

I. SKYRME INTERACTION - FINITE TEMPERATURE

A. Potential Matrix Element

Interaction Matrix:

$$\begin{aligned}
V_{ij} = & t_0(1 + x_0 P_\sigma) \delta(\mathbf{r}_{ij}) + \frac{1}{2} t_1(1 + x_1 P_\sigma) \frac{1}{\hbar^2} [\overleftarrow{p}_{ij}^{\dagger 2} \delta(\mathbf{r}_{ij}) + \delta(\mathbf{r}_{ij}) \overrightarrow{p}_{ij}^2] \\
& + t_2(1 + x_2 P_\sigma) \frac{1}{\hbar^2} \overleftarrow{\mathbf{p}}_{ij}^\dagger \cdot \delta(\mathbf{r}_{ij}) \overrightarrow{\mathbf{p}}_{ij} + \frac{1}{6} t_3(1 + x_3 P_\sigma) \rho^\alpha(\mathbf{r}) \delta(\mathbf{r}_{ij}) \\
& + \frac{i}{\hbar^2} W_0(\sigma_i + \sigma_j) \cdot \overleftarrow{\mathbf{p}}_{ij}^\dagger \times \delta(\mathbf{r}_{ij}) \overrightarrow{\mathbf{p}}_{ij} \\
& + \frac{1}{4} t_4(1 + x_4 P_\sigma) \frac{1}{\hbar^2} [\overleftarrow{p}_{ij}^{\dagger 2} \rho^\beta(\mathbf{r}) \delta(\mathbf{r}_{ij}) + \delta(\mathbf{r}_{ij}) \rho^\beta(\mathbf{r}) \overrightarrow{p}_{ij}^2] \\
& + t_5(1 + x_5 P_\sigma) \frac{1}{\hbar^2} \overleftarrow{\mathbf{p}}_{ij} \cdot \rho^\gamma(\mathbf{r}) \delta(\mathbf{r}_{ij}) \overrightarrow{\mathbf{p}}_{ij}
\end{aligned} \tag{1}$$

where, $\mathbf{r}_{ij} = \frac{\mathbf{r}_i - \mathbf{r}_j}{2}$, $\mathbf{r} = \frac{\mathbf{r}_i + \mathbf{r}_j}{2}$, $P_\sigma = \frac{1}{2}(1 + \sigma_1 \cdot \sigma_2)$, $\mathbf{p}_{ij} = -i\hbar \frac{\nabla_i - \nabla_j}{2}$, $\rho(\mathbf{r}) = \rho_p(\mathbf{r}) + \rho_n(\mathbf{r})$ and the arrows show the direction on which the momentum operators act.

B. Energy Density

Assume time reversal invariance. The Energy of the ground state can be written as integration over an energy density functional, $E_{HFB} = \int d^r \varepsilon_{HFB}$

$$\begin{aligned}
\varepsilon_{HFB} = & \sum_{t=n,p} \frac{\hbar^2}{2M_q} \tau_q + \frac{1}{2} t_0 \left[\left(1 + \frac{1}{2} x_0\right) \rho^2 - \left(\frac{1}{2} + x_0\right) \sum_{q=n,p} \rho_q^2 \right] \\
& + \frac{1}{4} t_1 \left[\left(1 + \frac{1}{2} x_1\right) (\rho\tau + \frac{3}{4} (\nabla\rho)^2) - \left(\frac{1}{2} + x_1\right) \sum_{q=n,p} (\rho_q \tau_q + \frac{3}{4} (\nabla\rho_q)^2) \right] \\
& + \frac{1}{4} t_2 \left[\left(1 + \frac{1}{2} x_2\right) (\rho\tau + \frac{3}{4} (\nabla\rho)^2) + \left(\frac{1}{2} + x_2\right) \sum_{q=n,p} (\rho_q \tau_q - \frac{1}{4} (\nabla\rho_q)^2) \right] \\
& + \frac{1}{12} t_3 \rho^\alpha \left[\left(1 + \frac{1}{2} x_3\right) \rho^2 - \left(\frac{1}{2} + x_3\right) \sum_{q=n,p} \rho_q^2 \right] \\
& + \frac{1}{4} t_4 \left[\left(1 + \frac{1}{2} x_4\right) (\rho\tau + \frac{3}{4} (\nabla\rho)^2) - \left(\frac{1}{2} + x_4\right) \sum_{q=n,p} (\rho_q \tau_q + \frac{3}{4} (\nabla\rho_q)^2) \right] \rho^\beta \\
& + \frac{\beta}{8} t_4 \left[\left(1 + \frac{1}{2} x_4\right) \rho (\nabla\rho)^2 - \left(\frac{1}{2} + x_4\right) \nabla\rho \cdot \sum_{q=n,p} \rho_q \nabla\rho_q \right] \rho^{\beta-1} \\
& + \frac{1}{4} t_5 \left[\left(1 + \frac{1}{2} x_5\right) (\rho\tau + \frac{3}{4} (\nabla\rho)^2) + \left(\frac{1}{2} + x_5\right) \sum_{q=n,p} (\rho_q \tau_q - \frac{1}{4} (\nabla\rho_q)^2) \right] \rho^\gamma \\
& - \frac{1}{16} (t_1 x_1 + t_2 x_2) J^2 + \frac{1}{16} (t_1 - t_2) \sum_{q=n,p} J_q^2 \\
& - \frac{1}{16} (t_4 x_4 \rho^\beta + t_5 x_5 \rho^\gamma) J^2 + \frac{1}{16} (t_4 \rho^\beta - t_5 \rho^\gamma) \sum_{q=n,p} J_q^2 \\
& + \frac{1}{2} W_0 (\mathbf{J} \cdot \nabla\rho + \sum_{q=n,p} \mathbf{J}_q \cdot \nabla\rho_q)
\end{aligned} \tag{2}$$

where,

$$\begin{aligned}
\rho &= 2 \int \frac{d^3 k}{(2\pi\hbar)^3} n(k) \\
\tau &= 2 \int \frac{d^3 k}{(2\pi\hbar)^3} k^2 n(k) \\
\mathbf{J} &= \int \frac{d^3 k}{(2\pi\hbar)^3} \mathbf{k} \times \sum_{s,s'} \langle s | \boldsymbol{\sigma} | s' \rangle n(k)
\end{aligned} \tag{3}$$

The different terms can be grouped together in simpler notation:

$$\begin{aligned}
\varepsilon = & \sum_{t=n,p} \frac{\hbar^2}{2M_q} \tau_q + \frac{1}{4} t_0 \left[(2+x_0) \rho^2 - (1+2x_0) \sum_{q=n,p} \rho_q^2 \right] \\
& + \frac{1}{8} \left[a(\rho\tau + \frac{3}{4}(\nabla\rho)^2) + 2b \sum_{q=n,p} (\rho_q \tau_q + \frac{3}{4}(\nabla\rho_q)^2) \right] \\
& + \frac{1}{24} t_3 \rho^\alpha \left[(2+x_3) \rho^2 - (1+2x_3) \sum_{q=n,p} \rho_q^2 \right] \\
& + \frac{1}{8} t_4 \left[(2+x_4)(\rho\tau + \frac{3}{4}(\nabla\rho)^2) - (1+2x_4) \sum_{q=n,p} (\rho_q \tau_q + \frac{3}{4}(\nabla\rho_q)^2) \right] \rho^\beta \\
& + \frac{\beta}{16} t_4 \left[(2+x_4) \rho(\nabla\rho)^2 - (1+2x_4) \nabla\rho \cdot \sum_{q=n,p} \rho_q \nabla\rho_q \right] \rho^{\beta-1} \\
& + \frac{1}{8} t_5 \left[(2+x_5)(\rho\tau + \frac{3}{4}(\nabla\rho)^2) + (1+2x_5) \sum_{q=n,p} (\rho_q \tau_q - \frac{1}{4}(\nabla\rho_q)^2) \right] \rho^\gamma \\
& - \frac{1}{16} (t_1 x_1 + t_2 x_2) J^2 + \frac{1}{16} (t_1 - t_2) \sum_{q=n,p} J_q^2 \\
& - \frac{1}{16} (t_4 x_4 \rho^\beta + t_5 x_5 \rho^\gamma) J^2 + \frac{1}{16} (t_4 \rho^\beta - t_5 \rho^\gamma) \sum_{q=n,p} J_q^2 \\
& + \frac{1}{2} W_0 (\mathbf{J} \cdot \nabla\rho + \sum_{q=n,p} \mathbf{J}_q \cdot \nabla\rho_q)
\end{aligned} \tag{4}$$

where, $a = t_1(x_1 + 2) + t_2(x_2 + 2)$, $b = \frac{1}{2}[t_2(2x_2 + 1) - t_1(2x_1 + 1)]$.

In uniform matter $\nabla\rho = 0$:

$$\begin{aligned}
\varepsilon = & \sum_{t=n,p} \frac{\hbar^2}{2M_q} \tau_q + \frac{1}{4} t_0 \left[(2+x_0) \rho^2 - (1+2x_0) \sum_{q=n,p} \rho_q^2 \right] \\
& + \frac{1}{8} \left[a \rho \tau + 2b \sum_{q=n,p} \rho_q \tau_q \right] \\
& + \frac{1}{24} t_3 \rho^\alpha \left[(2+x_3) \rho^2 - (1+2x_3) \sum_{q=n,p} \rho_q^2 \right] \\
& + \frac{1}{8} t_4 \left[(2+x_4) \rho \tau - (1+2x_4) \sum_{q=n,p} \rho_q \tau_q \right] \rho^\beta \\
& + \frac{1}{8} t_5 \left[(2+x_5) \rho \tau + (1+2x_5) \sum_{q=n,p} \rho_q \tau_q \right] \rho^\gamma \\
& - \frac{1}{16} (t_1 x_1 + t_2 x_2) J^2 + \frac{1}{16} (t_1 - t_2) \sum_{q=n,p} J_q^2 \\
& - \frac{1}{16} (t_4 x_4 \rho^\beta + t_5 x_5 \rho^\gamma) J^2 + \frac{1}{16} (t_4 \rho^\beta - t_5 \rho^\gamma) \sum_{q=n,p} J_q^2
\end{aligned} \tag{5}$$

In unpolarized matter, $\mathbf{J} = 0$:

$$\begin{aligned}
\varepsilon = & \sum_{t=n,p} \frac{\hbar^2}{2M_q} \tau_q + \frac{1}{4} t_0 \left[(2+x_0) \rho^2 - (1+2x_0) \sum_{q=n,p} \rho_q^2 \right] \\
& + \frac{1}{8} \left[a \rho \tau + 2b \sum_{q=n,p} \rho_q \tau_q \right] \\
& + \frac{1}{24} t_3 \rho^\alpha \left[(2+x_3) \rho^2 - (1+2x_3) \sum_{q=n,p} \rho_q^2 \right] \\
& + \frac{1}{8} t_4 \left[(2+x_4) \rho \tau - (1+2x_4) \sum_{q=n,p} \rho_q \tau_q \right] \rho^\beta \\
& + \frac{1}{8} t_5 \left[(2+x_5) \rho \tau + (1+2x_5) \sum_{q=n,p} \rho_q \tau_q \right] \rho^\gamma
\end{aligned} \tag{6}$$

Energy per bayon, $\mathcal{E} \equiv \varepsilon/rho$:

$$\begin{aligned}
\mathcal{E} = & \sum_{t=n,p} \frac{\hbar^2}{2M_q} \frac{\tau_q}{\rho} + \frac{1}{4} t_0 \left[(2+x_0)\rho - (1+2x_0) \sum_{q=n,p} \frac{\rho_q^2}{\rho} \right] \\
& + \frac{1}{8} \left[a\tau + 2b \sum_{q=n,p} \frac{\rho_q \tau_q}{\rho} \right] \\
& + \frac{1}{24} t_3 \rho^\alpha \left[(2+x_3)\rho - (1+2x_3) \sum_{q=n,p} \frac{\rho_q^2}{\rho} \right] \\
& + \frac{1}{8} t_4 \left[(2+x_4)\tau - (1+2x_4) \sum_{q=n,p} \frac{\rho_q \tau_q}{\rho} \right] \rho^\beta \\
& + \frac{1}{8} t_5 \left[(2+x_5)\tau + (1+2x_5) \sum_{q=n,p} \frac{\rho_q \tau_q}{\rho} \right] \rho^\gamma
\end{aligned} \tag{7}$$

In terms of proton fraction, $y = \frac{\rho_p}{\rho_p + \rho_n}$:

$$\begin{aligned}
\mathcal{E} = & \sum_{t=n,p} \frac{\hbar^2}{2M_q} \frac{\tau_q}{\rho} + \frac{1}{4} t_0 \left[(2+x_0) - (1+2x_0)[y^2 + (1-y)^2] \right] \rho \\
& + \frac{1}{8} \left[a\tau + 2b[y\tau_p + (1-y)\tau_n] \right] \\
& + \frac{1}{24} t_3 \rho^{\alpha+1} \left[(2+x_3) - (1+2x_3)[y^2 + (1-y)^2] \right] \\
& + \frac{1}{8} t_4 \left[(2+x_4)\tau - (1+2x_4)[y\tau_p + (1-y)\tau_n] \right] \rho^\beta \\
& + \frac{1}{8} t_5 \left[(2+x_5)\tau + (1+2x_5)[y\tau_p + (1-y)\tau_n] \right] \rho^\gamma
\end{aligned} \tag{8}$$

II. SINGLE PARTICLE PROPERTIES

From the energy density the single particle spectrum can be derived. By performing functional variation of the energy density with respect to the single particle wavefunction, a modified Schrodinger equation can be derived:

$$\delta\varepsilon_i = \left[\frac{\delta\varepsilon_i}{\delta\tau_i} + \frac{\delta\varepsilon_i}{\delta\rho_i} \right] \delta\phi_i = \epsilon_i \delta\phi_i \tag{9}$$

Since the Skyrme potential is at mostly quadratic in momenta with nonlinaer density dependence, its effect is exactly included by effective mass and mean field shift(residual interaction), both density dependent:

$$\begin{aligned}\epsilon_i(k) &= \frac{\hbar^2 k^2}{2M_i^*} + U_i \\ \frac{\hbar^2}{2M_q^*} &\equiv \frac{\partial \varepsilon}{\partial \tau_q} \\ U_i &\equiv \frac{\partial \varepsilon}{\partial \rho_i}\end{aligned}\tag{10}$$

From eq. 6 the effective baryon masses:

$$\begin{aligned}M_p^*/M &= \left\{ 1 + \frac{M}{4\hbar^2} \rho [a + 2 y b + t_4[(2 + x_4) - (1 + 2x_4) y] \rho^\beta + t_5[2 + x_5 + (1 + 2x_5) y] \rho^\gamma] \right\}^{-1} \\ M_n^*/M &= \left\{ 1 + \frac{M}{4\hbar^2} \rho [a + 2 (1 - y) b + t_4[(2 + x_4) - (1 + 2x_4) (1 - y)] \rho^\beta + t_5[2 + x_5 + (1 + 2x_5) (1 - y)] \rho^\gamma] \right\}^{-1}\end{aligned}\tag{11}$$

and the residual potentials:

$$\begin{aligned}U_p &= \frac{1}{8}(2b \tau_p + a \tau) + \frac{1}{2}t_0[(2 + x_0) - (1 + 2x_0) y] \rho \\ &\quad + \frac{1}{24}t_3 \left[4 + \alpha - 2y(1 - (1 - y)\alpha) + x_3(1 - 2y)[2 - (1 - 2y)\alpha] \right] \rho^{\alpha+1} \\ U_n &= \frac{1}{8}(2b \tau_n + a \tau) + \frac{1}{2}t_0[(1 - x_0) + (1 + 2x_0)y] \rho \\ &\quad + \frac{1}{24}t_3 \left[2 + \alpha + 2y(1 + \alpha - y\alpha) - x_3(1 - 2y)[2 + (1 - 2y)\alpha] \right] \rho^{\alpha+1}\end{aligned}\tag{12}$$

III. T=0 DFT

At $T = 0$, there are simple relation that can be drawn between the 2 integrations in Fourier space since the occupation number is a step function:

$$\begin{aligned}
\rho_q &= 2 \int \frac{d^3k}{(2\pi\hbar)^3} \theta(k_{F,q} - k) = \frac{k_{F,q}^3}{3\pi^2\hbar^3} \\
\tau_q &= 2 \int \frac{d^3k}{(2\pi\hbar)^3} (k/\hbar)^2 \theta(k_{F,q} - k) = \frac{k_{F,q}^5}{5\pi^2\hbar^5} \rightarrow \\
\tau &= \frac{3}{5} (3\pi^2)^{2/3} \rho_q^{5/3}, H_n(y) = 2^{n-1} [y^n + (1-y)^n], y = \rho_p/\rho \\
\tau &= \tau_p + \tau_n = \frac{3}{5} \left(\frac{3\pi^2}{2}\right)^{2/3} H_{5/3}(y) \rho^{5/3}
\end{aligned} \tag{13}$$

So,

$$\begin{aligned}
\mathcal{E}_0 &= \frac{3\hbar^2}{10M} \left(\frac{3\pi^2}{2}\right)^{2/3} \rho^{2/3} H_{5/3}(y) + \frac{1}{8} t_0 \rho \left[2(2+x_0) - (1+2x_0) H_2(y) \right] \\
&+ \frac{3}{40} \left(\frac{3\pi^2}{2}\right)^{2/3} \left[a H_{5/3}(y) + b H_{8/5}(y) \right] \rho^{5/3} \\
&+ \frac{1}{48} t_3 \rho^{\alpha+1} \left[2(2+x_3) - (1+2x_3) H_2(y) \right] \\
&+ \frac{3}{40} \left(\frac{3\pi^2}{2}\right)^{2/3} t_4 \rho^{\beta+5/3} \left[(2+x_4) H_{5/3}(y) - \left(\frac{1}{2} + x_4\right) H_{8/3}(y) \right] \\
&+ \frac{3}{40} \left(\frac{3\pi^2}{2}\right)^{2/3} t_5 \rho^{\gamma+5/3} \left[(2+x_5) H_{5/3}(y) + \left(\frac{1}{2} + x_5\right) H_{8/3}(y) \right]
\end{aligned} \tag{14}$$

with $H_n(y) = 2^{n-1} [y^n + (1-y)^n]$. A common choice is to set $M = 1/2(M_n + M_p)$, or use the individual value for each species. In compact notation,

$$\mathcal{E}_0 = C(y) \rho^{2/3} + A(y) \rho + B(y) \rho^{\alpha+1} + D(y) \rho^{5/3} + G(y) \rho^{\beta+5/3} + K(y) \rho^{\gamma+5/3} \tag{15}$$

By comparing the 2 expressions, the following relations can be easily deduced:

$$\begin{aligned}
C(y) &= \frac{3\hbar^2}{10M_y} \left(\frac{3\pi^2}{2}\right)^{2/3} H_{5/3}(y) \\
A(y) &= \frac{1}{8} t_0 \left[2(2 + x_0) - (1 + 2x_0) H_2(y) \right] \\
B(y) &= \frac{1}{48} t_3 \left[2(2 + x_3) - (1 + 2x_3) H_2(y) \right] \\
D(y) &= \frac{3}{40} \left(\frac{3\pi^2}{2}\right)^{2/3} \left[a H_{5/3}(y) + b H_{8/5}(y) \right] \\
G(y) &= \frac{3}{40} \left(\frac{3\pi^2}{2}\right)^{2/3} t_4 \left[(2 + x_4) H_{5/3}(y) - \left(\frac{1}{2} + x_4\right) H_{8/3}(y) \right] \\
K(y) &= \frac{3}{40} \left(\frac{3\pi^2}{2}\right)^{2/3} t_5 \left[(2 + x_5) H_{5/3}(y) + \left(\frac{1}{2} + x_5\right) H_{8/3}(y) \right]
\end{aligned} \tag{16}$$

For pure neutron matter and symmetric matter the kinetic coefficient is

$$\begin{aligned}
C_n = C(0) &= \frac{3\hbar^2}{10M_n} \left(\frac{3\pi^2}{2}\right)^{2/3} H_{5/3}(0) = 118.995 \text{ Mev fm}^2 \approx 119 \text{ Mev fm}^2 \\
C_{sym} = C(1/2) &= \frac{3\hbar^2}{5(M_n + M_p)} \left(\frac{3\pi^2}{2}\right)^{2/3} H_{5/3}(1/2) = 75.0139 \text{ Mev fm}^2 \approx 75 \text{ Mev fm}^2
\end{aligned} \tag{17}$$

The effective mass is due to the terms dependent on kinetic energy:

$$\begin{aligned}
\tau^{T=0}(\rho, y) &\equiv \frac{3\hbar^2}{10M^*} \left(\frac{3\pi^2}{2}\right)^{2/3} \rho^{2/3} H_{5/3}(y) \\
&= \frac{3\hbar^2}{10M} \left(\frac{3\pi^2}{2}\right)^{2/3} \rho^{2/3} H_{5/3}(y) + \frac{3}{40} \left(\frac{3\pi^2}{2}\right)^{2/3} \left[a H_{5/3}(y) + b H_{8/5}(y) \right] \rho^{5/3} \\
&\quad + \frac{3}{40} \left(\frac{3\pi^2}{2}\right)^{2/3} t_4 \rho^{\beta+5/3} \left[(2 + x_4) H_{5/3}(y) - \left(\frac{1}{2} + x_4\right) H_{8/3}(y) \right] \\
&\quad + \frac{3}{40} \left(\frac{3\pi^2}{2}\right)^{2/3} t_5 \rho^{\gamma+5/3} \left[(2 + x_5) H_{5/3}(y) + \left(\frac{1}{2} + x_5\right) H_{8/3}(y) \right] \\
&\equiv C(y) \rho^{2/3} \left[1 + (D(y) \rho + G(y) \rho^{\beta+1} + K(y) \rho^{\gamma+1}) / C(y) \right]
\end{aligned} \tag{18}$$

Thus,

$$\begin{aligned}
M^*/M &= \left\{ 1 + \frac{M}{4H_{5/3}(y)\hbar^2} \left[\rho[aH_{5/3}(y) + bH_{8/5}(y)] + \rho^{\beta+1}t_4[(2+x_4)H_{5/3}(y) - (\frac{1}{2} + x_4)H_{8/3}(y)] \right. \right. \\
&\quad \left. \left. + \rho^{\gamma+1}t_5[(2+x_5)H_{5/3}(y) - (\frac{1}{2} + x_5)H_{8/3}(y)] \right] \right\}^{-1} \\
&= \frac{C(y)}{C(y) + D(y)\rho + G(y)\rho^{\beta+1} + K(y)\rho^{\gamma+1}}
\end{aligned} \tag{19}$$

Also, the thermodynamic pressure:

$$\begin{aligned}
\mathcal{P}^{T=0}(\rho, y) &= \rho^2 \frac{\partial \mathcal{E}^{T=0}}{\partial \rho} \\
&= \frac{2}{3}C(y)\rho^{5/3} + A(y)\rho^2 + (\alpha + 1)B(y)\rho^{\alpha+1} + \frac{5}{3}D(y)\rho^{8/3} + \beta G(y)\rho^{\beta+8/3} + \gamma K(y)\rho^{\gamma+8/3}
\end{aligned} \tag{20}$$

IV. LATTIMER - SCHWESTY NOTATION

The standard parametrization:

$$\begin{aligned}
\varepsilon &= \frac{\hbar^2 \tau_n}{2M_n} + \frac{\hbar^2 \tau_p}{2M_p} + \frac{1}{8}[a(\rho_n + \rho_p)(\tau_n + \tau_p) + 2b(\rho_n \tau_n + \rho_p \tau_p)] + \frac{1}{4}t_0[(2+x_0)(\rho_n + \rho_p)^2 - (1+2x_0)(\rho_n^2 + \rho_p^2)] \\
&\quad + \frac{1}{24}t_3(\rho_n + \rho_p)^\alpha[(2+x_3)(\rho_n + \rho_p)^2 - (1+2x_3)(\rho_n^2 + \rho_p^2)]
\end{aligned} \tag{21}$$

$$\begin{aligned}
a &= \frac{4\hbar^2}{M}(F + G) \\
b &= -\frac{4\hbar^2}{M}G \\
t_0 &= \frac{8}{3}(A + B) \\
x_0 &= -\frac{1}{2}\frac{A - 2B}{A + B} \\
t_3 &= 16(C + D) \\
x_3 &= -\frac{1}{2}\frac{C - D}{C + D} \\
\alpha &= \delta - 1
\end{aligned} \tag{22}$$

$$\begin{aligned}
a &= \frac{4\hbar^2}{M}(F + G) \\
b &= -\frac{4\hbar^2}{M}G \\
t_0 &= \frac{8}{3}(A + B) \\
x_0 &= -\frac{1}{2}\frac{A - 2B}{A + B} \\
t_3 &= 16(C + D) \\
x_3 &= -\frac{1}{2}\frac{C - D}{C + D} \\
\alpha &= \delta - 1
\end{aligned} \tag{23}$$

$$\begin{aligned}
\varepsilon = & 4B\rho_n\rho_p + A(\rho_n + \rho_p)^2 + (\rho_n + \rho_p)^{\delta-1}[4D\rho_n\rho_p + C(\rho_n + \rho_p)^2] \\
& + \frac{5(\frac{2}{3})^{2/3}}{3\pi^{4/3}}\alpha_S[M(\frac{\tau_n}{M_n} + \frac{\tau_p}{M_p}) + F(\rho_n + \rho_p)(\tau_n + \tau_p) - G(\rho_n - \rho_p)(\tau_n - \tau_p)]
\end{aligned} \tag{24}$$

The standard parametrization for $T = 0$, $(M_n, M_p) \rightarrow M = \frac{1}{2}(M_n + M_p)$:

$$\begin{aligned}
\varepsilon = & \frac{3\hbar^2}{10M}(\frac{3\pi^2}{2})^{2/3}\rho^{5/3}H_{5/3}(y) + \frac{1}{8}t_0\rho^2\left[2(2 + x_0) - (1 + 2x_0)H_2(y)\right] \\
& + \frac{3}{40}(\frac{3\pi^2}{2})^{2/3}\left[aH_{5/3}(y) + bH_{8/5}(y)\right]\rho^{8/3} \\
& + \frac{1}{48}t_3\rho^{\alpha+2}\left[2(2 + x_3) - (1 + 2x_3)H_2(y)\right]
\end{aligned} \tag{25}$$

By comparing the expression in these notes with the ones from Lattimer parametrization, the energy density is:

$$\varepsilon = \alpha_S\rho^{5/3}H_{5/3}(y) + [A + B(2 - H_2(y))]\rho^2 + [C + D(2 - H_2(y))]\rho\delta + \alpha_S\rho^{5/3}[(F + G)H_{5/3} - GH_{8/3}(y)] \tag{26}$$

where, $\alpha_S = \frac{3\hbar^2}{10M}(\frac{3}{2}\pi^2)^{2/3}$

A. Skyrme parametrization from Saturation Observables

Given the following set of physical observables,

$$\begin{aligned}
E_0 = \varepsilon \Big|_{\rho_0, y=1/2} &= (A + B)\rho_0^2 + (C + D)\rho_0^{\delta+1} + \alpha_S \rho_0^{5/3} (1 + F\rho_0) \\
P = \rho^2 \frac{d(\varepsilon/\rho)}{d\rho} \Big|_{\rho_0, y=1/2} &= \frac{2}{3} \alpha_S \rho_0^{5/3} + (A + B)\rho_0^2 + \frac{5}{3} F \alpha_S \rho_0^{8/3} + (C + D)\delta \rho_0^{1+\delta} = 0 \\
(M^*/M) = \frac{d\varepsilon}{d\tau} \Big|_{\rho_0, y=1/2} &= (1 + F\rho_0)^{-1} \\
K_m = 9\rho^2 \frac{d^2(\varepsilon/\rho)}{d\rho^2} \Big|_{\rho_0, y=1/2} &= -2\alpha_S \rho_0^{2/3} + 10F\alpha_S \rho_0^{5/3} + 9(C + D)(\delta - 1)\delta \rho_0^\delta \\
S = \frac{1}{8} \frac{d^2(\varepsilon/\rho)}{dy^2} \Big|_{\rho_0, y=1/2} &= \frac{5}{9} \alpha_S \rho_0^{2/3} - B\rho_0 + \frac{5}{9} (F - 3G)\alpha_S \rho_0^{5/3} - D\rho_0^\delta \\
L = 3\rho \frac{dS}{d\rho} \Big|_{\rho_0, y=1/2} &= \frac{10}{9} \alpha_S \rho_0^{2/3} - 3B\rho_0 + \frac{25}{9} (F - 3G)\alpha_S \rho_0^{5/3} - 3D\delta \rho_0^\delta \\
K_s = 9\rho^2 \frac{d^2 S}{d\rho^2} \Big|_{\rho_0, y=1/2} &= -\frac{10}{9} \alpha_S \rho_0^{2/3} + \frac{50}{9} (F - 3G)\alpha_S \rho_0^{5/3} - 9D(\delta - 1)\delta \rho_0^\delta
\end{aligned} \tag{27}$$

the skyrme parameters can be found as follows,

$$\begin{aligned}
F &= \frac{(M^*/M)^{-1} - 1}{\rho_0} \\
\delta &= \frac{K_m + 2\rho_0^{2/3}(1 - 5F\rho_0)\alpha_S}{3\rho_0^{2/3}(1 - 2F\rho_0)\alpha_S - 9E_0} \\
G &= \frac{9K_s - 27(L - 3S)\delta + 5\rho_0^{2/3}\alpha_S[2 - 3\delta + 2F\rho_0(3\delta - 5)]}{30\rho_0^{5/3}\alpha_S(3\delta - 5)} \\
D &= \frac{5(3L - 9S + \rho_0^{2/3}\alpha_S) - 3K_s}{9(5 - 8\delta + 3\delta^2)\rho_0^\delta} \\
C &= \frac{\rho_0^{2/3}(1 - 2F\rho_0)\alpha_S - 3E_0}{3(\delta - 1)\rho_0^\delta} - D \\
B &= \frac{L(6 + 9\delta) + 5(\rho_0^{2/3}\alpha_S(3\delta - 2) - 9S\delta) - 3K_s}{18\rho_0(\delta - 1)} \\
A &= -\left[\frac{2}{3}\alpha_S\rho^{-1/3} + B + \frac{5}{3}F\alpha_S\rho^{2/3} + (C + D)\delta\rho^{\delta-1}\right]
\end{aligned} \tag{28}$$

V. FINITE TEMPERATURE DFT

In order to obtain the relationship between density and chemical potential, the following set of coupled equations need to be solved self-consistently:

$$\begin{aligned}
f_k &= \left[1 + e^{\left(\frac{k^2}{2M^*} + U - \mu \right) / T} \right]^{-1} \\
\rho &= \sum_{s, is} \int \frac{d^3 k}{(2\pi)^3} f_k \\
\tau &= \sum_{s, is} \int \frac{d^3 k}{(2\pi)^3} k^2 f_k \\
E &\equiv E(\rho, \tau)
\end{aligned} \tag{29}$$

where, a sum over all discrete quantum numbers is performed (spin and isospin). And from the energy density functional, the mean field parameters can be derived,

$$\begin{aligned}
M^* &= \frac{1}{2} \left(\frac{\delta E}{\delta \tau} \right)^{-1} \\
U &= \frac{\delta E}{\delta \rho}
\end{aligned} \tag{30}$$

The chemical potential can be found by inverting the expression for the density.

VI. THERMODYNAMIC POTENTIALS

Since the effect of phenomenological mean field models can be incorporated into (M^*, U) which are density dependent for Skyrme, and also temperature dependent for RMF, the thermodynamic properties of assymetric matter at finite temperature can be expressed by fermi integrals of ‘modified’ non-interacting fermi gases. The single particle spectrum and dustribution fuction:

$$\begin{aligned}
\xi_i &= \frac{k^2}{2M_i^*} + U_i \\
F_i &= \left\{ \exp \left[\frac{\xi_i - \mu_i}{T} \right] + 1 \right\}^{-1} = \left\{ \exp \left[\frac{\frac{k^2}{2M_i^*} - \eta_i}{T} \right] + 1 \right\}^{-1}, \quad \bar{F} = 1 - F
\end{aligned} \tag{31}$$

The density and kinetic density:

$$\begin{aligned}\rho_i &= \int_0^\infty \frac{dk}{\pi^2} k^2 F_i \\ \tau_i &= \int_0^\infty \frac{dk}{\pi^2} k^4 F_i\end{aligned}\tag{32}$$

The entropy density can be calculated from the distribution function:

$$\begin{aligned}S/V &= - \int_0^\infty \frac{dk}{\pi^2} k^2 [F_i \ln(F_i) + (1 - F_i) \ln(1 - F_i)] \\ &= - \int_0^\infty \frac{dk}{\pi^2} k^2 F_i \ln\left(\frac{F_i}{1 - F_i}\right) - \int_0^\infty \frac{dk}{\pi^2} k^2 \ln(1 - F_i) \\ &= - \int_0^\infty \frac{dk}{\pi^2} k^2 F_i \ln\left(\exp\left[\frac{-\xi_i + \mu_i}{T}\right]\right) - \left[\frac{k^3}{3\pi^2} \ln(1 - F_i)\right]_0^\infty + \int_0^\infty \frac{dk}{\pi^2} \frac{k^3}{3} \frac{k}{M_i^*} \frac{\exp\left[\frac{\xi_i - \mu_i}{T}\right]}{1 - F_i} \\ &= \left[\frac{1}{2M_i^* T} + \frac{1}{3M_i^* T}\right] \int_0^\infty \frac{dk}{\pi^2} k^4 F_i + \frac{U_i - \mu_i}{T} \int_0^\infty \frac{dk}{\pi^2} k^2 F_i \\ &= \frac{1}{T} \left[\frac{5\tau_i}{6M_i^*} + (U_i - \mu_i)\rho\right]\end{aligned}\tag{33}$$

From the first law of thermodynamics:

$$\begin{aligned}E_i &= TS_i + \mu_i N_i - P_i V \\ P_i &= \frac{TS_i + \mu_i N_i - E_i}{V} \\ &= \frac{5\tau_i}{6M_i^*} + U_i \rho_i - \frac{E_i}{V}\end{aligned}\tag{34}$$

VII. THERMODYNAMIC DERIVATIVES

Let,

$$\alpha_1 = \frac{\hbar^2}{2M}(F - G), \alpha_2 = \frac{\hbar^2}{2M}(G + F)\tag{35}$$

Then,

$$\begin{aligned}\frac{\hbar^2}{2M_i^*} &= \frac{\hbar^2}{2M} + \alpha_1 \rho_i + \alpha_2 \rho_{-i} \\ U_i &= \alpha_1 \tau_i + \alpha_2 \tau_{-i} + 2A\rho + 4B\rho_{-i} + \rho^\delta [C(1+\delta)\rho^2 + 4D\rho_{-i}[\rho - (1-\delta)\rho_i]]\end{aligned}\quad (36)$$

where, i denotes the isospin value. From expression above, the derivative of the effective mass in terms of the density can be found:

$$\begin{aligned}\partial_{\rho_i} M_r^* &= -2 \frac{M_r^{*2}}{\hbar^2} [\alpha_2 + (\alpha_1 - \alpha_2) \delta_{ir}] \\ &= -\rho_r M_r^* [F + G(1 - 2\delta_{ir})]\end{aligned}\quad (37)$$

The number density and kinetic density can be expressed in terms of the general fermi integration:

$$\begin{aligned}F_n(\eta) &= \int_0^\infty \frac{u^n}{e^{u-\eta} + 1} du, \quad \eta_i = (\mu_i - U_i)/T \\ \tau_r &= \frac{1}{2\pi^2} \left(\frac{2M_r^* T}{\hbar^2} \right)^{5/2} F_{3/2}(\eta_r) \\ \rho_r &= \frac{1}{2\pi^2} \left(\frac{2M_r^* T}{\hbar^2} \right)^{3/2} F_{1/2}(\eta_r)\end{aligned}\quad (38)$$

By inverting the expression for the density:

$$\eta_r = F_{1/2}^{-1} [2\pi^2 \rho_r \left(\frac{\hbar^2}{2M_r^* T} \right)^{3/2}] = F_{1/2}^{-1}(u_r)\quad (39)$$

And,

$$\begin{aligned}\partial_{\rho_i} u_r &= 2\pi^2 \left(\frac{\hbar^2}{2M_r^* T} \right)^{3/2} \left[\delta_{ir} + \frac{3\rho_r M_r^*}{\hbar^2} [\alpha_2 + (\alpha_1 - \alpha_2) \delta_{ir}] \right] \\ &= \frac{F_{1/2}(\eta_r)}{\rho_r} \left[\delta_{ir} + \frac{3\rho_r M_r^*}{\hbar^2} [\alpha_2 + (\alpha_1 - \alpha_2) \delta_{ir}] \right] \\ &= \frac{F_{1/2}(\eta_r)}{\rho_r} \left[\delta_{ir} + \frac{3\rho_r}{2} [F + G(1 - 2\delta_{ir})] \right]\end{aligned}\quad (40)$$

Also, $\partial_\eta F_n(\eta) = nF_{n-1}(\eta) \leftrightarrow \partial_u F_n^{-1}(u) = \frac{1}{nF_{n-1}(u)}$:

$$\begin{aligned}
\partial_{\rho_i} \eta_r &= d_u F_{1/2}^{-1}(u_r) \times \partial_{\rho_i} u_r \\
&= \frac{\partial_u F_{1/2}(u_r)}{\rho_r F_{-1/2}(\eta_r)} \left[\delta_{ir} + \frac{3\rho_r M_r^*}{\hbar^2} [\alpha_2 + (\alpha_1 - \alpha_2) \delta_{ir}] \right] \\
&= \frac{2F_{1/2}(u_r)}{\rho_r F_{-1/2}(\eta_r)} \left[\delta_{ir} + \frac{3\rho_r M_r^*}{\hbar^2} [\alpha_2 + (\alpha_1 - \alpha_2) \delta_{ir}] \right] \\
&\equiv \frac{G_r}{\rho_r} \left[\delta_{ir} + \frac{3\rho_r M_r^*}{\hbar^2} [\alpha_2 + (\alpha_1 - \alpha_2) \delta_{ir}] \right] \\
&= \frac{G_r}{\rho_r} \left[\delta_{ir} + \frac{3\rho_r}{2} [F + G(1 - 2\delta_{ir})] \right]
\end{aligned} \tag{41}$$

where, $G_r = \frac{2F_{1/2}(\eta_r)}{F_{-1/2}(\eta_r)}$. Thus,

$$\begin{aligned}
\partial_{\rho_i} \tau_r &= \frac{5}{4\pi^2 M_r^*} \left(\frac{2M_r^* T}{\hbar^2} \right)^{5/2} F_{3/2}(\eta_r) \partial_{\rho_i} M_r^* + \frac{3}{4\pi^2} \left(\frac{2M_r^* T}{\hbar^2} \right)^{3/2} F_{1/2}(\eta_r) \partial_{\rho_i} \eta_r \\
&= \frac{M_r^*}{\hbar^2} \left[3TG_r \delta_{ir} + \left(\frac{9M_r^*}{\hbar^2} T \rho_r G_r - 5\tau_r \right) [\alpha_2 + (\alpha_1 - \alpha_2) \delta_{ir}] \right] \\
&= 3TG_r \frac{M_r^*}{\hbar^2} \delta_{ir} + \frac{1}{2} \left(\frac{9M_r^*}{\hbar^2} T \rho_r G_r - 5\tau_r \right) [F + G(1 - 2\delta_{ir})]
\end{aligned} \tag{42}$$

$$\begin{aligned}
\partial_{\rho_i} \mu_r &= T \partial_{\rho_i} \eta_r + \partial_{\rho_i} U_r \\
&= \frac{TG_r}{\rho_r} \left[\delta_{ir} + \frac{3\rho_r M_r^*}{\hbar^2} [\alpha_2 + (\alpha_1 - \alpha_2) \delta_{ir}] \right] + \partial_{\rho_i} U_r
\end{aligned}$$