

1 Path Integral

1.

$$|0\rangle = \left(\prod_m \psi_m(\mathbf{x}, 0) \right) |s\rangle$$

$$\begin{aligned} |\chi\rangle &= \exp \left[\int \sum_m \psi_m^\dagger(\mathbf{x}, 0) \chi_m(\mathbf{x}) - \frac{1}{2} \chi_m^*(\mathbf{x}) \chi_m(\mathbf{x}) d^3x \right] |0\rangle \\ &= \exp \left[\int \left(\psi^\dagger \chi - \frac{1}{2} \chi^\dagger \chi \right) d^3x \right] |0\rangle \end{aligned}$$

$$\psi_m(\mathbf{x}, 0) |\chi\rangle = \exp \left[\int \sum_{i \neq m} \left(\psi_i^\dagger(\mathbf{x}, 0) \chi_m(\mathbf{x}) - \frac{1}{2} \chi_m^*(\mathbf{x}) \chi_m(\mathbf{x}) \right) d^3x \right] |0\rangle$$

$$\begin{aligned} &\psi_m(\mathbf{x}, 0) \left[\int \left(1 + \psi_m^\dagger(\mathbf{x}, 0) \chi_m(\mathbf{x}) - \frac{1}{2} \chi_m^*(\mathbf{x}) \chi_m(\mathbf{x}) \right) d^3x \right] |0\rangle \\ &= \exp \left[\int \sum_{i \neq m} \left(\psi_i^\dagger(\mathbf{x}, 0) \chi_i(\mathbf{x}) - \frac{1}{2} \chi_i^*(\mathbf{x}) \chi_i(\mathbf{x}) \right) d^3x \right] \end{aligned}$$

$$\begin{aligned} &(1 - \psi_m \psi_m^\dagger) \chi_m(\mathbf{x}) |0\rangle \\ &= \chi_m(\mathbf{x}) \exp \left[\int \sum_m \left(\psi_m^\dagger(\mathbf{x}, 0) \chi_m(\mathbf{x}) - \frac{1}{2} \chi_m^*(\mathbf{x}) \chi_m(\mathbf{x}) \right) d^3x \right] |0\rangle \\ &= \chi_m(\mathbf{x}) |\chi\rangle \end{aligned}$$

2.

$$|\chi\rangle = \exp \left[\int \left(\psi^\dagger \chi - \frac{1}{2} \chi^\dagger \chi \right) d^3x \right] |0\rangle$$

$$\langle \chi | = \langle 0 | \left[\exp \int \left(\chi^\dagger \psi - \frac{1}{2} \chi^\dagger \chi \right) d^3x \right]$$

$$\begin{aligned} \langle \chi | \chi \rangle &= \langle 0 | \exp \left(\int (\psi^\dagger \chi + \chi'^\dagger \psi - 1/2 \chi^\dagger \chi - 1/2 \chi'^\dagger \chi') d^3x \right) |0\rangle \\ &= \langle 0 | \int \prod_m \left(1 + \psi_m^\dagger \chi_m + \chi_m'^\dagger \psi_m - \frac{1}{2} \chi_m^\dagger \chi_m - \frac{1}{2} \chi_m'^\dagger \chi_m' + \psi_m^\dagger \chi_m \chi_m'^\dagger \psi_m \right) d^3x |0\rangle \\ &= \langle 0 | \int \prod_m \left(1 + \chi_m'^\dagger \chi_m - \frac{1}{2} \chi_m^\dagger \chi_m - \frac{1}{2} \chi_m'^\dagger \chi_m' \right) d^3x |0\rangle \\ &= \exp \left[\int \left(\chi'^\dagger \chi - \frac{1}{2} \chi'^\dagger \chi' - \frac{1}{2} \chi^\dagger \chi \right) d^3x \right] \end{aligned}$$