

## I. SKYRME INTERACTION - FINITE TEMPERATURE

### A. Potential Matrix Element

Interaction Matrix:

$$\begin{aligned}
V_{ij} = & t_0(1 + x_0 P_\sigma) \delta(\mathbf{r}_{ij}) + \frac{1}{2} t_1(1 + x_1 P_\sigma) \frac{1}{\hbar^2} [\overleftarrow{p}_{ij}^{\dagger 2} \delta(\mathbf{r}_{ij}) + \delta(\mathbf{r}_{ij}) \overrightarrow{p}_{ij}^2] \\
& + t_2(1 + x_2 P_\sigma) \frac{1}{\hbar^2} \overleftarrow{\mathbf{p}}_{ij}^\dagger \cdot \delta(\mathbf{r}_{ij}) \overrightarrow{\mathbf{p}}_{ij} + \frac{1}{6} t_3(1 + x_3 P_\sigma) \rho^\alpha(\mathbf{r}) \delta(\mathbf{r}_{ij}) \\
& + \frac{i}{\hbar^2} W_0(\sigma_i + \sigma_j) \cdot \overleftarrow{\mathbf{p}}_{ij}^\dagger \times \delta(\mathbf{r}_{ij}) \overrightarrow{\mathbf{p}}_{ij} \\
& + \frac{1}{4} t_4(1 + x_4 P_\sigma) \frac{1}{\hbar^2} [\overleftarrow{p}_{ij}^{\dagger 2} \rho^\beta(\mathbf{r}) \delta(\mathbf{r}_{ij}) + \delta(\mathbf{r}_{ij}) \rho^\beta(\mathbf{r}) \overrightarrow{p}_{ij}^2] \\
& + t_5(1 + x_5 P_\sigma) \frac{1}{\hbar^2} \overleftarrow{\mathbf{p}}_{ij} \cdot \rho^\gamma(\mathbf{r}) \delta(\mathbf{r}_{ij}) \overrightarrow{\mathbf{p}}_{ij}
\end{aligned} \tag{1}$$

where,  $\mathbf{r}_{ij} = \frac{\mathbf{r}_i - \mathbf{r}_j}{2}$ ,  $\mathbf{r} = \frac{\mathbf{r}_i + \mathbf{r}_j}{2}$ ,  $P_\sigma = \frac{1}{2}(1 + \sigma_1 \cdot \sigma_2)$ ,  $\mathbf{p}_{ij} = -i\hbar \frac{\nabla_i - \nabla_j}{2}$ ,  $\rho(\mathbf{r}) = \rho_p(\mathbf{r}) + \rho_n(\mathbf{r})$  and the arrows show the direction on which the momentum operators act.

## B. Energy Density

Assume time reversal invariance. The Energy of the ground state can be written as integration over an energy density functional,  $E_{HFB} = \int d^r \varepsilon_{HFB}$

$$\begin{aligned}
\varepsilon_{HFB} = & \sum_{t=n,p} \frac{\hbar^2}{2M_q} \tau_q + \frac{1}{2} t_0 \left[ \left(1 + \frac{1}{2} x_0\right) \rho^2 - \left(\frac{1}{2} + x_0\right) \sum_{q=n,p} \rho_q^2 \right] \\
& + \frac{1}{4} t_1 \left[ \left(1 + \frac{1}{2} x_1\right) (\rho\tau + \frac{3}{4} (\nabla\rho)^2) - \left(\frac{1}{2} + x_1\right) \sum_{q=n,p} (\rho_q \tau_q + \frac{3}{4} (\nabla\rho_q)^2) \right] \\
& + \frac{1}{4} t_2 \left[ \left(1 + \frac{1}{2} x_2\right) (\rho\tau + \frac{3}{4} (\nabla\rho)^2) + \left(\frac{1}{2} + x_2\right) \sum_{q=n,p} (\rho_q \tau_q - \frac{1}{4} (\nabla\rho_q)^2) \right] \\
& + \frac{1}{12} t_3 \rho^\alpha \left[ \left(1 + \frac{1}{2} x_3\right) \rho^2 - \left(\frac{1}{2} + x_3\right) \sum_{q=n,p} \rho_q^2 \right] \\
& + \frac{1}{4} t_4 \left[ \left(1 + \frac{1}{2} x_4\right) (\rho\tau + \frac{3}{4} (\nabla\rho)^2) - \left(\frac{1}{2} + x_4\right) \sum_{q=n,p} (\rho_q \tau_q + \frac{3}{4} (\nabla\rho_q)^2) \right] \rho^\beta \\
& + \frac{\beta}{8} t_4 \left[ \left(1 + \frac{1}{2} x_4\right) \rho (\nabla\rho)^2 - \left(\frac{1}{2} + x_4\right) \nabla\rho \cdot \sum_{q=n,p} \rho_q \nabla\rho_q \right] \rho^{\beta-1} \\
& + \frac{1}{4} t_5 \left[ \left(1 + \frac{1}{2} x_5\right) (\rho\tau + \frac{3}{4} (\nabla\rho)^2) + \left(\frac{1}{2} + x_5\right) \sum_{q=n,p} (\rho_q \tau_q - \frac{1}{4} (\nabla\rho_q)^2) \right] \rho^\gamma \\
& - \frac{1}{16} (t_1 x_1 + t_2 x_2) J^2 + \frac{1}{16} (t_1 - t_2) \sum_{q=n,p} J_q^2 \\
& - \frac{1}{16} (t_4 x_4 \rho^\beta + t_5 x_5 \rho^\gamma) J^2 + \frac{1}{16} (t_4 \rho^\beta - t_5 \rho^\gamma) \sum_{q=n,p} J_q^2 \\
& + \frac{1}{2} W_0 (\mathbf{J} \cdot \nabla\rho + \sum_{q=n,p} \mathbf{J}_q \cdot \nabla\rho_q)
\end{aligned} \tag{2}$$

where,

$$\begin{aligned}
\rho &= 2 \int \frac{d^3 k}{(2\pi\hbar)^3} n(k) \\
\tau &= 2 \int \frac{d^3 k}{(2\pi\hbar)^3} k^2 n(k) \\
\mathbf{J} &= \int \frac{d^3 k}{(2\pi\hbar)^3} \mathbf{k} \times \sum_{s,s'} \langle s | \boldsymbol{\sigma} | s' \rangle n(k)
\end{aligned} \tag{3}$$

The different terms can be grouped together in simpler notation:

$$\begin{aligned}
\varepsilon = & \sum_{t=n,p} \frac{\hbar^2}{2M_q} \tau_q + \frac{1}{4} t_0 \left[ (2+x_0) \rho^2 - (1+2x_0) \sum_{q=n,p} \rho_q^2 \right] \\
& + \frac{1}{8} \left[ a(\rho\tau + \frac{3}{4}(\nabla\rho)^2) + 2b \sum_{q=n,p} (\rho_q \tau_q + \frac{3}{4}(\nabla\rho_q)^2) \right] \\
& + \frac{1}{24} t_3 \rho^\alpha \left[ (2+x_3) \rho^2 - (1+2x_3) \sum_{q=n,p} \rho_q^2 \right] \\
& + \frac{1}{8} t_4 \left[ (2+x_4)(\rho\tau + \frac{3}{4}(\nabla\rho)^2) - (1+2x_4) \sum_{q=n,p} (\rho_q \tau_q + \frac{3}{4}(\nabla\rho_q)^2) \right] \rho^\beta \\
& + \frac{\beta}{16} t_4 \left[ (2+x_4) \rho(\nabla\rho)^2 - (1+2x_4) \nabla\rho \cdot \sum_{q=n,p} \rho_q \nabla\rho_q \right] \rho^{\beta-1} \\
& + \frac{1}{8} t_5 \left[ (2+x_5)(\rho\tau + \frac{3}{4}(\nabla\rho)^2) + (1+2x_5) \sum_{q=n,p} (\rho_q \tau_q - \frac{1}{4}(\nabla\rho_q)^2) \right] \rho^\gamma \\
& - \frac{1}{16} (t_1 x_1 + t_2 x_2) J^2 + \frac{1}{16} (t_1 - t_2) \sum_{q=n,p} J_q^2 \\
& - \frac{1}{16} (t_4 x_4 \rho^\beta + t_5 x_5 \rho^\gamma) J^2 + \frac{1}{16} (t_4 \rho^\beta - t_5 \rho^\gamma) \sum_{q=n,p} J_q^2 \\
& + \frac{1}{2} W_0 (\mathbf{J} \cdot \nabla\rho + \sum_{q=n,p} \mathbf{J}_q \cdot \nabla\rho_q)
\end{aligned} \tag{4}$$

where,  $a = t_1(x_1 + 2) + t_2(x_2 + 2)$ ,  $b = \frac{1}{2}[t_2(2x_2 + 1) - t_1(2x_1 + 1)]$ .

In uniform matter  $\nabla\rho = 0$ :

$$\begin{aligned}
\varepsilon = & \sum_{t=n,p} \frac{\hbar^2}{2M_q} \tau_q + \frac{1}{4} t_0 \left[ (2+x_0) \rho^2 - (1+2x_0) \sum_{q=n,p} \rho_q^2 \right] \\
& + \frac{1}{8} \left[ a \rho \tau + 2b \sum_{q=n,p} \rho_q \tau_q \right] \\
& + \frac{1}{24} t_3 \rho^\alpha \left[ (2+x_3) \rho^2 - (1+2x_3) \sum_{q=n,p} \rho_q^2 \right] \\
& + \frac{1}{8} t_4 \left[ (2+x_4) \rho \tau - (1+2x_4) \sum_{q=n,p} \rho_q \tau_q \right] \rho^\beta \\
& + \frac{1}{8} t_5 \left[ (2+x_5) \rho \tau + (1+2x_5) \sum_{q=n,p} \rho_q \tau_q \right] \rho^\gamma \\
& - \frac{1}{16} (t_1 x_1 + t_2 x_2) J^2 + \frac{1}{16} (t_1 - t_2) \sum_{q=n,p} J_q^2 \\
& - \frac{1}{16} (t_4 x_4 \rho^\beta + t_5 x_5 \rho^\gamma) J^2 + \frac{1}{16} (t_4 \rho^\beta - t_5 \rho^\gamma) \sum_{q=n,p} J_q^2
\end{aligned} \tag{5}$$

In unpolarized matter,  $\mathbf{J} = 0$ :

$$\begin{aligned}
\varepsilon = & \sum_{t=n,p} \frac{\hbar^2}{2M_q} \tau_q + \frac{1}{4} t_0 \left[ (2+x_0) \rho^2 - (1+2x_0) \sum_{q=n,p} \rho_q^2 \right] \\
& + \frac{1}{8} \left[ a \rho \tau + 2b \sum_{q=n,p} \rho_q \tau_q \right] \\
& + \frac{1}{24} t_3 \rho^\alpha \left[ (2+x_3) \rho^2 - (1+2x_3) \sum_{q=n,p} \rho_q^2 \right] \\
& + \frac{1}{8} t_4 \left[ (2+x_4) \rho \tau - (1+2x_4) \sum_{q=n,p} \rho_q \tau_q \right] \rho^\beta \\
& + \frac{1}{8} t_5 \left[ (2+x_5) \rho \tau + (1+2x_5) \sum_{q=n,p} \rho_q \tau_q \right] \rho^\gamma
\end{aligned} \tag{6}$$

Energy per bayon,  $\mathcal{E} \equiv \varepsilon/rho$ :

$$\begin{aligned}
\mathcal{E} = & \sum_{t=n,p} \frac{\hbar^2}{2M_q} \frac{\tau_q}{\rho} + \frac{1}{4} t_0 \left[ (2+x_0)\rho - (1+2x_0) \sum_{q=n,p} \frac{\rho_q^2}{\rho} \right] \\
& + \frac{1}{8} \left[ a\tau + 2b \sum_{q=n,p} \frac{\rho_q \tau_q}{\rho} \right] \\
& + \frac{1}{24} t_3 \rho^\alpha \left[ (2+x_3)\rho - (1+2x_3) \sum_{q=n,p} \frac{\rho_q^2}{\rho} \right] \\
& + \frac{1}{8} t_4 \left[ (2+x_4)\tau - (1+2x_4) \sum_{q=n,p} \frac{\rho_q \tau_q}{\rho} \right] \rho^\beta \\
& + \frac{1}{8} t_5 \left[ (2+x_5)\tau + (1+2x_5) \sum_{q=n,p} \frac{\rho_q \tau_q}{\rho} \right] \rho^\gamma
\end{aligned} \tag{7}$$

In terms of proton fraction,  $y = \frac{\rho_p}{\rho_p + \rho_n}$ :

$$\begin{aligned}
\mathcal{E} = & \sum_{t=n,p} \frac{\hbar^2}{2M_q} \frac{\tau_q}{\rho} + \frac{1}{4} t_0 \left[ (2+x_0) - (1+2x_0)[y^2 + (1-y)^2] \right] \rho \\
& + \frac{1}{8} \left[ a\tau + 2b[y\tau_p + (1-y)\tau_n] \right] \\
& + \frac{1}{24} t_3 \rho^{\alpha+1} \left[ (2+x_3) - (1+2x_3)[y^2 + (1-y)^2] \right] \\
& + \frac{1}{8} t_4 \left[ (2+x_4)\tau - (1+2x_4)[y\tau_p + (1-y)\tau_n] \right] \rho^\beta \\
& + \frac{1}{8} t_5 \left[ (2+x_5)\tau + (1+2x_5)[y\tau_p + (1-y)\tau_n] \right] \rho^\gamma
\end{aligned} \tag{8}$$

## II. SINGLE PARTICLE PROPERTIES

From the energy density the single particle spectrum can be derived. By performing functional variation of the energy density with respect to the single particle wavefunction, a modified Schrodinger equation can be derived:

$$\delta\varepsilon_i = \left[ \frac{\delta\varepsilon_i}{\delta\tau_i} + \frac{\delta\varepsilon_i}{\delta\rho_i} \right] \delta\phi_i = \epsilon_i \delta\phi_i \tag{9}$$

Since the Skyrme potential is at mostly quadratic in momenta with nonlinaer density dependence, its effect is exactly included by effective mass and mean field shift(residual interaction), both density dependent:

$$\begin{aligned}\epsilon_i(k) &= \frac{\hbar^2 k^2}{2M_i^*} + U_i \\ \frac{\hbar^2}{2M_q^*} &\equiv \frac{\partial \varepsilon}{\partial \tau_q} \\ U_i &\equiv \frac{\partial \varepsilon}{\partial \rho_i}\end{aligned}\tag{10}$$

From eq. 6 the effective baryon masses:

$$\begin{aligned}M_p^*/M &= \left\{ 1 + \frac{M}{4\hbar^2} \rho [a + 2 y b + t_4[(2 + x_4) - (1 + 2x_4) y] \rho^\beta + t_5[2 + x_5 + (1 + 2x_5) y] \rho^\gamma] \right\}^{-1} \\ M_n^*/M &= \left\{ 1 + \frac{M}{4\hbar^2} \rho [a + 2 (1 - y) b + t_4[(2 + x_4) - (1 + 2x_4) (1 - y)] \rho^\beta + t_5[2 + x_5 + (1 + 2x_5) (1 - y)] \rho^\gamma] \right\}^{-1}\end{aligned}\tag{11}$$

and the residual potentials:

$$\begin{aligned}U_p &= \frac{1}{8}(2b \tau_p + a \tau) + \frac{1}{2}t_0[(2 + x_0) - (1 + 2x_0) y] \rho \\ &\quad + \frac{1}{24}t_3 \left[ 4 + \alpha - 2y(1 - (1 - y)\alpha) + x_3(1 - 2y)[2 - (1 - 2y)\alpha] \right] \rho^{\alpha+1} \\ U_n &= \frac{1}{8}(2b \tau_n + a \tau) + \frac{1}{2}t_0[(1 - x_0) + (1 + 2x_0)y] \rho \\ &\quad + \frac{1}{24}t_3 \left[ 2 + \alpha + 2y(1 + \alpha - y\alpha) - x_3(1 - 2y)[2 + (1 - 2y)\alpha] \right] \rho^{\alpha+1}\end{aligned}\tag{12}$$

### III. T=0 DFT

At  $T = 0$ , there are simple relation that can be drawn between the 2 integrations in Fourier space since the occupation number is a step function:

$$\begin{aligned}
\rho_q &= 2 \int \frac{d^3k}{(2\pi\hbar)^3} \theta(k_{F,q} - k) = \frac{k_{F,q}^3}{3\pi^2\hbar^3} \\
\tau_q &= 2 \int \frac{d^3k}{(2\pi\hbar)^3} (k/\hbar)^2 \theta(k_{F,q} - k) = \frac{k_{F,q}^5}{5\pi^2\hbar^5} \rightarrow \\
\tau &= \frac{3}{5} (3\pi^2)^{2/3} \rho_q^{5/3}, H_n(y) = 2^{n-1} [y^n + (1-y)^n], y = \rho_p/\rho \\
\tau &= \tau_p + \tau_n = \frac{3}{5} \left(\frac{3\pi^2}{2}\right)^{2/3} H_{5/3}(y) \rho^{5/3}
\end{aligned} \tag{13}$$

So,

$$\begin{aligned}
\mathcal{E}_0 &= \frac{3\hbar^2}{10M} \left(\frac{3\pi^2}{2}\right)^{2/3} \rho^{2/3} H_{5/3}(y) + \frac{1}{8} t_0 \rho \left[ 2(2+x_0) - (1+2x_0) H_2(y) \right] \\
&+ \frac{3}{40} \left(\frac{3\pi^2}{2}\right)^{2/3} \left[ a H_{5/3}(y) + b H_{8/5}(y) \right] \rho^{5/3} \\
&+ \frac{1}{48} t_3 \rho^{\alpha+1} \left[ 2(2+x_3) - (1+2x_3) H_2(y) \right] \\
&+ \frac{3}{40} \left(\frac{3\pi^2}{2}\right)^{2/3} t_4 \rho^{\beta+5/3} \left[ (2+x_4) H_{5/3}(y) - \left(\frac{1}{2} + x_4\right) H_{8/3}(y) \right] \\
&+ \frac{3}{40} \left(\frac{3\pi^2}{2}\right)^{2/3} t_5 \rho^{\gamma+5/3} \left[ (2+x_5) H_{5/3}(y) + \left(\frac{1}{2} + x_5\right) H_{8/3}(y) \right]
\end{aligned} \tag{14}$$

with  $H_n(y) = 2^{n-1} [y^n + (1-y)^n]$ . A common choice is to set  $M = 1/2(M_n + M_p)$ , or use the individual value for each species. In compact notation,

$$\mathcal{E}_0 = C(y) \rho^{2/3} + A(y) \rho + B(y) \rho^{\alpha+1} + D(y) \rho^{5/3} + G(y) \rho^{\beta+5/3} + K(y) \rho^{\gamma+5/3} \tag{15}$$

By comparing the 2 expressions, the following relations can be easily deduced:

$$\begin{aligned}
C(y) &= \frac{3\hbar^2}{10M_y} \left(\frac{3\pi^2}{2}\right)^{2/3} H_{5/3}(y) \\
A(y) &= \frac{1}{8} t_0 \left[ 2(2 + x_0) - (1 + 2x_0) H_2(y) \right] \\
B(y) &= \frac{1}{48} t_3 \left[ 2(2 + x_3) - (1 + 2x_3) H_2(y) \right] \\
D(y) &= \frac{3}{40} \left(\frac{3\pi^2}{2}\right)^{2/3} \left[ a H_{5/3}(y) + b H_{8/5}(y) \right] \\
G(y) &= \frac{3}{40} \left(\frac{3\pi^2}{2}\right)^{2/3} t_4 \left[ (2 + x_4) H_{5/3}(y) - \left(\frac{1}{2} + x_4\right) H_{8/3}(y) \right] \\
K(y) &= \frac{3}{40} \left(\frac{3\pi^2}{2}\right)^{2/3} t_5 \left[ (2 + x_5) H_{5/3}(y) + \left(\frac{1}{2} + x_5\right) H_{8/3}(y) \right]
\end{aligned} \tag{16}$$

For pure neutron matter and symmetric matter the kinetic coefficient is

$$\begin{aligned}
C_n = C(0) &= \frac{3\hbar^2}{10M_n} \left(\frac{3\pi^2}{2}\right)^{2/3} H_{5/3}(0) = 118.995 \text{ Mev fm}^2 \approx 119 \text{ Mev fm}^2 \\
C_{sym} = C(1/2) &= \frac{3\hbar^2}{5(M_n + M_p)} \left(\frac{3\pi^2}{2}\right)^{2/3} H_{5/3}(1/2) = 75.0139 \text{ Mev fm}^2 \approx 75 \text{ Mev fm}^2
\end{aligned} \tag{17}$$

The effective mass is due to the terms dependent on kinetic energy:

$$\begin{aligned}
\tau^{T=0}(\rho, y) &\equiv \frac{3\hbar^2}{10M^*} \left(\frac{3\pi^2}{2}\right)^{2/3} \rho^{2/3} H_{5/3}(y) \\
&= \frac{3\hbar^2}{10M} \left(\frac{3\pi^2}{2}\right)^{2/3} \rho^{2/3} H_{5/3}(y) + \frac{3}{40} \left(\frac{3\pi^2}{2}\right)^{2/3} \left[ a H_{5/3}(y) + b H_{8/5}(y) \right] \rho^{5/3} \\
&\quad + \frac{3}{40} \left(\frac{3\pi^2}{2}\right)^{2/3} t_4 \rho^{\beta+5/3} \left[ (2 + x_4) H_{5/3}(y) - \left(\frac{1}{2} + x_4\right) H_{8/3}(y) \right] \\
&\quad + \frac{3}{40} \left(\frac{3\pi^2}{2}\right)^{2/3} t_5 \rho^{\gamma+5/3} \left[ (2 + x_5) H_{5/3}(y) + \left(\frac{1}{2} + x_5\right) H_{8/3}(y) \right] \\
&\equiv C(y) \rho^{2/3} \left[ 1 + (D(y) \rho + G(y) \rho^{\beta+1} + K(y) \rho^{\gamma+1}) / C(y) \right]
\end{aligned} \tag{18}$$



Thus,

$$\begin{aligned}
M^*/M &= \left\{ 1 + \frac{M}{4H_{5/3}(y)\hbar^2} \left[ \rho[aH_{5/3}(y) + bH_{8/5}(y)] + \rho^{\beta+1}t_4[(2+x_4)H_{5/3}(y) - (\frac{1}{2} + x_4)H_{8/3}(y)] \right. \right. \\
&\quad \left. \left. + \rho^{\gamma+1}t_5[(2+x_5)H_{5/3}(y) - (\frac{1}{2} + x_5)H_{8/3}(y)] \right] \right\}^{-1} \\
&= \frac{C(y)}{C(y) + D(y)\rho + G(y)\rho^{\beta+1} + K(y)\rho^{\gamma+1}}
\end{aligned} \tag{19}$$

Also, the thermodynamic pressure:

$$\begin{aligned}
\mathcal{P}^{T=0}(\rho, y) &= \rho^2 \frac{\partial \mathcal{E}^{T=0}}{\partial \rho} \\
&= \frac{2}{3}C(y)\rho^{5/3} + A(y)\rho^2 + (\alpha + 1)B(y)\rho^{\alpha+1} + \frac{5}{3}D(y)\rho^{8/3} + \beta G(y)\rho^{\beta+8/3} + \gamma K(y)\rho^{\gamma+8/3}
\end{aligned} \tag{20}$$

#### IV. LATTIMER - SCHWESTY NOTATION

The standard parametrization:

$$\begin{aligned}
\varepsilon &= \frac{\hbar^2\tau_n}{2M_n} + \frac{\hbar^2\tau_p}{2M_p} + \frac{1}{8}[a(\rho_n + \rho_p)(\tau_n + \tau_p) + 2b(\rho_n\tau_n + \rho_p\tau_p)] + \frac{1}{4}t_0[(2+x_0)(\rho_n + \rho_p)^2 - (1+2x_0)(\rho_n^2 + \rho_p^2)] \\
&\quad + \frac{1}{24}t_3(\rho_n + \rho_p)^\alpha[(2+x_3)(\rho_n + \rho_p)^2 - (1+2x_3)(\rho_n^2 + \rho_p^2)]
\end{aligned} \tag{21}$$

$$\begin{aligned}
a &= \frac{4\hbar^2}{M}(F + G) \\
b &= -\frac{4\hbar^2}{M}G \\
t_0 &= \frac{8}{3}(A + B) \\
x_0 &= -\frac{1}{2}\frac{A - 2B}{A + B} \\
t_3 &= 16(C + D) \\
x_3 &= -\frac{1}{2}\frac{C - D}{C + D} \\
\alpha &= \delta - 1
\end{aligned} \tag{22}$$

$$\begin{aligned}
a &= \frac{4\hbar^2}{M}(F + G) \\
b &= -\frac{4\hbar^2}{M}G \\
t_0 &= \frac{8}{3}(A + B) \\
x_0 &= -\frac{1}{2}\frac{A - 2B}{A + B} \\
t_3 &= 16(C + D) \\
x_3 &= -\frac{1}{2}\frac{C - D}{C + D} \\
\alpha &= \delta - 1
\end{aligned} \tag{23}$$

$$\begin{aligned}
\varepsilon = & 4B\rho_n\rho_p + A(\rho_n + \rho_p)^2 + (\rho_n + \rho_p)^{\delta-1}[4D\rho_n\rho_p + C(\rho_n + \rho_p)^2] \\
& + \frac{5(\frac{2}{3})^{2/3}}{3\pi^{4/3}}\alpha_S[M(\frac{\tau_n}{M_n} + \frac{\tau_p}{M_p}) + F(\rho_n + \rho_p)(\tau_n + \tau_p) - G(\rho_n - \rho_p)(\tau_n - \tau_p)]
\end{aligned} \tag{24}$$

The standard parametrization for  $T = 0$ ,  $(M_n, M_p) \rightarrow M = \frac{1}{2}(M_n + M_p)$ :

$$\begin{aligned}
\varepsilon = & \frac{3\hbar^2}{10M}(\frac{3\pi^2}{2})^{2/3}\rho^{5/3}H_{5/3}(y) + \frac{1}{8}t_0\rho^2\left[2(2 + x_0) - (1 + 2x_0)H_2(y)\right] \\
& + \frac{3}{40}(\frac{3\pi^2}{2})^{2/3}\left[aH_{5/3}(y) + bH_{8/5}(y)\right]\rho^{8/3} \\
& + \frac{1}{48}t_3\rho^{\alpha+2}\left[2(2 + x_3) - (1 + 2x_3)H_2(y)\right]
\end{aligned} \tag{25}$$

By comparing the expression in these notes with the ones from Lattimer parametrization, the energy density is:

$$\varepsilon = \alpha_S\rho^{5/3}H_{5/3}(y) + [A + B(2 - H_2(y))]\rho^2 + [C + D(2 - H_2(y))]\rho\delta + \alpha_S\rho^{5/3}[(F + G)H_{5/3} - GH_{8/3}(y)] \tag{26}$$

where,  $\alpha_S = \frac{3\hbar^2}{10M}(\frac{3}{2}\pi^2)^{2/3}$

### A. Skyrme parametrization from Saturation Observables

Given the following set of physical observables,

$$\begin{aligned}
E_0 = \varepsilon \Big|_{\rho_0, y=1/2} &= (A + B)\rho_0^2 + (C + D)\rho_0^{\delta+1} + \alpha_S \rho_0^{5/3} (1 + F\rho_0) \\
P = \rho^2 \frac{d(\varepsilon/\rho)}{d\rho} \Big|_{\rho_0, y=1/2} &= \frac{2}{3}\alpha_S \rho_0^{5/3} + (A + B)\rho_0^2 + \frac{5}{3}F\alpha_S \rho_0^{8/3} + (C + D)\delta\rho_0^{1+\delta} = 0 \\
(M^*/M) = \frac{d\varepsilon}{d\tau} \Big|_{\rho_0, y=1/2} &= (1 + F\rho_0)^{-1} \\
K_m = 9\rho^2 \frac{d^2(\varepsilon/\rho)}{d\rho^2} \Big|_{\rho_0, y=1/2} &= -2\alpha_S \rho_0^{2/3} + 10F\alpha_S \rho_0^{5/3} + 9(C + D)(\delta - 1)\delta\rho_0^\delta \\
S = \frac{1}{8} \frac{d^2(\varepsilon/\rho)}{dy^2} \Big|_{\rho_0, y=1/2} &= \frac{5}{9}\alpha_S \rho_0^{2/3} - B\rho_0 + \frac{5}{9}(F - 3G)\alpha_S \rho_0^{5/3} - D\rho_0^\delta \\
L = 3\rho \frac{dS}{d\rho} \Big|_{\rho_0, y=1/2} &= \frac{10}{9}\alpha_S \rho_0^{2/3} - 3B\rho_0 + \frac{25}{9}(F - 3G)\alpha_S \rho_0^{5/3} - 3D\delta\rho_0^\delta \\
K_s = 9\rho^2 \frac{d^2 S}{d\rho^2} \Big|_{\rho_0, y=1/2} &= -\frac{10}{9}\alpha_S \rho_0^{2/3} + \frac{50}{9}(F - 3G)\alpha_S \rho_0^{5/3} - 9D(\delta - 1)\delta\rho_0^\delta
\end{aligned} \tag{27}$$

the skyrme parameters can be found as follows,

$$\begin{aligned}
F &= \frac{(M^*/M)^{-1} - 1}{\rho_0} \\
\delta &= \frac{K_m + 2\rho_0^{2/3}(1 - 5F\rho_0)\alpha_S}{3\rho_0^{2/3}(1 - 2F\rho_0)\alpha_S - 9E_0} \\
G &= \frac{9K_s - 27(L - 3S)\delta + 5\rho_0^{2/3}\alpha_S[2 - 3\delta + 2F\rho_0(3\delta - 5)]}{30\rho_0^{5/3}\alpha_S(3\delta - 5)} \\
D &= \frac{5(3L - 9S + \rho_0^{2/3}\alpha_S) - 3K_s}{9(5 - 8\delta + 3\delta^2)\rho_0^\delta} \\
C &= \frac{\rho_0^{2/3}(1 - 2F\rho_0)\alpha_S - 3E_0}{3(\delta - 1)\rho_0^\delta} - D \\
B &= \frac{L(6 + 9\delta) + 5(\rho_0^{2/3}\alpha_S(3\delta - 2) - 9S\delta) - 3K_s}{18\rho_0(\delta - 1)} \\
A &= -\left[\frac{2}{3}\alpha_S \rho^{-1/3} + B + \frac{5}{3}F\alpha_S \rho^{2/3} + (C + D)\delta\rho^{\delta-1}\right]
\end{aligned} \tag{28}$$

## V. FINITE TEMPERATURE DFT

In order to obtain the relationship between density and chemical potential, the following set of coupled equations need to be solved self-consistently:

$$\begin{aligned}
f_k &= \left[ 1 + e^{\left( \frac{k^2}{2M^*} + U - \mu \right) / T} \right]^{-1} \\
\rho &= \sum_{s, is} \int \frac{d^3 k}{(2\pi)^3} f_k \\
\tau &= \sum_{s, is} \int \frac{d^3 k}{(2\pi)^3} k^2 f_k \\
E &\equiv E(\rho, \tau)
\end{aligned} \tag{29}$$

where, a sum over all discrete quantum numbers is performed (spin and isospin). And from the energy density functional, the mean field parameters can be derived,

$$\begin{aligned}
M^* &= \frac{1}{2} \left( \frac{\delta E}{\delta \tau} \right)^{-1} \\
U &= \frac{\delta E}{\delta \rho}
\end{aligned} \tag{30}$$

The chemical potential can be found by inverting the expression for the density.

## VI. THERMODYNAMIC POTENTIALS

Since the effect of phenomenological mean field models can be incorporated into  $(M^*, U)$  which are density dependent for Skyrme, and also temperature dependent for RMF, the thermodynamic properties of assymetric matter at finite temperature can be expressed by fermi integrals of ‘modified’ non-interacting fermi gases. The single particle spectrum and dustribution fuction:

$$\begin{aligned}
\xi_i &= \frac{k^2}{2M_i^*} + U_i \\
F_i &= \left\{ \exp \left[ \frac{\xi_i - \mu_i}{T} \right] + 1 \right\}^{-1} = \left\{ \exp \left[ \frac{\frac{k^2}{2M_i^*} - \eta_i}{T} \right] + 1 \right\}^{-1}, \quad \bar{F} = 1 - F
\end{aligned} \tag{31}$$

The density and kinetic density:

$$\begin{aligned}\rho_i &= \int_0^\infty \frac{dk}{\pi^2} k^2 F_i \\ \tau_i &= \int_0^\infty \frac{dk}{\pi^2} k^4 F_i\end{aligned}\tag{32}$$

The entropy density can be calculated from the distribution function:

$$\begin{aligned}S/V &= - \int_0^\infty \frac{dk}{\pi^2} k^2 [F_i \ln(F_i) + (1 - F_i) \ln(1 - F_i)] \\ &= - \int_0^\infty \frac{dk}{\pi^2} k^2 F_i \ln\left(\frac{F_i}{1 - F_i}\right) - \int_0^\infty \frac{dk}{\pi^2} k^2 \ln(1 - F_i) \\ &= - \int_0^\infty \frac{dk}{\pi^2} k^2 F_i \ln\left(\exp\left[\frac{-\xi_i + \mu_i}{T}\right]\right) - \left[\frac{k^3}{3\pi^2} \ln(1 - F_i)\right]_0^\infty + \int_0^\infty \frac{dk}{\pi^2} \frac{k^3}{3} \frac{k}{M_i^*} \frac{\exp\left[\frac{\xi_i - \mu_i}{T}\right]}{1 - F_i} \\ &= \left[\frac{1}{2M_i^* T} + \frac{1}{3M_i^* T}\right] \int_0^\infty \frac{dk}{\pi^2} k^4 F_i + \frac{U_i - \mu_i}{T} \int_0^\infty \frac{dk}{\pi^2} k^2 F_i \\ &= \frac{1}{T} \left[\frac{5\tau_i}{6M_i^*} + (U_i - \mu_i)\rho\right]\end{aligned}\tag{33}$$

From the first law of thermodynamics:

$$\begin{aligned}E_i &= TS_i + \mu_i N_i - P_i V \\ P_i &= \frac{TS_i + \mu_i N_i - E_i}{V} \\ &= \frac{5\tau_i}{6M_i^*} + U_i \rho_i - \frac{E_i}{V}\end{aligned}\tag{34}$$

## VII. THERMODYNAMIC DERIVATIVES

Let,

$$\alpha_1 = \frac{\hbar^2}{2M}(F - G), \alpha_2 = \frac{\hbar^2}{2M}(G + F)\tag{35}$$

Then,

$$\frac{\hbar^2}{2M_i^*} = \frac{\hbar^2}{2M} + \alpha_1 n_i + \alpha_2 n_{-i}\tag{36}$$

where,  $i$  denotes the isospin value. From expression above, the derivative of the effective mass in terms of the density can be found:

$$\partial_{n_i} M_r^* = -2 \frac{M_r^{*2}}{\hbar^2} [\alpha_2 + (\alpha_1 - \alpha_2) \delta_{ir}] \quad (37)$$

The density and kinetic density can be expressed in terms of the general fermi integration:

$$\begin{aligned} F_n(\eta) &= \int_0^\infty \frac{u^n}{e^{u-\eta} + 1} du, \quad \eta_i = (\mu_i - U_i)/T, \quad U_i = \frac{\delta \mathcal{E}}{\delta n_i} \\ n_i &= \frac{1}{2\pi^2} \left( \frac{2M_r^* T}{\hbar^2} \right)^{3/2} F_{1/2}(\eta_i) \leftrightarrow \eta_i = F_{1/2}^{-1} \left[ 2\pi^2 n_i \left( \frac{\hbar^2}{2M_r^* T} \right)^{3/2} \right] = F_{1/2}^{-1}(u_i) \end{aligned} \quad (38)$$

And,

$$\begin{aligned} \partial_{n_i} u_r &= 2\pi^2 \left( \frac{\hbar^2}{2M_r^* T} \right)^{3/2} \left[ \delta_{ir} + \frac{3n_r M_r^*}{\hbar^2} [\alpha_2 + (\alpha_1 - \alpha_2) \delta_{ir}] \right] \\ &= \frac{F_{1/2}(\eta_r)}{n_r} \left[ \delta_{ir} + \frac{3n_r M_r^*}{\hbar^2} [\alpha_2 + (\alpha_1 - \alpha_2) \delta_{ir}] \right] \end{aligned} \quad (39)$$

Also,  $\partial_\eta F_n(\eta) = n F_{n-1}(\eta) \leftrightarrow \partial_u F_n^{-1}(u) = \frac{1}{n F_{n-1}(u)}$ :

$$\begin{aligned} \partial_{n_i} \eta_r &= d_u F_{1/2}^{-1}(u_r) \times \partial_{n_i} u_r \\ &= \frac{\partial_u F_{1/2}(u_r)}{n_r F_{-1/2}(\eta_r)} \left[ \delta_{ir} + \frac{3n_r M_r^*}{\hbar^2} [\alpha_2 + (\alpha_1 - \alpha_2) \delta_{ir}] \right] \\ &= \frac{2F_{1/2}(u_r)}{n_r F_{-1/2}(\eta_r)} \left[ \delta_{ir} + \frac{3n_r M_r^*}{\hbar^2} [\alpha_2 + (\alpha_1 - \alpha_2) \delta_{ir}] \right] \\ &\equiv \frac{G}{n_r} \left[ \delta_{ir} + \frac{3n_r M_r^*}{\hbar^2} [\alpha_2 + (\alpha_1 - \alpha_2) \delta_{ir}] \right] \end{aligned} \quad (40)$$

Thus,

$$\begin{aligned} \partial_{n_i} \mu_r &= T \partial_{n_i} \eta_r + \partial_{n_i} U_r \\ &= \frac{TG}{n_r} \left[ \delta_{ir} + \frac{3n_r M_r^*}{\hbar^2} [\alpha_2 + (\alpha_1 - \alpha_2) \delta_{ir}] \right] + \partial_{n_i} U_r \end{aligned} \quad (41)$$