

I. SKYRME INTERACTION - FINITE TEMPERATURE

A. Potential Matrix Element

Interaction Matrix:

$$\begin{aligned}
V_{ij} = & t_0(1 + x_0 P_\sigma) \delta(\mathbf{r}_{ij}) + \frac{1}{2} t_1(1 + x_1 P_\sigma) \frac{1}{\hbar^2} [\overleftarrow{p}_{ij}^{\dagger 2} \delta(\mathbf{r}_{ij}) + \delta(\mathbf{r}_{ij}) \overrightarrow{p}_{ij}^2] \\
& + t_2(1 + x_2 P_\sigma) \frac{1}{\hbar^2} \overleftarrow{\mathbf{p}}_{ij}^\dagger \cdot \delta(\mathbf{r}_{ij}) \overrightarrow{\mathbf{p}}_{ij} + \frac{1}{6} t_3(1 + x_3 P_\sigma) \rho^\alpha(\mathbf{r}) \delta(\mathbf{r}_{ij}) \\
& + \frac{i}{\hbar^2} W_0(\sigma_i + \sigma_j) \cdot \overleftarrow{\mathbf{p}}_{ij}^\dagger \times \delta(\mathbf{r}_{ij}) \overrightarrow{\mathbf{p}}_{ij} \\
& + \frac{1}{4} t_4(1 + x_4 P_\sigma) \frac{1}{\hbar^2} [\overleftarrow{p}_{ij}^{\dagger 2} \rho^\beta(\mathbf{r}) \delta(\mathbf{r}_{ij}) + \delta(\mathbf{r}_{ij}) \rho^\beta(\mathbf{r}) \overrightarrow{p}_{ij}^2] \\
& + t_5(1 + x_5 P_\sigma) \frac{1}{\hbar^2} \overleftarrow{\mathbf{p}}_{ij} \cdot \rho^\gamma(\mathbf{r}) \delta(\mathbf{r}_{ij}) \overrightarrow{\mathbf{p}}_{ij}
\end{aligned} \tag{1}$$

where, $\mathbf{r}_{ij} = \frac{\mathbf{r}_i - \mathbf{r}_j}{2}$, $\mathbf{r} = \frac{\mathbf{r}_i + \mathbf{r}_j}{2}$, $P_\sigma = \frac{1}{2}(1 + \sigma_1 \cdot \sigma_2)$, $\mathbf{p}_{ij} = -i\hbar \frac{\nabla_i - \nabla_j}{2}$, $\rho(\mathbf{r}) = \rho_p(\mathbf{r}) + \rho_n(\mathbf{r})$ and the arrows show the direction on which the momentum operators act.

B. Energy Density

Assume time reversal invariance. The Energy of the ground state can be written as integration over an energy density functional, $E_{HFB} = \int d^r \varepsilon_{HFB}$

$$\begin{aligned}
\varepsilon_{HFB} = & \sum_{t=n,p} \frac{\hbar^2}{2M_q} \tau_q + \frac{1}{2} t_0 \left[\left(1 + \frac{1}{2} x_0\right) \rho^2 - \left(\frac{1}{2} + x_0\right) \sum_{q=n,p} \rho_q^2 \right] \\
& + \frac{1}{4} t_1 \left[\left(1 + \frac{1}{2} x_1\right) (\rho\tau + \frac{3}{4} (\nabla\rho)^2) - \left(\frac{1}{2} + x_1\right) \sum_{q=n,p} (\rho_q \tau_q + \frac{3}{4} (\nabla\rho_q)^2) \right] \\
& + \frac{1}{4} t_2 \left[\left(1 + \frac{1}{2} x_2\right) (\rho\tau + \frac{3}{4} (\nabla\rho)^2) + \left(\frac{1}{2} + x_2\right) \sum_{q=n,p} (\rho_q \tau_q - \frac{1}{4} (\nabla\rho_q)^2) \right] \\
& + \frac{1}{12} t_3 \rho^\alpha \left[\left(1 + \frac{1}{2} x_3\right) \rho^2 - \left(\frac{1}{2} + x_3\right) \sum_{q=n,p} \rho_q^2 \right] \\
& + \frac{1}{4} t_4 \left[\left(1 + \frac{1}{2} x_4\right) (\rho\tau + \frac{3}{4} (\nabla\rho)^2) - \left(\frac{1}{2} + x_4\right) \sum_{q=n,p} (\rho_q \tau_q + \frac{3}{4} (\nabla\rho_q)^2) \right] \rho^\beta \\
& + \frac{\beta}{8} t_4 \left[\left(1 + \frac{1}{2} x_4\right) \rho (\nabla\rho)^2 - \left(\frac{1}{2} + x_4\right) \nabla\rho \cdot \sum_{q=n,p} \rho_q \nabla\rho_q \right] \rho^{\beta-1} \\
& + \frac{1}{4} t_5 \left[\left(1 + \frac{1}{2} x_5\right) (\rho\tau + \frac{3}{4} (\nabla\rho)^2) + \left(\frac{1}{2} + x_5\right) \sum_{q=n,p} (\rho_q \tau_q - \frac{1}{4} (\nabla\rho_q)^2) \right] \rho^\gamma \\
& - \frac{1}{16} (t_1 x_1 + t_2 x_2) J^2 + \frac{1}{16} (t_1 - t_2) \sum_{q=n,p} J_q^2 \\
& - \frac{1}{16} (t_4 x_4 \rho^\beta + t_5 x_5 \rho^\gamma) J^2 + \frac{1}{16} (t_4 \rho^\beta - t_5 \rho^\gamma) \sum_{q=n,p} J_q^2 \\
& + \frac{1}{2} W_0 (\mathbf{J} \cdot \nabla\rho + \sum_{q=n,p} \mathbf{J}_q \cdot \nabla\rho_q)
\end{aligned} \tag{2}$$

where,

$$\begin{aligned}
\rho &= 2 \int \frac{d^3 k}{(2\pi\hbar)^3} n(k) \\
\tau &= 2 \int \frac{d^3 k}{(2\pi\hbar)^3} k^2 n(k) \\
\mathbf{J} &= \int \frac{d^3 k}{(2\pi\hbar)^3} \mathbf{k} \times \sum_{s,s'} \langle s | \boldsymbol{\sigma} | s' \rangle n(k)
\end{aligned} \tag{3}$$

The different terms can be grouped together in simpler notation:

$$\begin{aligned}
\varepsilon = & \sum_{t=n,p} \frac{\hbar^2}{2M_q} \tau_q + \frac{1}{4} t_0 \left[(2+x_0) \rho^2 - (1+2x_0) \sum_{q=n,p} \rho_q^2 \right] \\
& + \frac{1}{8} \left[a(\rho\tau + \frac{3}{4}(\nabla\rho)^2) + 2b \sum_{q=n,p} (\rho_q \tau_q + \frac{3}{4}(\nabla\rho_q)^2) \right] \\
& + \frac{1}{24} t_3 \rho^\alpha \left[(2+x_3) \rho^2 - (1+2x_3) \sum_{q=n,p} \rho_q^2 \right] \\
& + \frac{1}{8} t_4 \left[(2+x_4)(\rho\tau + \frac{3}{4}(\nabla\rho)^2) - (1+2x_4) \sum_{q=n,p} (\rho_q \tau_q + \frac{3}{4}(\nabla\rho_q)^2) \right] \rho^\beta \\
& + \frac{\beta}{16} t_4 \left[(2+x_4) \rho(\nabla\rho)^2 - (1+2x_4) \nabla\rho \cdot \sum_{q=n,p} \rho_q \nabla\rho_q \right] \rho^{\beta-1} \\
& + \frac{1}{8} t_5 \left[(2+x_5)(\rho\tau + \frac{3}{4}(\nabla\rho)^2) + (1+2x_5) \sum_{q=n,p} (\rho_q \tau_q - \frac{1}{4}(\nabla\rho_q)^2) \right] \rho^\gamma \\
& - \frac{1}{16} (t_1 x_1 + t_2 x_2) J^2 + \frac{1}{16} (t_1 - t_2) \sum_{q=n,p} J_q^2 \\
& - \frac{1}{16} (t_4 x_4 \rho^\beta + t_5 x_5 \rho^\gamma) J^2 + \frac{1}{16} (t_4 \rho^\beta - t_5 \rho^\gamma) \sum_{q=n,p} J_q^2 \\
& + \frac{1}{2} W_0 (\mathbf{J} \cdot \nabla\rho + \sum_{q=n,p} \mathbf{J}_q \cdot \nabla\rho_q)
\end{aligned} \tag{4}$$

where, $a = t_1(x_1 + 2) + t_2(x_2 + 2)$, $b = \frac{1}{2}[t_2(2x_2 + 1) - t_1(2x_1 + 1)]$.

In uniform matter $\nabla\rho = 0$:

$$\begin{aligned}
\varepsilon = & \sum_{t=n,p} \frac{\hbar^2}{2M_q} \tau_q + \frac{1}{4} t_0 \left[(2+x_0) \rho^2 - (1+2x_0) \sum_{q=n,p} \rho_q^2 \right] \\
& + \frac{1}{8} \left[a \rho \tau + 2b \sum_{q=n,p} \rho_q \tau_q \right] \\
& + \frac{1}{24} t_3 \rho^\alpha \left[(2+x_3) \rho^2 - (1+2x_3) \sum_{q=n,p} \rho_q^2 \right] \\
& + \frac{1}{8} t_4 \left[(2+x_4) \rho \tau - (1+2x_4) \sum_{q=n,p} \rho_q \tau_q \right] \rho^\beta \\
& + \frac{1}{8} t_5 \left[(2+x_5) \rho \tau + (1+2x_5) \sum_{q=n,p} \rho_q \tau_q \right] \rho^\gamma \\
& - \frac{1}{16} (t_1 x_1 + t_2 x_2) J^2 + \frac{1}{16} (t_1 - t_2) \sum_{q=n,p} J_q^2 \\
& - \frac{1}{16} (t_4 x_4 \rho^\beta + t_5 x_5 \rho^\gamma) J^2 + \frac{1}{16} (t_4 \rho^\beta - t_5 \rho^\gamma) \sum_{q=n,p} J_q^2
\end{aligned} \tag{5}$$

In unpolarized matter, $\mathbf{J} = 0$:

$$\begin{aligned}
\varepsilon = & \sum_{t=n,p} \frac{\hbar^2}{2M_q} \tau_q + \frac{1}{4} t_0 \left[(2+x_0) \rho^2 - (1+2x_0) \sum_{q=n,p} \rho_q^2 \right] \\
& + \frac{1}{8} \left[a \rho \tau + 2b \sum_{q=n,p} \rho_q \tau_q \right] \\
& + \frac{1}{24} t_3 \rho^\alpha \left[(2+x_3) \rho^2 - (1+2x_3) \sum_{q=n,p} \rho_q^2 \right] \\
& + \frac{1}{8} t_4 \left[(2+x_4) \rho \tau - (1+2x_4) \sum_{q=n,p} \rho_q \tau_q \right] \rho^\beta \\
& + \frac{1}{8} t_5 \left[(2+x_5) \rho \tau + (1+2x_5) \sum_{q=n,p} \rho_q \tau_q \right] \rho^\gamma
\end{aligned} \tag{6}$$

Energy per bayon, $\mathcal{E} \equiv \varepsilon/rho$:

$$\begin{aligned}
\mathcal{E} = & \sum_{t=n,p} \frac{\hbar^2}{2M_q} \frac{\tau_q}{\rho} + \frac{1}{4} t_0 \left[(2+x_0)\rho - (1+2x_0) \sum_{q=n,p} \frac{\rho_q^2}{\rho} \right] \\
& + \frac{1}{8} \left[a\tau + 2b \sum_{q=n,p} \frac{\rho_q \tau_q}{\rho} \right] \\
& + \frac{1}{24} t_3 \rho^\alpha \left[(2+x_3)\rho - (1+2x_3) \sum_{q=n,p} \frac{\rho_q^2}{\rho} \right] \\
& + \frac{1}{8} t_4 \left[(2+x_4)\tau - (1+2x_4) \sum_{q=n,p} \frac{\rho_q \tau_q}{\rho} \right] \rho^\beta \\
& + \frac{1}{8} t_5 \left[(2+x_5)\tau + (1+2x_5) \sum_{q=n,p} \frac{\rho_q \tau_q}{\rho} \right] \rho^\gamma
\end{aligned} \tag{7}$$

In terms of proton fraction, $y = \frac{\rho_p}{\rho_p + \rho_n}$:

$$\begin{aligned}
\mathcal{E} = & \sum_{t=n,p} \frac{\hbar^2}{2M_q} \frac{\tau_q}{\rho} + \frac{1}{4} t_0 \left[(2+x_0) - (1+2x_0)[y^2 + (1-y)^2] \right] \rho \\
& + \frac{1}{8} \left[a\tau + 2b[y\tau_p + (1-y)\tau_n] \right] \\
& + \frac{1}{24} t_3 \rho^{\alpha+1} \left[(2+x_3) - (1+2x_3)[y^2 + (1-y)^2] \right] \\
& + \frac{1}{8} t_4 \left[(2+x_4)\tau - (1+2x_4)[y\tau_p + (1-y)\tau_n] \right] \rho^\beta \\
& + \frac{1}{8} t_5 \left[(2+x_5)\tau + (1+2x_5)[y\tau_p + (1-y)\tau_n] \right] \rho^\gamma
\end{aligned} \tag{8}$$

II. SINGLE PARTICLE PROPERTIES

From the energy density the single particle spectrum can be derived. By performing functional variation of the energy density with respect to the single particle wavefunction, a modified Schrodinger equation can be derived:

$$\delta\varepsilon_i = \left[\frac{\delta\varepsilon_i}{\delta\tau_i} + \frac{\delta\varepsilon_i}{\delta\rho_i} \right] \delta\phi_i = \epsilon_i \delta\phi_i \tag{9}$$

Since the Skyrme potential is at mostly quadratic in momenta with nonlineaer density dependence, its effect is exactly included by effective mass and mean field shift(residual interaction), both density dependent:

$$\begin{aligned}\epsilon_i(k) &= \frac{\hbar^2 k^2}{2M_i^*} + U_i \\ \frac{\hbar^2}{2M_q^*} &\equiv \frac{\partial \varepsilon}{\partial \tau_q} \\ U_i &\equiv \frac{\partial \varepsilon}{\partial \rho_i}\end{aligned}\tag{10}$$

From eq. 6 the effective baryon masses:

$$\begin{aligned}M_p^*/M &= \left\{ 1 + \frac{M}{4\hbar^2} \rho [a + 2 y b + t_4[(2 + x_4) - (1 + 2x_4) y] \rho^\beta + t_5[2 + x_5 + (1 + 2x_5) y] \rho^\gamma] \right\}^{-1} \\ M_n^*/M &= \left\{ 1 + \frac{M}{4\hbar^2} \rho [a + 2 (1 - y) b + t_4[(2 + x_4) - (1 + 2x_4) (1 - y)] \rho^\beta + t_5[2 + x_5 + (1 + 2x_5) (1 - y)] \rho^\gamma] \right\}^{-1}\end{aligned}\tag{11}$$

and the residual potentials:

$$\begin{aligned}U_p &= \frac{1}{8}(2b \tau_p + a \tau) + \frac{1}{2}t_0[(2 + x_0) - (1 + 2x_0) y] \rho \\ &\quad + \frac{1}{24}t_3 \left[4 + \alpha - 2y(1 - (1 - y)\alpha) + x_3(1 - 2y)[2 - (1 - 2y)\alpha] \right] \rho^{\alpha+1} \\ U_n &= \frac{1}{8}(2b \tau_n + a \tau) + \frac{1}{2}t_0[(1 - x_0) + (1 + 2x_0)y] \rho \\ &\quad + \frac{1}{24}t_3 \left[2 + \alpha + 2y(1 + \alpha - y\alpha) - x_3(1 - 2y)[2 + (1 - 2y)\alpha] \right] \rho^{\alpha+1}\end{aligned}\tag{12}$$

III. T=0 DFT

At $T = 0$, there are simple relation that can be drawn between the 2 integrations in Fourier space since the occupation number is a step function:

$$\begin{aligned}
\rho_q &= 2 \int \frac{d^3k}{(2\pi\hbar)^3} \theta(k_{F,q} - k) = \frac{k_{F,q}^3}{3\pi^2\hbar^3} \\
\tau_q &= 2 \int \frac{d^3k}{(2\pi\hbar)^3} (k/\hbar)^2 \theta(k_{F,q} - k) = \frac{k_{F,q}^5}{5\pi^2\hbar^5} \rightarrow \\
\tau &= \frac{3}{5} (3\pi^2)^{2/3} \rho_q^{5/3}, H_n(y) = 2^{n-1} [y^n + (1-y)^n], y = \rho_p/\rho \\
\tau &= \tau_p + \tau_n = \frac{3}{5} \left(\frac{3\pi^2}{2}\right)^{2/3} H_{5/3}(y) \rho^{5/3}
\end{aligned} \tag{13}$$

So,

$$\begin{aligned}
\mathcal{E}_0 &= \frac{3\hbar^2}{10M} \left(\frac{3\pi^2}{2}\right)^{2/3} \rho^{2/3} H_{5/3}(y) + \frac{1}{8} t_0 \rho \left[2(2+x_0) - (1+2x_0) H_2(y) \right] \\
&+ \frac{3}{40} \left(\frac{3\pi^2}{2}\right)^{2/3} \left[a H_{5/3}(y) + b H_{8/5}(y) \right] \rho^{5/3} \\
&+ \frac{1}{48} t_3 \rho^{\alpha+1} \left[2(2+x_3) - (1+2x_3) H_2(y) \right] \\
&+ \frac{3}{40} \left(\frac{3\pi^2}{2}\right)^{2/3} t_4 \rho^{\beta+5/3} \left[(2+x_4) H_{5/3}(y) - \left(\frac{1}{2} + x_4\right) H_{8/3}(y) \right] \\
&+ \frac{3}{40} \left(\frac{3\pi^2}{2}\right)^{2/3} t_5 \rho^{\gamma+5/3} \left[(2+x_5) H_{5/3}(y) + \left(\frac{1}{2} + x_5\right) H_{8/3}(y) \right]
\end{aligned} \tag{14}$$

with $H_n(y) = 2^{n-1} [y^n + (1-y)^n]$. A common choice is to set $M = 1/2(M_n + M_p)$, or use the individual value for each species. In compact notation,

$$\mathcal{E}_0 = C(y) \rho^{2/3} + A(y) \rho + B(y) \rho^{\alpha+1} + D(y) \rho^{5/3} + G(y) \rho^{\beta+5/3} + K(y) \rho^{\gamma+5/3} \tag{15}$$

By comparing the 2 expressions, the following relations can be easily deduced:

$$\begin{aligned}
C(y) &= \frac{3\hbar^2}{10M_y} \left(\frac{3\pi^2}{2}\right)^{2/3} H_{5/3}(y) \\
A(y) &= \frac{1}{8} t_0 \left[2(2 + x_0) - (1 + 2x_0) H_2(y) \right] \\
B(y) &= \frac{1}{48} t_3 \left[2(2 + x_3) - (1 + 2x_3) H_2(y) \right] \\
D(y) &= \frac{3}{40} \left(\frac{3\pi^2}{2}\right)^{2/3} \left[a H_{5/3}(y) + b H_{8/5}(y) \right] \\
G(y) &= \frac{3}{40} \left(\frac{3\pi^2}{2}\right)^{2/3} t_4 \left[(2 + x_4) H_{5/3}(y) - \left(\frac{1}{2} + x_4\right) H_{8/3}(y) \right] \\
K(y) &= \frac{3}{40} \left(\frac{3\pi^2}{2}\right)^{2/3} t_5 \left[(2 + x_5) H_{5/3}(y) + \left(\frac{1}{2} + x_5\right) H_{8/3}(y) \right]
\end{aligned} \tag{16}$$

For pure neutron matter and symmetric matter the kinetic coefficient is

$$\begin{aligned}
C_n = C(0) &= \frac{3\hbar^2}{10M_n} \left(\frac{3\pi^2}{2}\right)^{2/3} H_{5/3}(0) = 118.995 \text{ Mev fm}^2 \approx 119 \text{ Mev fm}^2 \\
C_{sym} = C(1/2) &= \frac{3\hbar^2}{5(M_n + M_p)} \left(\frac{3\pi^2}{2}\right)^{2/3} H_{5/3}(1/2) = 75.0139 \text{ Mev fm}^2 \approx 75 \text{ Mev fm}^2
\end{aligned} \tag{17}$$

The effective mass is due to the terms dependent on kinetic energy:

$$\begin{aligned}
\tau^{T=0}(\rho, y) &\equiv \frac{3\hbar^2}{10M^*} \left(\frac{3\pi^2}{2}\right)^{2/3} \rho^{2/3} H_{5/3}(y) \\
&= \frac{3\hbar^2}{10M} \left(\frac{3\pi^2}{2}\right)^{2/3} \rho^{2/3} H_{5/3}(y) + \frac{3}{40} \left(\frac{3\pi^2}{2}\right)^{2/3} \left[a H_{5/3}(y) + b H_{8/5}(y) \right] \rho^{5/3} \\
&\quad + \frac{3}{40} \left(\frac{3\pi^2}{2}\right)^{2/3} t_4 \rho^{\beta+5/3} \left[(2 + x_4) H_{5/3}(y) - \left(\frac{1}{2} + x_4\right) H_{8/3}(y) \right] \\
&\quad + \frac{3}{40} \left(\frac{3\pi^2}{2}\right)^{2/3} t_5 \rho^{\gamma+5/3} \left[(2 + x_5) H_{5/3}(y) + \left(\frac{1}{2} + x_5\right) H_{8/3}(y) \right] \\
&\equiv C(y) \rho^{2/3} \left[1 + (D(y) \rho + G(y) \rho^{\beta+1} + K(y) \rho^{\gamma+1}) / C(y) \right]
\end{aligned} \tag{18}$$

Thus,

$$\begin{aligned}
M^*/M &= \left\{ 1 + \frac{M}{4H_{5/3}(y)\hbar^2} \left[\rho[aH_{5/3}(y) + bH_{8/5}(y)] + \rho^{\beta+1}t_4[(2+x_4)H_{5/3}(y) - (\frac{1}{2} + x_4)H_{8/3}(y)] \right. \right. \\
&\quad \left. \left. + \rho^{\gamma+1}t_5[(2+x_5)H_{5/3}(y) - (\frac{1}{2} + x_5)H_{8/3}(y)] \right] \right\}^{-1} \\
&= \frac{C(y)}{C(y) + D(y)\rho + G(y)\rho^{\beta+1} + K(y)\rho^{\gamma+1}}
\end{aligned} \tag{19}$$

Also, the thermodynamic pressure:

$$\begin{aligned}
\mathcal{P}^{T=0}(\rho, y) &= \rho^2 \frac{\partial \mathcal{E}^{T=0}}{\partial \rho} \\
&= \frac{2}{3}C(y)\rho^{5/3} + A(y)\rho^2 + (\alpha + 1)B(y)\rho^{\alpha+1} + \frac{5}{3}D(y)\rho^{8/3} + \beta G(y)\rho^{\beta+8/3} + \gamma K(y)\rho^{\gamma+8/3}
\end{aligned} \tag{20}$$

IV. LATTIMER - SCHWESTY NOTATION

The standard parametrization:

$$\begin{aligned}
\varepsilon &= \frac{\hbar^2 \tau_n}{2M_n} + \frac{\hbar^2 \tau_p}{2M_p} + \frac{1}{8}[a(\rho_n + \rho_p)(\tau_n + \tau_p) + 2b(\rho_n \tau_n + \rho_p \tau_p)] + \frac{1}{4}t_0[(2+x_0)(\rho_n + \rho_p)^2 - (1+2x_0)(\rho_n^2 + \rho_p^2)] \\
&\quad + \frac{1}{24}t_3(\rho_n + \rho_p)^\alpha[(2+x_3)(\rho_n + \rho_p)^2 - (1+2x_3)(\rho_n^2 + \rho_p^2)]
\end{aligned} \tag{21}$$

$$\begin{aligned}
a &= \frac{4\hbar^2}{M}(F + G) \\
b &= -\frac{4\hbar^2}{M}G \\
t_0 &= \frac{8}{3}(A + B) \\
x_0 &= -\frac{1}{2}\frac{A - 2B}{A + B} \\
t_3 &= 16(C + D) \\
x_3 &= -\frac{1}{2}\frac{C - D}{C + D} \\
\alpha &= \delta - 1
\end{aligned} \tag{22}$$

$$\begin{aligned}
a &= \frac{4\hbar^2}{M}(F + G) \\
b &= -\frac{4\hbar^2}{M}G \\
t_0 &= \frac{8}{3}(A + B) \\
x_0 &= -\frac{1}{2}\frac{A - 2B}{A + B} \\
t_3 &= 16(C + D) \\
x_3 &= -\frac{1}{2}\frac{C - D}{C + D} \\
\alpha &= \delta - 1
\end{aligned} \tag{23}$$

$$\begin{aligned}
\varepsilon = & 4B\rho_n\rho_p + A(\rho_n + \rho_p)^2 + (\rho_n + \rho_p)^{\delta-1}[4D\rho_n\rho_p + C(\rho_n + \rho_p)^2] \\
& + \frac{5(\frac{2}{3})^{2/3}}{3\pi^{4/3}}\alpha_S[M(\frac{\tau_n}{M_n} + \frac{\tau_p}{M_p}) + F(\rho_n + \rho_p)(\tau_n + \tau_p) - G(\rho_n - \rho_p)(\tau_n - \tau_p)]
\end{aligned} \tag{24}$$

The standard parametrization for $T = 0$, $(M_n, M_p) \rightarrow M = \frac{1}{2}(M_n + M_p)$:

$$\begin{aligned}
\varepsilon = & \frac{3\hbar^2}{10M}(\frac{3\pi^2}{2})^{2/3}\rho^{5/3}H_{5/3}(y) + \frac{1}{8}t_0\rho^2\left[2(2 + x_0) - (1 + 2x_0)H_2(y)\right] \\
& + \frac{3}{40}(\frac{3\pi^2}{2})^{2/3}\left[aH_{5/3}(y) + bH_{8/5}(y)\right]\rho^{8/3} \\
& + \frac{1}{48}t_3\rho^{\alpha+2}\left[2(2 + x_3) - (1 + 2x_3)H_2(y)\right]
\end{aligned} \tag{25}$$

By comparing the expression in these notes with the ones from Lattimer parametrization, the energy density is:

$$\varepsilon = \alpha_S\rho^{5/3}H_{5/3}(y) + [A + B(2 - H_2(y))]\rho^2 + [C + D(2 - H_2(y))]\rho\delta + \alpha_S\rho^{5/3}[(F + G)H_{5/3} - GH_{8/3}(y)] \tag{26}$$

where, $\alpha_S = \frac{3\hbar^2}{10M}(\frac{3}{2}\pi^2)^{2/3}$

A. Skyrme parametrization from Saturation Observables

Given the following set of physical observables,

$$\begin{aligned}
E_0 = \varepsilon \Big|_{\rho_0, y=1/2} &= (A + B)\rho_0^2 + (C + D)\rho_0^{\delta+1} + \alpha_S \rho_0^{5/3} (1 + F\rho_0) \\
P = \rho^2 \frac{d(\varepsilon/\rho)}{d\rho} \Big|_{\rho_0, y=1/2} &= \frac{2}{3} \alpha_S \rho_0^{5/3} + (A + B)\rho_0^2 + \frac{5}{3} F \alpha_S \rho_0^{8/3} + (C + D)\delta \rho_0^{1+\delta} = 0 \\
(M^*/M) = \frac{d\varepsilon}{d\tau} \Big|_{\rho_0, y=1/2} &= (1 + F\rho_0)^{-1} \\
K_m = 9\rho^2 \frac{d^2(\varepsilon/\rho)}{d\rho^2} \Big|_{\rho_0, y=1/2} &= -2\alpha_S \rho_0^{2/3} + 10F\alpha_S \rho_0^{5/3} + 9(C + D)(\delta - 1)\delta \rho_0^\delta \\
S = \frac{1}{8} \frac{d^2(\varepsilon/\rho)}{dy^2} \Big|_{\rho_0, y=1/2} &= \frac{5}{9} \alpha_S \rho_0^{2/3} - B\rho_0 + \frac{5}{9} (F - 3G)\alpha_S \rho_0^{5/3} - D\rho_0^\delta \\
L = 3\rho \frac{dS}{d\rho} \Big|_{\rho_0, y=1/2} &= \frac{10}{9} \alpha_S \rho_0^{2/3} - 3B\rho_0 + \frac{25}{9} (F - 3G)\alpha_S \rho_0^{5/3} - 3D\delta \rho_0^\delta \\
K_s = 9\rho^2 \frac{d^2 S}{d\rho^2} \Big|_{\rho_0, y=1/2} &= -\frac{10}{9} \alpha_S \rho_0^{2/3} + \frac{50}{9} (F - 3G)\alpha_S \rho_0^{5/3} - 9D(\delta - 1)\delta \rho_0^\delta
\end{aligned} \tag{27}$$

the skyrme parameters can be found as follows,

$$\begin{aligned}
F &= \frac{(M^*/M)^{-1} - 1}{\rho_0} \\
\delta &= \frac{K_m + 2\rho_0^{2/3}(1 - 5F\rho_0)\alpha_S}{3\rho_0^{2/3}(1 - 2F\rho_0)\alpha_S - 9E_0} \\
G &= \frac{9K_s - 27(L - 3S)\delta + 5\rho_0^{2/3}\alpha_S[2 - 3\delta + 2F\rho_0(3\delta - 5)]}{30\rho_0^{5/3}\alpha_S(3\delta - 5)} \\
D &= \frac{5(3L - 9S + \rho_0^{2/3}\alpha_S) - 3K_s}{9(5 - 8\delta + 3\delta^2)\rho_0^\delta} \\
C &= \frac{\rho_0^{2/3}(1 - 2F\rho_0)\alpha_S - 3E_0}{3(\delta - 1)\rho_0^\delta} - D \\
B &= \frac{L(6 + 9\delta) + 5(\rho_0^{2/3}\alpha_S(3\delta - 2) - 9S\delta) - 3K_s}{18\rho_0(\delta - 1)} \\
A &= -\left[\frac{2}{3}\alpha_S\rho^{-1/3} + B + \frac{5}{3}F\alpha_S\rho^{2/3} + (C + D)\delta\rho^{\delta-1}\right]
\end{aligned} \tag{28}$$

V. FINITE TEMPERATURE DFT

In order to obtain the relationship between density and chemical potential, the following set of coupled equations need to be solved self-consistently:

$$\begin{aligned}
f_k &= \left[1 + e^{\left(\frac{k^2}{2M^*} + U - \mu \right) / T} \right]^{-1} \\
\rho &= \sum_{s, is} \int \frac{d^3 k}{(2\pi)^3} f_k \\
\tau &= \sum_{s, is} \int \frac{d^3 k}{(2\pi)^3} k^2 f_k \\
E &\equiv E(\rho, \tau)
\end{aligned} \tag{29}$$

where, a sum over all discrete quantum numbers is performed (spin and isospin). And from the energy density functional, the mean field parameters can be derived,

$$\begin{aligned}
M^* &= \frac{1}{2} \left(\frac{\delta E}{\delta \tau} \right)^{-1} \\
U &= \frac{\delta E}{\delta \rho}
\end{aligned} \tag{30}$$

The chemical potential can be found by inverting the expression for the density.

VI. THERMODYNAMIC POTENTIALS

Since the effect of phenomenological mean field models can be incorporated into (M^*, U) which are density dependent for Skyrme, and also temperature dependent for RMF, the thermodynamic properties of assymetric matter at finite temperature can be expressed by fermi integrals of ‘modified’ non-interacting fermi gases. The single particle spectrum and dustribution fuction:

$$\begin{aligned}
\xi_i &= \frac{k^2}{2M_i^*} + U_i \\
F_i &= \left\{ \exp \left[\frac{\xi_i - \mu_i}{T} \right] + 1 \right\}^{-1} = \left\{ \exp \left[\frac{\frac{k^2}{2M_i^*} - \eta_i}{T} \right] + 1 \right\}^{-1}, \quad \bar{F} = 1 - F
\end{aligned} \tag{31}$$

The density and kinetic density:

$$\begin{aligned}\rho_i &= \int_0^\infty \frac{dk}{\pi^2} k^2 F_i \\ \tau_i &= \int_0^\infty \frac{dk}{\pi^2} k^4 F_i\end{aligned}\tag{32}$$

The entropy density can be calculated from the distribution function:

$$\begin{aligned}S/V &= - \int_0^\infty \frac{dk}{\pi^2} k^2 [F_i \ln(F_i) + (1 - F_i) \ln(1 - F_i)] \\ &= - \int_0^\infty \frac{dk}{\pi^2} k^2 F_i \ln\left(\frac{F_i}{1 - F_i}\right) - \int_0^\infty \frac{dk}{\pi^2} k^2 \ln(1 - F_i) \\ &= - \int_0^\infty \frac{dk}{\pi^2} k^2 F_i \ln\left(\exp\left[\frac{-\xi_i + \mu_i}{T}\right]\right) - \left[\frac{k^3}{3\pi^2} \ln(1 - F_i)\right]_0^\infty + \int_0^\infty \frac{dk}{\pi^2} \frac{k^3}{3} \frac{k}{M_i^*} \frac{\exp\left[\frac{\xi_i - \mu_i}{T}\right]}{1 - F_i} \\ &= \left[\frac{1}{2M_i^* T} + \frac{1}{3M_i^* T}\right] \int_0^\infty \frac{dk}{\pi^2} k^4 F_i + \frac{U_i - \mu_i}{T} \int_0^\infty \frac{dk}{\pi^2} k^2 F_i \\ &= \frac{1}{T} \left[\frac{5\tau_i}{6M_i^*} + (U_i - \mu_i)\rho\right]\end{aligned}\tag{33}$$

From the first law of thermodynamics:

$$\begin{aligned}E_i &= TS_i + \mu_i N_i - P_i V \\ P_i &= \frac{TS_i + \mu_i N_i - E_i}{V} \\ &= \frac{5\tau_i}{6M_i^*} + U_i \rho_i - \frac{E_i}{V}\end{aligned}\tag{34}$$

VII. THERMODYNAMIC DERIVATIVES

Let,

$$\alpha_1 = \frac{\hbar^2}{2M}(F - G), \alpha_2 = \frac{\hbar^2}{2M}(G + F)\tag{35}$$

Then,

$$\begin{aligned}
\frac{\hbar^2}{2M_i^*} &= \frac{\hbar^2}{2M} + \alpha_1 \rho_i + \alpha_2 \rho_{-i} \rightarrow \\
M_i^* &= M [1 + F\rho - \text{sgn}(i)G(\rho_n - \rho_p)]^{-1} \\
U_i &= \alpha_1 \tau_i + \alpha_2 \tau_{-i} + 2A\rho + 4B\rho_{-i} + C(1 + \delta)\rho^\delta + 4D\rho_{-i}(\rho_{-i} + \delta\rho_i)\rho^{\delta-2} \\
\rho &= \rho_n + \rho_p
\end{aligned} \tag{36}$$

where, i denotes the isospin value. The set of independent parameters which is used for the 1st part of this section is (ρ_n, ρ_p, T) . From expressions above, the partial derivatives can be found:

$$\begin{aligned}
\partial_{\rho_i} M_r^* &= -\frac{M_r^{*2}}{M} [F + G(1 - 2\delta_{ir})] \\
&= -M [F + G(1 - 2\delta_{ir})] [1 + F\rho - \text{sgn}(i)G(\rho_n - \rho_p)]^{-2} \\
\partial_T M_r^* &= 0 \\
\partial_{\rho_i} U_r &= \alpha_1 \partial_{\rho_i} \tau_r + \alpha_2 \partial_{\rho_i} \tau_{-r} + 2A + 4B(1 - \delta_{ir}) + C\delta(1 + \delta)\rho^{\delta-1} \\
&\quad + 4D\rho^{\delta-3} [(\delta - 1)\rho_{-r}(2\rho_{-r} + \delta\rho_r)\delta_{ir} + (1 - \delta_{-ir})[\delta(\rho_{-r}^2 + \rho_r^2) + \rho_{-r}\rho_r(2 + \delta(\delta - 1))]] \\
\partial_T U_r &= \alpha_1 \partial_T \tau_r + \alpha_2 \partial_T \tau_{-r}
\end{aligned} \tag{37}$$

The number density and kinetic density can be expressed in terms of the general fermi integration:

$$\begin{aligned}
F_n(\eta) &= \int_0^\infty \frac{u^n}{e^{u-\eta} + 1} du, \quad \eta_i = (\mu_i - U_i)/T \\
\tau_r &= \frac{1}{2\pi^2} \left(\frac{2M_r^* T}{\hbar^2} \right)^{5/2} F_{3/2}(\eta_r) \\
\rho_r &= \frac{1}{2\pi^2} \left(\frac{2M_r^* T}{\hbar^2} \right)^{3/2} F_{1/2}(\eta_r)
\end{aligned} \tag{38}$$

By inverting the expression for the density:

$$\begin{aligned}
\eta_r &= F_{1/2}^{-1} \left[2\pi^2 \rho_r \left(\frac{\hbar^2}{2M_r^* T} \right)^{3/2} \right] \equiv F_{1/2}^{-1} [\eta_r^{(-1)}] \leftrightarrow \\
\eta_r &= F_{1/2}^{-1} [\eta_r^{(-1)}], \quad \eta_r^{(-1)} = 2\pi^2 \rho_r \left(\frac{\hbar^2}{2M_r^* T} \right)^{3/2} = F_{1/2} [\eta_r]
\end{aligned} \tag{39}$$

And,

$$\begin{aligned}
\partial_{\rho_i} \eta_r^{(-1)} &= \left[\frac{\delta_{ir}}{\rho_r} - \frac{3}{2} \frac{\partial_{\rho_i} M_r^*}{M_r^*} \right] \eta_r^{(-1)} \\
&= \left[\frac{\delta_{ir}}{\rho_r} + \frac{3}{2} \frac{M_r^*}{M} [F + G(1 - 2\delta_{ir})] \right] \\
&= \frac{F_{1/2}(\eta_r)}{\rho_r} \left[\delta_{ir} + \frac{3\rho_r}{2} \frac{M_r^*}{M} [F + G(1 - 2\delta_{ir})] \right] \\
\partial_T \eta_r^{(-1)} &= -\frac{3}{2} \frac{\eta_r^{(-1)}}{T} \\
&= -\frac{3}{2} \frac{F_{1/2}(\eta_r)}{T}
\end{aligned} \tag{40}$$

Also, $\partial_\eta F_n(\eta) = nF_{n-1}(\eta) \leftrightarrow \partial_u F_n^{-1}(u) = \frac{1}{nF_{n-1}(u)}$:

$$\begin{aligned}
\partial_{\rho_i} \eta_r &= \frac{\partial_{\rho_i} \eta_r^{(-1)}}{\partial_{\eta_r} F_{1/2}(\eta_r)} \\
&= \frac{2F_{1/2}(\eta_r)}{F_{-1/2}(\eta_r) \rho_r} \left[\delta_{ir} + \frac{3\rho_r}{2} \frac{M_r^*}{M} [F + G(1 - 2\delta_{ir})] \right] \\
&= \frac{G_r}{\rho_r} \left[\delta_{ir} + \frac{3\rho_r}{2} \frac{M_r^*}{M} [F + G(1 - 2\delta_{ir})] \right] \\
\partial_T \eta_r &= \frac{\partial_T \eta_r^{(-1)}}{\partial_{\eta_r} F_{1/2}(\eta_r)} \\
&= -\frac{3}{2T} \frac{2F_{1/2}(\eta_r)}{F_{-1/2}(\eta_r)} \\
&= -\frac{3}{2} \frac{G_r}{T}
\end{aligned} \tag{41}$$

where, $G_r = \frac{2F_{1/2}(\eta_r)}{F_{-1/2}(\eta_r)}$. Thus,

$$\begin{aligned}
\partial_{\rho_i} \tau_r &= \frac{5}{2} \tau_r \frac{\partial_{\rho_i} M_r^*}{M_r^*} + \frac{3}{2} \left(\frac{2M_r^* T}{\hbar^2} \right) \rho_r \partial_{\rho_i} \eta_r \\
&= -\frac{5}{2} \tau_r \frac{M_r^*}{M} [F + G(1 - 2\delta_{ir})] + \frac{3}{2} \left(\frac{2M_r^* T}{\hbar^2} \right) G_r \left[\delta_{ir} + \frac{3\rho_r}{2} \frac{M_r^*}{M} [F + G(1 - 2\delta_{ir})] \right] \\
&= 3TG_r \frac{M_r^*}{\hbar^2} \delta_{ir} + \frac{1}{2} \frac{M_r^*}{M} \left(\frac{9M_r^*}{\hbar^2} T \rho_r G_r - 5\tau_r \right) [F + G(1 - 2\delta_{ir})] \\
\partial_T \tau_r &= \frac{5}{4\pi^2 T} \left(\frac{2M_r^* T}{\hbar^2} \right)^{5/2} F_{3/2}(\eta_r) + \frac{1}{2\pi^2} \left(\frac{2M_r^* T}{\hbar^2} \right)^{5/2} \partial_{\eta_r} F_{3/2}(\eta_r) \partial_T \eta_r \\
&= \frac{5}{2T} \tau_r - \frac{9}{8\pi^2 T} \left(\frac{2M_r^* T}{\hbar^2} \right)^{5/2} F_{1/2}(\eta_r) G_r \\
&= \frac{5}{2T} \tau_r - \frac{9}{2} \frac{M_r^*}{\hbar^2} G_r \rho_r
\end{aligned} \tag{42}$$

Now, we are ready to find the partial derivatives of the chemical potential,

$$\begin{aligned}
\partial_{\rho_i} \mu_r &= T \partial_{\rho_i} \eta_r + \partial_{\rho_i} U_r \\
&= \frac{T G_r}{\rho_r} \left[\delta_{ir} + \frac{3\rho_r M_r^*}{\hbar^2} [\alpha_2 + (\alpha_1 - \alpha_2) \delta_{ir}] \right] + \partial_{\rho_i} U_r \\
&= \frac{T G_r}{\rho_r} \left[\delta_{ir} + \frac{3}{2} \rho_r [F + G(1 - 2\delta_{ir})] \right] + \partial_{\rho_i} U_r \\
\partial_T \mu_r &= \eta_r + T \partial_T \eta_r + \partial_T U_r \\
&= \eta_r - \frac{3}{2} G_r + \alpha_1 \partial_T \tau_r + \alpha_2 \partial_T \tau_{-r} \\
&= \eta_r - \frac{3}{2} G_r + \frac{5}{2T} (\alpha_1 \tau_r + \alpha_2 \tau_{-r}) - \frac{9}{2} \left(\alpha_1 \frac{M_r^*}{\hbar^2} \rho_r G_r + \alpha_2 \frac{M_{-r}^*}{\hbar^2} \rho_{-r} G_{-r} \right) \\
&= \eta_r - \frac{3}{2} G_r + \sum_i \left(\frac{5\tau_i}{2T} - \frac{9M_i^*}{2\hbar^2} \rho_i G_i \right) [\alpha_2 + (\alpha_1 - \alpha_2)] \\
&= \eta_r - \frac{3}{2} G_r + \sum_i \left(\frac{5\tau_i \hbar^2}{4T M_i^*} - \frac{9}{4} \rho_i G_i \right) [F + G(1 - 2\delta_{ir})]
\end{aligned} \tag{43}$$

The set of independent variables we will use in our computations is (ρ, Y, T) ,

where $Y = Y_n - Y_p = \frac{\rho_n - \rho_p}{\rho}$:

$$\begin{aligned}
\partial_\rho X &= \partial_{\rho_n} X + \partial_{\rho_p} X, \\
\partial_Y X &= \frac{1}{\rho} (\partial_{\rho_n} X - \partial_{\rho_p} X)
\end{aligned} \tag{44}$$

Thus,

$$\begin{aligned}
\partial_\rho M_r^* &= -\frac{2M_r^{*2}}{\hbar^2}(\alpha_1 + \alpha_2) = -2\frac{M_r^{*2}}{M}F \\
\partial_\rho U_r &= 4(A + B) + 2C\delta(1 + \delta)\rho^{\delta-1} + 4D\rho^{\delta-3}[\delta\rho_r^2 + (3\delta - 2)\rho_{-r}^2 + 2[1 + \delta(\delta - 1)\rho_r\rho_{-r}]] \\
&\quad + (\alpha_1 + \alpha_2)\partial_\rho(\tau_r + \tau_{-r}) \\
&= 4(A + B) + 2C\delta(1 + \delta)\rho^{\delta-1} + 4D\rho^{\delta-3}[\delta\rho_r^2 + (3\delta - 2)\rho_{-r}^2 + 2[1 + \delta(\delta - 1)\rho_r\rho_{-r}]] \\
&\quad + \sum_r \left[3TG_r \frac{M_r^*}{\hbar^2} + F\left(\frac{9M_r^*}{\hbar^2}T\rho_r G_r - 5\tau_r\right) \right] \\
\partial_\rho \tau_r &= \frac{3G_r M_r^* T}{\hbar^2}(1 + 3F\rho_r) - 5F\tau_r \\
\partial_\rho \eta_r &= \frac{G_r}{\rho_r} + 3F\rho_r \\
\partial_Y M_r^* &= -\text{sgn}(r)\frac{2M_r^*}{\rho\hbar^2}(\alpha_1 - \alpha_2), \quad \text{sgn}(n) = -1, \quad \text{sgn}(p) = 1 \\
\partial_Y U_r &= \text{sgn}(r)\left[4\frac{B}{\rho} - 4D\rho^{\delta-3}[(\delta - 2)\rho_p - \delta\rho_n] \right] + \alpha_1 \frac{\partial_Y \tau_r}{\rho} + \alpha_2 \frac{\partial_Y \tau_{-r}}{\rho} \\
&= \text{sgn}(r)\left[4\frac{B}{\rho} - 4D\rho^{\delta-3}[(\delta - 2)\rho_p - \delta\rho_n] - \frac{3G_r M_r T}{\rho\hbar^2} + G\left(\frac{G_r M_r Y_r T}{\hbar^2} - 5\tau_r\right) \right], \quad Y_r = \frac{1 - \text{sgn}(r)Y}{2} \\
\partial_Y \tau_r &= -\text{sgn}(r)\left[\frac{3G_r T M_r}{\hbar^2}(1 - 3G\rho_r) - 5G\tau_r \right] \\
\partial_Y \eta_r &= -\text{sgn}(r)\left[\frac{G_r}{\rho_r} - 3G\rho_r \right]
\end{aligned} \tag{45}$$

From the expressions above the derivatives of the μ_r can be derived:

$$\begin{aligned}
\partial_\rho \mu_r &= \frac{TG_r}{\rho_r}(1 + 3\rho_r F) + \partial_\rho U_r, \\
\partial_Y \mu_r &= -\text{sgn}(r)\frac{TG_r}{\rho\rho_r}(1 - 3\rho_r G) + \partial_Y U_r
\end{aligned} \tag{46}$$

And, the chemical potential we will use is $\mu = \mu_n - \mu_p$:

$$\begin{aligned}
\partial_\rho \mu &= \partial_{\rho_n} \mu + \partial_{\rho_p} \mu \\
&= \frac{5FG\hbar^2}{M}(\tau_n - \tau_p) + \frac{3GT}{M} \left[G_p M_p^* (1 + 3F\rho_p) - G_n M_n^* (1 + 3F\rho_n) \right] \\
&\quad + 3FT(G_n - G_p) + \frac{2T}{(1-Y^2)\rho} [(G_n - G_p) - (G_n + G_p)Y] - 8D(\delta - 1)Y\rho^{\delta-1} \\
\partial_{\rho Y} \mu &= \partial_Y \left[\frac{5FG\hbar^2}{M}(\tau_n - \tau_p) + \frac{3GT}{M} \left[G_p M_p^* (1 + 3F\rho_p) - G_n M_n^* (1 + 3F\rho_n) \right] \right. \\
&\quad \left. + 3FT(G_n - G_p) + \frac{2T}{(1-Y^2)\rho} [(G_n - G_p) - (G_n + G_p)Y] - 8D(\delta - 1)Y\rho^{\delta-1} \right] \\
&= \frac{1}{\rho} \sum_r \left[\frac{G_r T}{\rho_r^2} [J_r + 3F(1 + J_r)\rho_r] \right. \\
&\quad \left. + 3G \frac{G_r M_r T}{M \rho_r} (1 + J_r + F\rho_r + 3FJ_r \rho_r) + 3GT(1 + J_r + 3FJ_r \rho_r + 3F\rho_r) \right. \\
&\quad \left. - 3G^2 G_r \frac{M_r}{M} T [15F\rho_r + 2\frac{M_r^*}{M} (1 + 3F\rho_r)] + 9G^2 T(1 + J_r) \frac{M_r^*}{M} \rho_r (1 + 3F\rho_r) + G^2 F \frac{25\hbar^2 \tau_r}{M} \right] \\
&\quad - 16d(\delta - 1)\rho^{\delta-3} \\
\partial_T \mu &= \partial_T \mu_n - \partial_T \mu_p \\
&= (\eta_n - \eta_p) - \frac{3}{2}(G_n - G_p) + \frac{9}{2}G(G_n \rho_n - G_p \rho_p) - \frac{5G\hbar^2}{T} \left(\frac{\tau_n}{M_n^*} - \frac{\tau_p}{M_p^*} \right)
\end{aligned} \tag{47}$$

For higher order thermodynamic derivatives, the following calculation will be needed:

$$\begin{aligned}
\partial_{\eta_r} G_r(\eta_r) &= 1 + J_r(\eta_r) \\
J_r(\eta_r) &= \frac{F_{-3/2}(\eta_r) F_{1/2}(\eta_r)}{F_{-1/2}(\eta_r)}
\end{aligned} \tag{48}$$

The entropy per baryon is calculated in the previous section and it is $\rho s = \sum_r (\frac{5\hbar^2\tau_r}{6M_r^*T} - \rho_r\eta_r)$:

$$\begin{aligned}
\rho\partial_T s &= \sum_r \frac{5\hbar^2}{6M_r^*T} (\partial_T\tau_r - \tau_r/T) - \rho_r\partial_T\eta_r \\
&= \frac{1}{4T} \sum_r \left(\frac{5\hbar^2\tau_r}{TM_r^*} - 9\rho_r G_r \right) \\
s + \rho\partial_\rho s &= \sum_r \left[\frac{5\hbar^2}{6M_r^*T} (\partial_\rho\tau_r - \tau_r\partial_\rho M_r^*/M_r^*) - y_r\eta_r - \rho_r\partial_\rho\eta_r \right] \\
&= \sum_r \left[\frac{3}{2}G_r + 3F\rho_r\left(\frac{5}{2}G_r - \rho_r\right) + \frac{5\hbar^2\tau_r}{6M_r^*T} F\left(2\frac{M_r^*}{M} - 5\right) - y_r\eta_r \right] \leftrightarrow \\
\rho\partial_\rho s &= \sum_r \left[\frac{3}{2}G_r(1 + 5F\rho_r) - 3F\rho_r^2 - \frac{5\hbar^2\tau_r}{6MT} \left[\frac{M}{M_r^*} \left(\frac{1}{\rho} + 5F \right) - 2F \right] \right] \leftrightarrow \\
\partial_\rho s + \rho\partial_{\rho\rho} s &= \sum_r \left[\frac{3}{2} \left[G_r + 5FG_r y_r - 4F y_r \rho_r + (1 + J_r) \left(\frac{G_r}{\rho_r} + 3F\rho_r \right) (1 + 5F\rho_r) \right] \right. \\
&\quad \left. + \frac{5\hbar^2\tau_r}{6MT} \left[5F^2 \left(5\frac{M_r^*}{M} - 4 \right) + \frac{F}{\rho} \left(5\frac{M}{M_r^*} - 2 \right) + \frac{M}{M_r^*\rho^2} \right] - \frac{5}{2} \frac{M_r^*}{M} G_r (1 + 3F\rho_r) \left[\frac{M}{M_r^*} \left(\frac{1}{\rho} + 5F \right) - 2F \right] \right] \\
\rho\partial_{\rho\rho} s &= \sum_r \left[\frac{3}{2} G_r (1 + 5F y_r) - \frac{3}{2} G_r (5F y_r + \frac{1}{\rho}) - 3F y_r \rho_r + \frac{3}{2} (1 + J_r) \left(\frac{G_r}{\rho_r} + 3F\rho_r \right) (1 + 5F\rho_r) \right. \\
&\quad \left. - \frac{5}{2} G_r (1 + 3F\rho_r) \left(\frac{1}{\rho} + 5F - 2F \frac{M_r^*}{M} \right) + \frac{5\hbar^2\tau_r}{6M_r^*T\rho^2} \left[2 - 4F \frac{M_r^*}{M} \rho (1 + 5F\rho) + 5F\rho(2 + 5F\rho) \right] \right] \\
\rho\partial_{T\rho} s &= \frac{3}{4T} \sum_r \left[(2 - 4y_r - J_r) G_r + 3\rho_r F \left[5G_r - 3(1 + J_r)\rho_r \right] + \frac{5\hbar^2\tau_r}{TM_r^*} \left[F \left(2\frac{M_r^*}{M} - 5 \right) - \frac{1}{3\rho} \right] \right] \\
-\partial_{TT} s &= \partial_T \left[\frac{1}{4T\rho} \sum_{r=n,p} \left(\frac{5\hbar^2\tau_r}{TM_r^*} - 9\rho_r G_r \right) \right] \\
&= \frac{1}{4T\rho} \sum_{r=n,p} \left[\frac{5\hbar^2(\partial_T\tau_r - 2\tau_r/T)}{TM_r^*} - 9\rho_r(\partial_T G_r - G_r/T) \right] \\
&= \frac{1}{4T\rho} \sum_{r=n,p} \left[\frac{25\hbar^2\tau_r}{2M_r^*T^2} - 9\rho_r \left(\frac{5}{2} G_r/T + \partial_T G_r - G_r/T \right) \right] \\
&= \frac{1}{4T\rho} \sum_{r=n,p} \left[\frac{15\hbar^2\tau_r}{2M_r^*T} - 9\rho_r \left[\frac{5}{2} G_r/T - G_r/T - \frac{3}{2} G_r/T(1 + J_r) \right] \right] \\
&= \frac{3}{8T^2\rho} \sum_{r=n,p} \left[\frac{5\hbar^2\tau_r}{M_r^*} - 9\rho_r G_r J_r \right]
\end{aligned} \tag{49}$$

Thus, we can give analytical expressions for all independent 2^{nd} derivatives of the free energy per

baryon (see NSE notes). The notation for number density is n in NSE notes and ρ so far here, everything else is the same. Here we switch to match NSE notation.

$$\begin{aligned}
\partial_{TT}f &= -\partial_{Ts} = -\frac{1}{4Tn} \sum_{r=n,p} \left(\frac{5\hbar^2\tau_r}{TM_r^*} - 9n_rG_r \right) \\
\partial_{nY}f &= \partial_{n\mu} = \frac{5FG\hbar^2}{M}(\tau_n - \tau_p) + \frac{3GT}{M} \left[G_pM_p^*(1 + 3Fn_p) - G_nM_n^*(1 + 3Fn_n) \right] \\
&\quad + 3FT(G_n - G_p) + \frac{2T}{(1 - Y^2)n} [(G_n - G_p) - (G_n + G_p)Y] - 8D(\delta - 1)Yn^{\delta-1} \quad (50) \\
\partial_{nT}f &= s/n + \partial_T\mu = \frac{1}{n^2} \sum_r \left(\frac{5\hbar^2\tau_r}{6M_r^*T} - n_r\eta_r \right) + (\eta_n - \eta_p) - \frac{3}{2}(G_n - G_p) \\
&\quad - \left[\frac{5\hbar^2}{2T} \left[\left(\frac{\tau_n}{M_n^*} - \frac{\tau_p}{M_p^*} \right) - \frac{9}{2}(n_n\tau_n - n_p\tau_p) \right] \right]
\end{aligned}$$

For 3^{rd} order derivatives, the expressions above in connections with NSE notes can be used to obtain analytical results:

$$\begin{aligned}
\partial_{TTT}f &= -\partial_{TT}s \\
&= \frac{3}{8T^2\rho} \sum_{r=n,p} \left[\frac{5\hbar^2\tau_r}{M_r^*} - 9\rho_r G_r J_r \right] \\
\partial_{\{TTn\}}f &= -\partial_{\{Tn\}}s \\
&= -\frac{3}{4Tn} \sum_r \left[(2 - 4y_r - J_r)G_r + 3n_r F \left[5G_r - 3(1 + J_r)n_r \right] + \frac{5\hbar^2\tau_r}{TM_r^*} \left[F \left(2\frac{M_r^*}{M} - 5 \right) - \frac{1}{3n} \right] \right] \\
\partial_{\{nnT\}}f &= -\partial_{nn}s \\
&= -\frac{1}{n} \sum_r \left[\frac{3}{2}G_r(1 + 5Fy_r) - \frac{3}{2}G_r(5Fy_r + \frac{1}{n}) - 3Fy_r n_r + \frac{3}{2}(1 + J_r) \left(\frac{G_r}{n_r} + 3Fn_r \right) (1 + 5Fn_r) \right. \\
&\quad \left. - \frac{5}{2}G_r(1 + 3Fn_r) \left(\frac{1}{n} + 5F - 2F\frac{M_r^*}{M} \right) + \frac{5\hbar^2\tau_r}{6M_r^*Tn^2} \left[2 - 4F\frac{M_r^*}{M}n(1 + 5Fn) + 5Fn(2 + 5Fn) \right] \right] \\
\partial_{\{nnY\}}f &= \partial_{\{nnY\}}\mu = \frac{1}{\rho} \sum_r \left[\frac{G_r T}{\rho_r^2} \left[J_r + 3F(1 + J_r)\rho_r \right] \right. \\
&\quad + 3G\frac{G_r M_r T}{M\rho_r} (1 + J_r + F\rho_r + 3FJ_r\rho_r) + 3GT(1 + J_r + 3FJ_r\rho_r + 3F\rho_r) \\
&\quad - 3G^2G_r\frac{M_r}{M}T \left[15F\rho_r + 2\frac{M_r^*}{M}(1 + 3F\rho_r) \right] + 9G^2T(1 + J_r)\frac{M_r^*}{M}\rho_r(1 + 3F\rho_r) + G^2F\frac{25\hbar^2\tau_r}{M} \\
&\quad \left. - 16d(\delta - 1)\rho^{\delta-3} \right]
\end{aligned} \tag{51}$$