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M/W 4:00pm - 5:00pm

### Project 1 : Pseudocode

#### The Left to Right Algorithm

**Input:** An unsorted array (before) the size of  $2n$

**Output:** A fully sorted swapped array (before) with the number of swaps that occurred (numOfSwaps)

```
// Algorithm that sorts disks using the left-to-right algorithm.  
// disk_state: The state of one row of disks, essentially the array  
sorted_disks sort_left_to_right(const disk_state& before) {  
  
    int numOfPasses = 0;  
    int index = 0;  
    int numOfSwaps = 0;  
  
    for numOfPasses between 0 to  $2n-1$ :  
        // The last element at index should be in proper order  
        for index between 0 to  $2n-\text{numOfPasses}-1$ :  
            if before[index] is out of order from before[index+1] then:  
                add the numOfSwaps by one  
                swap(index)  
  
    // returns the "row of disks", the amount of swaps that occurred  
    return sorted_disks(before, numOfSwaps);  
}
```

## The Lawnmower Algorithm

**Input:** An unsorted array (before) the size of  $2n$

**Output:** A fully sorted swapped array (before) with the number of swaps that occurred (numOfSwaps)

*// Algorithm that sorts disks using the Lawnmower Algorithm*

*//disk\_state: The state of the single row array of disks*  
sorted\_disks sort\_lawnmower(const disk\_state& before) {

    int numOfPasses = 0;  
    int indexer = 0;  
    int numOfSwaps = 0;

**for** numOfPasses **between** 0 to  $2n-1$ :

    int secondIndexer =  $2n-1$ ; *// The secondIndexer will always start at the end of the array*

*// If the numOfPasses is even, then the swaps will go LEFT to RIGHT*

**if** numOfPasses%2 == 0:

*// The last element at index should be in proper order*

**for** indexer **between** 0 to  $2n-\text{numOfPasses}-1$ :

**if** before[indexer] is out of order from before[indexer+1] **then**:

**add** the numOfSwaps by one

**swap**(indexer)

*// For every other pass through, the numOfPasses will be odd and therefore will be at the rhs of the*

*// array. If the numOfPasses is odd, then the swaps will go RIGHT to LEFT*

**else**:

*// From Right to Left*

**for** secondIndexer **between**  $2n-\text{numOfPasses}-1$  to 0:

**if** before[secondIndexer] is out of order from before[secondIndexer-1] **then**:

**add** the numOfSwaps by one

**swap**(secondIndexer-1)

    return sorted\_disks(before, numOfSwaps);

}

# Project 1 : Mathematical Analysis - Calculating Step Count and Time Complexity

## The Left to Right Algorithm

Note: For simplicity with calculations, I have changed the variable names within my code.

**Input:** An unsorted array (before) the size of  $2n$

**Output:** A fully sorted swapped array (before) with the number of swaps that occurred ( $c$ )

```
sorted_disks sort_left_to_right(const disk_state& before) {
```

```
    int i = 0; ①
```

```
    int k = 0; ①
```

```
    int c = 0; ①
```

```
    for (i = 0; i < 2n-1; i++) { ((2n-1)-o+1) = (2n)
```

```
        for (k = 0; k < 2n - i - 1; k++) { ((2n - i - 1) - o + 1) = (2n - i)
```

```
        if (before.get(k) == DISK_DARK && before.get(k+1) == DISK_LIGHT) { (3) + 3 = b }
```

```
            c++; (2) > 3
```

```
            swap(k); ①
```

```
}
```

```
}
```

```
return sorted_disks(before, c);
```

```
}
```

To prove  $24n^2 - 6n \in O(n^2)$

$$C = 30$$

$$= 24n^2 - 6n \leq 30n^2 \quad \forall n \geq n_0$$

$$-30n^2 \qquad \qquad \qquad -30n^2$$

$$= 30n^2 - 24 + 6n \leq 0$$

$$6n^2 + 6n \leq 0 \quad \checkmark$$

$$= (b)(2n - i)$$

$$= 12n - bi$$

$$= \sum_{i=0}^{2n} 12n - bi \Rightarrow 6 \sum_{i=0}^{2n} 2n - i$$

$$= 6 \sum_{i=0}^{2n} 2n - 6 \sum_{i=0}^{2n} i$$

$$= 6 \sum_{i=0}^{2n} 2n - 6 \left( \frac{2n(2n+1)}{2} \right)$$

$$= 6 \sum_{i=0}^{2n} 2n - 6 \left( \frac{4n^2+2n}{2} \right)$$

$$= 6(2n+1)(2n) - 6(2n+n)$$

$$= 6(4n^2+2n) - 12n^2 + 6n$$

$$= 24n^2 + 12n - 12n - 6n$$

$$= \underline{\underline{24n^2 - 6n}}$$

## The Lawnmower Algorithm

**Input:** An unsorted array (before) the size of  $2n$

**Output:** A fully sorted swapped array (before) with the number of swaps that occurred ( $c$ )

```
sorted_disks sort_lawnmower(const disk_state& before) {
```

```
    int i = 0; (1)
```

```
    int k = 0; (1)
```

```
    int c = 0; (1)
```

```
    for (i = 0; i < 2n-1; i++) { ((2n-1) - 0 + 1) = 2n
```

```
        int j = 2n-1; (2)
```

```
        if (i%2 == 0){ (2)
```

```
            for (k = 0; k < 2n - i - 1; k++) { ((2n-i-1) - 0 + 1) = (2n-i)
```

S1  
is larger

```
            if (before.get(k) == DISK_DARK && before.get(k+1) == DISK_LIGHT){ (3) + 3 = 6  
                c++; (2) > 3  
                swap(k); (1) > 3  
            }
```

$$\begin{aligned} &= 6(2n-i) \\ &= (12n-6i) \end{aligned}$$

```
        else{
```

$$\uparrow = ((2n-i-1)-(2n-1)+1) \Rightarrow 2 + 12n - 6i = S1$$

```
            for (j; 2n - i - 1 < j; j--) { = (2n-i-1) - 2n+1 + 1 = (i-1)
```

```
            if (before.get(j) == DISK_LIGHT && before.get(j-1) == DISK_DARK){ (3) + 3 = 6
```

```
                c++; (2) > 3  
                swap(j-1); (1) > 3
```

$$= (6)(i-1)$$

$$= (6i - 6)$$

$$= 2 + 6i - 6$$

$$\Rightarrow 6i - 4 = S2$$

```
}
```

```
return sorted_disks(before, c);
```

```
}
```

cont on next page ...

Since the if/else statement takes into consideration the larger sc of S1 if S2, we only use S1.

$$\begin{aligned}
 &= \sum_{i=0}^{2n} 2 + 12n - 6i \\
 &= 2 \sum_{i=0}^{2n} 1 + 2 \sum_{i=0}^{2n} 6n - 2 \sum_{i=0}^{2n} 3i \\
 &= 2 \sum_{i=0}^{2n} 1 + 6 \sum_{i=0}^{2n} 2n - 6 \left( \frac{2n(2n+1)}{2} \right) \\
 &= 2(2n+1) + 6(2n+1)(2n) - 6 \left( \frac{2n(2n+1)}{2} \right) \\
 &= 4n+2 + 6(4n^2+2n) - 6(2n+n) \\
 &= 4n+2 + 24n^2+12n - (12n+6n) \\
 &= 4n+2 + 24n^2+12n - 6n \\
 &= 24n^2+4n+12n-6n+2 \\
 &= \underline{\underline{24n^2+10n+2}}
 \end{aligned}$$

to prove  $24n^2 + 10n + 2 \in O(n^2)$

$$c = 34$$

$$\begin{aligned}
 &= 24n^2 + 10n + 2 \leq 34n^2 \quad \forall n \geq n_0 \\
 &\quad -34n^2 \quad \quad \quad -34n^2 \\
 &= 34n^2 - 24n^2 - 10n - 2 \leq 0 \\
 &= 10n^2 - 10n - 2 \leq 0 \quad \checkmark
 \end{aligned}$$