1 Path Integral

1.

$$
\begin{aligned}
& \int_{-\infty}^{\infty} \exp\left(\sum_i -r_i x_i^2 + c_i x_i\right) \prod_{i=1}^{N} dx_i \\
= & \prod_i \int_{-\infty}^{\infty} \exp\left(-r_i x_i^2 + c_i x_i\right) dx_i \\
= & \int_{-\infty}^{\infty} \prod_i \left(\exp\left(-r_i (x_i - \frac{c_i}{2r_i})^2\right) \exp\left(\frac{c_i^2}{4r_i}\right) dx_i\right) \\
= & \prod_{i=1}^{N} \sqrt{\frac{\pi}{r_i}} \exp\left(\frac{1}{4} \sum_i \frac{c_i^2}{r_i}\right)
\end{aligned}
$$

2. The matrix form of

$$
\begin{aligned}
& \int_{-\infty}^{\infty} \exp\left(\sum_i (-i a_i x_i^2 + i b_i x_i) \prod_i dx_i\right) \\
= & \prod_{i=1}^{N} \sqrt{\frac{\pi}{i a_i}} \exp\left(\frac{i}{4} \sum_i \frac{b_i^2}{a_i}\right)
\end{aligned}
$$

is

$$
\int_{-\infty}^{\infty} \exp\left(-i X^T A X + i B X\right) \prod_{i=1}^{N} dx_i = \sqrt{\frac{\pi^N}{\det(iA)}} \exp\left(\frac{i}{4} B^T A^{-1} B\right)
$$

We know that $A = O^T S O$,

$$
\begin{aligned}
& \int_{-\infty}^{\infty} \exp\left(-i X^T O^T S O X + i B O^T O X\right) \prod_{i=1}^{N} dx_i \\
= & \sqrt{\frac{\pi^N}{\det(S)}} \exp\left(\frac{i}{4} B^T O^T O A^{-1} O^T O B\right)
\end{aligned}
$$

Since $Y = OX$, $D = OB$,

$$
\int_{-\infty}^{\infty} \exp\left(-i Y^T S Y + i D^T Y\right) \prod_{i=1}^{N} dy_i = \sqrt{\frac{\pi^N}{\det(S)}} \exp\left(\frac{i}{4} D^T S^{-1} D\right)
$$

3. Denote

$$
I = \int_{-\infty}^{\infty} \exp\left(-i Y^T S Y + i D^T Y\right) \prod_{i}^{N} dy_i
$$

Variation of I

$$\delta I = \int_{-\infty}^{\infty} \left[\exp\left(-i(Y+\delta Y)^T S(Y+\delta Y) + iD^T(Y+\delta Y)\right)\right.$$
$$\left. - \exp\left(-iY^T SY + iD^T Y\right)\right] \prod_i^N dy_i$$
$$= \int_{-\infty}^{\infty} \exp\left(-iY^T SY + iD^T Y\right) \left[\exp\left(-i(2Y^T S - D^T)\delta Y\right) - 1\right] \prod_i^N dy_i$$

To make I stationary, $\delta I = 0$, that is

$$2\bar{Y}^T D - D^T = 0$$

Then we have

$$\bar{Y} = \frac{1}{2} S^{-1} D$$

At its stationary point, the integrand of I becomes

$$\exp\left[-i\left(\frac{1}{2}S^{-1}D\right)^T S\left(\frac{1}{2}S^{-1}D\right) + iD\left(\frac{1}{2}S^{-1}D\right)\right]$$
$$= \exp\left(\frac{1}{4} i D^T S^{-1} D\right)$$

which is different from the integral result of I with a prefactor $\sqrt{\pi^N / \det(iS)}$.

4. Denote

$$I = \int_{-\infty}^{\infty} \exp\left(-Y^T SY + D^T Y\right) \prod_i^N dy_i$$

Then

$$\delta I = \int_{-\infty}^{\infty} \left[\exp\left(-(Y+\delta Y)^T S(Y+\delta Y) + D^T(Y+\delta Y)\right)\right.$$
$$\left. - \exp\left(-Y^T SY + D^T Y\right)\right] \prod_i^N dy_i$$
$$= \int_{-\infty}^{\infty} \exp\left(-Y^T SY + D^T Y\right) \left[\exp\left(-(2Y^T S - D^T)\delta Y\right) - 1\right] \prod_i^N dy_i$$

To make I stationary,

$$2\bar{Y}^T S - D^T = 0.$$

i.e.,

$$\bar{Y} = \frac{1}{2}S^{-1}D$$

Put $\bar{Y} = \frac{1}{2}S^{-1}D$ into the integrand of I, we get

$$\begin{aligned}
&\exp\left[-\left(\frac{1}{2}S^{-1}D\right)^T S\frac{1}{2}S^{-1}D + D^T\frac{1}{2}S^{-1}D\right] \\
=\ &\exp\left(-\frac{1}{4}D^TS^{-1}SS^{-1}D + \frac{1}{2}D^TS^{-1}D\right) \\
=\ &\exp\left(\frac{1}{4}D^TS^{-1}D\right)
\end{aligned}$$

which is the same as the integral result of I apart from a prefactor.

5.

$$\begin{aligned}
\langle q|e^{-itH}\rangle &= \int\int dp'dp''\langle q|p'\rangle\langle p'|e^{-ip^2/(2m\hbar)t}|p''\rangle\langle p''|0\rangle \\
&= \int dp'\frac{1}{(2\hbar\pi)^3}e^{-\frac{-it}{2m\hbar}p'^2}e^{iqp/\hbar} \\
&= \frac{1}{(2\pi\hbar)^3}\sqrt{\frac{\pi^3}{(it/(2m\hbar))}}e^{2mq^2/(2\hbar t)} \\
&= \left(\frac{m}{2\pi i\hbar t}\right)^{3/2}e^{imq^2/(2\hbar t)}
\end{aligned}$$

6.

7.

$$\begin{aligned}
S[q] &= \int_0^t\left(\frac{1}{2}m\dot{q}^2 - \frac{1}{2}m\omega^2q^2\right)dt' \\
&= \int_0^t\frac{1}{2}m\left((-\omega q'\sin\omega t' + \dot{q}_0\cos\omega t')^2 - \omega^2(q'\cos\omega t' + \frac{\dot{q}^2}{\omega}\sin\omega t')^2\right)dt' \\
&= \frac{m\omega}{2\sin(\omega t)}\left((q'^2+q''^2)\cos\omega t - 2q'q''\right)
\end{aligned}$$

8.

$$
\begin{aligned}
S[\delta q] &= \int_0^t dt' \left(\frac{1}{2}m\left(\sum_{n=1} a_n n\pi/t \cos\frac{n\pi t'}{t}\right)^2 - \frac{1}{2}m\omega^2\left(\sum_{n=1} a_n \sin\frac{n\pi t'}{t}\right)^2\right) \\
&= \int_0^t dt' \left(\frac{1}{2}m\sum_{n=1} a_n^2 \frac{n^2\pi^2}{t^2}\cos^2\frac{n\pi t'}{t} - \frac{1}{2}m\omega^2\sum_{n=0} a_n^2 \sin^2\frac{n\pi t'}{t}\right) \\
&= \sum_{n=1}\frac{1}{2}ma_n^2\int_0^t dt'\left(\frac{n^2\pi^2}{t^2}\cos^2\frac{n\pi t'}{t} - \omega^2\sin^2\frac{n\pi t'}{t}\right) \\
&= \sum_{n=1}\frac{mt}{4}a_n^2\left(n^2\pi^2/t^2 - \omega^2\right)
\end{aligned}
$$

9. When $q' = 0$ and $q'' = q$, it becomes

$$
\langle q|e^{-itH/\hbar}|0\rangle = \sqrt{\frac{m\omega}{2\pi i\hbar\sin\omega t}}\exp\left[i\frac{m\omega[q^2\cos\omega t]}{2\hbar\sin\omega t}\right]
$$

In the limit of $t \to 0$, the trigonometric functions used in our calculation becomes $\sin\omega t \to \omega t$ and $\cos\omega t \to 1$.

$$
\lim_{t\to 0}\langle q|e^{-itH/\hbar}|0\rangle = \sqrt{\frac{m}{2\pi i\hbar t}}\exp\left(\frac{imq^2}{2\hbar t}\right)
$$

10.

11.

$$
\begin{aligned}
S_e[q] &= \int_0^\beta \left[\frac{1}{2}m\dot{q}^2 + \frac{1}{2}m\omega^2q^2\right]dt \\
&= \int_0^\beta \frac{1}{2}m\left[(A\omega e^{\omega t} - B\omega e^{-\omega t})^2 + \omega(Ae^{\omega t} + Be^{-\omega t})^2\right] \\
&= \frac{1}{2}m\omega^2\int_0^\beta 2(A^2e^{2\omega t} + B^2e^{-2\omega t})dt \\
&= m\omega^2\left[A^2(e^{2\omega t} - 1) - B^2(e^{-2\omega t} - 1)\right]
\end{aligned}
$$

12.

$$
\begin{aligned}
S_0[\phi] &= \int \frac{1}{2}[-\partial_a\phi(x)\partial^a\phi(x) - m^2\phi(x)]d^4x \\
&= \int \frac{1}{2}\left[-\int ip_a e^{ip'x}\tilde{\phi}(p')\frac{1}{(2\pi)^4}d^4p' \int \left(-ip_a e^{-ip''x}\tilde{\phi}(-p'')\right)\frac{d^4p''}{(2\pi)^4}\right. \\
&\quad \left. -m^2 \int\int \frac{dp'}{(2\pi)^4}\frac{dp''}{(2\pi)^4}e^{ip'-p''}x\tilde{\phi}(p')\tilde{\phi}(-p'')\right] d^4x \\
&= -\int d^4x e^{i(p'-p'')x}\int \frac{1}{2}(p^2+m^2)\tilde{\phi}(p')\tilde{\phi}(-p'')\frac{d^4p'}{(2\pi)^4}\frac{d^4p''}{(2\pi)^4} \\
&= -\delta(p'-p'')\int \frac{1}{2}(p^2+m^2)\tilde{\phi}(p')\tilde{\phi}(-p'')\frac{d^4p'}{(2\pi)^4}\frac{d^4p''}{(2\pi)^4} \\
&= -\frac{1}{2}\int |\tilde{\phi}(p)|^2(p^2+m^2)\frac{d^4p}{(2\pi)^4}
\end{aligned}
$$

13.

$$
\begin{aligned}
&\lim_{\epsilon\to 0+}\epsilon\int_{-\infty}^{\infty} e^{-\epsilon|t|}dt \\
&= \lim_{\epsilon\to 0+}\left(\epsilon\int_{-\infty}^{0} f(t)e^{\epsilon t}dt + \epsilon\int_{0}^{\infty} f(t)e^{-\epsilon t}dt\right) \\
&= \lim_{\epsilon\to 0+}\left(\int_{-\infty}^{0} f(t)de^{\epsilon t} - \int_{0}^{\infty} f(t)de^{-\epsilon t}\right) \\
&= \lim_{\epsilon\to 0+}\left(f(t)e^{\epsilon t}|_{-\infty}^{0} - \int_{-\infty}^{0} e^{\epsilon t}df(t) - f(t)e^{-\epsilon t}|_{0}^{\infty} + \int_{0}^{\infty} e^{-\epsilon t}df(t)\right) \\
&= \lim_{\epsilon\to 0}\left(2f(0) + \int_{0}^{\infty} e^{-\epsilon t}df(t) - \int_{-\infty}^{0} e^{\epsilon t}df(t)\right) \\
&= 2f(0) + f(\infty) - f(0) - f(0) + f(-\infty) \\
&= f(\infty) + f(-\infty)
\end{aligned}
$$

14. Check this problem.

Fourier transform of $\phi(\vec{x},t)$ and $\phi(p)$ are

$$
\tilde{\phi}(\vec{p},t) = \int e^{-i\vec{p}\cdot\vec{x}}\phi(\vec{x},t)d^3x
$$

$$
\phi(\vec{x},t) = \int e^{i\vec{p}\cdot\vec{x}}e^{-ip_0t}\phi(p')\frac{d^4p'}{(2\pi)^4}.
$$

Then we have

$$
\begin{aligned}
\tilde{\phi}(\vec{p},t) &= \iint e^{-i\vec{p}\cdot\vec{x}}e^{-i\vec{p}'\cdot\vec{x}}e^{-ip_0t}\phi(p')\frac{d^4}{(2\pi)^4}d^3x \\
&= \iint e^{-i(\vec{p}-\vec{p}')\cdot\vec{x}}e^{-ip_0t}\phi(p')\frac{d^4p'}{(2\pi)^4}d^3x.
\end{aligned}
$$

15.

$$\begin{aligned}
S_0[\phi,\epsilon,j] =& -\frac{1}{2}\int\left[|\tilde{\phi}(p)|^2(p^2+m^2-i\epsilon)-\tilde{j}^*(p)\tilde{\phi}(p)-\tilde{\phi}^*(p)\tilde{j}(p)\right]\frac{d^4p}{(2\pi)^4}\\
=& -\frac{1}{2}\int\left[\left(\tilde{\psi}(p)+\frac{\tilde{j}(p)}{p^2+m^2-i\epsilon}\right)^*\left(\tilde{\psi}(p)+\frac{\tilde{j}(p)}{p^2+m^2-i\epsilon}\right)(p^2+m^2-i\epsilon)\right.\\
&\left.-\tilde{j}^*(p)\left(\tilde{\psi}(p)+\frac{\tilde{j}(p)}{p^2+m^2-i\epsilon}\right)-\left(\tilde{\psi}^*(p)+\frac{\tilde{j}^*(p)}{p^2+m^2+i\epsilon}\right)\tilde{j}(p)\right]\frac{d^4}{(2\pi)^4}\\
=& -\frac{1}{2}\int\left[|\tilde{\psi}(p)|^2(p^2+m^2-i\epsilon)-\frac{\tilde{j}^*(p)\tilde{j}(p)}{p^2+m^2-i\epsilon}\right]\frac{d^4p}{(2\pi)^4}\\
=& S_0[\psi,\epsilon]+\frac{1}{2}\int\frac{\tilde{j}^*(p)\tilde{j}(p)}{p^2+m^2-i\epsilon}\frac{d^4p}{(2\pi)^4}
\end{aligned}$$

16.

$$\begin{aligned}
Z_0[j] &= \frac{\int\exp\left[i\int j(x)\phi(x)d^4x\right]e^{iS_0[\phi,\epsilon]}D\phi}{\int e^{iS_0[\phi,\epsilon]}D\phi}\\
&= \frac{\int e^{iS_0[\phi,\epsilon,j]}D\phi}{\int e^{iS_0[\phi,\epsilon]}D\phi}\\
&= \frac{\int e^{iS_0[\psi,\epsilon]}D\psi\cdot e^{\frac{i}{2}\int\frac{\tilde{j}^*(p)\tilde{j}(p)}{p^2+m^2-i\epsilon}\frac{d^4p}{(2\pi)^4}}}{\int e^{iS_0[\psi,\epsilon]}D\psi}\\
&= \exp\left[\frac{i}{2}\int\frac{|\tilde{j}(p)|^2}{p^2+m^2-i\epsilon}\frac{d^4p}{(2\pi)^4}\right]
\end{aligned}$$

17. Applying

$$\begin{aligned}
\tilde{j}(p) &= \int e^{-ipx}j(x)d^4x\\
\tilde{j}^*(p) &= \int e^{ipx'}j(x')d^4x'
\end{aligned}$$

to $Z_0[j]$, we get

$$\begin{aligned}
Z_0[j] &= \exp\left[\frac{i}{2}\int\frac{\iint d^4xd^4x'e^{ip(x-x')}j(x)j(x')}{p^2+m^2-i\epsilon}\frac{d^4p}{(2\pi)^4}\right]\\
&= \exp\left[\frac{i}{2}\int j(x)\Delta(x-x')j(x')d^4xd^4x'\right],
\end{aligned}$$

in which $\Delta(x-x')$ is the Feynmann's propagrator.

18.

$$
\begin{aligned}
&\frac{1}{i^4}\frac{\delta^4 Z_0[j]}{\delta_j(x_1)\delta_j(x_2)\delta_j(x_3)\delta_j(x_4)}\bigg|_{j=0} \\
=\ &\bigg(Z_0[j]\int d^4x' d^4x'' d^4x''' d^4x'''' \Delta(x_4-x')\Delta(x_3-x'')\Delta(x_2-x''')\Delta(x_1-x'''') \\
&j(x')j(x'')j(x''')j(x'''') \\
&+\frac{1}{i}Z_0[j]\Delta(x_3-x_4)\int d^4x'd^4x''\Delta(x_1-x')\Delta(x_1-x'')j(x')j(x'') \\
&+\frac{1}{i}Z_0[j]\Delta(x_2-x_4)\int d^4x'd^4x''\Delta(x_3-x')\Delta(x_1-x'')j(x')j(x'') \\
&+\frac{1}{i}Z_0[j]\Delta(x_1-x_4)\int d^4x'd^4x''\Delta(x_3-x')\Delta(x_2-x'')j(x')j(x'') \\
&+\frac{1}{i}Z_0[j]\Delta(x_2-x_3)\int d^4x'd^4x''\Delta(x_3-x')\Delta(x_2-x'')j(x')j(x'') \\
&+\frac{1}{i}Z_0[j]\Delta(x_1-x_3)\int d^4x'd^4x''\Delta(x_2-x')\Delta(x_4-x'')j(x')j(x'') \\
&+i^2Z_0[j]\Delta(x_2-x_3)\Delta(x_1-x_4)+i^2Z_0[j]\Delta(x_1-x_3)\Delta(x_2-x_4) \\
&+\frac{1}{i}Z_0[j]\Delta(x_1-x_2)\int d^4x'd^4x''\Delta(x_3-x')\Delta(x_4-x'')j(x')j(x'') \\
&\frac{1}{i^2}Z_0[j]\Delta(x_3-x_4)\Delta(x_1-x_2)\bigg)\bigg|_{j=0} \\
=\ &-\Delta(x_1-x_4)\Delta(x_2-x_3)-\Delta(x_2-x_4)\Delta(x_1-x_3)-\Delta(x_1-x_2)\Delta(x_3-x_4)
\end{aligned}
$$

19.

$$
\begin{aligned}
&\frac{1}{2}\int\left(\boldsymbol{\nabla}\Delta^{-1}j^0\right)^2 d^4x \\
=\ &\frac{1}{2}\int\left[\boldsymbol{\nabla}\left(-\int\frac{1}{4\pi}\frac{j^0(x')}{|x-x'|}d^3x'\right)\right]^2 d^4x \\
=\ &\frac{1}{2}\iiint\frac{1}{4\pi}\frac{j(x')}{|x-x'|}\nabla^2\frac{j(x'')}{|x'-x''|}d^3x'd^3x''d^4x \\
=\ &\frac{1}{2}\iiint\frac{1}{4\pi}\frac{j^)(x)j^0(x)}{|x-x'|}\delta(x'-x'')d^3xd^3x''d^4x \\
=\ &\frac{1}{2}\iiint\frac{j^0(x)j^0(x')}{4\pi|x-x'|}d^3x'd^3xdt \\
=\ &\int V_c dt
\end{aligned}
$$

That is

$$
V_c=\frac{1}{2}\int\frac{j^0(\boldsymbol{x},t)j^0(\boldsymbol{y},t)}{4\pi|\boldsymbol{x}-\boldsymbol{y}|}d^3xd^3y
$$

20.

$$
\begin{aligned}
S_0 = & \int d^4x \left(-\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + J^\mu A_\mu\right) \\
= & \frac{1}{2}\int \frac{d^4k}{(2\pi)^4}\left[-\tilde{A}_\mu(k)(k^2 g^{\mu\nu} - k^\mu k^\nu)\tilde{A}_\nu(-k) + \tilde{J}^\mu(k)\tilde{A}_\mu(-k) + \tilde{J}^\mu(-k)\tilde{A}_\mu(k)\right]
\end{aligned}
$$

in which $k^2 g^{\mu\nu - k^\mu k^\nu} = k^2 g^{\mu\nu}(k)$j.

Then we have

$$
\begin{aligned}
Z_0 = & \int DAe^{iS_0} \\
= & \exp\left[\frac{i}{2}\int \frac{d^4k}{(2\pi)^4}\tilde{J}_\mu(k)\frac{j^{\mu\nu}(k)}{k^2 - i\epsilon}\tilde{J}_\nu(-k)\right]
\end{aligned}
$$

Finally

$$
\begin{aligned}
& \langle 0|\mathcal{T}[A_\mu(x)A_\nu(y)]|0\rangle \\
= & \int \frac{y_{\mu\nu}}{k^2 - i\epsilon}e^{ik(x-y)}\frac{d^4k}{(2\pi)^4}
\end{aligned}
$$

21. Using

$$
\begin{aligned}
|\theta\rangle = & \exp\left(\psi^\dagger\theta - \frac{1}{2}\theta^*\theta\right)|0\rangle \\
= & \left(1 + \psi^\dagger\theta - \frac{1}{2}\theta^*\theta\right)|0\rangle
\end{aligned}
$$

and

$$
\langle\xi| = \langle 0|\left(1 + \xi^*\psi - \frac{1}{2}\xi^*\xi\right),
$$

we get

$$
\begin{aligned}
\langle\xi|\theta\rangle = & \langle 0|\left(1 + \xi^*\psi - \frac{1}{2}\xi^*\xi\right)\left(1 + \psi^\dagger\theta - \frac{1}{2}\theta^*\theta\right)|0\rangle \\
= & \langle 0|1 + \xi^*\psi\psi^\dagger\theta - \frac{1}{2}\xi^*\xi - \frac{1}{2}\theta^*\theta + \frac{1}{4}\xi^*\xi\theta^*\theta|0\rangle \\
= & \langle 0|1 + \xi^*\theta - \frac{1}{2}\xi^*\xi - \frac{1}{2}\theta^*\theta + \frac{1}{4}\xi^*\xi\theta^*\theta|0\rangle \\
= & \exp\left(\xi^*\theta - \frac{1}{2}(\xi^*\xi + \theta^*\theta)\right)
\end{aligned}
$$

22.

$$
\begin{aligned}
&\int |\theta\rangle\langle\theta| d\theta^* d\theta \\
&= \left(1 + \psi^\dagger\theta - \frac{1}{2}\theta^*\theta\right) |0\rangle\langle 0| \left(1 + \theta^*\psi - \frac{1}{2}\theta^*\theta\right) d\theta^* d\theta \\
&= \int \left(|0\rangle\langle 0|\theta|1\rangle\langle 0| - \frac{1}{2}\theta^*\theta|0\rangle\langle 0| + \theta\theta^*|1\rangle\langle 1| - \frac{1}{2}\theta^*\theta|0\rangle\langle 0|\right) d\theta^* d\theta \\
&= |0\rangle\langle 0| + |1\rangle\langle 1| \\
&= I
\end{aligned}
$$

23. We can expand the state

$$
\begin{aligned}
|\theta\rangle &= \exp\left(\sum_{k=1}^{n} \psi_k^\dagger\theta_k - \frac{1}{2}\theta_k^*\theta_k\right)|0\rangle \\
&= \left[\prod_{k=1}^{n}\left(1 + \psi_k^\dagger\theta_k - \frac{1}{2}\theta_k^*\theta_k\right)\right]|0\rangle
\end{aligned}
$$

The ground state is

$$
|0\rangle = \left(\prod_{k=1}^{n}\psi_k\right)|s\rangle
$$

Then

$$
\begin{aligned}
\psi_k|\theta\rangle &= \prod_{i\neq k}^{n}\left(1 + \psi_i^\dagger\theta_i - \frac{1}{2}\theta_i^*\theta_i\right)\psi_k\left(1 + \psi_k^\dagger\theta_k - \frac{1}{2}\theta_k^*\theta_k\right)|0\rangle \\
&= \prod_{i\neq k}^{n}\left(1 + \psi_i^\dagger\theta_i - \frac{1}{2}\theta_i^*\theta_i\right)\psi_k\psi_k^\dagger\theta_k|0\rangle \\
&= \prod_{i\neq k}^{n}\left(1 + \psi_i^\dagger\theta_i - \frac{1}{2}\theta_i^*\theta_i\right)\left(1 - \psi_k^\dagger\psi_k\right)\theta_k|0\rangle \\
&= \theta_k\prod_{k=1}^{n}\left(1 + \psi_k^\dagger\theta_k - \frac{1}{2}\theta_k^*\theta_k\right)|0\rangle \\
&= \theta_k|\theta\rangle
\end{aligned}
$$

24.

$$
|0\rangle = \left(\prod_m \psi_m(\boldsymbol{x}, 0)\right)|s\rangle
$$

$$
\begin{aligned}
|\chi\rangle &= \exp\left[\int \sum_m \psi_m^\dagger(\boldsymbol{x},0)\chi_m(\boldsymbol{x}) - \frac{1}{2}\chi_m^*(\boldsymbol{x})\chi_m(\boldsymbol{x}) d^3x\right]|0\rangle \\
&= \exp\left[\int \left(\psi^\dagger\chi - \frac{1}{2}\chi^\dagger\chi\right) d^3x\right]|0\rangle
\end{aligned}
$$

$$
\psi_m(\boldsymbol{x},0)|\chi\rangle = \exp\left[\int \sum_{i\neq m}\left(\psi_i^\dagger(\boldsymbol{x},0)\chi_m(\boldsymbol{x}) - \frac{1}{2}\chi_m^*(\boldsymbol{x})\chi_m(\boldsymbol{x})\right) d^3x\right]|0\rangle
$$

$$
\begin{aligned}
&\psi_m(\boldsymbol{x},0)\left[\int \left(1+\psi_m^\dagger(\boldsymbol{x},0)\chi_m(\boldsymbol{x}) - \frac{1}{2}\chi_m^*(\boldsymbol{x})\chi_m(\boldsymbol{x})\right) d^3x\right]|0\rangle \\
&= \exp\left[\int \sum_{i\neq m}\left(\psi_i^\dagger(\boldsymbol{x},0)\chi_i(\boldsymbol{x}) - \frac{1}{2}\chi_i^*(\boldsymbol{x})\chi_i(\boldsymbol{x})\right) d^3x\right]
\end{aligned}
$$

$$
\begin{aligned}
&(1-\psi_m\psi_m^\dagger)\chi_m(\boldsymbol{x})|0\rangle \\
&= \chi_m(\boldsymbol{x})\exp\left[\int \sum_m \left(\psi_m^\dagger(\boldsymbol{x},0)\chi_m(\boldsymbol{x}) - \frac{1}{2}\chi_m^*(\boldsymbol{x})\chi_m\right) d^3x\right]|0\rangle \\
&= \chi_m(\boldsymbol{x})|\chi\rangle
\end{aligned}
$$

25.

$$
|\chi\rangle = \exp\left[\int \left(\psi^\dagger\chi - \frac{1}{2}\chi^\dagger\chi\right) d^3x\right]|0\rangle
$$

$$
\langle\chi| = \langle 0|\left[\exp\int \left(\chi^\dagger\psi - \frac{1}{2}\chi^\dagger\chi\right) d^3x\right]
$$

$$
\begin{aligned}
\langle\chi|\chi\rangle &= \langle 0|\exp\left(\int (\psi^\dagger\chi + \chi'^\dagger\psi - 1/2\chi^\dagger\chi - 1/2\chi'\dagger\chi')d^3x\right)|0\rangle \\
&= \langle 0|\int\prod_m\left(1+\psi_m^\dagger\chi_m + \chi_m'^\dagger\psi_m - \frac{1}{2}\chi_m^\dagger\chi_m - \frac{1}{2}\chi_m'^\dagger\chi_m' + \psi_m^\dagger\chi_m\chi_m'^\dagger\psi_m\right) d^3x|0\rangle \\
&= \langle 0|\int\prod_m\left(1+\chi_m'^\dagger\chi_m - \frac{1}{2}\chi_m^\dagger\chi_m - \frac{1}{2}\chi_m'^\dagger\chi_m'\right) d^3x|0\rangle \\
&= \exp\left[\int\left(\chi'^\dagger\chi - \frac{1}{2}\chi'^\dagger\chi' - \frac{1}{2}\chi^\dagger\chi\right) d^3x\right]
\end{aligned}
$$