3

Get a relation for θ_T:

- θ_T is the angle between the tangent and the x-axis, hence $\tan\theta_T = \frac{\partial f}{\partial x}$. (from that can get the $f(x)$)

Use (4), (5), (6):

$$\begin{cases} n_a \sin\theta_1 = n_p \sin\theta_2 \\ \theta_1 = -\frac{\pi}{2} + \theta_T + \theta_o \\ \theta_2 = \theta_T + \theta_c - \frac{\pi}{2} \end{cases}$$

, let $n_1 = n_{air} = n_a$
$n_2 = n_{prism} = n_p$

$$\Rightarrow \frac{n_a}{n_p}\left[\sin\left(-\frac{\pi}{2} + \theta_T + \theta_o\right)\right] = \sin\left(\theta_T + \theta_c - \frac{\pi}{2}\right)$$

Use $\sin(A+B) = \sin A \cos B + \sin B \cos A$ in both sides.

Trigonometry

$$\Rightarrow \frac{n_a}{n_p}\left[\underbrace{\sin\left(\theta_o - \frac{\pi}{2}\right)}_{-\cos(\theta_o)} \cos\theta_T + \sin\theta_T \underbrace{\cos\left(\theta_o - \frac{\pi}{2}\right)}_{\sin\theta_o}\right] =$$

$$\left[\sin\theta_T \underbrace{\cos\left(\theta_c - \frac{\pi}{2}\right)}_{\sin\theta_c} + \underbrace{\sin\left(\theta_c - \frac{\pi}{2}\right)}_{-\cos(\theta_c)} \cos\theta_T\right]$$

$$\Rightarrow \frac{n_a}{n_p}\left[-\cos\theta_o \cos\theta_T + \sin\theta_T \sin\theta_o\right] = \sin\theta_T \sin\theta_c - \cos\theta_c \cos\theta_T$$

$$\Rightarrow \cos\theta_T\left[-\frac{n_a}{n_p}\cos\theta_o + \cos\theta_c\right] = \sin\theta_T\left[\sin\theta_c - \frac{n_a}{n_p}\sin\theta_o\right]$$

$$\Rightarrow \boxed{\tan\theta_T = \frac{\cos\theta_c - \frac{n_a}{n_p}\cos\theta_o}{\sin\theta_c - \frac{n_a}{n_p}\sin\theta_o}} \quad (7).$$