Let m_x, m_y be the minimum and M_x, M_y be the maximum x and y values of the target area respectively. It is assumed than $m_x, M_x > 0$ and $m_y, M_y < 0$.

With initial y velocity j, after n steps, the y position of the probe, y, is given by

$$\begin{aligned} y &= \sum_{r=0}^{n-1}(j-r) \\ &= nj - \sum_{r=0}^{n-1} r \\ &= nj - \frac{n(n-1)}{2} \\ &= \frac{-n^2 + n(2j+1)}{2} \end{aligned}$$

The time at which this intersects the target area is given by

$$\begin{aligned} & m_y \leq y \leq M_y \\ \iff & m_y \leq \frac{-n^2 + n(2j+1)}{2} \leq M_y \\ \iff & 2m_y \leq -n^2 + n(2j+1) \leq 2M_y \\ \iff & 2m_y \leq -(n - \frac{2j+1}{2})^2 + (\frac{2j+1}{2})^2 \leq 2M_y \\ \iff & 2m_y - (\frac{2j+1}{2})^2 \leq -(n - \frac{2j+1}{2})^2 \leq 2M_y - (\frac{2j+1}{2})^2 \\ \iff & (\frac{2j+1}{2})^2 - 2m_y \geq (n - \frac{2j+1}{2})^2 \geq (\frac{2j+1}{2})^2 - 2M_y \\ \iff & \sqrt{(\frac{2j+1}{2})^2 - 2m_y} \geq n - \frac{2j+1}{2} \geq \sqrt{(\frac{2j+1}{2})^2 - 2M_y} \\ \iff & \frac{2j+1}{2} + \sqrt{(\frac{2j+1}{2})^2 - 2m_y} \geq n \geq \frac{2j+1}{2} + \sqrt{(\frac{2j+1}{2})^2 - 2M_y} \end{aligned}$$

With initial x velocity $i \geq 0$, the x position after n steps, x, is given by

$$\begin{aligned} x &= \begin{cases} \sum_{r=0}^{n-1}(i-r) & n \leq i \\ \sum_{r=0}^{i-1}(i-r) & n > i \end{cases} \\ &= \begin{cases} \sum_{r=1}^{i} r - \sum_{r=1}^{n-1} r & n \leq i \\ \sum_{r=1}^{i} r & n > i \end{cases} \\ &= \begin{cases} \frac{i^2+i}{2} - \frac{n^2-n}{2} & n \leq i \\ \frac{i^2+i}{2} & n > i \end{cases} \\ &= \begin{cases} \frac{(i+n)(i-n+1)}{2} & n \leq i \\ \frac{i^2+i}{2} & n > i \end{cases} \end{aligned}$$