psi^\dagger\theta - \tfrac{1}{2}\theta^*\theta)|0\rangle\langle 0|(1+\theta^*\psi - \tfrac{1}{2}\theta^*\theta)$$
$$= |0\rangle\langle 0|(1+\theta^*\psi - \tfrac{1}{2}\theta^*\theta + \psi^\dagger\theta + \psi^\dagger\theta\theta^*\psi - \tfrac{1}{2}\psi^\dagger\theta\theta^*\theta$$
$$- \tfrac{1}{2}\theta^*\theta - \tfrac{1}{2}\theta^*\theta\theta^*\psi + \tfrac{1}{4}\theta^*\theta\theta^*\theta)$$
$$= |0\rangle\langle 0|(1+\theta^*\psi - \theta^*\theta + \psi^\dagger\theta + \psi^\dagger\theta\theta^*\psi)$$

$$\int (1+\theta^*\psi - \theta^*\theta + \psi^\dagger\theta + \psi^\dagger\theta\theta^*\psi)\,d\theta^* d\theta$$
$$= \int (1 - \psi\theta^* + \theta\theta^* + \psi^\dagger\theta + \psi^\dagger\psi\theta\theta^*)\,d\theta^* d\theta$$
$$= 1 + \psi^\dagger\psi \Rightarrow \int |\theta\rangle\langle\theta|\,d\theta^* d\theta = |0\rangle\langle 0|(1+\psi^\dagger\psi)$$
$$= |0\rangle\langle 0| + |1\rangle\langle 1|$$

21. Derive the eigenvalue equation (17.198) from the definition (17.196) & (17.197) of the eigenstate $|\theta\rangle$ and the anti-commutation relations (17.194 & 17.195).

It is easy to verify the following commutation relations.

$$[\psi_k, 1+\psi_j^{\dagger}\theta_j - \tfrac{1}{2}\theta_j^{*}\theta_j] = \delta_{kj} = [\theta_k, 1+\psi_j^{*}\theta_j - \tfrac{1}{2}\theta_j^{*}\theta_j]$$

$$[1+\psi_k^{\dagger}\theta_k - \tfrac{1}{2}\theta_k^{*}\theta_k, 1+\psi_j^{\dagger}\theta_j - \tfrac{1}{2}\theta_j^{*}\theta_j] = \delta_{kj}$$

Σ

so we can get

$$\psi_k|\theta\rangle = \psi_k \prod_{j=1}^{n}(1+\psi_j^{\dagger}\theta_j - \tfrac{1}{2}\theta_j^{*}\theta_j)|0\rangle$$

$$= \psi_k \prod_{j=1}^{k-1}(1+\psi_j^{\dagger}\theta_j - \tfrac{1}{2}\theta_j^{*}\theta_j)\prod_{j=k+1}^{n}(1+\psi_j^{\dagger}\theta_j - \tfrac{1}{2}\theta_j^{*}\theta_j)$$

$$(1+\psi_k^{\dagger}\theta_k - \tfrac{1}{2}\theta_k^{*}\theta_k)|0\rangle$$

$$= \prod_{j=1}^{k-1}(1+\psi_j^{\dagger}\theta_j - \tfrac{1}{2}\theta_j^{*}\theta_j)\prod_{j=k+1}^{n}(1+\psi_j^{\dagger}\theta_j - \tfrac{1}{2}\theta_j^{*}\theta_j)$$

$$\cdot\, \psi_k(1+\psi_k^{\dagger}\theta_k - \tfrac{1}{2}\theta_k^{*}\theta_k)|0\rangle$$

according to (17.186) we have

$$\psi_k(1+\psi_k^{\dagger}\theta_k - \tfrac{1}{2}\theta_k^{*}\theta_k)|0\rangle = \psi_k|\theta_k\rangle = \theta_k|0\rangle$$

$$= \theta_k|\theta_k\rangle$$

$$= \theta_k(1+\psi_k^{\dagger}\theta_k - \tfrac{1}{2}\theta_k^{*}\theta_k)|0\rangle$$

put it into the above formula we can get

$$\psi_k|\theta\rangle = \prod_{j=1}^{k-1}(1+\psi_j^+\theta_j - \tfrac{1}{2}\theta_j^*\theta_j)\prod_{j=k+1}^{n}(1+\psi_j^+\theta_j - \tfrac{1}{2}\theta_j^*\theta_j)\theta_k(1+\psi_k^+\theta_k - \tfrac{1}{2}\theta_k^*\theta_k)|0\rangle$$

$$= \theta_k|\theta\rangle$$

22. Derive the eigen value relation (17.211) for the Fermi Field $\psi_m(\vec{x},t)$ from the anti-commutation relations (17.207 & 17.208) and the definitions (17.209 & 17.210).

following the same way as Problem 21. we can easily verify the following commutation relations

$$[\psi_m(\vec{x},0), \exp\{\int[\psi_n^+(\vec{x}',0)\chi_n(\vec{x}') - \tfrac{1}{2}\chi_n^*(\vec{x}')\chi_n(\vec{x}')]d^3x'\}] = \delta_{mn}\delta(\vec{x}-\vec{x}')$$

$$= [\chi_m(\vec{x}), \exp\{\int[\psi_n^+(\vec{x},0)\chi_n(\vec{x}) - \tfrac{1}{2}\chi_n^*(\vec{x})\chi_n(\vec{x})\, d^3x]\}]$$

$$[\exp\{\int[\psi_n^+(\vec{x},0)\chi_n(\vec{x}) - \tfrac{1}{2}\chi_n^*(\vec{x})\chi_n(\vec{x})]d^3x\}, \exp\{\int[\psi_m^+(\vec{x}',0)\chi_m(\vec{x}') - \tfrac{1}{2}\chi_m^*(\vec{x}')\chi_m(\vec{x}')]d^3x'\}] = \delta_{mn}\delta(\vec{x}-\vec{x}')$$

so using this commutation relations we can easily get

$$\psi_m(\vec{x},0)|\chi\rangle = \chi_m(\vec{x})|\chi\rangle$$

23. Derive the formula (17.212) for the inner producto from

the definition (17.210) of the ket $|\chi\rangle$.

$$\langle\chi'|\chi\rangle = \langle 0|\left[1+\int(\chi'^{\dagger}\psi-\tfrac{1}{2}\chi'^{\dagger}\chi')d^3x'\right]\left[1+\int(\psi^{\dagger}\chi-\tfrac{1}{2}\chi^{\dagger}\chi)d^3x\right]|0\rangle$$

$$=\langle 0|\ 1+\int(\chi'^{\dagger}\psi-\tfrac{1}{2}\chi'^{\dagger}\chi')d^3x'+\int(\chi^{\dagger}\psi-\tfrac{1}{2}\chi^{\dagger}\chi)d^3x$$

$$+\iint d^3x\,d^3x'\,(\chi'^{\dagger}\psi-\tfrac{1}{2}\chi'^{\dagger}\chi')(\psi^{\dagger}\chi-\tfrac{1}{2}\chi^{\dagger}\chi)|0\rangle$$

$$=\langle 0|\ 1-\tfrac{1}{2}\int(\chi'^{\dagger}\chi'+\chi^{\dagger}\chi)\,d^3x+\iint d^3x\,d^3x'\,(\chi'^{\dagger}\psi\psi^{\dagger}\chi+\tfrac{1}{4}\chi'^{\dagger}\chi'\chi^{\dagger}\chi)|0\rangle$$

$$=\langle 0|\ 1+\int\chi'^{\dagger}\chi\,d^3x-\tfrac{1}{2}\int(\chi'^{\dagger}\chi'+\chi^{\dagger}\chi)d^3x)|0\rangle$$

$$=\exp\left[\int(\chi'^{\dagger}\chi-\tfrac{1}{2}\chi'^{\dagger}\chi'-\tfrac{1}{2}\chi^{\dagger}\chi)\,d^3x\right]$$