Telecom Customer Churn

Introduction

In this report, Kaplan-Meier estimate and Cox proportional hazards method are used on the dataset churn available in package BDgraph contains the length of time the customer was with the company (Account.Length), whether they churned (Churn) and a number of other covariates for 3333 customers.

The `churn` dataset, as a data frame, contains *3333* rows (customers) and *20* columns (variables/features). The *20* variables are:

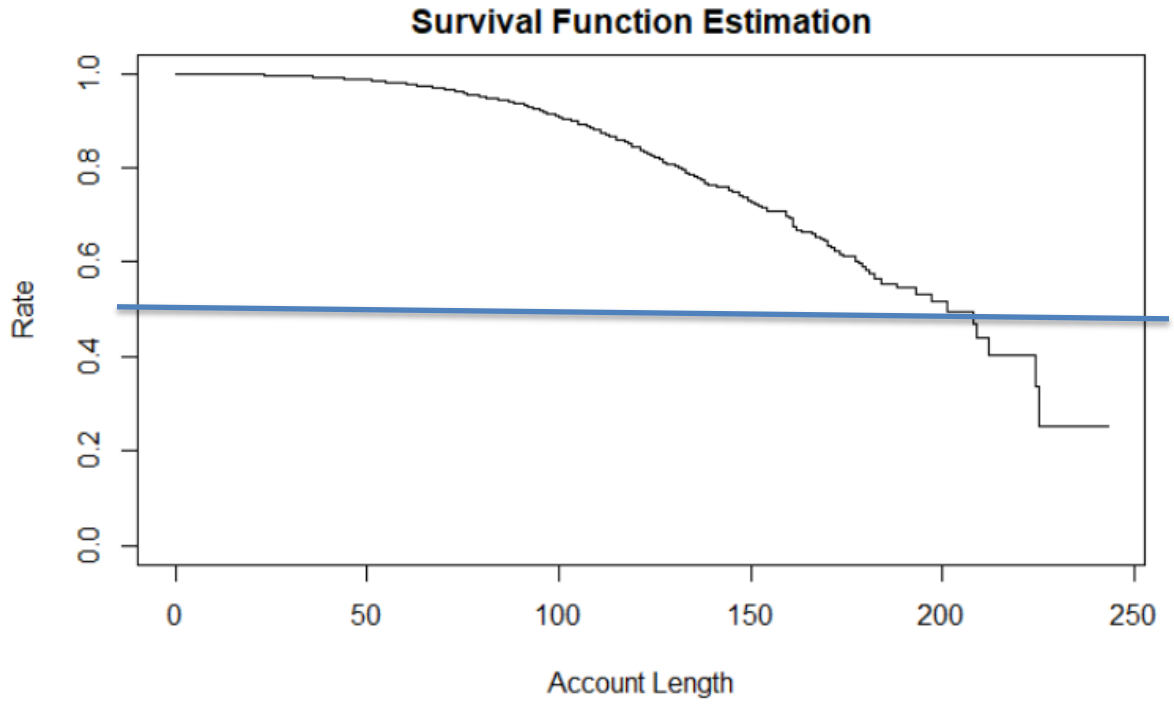
- `State`: Categorical, for the *50* states and the District of Columbia.
- `Account.Length`: count, how long account has been active.
- `Area.Code`: Categorical.
- `Int.l.Plan`: Categorical, yes or no, international plan.
- `VMail.Plan`: Categorical, yes or no, voice mail plan.
- `VMail.Message`: Count, number of voice mail messages.
- `Day.Mins`: Continuous, minutes customer used service during the day.
- `Day.Calls`: Count, total number of calls during the day.
- `Day.Charge`: Continuous, total charge during the day.
- `Eve.Mins`: Continuous, minutes customer used service during the evening.
- `Eve.Calls`: Count, total number of calls during the evening.
- `Eve.Charge`: Continuous, total charge during the evening.
- `Night.Mins`: Continuous, minutes customer used service during the night.
- `Night.Calls`: Count, total number of calls during the night.
- `Night.Charge`: Continuous, total charge during the night.
- `Intl.Mins`: Continuous, minutes customer used service to make international calls.
- `Intl.Calls`: Count, total number of international calls.
- `Intl.Charge`: Continuous, total international charge.
- `CustServ.Calls`: Count, number of calls to customer service.
- `Churn`: Categorical, True or False. Indicator of whether the customer has left the company (True or False).

The next section is the detail of both methods and survival analysis for the data.

Fitting Both Models

(a) Construct a Kaplan-Meier estimate of the survival function for time-to-churn and plot the estimate of the survival function.

```
# Fit Kaplan-Meier Curve
churn$Churn=ifelse(churn$Churn=="True",1,0)
# Set up Surv() object
survdat <- Surv(time=churn[,2],event=churn[,20])
# Fit the model to the data
fit <- survfit(survdat~1,se=FALSE)
#summary(fit)
plot(fit, main= "Survival Function Estimation", xlab= "Account Length", ylab= "Rate")
```



From the plot, we can see the survival rate is decreasing as the account length is increasing.

(b) Estimate the median time-to-churn.

The central value of the time to churn is 201. Furthermore, the previous plot shows that from account length around 200, the survival rate starts to grow smaller than 50%. Means that if the account length is more than 201, the company needs to do something to keep the customer stays.

```
print(fit)
```

```
## Call: survfit(formula = survdat ~ 1, se = FALSE)
##
##       n events median
##    3333    483    201
```

(c) Use the Cox proportional hazards method to study the impact of the customer covariates on the time-to-churn and develop a parsimonious model.

```
# Fit the Cox PH model to the data
fit <- coxph(Surv(Account.Length,Churn)~.,data=churn)

summary(fit)

## Call:
## coxph(formula = Surv(Account.Length, Churn) ~ ., data = churn)
##
##   n= 3333, number of events= 483
##
##                     coef  exp(coef)   se(coef)      z Pr(>|z|)
## StateAL         3.727e-01  1.452e+00  6.815e-01  0.547 0.584509
## StateAR         7.263e-01  2.067e+00  6.578e-01  1.104 0.269536
## StateAZ         3.591e-01  1.432e+00  7.679e-01  0.468 0.640011
## StateCA         1.501e+00  4.484e+00  6.710e-01  2.236 0.025333 *
## StateCO         3.684e-01  1.445e+00  6.714e-01  0.549 0.583169
## StateCT         9.994e-01  2.717e+00  6.487e-01  1.541 0.123376
## StateDC         5.992e-01  1.821e+00  7.352e-01  0.815 0.415027
## StateDE         9.039e-01  2.469e+00  6.734e-01  1.342 0.179495
## StateFL         6.055e-01  1.832e+00  6.818e-01  0.888 0.374456
## StateGA         7.639e-01  2.147e+00  6.832e-01  1.118 0.263471
## StateHI        -1.553e-01  8.562e-01  8.200e-01 -0.189 0.849830
## StateIA         6.270e-01  1.872e+00  8.220e-01  0.763 0.445642
## StateID         9.225e-01  2.516e+00  6.724e-01  1.372 0.170049
## StateIL         2.504e-01  1.285e+00  7.335e-01  0.341 0.732838
## StateIN         8.284e-01  2.290e+00  6.709e-01  1.235 0.216887
## StateKS         9.738e-01  2.648e+00  6.441e-01  1.512 0.130580
## StateKY         1.377e+00  3.965e+00  6.829e-01  2.017 0.043688 *
## StateLA         4.333e-01  1.542e+00  7.687e-01  0.564 0.572958
## StateMA         1.335e+00  3.802e+00  6.545e-01  2.040 0.041311 *
## StateMD         9.443e-01  2.571e+00  6.314e-01  1.496 0.134760
## StateME         1.257e+00  3.515e+00  6.487e-01  1.938 0.052649 .
## StateMI         1.252e+00  3.497e+00  6.344e-01  1.973 0.048456 *
## StateMN         5.042e-01  1.656e+00  6.498e-01  0.776 0.437808
## StateMO         4.839e-01  1.622e+00  6.955e-01  0.696 0.486547
## StateMS         1.210e+00  3.353e+00  6.414e-01  1.886 0.059262 .
## StateMT         1.612e+00  5.013e+00  6.418e-01  2.512 0.012010 *
## StateNC         1.687e-01  1.184e+00  6.618e-01  0.255 0.798794
## StateND         1.417e-01  1.152e+00  7.112e-01  0.199 0.842124
## StateNE         3.729e-01  1.452e+00  7.373e-01  0.506 0.613038
## StateNH         1.024e+00  2.784e+00  6.713e-01  1.525 0.127203
## StateNJ         1.453e+00  4.275e+00  6.297e-01  2.307 0.021041 *
## StateNM         3.356e-01  1.399e+00  7.127e-01  0.471 0.637719
## StateNV         1.187e+00  3.278e+00  6.396e-01  1.856 0.063450 .
## StateNY         7.184e-01  2.051e+00  6.377e-01  1.127 0.259918
## StateOH         8.039e-01  2.234e+00  6.629e-01  1.213 0.225212
## StateOK         6.394e-01  1.895e+00  6.744e-01  0.948 0.343108
## StateOR         7.061e-01  2.026e+00  6.564e-01  1.076 0.282024
## StatePA         1.153e+00  3.168e+00  6.791e-01  1.698 0.089500 .
## StateRI         2.414e-01  1.273e+00  7.100e-01  0.340 0.733853
## StateSC         1.166e+00  3.210e+00  6.472e-01  1.802 0.071541 .
```

```
## StateSD          7.118e-01  2.038e+00  6.808e-01  1.046 0.295734
## StateTN          8.526e-01  2.346e+00  7.342e-01  1.161 0.245514
## StateTX          1.101e+00  3.007e+00  6.295e-01  1.749 0.080331 .
## StateUT          8.669e-01  2.379e+00  6.628e-01  1.308 0.190896
## StateVA         -3.732e-01  6.885e-01  7.339e-01 -0.508 0.611116
## StateVT          3.576e-01  1.430e+00  6.831e-01  0.524 0.600624
## StateWA          1.180e+00  3.255e+00  6.409e-01  1.842 0.065517 .
## StateWI          4.867e-01  1.627e+00  6.941e-01  0.701 0.483203
## StateWV          6.826e-01  1.979e+00  6.619e-01  1.031 0.302395
## StateWY          2.866e-01  1.332e+00  6.710e-01  0.427 0.669252
## Area.Code        4.925e-04  1.000e+00  1.105e-03  0.446 0.655668
## Int.l.Planyes    1.329e+00  3.777e+00  1.094e-01 12.152  < 2e-16 ***
## VMail.Planyes   -2.304e+00  9.988e-02  5.165e-01 -4.460 8.19e-06 ***
## VMail.Message    4.801e-02  1.049e+00  1.590e-02  3.020 0.002527 **
## Day.Mins        -3.636e-01  6.952e-01  2.727e+00 -0.133 0.893932
## Day.Calls        1.262e-03  1.001e+00  2.299e-03  0.549 0.582966
## Day.Charge       2.193e+00  8.959e+00  1.604e+01  0.137 0.891270
## Eve.Mins         3.242e-01  1.383e+00  1.415e+00  0.229 0.818755
## Eve.Calls        1.872e-03  1.002e+00  2.413e-03  0.776 0.437909
## Eve.Charge      -3.759e+00  2.332e-02  1.665e+01 -0.226 0.821364
## Night.Mins      -3.205e-01  7.258e-01  7.682e-01 -0.417 0.676546
## Night.Calls      1.513e-03  1.002e+00  2.457e-03  0.616 0.537914
## Night.Charge     7.186e+00  1.320e+03  1.707e+01  0.421 0.673777
## Intl.Mins       -4.607e+00  9.980e-03  4.510e+00 -1.021 0.307019
## Intl.Calls      -7.837e-02  9.246e-01  2.179e-02 -3.596 0.000323 ***
## Intl.Charge      1.724e+01  3.081e+07  1.670e+01  1.032 0.301906
## CustServ.Calls   3.318e-01  1.393e+00  2.947e-02 11.257  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Concordance= 0.786  (se = 0.011 )
## Likelihood ratio test= 523.6  on 67 df,   p=<2e-16
## Wald test            = 508.3  on 67 df,   p=<2e-16
## Score (logrank) test = 566.6  on 67 df,   p=<2e-16
```

The model is significant based on the overall tests (likelihood ratio, Wald and score tests).

In the multivariate Cox analysis, the covariates State, Int.l.Planyes, VMail.Planyes, VMail.Message, Intl.Calls and CustServ.Calls remain significant ($p < 0.05$). However, the other covariates fail to be significant.

Now, let's study further the covariates (note that each state is not further explained)

First, covariate state is important by seeing its significance. Then, for the covariates with positive coef such as Int.l.Planyes, VMail.Message, and CustServ.Calls; They have ex(coef) less than 1, means they contribute to increase the hazard and decrease the survival rate or time to churn. Having

Meanwhile, for the covariates with negative coef such as VMail.Planyes and Intl.Calls; They have ex(coef) more than 1, means they contribute to decrease the hazard and increase the survival rate or time to churn.

The comment below the chosen categorical covariates will show what will happen to survival rate or time to churn when other covariates are constant.

```
##                    coef  exp(coef)   se(coef)      z Pr(>|z|)
## Int.l.Planyes   1.329e+00  3.777e+00  1.094e-01 12.152  < 2e-16 ***
```

Having international plan will increase the hazard ratio and decrease the survival rate

```
##                       coef  exp(coef)   se(coef)      z Pr(>|z|)
## VMail.Planyes  -2.304e+00  9.988e-02  5.165e-01 -4.460 8.19e-06 ***
```
Having voice mail plan will decrease the hazard ratio and increase the survival rate

And for numerical covariates:
```
##                       coef  exp(coef)   se(coef)      z Pr(>|z|)
## VMail.Message   4.801e-02  1.049e+00  1.590e-02  3.020 0.002527 **
## CustServ.Calls  3.318e-01  1.393e+00  2.947e-02 11.257  < 2e-16 ***
```

The increasing one unit of VMail.Message or CustServ.Calls will increase the hazard ratio and decrease the survival rate
```
##                       coef  exp(coef)   se(coef)      z Pr(>|z|)
## Intl.Calls     -7.837e-02  9.246e-01  2.179e-02 -3.596 0.000323 ***
```
The increasing one unit of Intl.Calls will decrease the hazard ratio and increase the survival rate

Develop a parsimonious model with step function

```
obj<-step(fit);summary(obj)
```

```
## Call:
## coxph(formula = Surv(Account.Length, Churn) ~ Int.l.Plan + VMail.Plan +
##     VMail.Message + Day.Charge + Eve.Mins + Night.Charge + Intl.Calls +
##     Intl.Charge + CustServ.Calls, data = churn)
##
##   n= 3333, number of events= 483
##
##                       coef  exp(coef)   se(coef)      z Pr(>|z|)
## Int.l.Planyes   1.1903028  3.2880768  0.1028263 11.576  < 2e-16 ***
## VMail.Planyes  -2.1293515  0.1189144  0.4783699 -4.451 8.54e-06 ***
## VMail.Message   0.0432333  1.0441815  0.0147539  2.930 0.003386 **
## Day.Charge      0.0540627  1.0555507  0.0050891 10.623  < 2e-16 ***
## Eve.Mins        0.0041883  1.0041971  0.0008873  4.720 2.35e-06 ***
## Night.Charge    0.0684319  1.0708277  0.0198433  3.449 0.000563 ***
## Intl.Calls     -0.0701161  0.9322856  0.0209824 -3.342 0.000833 ***
## Intl.Charge     0.2025062  1.2244677  0.0615924  3.288 0.001010 **
## CustServ.Calls  0.3037545  1.3549364  0.0271930 11.170  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##                exp(coef) exp(-coef) lower .95 upper .95
## Int.l.Planyes     3.2881     0.3041   2.68792    4.0222
## VMail.Planyes     0.1189     8.4094   0.04656    0.3037
## VMail.Message     1.0442     0.9577   1.01442    1.0748
## Day.Charge        1.0556     0.9474   1.04507    1.0661
## Eve.Mins          1.0042     0.9958   1.00245    1.0059
## Night.Charge      1.0708     0.9339   1.02998    1.1133
## Intl.Calls        0.9323     1.0726   0.89472    0.9714
## Intl.Charge       1.2245     0.8167   1.08522    1.3816
## CustServ.Calls    1.3549     0.7380   1.28461    1.4291
##
## Concordance= 0.772  (se = 0.012 )
## Likelihood ratio test= 440.4  on 9 df,   p=<2e-16
## Wald test            = 469.1  on 9 df,   p=<2e-16
## Score (logrank) test = 497  on 9 df,   p=<2e-16
```

The new model is significant based on the overall tests (likelihood ratio, Wald and score tests).

In the multivariate Cox analysis, the covariates Int.l.Planyes, VMail.Planyes, VMail.Message, Day.Charge, Eve.Mins, Night.Charge, Intl.Calls, Intl.Charge and CustServ.Calls remain significant ($p < 0.05$).

The comment below the chosen categorical covariates will show what will happen to survival rate or time to churn when other covariates are constant.

```
##                      coef  exp(coef)   se(coef)      z Pr(>|z|)
## Int.l.Planyes   1.1903028  3.2880768  0.1028263 11.576  < 2e-16 ***
```

Having international plan will increase the hazard ratio and decrease the survival rate

```
##                      coef  exp(coef)   se(coef)      z Pr(>|z|)
## VMail.Planyes  -2.1293515  0.1189144  0.4783699 -4.451 8.54e-06 ***
```

Having voice mail plan will decrease the hazard ratio and increase the survival rate

And for numerical covariates:

```
##                      coef  exp(coef)   se(coef)      z Pr(>|z|)

## VMail.Message   0.0432333  1.0441815  0.0147539  2.930 0.003386 **
## Day.Charge      0.0540627  1.0555507  0.0050891 10.623  < 2e-16 ***
## Eve.Mins        0.0041883  1.0041971  0.0008873  4.720 2.35e-06 ***
## Night.Charge    0.0684319  1.0708277  0.0198433  3.449 0.000563 ***
## Intl.Charge     0.2025062  1.2244677  0.0615924  3.288 0.001010 **
## CustServ.Calls  0.3037545  1.3549364  0.0271930 11.170  < 2e-16 ***
```

The increasing one unit of covariate VMail.Message, Day.Charge, Eve.Mins, Night.Charge, Intl.Calls, Intl.Charge or CustServ.Calls will increase the hazard ratio and decrease the survival rate

```
##                      coef  exp(coef)   se(coef)      z Pr(>|z|)
## Intl.Calls     -0.0701161  0.9322856  0.0209824 -3.342 0.000833 ***
```

The increasing one unit of Intl.Calls will decrease the hazard ratio and increase the survival rate

(d) Summary of analysis.

To increase the survival rate, the company should prepare the long-term plan for the customer. So, when the account length is reaching 200 and growing from that length, the customer will stay.

Moreover, the modification of international plan product, should be made to tackle the reason why it decreases the time to churn. While the voice mail plan should be maintained since the customer who has it tends to stay.

Also, based on the result, the number of voice mail message, total charge during the day, minutes customer used service during the evening, total charge during the night, total number of international calls, total international charge, and number of calls to customer services are decreasing the time to churn. Then, I suggest the company should reassess their product's price related with charges or maybe what are the issues whenever the customer calls. On the other hand, the total number of international calls will increase the time to churn, the quality of this service should be maintained.

Appendices

```
# Fit the Cox PH model to the data
fit <- coxph(Surv(Account.Length,Churn)~.,data=churn)

summary(fit)

## Call:
## coxph(formula = Surv(Account.Length, Churn) ~ ., data = churn)
##
##   n= 3333, number of events= 483
##
##                      coef  exp(coef)   se(coef)      z Pr(>|z|)
## StateAL         3.727e-01  1.452e+00  6.815e-01  0.547 0.584509
## StateAR         7.263e-01  2.067e+00  6.578e-01  1.104 0.269536
## StateAZ         3.591e-01  1.432e+00  7.679e-01  0.468 0.640011
## StateCA         1.501e+00  4.484e+00  6.710e-01  2.236 0.025333 *
## StateCO         3.684e-01  1.445e+00  6.714e-01  0.549 0.583169
## StateCT         9.994e-01  2.717e+00  6.487e-01  1.541 0.123376
## StateDC         5.992e-01  1.821e+00  7.352e-01  0.815 0.415027
## StateDE         9.039e-01  2.469e+00  6.734e-01  1.342 0.179495
## StateFL         6.055e-01  1.832e+00  6.818e-01  0.888 0.374456
## StateGA         7.639e-01  2.147e+00  6.832e-01  1.118 0.263471
## StateHI        -1.553e-01  8.562e-01  8.200e-01 -0.189 0.849830
## StateIA         6.270e-01  1.872e+00  8.220e-01  0.763 0.445642
## StateID         9.225e-01  2.516e+00  6.724e-01  1.372 0.170049
## StateIL         2.504e-01  1.285e+00  7.335e-01  0.341 0.732838
## StateIN         8.284e-01  2.290e+00  6.709e-01  1.235 0.216887
## StateKS         9.738e-01  2.648e+00  6.441e-01  1.512 0.130580
## StateKY         1.377e+00  3.965e+00  6.829e-01  2.017 0.043688 *
## StateLA         4.333e-01  1.542e+00  7.687e-01  0.564 0.572958
## StateMA         1.335e+00  3.802e+00  6.545e-01  2.040 0.041311 *
## StateMD         9.443e-01  2.571e+00  6.314e-01  1.496 0.134760
## StateME         1.257e+00  3.515e+00  6.487e-01  1.938 0.052649 .
## StateMI         1.252e+00  3.497e+00  6.344e-01  1.973 0.048456 *
## StateMN         5.042e-01  1.656e+00  6.498e-01  0.776 0.437808
## StateMO         4.839e-01  1.622e+00  6.955e-01  0.696 0.486547
## StateMS         1.210e+00  3.353e+00  6.414e-01  1.886 0.059262 .
## StateMT         1.612e+00  5.013e+00  6.418e-01  2.512 0.012010 *
## StateNC         1.687e-01  1.184e+00  6.618e-01  0.255 0.798794
## StateND         1.417e-01  1.152e+00  7.112e-01  0.199 0.842124
## StateNE         3.729e-01  1.452e+00  7.373e-01  0.506 0.613038
## StateNH         1.024e+00  2.784e+00  6.713e-01  1.525 0.127203
## StateNJ         1.453e+00  4.275e+00  6.297e-01  2.307 0.021041 *
## StateNM         3.356e-01  1.399e+00  7.127e-01  0.471 0.637719
## StateNV         1.187e+00  3.278e+00  6.396e-01  1.856 0.063450 .
## StateNY         7.184e-01  2.051e+00  6.377e-01  1.127 0.259918
## StateOH         8.039e-01  2.234e+00  6.629e-01  1.213 0.225212
## StateOK         6.394e-01  1.895e+00  6.744e-01  0.948 0.343108
## StateOR         7.061e-01  2.026e+00  6.564e-01  1.076 0.282024
## StatePA         1.153e+00  3.168e+00  6.791e-01  1.698 0.089500 .
## StateRI         2.414e-01  1.273e+00  7.100e-01  0.340 0.733853
## StateSC         1.166e+00  3.210e+00  6.472e-01  1.802 0.071541 .
## StateSD         7.118e-01  2.038e+00  6.808e-01  1.046 0.295734
```

```
## StateTN          8.526e-01  2.346e+00  7.342e-01  1.161 0.245514
## StateTX          1.101e+00  3.007e+00  6.295e-01  1.749 0.080331 .
## StateUT          8.669e-01  2.379e+00  6.628e-01  1.308 0.190896
## StateVA         -3.732e-01  6.885e-01  7.339e-01 -0.508 0.611116
## StateVT          3.576e-01  1.430e+00  6.831e-01  0.524 0.600624
## StateWA          1.180e+00  3.255e+00  6.409e-01  1.842 0.065517 .
## StateWI          4.867e-01  1.627e+00  6.941e-01  0.701 0.483203
## StateWV          6.826e-01  1.979e+00  6.619e-01  1.031 0.302395
## StateWY          2.866e-01  1.332e+00  6.710e-01  0.427 0.669252
## Area.Code        4.925e-04  1.000e+00  1.105e-03  0.446 0.655668
## Int.l.Planyes    1.329e+00  3.777e+00  1.094e-01 12.152  < 2e-16 ***
## VMail.Planyes   -2.304e+00  9.988e-02  5.165e-01 -4.460 8.19e-06 ***
## VMail.Message    4.801e-02  1.049e+00  1.590e-02  3.020 0.002527 **
## Day.Mins        -3.636e-01  6.952e-01  2.727e+00 -0.133 0.893932
## Day.Calls        1.262e-03  1.001e+00  2.299e-03  0.549 0.582966
## Day.Charge       2.193e+00  8.959e+00  1.604e+01  0.137 0.891270
## Eve.Mins         3.242e-01  1.383e+00  1.415e+00  0.229 0.818755
## Eve.Calls        1.872e-03  1.002e+00  2.413e-03  0.776 0.437909
## Eve.Charge      -3.759e+00  2.332e-02  1.665e+01 -0.226 0.821364
## Night.Mins      -3.205e-01  7.258e-01  7.682e-01 -0.417 0.676546
## Night.Calls      1.513e-03  1.002e+00  2.457e-03  0.616 0.537914
## Night.Charge     7.186e+00  1.320e+03  1.707e+01  0.421 0.673777
## Intl.Mins       -4.607e+00  9.980e-03  4.510e+00 -1.021 0.307019
## Intl.Calls      -7.837e-02  9.246e-01  2.179e-02 -3.596 0.000323 ***
## Intl.Charge      1.724e+01  3.081e+07  1.670e+01  1.032 0.301906
## CustServ.Calls   3.318e-01  1.393e+00  2.947e-02 11.257  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##                exp(coef) exp(-coef) lower .95 upper .95
## StateAL        1.452e+00  6.889e-01 3.817e-01 5.520e+00
## StateAR        2.067e+00  4.837e-01 5.695e-01 7.505e+00
## StateAZ        1.432e+00  6.983e-01 3.180e-01 6.450e+00
## StateCA        4.484e+00  2.230e-01 1.204e+00 1.670e+01
## StateCO        1.445e+00  6.918e-01 3.877e-01 5.389e+00
## StateCT        2.717e+00  3.681e-01 7.619e-01 9.687e+00
## StateDC        1.821e+00  5.492e-01 4.310e-01 7.692e+00
## StateDE        2.469e+00  4.050e-01 6.597e-01 9.241e+00
## StateFL        1.832e+00  5.458e-01 4.816e-01 6.971e+00
## StateGA        2.147e+00  4.658e-01 5.627e-01 8.190e+00
## StateHI        8.562e-01  1.168e+00 1.716e-01 4.272e+00
## StateIA        1.872e+00  5.342e-01 3.737e-01 9.376e+00
## StateID        2.516e+00  3.975e-01 6.735e-01 9.396e+00
## StateIL        1.285e+00  7.785e-01 3.050e-01 5.409e+00
## StateIN        2.290e+00  4.367e-01 6.148e-01 8.527e+00
## StateKS        2.648e+00  3.776e-01 7.493e-01 9.358e+00
## StateKY        3.965e+00  2.522e-01 1.040e+00 1.512e+01
## StateLA        1.542e+00  6.484e-01 3.419e-01 6.958e+00
## StateMA        3.802e+00  2.630e-01 1.054e+00 1.371e+01
## StateMD        2.571e+00  3.890e-01 7.459e-01 8.862e+00
## StateME        3.515e+00  2.845e-01 9.857e-01 1.254e+01
## StateMI        3.497e+00  2.860e-01 1.009e+00 1.212e+01
## StateMN        1.656e+00  6.040e-01 4.633e-01 5.917e+00
## StateMO        1.622e+00  6.164e-01 4.151e-01 6.341e+00
## StateMS        3.353e+00  2.982e-01 9.538e-01 1.179e+01
```

```
## StateMT        5.013e+00  1.995e-01 1.425e+00 1.764e+01
## StateNC        1.184e+00  8.448e-01 3.236e-01 4.331e+00
## StateND        1.152e+00  8.679e-01 2.858e-01 4.645e+00
## StateNE        1.452e+00  6.888e-01 3.423e-01 6.159e+00
## StateNH        2.784e+00  3.592e-01 7.469e-01 1.038e+01
## StateNJ        4.275e+00  2.339e-01 1.244e+00 1.469e+01
## StateNM        1.399e+00  7.149e-01 3.460e-01 5.655e+00
## StateNV        3.278e+00  3.051e-01 9.357e-01 1.148e+01
## StateNY        2.051e+00  4.875e-01 5.878e-01 7.158e+00
## StateOH        2.234e+00  4.476e-01 6.094e-01 8.192e+00
## StateOK        1.895e+00  5.276e-01 5.054e-01 7.107e+00
## StateOR        2.026e+00  4.936e-01 5.597e-01 7.334e+00
## StatePA        3.168e+00  3.156e-01 8.370e-01 1.199e+01
## StateRI        1.273e+00  7.855e-01 3.166e-01 5.119e+00
## StateSC        3.210e+00  3.115e-01 9.028e-01 1.142e+01
## StateSD        2.038e+00  4.908e-01 5.366e-01 7.737e+00
## StateTN        2.346e+00  4.263e-01 5.563e-01 9.891e+00
## StateTX        3.007e+00  3.326e-01 8.755e-01 1.033e+01
## StateUT        2.379e+00  4.203e-01 6.491e-01 8.723e+00
## StateVA        6.885e-01  1.452e+00 1.634e-01 2.902e+00
## StateVT        1.430e+00  6.994e-01 3.749e-01 5.454e+00
## StateWA        3.255e+00  3.072e-01 9.270e-01 1.143e+01
## StateWI        1.627e+00  6.147e-01 4.174e-01 6.341e+00
## StateWV        1.979e+00  5.053e-01 5.408e-01 7.242e+00
## StateWY        1.332e+00  7.508e-01 3.576e-01 4.961e+00
## Area.Code      1.000e+00  9.995e-01 9.983e-01 1.003e+00
## Int.l.Planyes  3.777e+00  2.647e-01 3.048e+00 4.680e+00
## VMail.Planyes  9.988e-02  1.001e+01 3.629e-02 2.749e-01
## VMail.Message  1.049e+00  9.531e-01 1.017e+00 1.082e+00
## Day.Mins       6.952e-01  1.438e+00 3.318e-03 1.456e+02
## Day.Calls      1.001e+00  9.987e-01 9.968e-01 1.006e+00
## Day.Charge     8.959e+00  1.116e-01 1.988e-13 4.037e+14
## Eve.Mins       1.383e+00  7.231e-01 8.638e-02 2.214e+01
## Eve.Calls      1.002e+00  9.981e-01 9.971e-01 1.007e+00
## Eve.Charge     2.332e-02  4.289e+01 1.578e-16 3.446e+12
## Night.Mins     7.258e-01  1.378e+00 1.610e-01 3.271e+00
## Night.Calls    1.002e+00  9.985e-01 9.967e-01 1.006e+00
## Night.Charge   1.320e+03  7.573e-04 3.901e-12 4.470e+17
## Intl.Mins      9.980e-03  1.002e+02 1.446e-06 6.890e+01
## Intl.Calls     9.246e-01  1.082e+00 8.860e-01 9.650e-01
## Intl.Charge    3.081e+07  3.246e-08 1.867e-07 5.083e+21
## CustServ.Calls 1.393e+00  7.177e-01 1.315e+00 1.476e+00
##
## Concordance= 0.786  (se = 0.011 )
## Likelihood ratio test= 523.6  on 67 df,   p=<2e-16
## Wald test            = 508.3  on 67 df,   p=<2e-16
## Score (logrank) test = 566.6  on 67 df,   p=<2e-16
```

```
## Start:  AIC=6461.78
## Surv(Account.Length, Churn) ~ State + Area.Code + Int.l.Plan +
##     VMail.Plan + VMail.Message + Day.Mins + Day.Calls + Day.Charge +
##     Eve.Mins + Eve.Calls + Eve.Charge + Night.Mins + Night.Calls +
##     Night.Charge + Intl.Mins + Intl.Calls + Intl.Charge + CustServ.Calls
##
```

```
##                  Df    AIC
## - State          50 6442.3
## - Day.Mins        1 6459.8
## - Day.Charge      1 6459.8
## - Eve.Charge      1 6459.8
## - Eve.Mins        1 6459.8
## - Night.Mins      1 6460.0
## - Night.Charge    1 6460.0
## - Area.Code       1 6460.0
## - Day.Calls       1 6460.1
## - Night.Calls     1 6460.2
## - Eve.Calls       1 6460.4
## - Intl.Mins       1 6460.8
## - Intl.Charge     1 6460.8
## <none>              6461.8
## - VMail.Message   1 6468.8
## - Intl.Calls      1 6473.5
## - VMail.Plan      1 6481.5
## - CustServ.Calls  1 6575.3
## - Int.l.Plan      1 6584.3
##
## Step:  AIC=6442.27
## Surv(Account.Length, Churn) ~ Area.Code + Int.l.Plan + VMail.Plan +
##     VMail.Message + Day.Mins + Day.Calls + Day.Charge + Eve.Mins +
##     Eve.Calls + Eve.Charge + Night.Mins + Night.Calls + Night.Charge +
##     Intl.Mins + Intl.Calls + Intl.Charge + CustServ.Calls
##
##                  Df    AIC
## - Night.Mins      1 6440.3
## - Night.Charge    1 6440.3
## - Day.Mins        1 6440.4
## - Day.Charge      1 6440.4
## - Area.Code       1 6440.4
## - Night.Calls     1 6440.6
## - Day.Calls       1 6440.6
## - Eve.Charge      1 6440.7
## - Eve.Mins        1 6440.7
## - Eve.Calls       1 6440.7
## - Intl.Mins       1 6441.2
## - Intl.Charge     1 6441.3
## <none>              6442.3
## - VMail.Message   1 6448.6
## - Intl.Calls      1 6453.1
## - VMail.Plan      1 6461.6
## - CustServ.Calls  1 6550.7
## - Int.l.Plan      1 6551.4
##
## Step:  AIC=6440.27
## Surv(Account.Length, Churn) ~ Area.Code + Int.l.Plan + VMail.Plan +
##     VMail.Message + Day.Mins + Day.Calls + Day.Charge + Eve.Mins +
##     Eve.Calls + Eve.Charge + Night.Calls + Night.Charge + Intl.Mins +
##     Intl.Calls + Intl.Charge + CustServ.Calls
##
##                  Df    AIC
## - Day.Mins        1 6438.4
## - Day.Charge      1 6438.4
## - Area.Code       1 6438.4
## - Night.Calls     1 6438.6
## - Day.Calls       1 6438.6
## - Eve.Charge      1 6438.7
## - Eve.Mins        1 6438.7
```

```
## - Eve.Calls       1 6438.7
## - Intl.Mins       1 6439.2
## - Intl.Charge     1 6439.3
## <none>              6440.3
## - VMail.Message   1 6446.7
## - Night.Charge    1 6449.9
## - Intl.Calls      1 6451.1
## - VMail.Plan      1 6459.7
## - CustServ.Calls  1 6548.7
## - Int.l.Plan      1 6549.4
##
## Step:  AIC=6438.38
## Surv(Account.Length, Churn) ~ Area.Code + Int.l.Plan + VMail.Plan +
##     VMail.Message + Day.Calls + Day.Charge + Eve.Mins + Eve.Calls +
##     Eve.Charge + Night.Calls + Night.Charge + Intl.Mins + Intl.Calls +
##     Intl.Charge + CustServ.Calls
##
##                  Df    AIC
## - Area.Code       1 6436.5
## - Night.Calls     1 6436.7
## - Day.Calls       1 6436.7
## - Eve.Charge      1 6436.8
## - Eve.Mins        1 6436.8
## - Eve.Calls       1 6436.8
## - Intl.Mins       1 6437.4
## - Intl.Charge     1 6437.4
## <none>              6438.4
## - VMail.Message   1 6444.7
## - Night.Charge    1 6448.1
## - Intl.Calls      1 6449.1
## - VMail.Plan      1 6457.7
## - CustServ.Calls  1 6546.8
## - Int.l.Plan      1 6547.4
## - Day.Charge      1 6550.4
##
## Step:  AIC=6436.53
## Surv(Account.Length, Churn) ~ Int.l.Plan + VMail.Plan + VMail.Message +
##     Day.Calls + Day.Charge + Eve.Mins + Eve.Calls + Eve.Charge +
##     Night.Calls + Night.Charge + Intl.Mins + Intl.Calls + Intl.Charge +
##     CustServ.Calls
##
##                  Df    AIC
## - Night.Calls     1 6434.8
## - Day.Calls       1 6434.9
## - Eve.Charge      1 6434.9
## - Eve.Mins        1 6434.9
## - Eve.Calls       1 6435.0
## - Intl.Mins       1 6435.5
## - Intl.Charge     1 6435.6
## <none>              6436.5
## - VMail.Message   1 6442.9
## - Night.Charge    1 6446.3
## - Intl.Calls      1 6447.2
## - VMail.Plan      1 6455.9
## - CustServ.Calls  1 6544.9
## - Int.l.Plan      1 6546.3
## - Day.Charge      1 6548.5
##
## Step:  AIC=6434.84
## Surv(Account.Length, Churn) ~ Int.l.Plan + VMail.Plan + VMail.Message +
##     Day.Calls + Day.Charge + Eve.Mins + Eve.Calls + Eve.Charge +
```

```
##     Night.Charge + Intl.Mins + Intl.Calls + Intl.Charge + CustServ.Calls
##
##                  Df    AIC
## - Day.Calls       1 6433.2
## - Eve.Charge      1 6433.2
## - Eve.Mins        1 6433.2
## - Eve.Calls       1 6433.3
## - Intl.Mins       1 6433.8
## - Intl.Charge     1 6433.9
## <none>              6434.8
## - VMail.Message   1 6441.2
## - Night.Charge    1 6444.5
## - Intl.Calls      1 6445.5
## - VMail.Plan      1 6454.3
## - CustServ.Calls  1 6542.9
## - Int.l.Plan      1 6545.3
## - Day.Charge      1 6547.2
##
## Step:  AIC=6433.18
## Surv(Account.Length, Churn) ~ Int.l.Plan + VMail.Plan + VMail.Message +
##     Day.Charge + Eve.Mins + Eve.Calls + Eve.Charge + Night.Charge +
##     Intl.Mins + Intl.Calls + Intl.Charge + CustServ.Calls
##
##                  Df    AIC
## - Eve.Calls       1 6431.6
## - Eve.Charge      1 6431.6
## - Eve.Mins        1 6431.6
## - Intl.Mins       1 6432.2
## - Intl.Charge     1 6432.2
## <none>              6433.2
## - VMail.Message   1 6439.3
## - Night.Charge    1 6443.1
## - Intl.Calls      1 6444.0
## - VMail.Plan      1 6452.3
## - CustServ.Calls  1 6541.1
## - Int.l.Plan      1 6543.4
## - Day.Charge      1 6545.8
##
## Step:  AIC=6431.57
## Surv(Account.Length, Churn) ~ Int.l.Plan + VMail.Plan + VMail.Message +
##     Day.Charge + Eve.Mins + Eve.Charge + Night.Charge + Intl.Mins +
##     Intl.Calls + Intl.Charge + CustServ.Calls
##
##                  Df    AIC
## - Eve.Charge      1 6429.9
## - Eve.Mins        1 6430.0
## - Intl.Mins       1 6430.6
## - Intl.Charge     1 6430.6
## <none>              6431.6
## - VMail.Message   1 6437.6
## - Night.Charge    1 6441.5
## - Intl.Calls      1 6442.2
## - VMail.Plan      1 6450.6
## - CustServ.Calls  1 6539.2
## - Int.l.Plan      1 6541.4
## - Day.Charge      1 6544.5
##
## Step:  AIC=6429.95
## Surv(Account.Length, Churn) ~ Int.l.Plan + VMail.Plan + VMail.Message +
##     Day.Charge + Eve.Mins + Night.Charge + Intl.Mins + Intl.Calls +
##     Intl.Charge + CustServ.Calls
```

```
##
##                  Df    AIC
## - Intl.Mins       1 6429.0
## - Intl.Charge     1 6429.0
## <none>              6429.9
## - VMail.Message   1 6435.9
## - Night.Charge    1 6439.9
## - Intl.Calls      1 6440.5
## - VMail.Plan      1 6448.9
## - Eve.Mins        1 6450.7
## - CustServ.Calls  1 6537.6
## - Int.l.Plan      1 6539.5
## - Day.Charge      1 6543.2
##
## Step:  AIC=6428.96
## Surv(Account.Length, Churn) ~ Int.l.Plan + VMail.Plan + VMail.Message +
##     Day.Charge + Eve.Mins + Night.Charge + Intl.Calls + Intl.Charge +
##     CustServ.Calls
##
##                  Df    AIC
## <none>              6429.0
## - VMail.Message   1 6435.4
## - Intl.Charge     1 6437.9
## - Intl.Calls      1 6438.8
## - Night.Charge    1 6438.8
## - VMail.Plan      1 6448.6
## - Eve.Mins        1 6449.2
## - CustServ.Calls  1 6536.0
## - Int.l.Plan      1 6537.6
## - Day.Charge      1 6542.5
```

The code in this report is adapted from:
STAT40810-Stochastic Models (online)-2019/20 Summer (with additional modification).