I. SKYRME INTERACTION - FINITE TEMPERATURE

A. Potential Matrix Element

Interaction Matrix:

$$\begin{aligned}
V_{ij} =& t_0(1+x_0P_\sigma)\delta(\mathbf{r}_{ij}) + \frac{1}{2}t_1(1+x_1P_\sigma)\frac{1}{\hbar^2}[\overleftarrow{p^\dagger}^2_{ij}\delta(\mathbf{r}_{ij}) + \delta(\mathbf{r}_{ij})\overrightarrow{p}^2_{ij}] \\
&+ t_2(1+x_2P_\sigma)\frac{1}{\hbar^2}\overleftarrow{\mathbf{p}^\dagger}_{ij}\cdot\delta(\mathbf{r}_{ij})\overrightarrow{\mathbf{p}}_{ij} + \frac{1}{6}t_3(1+x_3P_\sigma)\rho^\alpha(\mathbf{r})\delta(\mathbf{r}_{ij}) \\
&+ \frac{i}{\hbar^2}W_0(\sigma_i+\sigma_j)\cdot\overleftarrow{\mathbf{p}^\dagger}_{ij}\times\delta(\mathbf{r}_{ij})\overrightarrow{\mathbf{p}}_{ij} \\
&+ \frac{1}{4}t_4(1+x_4P_\sigma)\frac{1}{\hbar^2}[\overleftarrow{p^\dagger}^2_{ij}\rho^\beta(\mathbf{r})\delta(\mathbf{r}_{ij}) + \delta(\mathbf{r}_{ij})\rho^\beta(\mathbf{r})\overrightarrow{p}^2_{ij}] \\
&+ t_5(1+x_5P_\sigma)\frac{1}{\hbar^2}\overleftarrow{\mathbf{p}}_{ij}\cdot\rho^\gamma(\mathbf{r})\delta(\mathbf{r}_{ij})\overrightarrow{\mathbf{p}}_{ij}
\end{aligned} \tag{1}$$

where, $\mathbf{r}_{ij} = \frac{\mathbf{r}_i-\mathbf{r}_j}{2},\ \mathbf{r} = \frac{\mathbf{r}_i+\mathbf{r}_j}{2},\ P_\sigma = \frac{1}{2}(1+\sigma_1\cdot\sigma_2),\ \mathbf{p}_{ij} = -i\hbar\frac{\nabla_i-\nabla_j}{2},\ \rho(\mathbf{r}) = \rho_p(\mathbf{r})+\rho_n(\mathbf{r})$ and the arrows show the direction on which the momentum operators act.

B. Energy Density

Assume time reversal invariance. The Energy of the ground state can be written as integration over an energy density functional, $E_{HFB} = \int d^r \varepsilon_{HFB}$

$$
\begin{aligned}
\varepsilon_{HFB} = & \sum_{t=n,p} \frac{\hbar^2}{2M_q}\tau_q + \frac{1}{2}t_0\Big[(1+\frac{1}{2}x_0)\rho^2 - (\frac{1}{2}+x_0)\sum_{q=n,p}\rho_q^2\Big] \\
& + \frac{1}{4}t_1\Big[(1+\frac{1}{2}x_1)(\rho\tau + \frac{3}{4}(\nabla\rho)^2) - (\frac{1}{2}+x_1)\sum_{q=n,p}(\rho_q\tau_q + \frac{3}{4}(\nabla\rho_q)^2)\Big] \\
& + \frac{1}{4}t_2\Big[(1+\frac{1}{2}x_2)(\rho\tau + \frac{3}{4}(\nabla\rho)^2) + (\frac{1}{2}+x_2)\sum_{q=n,p}(\rho_q\tau_q - \frac{1}{4}(\nabla\rho_q)^2)\Big] \\
& + \frac{1}{12}t_3\rho^\alpha\Big[(1+\frac{1}{2}x_3)\rho^2 - (\frac{1}{2}+x_3)\sum_{q=n,p}\rho_q^2\Big] \\
& + \frac{1}{4}t_4\Big[(1+\frac{1}{2}x_4)(\rho\tau + \frac{3}{4}(\nabla\rho)^2) - (\frac{1}{2}+x_4)\sum_{q=n,p}(\rho_q\tau_q + \frac{3}{4}(\nabla\rho_q)^2)\Big]\rho^\beta \\
& + \frac{\beta}{8}t_4\Big[(1+\frac{1}{2}x_4)\rho(\nabla\rho)^2 - (\frac{1}{2}+x_4)\nabla\rho\cdot\sum_{q=n,p}\rho_q\nabla\rho_q\Big]\rho^{\beta-1} \\
& + \frac{1}{4}t_5\Big[(1+\frac{1}{2}x_5)(\rho\tau + \frac{3}{4}(\nabla\rho)^2) + (\frac{1}{2}+x_5)\sum_{q=n,p}(\rho_q\tau_q - \frac{1}{4}(\nabla\rho_q)^2)\Big]\rho^\gamma \\
& - \frac{1}{16}(t_1x_1 + t_2x_2)J^2 + \frac{1}{16}(t_1 - t_2)\sum_{q=n,p}J_q^2 \\
& - \frac{1}{16}(t_4x_4\rho^\beta + t_5x_5\rho^\gamma)J^2 + \frac{1}{16}(t_4\rho^\beta - t_5\rho^\beta)\sum_{q=n,p}J_q^2 \\
& + \frac{1}{2}W_0(\mathbf{J}\cdot\nabla\rho + \sum_{q=n,p}\mathbf{J}_q\cdot\nabla\rho_q)
\end{aligned}
\tag{2}
$$

where,

$$
\begin{aligned}
\rho &= 2\int \frac{d^3k}{(2\pi\hbar)^3} n(k) \\
\tau &= 2\int \frac{d^3k}{(2\pi\hbar)^3} k^2 n(k) \\
\mathbf{J} &= \int \frac{d^3k}{(2\pi\hbar)^3} \mathbf{k} \times \sum_{s,s'} \langle s|\boldsymbol{\sigma}|s'\rangle\ n(k)
\end{aligned}
\tag{3}
$$

The different terms can be grouped together in simpler notation:

$$\begin{aligned}
\varepsilon = &\sum_{t=n,p} \frac{\hbar^2}{2M_q}\tau_q + \frac{1}{4}t_0\Big[(2+x_0)\rho^2 - (1+2x_0)\sum_{q=n,p}\rho_q^2\Big] \\
&+ \frac{1}{8}\Big[a(\rho\tau + \frac{3}{4}(\nabla\rho)^2) + 2b\sum_{q=n,p}(\rho_q\tau_q + \frac{3}{4}(\nabla\rho_q)^2)\Big] \\
&+ \frac{1}{24}t_3\rho^\alpha\Big[(2+x_3)\rho^2 - (1+2x_3)\sum_{q=n,p}\rho_q^2\Big] \\
&+ \frac{1}{8}t_4\Big[(2+x_4)(\rho\tau + \frac{3}{4}(\nabla\rho)^2) - (1+2x_4)\sum_{q=n,p}(\rho_q\tau_q + \frac{3}{4}(\nabla\rho_q)^2)\Big]\rho^\beta \\
&+ \frac{\beta}{16}t_4\Big[(2+x_4)\rho(\nabla\rho)^2 - (1+2x_4)\nabla\rho\cdot\sum_{q=n,p}\rho_q\nabla\rho_q\Big]\rho^{\beta-1} \\
&+ \frac{1}{8}t_5\Big[(2+x_5)(\rho\tau + \frac{3}{4}(\nabla\rho)^2) + (1+2x_5)\sum_{q=n,p}(\rho_q\tau_q - \frac{1}{4}(\nabla\rho_q)^2)\Big]\rho^\gamma \\
&- \frac{1}{16}(t_1x_1 + t_2x_2)J^2 + \frac{1}{16}(t_1 - t_2)\sum_{q=n,p}J_q^2 \\
&- \frac{1}{16}(t_4x_4\rho^\beta + t_5x_5\rho^\gamma)J^2 + \frac{1}{16}(t_4\rho^\beta - t_5\rho^\beta)\sum_{q=n,p}J_q^2 \\
&+ \frac{1}{2}W_0(\mathbf{J}\cdot\nabla\rho + \sum_{q=n,p}\mathbf{J}_q\cdot\nabla\rho_q)
\end{aligned} \tag{4}$$

where, $a = t_1(x_1+2) + t_2(x_2+2)$, $b = \frac{1}{2}[t_2(2x_2+1) - t_1(2x_1+1)]$.

In uniform matter $\nabla\rho = 0$:

$$
\begin{aligned}
\varepsilon = & \sum_{t=n,p} \frac{\hbar^2}{2M_q}\tau_q + \frac{1}{4}t_0\Big[(2+x_0)\rho^2 - (1+2x_0)\sum_{q=n,p}\rho_q^2\Big] \\
& + \frac{1}{8}\Big[a\rho\tau + 2b\sum_{q=n,p}\rho_q\tau_q\Big] \\
& + \frac{1}{24}t_3\rho^\alpha\Big[(2+x_3)\rho^2 - (1+2x_3)\sum_{q=n,p}\rho_q^2\Big] \\
& + \frac{1}{8}t_4\Big[(2+x_4)\rho\tau - (1+2x_4)\sum_{q=n,p}\rho_q\tau_q\Big]\rho^\beta \\
& + \frac{1}{8}t_5\Big[(2+x_5)\rho\tau + (1+2x_5)\sum_{q=n,p}\rho_q\tau_q\Big]\rho^\gamma \\
& - \frac{1}{16}(t_1x_1 + t_2x_2)J^2 + \frac{1}{16}(t_1 - t_2)\sum_{q=n,p}J_q^2 \\
& - \frac{1}{16}(t_4x_4\rho^\beta + t_5x_5\rho^\gamma)J^2 + \frac{1}{16}(t_4\rho^\beta - t_5\rho^\beta)\sum_{q=n,p}J_q^2
\end{aligned}
\tag{5}
$$

In unpolarized matter, $\mathbf{J} = 0$:

$$
\begin{aligned}
\varepsilon = & \sum_{t=n,p} \frac{\hbar^2}{2M_q}\tau_q + \frac{1}{4}t_0\Big[(2+x_0)\rho^2 - (1+2x_0)\sum_{q=n,p}\rho_q^2\Big] \\
& + \frac{1}{8}\Big[a\rho\tau + 2b\sum_{q=n,p}\rho_q\tau_q\Big] \\
& + \frac{1}{24}t_3\rho^\alpha\Big[(2+x_3)\rho^2 - (1+2x_3)\sum_{q=n,p}\rho_q^2\Big] \\
& + \frac{1}{8}t_4\Big[(2+x_4)\rho\tau - (1+2x_4)\sum_{q=n,p}\rho_q\tau_q\Big]\rho^\beta \\
& + \frac{1}{8}t_5\Big[(2+x_5)\rho\tau + (1+2x_5)\sum_{q=n,p}\rho_q\tau_q\Big]\rho^\gamma
\end{aligned}
\tag{6}
$$

Energy per bayon, $\mathcal{E} \equiv \varepsilon/rho$:

$$\begin{aligned}\mathcal{E} = &\sum_{t=n,p} \frac{\hbar^2}{2M_q}\frac{\tau_q}{\rho} + \frac{1}{4}t_0\Big[(2+x_0)\rho - (1+2x_0)\sum_{q=n,p}\frac{\rho_q^2}{\rho}\Big] \\ &+ \frac{1}{8}\Big[a\tau + 2b\sum_{q=n,p}\frac{\rho_q\tau_q}{\rho}\Big] \\ &+ \frac{1}{24}t_3\rho^\alpha\Big[(2+x_3)\rho - (1+2x_3)\sum_{q=n,p}\frac{\rho_q^2}{\rho}\Big] \\ &+ \frac{1}{8}t_4\Big[(2+x_4)\tau - (1+2x_4)\sum_{q=n,p}\frac{\rho_q\tau_q}{\rho}\Big]\rho^\beta \\ &+ \frac{1}{8}t_5\Big[(2+x_5)\tau + (1+2x_5)\sum_{q=n,p}\frac{\rho_q\tau_q}{\rho}\Big]\rho^\gamma\end{aligned} \tag{7}$$

In terms of proton fraction, $y = \frac{\rho_p}{\rho_p+\rho_n}$:

$$\begin{aligned}\mathcal{E} = &\sum_{t=n,p} \frac{\hbar^2}{2M_q}\frac{\tau_q}{\rho} + \frac{1}{4}t_0\Big[(2+x_0) - (1+2x_0)[y^2+(1-y)^2]\Big]\rho \\ &+ \frac{1}{8}\Big[a\tau + 2b[y\tau_p + (1-y)\tau_n]\Big] \\ &+ \frac{1}{24}t_3\rho^{\alpha+1}\Big[(2+x_3) - (1+2x_3)[y^2+(1-y)^2]\Big] \\ &+ \frac{1}{8}t_4\Big[(2+x_4)\tau - (1+2x_4)[y\tau_p + (1-y)\tau_n]\Big]\rho^\beta \\ &+ \frac{1}{8}t_5\Big[(2+x_5)\tau + (1+2x_5)[y\tau_p + (1-y)\tau_n]\Big]\rho^\gamma\end{aligned} \tag{8}$$

II. SINGLE PARTICLE PROPERTIES

From the energy density the single particle spectrum can be derived. By performing functional variation of the energy density with repsect to the single particle wavefunction, a modified Schrodinger equation can be derived:

$$\delta\varepsilon_i = [\frac{\delta\varepsilon_i}{\delta\tau_i} + \frac{\delta\varepsilon_i}{\delta\rho_i}]\delta\phi_i = \epsilon_i\delta\phi_i \tag{9}$$

Since the Skyrme potential is at mostly quadratic in momenta with nonlinaer density dependence, its effect is exactly included by effective mass and mean field shift(residual interaction), both density dependent:

$$\begin{aligned}
\epsilon_i(k) =& \frac{\hbar^2 k^2}{2M_i^*} + U_i \\
\frac{\hbar^2}{2M_q^*} \equiv& \frac{\partial \varepsilon}{\partial \tau_q} \\
U_i \equiv& \frac{\partial \varepsilon}{\partial \rho_i}
\end{aligned} \tag{10}$$

From eq. 6 the effective baryon masses:

$$\begin{aligned}
M_p^*/M =& \Big\{ 1 + \frac{M\ \rho}{4\hbar^2} \big[a + 2\ y\ b + t_4[(2+x_4) - (1+2x_4)\ y]\rho^\beta + t_5[2+x_5+(1+2x_5)\ y]\rho^\gamma \big] \Big\}^{-1} \\
M_n^*/M =& \Big\{ 1 + \frac{M\ \rho}{4\hbar^2} \big[a + 2\ (1-y)\ b + t_4[(2+x_4) - (1+2x_4)\ (1-y)]\rho^\beta + t_5[2+x_5+(1+2x_5)\ (1-y)]\rho^\gamma \big] \Big\}^{-1}
\end{aligned} \tag{11}$$

and the residual potentials:

$$\begin{aligned}
U_p =& \frac{1}{8}(2b\ \tau_p + a\ \tau) + \frac{1}{2} t_0[(2+x_0) - (1+2x_0)\ y]\rho \\
&+ \frac{1}{24} t_3 \Big[4 + \alpha - 2y(1-(1-y)\alpha) + x_3(1-2y)[2-(1-2y)\alpha] \Big] \rho^{\alpha+1} \\
U_n =& \frac{1}{8}(2b\ \tau_n + a\ \tau) + \frac{1}{2} t_0[(1-x_0) + (1+2x_0)y]\rho \\
&+ \frac{1}{24} t_3 \Big[2 + \alpha + 2y(1+\alpha-y\alpha) - x_3(1-2y)[2+(1-2y)\alpha] \Big] \rho^{\alpha+1}
\end{aligned} \tag{12}$$

III. T=0 DFT

At $T = 0$, there are simple relation that can be drawn between the 2 integrations in Fourier space since the occuppation number is a step function:

$$
\begin{aligned}
\rho_q &= 2\int \frac{d^3k}{(2\pi\hbar)^3}\theta(k_{F,q}-k) = \frac{k_{F,q}^3}{3\pi^2\hbar^3} \\
\tau_q &= 2\int \frac{d^3k}{(2\pi\hbar)^3}(k/\hbar)^2\theta(k_{F,q}-k) = \frac{k_{F,q}^5}{5\pi^2\hbar^5} \rightarrow \\
\tau &= \frac{3}{5}(3\pi^2)^{2/3}\rho_q^{5/3}, H_n(y) = 2^{n-1}[y^n + (1-y)^n], y = \rho_p/\rho \\
\tau &= \tau_p + \tau_n = \frac{3}{5}(\frac{3\pi^2}{2})^{2/3}H_{5/3}(y)\rho^{5/3}
\end{aligned}
\tag{13}
$$

So,

$$
\begin{aligned}
\mathcal{E}_0 = & \frac{3\hbar^2}{10M}(\frac{3\pi^2}{2})^{2/3}\rho^{2/3}H_{5/3}(y) + \frac{1}{8}t_0\rho\Big[2(2+x_0) - (1+2x_0)H_2(y)\Big] \\
& + \frac{3}{40}(\frac{3\pi^2}{2})^{2/3}\Big[aH_{5/3}(y) + bH_{8/5}(y)\Big]\rho^{5/3} \\
& + \frac{1}{48}t_3\rho^{\alpha+1}\Big[2(2+x_3) - (1+2x_3)H_2(y)\Big] \\
& + \frac{3}{40}(\frac{3\pi^2}{2})^{2/3}t_4\rho^{\beta+5/3}\Big[(2+x_4)H_{5/3}(y) - (\frac{1}{2}+x_4)H_{8/3}(y)\Big] \\
& + \frac{3}{40}(\frac{3\pi^2}{2})^{2/3}t_5\rho^{\gamma+5/3}\Big[(2+x_5)H_{5/3}(y) + (\frac{1}{2}+x_5)H_{8/3}(y)\Big]
\end{aligned}
\tag{14}
$$

with $H_n(y) = 2^{n-1}[y^n + (1-y)^n]$. A common choice is to set $M = 1/2(M_n + M_p)$, or use the individual value for each species. In compact notation,

$$
\mathcal{E}_0 = C(y)\rho^{2/3} + A(y)\rho + B(y)\rho^{\alpha+1} + D(y)\rho^{5/3} + G(y)\rho^{\beta+5/3} + K(y)\rho^{\gamma+5/3} \tag{15}
$$

By comparing the 2 expressions, the following relations can be easily deduced:

$$
\begin{aligned}
C(y) =& \frac{3\hbar^2}{10M_y}(\frac{3\pi^2}{2})^{2/3}H_{5/3}(y) \\
A(y) =& \frac{1}{8}t_0\Big[2(2+x_0)-(1+2x_0)H_2(y)\Big] \\
B(y) =& \frac{1}{48}t_3\Big[2(2+x_3)-(1+2x_3)H_2(y)\Big] \\
D(y) =& \frac{3}{40}(\frac{3\pi^2}{2})^{2/3}\Big[aH_{5/3}(y)+bH_{8/5}(y)\Big] \\
G(y) =& \frac{3}{40}(\frac{3\pi^2}{2})^{2/3}t_4\Big[(2+x_4)H_{5/3}(y)-(\frac{1}{2}+x_4)H_{8/3}(y)\Big] \\
K(y) =& \frac{3}{40}(\frac{3\pi^2}{2})^{2/3}t_5\Big[(2+x_5)H_{5/3}(y)+(\frac{1}{2}+x_5)H_{8/3}(y)\Big]
\end{aligned}
\tag{16}
$$

For pure neutron matter and symmetric matter the kinetic coefficient is

$$
\begin{aligned}
C_n =& C(0) = \frac{3\hbar^2}{10M_n}(\frac{3\pi^2}{2})^{2/3}H_{5/3}(0) = 118.995 \text{ Mev fm}^2 \approx 119 \text{ Mev fm}^2 \\
C_{sym} =& C(1/2) = \frac{3\hbar^2}{5(M_n+M_p)}(\frac{3\pi^2}{2})^{2/3}H_{5/3}(1/2) = 75.0139 \text{ Mev fm}^2 \approx 75 \text{ Mev fm}^2
\end{aligned}
\tag{17}
$$

The effective mass is due to the terms dependent on kinetic energy:

$$
\begin{aligned}
\tau^{T=0}(\rho,y) \equiv& \frac{3\hbar^2}{10M^*}(\frac{3\pi^2}{2})^{2/3}\rho^{2/3}H_{5/3}(y) \\
=& \frac{3\hbar^2}{10M}(\frac{3\pi^2}{2})^{2/3}\rho^{2/3}H_{5/3}(y) + \frac{3}{40}(\frac{3\pi^2}{2})^{2/3}\Big[aH_{5/3}(y)+bH_{8/5}(y)\Big]\rho^{5/3} \\
&+ \frac{3}{40}(\frac{3\pi^2}{2})^{2/3}t_4\rho^{\beta+5/3}\Big[(2+x_4)H_{5/3}(y)-(\frac{1}{2}+x_4)H_{8/3}(y)\Big] \\
&+ \frac{3}{40}(\frac{3\pi^2}{2})^{2/3}t_5\rho^{\gamma+5/3}\Big[(2+x_5)H_{5/3}(y)+(\frac{1}{2}+x_5)H_{8/3}(y)\Big] \\
\equiv& C(y)\rho^{2/3}\Big[1+(D(y)\rho+G(y)\rho^{\beta+1}+K(y)\rho^{\gamma+1})/C(y)\Big]
\end{aligned}
\tag{18}
$$

Thus,

$$
\begin{aligned}
M^*/M =& \{1+\frac{M}{4H_{5/3}(y)\hbar^2}\Big[\rho[aH_{5/3}(y)+bH_{8/5}(y)]+\rho^{\beta+1}t_4[(2+x_4)H_{5/3}(y)-(\frac{1}{2}+x_4)H_{8/3}(y)] \\
&+\rho^{\gamma+1}t_5[(2+x_5)H_{5/3}(y)-(\frac{1}{2}+x_5)H_{8/3}(y)]\Big]\}^{-1} \\
=&\frac{C(y)}{C(y)+D(y)\rho+G(y)\rho^{\beta+1}+K(y)\rho^{\gamma+1}}
\end{aligned} \tag{19}
$$

Also, the thermodynamic pressure:

$$
\begin{aligned}
\mathcal{P}^{T=0}(\rho,y) =& \rho^2\frac{\partial\mathcal{E}^{T=0}}{\partial\rho} \\
=&\frac{2}{3}C(y)\rho^{5/3}+A(y)\rho^2+(\alpha+1)B(y)\rho^{\alpha+1}+\frac{5}{3}D(y)\rho^{8/3}+\beta G(y)\rho^{\beta+8/3}+\gamma K(y)\rho^{\gamma+8/3}
\end{aligned} \tag{20}
$$

IV. LATTIMER - SCHWESTY NOTATION

The standard parametrization:

$$
\begin{aligned}
\varepsilon =&\frac{\hbar^2\tau_n}{2M_n}+\frac{\hbar^2\tau_p}{2tM_p}+\frac{1}{8}[a(\rho_n+\rho_p)(\tau_n+\tau_p)+2b(\rho_n\tau_n+\rho_p\tau_p)]+\frac{1}{4}t_0[(2+x_0)(\rho_n+\rho_p)^2-(1+2x_0)(\rho_n^2+\rho_p^2)] \\
&+\frac{1}{24}t_3(\rho_n+\rho_p)^\alpha[(2+x_3)(\rho_n+\rho_p)^2-(1+2x_3)(\rho_n^2+\rho_p^2)]
\end{aligned} \tag{21}
$$

$$
\begin{aligned}
a =&\frac{4\hbar^2}{M}(F+G) \\
b =&-\frac{4\hbar^2}{M}G \\
t_0 =&\frac{8}{3}(A+B) \\
x_0 =&-\frac{1}{2}\frac{A-2B}{A+B} \\
t_3 =&16(C+D) \\
x_3 =&-\frac{1}{2}\frac{C-D}{C+D} \\
\alpha =&\delta-1
\end{aligned} \tag{22}
$$

$$
\begin{aligned}
a =&\frac{4\hbar^2}{M}(F+G) \\
b =&-\frac{4\hbar^2}{M}G \\
t_0 =&\frac{8}{3}(A+B) \\
x_0 =&-\frac{1}{2}\frac{A-2B}{A+B} \\
t_3 =&16(C+D) \\
x_3 =&-\frac{1}{2}\frac{C-D}{C+D} \\
\alpha =&\delta-1
\end{aligned} \tag{23}
$$

$$\begin{aligned}\varepsilon =&4B\rho_n\rho_p + A(\rho_n+\rho_p)^2 + (\rho_n+\rho_p)^{\delta-1}[4D\rho_n\rho_p + C(\rho_n+\rho_p)^2]\\&+\frac{5(\frac{2}{3})^{2/3}}{3\pi^{4/3}}\alpha_S[M(\frac{\tau_n}{M_n}+\frac{\tau_p}{M_p}) + F(\rho_n+\rho_p)(\tau_n+\tau_p) - G(\rho_n-\rho_p)(\tau_n-\tau_p)]\end{aligned} \tag{24}$$

The standard parametrization for $T=0,\ (M_n, M_p) \rightarrow M = \frac{1}{2}(M_n+M_p)$:

$$\begin{aligned}\varepsilon =& \frac{3\hbar^2}{10M}(\frac{3\pi^2}{2})^{2/3}\rho^{5/3}H_{5/3}(y) + \frac{1}{8}t_0\rho^2\Big[2(2+x_0)-(1+2x_0)H_2(y)\Big]\\&+\frac{3}{40}(\frac{3\pi^2}{2})^{2/3}\Big[aH_{5/3}(y)+bH_{8/5}(y)\Big]\rho^{8/3}\\&+\frac{1}{48}t_3\rho^{\alpha+2}\Big[2(2+x_3)-(1+2x_3)H_2(y)\Big]\end{aligned} \tag{25}$$

By comparing the expression in these notes with the ones from Lattimer parametrization, the energy density is:

$$\varepsilon = \alpha_S\rho^{5/3}H_{5/3}(y) + [A+B(2-H_2(y))]\rho^2 + [C+D(2-H_2(y))]\rho\delta + \alpha_S\rho^{5/3}[(F+G)H_{5/3} - GH_{8/3}(y)] \tag{26}$$

where, $\alpha_S = \frac{3\hbar^2}{10M}(\frac{3}{2}\pi^2)^{2/3}$

A. Skyrme parametrization from Saturation Observables

Given the following set of physical observables,

$$\begin{aligned}
E_0 =&\varepsilon\Big|_{\rho_0,y=1/2} = (A+B)\rho_0^2 + (C+D)\rho_0^{\delta+1} + \alpha_S\rho_0^{5/3}(1+F\rho_0) \\
P =&\rho^2\frac{d(\varepsilon/\rho)}{d\rho}\Big|_{\rho_0,y=1/2} = \frac{2}{3}\alpha_S\rho_0^{5/3} + (A+B)\rho_0^2 + \frac{5}{3}F\alpha_S\rho_0^{8/3} + (C+D)\delta\rho_0^{1+\delta} = 0 \\
(M^*/M) =&\frac{d\varepsilon}{d\tau}\Big|_{\rho_0,y=1/2} = (1+F\rho_0)^{-1} \\
K_m =&9\rho^2\frac{d^2(\varepsilon/\rho)}{d\rho^2}\Big|_{\rho_0,y=1/2} = -2\alpha_S\rho_0^{2/3} + 10F\alpha_S\rho_0^{5/3} + 9(C+D)(\delta-1)\delta\rho_0^{\delta} \\
S =&\frac{1}{8}\frac{d^2(\varepsilon/\rho)}{dy^2}\Big|_{\rho_0,y=1/2} = \frac{5}{9}\alpha_S\rho_0^{2/3} - B\rho_0 + \frac{5}{9}(F-3G)\alpha_S\rho_0^{5/3} - D\rho_0^{\delta} \\
L =&3\rho\frac{dS}{\rho}\Big|_{\rho_0,y=1/2} = \frac{10}{9}\alpha_S\rho_0^{2/3} - 3B\rho_0 + \frac{25}{9}(F-3G)\alpha_S\rho_0^{5/3} - 3D\delta\rho_0^{\delta} \\
K_s =&9\rho^2\frac{d^2S}{d\rho^2}\Big|_{\rho_0,y=1/2} = -\frac{10}{9}\alpha_S\rho_0^{2/3} + \frac{50}{9}(F-3G)\alpha_S\rho_0^{5/3} - 9D(\delta-1)\delta\rho_0^{\delta}
\end{aligned} \tag{27}$$

the skyrme parameters can be found as follows,

$$\begin{aligned}
F =&\frac{(M^*/M)^{-1}-1}{\rho_0} \\
\delta =&\frac{K_m + 2\rho_0^{2/3}(1-5F\rho_0)\alpha_S}{3\rho_0^{2/3}(1-2F\rho_0)\alpha_S - 9E_0} \\
G =&\frac{9K_S - 27(L-3S)\delta + 5\rho_0^{2/3}\alpha_S[2-3\delta+2F\rho_0(3\delta-5)]}{30\rho_0^{5/3}\alpha_S(3\delta-5)} \\
D =&\frac{5(3L-9S+\rho_0^{2/3}\alpha_S) - 3K_S}{9(5-8\delta+3\delta^2)\rho_0^{\delta}} \\
C =&\frac{\rho_0^{2/3}(1-2F\rho_0)\alpha_S - 3E_0}{3(\delta-1)\rho_0^{\delta}} - D \\
B =&\frac{L(6+9\delta) + 5(\rho_0^{2/3}\alpha_S(3\delta-2) - 9S\delta) - 3K_S}{18\rho_0(\delta-1)} \\
A =&-[\frac{2}{3}\alpha_S\rho^{-1/3} + B + \frac{5}{3}F\alpha_S\rho^{2/3} + (C+D)\delta\rho^{\delta-1}]
\end{aligned} \tag{28}$$

V. FINITE TEMPERATURE DFT

In order to obtain the relationship between density and chemical potential, the following set of coupled equations need to be solved self-consistenly:

$$
\begin{aligned}
f_k &= \left[1 + e^{\left(\frac{k^2}{2M^*} + U - \mu\right)/T}\right]^{-1} \\
\rho &= \sum_{s,is} \int \frac{d^3k}{(2\pi)^3} f_k \\
\tau &= \sum_{s,is} \int \frac{d^3k}{(2\pi)^3} k^2 f_k \\
E &\equiv E(\rho, \tau)
\end{aligned}
\tag{29}
$$

where, a sum over all discrete quantum numbers is performed (spin and isospin). And from the energy density functional, the mean field parameters can be derived,

$$
\begin{aligned}
M^* &= \frac{1}{2}\left(\frac{\delta E}{\delta \tau}\right)^{-1} \\
U &= \frac{\delta E}{\delta \rho}
\end{aligned}
\tag{30}
$$

The chemical potential can be found by inverting the expression for the density.

VI. THERMODYNAMIC POTENTIALS

Since the effect of phenomenological mean field models can be incorporated into (M^*, U) which are density dependent for Skyrme, and also temperature dependent for RMF, the thermodynamic properties of assymetric matter at finite temperature can be expressed by fermi integrals of 'modified' non-interacting fermi gases. The single particle spectrum and dustribution fucntion:

$$
\begin{aligned}
\xi_i =& \frac{k^2}{2M_i^*} + U_i \\
F_i =& \left\{ \exp\left[\frac{\xi_i - \mu_i}{T}\right] + 1 \right\}^{-1} = \left\{ \exp\left[\frac{\frac{k^2}{2M_i^*} - \eta_i}{T}\right] + 1 \right\}^{-1}, \ \overline{F} = 1 - F
\end{aligned}
\tag{31}
$$

The density and kinetic density:

$$\begin{aligned}\rho_i &= \int_0^\infty \frac{dk}{\pi^2} k^2 F_i \\ \tau_i &= \int_0^\infty \frac{dk}{\pi^2} k^4 F_i\end{aligned} \tag{32}$$

The entropy density can be calcualted fro mthe distribution function:

$$\begin{aligned}S/V =& -\int_0^\infty \frac{dk}{\pi^2}\ k^2 \big[F_i \ln(F_i) + (1-F_i)\ln(1-F_i)\big] \\ =& -\int_0^\infty \frac{dk}{\pi^2}\ k^2 F_i \ln(\frac{F_i}{1-F_i}) - \int_0^\infty \frac{dk}{\pi^2}\ k^2 \ln(1-F_i) \\ =& -\int_0^\infty \frac{dk}{\pi^2}\ k^2 F_i \ln(\exp\big[\frac{-\xi_i+\mu_i}{T}\big]) - [\frac{k^3}{3\pi^2}\ln(1-F_i)]\Big|_0^\infty + \int_0^\infty \frac{dk}{\pi^2}\ \frac{k^3}{3}\frac{k}{M_i^*}\frac{\exp\big[\frac{\xi_i-\mu_i}{T}\big]}{1-F_i} \\ =&[\frac{1}{2M_i^*T} + \frac{1}{3M_i^*T}] \int_0^\infty \frac{dk}{\pi^2}\ k^4 F_i + \frac{U_i-\mu_i}{T}\ \int_0^\infty \frac{dk}{\pi^2}\ k^2 F_i \\ =&\frac{1}{T}[\frac{5\tau_i}{6M_i^*} + (U_i-\mu_i)\rho]\end{aligned} \tag{33}$$

From the first law of thermodynamics:

$$\begin{aligned}E_i =& TS_i + \mu_i N_i - P_i V \\ P_i =& \frac{TS_i + \mu_i N_i - E_i}{V} \\ =& \frac{5\tau_i}{6M_i^*} + U_i\rho_i - \frac{E_i}{V}\end{aligned} \tag{34}$$

VII. THERMODYNAMIC DERIVATIVES

Let,

$$\alpha_1 = \frac{\hbar^2}{2M}(F-G), \alpha_2 = \frac{\hbar^2}{2M}(G+F) \tag{35}$$

Then,

$$
\begin{aligned}
\frac{\hbar^2}{2M_i^*} =& \frac{\hbar^2}{2M} + \alpha_1\rho_i + \alpha_2\rho_{-i} \rightarrow \\
M_i^* =& M\big[1 + F\rho - \mathrm{sgn}(i)G(\rho_n - \rho_p)\big]^{-1} \\
U_i =& \alpha_1\tau_i + \alpha_2\tau_{-i} + 2A\rho + 4B\rho_{-i} + C(1+\delta)\rho^\delta + 4D\rho_{-i}(\rho_{-i} + \delta\rho_i)\rho^{\delta-2} \\
\rho =& \rho_n + \rho_p
\end{aligned}
\tag{36}
$$

where, i denotes the isospin value. The set of independent parameters which is used for the 1^{st} part of thsi section is (ρ_n, ρ_p, T). From epxressions above, the partial derivatives can be found:

$$
\begin{aligned}
\partial_{\rho_i} M_r^* =& -\frac{M_r^{*2}}{M}\big[F + G(1 - 2\delta_{ir})\big] \\
=& -M\big[F + G(1-2\delta_{ir})\big]\big[1 + F\rho - \mathrm{sgn}(i)G(\rho_n - \rho_p)\big]^{-2} \\
\partial_T M_r^* =& 0 \\
\partial_{\rho_i} U_r =& \alpha_1\partial_{\rho_i}\tau_r + \alpha_2\partial_{\rho_i}\tau_{-r} + 2A + 4B(1-\delta_{ir}) + C\delta(1+\delta)\rho^{\delta-1} \\
&+ 4D\rho^{\delta-3}\big[(\delta-1)\rho_{-r}(2\rho_{-r} + \delta\rho_r)\delta_{ir} + (1-\delta_{-ir})[\delta(\rho_{-r}^2 + \rho_r^2) + \rho_{-r}\rho_r(2 + \delta(\delta-1))]\big] \\
\partial_T U_r =& \alpha_1\partial_T\tau_r + \alpha_2\partial_T\tau_{-r}
\end{aligned}
\tag{37}
$$

The number density and kinetic density can be expressed in terms of the general fermi integration:

$$
\begin{aligned}
F_n(\eta) = \int_0^\infty \frac{u^n}{e^{u-\eta}+1}du,\ \ \eta_i = (\mu_i - U_i)/T \\
\tau_r = \frac{1}{2\pi^2}(\frac{2M_r^*T}{\hbar^2})^{5/2}F_{3/2}(\eta_r) \\
\rho_r = \frac{1}{2\pi^2}(\frac{2M_r^*T}{\hbar^2})^{3/2}F_{1/2}(\eta_r)
\end{aligned}
\tag{38}
$$

By inverting the expression for the density:

$$
\begin{aligned}
\eta_r = F_{1/2}^{-1}\big[2\pi^2\rho_r(\frac{\hbar^2}{2M_r^*T})^{3/2}\big] \equiv F_{1/2}^{-1}\big[\eta_r^{(-1)}\big] \leftrightarrow \\
\eta_r = F_{1/2}^{-1}\big[\eta_r^{(-1)}\big],\ \eta_r^{(-1)} = 2\pi^2\rho_r(\frac{\hbar^2}{2M_r^*T})^{3/2} = F_{1/2}\big[\eta_r\big]
\end{aligned}
\tag{39}
$$

And,

$$
\begin{aligned}
\partial_{\rho_i}\eta_r^{(-1)} =& \Big[\frac{\delta_{ir}}{\rho_r} - \frac{3}{2}\frac{\partial_{\rho_i}M_r^*}{M_r^*}\Big]\eta_r^{(-1)} \\
=& \Big[\frac{\delta_{ir}}{\rho_r} + \frac{3}{2}\frac{M_r^*}{M}\big[F + G(1-2\delta_{ir})\big]\Big] \\
=& \frac{F_{1/2}(\eta_r)}{\rho_r}\Big[\delta_{ir} + \frac{3\rho_r}{2}\frac{M_r^*}{M}\big[F + G(1-2\delta_{ir})\big]\Big] \\
\partial_T\eta_r^{(-1)} =& -\frac{3}{2}\frac{\eta_r^{(-1)}}{T} \\
=& -\frac{3}{2}\frac{F_{1/2}(\eta_r)}{T}
\end{aligned}
\tag{40}
$$

Also, $\partial_\eta F_n(\eta) = nF_{n-1}(\eta) \leftrightarrow \partial_u F_n^{-1}(u) = \frac{1}{nF_{n-1}(u)}$:

$$
\begin{aligned}
\partial_{\rho_i}\eta_r =& \frac{\partial_{\rho_i}\eta_r^{(-1)}}{\partial_{\eta_r}F_{1/2}(\eta_r)} \\
=& \frac{2F_{1/2}(\eta_r)}{F_{-1/2}(\eta_r)\rho_r}\Big[\delta_{ir} + \frac{3\rho_r}{2}\frac{M_r^*}{M}\big[F + G(1-2\delta_{ir})\big]\Big] \\
=& \frac{G_r}{\rho_r}\Big[\delta_{ir} + \frac{3\rho_r}{2}\frac{M_r^*}{M}\big[F + G(1-2\delta_{ir})\big]\Big] \\
\partial_T\eta_r =& \frac{\partial_T\eta_r^{(-1)}}{\partial_{\eta_r}F_{1/2}(\eta_r)} \\
=& -\frac{3}{2T}\frac{2F_{1/2}(\eta_r)}{F_{-1/2}(\eta_r)} \\
=& -\frac{3}{2}\frac{G_r}{T}
\end{aligned}
\tag{41}
$$

where, $G_r = \frac{2F_{1/2}(\eta_r)}{F_{-1/2}(\eta_r)}$. Thus,

$$\begin{aligned}
\partial_{\rho_i}\tau_r =& \frac{5}{2}\tau_r\frac{\partial_{\rho_i}M_r^*}{M_r^*} + \frac{3}{2}(\frac{2M_r^*T}{\hbar^2})\rho_r\partial_{\rho_i}\eta_r \\
=& -\frac{5}{2}\tau_r\frac{M_r^*}{M}\big[F + G(1-2\delta_{ir})\big] + \frac{3}{2}(\frac{2M_r^*T}{\hbar^2})G_r\Big[\delta_{ir} + \frac{3\rho_r}{2}\frac{M_r^*}{M}\big[F + G(1-2\delta_{ir})\big]\Big] \\
=& 3TG_r\frac{M_r^*}{\hbar^2}\delta_{ir} + \frac{1}{2}\frac{M_r^*}{M}(\frac{9M_r^*}{\hbar^2}T\rho_rG_r - 5\tau_r)\big[F + G(1-2\delta_{ir})\big] \\
\partial_T\tau_r =& \frac{5}{4\pi^2T}(\frac{2M_r^*T}{\hbar^2})^{5/2}F_{3/2}(\eta_r) + \frac{1}{2\pi^2}(\frac{2M_r^*T}{\hbar^2})^{5/2}\partial_{\eta_r}F_{3/2}(\eta_r)\partial_T\eta_r \\
=& \frac{5}{2T}\tau_r - \frac{9}{8\pi^2T}(\frac{2M_r^*T}{\hbar^2})^{5/2}F_{1/2}(\eta_r)G_r \\
=& \frac{5}{2T}\tau_r - \frac{9}{2}\frac{M_r^*}{\hbar^2}G_r\rho_r
\end{aligned} \tag{42}$$

Now, we are ready to find the partial derivatives of the chemical potential,

$$\begin{aligned}
\partial_{\rho_i}\mu_r =& T\partial_{\rho_i}\eta_r + \partial_{\rho_i}U_r \\
=& \frac{TG_r}{\rho_r}\Big[\delta_{ir} + \frac{3\rho_rM_r^*}{\hbar^2}\big[\alpha_2 + (\alpha_1-\alpha_2)\delta_{ir}\big]\Big] + \partial_{\rho_i}U_r \\
=& \frac{TG_r}{\rho_r}\Big[\delta_{ir} + \frac{3}{2}\rho_r\big[F + G(1-2\delta_{ir})\big]\Big] + \partial_{\rho_i}U_r \\
\partial_T\mu_r =& \eta_r + T\partial_T\eta_r + \partial_TU_r \\
=& \eta_r - \frac{3}{2}G_r + \alpha_1\partial_T\tau_r + \alpha_2\partial_T\tau_{-r} \\
=& \eta_r - \frac{3}{2}G_r + \frac{5}{2T}(\alpha_1\tau_r + \alpha_2\tau_{-r}) - \frac{9}{2}(\alpha_1\frac{M_r^*}{\hbar^2}\rho_rG_r + \alpha_2\frac{M_{-r}^*}{\hbar^2}\rho_{-r}G_{-r}) \\
=& \eta_r - \frac{3}{2}G_r + \sum_i(\frac{5\tau_i}{2T} - \frac{9M_i^*}{2\hbar^2}\rho_iG_i)\big[\alpha_2 + (\alpha_1-\alpha_2)\big] \\
=& \eta_r - \frac{3}{2}G_r + \sum_i(\frac{5\tau_i\hbar^2}{4TM_i^*} - \frac{9}{4}\rho_iG_i)\big[F + G(1-2\delta_{ir})\big]
\end{aligned} \tag{43}$$

The set of independent variables we will use in our computations is (ρ, Y, T),

where $Y = Y_n - Y_p = \frac{\rho_n - \rho_p}{\rho}$:

$$\begin{aligned}
\partial_\rho X =& \partial_{\rho_n}X + \partial_{\rho_p}X, \\
\partial_Y X =& \frac{1}{\rho}(\partial_{\rho_n}X - \partial_{\rho_p}X)
\end{aligned} \tag{44}$$

Thus,

$$
\begin{aligned}
\partial_\rho M_r^* =& -\frac{2M_r^{*2}}{\hbar^2}(\alpha_1+\alpha_2) = -2\frac{M_r^{*2}}{M}F \\
\partial_\rho U_r =& 4(A+B) + 2C\delta(1+\delta)\rho^{\delta-1} + 4D\rho^{\delta-3}\big[\delta\rho_r^2 + (3\delta-2)\rho_{-r}^2 + 2[1+\delta(\delta-1)\rho_r\rho_{-r}]\big] \\
& + (\alpha_1+\alpha_2)\partial_\rho(\tau_r+\tau_{-r}) \\
=& 4(A+B) + 2C\delta(1+\delta)\rho^{\delta-1} + 4D\rho^{\delta-3}\big[\delta\rho_r^2 + (3\delta-2)\rho_{-r}^2 + 2[1+\delta(\delta-1)\rho_r\rho_{-r}]\big] \\
& + \sum_r \big[3TG_r\frac{M_r^*}{\hbar^2} + F(\frac{9M_r^*}{\hbar^2}T\rho_r G_r - 5\tau_r)\big] \\
\partial_\rho \tau_r =& \frac{3G_r M_r^* T}{\hbar^2}(1+3F\rho_r) - 5F\tau_r \\
\partial_\rho \eta_r =& \frac{G_r}{\rho_r} + 3F\rho_r \\
\partial_Y M_r^* =& -\operatorname{sgn}(r)\frac{2M_r^*}{\rho\hbar^2}(\alpha_1-\alpha_2),\ \operatorname{sgn}(n)=-1,\ \operatorname{sgn}(p)=1 \\
\partial_Y U_r =& \operatorname{sgn}(r)\big[4\frac{B}{\rho} - 4D\rho^{\delta-3}[(\delta-2)\rho_p - \delta\rho_n]\big] + \alpha_1\frac{\partial_Y\tau_r}{\rho} + \alpha_2\frac{\partial_Y\tau_{-r}}{\rho} \\
=& \operatorname{sgn}(r)\big[4\frac{B}{\rho} - 4D\rho^{\delta-3}[(\delta-2)\rho_p - \delta\rho_n] - \frac{3G_r M_r T}{\rho\hbar^2} + G(\frac{G_r M_r Y_r T}{\hbar^2} - 5\tau_r)\big],\ Y_r = \frac{1-\operatorname{sgn}(r)Y}{2} \\
\partial_Y \tau_r =& -\operatorname{sgn}(r)\big[\frac{3G_r T M_r}{\hbar^2}(1-3G\rho_r) - 5G\tau_r\big] \\
\partial_Y \eta_r =& -\operatorname{sgn}(r)\big[\frac{G_r}{\rho_r} - 3G\rho_r\big]
\end{aligned}
\tag{45}
$$

From the expressions above the derivatives of the μ_r can be derived:

$$
\begin{aligned}
\partial_\rho \mu_r &= \frac{TG_r}{\rho_r}(1+3\rho_r F) + \partial_\rho U_r, \\
\partial_Y \mu_r &= -\operatorname{sgn}(r)\frac{TG_r}{\rho\rho_r}(1-3\rho_r G) + \partial_Y U_r
\end{aligned}
\tag{46}
$$

And, the chemical potential we will use is $\mu = \mu_n - \mu_p$:

$$
\begin{aligned}
\partial_\rho \mu =& \partial_{\rho_n}\mu + \partial_{\rho_p}\mu \\
=& \frac{5FG\hbar^2}{M}(\tau_n - \tau_p) + \frac{3GT}{M}\Big[G_p M_p^*(1+3F\rho_p) - G_n M_n^*(1+3F\rho_n)\Big] \\
&+ 3FT(G_n - G_p) + \frac{2T}{(1-Y^2)\rho}[(G_n - G_p) - (G_n+G_p)Y] - 8D(\delta-1)Y\rho^{\delta-1} \\
\partial_{\rho Y}\mu =& \partial_Y\Big[\frac{5FG\hbar^2}{M}(\tau_n - \tau_p) + \frac{3GT}{M}\Big[G_p M_p^*(1+3F\rho_p) - G_n M_n^*(1+3F\rho_n)\Big] \\
&+ 3FT(G_n - G_p) + \frac{2T}{(1-Y^2)\rho}[(G_n - G_p) - (G_n+G_p)Y] - 8D(\delta-1)Y\rho^{\delta-1}\Big] \\
=& \frac{1}{\rho}\sum_r \Big[\frac{G_r T}{{\rho_r}^2}\big[J_r + 3F(1+J_r)\rho_r\big] \\
&+ 3G\frac{G_r M_r T}{M\rho_r}(1 + J_r + F\rho_r + 3FJ_r\rho_r) + 3GT(1 + J_r + 3FJ_r\rho_r + 3F\rho_r) \\
&- 3G^2 G_r \frac{M_r}{M} T\big[15F\rho_r + 2\frac{M_r^*}{M}(1+3F\rho_r)\big] + 9G^2 T(1+J_r)\frac{M_r^*}{M}\rho_r(1+3F\rho_r) + G^2 F\frac{25\hbar^2\tau_r}{M}\Big] \\
&- 16d(\delta-1)\rho^{\delta-3} \\
\partial_T \mu =& \partial_T \mu_n - \partial_T \mu_p \\
=& (\eta_n - \eta_p) - \frac{3}{2}(G_n - G_p) + \frac{9}{2}G(G_n\rho_n - G_p\rho_p) - \frac{5G\hbar^2}{T}(\frac{\tau_n}{M_n^*} - \frac{\tau_p}{M_p^*})
\end{aligned}
\tag{47}
$$

For higher order thermodynamic derivatives, the following calculation will be needed:

$$
\begin{aligned}
\partial_{\eta_r} G_r(\eta_r) =& 1 + J_r(\eta_r) \\
J_r(\eta_r) =& \frac{F_{-3/2}(\eta_r)F_{1/2}(\eta_r)}{F_{-1/2}(\eta_r)}
\end{aligned}
\tag{48}
$$

The entropy per baryon is calculated in the previous section and it is $\rho s = \sum_r (\frac{5\hbar^2\tau_r}{6M_r^*T} - \rho_r\eta_r)$:

$$
\begin{aligned}
\rho\partial_T s =& \sum_r \frac{5\hbar^2}{6M_r^*T}(\partial_T\tau_r - \tau_r/T) - \rho_r\partial_T\eta_r) \\
=& \frac{1}{4T}\sum_r (\frac{5\hbar^2\tau_r}{TM_r^*} - 9\rho_r G_r) \\
s + \rho\partial_\rho s =& \sum_r \Big[\frac{5\hbar^2}{6M_r^*T}(\partial_\rho\tau_r - \tau_r\partial_\rho M_r^*/M_r^*) - y_r\eta_r - \rho_r\partial_\rho\eta_r)\Big] \\
=& \sum_r \Big[\frac{3}{2}G_r + 3F\rho_r(\frac{5}{2}G_r - \rho_r) + \frac{5\hbar^2\tau_r}{6M_r^*T}F(2\frac{M_r^*}{M} - 5) - y_r\eta_r\Big] \leftrightarrow \\
\rho\partial_\rho s =& \sum_r \Big[\frac{3}{2}G_r(1+5F\rho_r) - 3F\rho_r^2 - \frac{5\hbar^2\tau_r}{6MT}\big[\frac{M}{M_r^*}(\frac{1}{\rho} + 5F) - 2F\big]\Big] \leftrightarrow \\
\partial_\rho s + \rho\partial_{\rho\rho} s =& \sum_r \Big[\frac{3}{2}\big[G_r + 5FG_r y_r - 4Fy_r\rho_r + (1+J_r)(\frac{G_r}{\rho_r} + 3F\rho_r)(1+5F\rho_r)\big] \\
& + \frac{5\hbar^2\tau_r}{6MT}\big[5F^2(5\frac{M_r^*}{M} - 4) + \frac{F}{\rho}(5\frac{M}{M_r^*} - 2) + \frac{M}{M_r^*\rho^2}\big] - \frac{5}{2}\frac{M_r^*}{M}G_r(1+3F\rho_r)\big[\frac{M}{M_r^*}(\frac{1}{\rho} + 5F) - 2F\big]\Big] \\
\rho\partial_{\rho\rho} s =& \sum_r \Big[\frac{3}{2}G_r(1+5Fy_r) - \frac{3}{2}G_r(5Fy_r + \frac{1}{\rho}) - 3Fy_r\rho_r + \frac{3}{2}(1+J_r)(\frac{G_r}{\rho_r} + 3F\rho_r)(1+5F\rho_r) \\
& - \frac{5}{2}G_r(1+3F\rho_r)(\frac{1}{\rho} + 5F - 2F\frac{M_r^*}{M}) + \frac{5\hbar^2\tau_r}{6M_r^*T\rho^2}\big[2 - 4F\frac{M_r^*}{M}\rho(1+5F\rho) + 5F\rho(2+5F\rho)\big]\Big] \\
\rho\partial_{T\rho} s =& \frac{3}{4T}\sum_r \Big[(2 - 4y_r - J_r)G_r + 3\rho_r F\big[5G_r - 3(1+J_r)\rho_r\big] + \frac{5\hbar^2\tau_r}{TM_r^*}\big[F(2\frac{M_r^*}{M} - 5) - \frac{1}{3\rho}\big]\Big] \\
-\partial_{TT} s =& \partial_T\Big[\frac{1}{4T\rho}\sum_{r=n,p}(\frac{5\hbar^2\tau_r}{TM_r^*} - 9\rho_r G_r)\Big] \\
=& \frac{1}{4T\rho}\sum_{r=n,p}\big[\frac{5\hbar^2(\partial_T\tau_r - 2\tau_r/T)}{TM_r^*} - 9\rho_r(\partial_T G_r - G_r/T)\big] \\
=& \frac{1}{4T\rho}\sum_{r=n,p}\big[\frac{25\hbar^2\tau_r}{2M_r^*T^2} - 9\rho_r(\frac{5}{2}G_r/T + \partial_T G_r - G_r/T)\big] \\
=& \frac{1}{4T\rho}\sum_{r=n,p}\big[\frac{15\hbar^2\tau_r}{2M_r^*T} - 9\rho_r\big[\frac{5}{2}G_r/T - G_r/T - \frac{3}{2}G_r/T(1+J_r)\big]\big] \\
=& \frac{3}{8T^2\rho}\sum_{r=n,p}\big[\frac{5\hbar^2\tau_r}{M_r^*} - 9\rho_r G_r J_r\big]
\end{aligned}
\tag{49}
$$

Thus, we can give analytical expressions for all independent 2^{nd} derivatives of the free energy per baryon (see NSE notes). The notation for number density is n in NSE notes and ρ so far here, everything else is the same. Here we switch to match NSE notation.

$$\boxed{\begin{aligned}
\partial_{TT} f =& -\partial_T s = -\frac{1}{4Tn}\sum_{r=n,p}(\frac{5\hbar^2\tau_r}{TM_r^*} - 9n_r G_r) \\
\partial_{nY} f =& \partial_n \mu = \frac{5FG\hbar^2}{M}(\tau_n - \tau_p) + \frac{3GT}{M}\Big[G_p M_p^*(1+3Fn_p) - G_n M_n^*(1+3Fn_n)\Big] \\
& + 3FT(G_n - G_p) + \frac{2T}{(1-Y^2)n}[(G_n - G_p) - (G_n + G_p)Y] - 8D(\delta - 1)Yn^{\delta-1} \\
\partial_{nT} f =& s/n + \partial_T \mu = \frac{1}{n^2}\sum_r(\frac{5\hbar^2\tau_r}{6M_r^* T} - n_r\eta_r) + (\eta_n - \eta_p) - \frac{3}{2}(G_n - G_p) \\
& - \Big[\frac{5\hbar^2}{2T}[(\frac{\tau_n}{M_n^*} - \frac{\tau_p}{M_p^*}) - \frac{9}{2}(n_n\tau_n - n_p\tau_p)]\Big]
\end{aligned}} \tag{50}$$

For 3^{rd} order derivatives, the expressions above in connections with NSE notes can be used to obtain analytical results:

$$
\begin{aligned}
\partial_{TTT} f =& -\partial_{TT} s \\
=& \frac{3}{8T^2\rho} \sum_{r=n,p} \big[\frac{5\hbar^2\tau_r}{M_r^*} - 9\rho_r G_r J_r\big] \\
\partial_{\{TTn\}} f =& -\partial_{\{Tn\}} s \\
=& -\frac{3}{4Tn} \sum_r \Big[(2-4y_r-J_r)G_r + 3n_r F\big[5G_r - 3(1+J_r)n_r\big] + \frac{5\hbar^2\tau_r}{TM_r^*}\big[F(2\frac{M_r^*}{M}-5) - \frac{1}{3n}\big]\Big] \\
\partial_{\{nnT\}} f =& -\partial_{nn} s \\
=& -\frac{1}{n} \sum_r \Big[\frac{3}{2}G_r(1+5Fy_r) - \frac{3}{2}G_r(5Fy_r + \frac{1}{n}) - 3Fy_r n_r + \frac{3}{2}(1+J_r)(\frac{G_r}{n_r} + 3Fn_r)(1+5Fn_r) \\
& - \frac{5}{2}G_r(1+3Fn_r)(\frac{1}{n} + 5F - 2F\frac{M_r^*}{M}) + \frac{5\hbar^2\tau_r}{6M_r^*Tn^2}\big[2 - 4F\frac{M_r^*}{M}n(1+5Fn) + 5Fn(2+5Fn)\big]\Big] \\
\partial_{\{nnY\}} f =& \partial_{\{nnY\}}\mu = \frac{1}{\rho} \sum_r \Big[\frac{G_r T}{{\rho_r}^2}\big[J_r + 3F(1+J_r)\rho_r\big] \\
& + 3G\frac{G_r M_r T}{M\rho_r}(1 + J_r + F\rho_r + 3FJ_r\rho_r) + 3GT(1 + J_r + 3FJ_r\rho_r + 3F\rho_r) \\
& - 3G^2 G_r \frac{M_r}{M} T\big[15F\rho_r + 2\frac{M_r^*}{M}(1+3F\rho_r)\big] + 9G^2T(1+J_r)\frac{M_r^*}{M}\rho_r(1+3F\rho_r) + G^2F\frac{25\hbar^2\tau_r}{M}\Big] \\
& - 16d(\delta-1)\rho^{\delta-3}
\end{aligned}
\tag{51}
$$