Exercises

17.1 Derive the multiple gaussian integral (17.8) from (5.178).

17.2 Derive the multiple gaussian integral (17.12) from (5.177).

17.3 Show that the vector $\overline{Y}$ that makes the argument of the multiple gaussian integral (17.12) stationary is given by (17.13), and that the multiple gaussian integral (17.12) is equal to its exponential evaluated at its stationary point $\overline{Y}$ apart from a prefactor involving the determinant $\det iS$.

17.4 Repeat the previous problem for the multiple gaussian integral (17.11).

17.5 Insert a complete set of momentum dyadics $|p\rangle\langle p|$, use the inner product $\langle q|p\rangle = \exp(iqp)/\sqrt{2\pi}$, do the resulting Fourier transform, and so verify the free-particle path integral (17.54).

17.6 By taking the non-relativistic limit of the formula (11.313) for the action of a relativistic particle of mass m and charge q, derive the expression (17.55) for the action a non-relativistic particle in an electromagnetic field with no scalar potential.

17.7 Show that for the hamiltonian (17.60) of the simple harmonic oscillator the action $S[q_c]$ of the classical path is (17.67).

17.8 Show that the harmonic-oscillator action of the loop (17.68) is (17.69).

17.9 Show that the harmonic-oscillator amplitude (17.72) for $q' = 0$ and $q'' = q$ reduces as $t \to 0$ to the one-dimensional version of the free-particle amplitude (17.54).

17.10 Work out the path-integral formula for the amplitude for a mass m to fall to the ground from height h in a gravitational field of local acceleration g to lowest order and then including loops. Hint: use the technique of section 17.6.

17.11 Show that the action (17.74) of the stationary solution (17.77) is (17.79).

17.12 Derive formula (17.132) for the action $S_0[\phi]$ from (17.130 & 17.131).

17.13 Derive identity (17.136). Split the time integral at $t = 0$ into two halves, use

$$\epsilon\, e^{\pm\epsilon t} = \pm\,\frac{d}{dt}\,e^{\pm\epsilon t} \tag{17.256}$$

and then integrate each half by parts.

17.14 Derive the third term in equation (17.138) from its second term.

17.15 Derive equation (17.147) from equations (17.144, 17.147, & 17.146).

17.16 Derive the formula (17.148) for $Z_0[j]$ from the expression (17.147) for $S_0[\phi, \epsilon, j]$.

17.17 Derive equations (17.149 & 17.150) from formula (17.148).

17.18 Derive equation (17.154) from the formula (17.149) for $Z_0[j]$.

17.19 Show that the time integral of the Coulomb term (17.159) is the negative of the term that is quadratic in j^0 in the number F defined by (17.164).

17.20 By following steps analogous to those the led to (17.150), derive the formula (17.177) for the photon propagator in Feynman's gauge.

17.21 Derive expression (17.192) for the inner product $\langle\zeta|\theta\rangle$.

17.22 Derive the representation (17.195) of the identity operator I for a single fermionic degree of freedom from the rules (17.182 & 17.185) for Grassmann integration and the anti-commutation relations (17.178 & 17.184).

17.23 Derive the eigenvalue equation (17.200) from the definition (17.198 & 17.199) of the eigenstate $|\theta\rangle$ and the anti-commutation relations (17.196 & 17.197).

17.24 Derive the eigenvalue relation (17.213) for the Fermi field $\psi_m(\boldsymbol{x}, t)$ from the anti-commutation relations (17.209 & 17.210) and the definitions (17.211 & 17.212).

17.25 Derive the formula (17.214) for the inner product from the definition (17.212) of the ket $|\chi\rangle$.