Optimizing the ascent trajectory for an orbital class launch vehicle

Final project in SI1336

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1 Parameters

2 The model

To simulate the ascent of the rocket, a model of the physics involved is required. In this model, it assumed that only main three forces are acting on the rocket: its thrust T, aerodynamic drag D and gravity G. All three of these forces need approximations in order to run the simulation with reasonable result.

2.1 Coordinate system

Let $\hat{r}$ denote the normalized radial vector. Let furthermore $\hat{z}$ be oriented according to the right-hand rule relative to the direction of travel. Then, the tangential vector is $\hat{t} = \hat{r} \times \hat{z}$.

2.2 State vectors

2.3 Gravity - G

Gravity is modelled based on the Newtonian formulation, resulting in a force

$$\vec{G}(r) = -\frac{\mu}{r^2}\hat{r}$$

where r is the distance to the center of Earth from the rocket, and $\mu \approx 3.986 \cdot 10^{14} m^3 s^2$ is the standard gravitational parameter for Earth.

2.4 Aerodynamic drag - D

To model aerodynamic drag, it is assumed that the atmosphere moves at a velocity, independent of radius

$$\vec{v}_{atm}(\vec{r}) = v_{surf} \cdot \hat{t}(\vec{r})$$

This allows the definiton of the wind-relative velocity

$$\vec{v}_{atm,rel}(\vec{r}, t) = \vec{v}(t) - \vec{v}_{atm}(\vec{r})$$

2.5 Thrust - D