Let m_x, m_y be the minimum and M_x, M_y be the maximum x and y values of the target area respectively. It is assumed than $m_x, M_x > 0$ and $m_y, M_y < 0$.

With initial y velocity j, after n steps, the y position of the probe, y, is given by

$$
\begin{aligned}
y &= \sum_{r=0}^{n-1}(j-r) \\
&= nj - \sum_{r=0}^{n-1} r \\
&= nj - \frac{n(n-1)}{2} \\
&= \frac{-n^2 + n(2j+1)}{2}
\end{aligned}
$$

The time at which this intersects the target area is given by

$$
\begin{aligned}
& m_y \leq y \leq M_y \\
\iff & \frac{2j+1-\sqrt{(2j+1)^2 - 8m_y}}{2} \\
& \leq y \leq \frac{2j+1-\sqrt{(2j+1)^2 - 8M_y}}{2}
\end{aligned}
$$

With initial x velocity $i \geq 0$, the x position after n steps, x, is given by

$$
\begin{aligned}
x &= \begin{cases} \sum_{r=0}^{n-1}(i-r) & n \leq i \\ \sum_{r=0}^{i-1}(i-r) & n > i \end{cases} \\
&= \begin{cases} \sum_{r=0}^{n-1}(i-r) & n \leq i \\ \sum_{r=1}^{i} r & n > i \end{cases} \\
&= \begin{cases} \frac{-n^2+(2i+1)n}{2} & n \leq i \\ \frac{i^2+i}{2} & n > i \end{cases}
\end{aligned}
$$

$$
\begin{aligned}
& m_x \leq \frac{-n^2 + (2i+1)n}{2} \leq M_x \\
\implies & 2m_x \leq -n^2 + 2in + n \leq 2M_x \\
\implies & \frac{2m_x}{n} \leq -n + 2i + 1 \leq \frac{2M_x}{n} \\
\implies & \frac{2m_x}{n} + n - 1 \leq 2i \leq \frac{2M_x}{n} + n - 1 \\
\implies & \frac{m_x}{n} + \frac{n-1}{2} \leq i \leq \frac{M_x}{n} + \frac{n-1}{2}
\end{aligned}
$$

$$
\begin{aligned}
& m_x \leq \frac{n^2+n}{2} \leq M_x \\
\implies & 2m_x \leq n^2+n \leq 2M_x \\
\implies & 2m_x \leq (n+\frac{1}{2})^2 - \frac{1}{4} \leq 2M_x \\
\implies & -\frac{1}{2} + \sqrt{2m_x + \frac{1}{4}} \leq \\
& n \leq -\frac{1}{2} + \sqrt{2M_x + \frac{1}{4}}
\end{aligned}
$$