I. SKYRME INTERACTION - FINITE TEMPERATURE

A. Potential Matrix Element

Interaction Matrix:

$$
\begin{aligned}
V_{ij} =& t_0(1+x_0P_\sigma)\delta(\mathbf{r}_{ij}) + \frac{1}{2}t_1(1+x_1P_\sigma)\frac{1}{\hbar^2}[\overleftarrow{p^{\dagger}}{}^2_{ij}\delta(\mathbf{r}_{ij}) + \delta(\mathbf{r}_{ij})\overrightarrow{p}{}^2_{ij}] \\
&+ t_2(1+x_2P_\sigma)\frac{1}{\hbar^2}\overleftarrow{\mathbf{p}^{\dagger}}_{ij}\cdot\delta(\mathbf{r}_{ij})\overrightarrow{\mathbf{p}}_{ij} + \frac{1}{6}t_3(1+x_3P_\sigma)\rho^{\alpha}(\mathbf{r})\delta(\mathbf{r}_{ij}) \\
&+ \frac{i}{\hbar^2}W_0(\sigma_i+\sigma_j)\cdot\overleftarrow{\mathbf{p}^{\dagger}}_{ij}\times\delta(\mathbf{r}_{ij})\overrightarrow{\mathbf{p}}_{ij} \\
&+ \frac{1}{4}t_4(1+x_4P_\sigma)\frac{1}{\hbar^2}[\overleftarrow{p^{\dagger}}{}^2_{ij}\rho^{\beta}(\mathbf{r})\delta(\mathbf{r}_{ij}) + \delta(\mathbf{r}_{ij})\rho^{\beta}(\mathbf{r})\overrightarrow{p}{}^2_{ij}] \\
&+ t_5(1+x_5P_\sigma)\frac{1}{\hbar^2}\overleftarrow{\mathbf{p}}_{ij}\cdot\rho^{\gamma}(\mathbf{r})\delta(\mathbf{r}_{ij})\overrightarrow{\mathbf{p}}_{ij}
\end{aligned} \tag{1}
$$

where, $\mathbf{r}_{ij} = \frac{\mathbf{r}_i - \mathbf{r}_j}{2}$, $\mathbf{r} = \frac{\mathbf{r}_i + \mathbf{r}_j}{2}$, $P_\sigma = \frac{1}{2}(1+\sigma_1\cdot\sigma_2)$, $\mathbf{p}_{ij} = -i\hbar\frac{\nabla_i - \nabla_j}{2}$, $\rho(\mathbf{r}) = \rho_p(\mathbf{r}) + \rho_n(\mathbf{r})$ and the arrows show the direction on which the momentum operators act.

B. Energy Density

Assume time reversal invariance. The Energy of the ground state can be written as integration over an energy density functional, $E_{HFB} = \int d^r \varepsilon_{HFB}$

$$
\begin{aligned}
\varepsilon_{HFB} = & \sum_{t=n,p} \frac{\hbar^2}{2M_q}\tau_q + \frac{1}{2}t_0\Big[(1+\frac{1}{2}x_0)\rho^2 - (\frac{1}{2}+x_0)\sum_{q=n,p}\rho_q^2\Big] \\
& + \frac{1}{4}t_1\Big[(1+\frac{1}{2}x_1)(\rho\tau + \frac{3}{4}(\nabla\rho)^2) - (\frac{1}{2}+x_1)\sum_{q=n,p}(\rho_q\tau_q + \frac{3}{4}(\nabla\rho_q)^2)\Big] \\
& + \frac{1}{4}t_2\Big[(1+\frac{1}{2}x_2)(\rho\tau + \frac{3}{4}(\nabla\rho)^2) + (\frac{1}{2}+x_2)\sum_{q=n,p}(\rho_q\tau_q - \frac{1}{4}(\nabla\rho_q)^2)\Big] \\
& + \frac{1}{12}t_3\rho^\alpha\Big[(1+\frac{1}{2}x_3)\rho^2 - (\frac{1}{2}+x_3)\sum_{q=n,p}\rho_q^2\Big] \\
& + \frac{1}{4}t_4\Big[(1+\frac{1}{2}x_4)(\rho\tau + \frac{3}{4}(\nabla\rho)^2) - (\frac{1}{2}+x_4)\sum_{q=n,p}(\rho_q\tau_q + \frac{3}{4}(\nabla\rho_q)^2)\Big]\rho^\beta \\
& + \frac{\beta}{8}t_4\Big[(1+\frac{1}{2}x_4)\rho(\nabla\rho)^2 - (\frac{1}{2}+x_4)\nabla\rho\cdot\sum_{q=n,p}\rho_q\nabla\rho_q\Big]\rho^{\beta-1} \\
& + \frac{1}{4}t_5\Big[(1+\frac{1}{2}x_5)(\rho\tau + \frac{3}{4}(\nabla\rho)^2) + (\frac{1}{2}+x_5)\sum_{q=n,p}(\rho_q\tau_q - \frac{1}{4}(\nabla\rho_q)^2)\Big]\rho^\gamma \\
& - \frac{1}{16}(t_1x_1 + t_2x_2)J^2 + \frac{1}{16}(t_1 - t_2)\sum_{q=n,p}J_q^2 \\
& - \frac{1}{16}(t_4x_4\rho^\beta + t_5x_5\rho^\gamma)J^2 + \frac{1}{16}(t_4\rho^\beta - t_5\rho^\beta)\sum_{q=n,p}J_q^2 \\
& + \frac{1}{2}W_0(\mathbf{J}\cdot\nabla\rho + \sum_{q=n,p}\mathbf{J}_q\cdot\nabla\rho_q)
\end{aligned} \tag{2}
$$

where,

$$
\begin{aligned}
\rho &= 2\int \frac{d^3k}{(2\pi\hbar)^3}n(k) \\
\tau &= 2\int \frac{d^3k}{(2\pi\hbar)^3}k^2 n(k) \\
\mathbf{J} &= \int \frac{d^3k}{(2\pi\hbar)^3}\mathbf{k}\times\sum_{s,s'}\langle s|\boldsymbol{\sigma}|s'\rangle\ n(k)
\end{aligned} \tag{3}
$$

The different terms can be grouped together in simpler notation:

$$
\begin{aligned}
\varepsilon = &\sum_{t=n,p} \frac{\hbar^2}{2M_q}\tau_q + \frac{1}{4}t_0\Big[(2+x_0)\rho^2 - (1+2x_0)\sum_{q=n,p}\rho_q^2\Big] \\
&+ \frac{1}{8}\Big[a(\rho\tau + \frac{3}{4}(\nabla\rho)^2) + 2b\sum_{q=n,p}(\rho_q\tau_q + \frac{3}{4}(\nabla\rho_q)^2)\Big] \\
&+ \frac{1}{24}t_3\rho^\alpha\Big[(2+x_3)\rho^2 - (1+2x_3)\sum_{q=n,p}\rho_q^2\Big] \\
&+ \frac{1}{8}t_4\Big[(2+x_4)(\rho\tau + \frac{3}{4}(\nabla\rho)^2) - (1+2x_4)\sum_{q=n,p}(\rho_q\tau_q + \frac{3}{4}(\nabla\rho_q)^2)\Big]\rho^\beta \\
&+ \frac{\beta}{16}t_4\Big[(2+x_4)\rho(\nabla\rho)^2 - (1+2x_4)\nabla\rho\cdot\sum_{q=n,p}\rho_q\nabla\rho_q\Big]\rho^{\beta-1} \\
&+ \frac{1}{8}t_5\Big[(2+x_5)(\rho\tau + \frac{3}{4}(\nabla\rho)^2) + (1+2x_5)\sum_{q=n,p}(\rho_q\tau_q - \frac{1}{4}(\nabla\rho_q)^2)\Big]\rho^\gamma \\
&- \frac{1}{16}(t_1x_1 + t_2x_2)J^2 + \frac{1}{16}(t_1 - t_2)\sum_{q=n,p}J_q^2 \\
&- \frac{1}{16}(t_4x_4\rho^\beta + t_5x_5\rho^\gamma)J^2 + \frac{1}{16}(t_4\rho^\beta - t_5\rho^\beta)\sum_{q=n,p}J_q^2 \\
&+ \frac{1}{2}W_0(\mathbf{J}\cdot\nabla\rho + \sum_{q=n,p}\mathbf{J}_q\cdot\nabla\rho_q)
\end{aligned}
\tag{4}
$$

where, $a = t_1(x_1+2) + t_2(x_2+2),\ b = \frac{1}{2}[t_2(2x_2+1) - t_1(2x_1+1)]$.

In uniform matter $\nabla\rho = 0$:

$$
\begin{aligned}
\varepsilon = & \sum_{t=n,p} \frac{\hbar^2}{2M_q}\tau_q + \frac{1}{4}t_0\Big[(2+x_0)\rho^2 - (1+2x_0)\sum_{q=n,p}\rho_q^2\Big] \\
& + \frac{1}{8}\Big[a\rho\tau + 2b\sum_{q=n,p}\rho_q\tau_q\Big] \\
& + \frac{1}{24}t_3\rho^\alpha\Big[(2+x_3)\rho^2 - (1+2x_3)\sum_{q=n,p}\rho_q^2\Big] \\
& + \frac{1}{8}t_4\Big[(2+x_4)\rho\tau - (1+2x_4)\sum_{q=n,p}\rho_q\tau_q\Big]\rho^\beta \\
& + \frac{1}{8}t_5\Big[(2+x_5)\rho\tau + (1+2x_5)\sum_{q=n,p}\rho_q\tau_q\Big]\rho^\gamma \\
& - \frac{1}{16}(t_1x_1 + t_2x_2)J^2 + \frac{1}{16}(t_1 - t_2)\sum_{q=n,p}J_q^2 \\
& - \frac{1}{16}(t_4x_4\rho^\beta + t_5x_5\rho^\gamma)J^2 + \frac{1}{16}(t_4\rho^\beta - t_5\rho^\beta)\sum_{q=n,p}J_q^2
\end{aligned}
\tag{5}
$$

In unpolarized matter, $\mathbf{J} = 0$:

$$
\begin{aligned}
\varepsilon = & \sum_{t=n,p} \frac{\hbar^2}{2M_q}\tau_q + \frac{1}{4}t_0\Big[(2+x_0)\rho^2 - (1+2x_0)\sum_{q=n,p}\rho_q^2\Big] \\
& + \frac{1}{8}\Big[a\rho\tau + 2b\sum_{q=n,p}\rho_q\tau_q\Big] \\
& + \frac{1}{24}t_3\rho^\alpha\Big[(2+x_3)\rho^2 - (1+2x_3)\sum_{q=n,p}\rho_q^2\Big] \\
& + \frac{1}{8}t_4\Big[(2+x_4)\rho\tau - (1+2x_4)\sum_{q=n,p}\rho_q\tau_q\Big]\rho^\beta \\
& + \frac{1}{8}t_5\Big[(2+x_5)\rho\tau + (1+2x_5)\sum_{q=n,p}\rho_q\tau_q\Big]\rho^\gamma
\end{aligned}
\tag{6}
$$

Energy per bayon, $\mathcal{E} \equiv \varepsilon/rho$:

$$\begin{aligned}\mathcal{E} = & \sum_{t=n,p} \frac{\hbar^2}{2M_q}\frac{\tau_q}{\rho} + \frac{1}{4}t_0\Big[(2+x_0)\rho - (1+2x_0)\sum_{q=n,p}\frac{\rho_q^2}{\rho}\Big] \\ & + \frac{1}{8}\Big[a\tau + 2b\sum_{q=n,p}\frac{\rho_q\tau_q}{\rho}\Big] \\ & + \frac{1}{24}t_3\rho^{\alpha}\Big[(2+x_3)\rho - (1+2x_3)\sum_{q=n,p}\frac{\rho_q^2}{\rho}\Big] \\ & + \frac{1}{8}t_4\Big[(2+x_4)\tau - (1+2x_4)\sum_{q=n,p}\frac{\rho_q\tau_q}{\rho}\Big]\rho^{\beta} \\ & + \frac{1}{8}t_5\Big[(2+x_5)\tau + (1+2x_5)\sum_{q=n,p}\frac{\rho_q\tau_q}{\rho}\Big]\rho^{\gamma}\end{aligned} \tag{7}$$

In terms of proton fraction, $y = \frac{\rho_p}{\rho_p+\rho_n}$:

$$\begin{aligned}\mathcal{E} = & \sum_{t=n,p} \frac{\hbar^2}{2M_q}\frac{\tau_q}{\rho} + \frac{1}{4}t_0\Big[(2+x_0) - (1+2x_0)[y^2+(1-y)^2]\Big]\rho \\ & + \frac{1}{8}\Big[a\tau + 2b[y\tau_p + (1-y)\tau_n]\Big] \\ & + \frac{1}{24}t_3\rho^{\alpha+1}\Big[(2+x_3) - (1+2x_3)[y^2+(1-y)^2]\Big] \\ & + \frac{1}{8}t_4\Big[(2+x_4)\tau - (1+2x_4)[y\tau_p + (1-y)\tau_n]\Big]\rho^{\beta} \\ & + \frac{1}{8}t_5\Big[(2+x_5)\tau + (1+2x_5)[y\tau_p + (1-y)\tau_n]\Big]\rho^{\gamma}\end{aligned} \tag{8}$$

II. SINGLE PARTICLE PROPERTIES

From the energy density the single particle spectrum can be derived. By performing functional variation of the energy density with repsect to the single particle wavefunction, a modified Schrodinger equation can be derived:

$$\delta\varepsilon_i = [\frac{\delta\varepsilon_i}{\delta\tau_i} + \frac{\delta\varepsilon_i}{\delta\rho_i}]\delta\phi_i = \epsilon_i\delta\phi_i \tag{9}$$

Since the Skyrme potential is at mostly quadratic in momenta with nonlinaer density dependence, its effect is exactly included by effective mass and mean field shift(residual interaction), both density dependent:

$$
\begin{aligned}
\epsilon_i(k) =& \frac{\hbar^2 k^2}{2M_i^*} + U_i \\
\frac{\hbar^2}{2M_q^*} \equiv& \frac{\partial \varepsilon}{\partial \tau_q} \\
U_i \equiv& \frac{\partial \varepsilon}{\partial \rho_i}
\end{aligned}
\tag{10}
$$

From eq. 6 the effective baryon masses:

$$
\begin{aligned}
M_p^*/M =& \left\{ 1 + \frac{M\ \rho}{4\hbar^2} \left[a + 2\ y\ b + t_4[(2+x_4) - (1+2x_4)\ y]\rho^\beta + t_5[2 + x_5 + (1+2x_5)\ y]\rho^\gamma \right] \right\}^{-1} \\
M_n^*/M =& \left\{ 1 + \frac{M\ \rho}{4\hbar^2} \left[a + 2\ (1-y)\ b + t_4[(2+x_4) - (1+2x_4)\ (1-y)]\rho^\beta + t_5[2 + x_5 + (1+2x_5)\ (1-y)]\rho^\gamma \right] \right\}^{-1}
\end{aligned}
\tag{11}
$$

and the residual potentials:

$$
\begin{aligned}
U_p =& \frac{1}{8}(2b\ \tau_p + a\ \tau) + \frac{1}{2} t_0[(2+x_0) - (1+2x_0)\ y]\rho \\
&+ \frac{1}{24} t_3 \Big[4 + \alpha - 2y(1 - (1-y)\alpha) + x_3(1-2y)[2 - (1-2y)\alpha] \Big] \rho^{\alpha+1} \\
U_n =& \frac{1}{8}(2b\ \tau_n + a\ \tau) + \frac{1}{2} t_0[(1-x_0) + (1+2x_0)y]\rho \\
&+ \frac{1}{24} t_3 \Big[2 + \alpha + 2y(1 + \alpha - y\alpha) - x_3(1-2y)[2 + (1-2y)\alpha] \Big] \rho^{\alpha+1}
\end{aligned}
\tag{12}
$$

III. T=0 DFT

At $T = 0$, there are simple relation that can be drawn between the 2 integrations in Fourier space since the occuppation number is a step function:

$$\begin{aligned} \rho_q &= 2\int \frac{d^3k}{(2\pi\hbar)^3}\theta(k_{F,q}-k) = \frac{k_{F,q}^3}{3\pi^2\hbar^3} \\ \tau_q &= 2\int \frac{d^3k}{(2\pi\hbar)^3}(k/\hbar)^2\theta(k_{F,q}-k) = \frac{k_{F,q}^5}{5\pi^2\hbar^5} \rightarrow \\ \tau &= \frac{3}{5}(3\pi^2)^{2/3}\rho_q^{5/3}, H_n(y) = 2^{n-1}[y^n + (1-y)^n], y = \rho_p/\rho \\ \tau &= \tau_p + \tau_n = \frac{3}{5}(\frac{3\pi^2}{2})^{2/3}H_{5/3}(y)\rho^{5/3} \end{aligned} \tag{13}$$

So,

$$\begin{aligned} \mathcal{E}_0 = & \frac{3\hbar^2}{10M}(\frac{3\pi^2}{2})^{2/3}\rho^{2/3}H_{5/3}(y) + \frac{1}{8}t_0\rho\Big[2(2+x_0) - (1+2x_0)H_2(y)\Big] \\ & + \frac{3}{40}(\frac{3\pi^2}{2})^{2/3}\Big[aH_{5/3}(y) + bH_{8/5}(y)\Big]\rho^{5/3} \\ & + \frac{1}{48}t_3\rho^{\alpha+1}\Big[2(2+x_3) - (1+2x_3)H_2(y)\Big] \\ & + \frac{3}{40}(\frac{3\pi^2}{2})^{2/3}t_4\rho^{\beta+5/3}\Big[(2+x_4)H_{5/3}(y) - (\frac{1}{2}+x_4)H_{8/3}(y)\Big] \\ & + \frac{3}{40}(\frac{3\pi^2}{2})^{2/3}t_5\rho^{\gamma+5/3}\Big[(2+x_5)H_{5/3}(y) + (\frac{1}{2}+x_5)H_{8/3}(y)\Big] \end{aligned} \tag{14}$$

with $H_n(y) = 2^{n-1}[y^n + (1-y)^n]$. A common choice is to set $M = 1/2(M_n + M_p)$, or use the individual value for each species. In compact notation,

$$\mathcal{E}_0 = C(y)\rho^{2/3} + A(y)\rho + B(y)\rho^{\alpha+1} + D(y)\rho^{5/3} + G(y)\rho^{\beta+5/3} + K(y)\rho^{\gamma+5/3} \tag{15}$$

By comparing the 2 expressions, the following relations can be easily deduced:

$$
\begin{aligned}
C(y) =& \frac{3\hbar^2}{10M_y}(\frac{3\pi^2}{2})^{2/3}H_{5/3}(y) \\
A(y) =& \frac{1}{8}t_0\Big[2(2+x_0)-(1+2x_0)H_2(y)\Big] \\
B(y) =& \frac{1}{48}t_3\Big[2(2+x_3)-(1+2x_3)H_2(y)\Big] \\
D(y) =& \frac{3}{40}(\frac{3\pi^2}{2})^{2/3}\Big[aH_{5/3}(y)+bH_{8/5}(y)\Big] \\
G(y) =& \frac{3}{40}(\frac{3\pi^2}{2})^{2/3}t_4\Big[(2+x_4)H_{5/3}(y)-(\frac{1}{2}+x_4)H_{8/3}(y)\Big] \\
K(y) =& \frac{3}{40}(\frac{3\pi^2}{2})^{2/3}t_5\Big[(2+x_5)H_{5/3}(y)+(\frac{1}{2}+x_5)H_{8/3}(y)\Big]
\end{aligned}
\tag{16}
$$

For pure neutron matter and symmetric matter the kinetic coefficient is

$$
\begin{aligned}
C_n =& C(0) = \frac{3\hbar^2}{10M_n}(\frac{3\pi^2}{2})^{2/3}H_{5/3}(0) = 118.995 \text{ Mev fm}^2 \approx 119 \text{ Mev fm}^2 \\
C_{sym} =& C(1/2) = \frac{3\hbar^2}{5(M_n+M_p)}(\frac{3\pi^2}{2})^{2/3}H_{5/3}(1/2) = 75.0139 \text{ Mev fm}^2 \approx 75 \text{ Mev fm}^2
\end{aligned}
\tag{17}
$$

The effective mass is due to the terms dependent on kinetic energy:

$$
\begin{aligned}
\tau^{T=0}(\rho,y) \equiv& \frac{3\hbar^2}{10M^*}(\frac{3\pi^2}{2})^{2/3}\rho^{2/3}H_{5/3}(y) \\
=& \frac{3\hbar^2}{10M}(\frac{3\pi^2}{2})^{2/3}\rho^{2/3}H_{5/3}(y)+\frac{3}{40}(\frac{3\pi^2}{2})^{2/3}\Big[aH_{5/3}(y)+bH_{8/5}(y)\Big]\rho^{5/3} \\
&+\frac{3}{40}(\frac{3\pi^2}{2})^{2/3}t_4\rho^{\beta+5/3}\Big[(2+x_4)H_{5/3}(y)-(\frac{1}{2}+x_4)H_{8/3}(y)\Big] \\
&+\frac{3}{40}(\frac{3\pi^2}{2})^{2/3}t_5\rho^{\gamma+5/3}\Big[(2+x_5)H_{5/3}(y)+(\frac{1}{2}+x_5)H_{8/3}(y)\Big] \\
\equiv& C(y)\rho^{2/3}\Big[1+(D(y)\rho+G(y)\rho^{\beta+1}+K(y)\rho^{\gamma+1})/C(y)\Big]
\end{aligned}
\tag{18}
$$

Thus,

$$
\begin{aligned}
M^*/M = \{1 + &\frac{M}{4H_{5/3}(y)\hbar^2}\Big[\rho[aH_{5/3}(y) + bH_{8/5}(y)] + \rho^{\beta+1}t_4[(2+x_4)H_{5/3}(y) - (\frac{1}{2}+x_4)H_{8/3}(y)] \\
&+ \rho^{\gamma+1}t_5[(2+x_5)H_{5/3}(y) - (\frac{1}{2}+x_5)H_{8/3}(y)]\Big]\}^{-1} \\
=&\frac{C(y)}{C(y)+D(y)\rho+G(y)\rho^{\beta+1}+K(y)\rho^{\gamma+1}}
\end{aligned}
\tag{19}
$$

Also, the thermodynamic pressure:

$$
\begin{aligned}
\mathcal{P}^{T=0}(\rho,y) &= \rho^2\frac{\partial\mathcal{E}^{T=0}}{\partial\rho} \\
&= \frac{2}{3}C(y)\rho^{5/3} + A(y)\rho^2 + (\alpha+1)B(y)\rho^{\alpha+1} + \frac{5}{3}D(y)\rho^{8/3} + \beta G(y)\rho^{\beta+8/3} + \gamma K(y)\rho^{\gamma+8/3}
\end{aligned}
\tag{20}
$$

IV. LATTIMER - SCHWESTY NOTATION

The standard parametrization:

$$
\begin{aligned}
\varepsilon =&\frac{\hbar^2\tau_n}{2M_n} + \frac{\hbar^2\tau_p}{2tM_p} + \frac{1}{8}[a(\rho_n+\rho_p)(\tau_n+\tau_p) + 2b(\rho_n\tau_n+\rho_p\tau_p)] + \frac{1}{4}t_0[(2+x_0)(\rho_n+\rho_p)^2 - (1+2x_0)(\rho_n^2+\rho_p^2)] \\
&+ \frac{1}{24}t_3(\rho_n+\rho_p)^\alpha[(2+x_3)(\rho_n+\rho_p)^2 - (1+2x_3)(\rho_n^2+\rho_p^2)]
\end{aligned}
\tag{21}
$$

$$
\begin{aligned}
a =&\frac{4\hbar^2}{M}(F+G) \\
b =&-\frac{4\hbar^2}{M}G \\
t_0 =&\frac{8}{3}(A+B) \\
x_0 =&-\frac{1}{2}\frac{A-2B}{A+B} \\
t_3 =&16(C+D) \\
x_3 =&-\frac{1}{2}\frac{C-D}{C+D} \\
\alpha =&\delta-1
\end{aligned}
\tag{22}
$$

$$
\begin{aligned}
a =&\frac{4\hbar^2}{M}(F+G) \\
b =&-\frac{4\hbar^2}{M}G \\
t_0 =&\frac{8}{3}(A+B) \\
x_0 =&-\frac{1}{2}\frac{A-2B}{A+B} \\
t_3 =&16(C+D) \\
x_3 =&-\frac{1}{2}\frac{C-D}{C+D} \\
\alpha =&\delta-1
\end{aligned}
\tag{23}
$$

$$
\begin{aligned}
\varepsilon =& 4B\rho_n\rho_p + A(\rho_n+\rho_p)^2 + (\rho_n+\rho_p)^{\delta-1}[4D\rho_n\rho_p + C(\rho_n+\rho_p)^2] \\
&+ \frac{5(\frac{2}{3})^{2/3}}{3\pi^{4/3}}\alpha_S[M(\frac{\tau_n}{M_n}+\frac{\tau_p}{M_p}) + F(\rho_n+\rho_p)(\tau_n+\tau_p) - G(\rho_n-\rho_p)(\tau_n-\tau_p)]
\end{aligned} \tag{24}
$$

The standard parametrization for $T = 0,\ (M_n, M_p) \to M = \frac{1}{2}(M_n + M_p)$:

$$
\begin{aligned}
\varepsilon =& \frac{3\hbar^2}{10M}(\frac{3\pi^2}{2})^{2/3}\rho^{5/3}H_{5/3}(y) + \frac{1}{8}t_0\rho^2\Big[2(2+x_0) - (1+2x_0)H_2(y)\Big] \\
&+ \frac{3}{40}(\frac{3\pi^2}{2})^{2/3}\Big[aH_{5/3}(y) + bH_{8/5}(y)\Big]\rho^{8/3} \\
&+ \frac{1}{48}t_3\rho^{\alpha+2}\Big[2(2+x_3) - (1+2x_3)H_2(y)\Big]
\end{aligned} \tag{25}
$$

By comparing the expression in these notes with the ones from Lattimer parametrization, the energy density is:

$$
\varepsilon = \alpha_S\rho^{5/3}H_{5/3}(y) + [A + B(2 - H_2(y))]\rho^2 + [C + D(2 - H_2(y))]\rho\delta + \alpha_S\rho^{5/3}[(F+G)H_{5/3} - GH_{8/3}(y)] \tag{26}
$$

where, $\alpha_S = \frac{3\hbar^2}{10M}(\frac{3}{2}\pi^2)^{2/3}$

A. Skyrme parametrization from Saturation Observables

Given the following set of physical observables,

$$
\begin{aligned}
E_0 =& \varepsilon\Big|_{\rho_0,y=1/2} = (A+B)\rho_0^2 + (C+D)\rho_0^{\delta+1} + \alpha_S\rho_0^{5/3}(1+F\rho_0) \\
P =& \rho^2\frac{d(\varepsilon/\rho)}{d\rho}\Big|_{\rho_0,y=1/2} = \frac{2}{3}\alpha_S\rho_0^{5/3} + (A+B)\rho_0^2 + \frac{5}{3}F\alpha_S\rho_0^{8/3} + (C+D)\delta\rho_0^{1+\delta} = 0 \\
(M^*/M) =& \frac{d\varepsilon}{d\tau}\Big|_{\rho_0,y=1/2} = (1+F\rho_0)^{-1} \\
K_m =& 9\rho^2\frac{d^2(\varepsilon/\rho)}{d\rho^2}\Big|_{\rho_0,y=1/2} = -2\alpha_S\rho_0^{2/3} + 10F\alpha_S\rho_0^{5/3} + 9(C+D)(\delta-1)\delta\rho_0^{\delta} \\
S =& \frac{1}{8}\frac{d^2(\varepsilon/\rho)}{dy^2}\Big|_{\rho_0,y=1/2} = \frac{5}{9}\alpha_S\rho_0^{2/3} - B\rho_0 + \frac{5}{9}(F-3G)\alpha_S\rho_0^{5/3} - D\rho_0^{\delta} \\
L =& 3\rho\frac{dS}{\rho}\Big|_{\rho_0,y=1/2} = \frac{10}{9}\alpha_S\rho_0^{2/3} - 3B\rho_0 + \frac{25}{9}(F-3G)\alpha_S\rho_0^{5/3} - 3D\delta\rho_0^{\delta} \\
K_s =& 9\rho^2\frac{d^2S}{d\rho^2}\Big|_{\rho_0,y=1/2} = -\frac{10}{9}\alpha_S\rho_0^{2/3} + \frac{50}{9}(F-3G)\alpha_S\rho_0^{5/3} - 9D(\delta-1)\delta\rho_0^{\delta}
\end{aligned}
\tag{27}
$$

the skyrme parameters can be found as follows,

$$
\begin{aligned}
F =& \frac{(M^*/M)^{-1}-1}{\rho_0} \\
\delta =& \frac{K_m + 2\rho_0^{2/3}(1-5F\rho_0)\alpha_S}{3\rho_0^{2/3}(1-2F\rho_0)\alpha_S - 9E_0} \\
G =& \frac{9K_S - 27(L-3S)\delta + 5\rho_0^{2/3}\alpha_S[2-3\delta+2F\rho_0(3\delta-5)]}{30\rho_0^{5/3}\alpha_S(3\delta-5)} \\
D =& \frac{5(3L-9S+\rho_0^{2/3}\alpha_S) - 3K_S}{9(5-8\delta+3\delta^2)\rho_0^{\delta}} \\
C =& \frac{\rho_0^{2/3}(1-2F\rho_0)\alpha_S - 3E_0}{3(\delta-1)\rho_0^{\delta}} - D \\
B =& \frac{L(6+9\delta) + 5(\rho_0^{2/3}\alpha_S(3\delta-2) - 9S\delta) - 3K_S}{18\rho_0(\delta-1)} \\
A =& -[\frac{2}{3}\alpha_S\rho^{-1/3} + B + \frac{5}{3}F\alpha_S\rho^{2/3} + (C+D)\delta\rho^{\delta-1}]
\end{aligned}
\tag{28}
$$

V. FINITE TEMPERATURE DFT

In order to obtain the relationship between density and chemical potential, the following set of coupled equations need to be solved self-consistenly:

$$
\begin{aligned}
f_k &= \left[1 + e^{\left(\frac{k^2}{2M^*} + U - \mu\right)/T}\right]^{-1} \\
\rho &= \sum_{s,is} \int \frac{d^3k}{(2\pi)^3} f_k \\
\tau &= \sum_{s,is} \int \frac{d^3k}{(2\pi)^3} k^2 f_k \\
E &\equiv E(\rho, \tau)
\end{aligned}
\tag{29}
$$

where, a sum over all discrete quantum numbers is performed (spin and isospin). And from the energy density functional, the mean field parameters can be derived,

$$
\begin{aligned}
M^* &= \frac{1}{2}\left(\frac{\delta E}{\delta \tau}\right)^{-1} \\
U &= \frac{\delta E}{\delta \rho}
\end{aligned}
\tag{30}
$$

The chemical potential can be found by inverting the expression for the density.

VI. THERMODYNAMIC POTENTIALS

Since the effect of phenomenological mean field models can be incorporated into (M^*, U) which are density dependent for Skyrme, and also temperature dependent for RMF, the thermodynamic properties of assymetric matter at finite temperature can be expressed by fermi integrals of 'modified' non-interacting fermi gases. The single particle spectrum and dustribution fucntion:

$$
\begin{aligned}
\xi_i =& \frac{k^2}{2M_i^*} + U_i \\
F_i =& \Big\{ \exp\big[\frac{\xi_i - \mu_i}{T}\big] + 1 \Big\}^{-1} = \Big\{ \exp\big[\frac{\frac{k^2}{2M_i^*} - \eta_i}{T}\big] + 1 \Big\}^{-1}, \ \overline{F} = 1 - F
\end{aligned}
\tag{31}
$$

The density and kinetic density:

$$\begin{aligned} \rho_i &= \int_0^\infty \frac{dk}{\pi^2} k^2 F_i \\ \tau_i &= \int_0^\infty \frac{dk}{\pi^2} k^4 F_i \end{aligned} \tag{32}$$

The entropy density can be calcualted fro mthe distribution function:

$$\begin{aligned} S/V &= -\int_0^\infty \frac{dk}{\pi^2}\ k^2 \big[F_i \ln(F_i) + (1-F_i)\ln(1-F_i)\big] \\ &= -\int_0^\infty \frac{dk}{\pi^2}\ k^2 F_i \ln(\frac{F_i}{1-F_i}) - \int_0^\infty \frac{dk}{\pi^2}\ k^2 \ln(1-F_i) \\ &= -\int_0^\infty \frac{dk}{\pi^2}\ k^2 F_i \ln(\exp\big[\frac{-\xi_i+\mu_i}{T}\big]) - [\frac{k^3}{3\pi^2}\ln(1-F_i)]\Big|_0^\infty + \int_0^\infty \frac{dk}{\pi^2}\ \frac{k^3}{3}\frac{k}{M_i^*}\frac{\exp\big[\frac{\xi_i-\mu_i}{T}\big]}{1-F_i} \\ &=[\frac{1}{2M_i^*T} + \frac{1}{3M_i^*T}] \int_0^\infty \frac{dk}{\pi^2}\ k^4 F_i + \frac{U_i-\mu_i}{T}\ \int_0^\infty \frac{dk}{\pi^2}\ k^2 F_i \\ &= \frac{1}{T}[\frac{5\tau_i}{6M_i^*} + (U_i-\mu_i)\rho] \end{aligned} \tag{33}$$

From the first law of thermodynamics:

$$\begin{aligned} E_i &= TS_i + \mu_i N_i - P_i V \\ P_i &= \frac{TS_i + \mu_i N_i - E_i}{V} \\ &= \frac{5\tau_i}{6M_i^*} + U_i\rho_i - \frac{E_i}{V} \end{aligned} \tag{34}$$

VII. THERMODYNAMIC DERIVATIVES

Let,

$$\alpha_1 = \frac{\hbar^2}{2M}(F-G), \alpha_2 = \frac{\hbar^2}{2M}(G+F) \tag{35}$$

Then,

$$\frac{\hbar^2}{2M_i^*} = \frac{\hbar^2}{2M} + \alpha_1 n_i + \alpha_2 n_{-i} \tag{36}$$

where, i denotes the isospin value. From epxression above, the derivative of the effective mass in terms of the density can be found:

$$\partial_{n_i} M_r^* = -2\frac{M_r^{*2}}{\hbar^2}\big[\alpha_2 + (\alpha_1 - \alpha_2)\delta_{ir}\big] \tag{37}$$

The density and kinetic density can be expressed in terms of the general fermi integration:

$$\begin{aligned} F_n(\eta) &= \int_0^\infty \frac{u^n}{e^{u-\eta}+1}du,\ \eta_i = (\mu_i - U_i)/T,\ U_i = \frac{\delta\mathcal{E}}{\delta n_i} \\ n_i =& \frac{1}{2\pi^2}(\frac{2M_r^*T}{\hbar^2})^{3/2}F_{1/2}(\eta_i) \leftrightarrow \eta_i = F_{1/2}^{-1}\big[2\pi^2 n_i(\frac{\hbar^2}{2M_r^*T})^{3/2}\big] = F_{1/2}^{-1}(u_i) \end{aligned} \tag{38}$$

And,

$$\begin{aligned} \partial_{n_i} u_r =& 2\pi^2(\frac{\hbar^2}{2M_r^*T})^{3/2}\Big[\delta_{ir} + \frac{3n_r M_r^*}{\hbar^2}\big[\alpha_2 + (\alpha_1 - \alpha_2)\delta_{ir}\big]\Big] \\ =& \frac{1}{n_r\ F_{1/2}(\eta_r)}\Big[\delta_{ir} + \frac{3n_r M_r^*}{\hbar^2}\big[\alpha_2 + (\alpha_1 - \alpha_2)\delta_{ir}\big]\Big] \end{aligned} \tag{39}$$

Also, $\partial_\eta F_n^{-1}(\eta) = nF_{n-1}(\eta) \leftrightarrow \partial_u F_n^{-1}(u) = (1/n)F_{n-1}^{-1}(u)$:

$$\begin{aligned} \partial_{n_i}\eta_r =& d_u F_{1/2}^{-1}(u_r) \times \partial_{n_i} u_r \\ =& \frac{\partial u F_{1/2}^{-1}(u_r)}{n_r\ F_{1/2}(\eta_r)}\Big[\delta_{ir} + \frac{3n_r M_r^*}{\hbar^2}\big[\alpha_2 + (\alpha_1 - \alpha_2)\delta_{ir}\big]\Big] \\ =& \frac{2F_{1/2}^{-1}(u_r)}{n_r\ F_{1/2}(\eta_r)}\Big[\delta_{ir} + \frac{3n_r M_r^*}{\hbar^2}\big[\alpha_2 + (\alpha_1 - \alpha_2)\delta_{ir}\big]\Big] \\ \equiv& \frac{G}{n_r}\Big[\delta_{ir} + \frac{3n_r M_r^*}{\hbar^2}\big[\alpha_2 + (\alpha_1 - \alpha_2)\delta_{ir}\big]\Big] \end{aligned} \tag{40}$$

Thus,

$$\begin{aligned} \partial_{n_i}\mu_r =& T\partial_{n_i}\eta_r + \partial_{n_i}U_r \\ =& \frac{TG}{n_r}\Big[\delta_{ir} + \frac{3n_r M_r^*}{\hbar^2}\big[\alpha_2 + (\alpha_1 - \alpha_2)\delta_{ir}\big]\Big] + \partial_{n_i}U_r \end{aligned} \tag{41}$$