What is Rationality in DM?

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Game Theory Course

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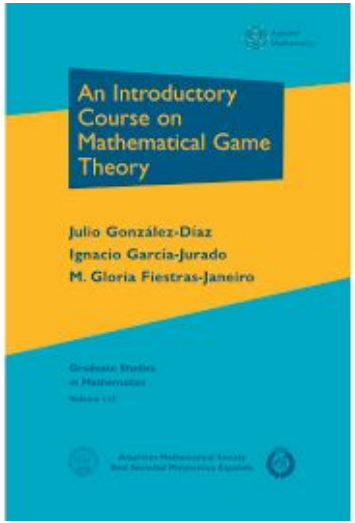
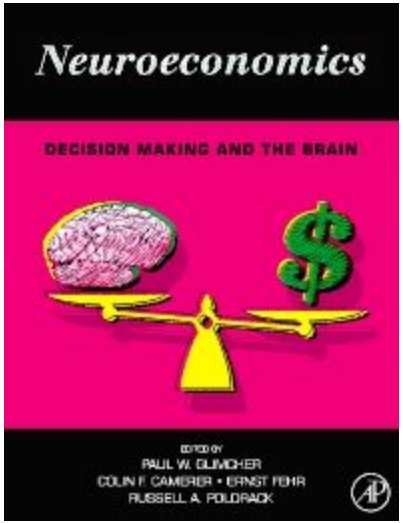
References and Acknowledges



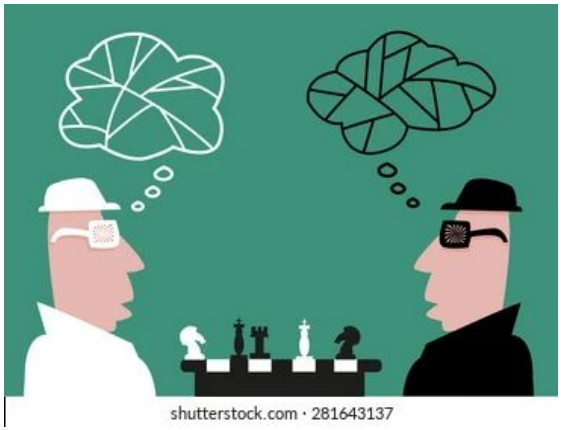


Game Theory

Game Theory and Game Definition

Game: The multi agent/player decision making problem under different settings like cooperation, competition, bargaining and coalition formation.

Game Theory: Modeling and analysis of situations which multi rational agent\player with different goals make decisions, and decision of each person effect the final result.



Our Expectation from Game Theory

Major Models of Decision-making

– Normative models

- Revolve around the concept of utility, or the overall value of a choice to the decision-maker.
- Prescriptive -- they specify what people ideally should do.
- Do not describe how people actually perform decision making tasks.

– Descriptive models

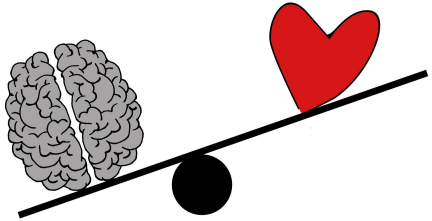
Predicting agents behavior

- Attempt to describe and model actual human decision-making.

Assumptions and Challenges

- Rationality
- DM under Strategic Uncertainty vs. DM under Uncertainty

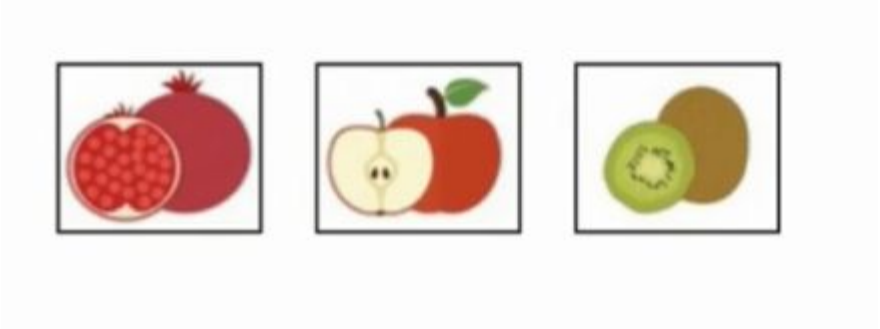
Player 1 \ Player 2	confess	don't confess
confess	(-6, -6)	(0, -10)
don't confess	(-10, 0)	(-1, -1)



Decision Theory

Utility/Decision Theory

Von Neumann–Morgenstern utility theory



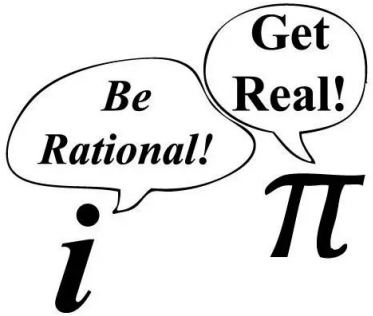
Mathematical Formulation

Preference Relation

$$x \succcurlyeq y$$

$$x \succ y \Leftrightarrow x \succcurlyeq y \text{ but not } y \succcurlyeq x$$

$$x \sim y \Leftrightarrow x \succcurlyeq y \text{ and } y \succcurlyeq x$$



What is rationality?

Rationality

- **Transitivity axiom**

$$\text{for all } x, y, z \quad x \succcurlyeq y \text{ and } y \succcurlyeq z \implies x \succcurlyeq z$$

- **Completeness axiom**

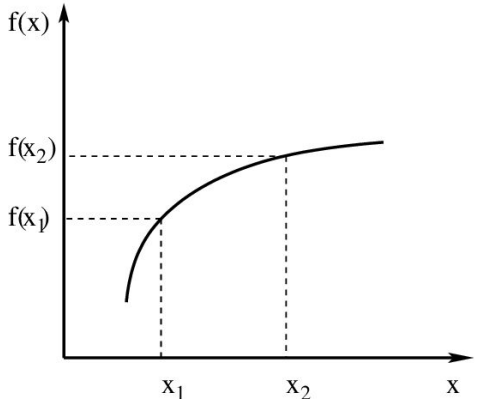
$$\text{for all } x, y \quad x \succcurlyeq y \text{ or } y \succcurlyeq x$$

Utility Function

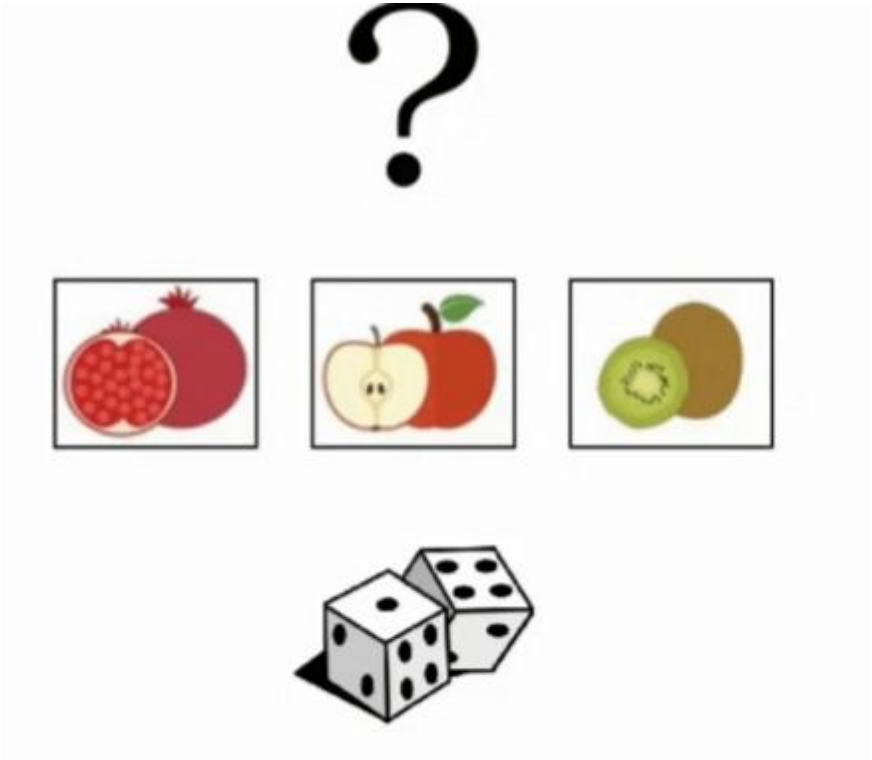
u: Action set —---> R

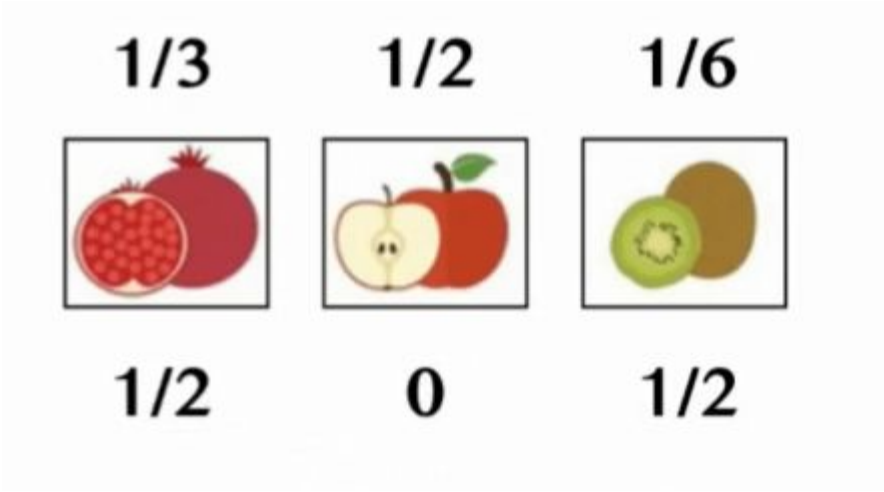
$$x \succcurlyeq y \quad \text{iff} \quad u(x) \geq u(y)$$

- Is always exists?
- Is Unique?
- Which utility function is more utilizing?



Stochastic Situations



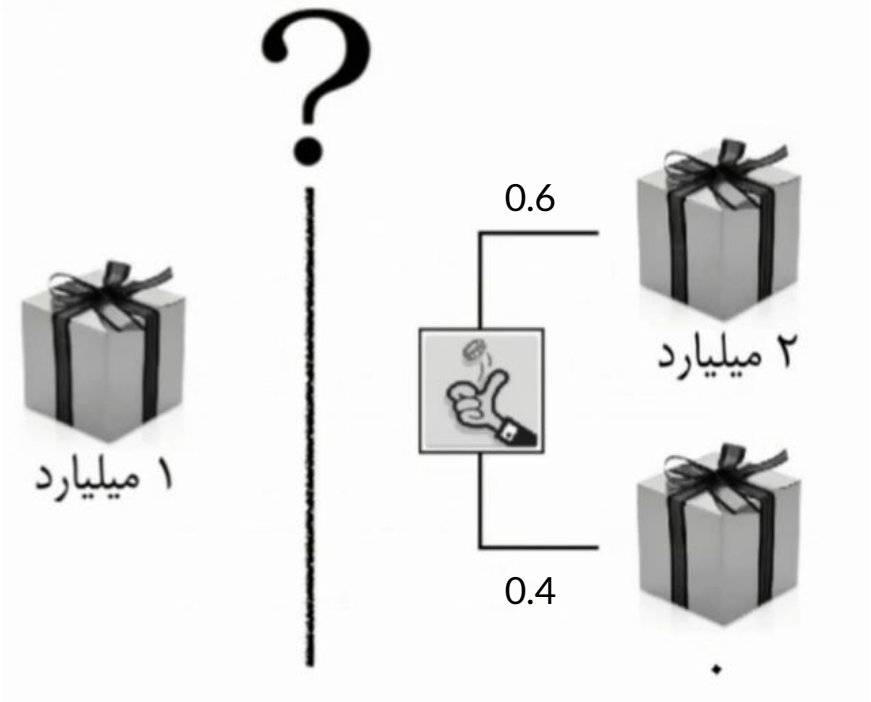


Stochastic Decision Making

Why Stochasticity?

- Unpredictability of other agents
- Uncertainty of the our world model

Expected utility maximization does not seem trivial.



Expected Utility Theorem

- **Independence(Substitution) axiom**

$$L \succcurlyeq L'$$

$$\text{iff}$$

$$\alpha L + (1-\alpha)L'' \succcurlyeq \alpha L' + (1-\alpha)L''$$

- **Continuity axiom**

$$L \prec L' \prec L''$$

$$\Downarrow$$

$$\exists \alpha \in (0,1) : \quad L' \sim \alpha L + (1-\alpha) L''$$

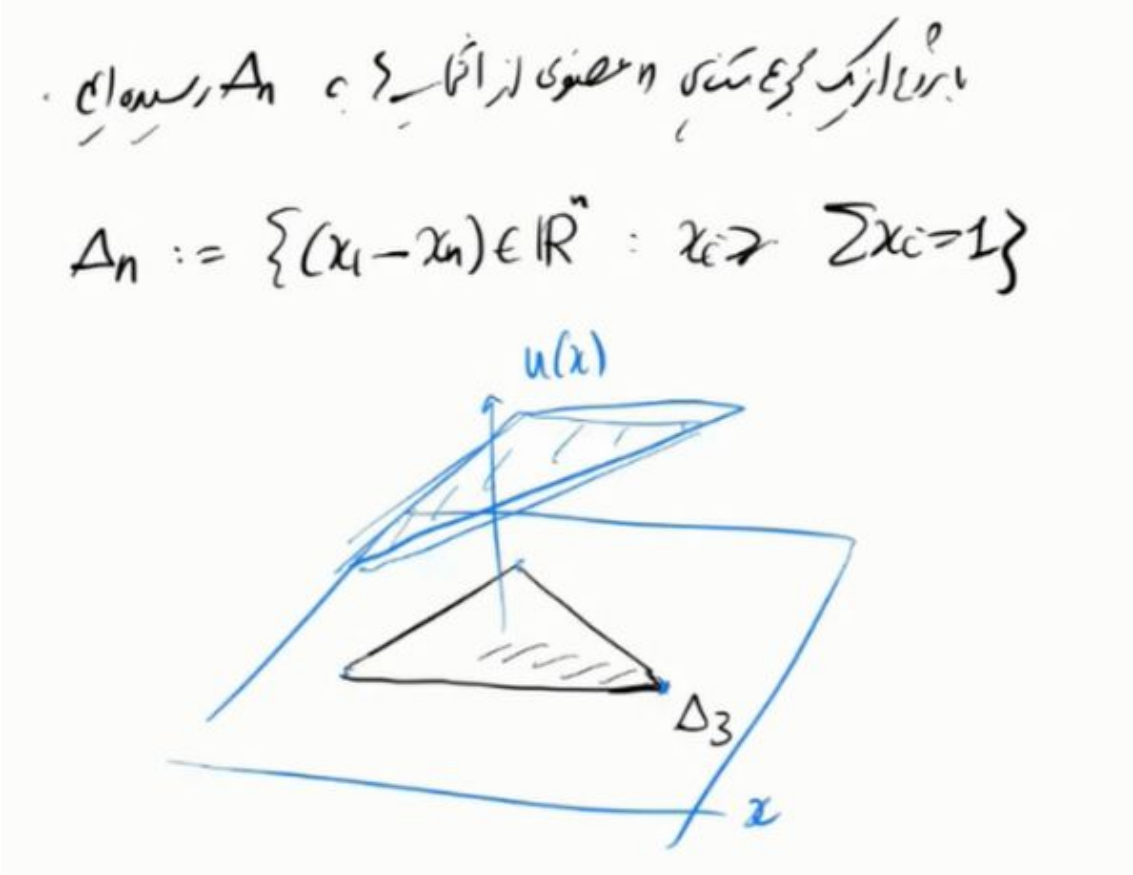
Expected Utility Theorem

اگر رابطه‌ی ترجیح معقول در اصل استقلال صدق کند نمایشی بر حسب یک تابع مطلوبیت خطی دارد!

اصل کمال + اصل تراگذری =: معقول بودن

تابع مطلوبیت خطی در حد آفین یکتاست!

Expected Utility Theorem



Prospect Theory

What is utility function?

P = 0.6

10

P = 0.4

-10

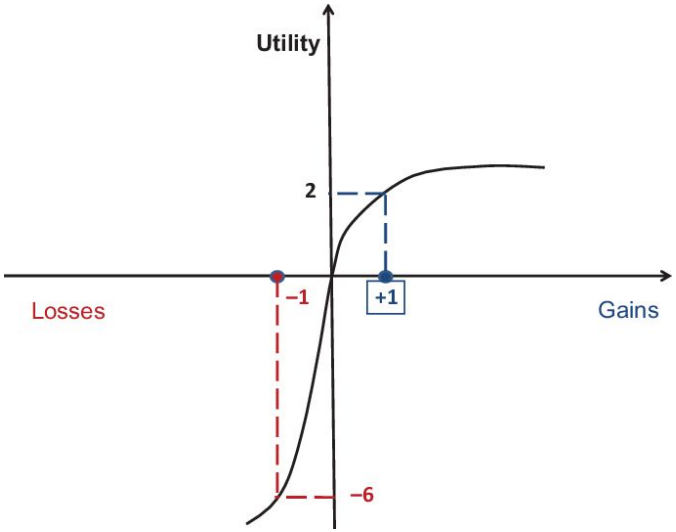
P = 0.8

0

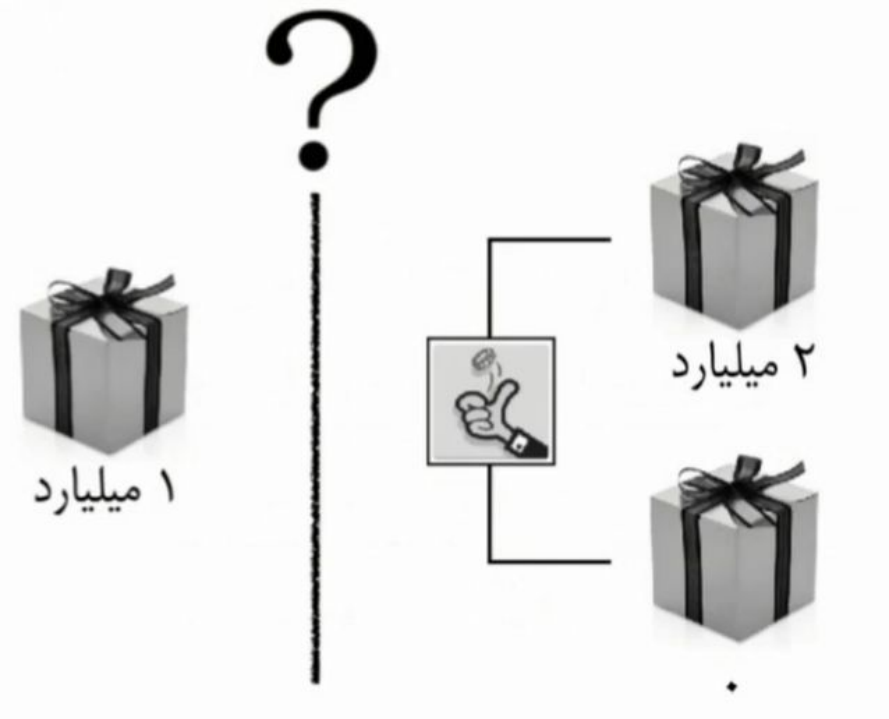
P = 0.2

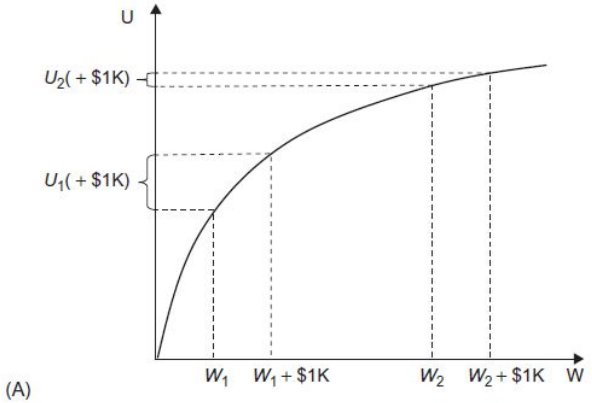
10

Loss Aversion

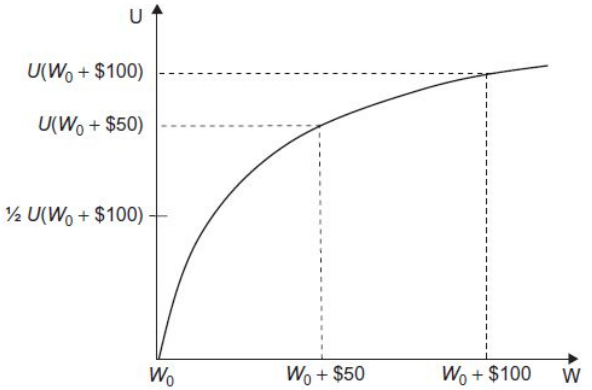


Reference Point - Concavity





(A)



(B)

Allais Paradox

Decision 1: Choose between (A) an 80% chance of \$4000; (B) \$3000 for sure.

Decision 2: Choose between (C) a 20% chance of \$4000; (D) a 25% chance of \$3000.

- **Independence(Substitution) axiom**

$C = {}^1\!/_4 A$ and $D = {}^1\!/_4 B$

$$L \succcurlyeq L'$$

iff

$$\alpha L + (1-\alpha)L'' \succcurlyeq \alpha L' + (1-\alpha)L''$$

Risk Attitudes

P

1-P

0

100

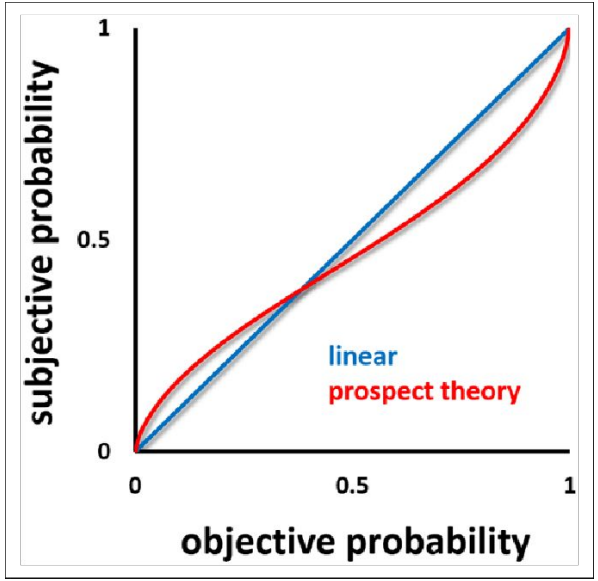
P = 1

C

Risk Attitudes

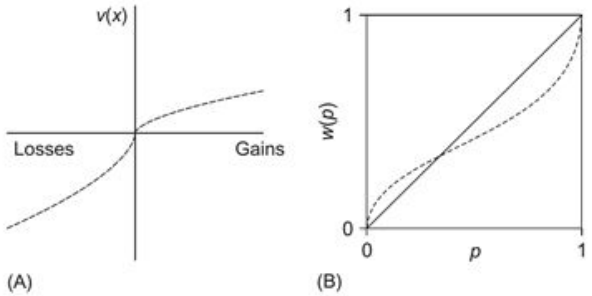
TABLE A.2A The Fourfold Pattern of Risk Attitudes

	Gains	Losses
Low probability	$c(\$100, .05) = \$14$	$c(-\$100, .05) = -\$8$
	Risk seeking	*Risk aversion*
High probability	$c(\$100, .95) = \$78$	$c(-\$100, .95) = -\$84$
	Risk aversion	*Risk seeking*



Prospect Theory

$$V(x,p) = w(p)v(x),$$



Utility function

$$v(x) = \begin{cases} x^{\alpha} & x \geq 0 \\ -\lambda(-x)^{\beta} & x < 0 \end{cases}$$

Probability weighting function

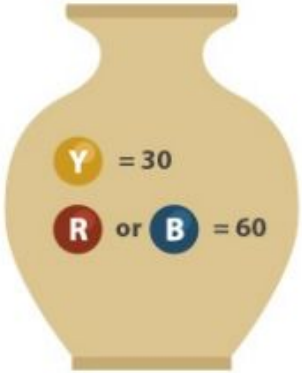
$$w(p) = \frac{\delta p^{\gamma}}{\delta p^{\gamma} + (1-p)^{\gamma}}$$

$$w(p) = \exp[-\delta(-\ln p)^{\gamma}],$$

3 main Contributions :

- Subjective probability
- Utility (Endowment effect → NeuroMarketing)
- Dynamic reference point (Framing effect)

Ellsberg paradox



Situation A

Bet Y or R

Most bet

Y

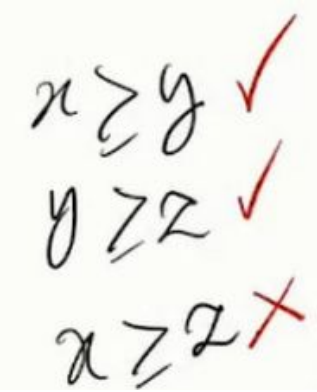
Situation B

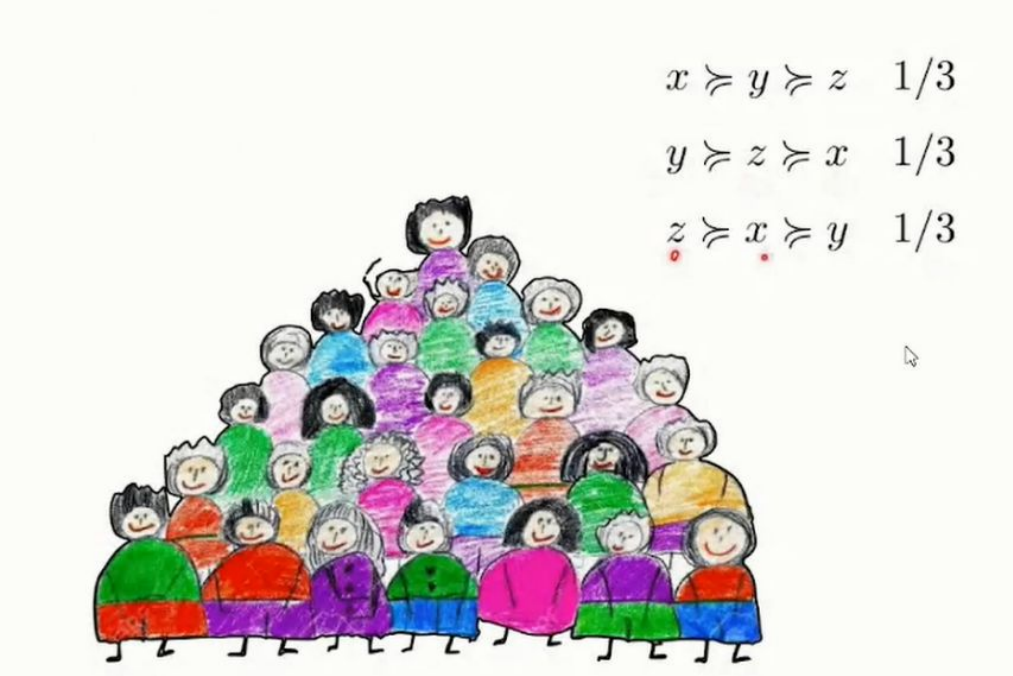
Bet R / B or Y / B

R / B

Social Rationality

- Consensus — Almost doesn't exists.
- Majority vote





Utilitarianism

Bargaining