

TDT4171 — Artificial Intelligence Methods

Assignment 4 - Making simple decisions

Erik Storås Sommer - 535006

February 2023

Exercise 1

a.

The first Bayes net ($\text{Wrapper} \rightarrow \text{Flavor} \leftarrow \text{Shape}$) can not correctly represent $P(\text{Flavor}, \text{Wrapper}, \text{Shape})$ because Wrapper and Shape are dependant of eachother.

The second Bayes net ($\text{Flavor} \leftarrow \text{Wrapper} \rightarrow \text{Shape} \rightarrow \text{Flavor}$) correctly represents the dependencies between the variables and also represents the interdependence between Flavor, Wrapper, and Shape, but it creates a cyclic graph, which is not desirable in a Bayesian network.

The third Bayes net ($\text{Wrapper} \leftarrow \text{Flavor} \rightarrow \text{Shape}$) represents the direct causal relationships between Wrapper, Flavor, and Shape, as well as the interdependence between the variables, and it does not create a cyclic graph, making it the most desirable representation of the variables.

b.

The best representation for this problem is the Bayes net (iii). This network is the most compact representation that captures the causal relationships between the variables, and it is also the easiest to use in deducing the numbers required by the conditional probability tables. In this network, the flavor of the candy is first determined, and then the wrapper and shape are selected based on the flavor. The conditional probability tables for this network can be easily computed from the information given in the story.

c.

Yes, network (i) asserts that Wrapper is independent of Shape. The directed edge between Wrapper and Flavor in this network represents a causal relationship, meaning that Wrapper is dependent on Flavor but not on Shape. The absence of an edge between Wrapper and Shape in this network represents that Wrapper and Shape are conditionally independent given Flavor.

d.

The probability that the flavor of the candy is strawberry, given that it is a round candy with a red wrapper, can be calculated using Bayes' Theorem as follows:

$$P(W = r) = P(W = r|F = s) \cdot P(F = s) + P(W = r|F = a) \cdot P(F = a)$$

where $W = r$ is the event that the wrapper is red, $F = s$ is the event that the flavor is strawberry, and $F = a$ is the event that the flavor is anchovy.

Applying the probabilities given in the story, we get:

$$P(W = r) = 0.7 \cdot 0.8 + 0.3 \cdot 0.1 = 0.59$$

The the probability that your candy has a red wrapper is 59 percent.

e.

Defining $S = r$ as the event that the shape is round, and $W = r$ as the event that the wrapper is red, we can calculate the probability that the flavor is strawberry, given that the shape is round and the wrapper is red, using Bayes' Theorem as follows:

$$\begin{aligned} P(F|S = r, W = r) &= \alpha P(S = r, W = r|F) \cdot P(F) \\ &= \alpha P(S = r|F) \cdot P(W = r|F) \cdot P(F) \\ &= \alpha(0.8 \cdot 0.8 \cdot 0.7, 0.1 \cdot 0.1 \cdot 0.3) \\ &= \alpha(0.448, 0.003) \end{aligned}$$

We see that the probability that the canys flavor is strawberry is > 99.9 percent higher than the probability that the flavor is anchovy.

f.

Denoting R as the event of a candy having a red wrapper, S as the event of a candy being round, and F as the event of a candy being strawberry flavored.

Using the Bayes' Rule, the probability of F given R and S can be calculated as:

$$P(F|R, S) = P(R|F, S) \cdot P(F) / (P(R|F, S) \cdot P(F) + P(R|\neg F, S) \cdot P(\neg F))$$

Where:

$$\begin{aligned} P(F = s) &= 0.7 \\ P(W = r|F = s, S = r) &= 0.8 \\ P(W = r|\neg F = s, S = R) &= 0 \\ P(\neg F = s) &= 0.3 \end{aligned}$$

Plugging in the values, we get:

$$P(F|R, S) = 0.8 \cdot 0.7 / (0.8 \cdot 0.7 + 0 \cdot 0.3) = 0.8 \cdot 0.7 / 0.7 = 0.8$$

Thus, the expected value of an unopened candy box can be calculated as:

$$\begin{aligned} E(Box) &= s \cdot P(F|R, S) + a \cdot (1 - P(F|R, S)) \\ E(Box) &= s \cdot 0.8 + a \cdot (1 - 0.8) = 0.8 \cdot s + 0.2 \cdot a \end{aligned}$$

Exercise 2

a.

To determine whether Mary would choose the certain payout of \$500 or the lottery, we need to compare their expected monetary values (EMVs). The EMV of the certain payout is simply the payout itself, which is \$500. The EMV of the lottery is given by the weighted average of its possible payouts, where the weights are the probabilities of each outcome:

EMV of the lottery:

$$EMV = 0.6 \cdot \$5000 + 0.4 \cdot \$0 = \$3000$$

We can then compare the EMVs of the two options by calculating the expected utility of each option, which is given by the exponential utility function:

Expected utility of the certain payout:

$$U(\$500) = -e^{-500/500} = -e^{-1} = -0.3679$$

Expected utility of the lottery:

$$U(\$3000) = -e^{-3000/500} = -e^{-6} = -0.0024$$

Since the expected utility of the lottery is higher than the expected utility of the certain payout, we can conclude that Mary would choose the lottery. This is because Mary acts rationally and seeks to maximize her expected utility, and the lottery has a higher expected utility than the certain payout.

b.

To find the value of R , we need to find the expected utility (EU) for each option and set them equal. If an individual is indifferent between the two options, their expected utilities would be the same.

The expected utility of receiving \$100 with certainty is:

$$EU1 = U(\$100) = -e^{-100/R}$$

The expected utility of participating in the lottery is:

$$EU2 = 0.5 * U(\$500) + 0.5 * U(0) = 0.5 * -e^{-500/R} + 0.5 * -e^{0/R} = 0.5 * -e^{-5} + 0.5 * -1$$

Setting $EU1 = EU2$, we get:

$$-e^{-100/R} = 0.5 * -e^{-5} + 0.5 * -1$$

Solving for R :

$$R \approx 255$$

So the value of R to 3 significant digits would be approximately 255.