

The basic rules of natural deduction:

	<i>introduction</i>	<i>elimination</i>
$\wedge$	$\frac{\phi \quad \psi}{\phi \wedge \psi} \wedge i$	$\frac{\phi \wedge \psi}{\phi} \wedge e_1 \quad \frac{\phi \wedge \psi}{\psi} \wedge e_2$
$\vee$	$\frac{\phi}{\phi \vee \psi} \vee i_1 \quad \frac{\psi}{\phi \vee \psi} \vee i_2$	$\frac{\phi \vee \psi \quad \boxed{\begin{array}{c c} \phi & \psi \\ \vdots & \vdots \\ x & x \end{array}}}{x} \vee e$
$\rightarrow$	$\frac{\boxed{\begin{array}{c} \phi \\ \vdots \\ \psi \end{array}}}{\phi \rightarrow \psi} \rightarrow i$	$\frac{\phi \quad \phi \rightarrow \psi}{\psi} \rightarrow e$
$\neg$	$\frac{\boxed{\begin{array}{c} \phi \\ \vdots \\ \perp \end{array}}}{\neg \phi} \neg i$	$\frac{\phi \quad \neg \phi}{\perp} \neg e$
$\perp$	(no introduction rule for $\perp$ )	$\frac{\perp}{\phi} \perp e$
$\neg\neg$		$\frac{\neg\neg \phi}{\phi} \neg\neg e$

Some useful derived rules:

$$\frac{\phi \rightarrow \psi \quad \neg \psi}{\neg \phi} \text{ MT}$$

$$\frac{\phi}{\neg\neg \phi} \neg\neg i$$

$$\frac{\boxed{\begin{array}{c} \neg \phi \\ \vdots \\ \perp \end{array}}}{\phi} \text{ PBC}$$

$$\frac{\phi \vee \neg \phi}{\phi \vee \neg \phi} \text{ LEM}$$

Figure 1.2. Natural deduction rules for propositional logic.