I. SKYRME FORCES AT FINITE TEMPERATURE

A. Potential Matrix Element

Interaction Matrix:

$$
\begin{aligned}
V_{ij} =& t_0(1+x_0P_\sigma)\delta(\mathbf{r}_{ij}) + \frac{1}{2}t_1(1+x_1P_\sigma)\frac{1}{\hbar^2}[\overleftarrow{p^\dagger}^2_{ij}\delta(\mathbf{r}_{ij}) + \delta(\mathbf{r}_{ij})\overrightarrow{p}^2_{ij}] \\
&+ t_2(1+x_2P_\sigma)\frac{1}{\hbar^2}\overleftarrow{\mathbf{p}^\dagger}_{ij}\cdot\delta(\mathbf{r}_{ij})\overrightarrow{\mathbf{p}}_{ij} + \frac{1}{6}t_3(1+x_3P_\sigma)\rho^\alpha(\mathbf{r})\delta(\mathbf{r}_{ij}) \\
&+ \frac{i}{\hbar^2}W_0(\sigma_i+\sigma_j)\cdot\overleftarrow{\mathbf{p}^\dagger}_{ij}\times\delta(\mathbf{r}_{ij})\overrightarrow{\mathbf{p}}_{ij} \\
&+ \frac{1}{4}t_4(1+x_4P_\sigma)\frac{1}{\hbar^2}[\overleftarrow{p^\dagger}^2_{ij}\rho^\beta(\mathbf{r})\delta(\mathbf{r}_{ij}) + \delta(\mathbf{r}_{ij})\rho^\beta(\mathbf{r})\overrightarrow{p}^2_{ij}] \\
&+ t_5(1+x_5P_\sigma)\frac{1}{\hbar^2}\overleftarrow{\mathbf{p}}_{ij}\cdot\rho^\gamma(\mathbf{r})\delta(\mathbf{r}_{ij})\overrightarrow{\mathbf{p}}_{ij}
\end{aligned} \tag{1}
$$

where, $\mathbf{r}_{ij} = \frac{\mathbf{r}_i - \mathbf{r}_j}{2},\ \mathbf{r} = \frac{\mathbf{r}_i+\mathbf{r}_j}{2},\ P_\sigma = \frac{1}{2}(1+\sigma_1\cdot\sigma_2),\ \mathbf{p}_{ij} = -i\hbar\frac{\nabla_i - \nabla_j}{2},\ \rho(\mathbf{r}) = \rho_p(\mathbf{r}) + \rho_n(\mathbf{r})$ and the arrows show the direction on which the momentum operators act.

B. Energy Density

Assume time reversal invariance. The Energy of the ground state can be written as integration over an energy density functional, $E_{HFB} = \int d^r \varepsilon_{HFB}$

$$
\begin{aligned}
\varepsilon_{HFB} = & \sum_{t=n,p} \frac{\hbar^2}{2M_q}\tau_q + \frac{1}{2}t_0\Big[(1+\frac{1}{2}x_0)\rho^2 - (\frac{1}{2}+x_0)\sum_{q=n,p}\rho_q^2\Big] \\
& + \frac{1}{4}t_1\Big[(1+\frac{1}{2}x_1)(\rho\tau + \frac{3}{4}(\nabla\rho)^2) - (\frac{1}{2}+x_1)\sum_{q=n,p}(\rho_q\tau_q + \frac{3}{4}(\nabla\rho_q)^2)\Big] \\
& + \frac{1}{4}t_2\Big[(1+\frac{1}{2}x_2)(\rho\tau + \frac{3}{4}(\nabla\rho)^2) + (\frac{1}{2}+x_2)\sum_{q=n,p}(\rho_q\tau_q - \frac{1}{4}(\nabla\rho_q)^2)\Big] \\
& + \frac{1}{12}t_3\rho^\alpha\Big[(1+\frac{1}{2}x_3)\rho^2 - (\frac{1}{2}+x_3)\sum_{q=n,p}\rho_q^2\Big] \\
& + \frac{1}{4}t_4\Big[(1+\frac{1}{2}x_4)(\rho\tau + \frac{3}{4}(\nabla\rho)^2) - (\frac{1}{2}+x_4)\sum_{q=n,p}(\rho_q\tau_q + \frac{3}{4}(\nabla\rho_q)^2)\Big]\rho^\beta \\
& + \frac{\beta}{8}t_4\Big[(1+\frac{1}{2}x_4)\rho(\nabla\rho)^2 - (\frac{1}{2}+x_4)\nabla\rho\cdot\sum_{q=n,p}\rho_q\nabla\rho_q\Big]\rho^{\beta-1} \\
& + \frac{1}{4}t_5\Big[(1+\frac{1}{2}x_5)(\rho\tau + \frac{3}{4}(\nabla\rho)^2) + (\frac{1}{2}+x_5)\sum_{q=n,p}(\rho_q\tau_q - \frac{1}{4}(\nabla\rho_q)^2)\Big]\rho^\gamma \\
& - \frac{1}{16}(t_1x_1 + t_2x_2)J^2 + \frac{1}{16}(t_1 - t_2)\sum_{q=n,p}J_q^2 \\
& - \frac{1}{16}(t_4x_4\rho^\beta + t_5x_5\rho^\gamma)J^2 + \frac{1}{16}(t_4\rho^\beta - t_5\rho^\beta)\sum_{q=n,p}J_q^2 \\
& + \frac{1}{2}W_0(\mathbf{J}\cdot\nabla\rho + \sum_{q=n,p}\mathbf{J}_q\cdot\nabla\rho_q)
\end{aligned} \tag{2}
$$

where,

$$
\begin{aligned}
\rho &= 2\int\frac{d^3k}{(2\pi\hbar)^3}n(k) \\
\tau &= 2\int\frac{d^3k}{(2\pi\hbar)^3}k^2 n(k) \\
\mathbf{J} &= \int\frac{d^3k}{(2\pi\hbar)^3}\mathbf{k}\times\sum_{s,s'}\langle s|\boldsymbol{\sigma}|s'\rangle\ n(k)
\end{aligned} \tag{3}
$$

The different terms can be grouped together in simpler notation:

$$
\begin{aligned}
\varepsilon = &\sum_{t=n,p} \frac{\hbar^2}{2M_q}\tau_q + \frac{1}{4}t_0\Big[(2+x_0)\rho^2 - (1+2x_0)\sum_{q=n,p}\rho_q^2\Big] \\
&+ \frac{1}{8}\Big[a(\rho\tau + \frac{3}{4}(\nabla\rho)^2) + 2b\sum_{q=n,p}(\rho_q\tau_q + \frac{3}{4}(\nabla\rho_q)^2)\Big] \\
&+ \frac{1}{24}t_3\rho^\alpha\Big[(2+x_3)\rho^2 - (1+2x_3)\sum_{q=n,p}\rho_q^2\Big] \\
&+ \frac{1}{8}t_4\Big[(2+x_4)(\rho\tau + \frac{3}{4}(\nabla\rho)^2) - (1+2x_4)\sum_{q=n,p}(\rho_q\tau_q + \frac{3}{4}(\nabla\rho_q)^2)\Big]\rho^\beta \\
&+ \frac{\beta}{16}t_4\Big[(2+x_4)\rho(\nabla\rho)^2 - (1+2x_4)\nabla\rho\cdot\sum_{q=n,p}\rho_q\nabla\rho_q\Big]\rho^{\beta-1} \\
&+ \frac{1}{8}t_5\Big[(2+x_5)(\rho\tau + \frac{3}{4}(\nabla\rho)^2) + (1+2x_5)\sum_{q=n,p}(\rho_q\tau_q - \frac{1}{4}(\nabla\rho_q)^2)\Big]\rho^\gamma \\
&- \frac{1}{16}(t_1x_1 + t_2x_2)J^2 + \frac{1}{16}(t_1 - t_2)\sum_{q=n,p}J_q^2 \\
&- \frac{1}{16}(t_4x_4\rho^\beta + t_5x_5\rho^\gamma)J^2 + \frac{1}{16}(t_4\rho^\beta - t_5\rho^\beta)\sum_{q=n,p}J_q^2 \\
&+ \frac{1}{2}W_0(\mathbf{J}\cdot\nabla\rho + \sum_{q=n,p}\mathbf{J}_q\cdot\nabla\rho_q)
\end{aligned}
\tag{4}
$$

where, $a = t_1(x_1+2) + t_2(x_2+2)$, $b = \frac{1}{2}[t_2(2x_2+1) - t_1(2x_1+1)]$.

In uniform matter $\nabla\rho = 0$:

$$
\begin{aligned}
\varepsilon = & \sum_{t=n,p} \frac{\hbar^2}{2M_q}\tau_q + \frac{1}{4}t_0\Big[(2+x_0)\rho^2 - (1+2x_0)\sum_{q=n,p}\rho_q^2\Big] \\
& + \frac{1}{8}\Big[a\rho\tau + 2b\sum_{q=n,p}\rho_q\tau_q\Big] \\
& + \frac{1}{24}t_3\rho^\alpha\Big[(2+x_3)\rho^2 - (1+2x_3)\sum_{q=n,p}\rho_q^2\Big] \\
& + \frac{1}{8}t_4\Big[(2+x_4)\rho\tau - (1+2x_4)\sum_{q=n,p}\rho_q\tau_q\Big]\rho^\beta \\
& + \frac{1}{8}t_5\Big[(2+x_5)\rho\tau + (1+2x_5)\sum_{q=n,p}\rho_q\tau_q\Big]\rho^\gamma \\
& - \frac{1}{16}(t_1x_1 + t_2x_2)J^2 + \frac{1}{16}(t_1 - t_2)\sum_{q=n,p}J_q^2 \\
& - \frac{1}{16}(t_4x_4\rho^\beta + t_5x_5\rho^\gamma)J^2 + \frac{1}{16}(t_4\rho^\beta - t_5\rho^\beta)\sum_{q=n,p}J_q^2
\end{aligned}
\tag{5}
$$

In unpolarized matter, $\mathbf{J} = 0$:

$$
\begin{aligned}
\varepsilon = & \sum_{t=n,p} \frac{\hbar^2}{2M_q}\tau_q + \frac{1}{4}t_0\Big[(2+x_0)\rho^2 - (1+2x_0)\sum_{q=n,p}\rho_q^2\Big] \\
& + \frac{1}{8}\Big[a\rho\tau + 2b\sum_{q=n,p}\rho_q\tau_q\Big] \\
& + \frac{1}{24}t_3\rho^\alpha\Big[(2+x_3)\rho^2 - (1+2x_3)\sum_{q=n,p}\rho_q^2\Big] \\
& + \frac{1}{8}t_4\Big[(2+x_4)\rho\tau - (1+2x_4)\sum_{q=n,p}\rho_q\tau_q\Big]\rho^\beta \\
& + \frac{1}{8}t_5\Big[(2+x_5)\rho\tau + (1+2x_5)\sum_{q=n,p}\rho_q\tau_q\Big]\rho^\gamma
\end{aligned}
\tag{6}
$$

Energy per bayon, $\mathcal{E} \equiv \varepsilon/rho$:

$$\begin{aligned}
\mathcal{E} = &\sum_{t=n,p} \frac{\hbar^2}{2M_q}\frac{\tau_q}{\rho} + \frac{1}{4}t_0\Big[(2+x_0)\rho - (1+2x_0)\sum_{q=n,p}\frac{\rho_q^2}{\rho}\Big] \\
&+ \frac{1}{8}\Big[a\tau + 2b\sum_{q=n,p}\frac{\rho_q\tau_q}{\rho}\Big] \\
&+ \frac{1}{24}t_3\rho^{\alpha}\Big[(2+x_3)\rho - (1+2x_3)\sum_{q=n,p}\frac{\rho_q^2}{\rho}\Big] \\
&+ \frac{1}{8}t_4\Big[(2+x_4)\tau - (1+2x_4)\sum_{q=n,p}\frac{\rho_q\tau_q}{\rho}\Big]\rho^{\beta} \\
&+ \frac{1}{8}t_5\Big[(2+x_5)\tau + (1+2x_5)\sum_{q=n,p}\frac{\rho_q\tau_q}{\rho}\Big]\rho^{\gamma}
\end{aligned} \tag{7}$$

In terms of proton fraction, $y = \frac{\rho_p}{\rho_p+\rho_n}$:

$$\begin{aligned}
\mathcal{E} = &\sum_{t=n,p} \frac{\hbar^2}{2M_q}\frac{\tau_q}{\rho} + \frac{1}{4}t_0\Big[(2+x_0) - (1+2x_0)[y^2+(1-y)^2]\Big]\rho \\
&+ \frac{1}{8}\Big[a\tau + 2b[y\tau_p + (1-y)\tau_n]\Big] \\
&+ \frac{1}{24}t_3\rho^{\alpha+1}\Big[(2+x_3) - (1+2x_3)[y^2+(1-y)^2]\Big] \\
&+ \frac{1}{8}t_4\Big[(2+x_4)\tau - (1+2x_4)[y\tau_p + (1-y)\tau_n]\Big]\rho^{\beta} \\
&+ \frac{1}{8}t_5\Big[(2+x_5)\tau + (1+2x_5)[y\tau_p + (1-y)\tau_n]\Big]\rho^{\gamma}
\end{aligned} \tag{8}$$

II. SINGLE PARTICLE PROPERTIES

From the energy density the single particle spectrum can be derived. Since the Skyrme potential is at mostly quadratic in momenta, its effect is exactly included by effective mass and mean field shift(residual interaction), both density dependent:

$$
\begin{aligned}
\epsilon_q(k) =& \frac{\hbar^2 k^2}{2M_q^*} + U_q \\
\frac{\hbar^2}{2M_q^*} =& \frac{\partial \varepsilon}{\partial \tau_q} \\
U_q =& \frac{\partial \varepsilon}{\partial \rho_q}\Big|_{\tau_{n=p}=0}
\end{aligned}
\tag{9}
$$

From eq. 6 the effective baryon masses:

$$
\begin{aligned}
M_p^*/M =& \Big\{1 + \frac{M\ \rho}{4\hbar^2}\big[a + 2\ y\ b + t_4[(2+x_4) - (1+2x_4)\ y]\rho^\beta + t_5[2 + x_5 + (1+2x_5)\ y]\rho^\gamma\big]\Big\}^{-1} \\
M_n^*/M =& \Big\{1 + \frac{M\ \rho}{4\hbar^2}\big[a + 2\ (1-y)\ b + t_4[(2+x_4) - (1+2x_4)\ (1-y)]\rho^\beta + t_5[2 + x_5 + (1+2x_5)\ (1-y)]\rho^\gamma\big]\Big\}^{-1}
\end{aligned}
\tag{10}
$$

and the residual potentials:

$$
\begin{aligned}
U_p =& \frac{1}{2}t_0[(2+x_0) - (1+2x_0)\ y]\rho + \frac{1}{24}t_3\Big[4 + \alpha - 2y(1-(1-y)\alpha) + x_3(1-2y)[2-(1-2y)\alpha]\Big]\rho^{\alpha+1} \\
U_n =& \frac{1}{2}t_0[(1-x_0) + (1+2x_0)y]\rho + \frac{1}{24}t_3\Big[2 + \alpha + 2y(1+\alpha-y\alpha) - x_3(1-2y)[2+(1-2y)\alpha]\Big]\rho^{\alpha+1}
\end{aligned}
\tag{11}
$$

For symmetric matter ($y = 1/2$):

$$
\begin{aligned}
M^*/M =& \Big\{1 + \frac{M}{4\hbar^2}\rho\big[a + b + \frac{3}{2}t_4\rho^\beta + t_5(\frac{5}{2} + 2x_5)\rho^\gamma\big]\Big\}^{-1} \\
U =& \frac{3}{4}t_0\rho + \frac{1}{16}t_3(\alpha+2)\rho^{\alpha+1}
\end{aligned}
\tag{12}
$$

For pure neutron matter (y = 0):

$$
\begin{aligned}
M^*/M =& \Big\{1 + \frac{M}{4\hbar^2}\rho\big[a + 2b + t_4(1-x_4)\rho^\beta + 3t_5(1+x_5)\rho^\gamma\big]\Big\}^{-1} \\
U =& \frac{1}{2}t_0(1-x_0)\rho + \frac{1}{24}t_3(1-x_3)(\alpha+2)\rho^{\alpha+1}
\end{aligned}
\tag{13}
$$

III. T=0 DFT

At $T = 0$, there are simple relation that can be drawn between the 2 integrations in Fourier space since the occuppation number is a step function:

$$
\begin{aligned}
\rho_q &= 2\int \frac{d^3k}{(2\pi\hbar)^3}\theta(k_{F,q}-k) = \frac{k_{F,q}^3}{3\pi^2\hbar^3} \\
\tau_q &= 2\int \frac{d^3k}{(2\pi\hbar)^3}(k/\hbar)^2\theta(k_{F,q}-k) = \frac{k_{F,q}^5}{5\pi^2\hbar^5} \rightarrow \\
\tau &= \frac{3}{5}(3\pi^2)^{2/3}\rho_q^{5/3}, H_n(y) = 2^{n-1}[y^n + (1-y)^n], y = \rho_p/\rho \\
\tau &= \tau_p + \tau_n = \frac{3}{5}(\frac{3\pi^2}{2})^{2/3}H_{5/3}(y)\rho^{5/3}
\end{aligned}
\tag{14}
$$

So,

$$
\begin{aligned}
\mathcal{E}_0 = & \frac{3\hbar^2}{10M}(\frac{3\pi^2}{2})^{2/3}\rho^{2/3}H_{5/3}(y) + \frac{1}{8}t_0\rho\Big[2(2+x_0) - (1+2x_0)H_2(y)\Big] \\
&+ \frac{3}{40}(\frac{3\pi^2}{2})^{2/3}\Big[aH_{5/3}(y) + bH_{8/5}(y)\Big]\rho^{5/3} \\
&+ \frac{1}{48}t_3\rho^{\alpha+1}\Big[2(2+x_3) - (1+2x_3)H_2(y)\Big] \\
&+ \frac{3}{40}(\frac{3\pi^2}{2})^{2/3}t_4\rho^{\beta+5/3}\Big[(2+x_4)H_{5/3}(y) - (\frac{1}{2}+x_4)H_{8/3}(y)\Big] \\
&+ \frac{3}{40}(\frac{3\pi^2}{2})^{2/3}t_5\rho^{\gamma+5/3}\Big[(2+x_5)H_{5/3}(y) + (\frac{1}{2}+x_5)H_{8/3}(y)\Big]
\end{aligned}
\tag{15}
$$

with $H_n(y) = 2^{n-1}[y^n + (1-y)^n]$. A common choice is to set $M = 1/2(M_n + M_p)$, or use the individual value for each species. In compact notation,

$$
\mathcal{E}_0 = C(y)\rho^{2/3} + A(y)\rho + B(y)\rho^{\alpha+1} + D(y)\rho^{5/3} + G(y)\rho^{\beta+5/3} + K(y)\rho^{\gamma+5/3} \tag{16}
$$

By comparing the 2 expressions, the following relations can be easily deduced:

$$
\begin{aligned}
C(y) =& \frac{3\hbar^2}{10M_y}(\frac{3\pi^2}{2})^{2/3}H_{5/3}(y) \\
A(y) =& \frac{1}{8}t_0\Big[2(2+x_0)-(1+2x_0)H_2(y)\Big] \\
B(y) =& \frac{1}{48}t_3\Big[2(2+x_3)-(1+2x_3)H_2(y)\Big] \\
D(y) =& \frac{3}{40}(\frac{3\pi^2}{2})^{2/3}\Big[aH_{5/3}(y)+bH_{8/5}(y)\Big] \\
G(y) =& \frac{3}{40}(\frac{3\pi^2}{2})^{2/3}t_4\Big[(2+x_4)H_{5/3}(y)-(\frac{1}{2}+x_4)H_{8/3}(y)\Big] \\
K(y) =& \frac{3}{40}(\frac{3\pi^2}{2})^{2/3}t_5\Big[(2+x_5)H_{5/3}(y)+(\frac{1}{2}+x_5)H_{8/3}(y)\Big]
\end{aligned}
\tag{17}
$$

For pure neutron matter and symmetric matter the kinetic coefficient is

$$
\begin{aligned}
C_n =& C(0) = \frac{3\hbar^2}{10M_n}(\frac{3\pi^2}{2})^{2/3}H_{5/3}(0) = 118.995 \text{ Mev fm}^2 \approx 119 \text{ Mev fm}^2 \\
C_{sym} =& C(1/2) = \frac{3\hbar^2}{5(M_n+M_p)}(\frac{3\pi^2}{2})^{2/3}H_{5/3}(1/2) = 75.0139 \text{ Mev fm}^2 \approx 75 \text{ Mev fm}^2
\end{aligned}
\tag{18}
$$

The effective mass is due to the terms dependent on kinetic energy:

$$
\begin{aligned}
\tau^{T=0}(\rho,y) \equiv& \frac{3\hbar^2}{10M^*}(\frac{3\pi^2}{2})^{2/3}\rho^{2/3}H_{5/3}(y) \\
=& \frac{3\hbar^2}{10M}(\frac{3\pi^2}{2})^{2/3}\rho^{2/3}H_{5/3}(y) + \frac{3}{40}(\frac{3\pi^2}{2})^{2/3}\Big[aH_{5/3}(y)+bH_{8/5}(y)\Big]\rho^{5/3} \\
&+ \frac{3}{40}(\frac{3\pi^2}{2})^{2/3}t_4\rho^{\beta+5/3}\Big[(2+x_4)H_{5/3}(y)-(\frac{1}{2}+x_4)H_{8/3}(y)\Big] \\
&+ \frac{3}{40}(\frac{3\pi^2}{2})^{2/3}t_5\rho^{\gamma+5/3}\Big[(2+x_5)H_{5/3}(y)+(\frac{1}{2}+x_5)H_{8/3}(y)\Big] \\
\equiv& C(y)\rho^{2/3}\Big[1+(D(y)\rho+G(y)\rho^{\beta+1}+K(y)\rho^{\gamma+1})/C(y)\Big]
\end{aligned}
\tag{19}
$$

Thus,

$$
\begin{aligned}
M^*/M = &\{1 + \frac{M}{4H_{5/3}(y)\hbar^2}\Big[\rho[aH_{5/3}(y) + bH_{8/5}(y)] + \rho^{\beta+1}t_4[(2+x_4)H_{5/3}(y) - (\frac{1}{2}+x_4)H_{8/3}(y)] \\
&+ \rho^{\gamma+1}t_5[(2+x_5)H_{5/3}(y) - (\frac{1}{2}+x_5)H_{8/3}(y)]\Big]\}^{-1} \\
=&\frac{C(y)}{C(y) + D(y)\rho + G(y)\rho^{\beta+1} + K(y)\rho^{\gamma+1}}
\end{aligned} \tag{20}
$$

Also, the thermodynamic pressure:

$$
\begin{aligned}
\mathcal{P}^{T=0}(\rho, y) &= \rho^2\frac{\partial\mathcal{E}^{T=0}}{\partial\rho} \\
&= \frac{2}{3}C(y)\rho^{5/3} + A(y)\rho^2 + (\alpha+1)B(y)\rho^{\alpha+1} + \frac{5}{3}D(y)\rho^{8/3} + \beta G(y)\rho^{\beta+8/3} + \gamma K(y)\rho^{\gamma+8/3}
\end{aligned} \tag{21}
$$

IV. LATTIMER - SCHWESTY NOTATION

The standard parametrization for T = 0:

$$
\begin{aligned}
\varepsilon = &\frac{3\hbar^2}{10M}(\frac{3\pi^2}{2})^{2/3}\rho^{5/3}H_{5/3}(y) + \frac{1}{8}t_0\rho^2\Big[2(2+x_0) - (1+2x_0)H_2(y)\Big] \\
&+ \frac{3}{40}(\frac{3\pi^2}{2})^{2/3}\Big[aH_{5/3}(y) + bH_{8/5}(y)\Big]\rho^{8/3} \\
&+ \frac{1}{48}t_3\rho^{\alpha+2}\Big[2(2+x_3) - (1+2x_3)H_2(y)\Big]
\end{aligned} \tag{22}
$$

By comparing the expression in these notes with the ones from "A Generalized Equation of State for Hot, Dense Matter", the following relations can be found between the 2 sets of parametrization:

$$\begin{aligned}
a =&\frac{4\hbar^2}{M}(F+G)\\
b =&-\frac{4\hbar^2}{M}G\\
t_0 =&\frac{8}{3}(A+B)\\
x_0 =&-\frac{1}{2}\frac{A-2B}{A+B}\\
t_3 =&16(C+D)\\
x_3 =&-\frac{1}{2}\frac{C-D}{C+D}\\
\alpha =&\delta-1
\end{aligned} \tag{23}$$

In terms of Lattimer parametrization, the energy density is:

$$\varepsilon = \alpha_S \rho^{5/3} H_{5/3}(y) + [A + B(2 - H_2(y))]\rho^2 + [C + D(2 - H_2(y))]\rho\delta + \alpha_S \rho^{5/3}[(F+G)H_{5/3} - GH_{8/3}(y)] \tag{24}$$

where, $\alpha_S = \frac{3\hbar^2}{10M}(\frac{3}{2}\pi^2)^{2/3}$

A. Skyrme parametrization from Saturation Observables

Given the following set of physical observables,

$$\begin{aligned}
E_0 =& \varepsilon\Big|_{\rho_0,y=1/2} \\
P =& \rho^2\frac{d(\varepsilon/\rho)}{d\rho}\Big|_{\rho_0,y=1/2} = 0 \\
(M^*/M) =& [1+F\rho+G\rho(1-2y)]^{-1}\Big|_{\rho_0,y=1/2} \\
K_m =& 9\rho^2\frac{d^2(\varepsilon/\rho)}{d\rho^2}\Big|_{\rho_0,y=1/2} \\
S =& \frac{1}{8}\frac{d^2(\varepsilon/rho)}{dy^2}\Big|_{\rho_0,y=1/2} \\
L =& 3\rho\frac{dS}{\rho}\Big|_{\rho_0,y=1/2} \\
K_s =& 9\rho^2\frac{d^2S}{d\rho^2}\Big|_{\rho_0,y=1/2}
\end{aligned} \tag{25}$$

the skyrme parameters can be found as follows,

$$\begin{aligned}
F =& \frac{(M^*/M)^{-1}-1}{\rho_0} \\
\delta =& \frac{K_m+2\rho_0^{2/3}(1-5F\rho_0)\alpha_S}{3\rho_0^{2/3}(1-2F\rho_0)\alpha_S-9E_0} \\
G =& \frac{9K_S-27(L-3S)\delta+5\rho_0^{2/3}\alpha_S[2-3\delta+2F\rho_0(3\delta-5)]}{30\rho_0^{5/3}\alpha_S(3\delta-5)} \\
D =& \frac{5(3L-9S+\rho_0^{2/3}\alpha_S)-3K_S}{9(5-8\delta+3\delta^2)\rho_0^{\delta}} \\
C =& \frac{\rho_0^{2/3}(1-2F\rho_0)\alpha_S-3E_0}{3(\delta-1)\rho_0^{\delta}}-D \\
B =& \frac{L(6+9\delta)+5(\rho_0^{2/3}\alpha_S(3\delta-2)-9S\delta)-3K_S}{18\rho_0(\delta-1)} \\
A =& -[\frac{2}{3}\alpha_S\rho^{-1/3}+B+\frac{5}{3}F\alpha_S\rho^{2/3}+(C+D)\delta\rho^{\delta-1}]
\end{aligned} \tag{26}$$

V. FINITE TEMPERATURE DFT

At $T > 0$, there is no simple relationship between the 2 integrals, but they can be given as functions of the nucleon chemical potentials:

$$
\begin{aligned}
\rho_q &= 2\int \frac{d^3k}{(2\pi\hbar)^3} f_q(k) \\
\tau_q &= 2\int \frac{d^3k}{(2\pi\hbar)^3} k^2 f_q(k) \\
\tau &= \sum_{q=n,p} \tau_q, \rho = \sum_{q=n,p} \rho_q \\
s_q &= -\frac{1}{\pi^2\hbar^3}\int_0^\infty k^2[f(k)\log f_q(k) - (1-f_q(k))\log(1-f_q(k))]dk;
\end{aligned}
\tag{27}
$$

And, in order to obtain the relationship between density and chemical potential, the following set of coupled equations need to be solved self-consistenly:

$$
\begin{aligned}
\rho_q &= 2\int \frac{d^3k}{(2\pi\hbar)^3} \frac{1}{1+\exp[(\frac{k^2}{2M_q} + V_q(\rho_q) - \mu_q)/T]} \\
V(\rho_q) &= 2\int \frac{d^3k}{(2\pi\hbar)^3} f_q(k)\langle \vec{k}, \vec{k}'|V|\vec{k}, \vec{k}'\rangle
\end{aligned}
\tag{28}
$$

where, the potential matrix element is shown in the begining of these notes. This complicates matters, but an easy alternative is to solve for the chemical potential from the density using the fact that the potential is quadratic and can be absorbed in effective mass and mean field shift (both desnity dependent):

$$
\rho_q = 2\int \frac{d^3k}{(2\pi\hbar)^3} \frac{1}{1+\exp[(\frac{k^2}{2M_q^*(\rho_q)} + U_q(\rho_q) - \mu_q)/T]}
\tag{29}
$$

For a given value of density, there will be one μ_q which is obtained by the solution to the equation above. The effective mass and mean field shift have been discussed previously.

VI. THERMODYNAMIC POTENTIALS

Since the effect of phenomenological mean field models can be incorporated into (M^*, U) which are density dependent for Skyrme, and also temperature dependent for RMF, the thermodynamic properties of assymetric matter at finite temperature can be expressed by fermi integrals of 'modified' non-interacting fermi gases:

$$
\begin{aligned}
F_i =& \Big\{ \exp\big[\frac{\xi_i - \mu_i}{T}\big] + 1 \Big\}^{-1}, \ \overline{F} = 1 - F, \ \xi_i = \frac{q^2}{2M_i^*} + U_i \\
\rho_i =& \frac{N_i}{V} = \int_0^\infty \frac{dq}{\pi^2} q^2 F \\
P_i =& -\frac{\Omega_i}{V} = \int_0^\infty \frac{dq}{\pi^2} q^2 \ln\Big[1 + \exp\big[\frac{-(\xi_i - \mu_i)}{T}\big]\Big] \\
\epsilon_i =& \frac{E_i}{V} = \int_0^\infty \frac{dq}{\pi^2} q^2 \xi_i \ F \\
s_i =& \frac{S_i}{V} = -\frac{1}{TV}\frac{\partial \Omega}{\partial T} = \frac{1}{T}\frac{\partial P_i}{\partial T} \\
=& \int_0^\infty \frac{dq}{\pi^2} \Big\{ \frac{(\xi_i - \mu_i)}{T} F + \ln\Big[1 + \exp\big[\frac{-(\xi_i - \mu_i)}{T}\big]\Big] \Big\} \\
=& \frac{1}{T}\big(\epsilon_i - \mu_i \rho_i + P_i\big)
\end{aligned}
\tag{30}
$$

where the subscript i refers to an isotropic particle species with spin degeneracy $g_S = 2$, and for assymetric the thermodynamic potentials are the sum of neutron and proton species.