Theoretical Models of Exoplanet Atmospheres

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Atmospheres, Atmospheres!

Do I look like I care about atmospheres?

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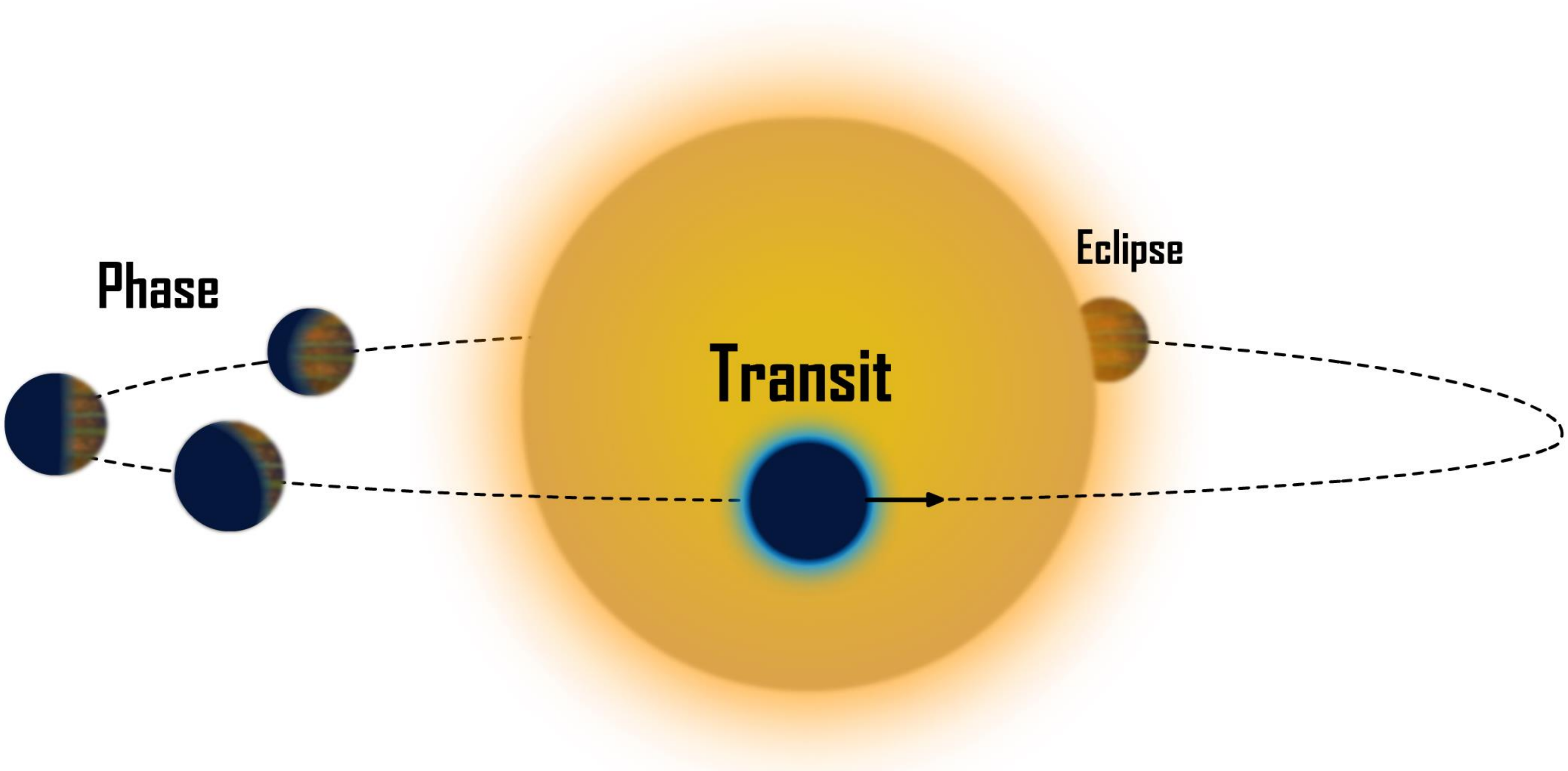
Image credit
ESO/M. Kornmesser

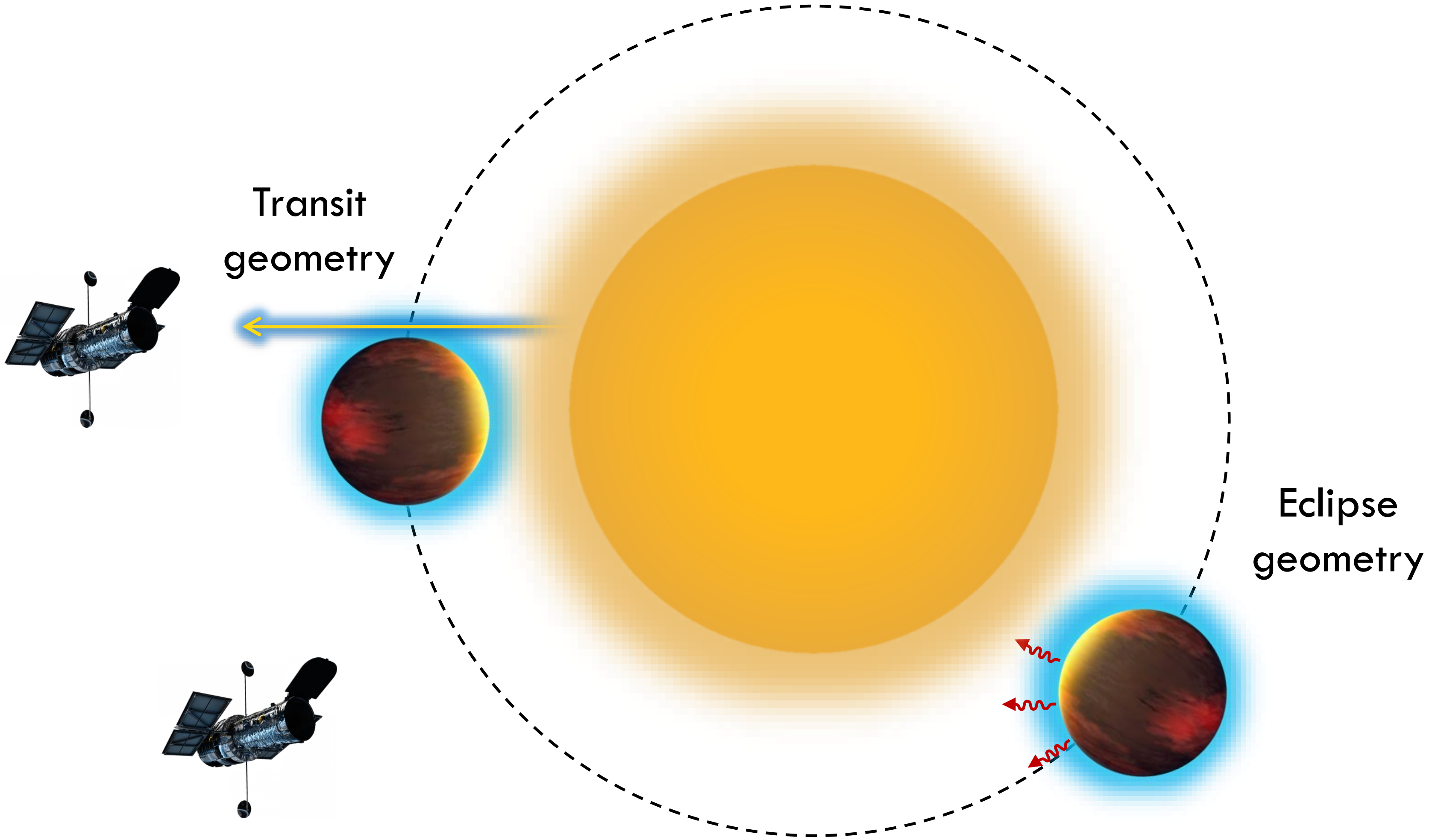
PART 1:

SPECTRA OF EXOPLANET ATMOSPHERES

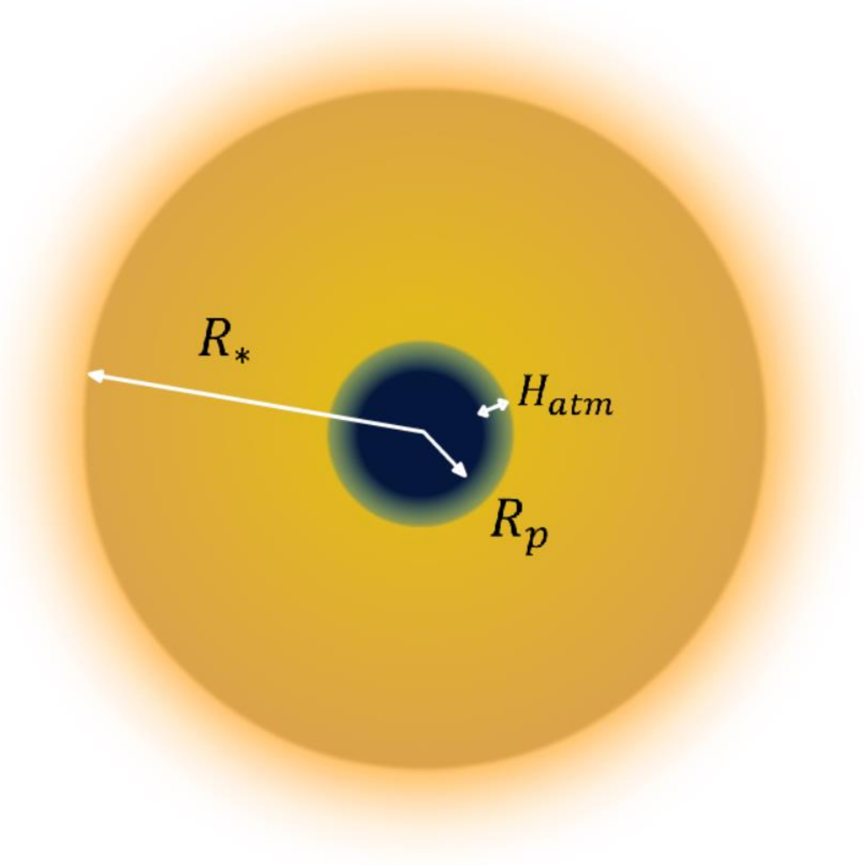
TECHNIQUES TO PROBE EXOPLANET ATMOSPHERES

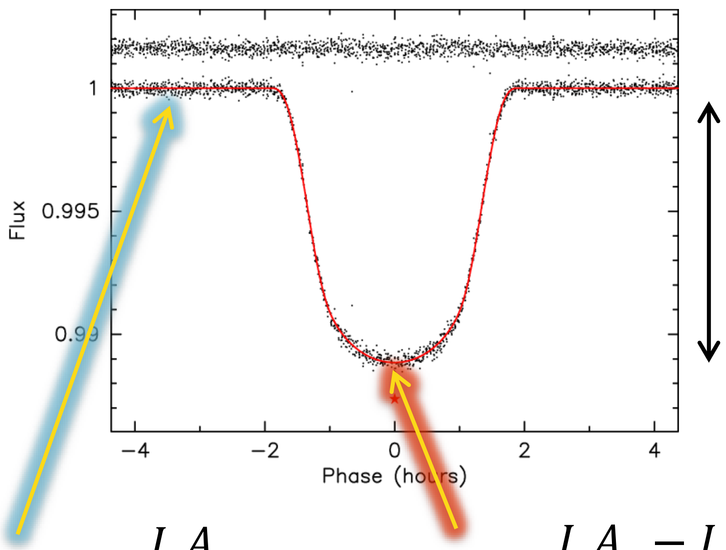


DIFFERENT SPECTRA PROBE DIFFERENT ATMOSPHERIC REGIONS



A TOY MODEL FOR TRANSMISSION SPECTRA





$$\delta_{trans} = \frac{F_{out} - F_{in}}{F_{out}}$$

$$F_{out} = \frac{I_* A_*}{d^2} \qquad F_{in} = \frac{I_* A_* - I_* A_p}{d^2} \qquad \rightarrow \quad \delta_{trans} = \frac{A_p}{A_*}$$

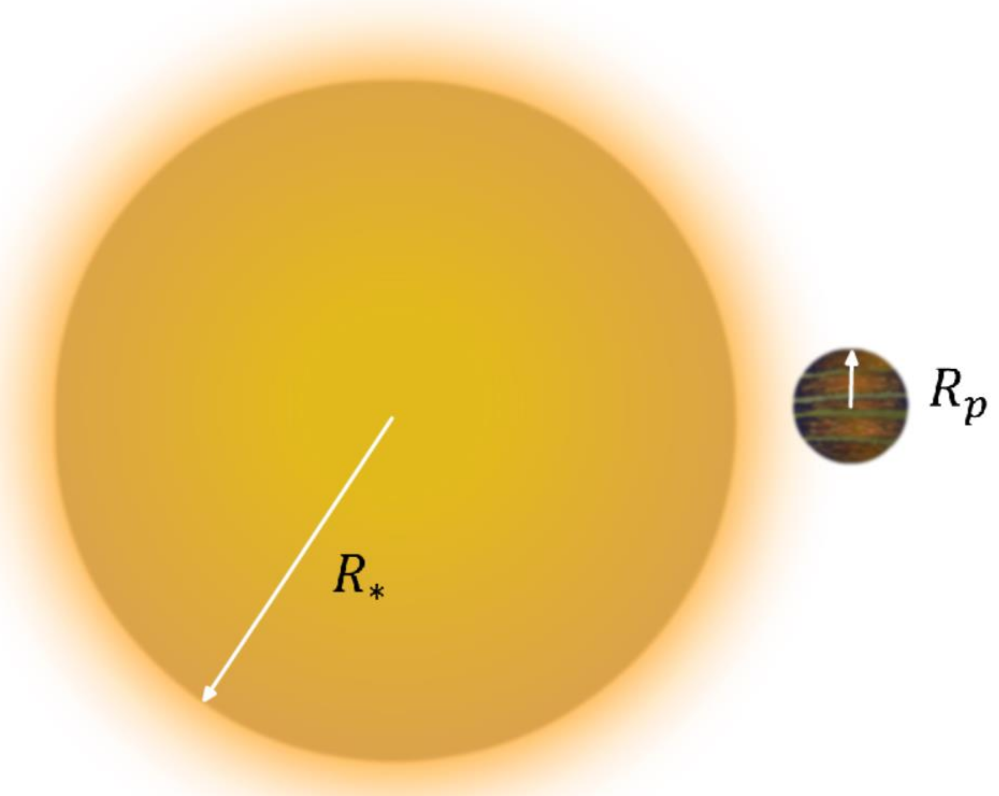
For a transparent atmosphere:

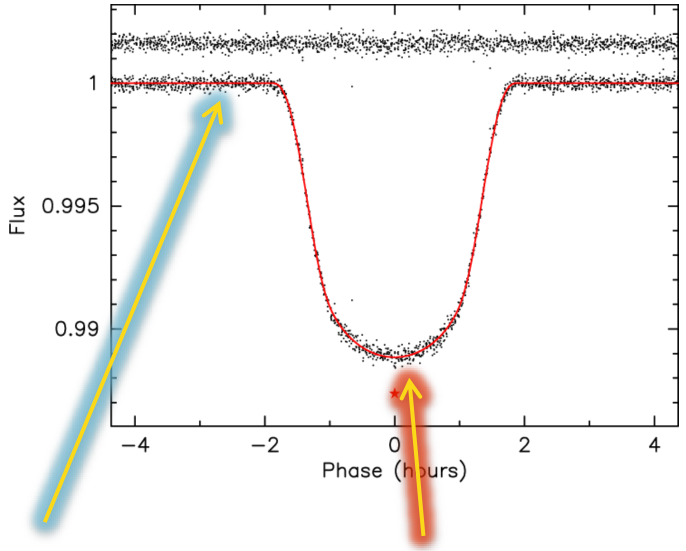
$$\delta_{trans} = \frac{R_p^2}{R_*^2}$$

For an opaque atmosphere:

$$\delta_{trans} = \frac{R_p^2}{R_*^2} + \left(\frac{2 R_p H_{atm}}{R_*^2} + \frac{H_{atm}^2}{R_*^2} \right) \approx \frac{R_p^2}{R_*^2} + \frac{2 R_p H_{atm}}{R_*^2}$$

A TOY MODEL FOR EMISSION SPECTRA





$$\delta_{eclipse} = \frac{F_{out} - F_{in}}{F_{out}}$$

$$F_{out} = F_p + F_* \qquad F_{in} = F_* \qquad \rightarrow \quad \delta_{eclipse} = \frac{F_p}{F_p + F_*} \approx \frac{F_p}{F_*}$$

Assume both planet and star emit as black bodies:

$$B_\lambda(T) = \frac{2hc^2}{\lambda^5}\left(\frac{1}{e^{\frac{hc}{\lambda k_B T}} - 1}\right)$$

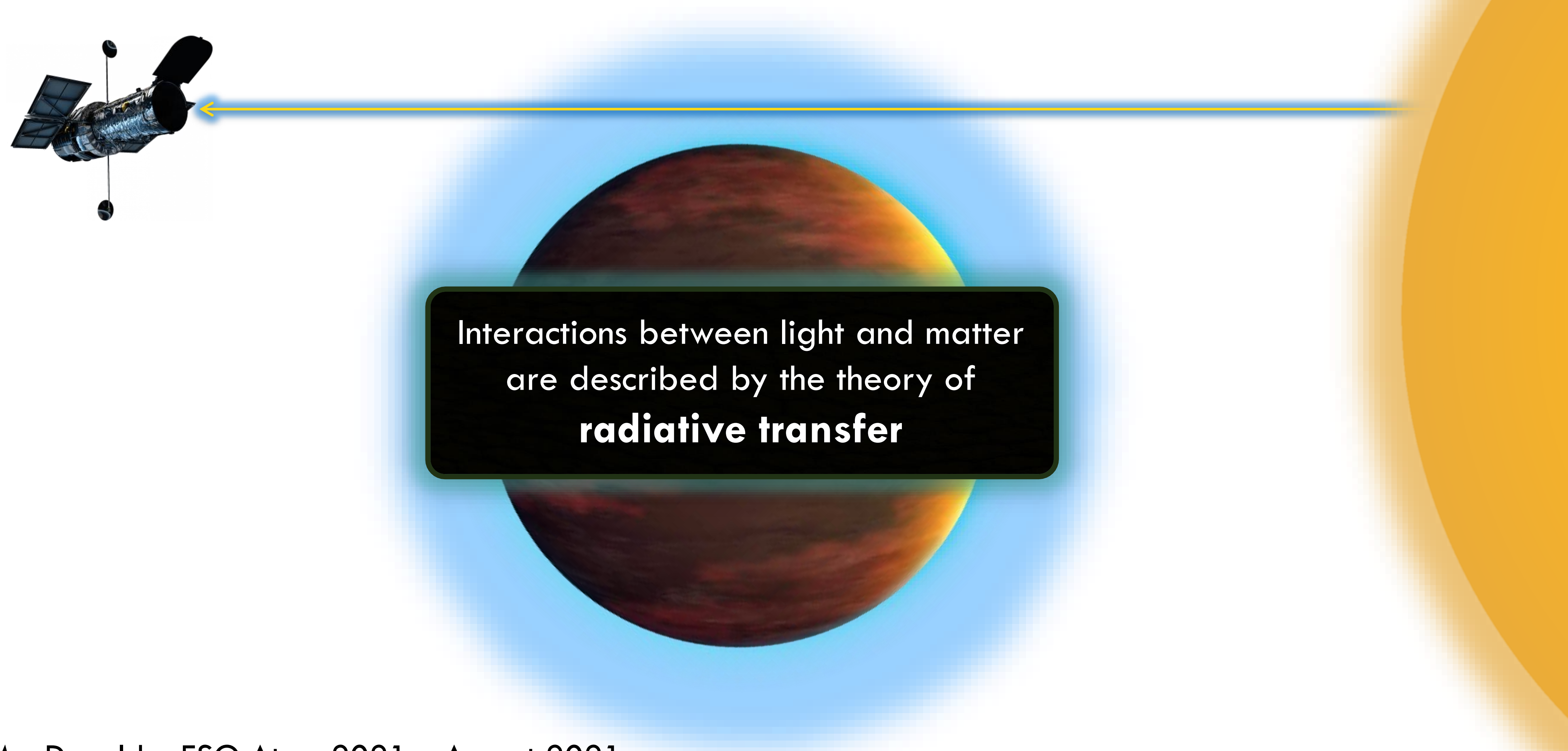
$$F_p = \frac{\pi B_\lambda(T_p) R_p^2}{d^2}$$

$$F_* = \frac{\pi B_\lambda(T_*) R_*^2}{d^2}$$

$$\therefore \; \delta_{eclipse} = \left[\frac{e^{\frac{hc}{\lambda k_B T_*}} - 1}{e^{\frac{hc}{\lambda k_B T_p}} - 1}\right]\left(\frac{R_p^2}{R_*^2}\right) \approx \frac{T_p}{T_*}\left(\frac{R_p^2}{R_*^2}\right)$$

$$\frac{hc}{\lambda k_B T_p} \ll 1$$

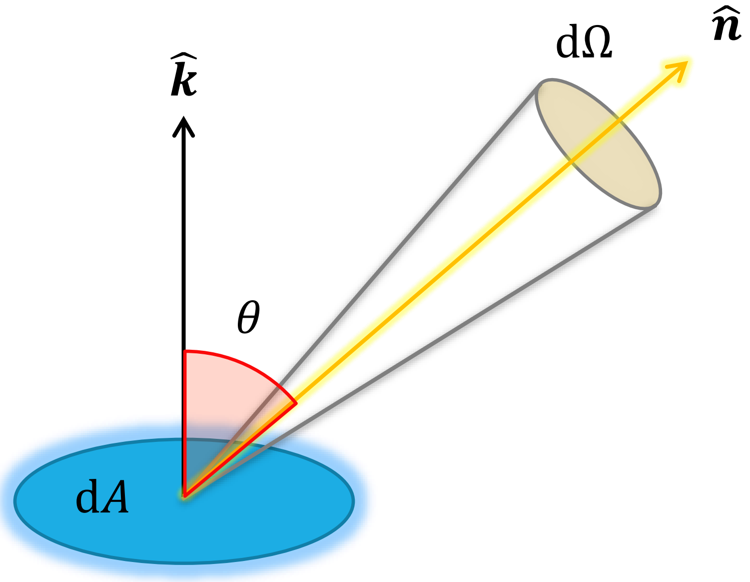
BUILDING A BETTER MODEL OF EXOPLANET SPECTRA



Interactions between light and matter are described by the theory of

radiative transfer

BASIC QUANTITIES OF RADIATIVE TRANSFER



A beam of light, travelling in direction $\hat{\boldsymbol{n}}$, represented as a cone with solid angle $d\Omega$, passes through a surface of area dA

The **intensity** encodes how much energy flows through the surface per time per wavelength:

$$dE_\lambda = I_\lambda \left(\hat{\boldsymbol{n}} \cdot \hat{\boldsymbol{k}} \right) d\Omega \, \mathrm{dA} \, \mathrm{d}\lambda \, dt$$

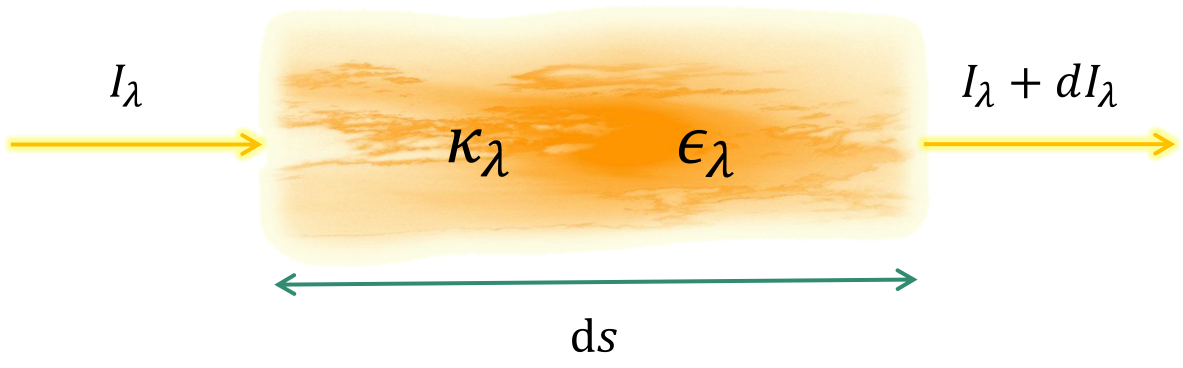
Intensity is a fundamental quantity of radiative transfer

In a vacuum, the intensity of a beam remains constant

We observe **flux**, which is the integrated intensity over all solid angles:

$$F_\lambda = \int I_\lambda \, \hat{\boldsymbol{n}} \cdot \hat{\boldsymbol{k}} \, d\Omega$$

INTERACTIONS BETWEEN LIGHT AND MATTER



A beam travelling through a medium can have its intensity changed

The **extinction coefficient** (κ_λ) encodes processes that remove photons from the beam:

$$dI_{\lambda,\, loss} = -\kappa_\lambda \, I_\lambda \, ds$$

The **emission coefficient** (ϵ_λ) encodes processes that add photons to the beam:

$$dI_{\lambda,\, gain} = \epsilon_\lambda \, ds$$

Combined effect:

$$\boxed{\frac{dI_\lambda}{ds} = -\kappa_\lambda I_\lambda + \epsilon_\lambda}$$

The equation of radiative transfer

THE RADIATIVE TRANSFER EQUATION

Now we can introduce two important quantities

The optical depth:

$$\tau_\lambda = \int \kappa_\lambda \, ds$$

The source function:

$$S_\lambda = \frac{\epsilon_\lambda}{\kappa_\lambda}$$

This yields the most common form of the radiative transfer equation

$$\frac{dI_\lambda}{ds} = -\kappa_\lambda I_\lambda + \epsilon_\lambda \quad \rightarrow \quad \frac{dI_\lambda}{d\tau_\lambda} = -I_\lambda + S_\lambda$$

In local thermodynamic equilibrium (LTE) [and when scattering is negligible], Kirchoff's law gives us

$$S_\lambda = B_\lambda$$

$$\boxed{\frac{dI_\lambda}{d\tau_\lambda} = -I_\lambda + B_\lambda}$$

RADIATIVE TRANSFER: TRANSMISSION SPECTRA

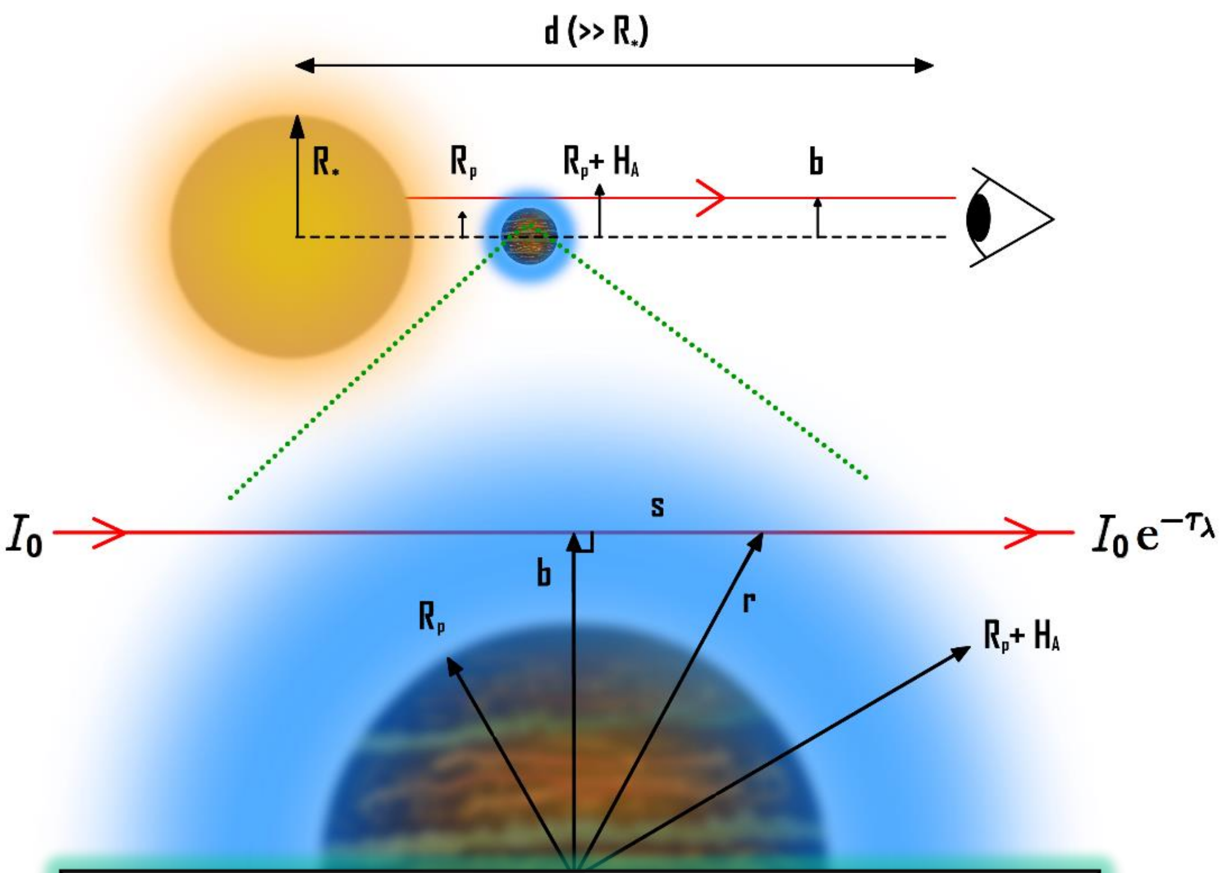
In transmission geometry, there is no emission into the beam:

$$\frac{dI_\lambda}{d\tau_\lambda} = -I_\lambda$$

$$I_\lambda(\tau_\lambda) = I_\lambda(0)\, e^{-\tau_\lambda}$$

The Beer-Lambert law

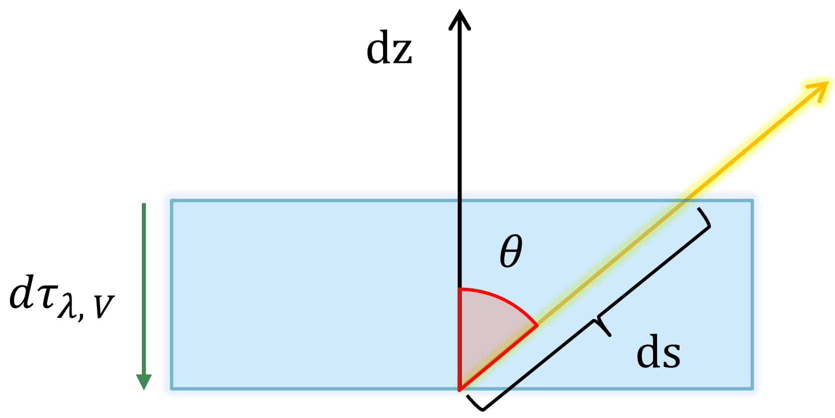
A transmission spectrum is a sum over the areas of atmospheric annuli, weighted by how strongly they absorb light $(1 - e^{-\tau_\lambda})$:



$$\delta_\lambda = \frac{R_p^2 + 2\int_{R_p}^{R_p+H_A} b\left(1 - e^{-\tau_\lambda(b)}\right) db \; - 2\int_0^{R_p} b e^{-\tau_\lambda(b)}\, db}{R_*^2}$$

MacDonald & Madhusudhan (2017)

RADIATIVE TRANSFER: EMISSION SPECTRA



Emission spectra usually consider the *vertical* optical depth instead of the path optical depth:

$$d\tau_{\lambda,V} = -\kappa_\lambda dz = -\kappa_\lambda \cos\theta \, ds$$

The radiative transfer equation for emission spectra then becomes:

$$\mu \frac{dI_\lambda}{d\tau_{\lambda,V}} = I_\lambda - B_\lambda$$

Solution for top-of-atmosphere intensity:

$$I_{\lambda,\mathrm{top}}(\mu) = \frac{1}{\mu}\int_0^\infty B_\lambda(\tau_{\lambda,V}) e^{-\frac{\tau_{\lambda,V}}{\mu}} d\tau_{\lambda,V}$$

Integrating over all emission angles gives us the planet flux:

$$F_{p,\lambda} = 2\pi \frac{R_p^2}{d^2} \int_0^1 \int_0^\infty B_\lambda(\tau_{\lambda,V}) e^{-\frac{\tau_{\lambda,V}}{\mu}} d\tau_{\lambda,V} \, d\mu$$

DETERMINING THE OPTICAL DEPTH

The optical depth is required to compute exoplanet spectra:

$$\tau_\lambda = \int \kappa_\lambda \, ds$$

Many different physical processes can contribute opacity to the extinction coefficient:

$$\kappa_\lambda = \kappa_{\lambda,\,\mathrm{chem}} + \kappa_{\lambda,\,\mathrm{Rayleigh}} + \kappa_{\lambda,\,\mathrm{pair}} + \kappa_{\lambda,\,\mathrm{aerosol}}$$

Let us focus on the first term ('regular absorption' by atoms, molecules, or ions):

$$\kappa_{\lambda,\,\mathrm{chem}} = \sum_{q=1}^{N_{\mathrm{species}}} n_q \, \sigma_{\lambda,\,\mathrm{abs},\,q}$$

$$X_q \, n_{\mathrm{tot}}$$

Absorption cross section
(see Part 2)

Mixing ratio
(composition)

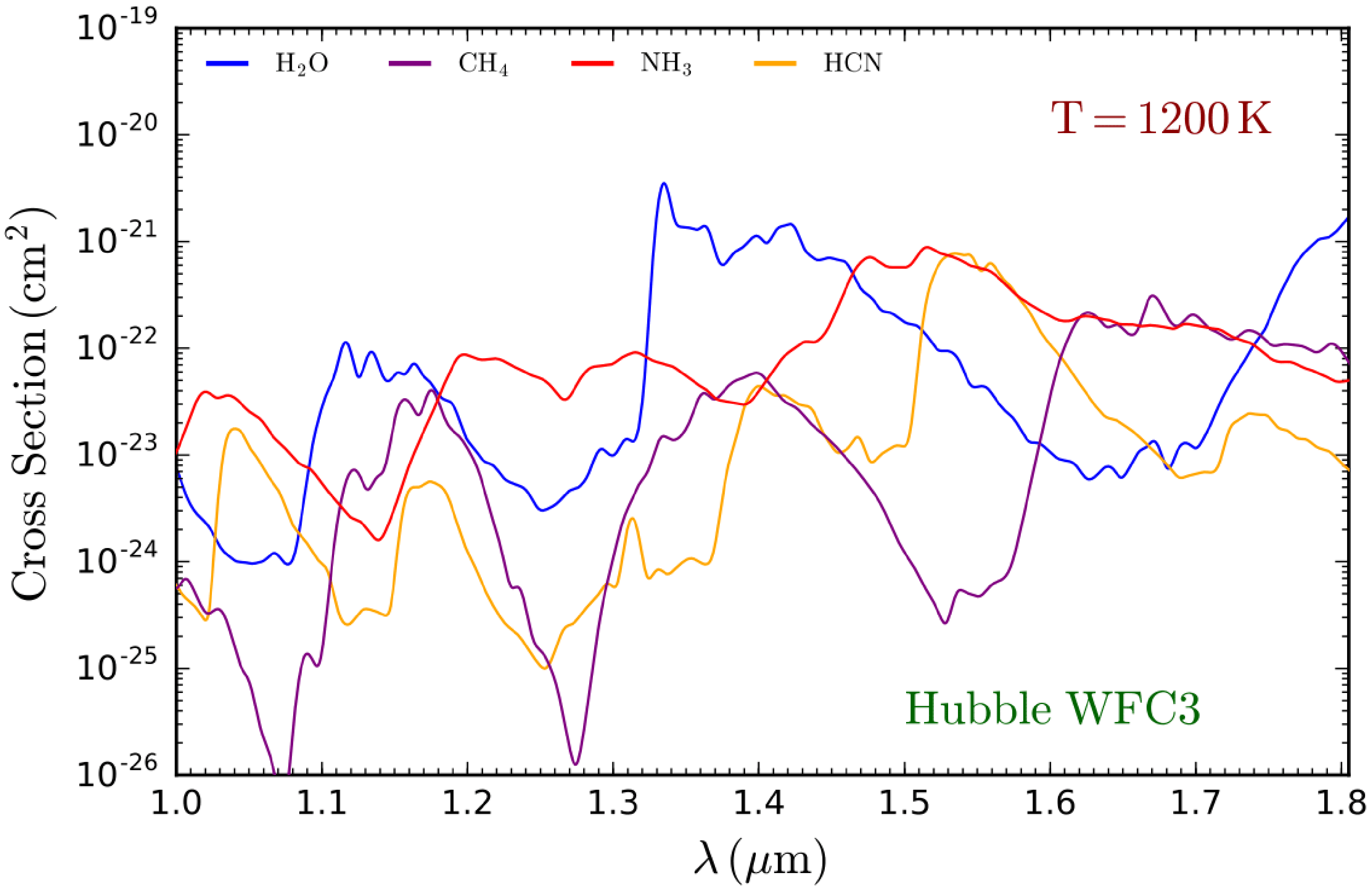
Number density
(see Part 3)

Tutorial 1: Sequences of spectra and atmospheric compositions

PART 2:

OPACITY IN EXOPLANET ATMOSPHERES

INTRODUCTION TO CROSS SECTIONS



Cross sections, σ_λ, determine how strongly a given chemical species absorbs (or scatters) light

They can be thought of as the 'effective area' of a chemical species

COMPUTATION OF CROSS SECTIONS

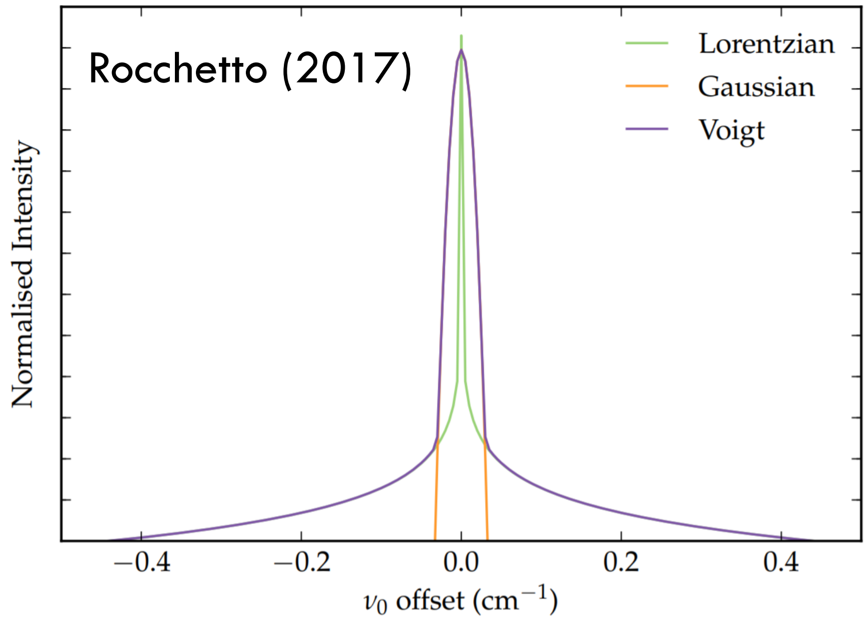
$$\sigma_\lambda(P,T) = \sum_j S_j(T)\, f(\lambda, \lambda_{0,j}, P, T)$$

Sum over many billions of quantum transitions

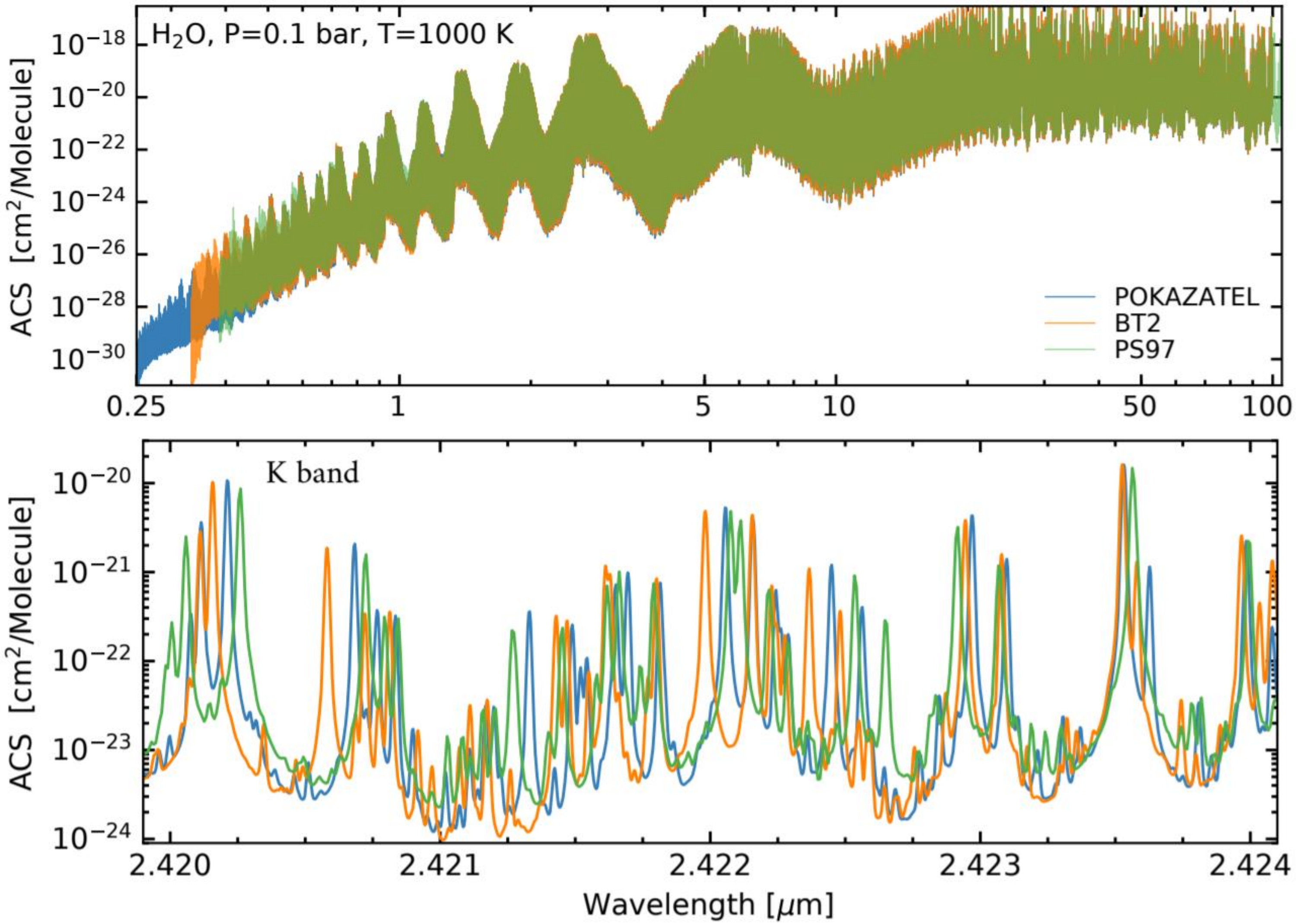
Tabulated for each transition in a **line list**

(ExoMol, HITEMP, HITRAN etc.)

Line profiles are normally assumed to be Voigt profiles

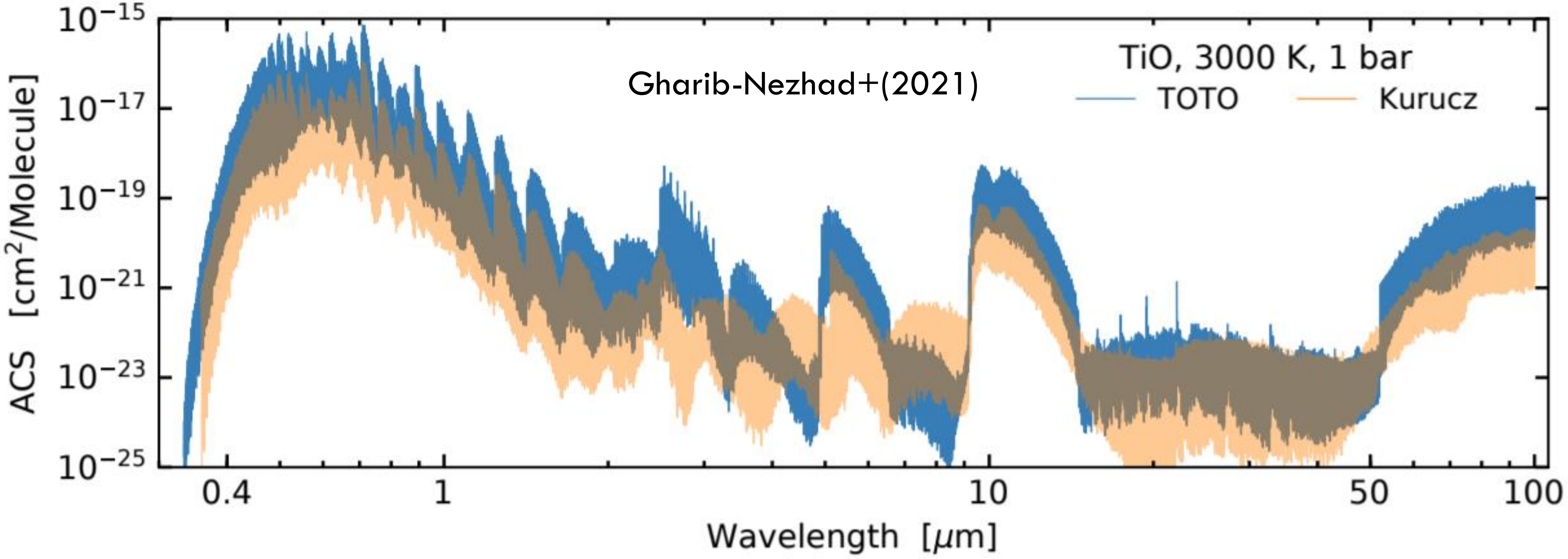


CROSS SECTION UNCERTAINTIES



Gharib-Nezhad+(2021)

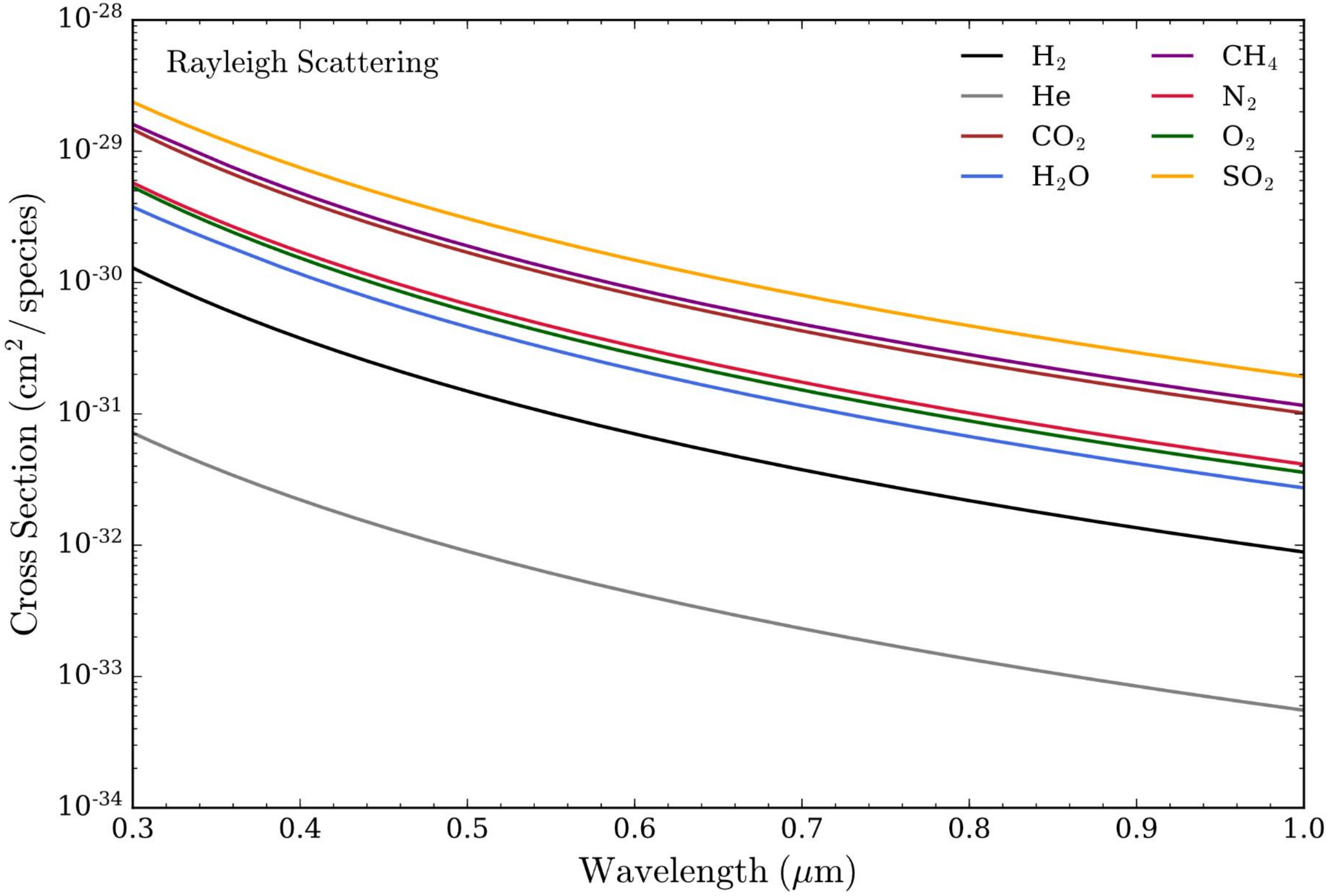
CROSS SECTION UNCERTAINTIES



The **choice of line list** is an important consideration that can **significantly alter cross sections**

Cross sections are fixed inputs to atmospheric models, so any **uncertainties in cross sections can propagate to model inferences**

RAYLEIGH SCATTERING

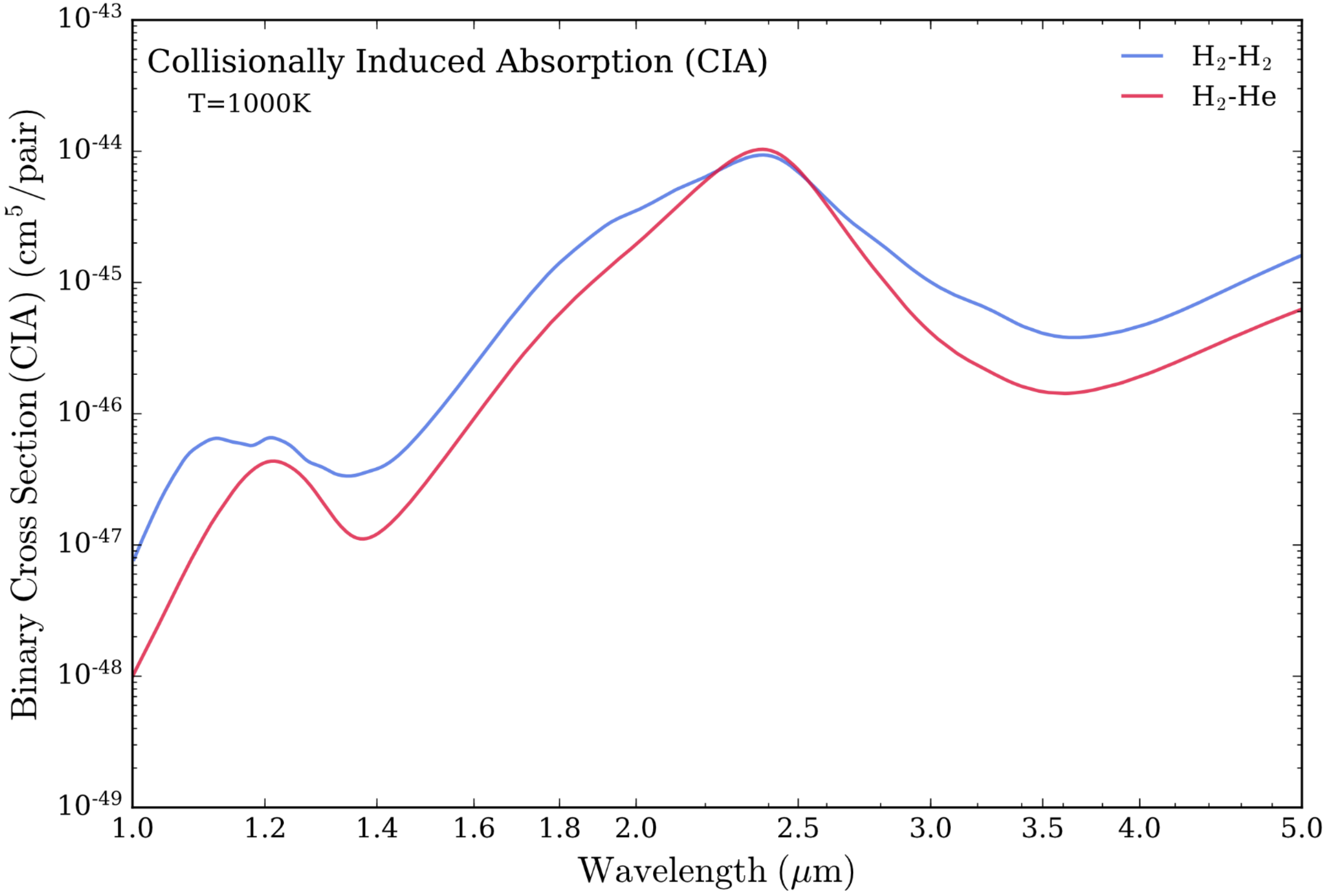


$$\kappa_{\lambda,\,\mathrm{Rayleigh}} = \sum_{q=1}^{N_{\mathrm{species}}} n_q \, \sigma_{\lambda,\,\mathrm{Rayleigh},\,q}$$

$$\sigma_{\lambda,\,\mathrm{Rayleigh},\,q} = \frac{128}{3} \pi^5 \alpha_q^2(\lambda) \, \lambda^{-4} \, F_{\mathrm{King},\,q}(\lambda)$$

Rayleigh scattering provides a continuum opacity that **can dominate at short wavelengths**

CONTINUUM (PAIR PROCESS) ABSORPTION

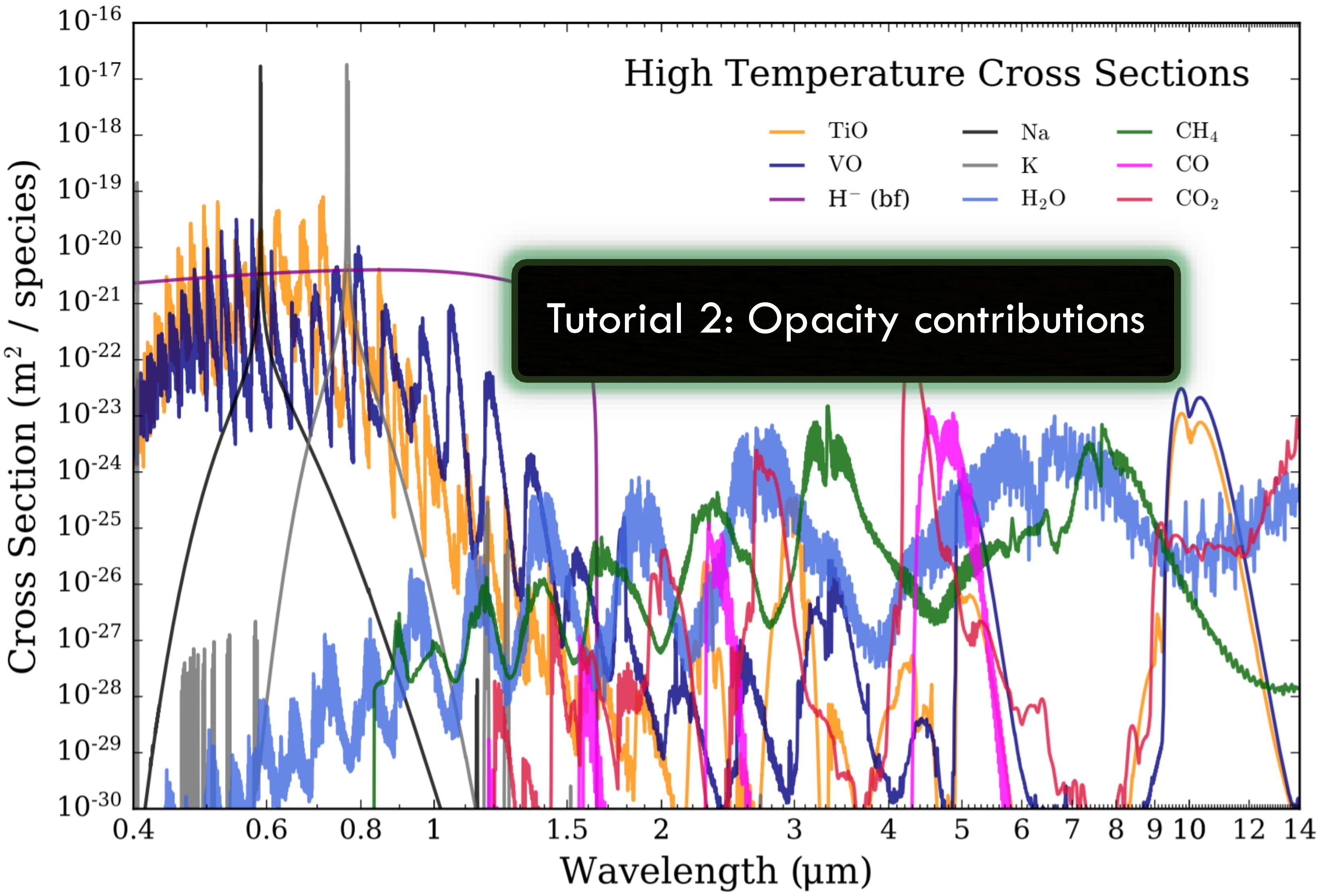


$$\kappa_{\lambda,\,\mathrm{pair}} = \sum_{\mathrm{pairs}} n_{q_1} n_{q_2} \sigma_{\lambda,\,\mathrm{pair},\,q}$$

Collision-induced absorption increases with n_{tot}^2, so this absorption dominates at high pressures

→ **sets the floor of a spectrum**

CROSS SECTION DATABASES



PART 3:

BUILDING A MODEL ATMOSPHERE

ATMOSPHERE MODELLING

To compute optical depths, we need to know the number densities in each atmospheric layer (n_{q}) and the physical spacing of each layer (dr)

$$\tau_\lambda = \int \kappa_\lambda \, ds \qquad \kappa_{\lambda,\,\mathrm{chem}} = \sum_{q=1}^{N_{\mathrm{species}}} n_q \, \sigma_{\lambda,\,\mathrm{abs},\,q}$$

The number densities are given by the **ideal gas law**:

$$n_q = X_q \frac{P}{k_B T(P)}$$

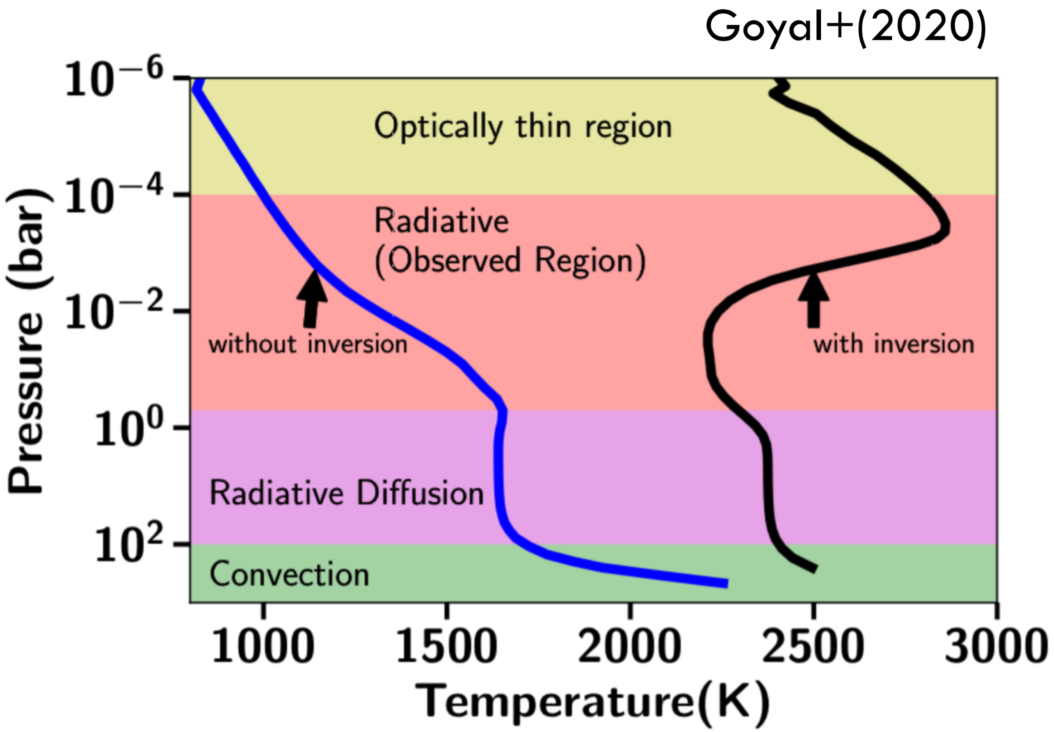
We need a prescription for $\boldsymbol{T(P)}$

The radial grid is determined by solving the equation of **hydrostatic equilibrium**:

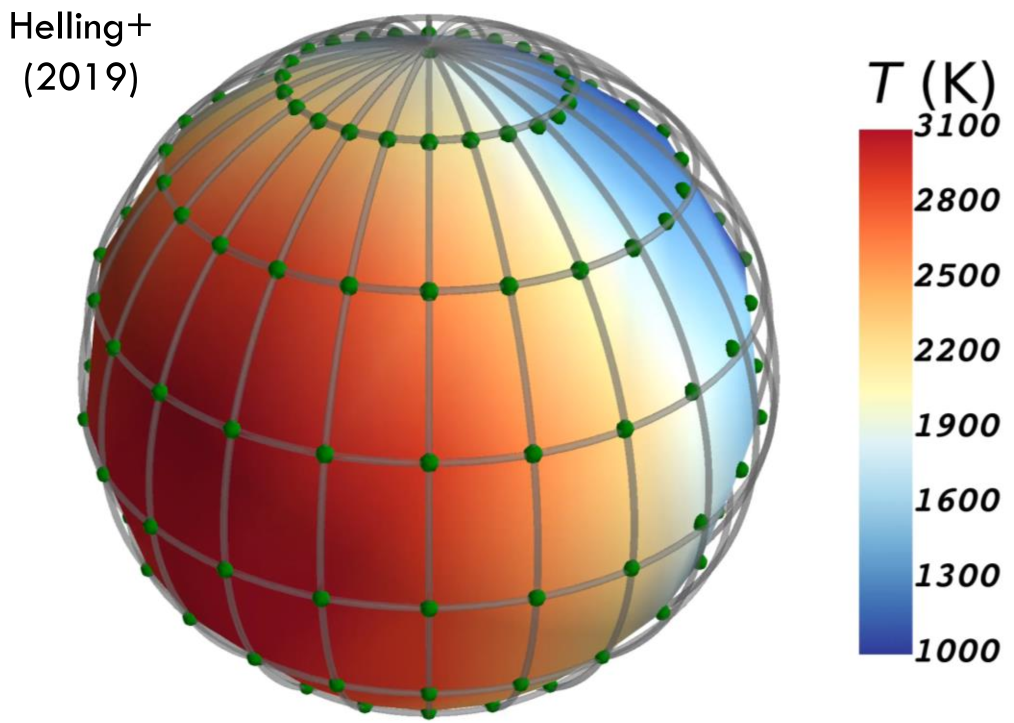
$$\frac{dP}{dr} = -\frac{G M_p \rho(r)}{r^2}$$

$$\rightarrow \quad r(P) = \left(\frac{1}{R_{p,\,\mathrm{deep}}} + \int_{P_{\mathrm{deep}}}^{P} \frac{k_B T(P)}{G M_p \mu_{mmw}} \frac{dP}{P} \right)^{-1}$$

ATMOSPHERE MODELLING: TEMPERATURE STRUCTURE

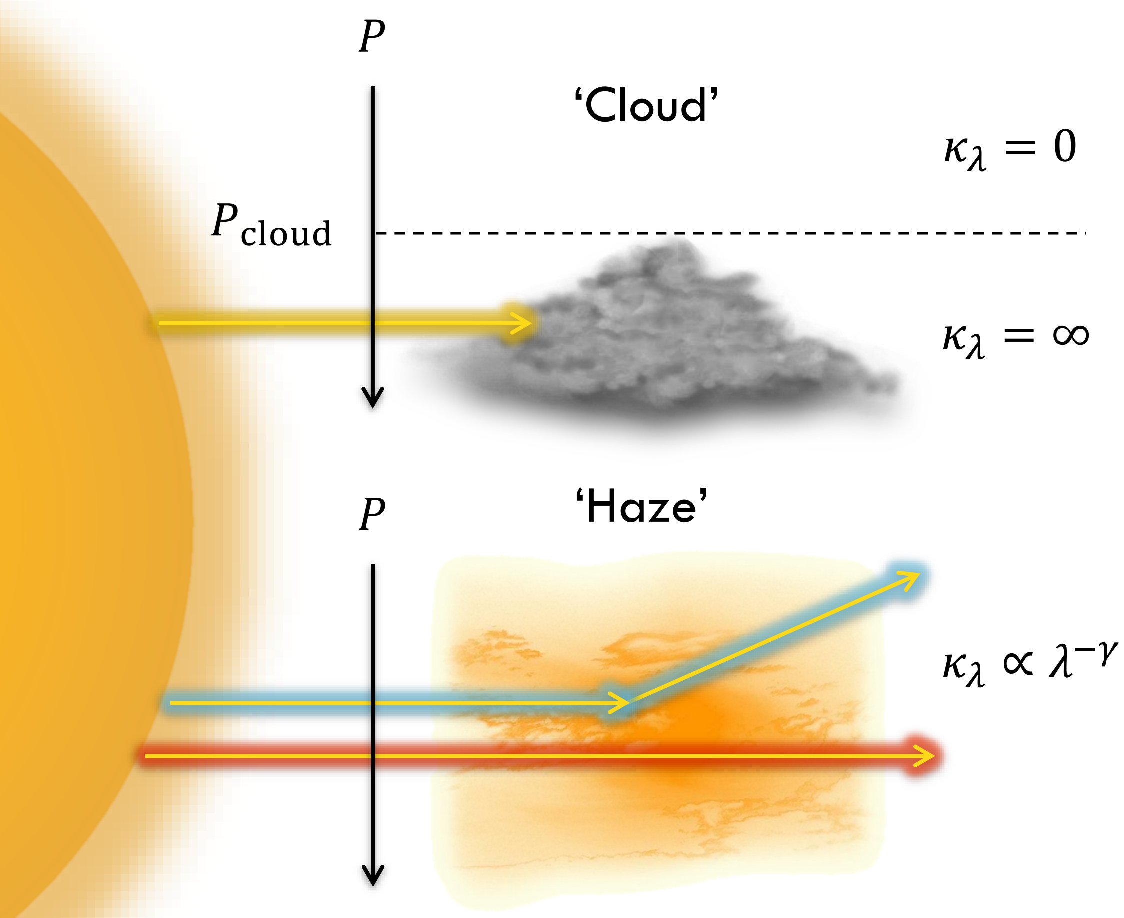


A **self-consistent P-T profile** can be computed by iteratively enforcing the condition of **radiative-convective equilibrium**



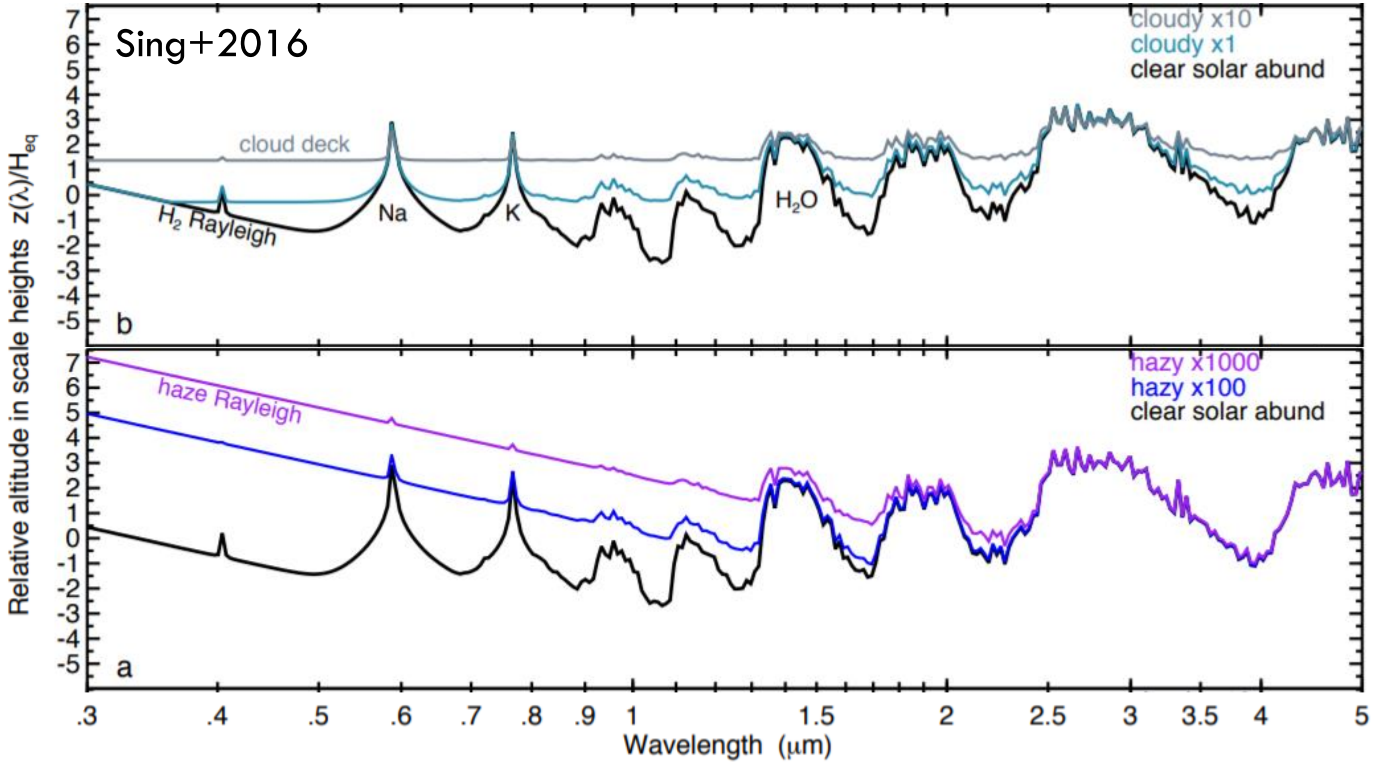
3D models can have temperatures and compositions that strongly vary with longitude, latitude, and altitude

ATMOSPHERE MODELLING: AEROSOLS



$$\tau_{\lambda,} = \int_0^\infty \kappa_\lambda \, ds$$

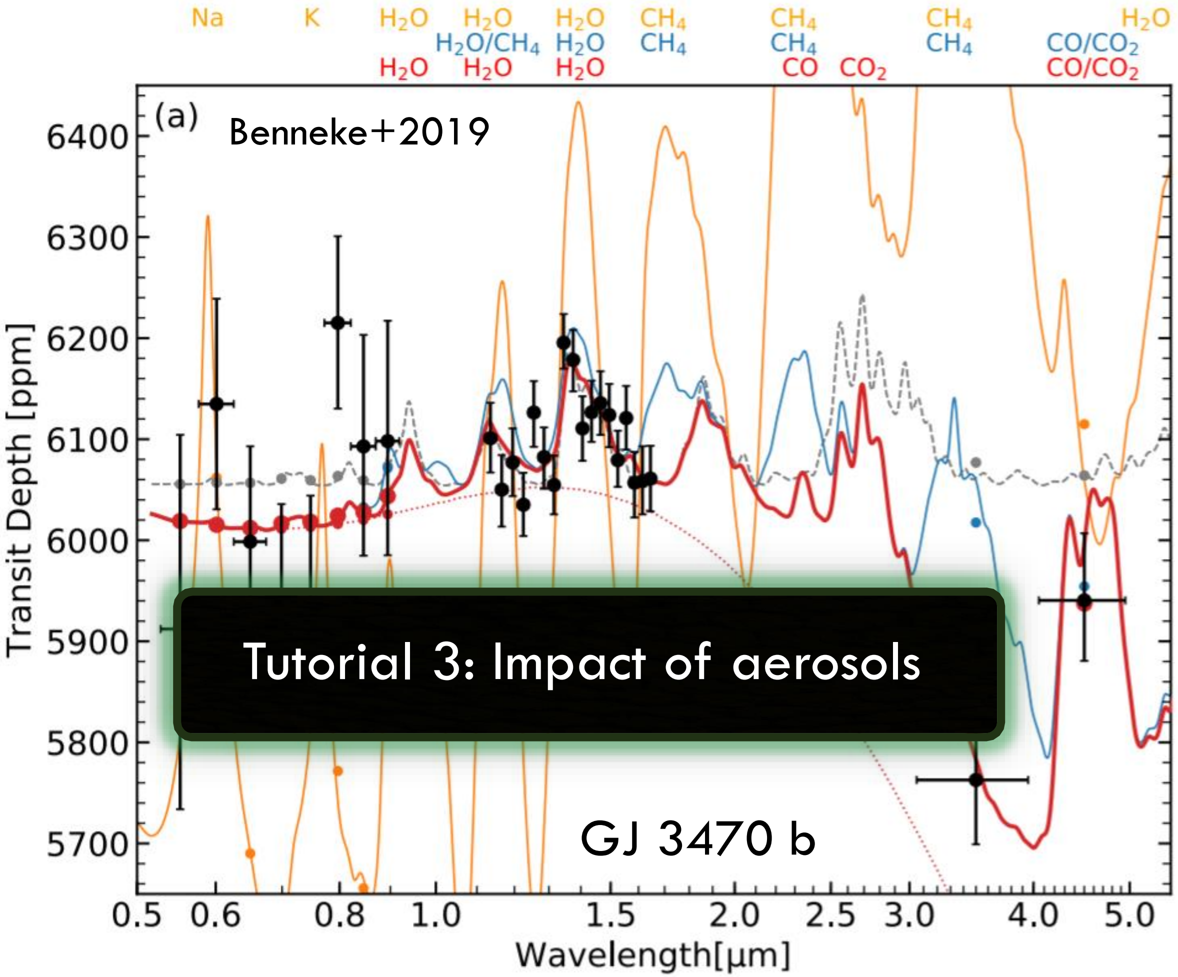
Impact of aerosols on transmission spectra



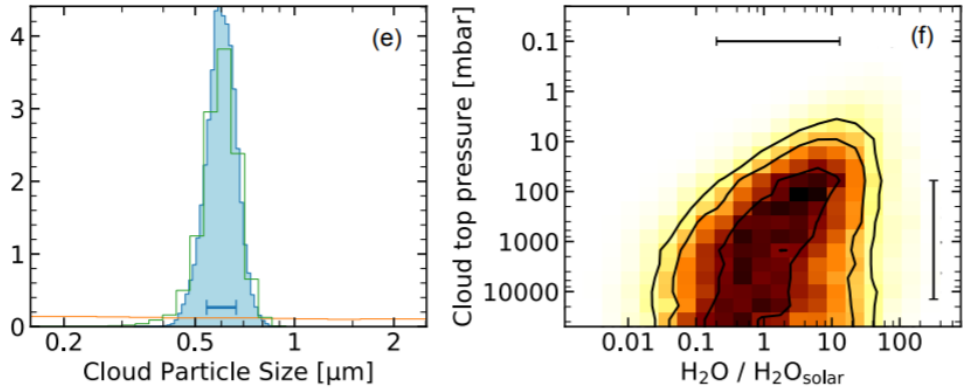
<u>Clouds</u> → **flat spectrum + muted features**

<u>Haze</u> → **strong scattering + non-Rayleigh slope**

MIE SCATTERING AEROSOLS



Tutorial 3: Impact of aerosols



Mie scattering predicts a **rapid drop off** in **aerosol extinction** for $\lambda \gtrsim 2\pi r$

$$\kappa_\lambda = n_0 e^{-\frac{z}{f H_{\mathrm{gas}}}} \int_0^\infty \pi r^2 Q_{\mathrm{ext}}\left(m, \frac{2\pi r}{\lambda}\right) P(r)\,dr$$

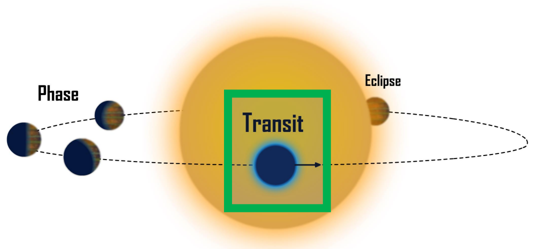
$$P(r) = \frac{1}{\sigma_g r \sqrt{2\pi}} e^{-\left(\frac{\ln r - \ln r_m}{\sqrt{2}\sigma_g}\right)^2}$$

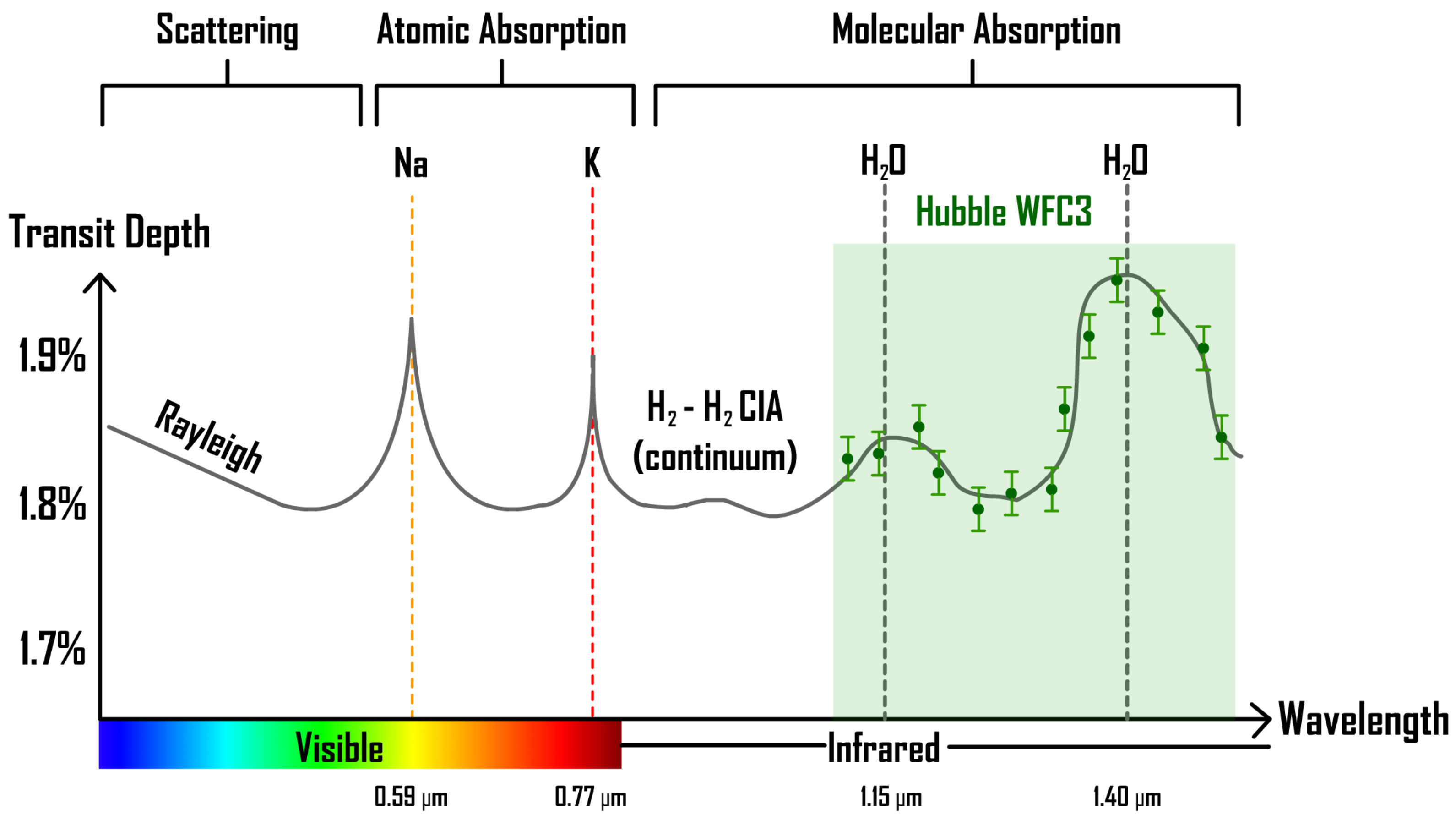
[Equations from Zhang+2019]

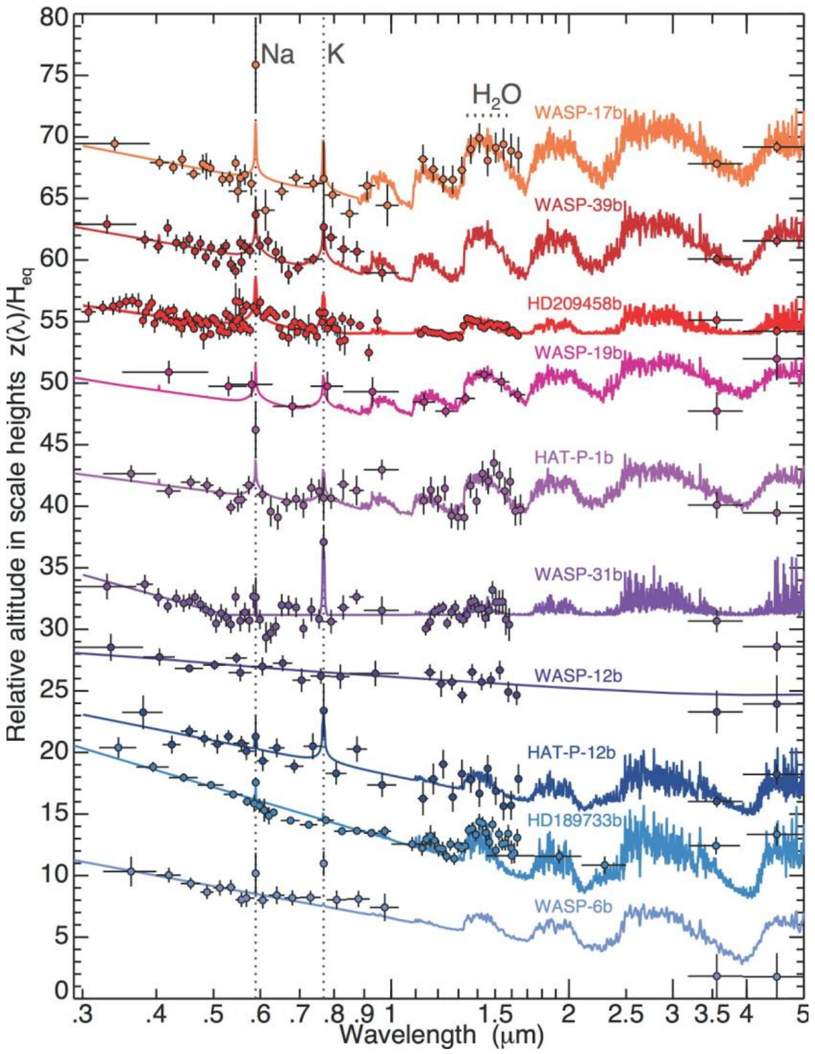
TAKEAWAYS:

TRANSMISSION AND EMISSION SPECTROSCOPY

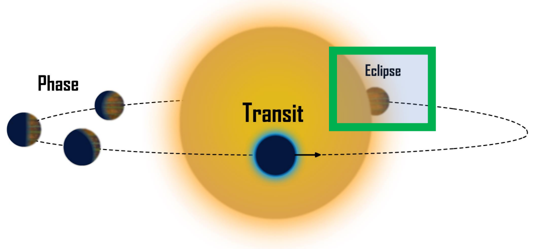
TRANSMISSION SPECTROSCOPY



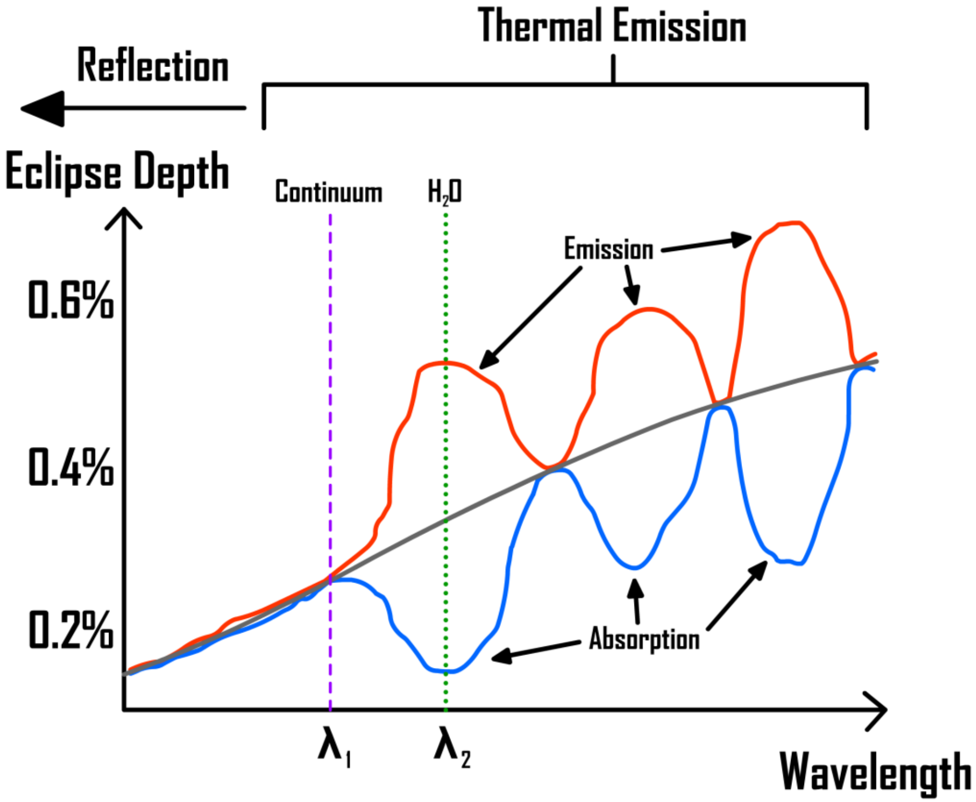


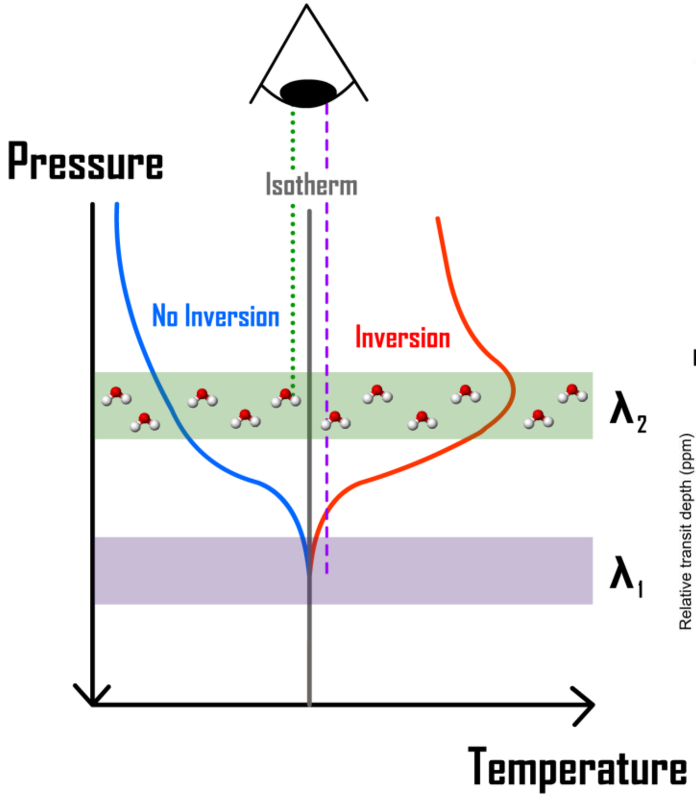


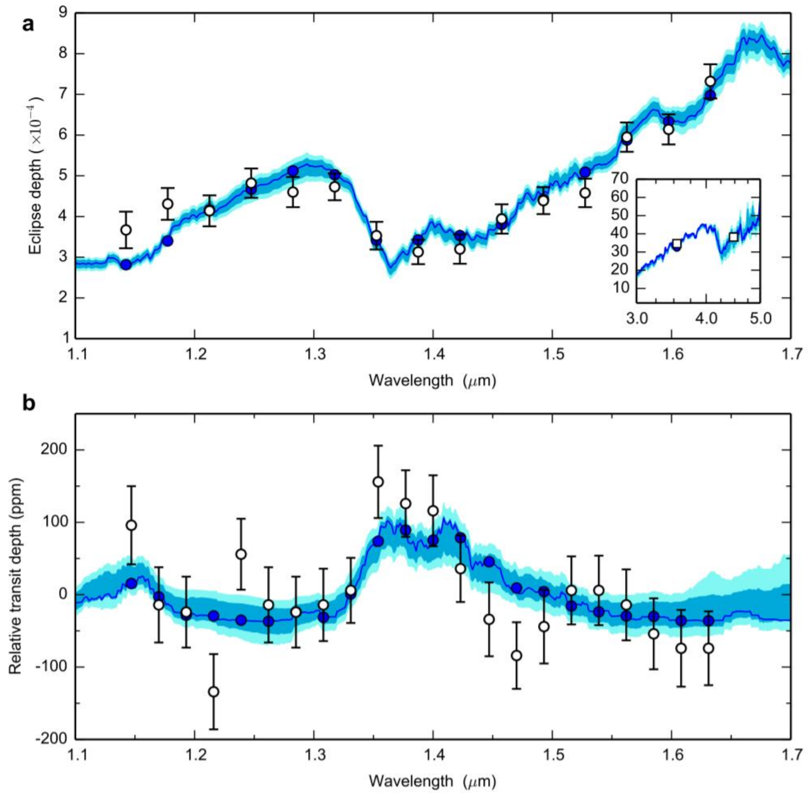
Sing+(2016)



EMISSION SPECTROSCOPY







Kreidberg+(2014)

QUESTIONS?

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