Day 04: Analyzing experimental data

Erin Rossiter

January 25, 2022

1. Tidy up

- Abigail, will you:
 - » remind me to take a break at 4:30-4:45
 - » jot notes on typos & email after class
- Note that HW4 will ask for a one-pager on your research topic, question, and hypothesis(es)
- Lecture part 1 and lab 1 (hypothesis testing)
- 3. Lecture part 2 and lab 2 (covariates in design)

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Recap

- The average treatment effect (ATE) is often an interesting, important, useful estimand.
- Difference-in-means is an unbiased estimator of the ATE assuming:
 - 1. Randomization of treatment
 - » $E[Y_i(1)|D_i=1]=E[Y_i(1)]$
 - » $E[Y_i(0)|D_i=0]=E[Y_i(0)]$
 - » We can estimate left-hand terms using our observed data!
 - 2. Excludability
 - 3. Noninterference
- What about uncertainty

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There's a true standard error: $SE(A\hat{T}E)$

- How much might our estimate vary across all hypothetical random assignments?
- Our design choices influence this!
 - » N size
 - » Potential outcome variability
 - » Randomization routine (blocking, clustering)
- Beneficial to reduce standard error in design phase

- DeclareDesign helps understand both true standard error and how we plan to estimate it
 - » Is our estimation strategy getting a standard error that's too big? Meaning, we'd fail to reject a false null? No stars when there should be?!

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✓ Uncertainty associated with the \hat{ATE}

Today → Hypothesis testing

- (Typical) null hypotheses of no average effect and hypothesis testing with approximate p-values based on assumptions about sampling distributions
- Sharp null hypotheses and randomization inference hypothesis testing with exact p-values

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Hypothesis testing

"Traditional" hypothesis testing

1. We state a null hypothesis

- e.g., H_0 : ATE = 0
- No average treatment effect
- The claim we'd like to reject
- 2. We choose a test statistic
 - e.g., t-value
- 3. Determine the distribution of the test statistic under the null
- A thought experiment. If the null is true, what data would we expect to see? (board)
- e.g., Student's t
- 4. Calculate the probability (*p*-value) of our test statistic under the null
- Continuing our thought experiment. If the null is true, how surprising is our data? (board)

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- Data from page 65 of GG
- Treatment is encouragement to make a charitable donation
- Outcome is how much money donated

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## Donation Treatment
## 1 500 1
## 2 100 1
## 3 100 1
## 4 50 1
## 5 25 1
## 6 25 1
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t-test by hand

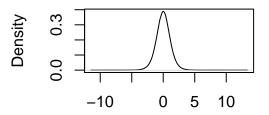
```
# 1. state null
h_not <- 0

# 2. choose test statistic (t-test with unequal variances)
s1 <- sd(df$Donation[df$Treatment == 0])^2
s2 <- sd(df$Donation[df$Treatment == 1])^2
t <- (70-h_not)/sqrt(s1/10 + s2/10)
t

## [1] 1.44776</pre>
```

t-test by hand

nsity.default(x = rand_data_unde



t-test by hand

```
# (from step 3)
plot(density(rand_data_under_null), main = "", xlab = "", cex =
# 4. how odd is our data (or something bigger) given the null?
abline(v = t, col = "red")
Density
          -10
                0 5 10
pt(t, df = 9.0558, lower.tail = F) #or...
## [1] 0.09070126
sum(rand data under null > t)/1000000
## [1] 0.09039
```

t-test in R using t.test()

```
t.test(df$Donation[df$Treatment == 1],
       df$Donation[df$Treatment == 0],
       var.equal = F, alternative = "greater")
##
##
   Welch Two Sample t-test
##
## data: df$Donation[df$Treatment == 1] and df$Donation[df$Trea
## t = 1.4478, df = 9.0558, p-value = 0.0907
## alternative hypothesis: true difference in means is greater t
## 95 percent confidence interval:
## -18.56994
                    Tnf
## sample estimates:
## mean of x mean of y
##
          80
                    10
```

- no one-sided tests
- allows for easy design-based estimation when you use blocking and clustering (see homework)
- I encourage this package since it's built with experiments in mind

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## Design: Standard

## Estimate Std. Error t value Pr(>|t|) CI Lower

## Treatment 70 48.35057 1.44776 0.1814026 -39.27398

## CI Upper DF

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t-test big picture

We're making assumptions about sampling distribution

- » sampling dist for DIM estimate follows t distribution
- Good assumption when.
 - » Outcomes distributed normally, or
 - » Outcome distributed non-normally, but big sample (invoke CLT for sampling distribution to be approximately normal)
- In general, we should be wary when we're using small and/or skewed samples
 - "Rules of thumb" are hard...

Next, an alternative approach that starts from a different null hypothesis. . .

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Randomization inference

$$H_0: \tau_i = Y_i(1) - Y_i(0) = 0 \quad \forall i$$

- Think, "no effect means no effect"
- Different than no average treatment effect
 - » No average treatment effect does not imply sharp null
- Example on board

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1. We state a null hypothesis

- $H_0: \tau_i = Y_i(1) Y_i(0) = 0 \quad \forall i$
- Or simply, $H_0: Y_i(1) = Y_i(0)$
- 2. We choose a test statistic
- Keep in mind, many to choose from
- Let's stick with DIV
- 3. Determine the distribution of the test statistic under the null
- This is where we shake things up! Example next.
- Null hypothesis is about individual rather than average effect
- We have a link between observed data and the potential outcomes.
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- 4. Calculate the probability (*p*-value) of the test statistic under the null.

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1. What do we know about potential outcomes from what we *observe*?

d_i	Y_i	$Y_i(0)$	$Y_i(1)$
0	1	?	?
0	4	?	?
0	2	?	?
0	1	?	?
1	5	?	?
1	7	?	?
1	3	?	?
1	3	?	?
	0 0 0 0 1 1	0 1 0 4 0 2 0 1 1 5 1 7 1 3	0 1 ? 0 4 ? 0 2 ? 0 1 ? 1 5 ? 1 7 ? 1 3 ?

- 1. What do we know about potential outcomes from what we *observe*?
- $-Y_{i}(0)|d_{i}=0$
- $-Y_i(1)|d_i=1$

		.,	1 ((0)	1 (1)
Unit	di	Y_i	$Y_i(0)$	$Y_i(1)$
1	0	1	1	?
2	0	4	4	?
3	0	2	2	?
4	0	1	1	?
5	1	5	?	5
6	1	7	?	7
7	1	3	?	3
8	1	3	?	3

2. What do we know about potential outcomes from what we assume?

Unit	di	Y_i	$Y_i(0)$	$Y_i(1)$
1	0	1	1	?
2	0	4	4	?
3	0	2	2	?
4	0	1	1	?
5	1	5	?	5
6	1	7	?	7
7	1	3	?	3
8	1	3	?	3

- 2. What do we know about potential outcomes from what we *assume*?
 - We can use the sharp null to fill in remaining potential outcomes
- $-Y_i(1)-Y_i(0)=0$

Unit	di	Yi	$Y_i(0)$	$Y_i(1)$
1	0	1	1	(1)
2	0	4	4	(4)
3	0	2	2	(2)
4	0	1	1	(1)
5	1	5	(5)	5
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- 3. Since we have *all* potential outcomes, we can calculate test-statistic for all hypothetical randomizations!!!
- What is this called?

- 3. Since we have *all* potential outcomes, we can calculate test-statistic for all hypothetical randomizations!!!
- What is this called? **Sampling distribution**
- Also called "randomization distribution"
- 4. Compare our test statistic to all possible test statistics under the null to get p-value.

Unit	di	Y_i	$Y_i(0)$	$Y_i(1)$
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```
df <- data.frame(Z = c(0,0,0,0,1,1,1,1),

Y = c(1,4,2,1,5,7,3,3))
```

- For randomization inference (often called RI), you need exact randomization procedure
- Why is this so important?

```
random_assignment
## Random assignment procedure: Complete random assignment
## Number of units: 8
## Number of treatment arms: 2
## The possible treatment categories are 0 and 1.
## The number of possible random assignments is 70.
## The probabilities of assignment are constant across units:
## prob_0 prob_1
## 0.5 0.5
```

- For randomization inference (often called RI), you need exact randomization procedure
- Why is this so important?

```
random_assignment <- randomizr::declare_ra(N = 8, m = 4)
random_assignment

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```

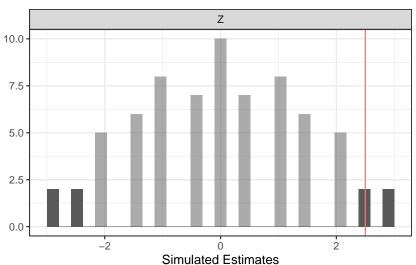
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plot(out)

Randomization Inference



31

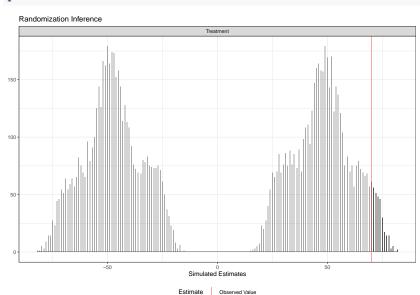
Let's re-do the GG t-test example

```
## Donation Treatment
## 1 500 1
## 2 100 1
## 3 100 1
## 4 50 1
## 5 25 1
## 6 25 1
```

```
random assignment \leftarrow randomizr::declare ra(N = 20)
random_assignment
## Random assignment procedure: Complete random assignment
## Number of units: 20
## Number of treatment arms: 2
## The possible treatment categories are 0 and 1.
## The number of possible random assignments is 184756.
  The probabilities of assignment are constant across units:
## prob_0 prob_1
##
      0.5
             0.5
```

```
## term estimate upper_p_value
## 1 Treatment 70 0.0347
```

plot(out)



Sharp null hypothesis of no effect

- Null of $Y_i(1) Y_i(0) = 0$ allows us to say $Y_i(1) = Y_i(0)$
 - » Then we can impute all potential outcomes
 - » Then we can find sampling distribution of our test stat by looking at all D_i
 - » Then we can get exact p-values!

Null hypothesis of no average effect

- Only allows us to say that $E[Y_i(1)] = E[Y_i(0)]$
 - » Tells us nothing about individual causal effects
 - » Requires assumption about sampling distribution, assumptions might not be met

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Randomization inference:

- good when assumptions about sampling distribution are worrisome
 - » small N
 - » skewed outcomes
- good when we have complicated randomization procedures
 » blocking, clustering, etc.
- it's simple and exact!
- but null hypothesis might be less interesting
- regardless, it forces you to take a moment to think carefully about what the null hypothesis is and how it should be tested

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Lab 1

Covariates in experimental design

Recall,

- Randomization addresses the problem of omitted variables that plagues observational data
- Therefore, covariates are not a primary concern for causal inference
 - » However, covariates can have important roles in design and analysis (or can be mis-used!)
 - » Incorporation of covariates is best though-out during the design of experiments

Roles

- » Rescaling the outcome
- » Regression adjustment
- » Balance tests
- » Block randomization (last time)

- Randomization addresses the problem of omitted variables that plagues observational data
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- We assume that they are fixed pre-treatment
 - » i.e., assignment to treatment and control doesn't change them
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$$Y_i = Y_i(0)(1 - d_i) + Y_i(1)d_i$$

Rearrange
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Rename
$$Y_i = \mu_{Y(0)} + [\mu_{Y(1)} - \mu_{Y(0)}]d_i + u_i$$

Rename
$$Y_i = a + bd_i + u_i$$

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- When X_i predicts the outcome, we reduce amount of unexplained variation in Y_i!
 - » Doing so reduces the standard error of \hat{b} like we talked about last time :)
- Restablishes balance if an unlucky randomization, administrative error, etc.

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- Helps to reduce variability in our estimate
- Including covariates that does not predict outcomes does nothing to the sampling variability of our estimates
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