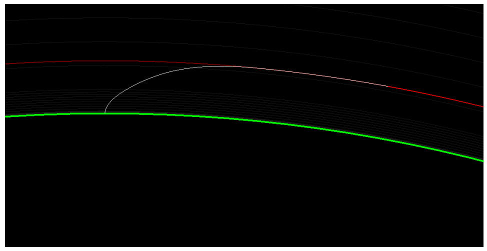

Optimizing the ascent trajectory for an orbital class launch vehicle

Final project in SI1336

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1 Project summary

This project formulates an approximate model of the conditions encountered for an orbital class launch vehicle during ascent into Low-Earth orbit, as well as a strategy for trajectory optimization for polar and equatorial launches.

The project is split into four major parts: the equations describing the rocket, the physics model, the ascent guidance algorithm as well as the optimization of parameters.

2 Intrinsic equations describing the rocket

2.1 Input parameters

The state of the rocket can be described with position, velocity and expended mass (m_e)

$$S = (\vec{r}, \vec{v}, m) = (x, y, z, v_x, v_y, v_z, m_e)$$

where m_e is the integral of exhaust mass flow (propulsion) over time

$$m_e(t) = \int_0^t \dot{m}(S, t)dt$$

The reason for including expended mass in the model is that it lets us derive the currently active stage, as well as the necessary variables of the rocket as a function of used propellant. This is done by considering each state as collection of parameters

Parameter	Description
$m_{prop,i}$	Propellant mass of stage
$m_{dry,i}$	Dry mass of stage (without propellant)
$F_{sl,i}$	Thrust at sea level pressure
$F_{vac,i}$	Thrust in vacuum pressure
$I_{sp,sl,i}$	Specific impulse at sea level
$I_{sp,vac,i}$	Specific impulse in vacuum pressure

such that each stage $i = 0, 1, 2...$ can be written

$$s_i = (m_{prop,i}, m_{dry,i}, F_{sl,i}, F_{vac,i}, I_{sp,sl,i}, I_{sp,vac,i})$$

Furthermore, a rocket typically has an aerodynamic shell (fairing) that is deployed around the edge of space (100km). This is modelled simply by the parameters

Parameter	Description
m_i	Mass of fairing
$h_{deployment,i}$	Altitude of deployment

such that each fairing $i = 0, 1, 2...$ can be written

$$f_i = (m_i, h_{deployment,i})$$

Lastly, the payload of the rocket is modelled as a mass $m_{payload}$.

2.2 Derived parameters

To run simulations of defined by the parameters in the previous section, we need to derive the current mass of the entire rocket. This is not as simple as subtracting m_e from the initial mass, as we want to model staging. Instead consider parameters driven by the currently active stage s_{active}.

2.2.1 Mass

If the propellant of stages before s_i is

$$m'_{prop,i} = \sum_{j=0}^{i-1} m_{prop,j}$$

then the currently active stage fulfill the criteria

$$m'_{prop,active} < m_e(t) < m'_{prop,active} + m_{prop,active}$$

As we are only considering a two stage launch vehicle, the active stage can be defined by

$$s_{active} = \begin{cases} s_0 & : m_e < m_{prop,0} \\ s_1 & : 0 < m_e - m_{prop,0} < m_{prop,1} \end{cases}$$

If the mass of stages after the stage s_i is

$$m'_{stages,i} = \sum_{j=i+1} m_{prop,j} + m_{dry,j}$$

This defines the current mass of stages, when s_i is active, as

$$m_{stagemass,i} = \left(m_e(t) - m'_{prop,i}\right) + m_{dry,i} + m'_{stages,i}$$

If h_{max} is the maximum altitude reached up until time t, then the mass of fairings currently on the rocket can be described by the sum

$$m_{fairings}(h_{max}) = \sum_{i=0} m_i(1 - H(h_{max} - h_{deployment,i}))$$

Thus, the total instantaneous mass for an active stage s_i is

$$m_i = m_{stagemass,i} + m_{fairings}(h_{max}) + m_{payload}$$

2.2.2 Thrust

Since the thrust of a rocket propulsion system is linear in the pressure difference between the exhaust and surrounding pressure, it is assumed in our model that the thrust F changes linearly between F_{sea} and F_{vacuum} with pressure p.

For a given active stage s_i, if pressure is written as a function of altitude h, thrust can be written as

$$F_i(p(h)) = F_{sl,i} + (F_{sl,i} - F_{vac,i})\frac{p(h)}{p(0)}$$

This only holds if the stage has remaining fuel ($m_e - m_{prop,i} > 0$) otherwise

$$F_i(p(h)) = 0$$

2.2.3 Mass flow

Mass flow is necessary to find $m_e(t)$, and is given in a general form by

$$\dot{m} = \frac{F}{g_0 I_{sp}}$$

If we assume mass flow to be constant through the propulsion system, then I_{sp} must share the same linear behaviour in pressure as thrust does. Thus, for an active stage s_i, let

$$I_{sp,i}(p(h)) = I_{sp,sl,i} + (I_{sp,sl,i} - I_{sp,vac,i})\frac{p(h)}{p(0)}$$

This gives the mass flow

$$\dot{m}_i(p(h)) = \frac{F_i(p(h))}{g_0 I_{sp,i}(p(h))}$$

Finally, to tie the equations together, we define

$$m(m_e) = m_i, \quad F(m_e, p(h)) = F_i(p(h)), \quad \dot{m}(m_e, p(h)) = \dot{m}_i(p(h))$$

where the active stage s_i is derived from a given m_e.

3 Physics model

To simulate the ascent of the rocket, a model of the physics involved is required.

3.1 Coordinate system

The simulation uses two coordinate systems, one cartesian and one kinematic. The cartesian system $\hat{x}, \hat{y}, \hat{z}$ is originated in the starting location and oriented such that $\hat{x}$ points towards the wanted direction of orbit, $\hat{y}$ points radially up, and $\hat{z} = \hat{x} \times \hat{y}$.

The kinematic system $\hat{r}, \hat{t}, \hat{z}$ follows the rocket and is oriented such that $\hat{r}$ is the normalized radial vector, $\hat{z}$ is the same as in the cartesian system, and the tangential vector is $\hat{t} = \hat{r} \times \hat{z}$. In a circular orbit, $\hat{t}$ is parallel to velocity $\vec{v}$.

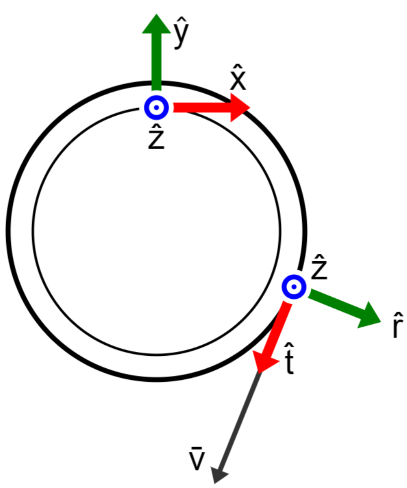

Figure 1: The instantaneous coordinates

3.2 Atmosphere

Produces $\rho(r)$

3.3 Forces

In this model, it assumed that three forces are acting on the rocket: thrust T, aerodynamic drag D and gravity G.

3.3.1 Gravity - G

Gravity is modelled based on the Newtonian formulation, resulting in a force

$$\vec{G}(\vec{r}, m_e) = -m(m_e)\frac{\mu}{r^2}\hat{r}$$

where r is the distance to the center of Earth from the rocket, and $\mu \approx 3.986 \cdot 10^{14} m^3 s^2$ is the standard gravitational parameter.

3.3.2 Aerodynamic drag - D

To model aerodynamic drag it first is assumed that the atmosphere, independently of radius, moves at a velocity:

$$\vec{v}_{atm}(\vec{r}) = v_{surf} \cdot \hat{t}$$

This allows the definition of the wind-relative velocity

$$\vec{v}_{atm,rel}(\vec{r}, \vec{v}) = \vec{v} - \vec{v}_{atm}(\vec{r})$$

Under the assumption that the the drag equation holds for the entirety of the ascent, then

$$\vec{D}(\vec{r}, \vec{v}, m_e) = -\frac{m(m_e) \cdot C_d \cdot A \cdot \rho(r)}{2} |\vec{v}_{atm,rel}(\vec{r}, \vec{v})|^2 \hat{v}$$

3.3.3 Thrust - T

To abstract the guidance algorithms from the physics model, it is assumed that guidance controls throttle as

$$\eta = \eta(\vec{r}, \vec{v}, m_e, t) \in [0, 1]$$

and angle of thrust (AoT) from the vertical $\hat{r}$ as

$$\theta = \theta(\vec{r}, \vec{v}, m_e, t) \in [0, \pi]$$

Throttle and AoT interact such that

$$\vec{T}(\vec{r}, \vec{v}, m_e, t) = \left(\cos\theta\hat{r} + \sin\theta\hat{t}\right) \cdot F(m_e, P) \cdot \eta$$

3.4 Differential equation

The combination of gravity, aerodynamic drag, and thrust give the equation

$$\vec{a}(\vec{r}, \vec{v}, m_e) = \frac{1}{m_i}\left(G(\vec{r}, m_e) + D(\vec{r}, \vec{v}, m_e) + T(\vec{r}, \vec{v}, m_e, t)\right)$$

4 Ascent guidance

The ascent guidance implemented in the simulation can be categorized into five phases:

- Liftoff
- Kickpitch
- Gravity turn
- Orbital insertion
- Terminal guidance

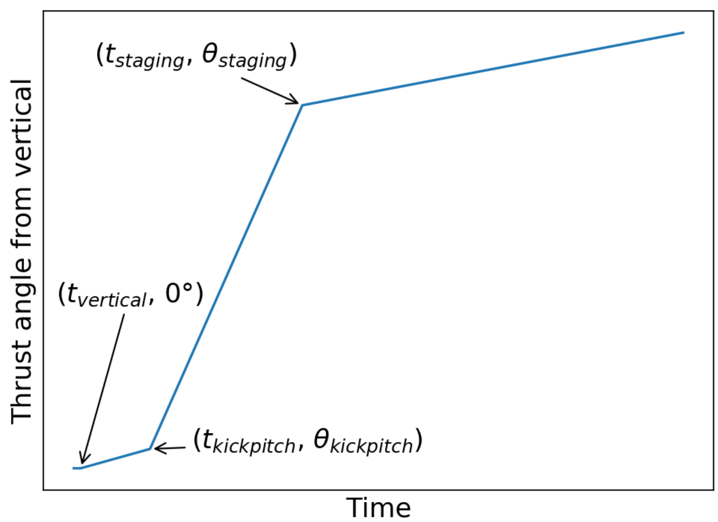

Figure 2: Typical thrust angle from vertical (intentionally without units)

4.1 Liftoff $(0 < t < t_{vertical})$

During this phase, the trajectory is entirely vertical - $\theta = 0$ and $\eta = 1$.

4.2 Kickpitch $(t_{vertical} < t < t_{kickpitch})$

During this phase, the trajectory starts pitching towards the horizon slightly. The angle θ is linearly interpolated between 0 and the defining parameter

$\theta_{kickpitch} = 4°$ according to

$$\theta(t) = \theta_{kickpitch} \frac{t - t_{vertical}}{t_{kickpitch} - t_{vertical}}$$

4.3 Gravity turn ($\dot{m}t_{kickpitch} < m_e(t) < m_{prop,0}$)

During the time between the end of kickpitch and the time of staging (when the expended mass is equal to the propellant in the first stage), the angle θ is linearly interpolated between $\theta_{kickpitch}$ and the defining parameter $\theta_{staging}$ according to

$$\theta(t) = \theta_{kickpitch} + (\theta_{staging} - \theta_{kickpitch}) \frac{m_e(t) - \dot{m}t_{kickpitch}}{m_{prop,0} - \dot{m}t_{kickpitch}}$$

4.4 Orbital insertion ($m_{prop,0} < m_e(t) < m_{prop,0} + m_{prop,1}$)

This phase lasts until terminal guidance is triggered. The angle θ is interpolated from $\theta_{staging}$ to 0 according to

$$\theta(t) = \theta_{staging} \left(1 - \frac{m_e(t) - m_{prop,0}}{m_{prop,1}}\right)$$

4.5 Terminal guidance

Terminal guidance is necessary to achieve a circular orbit within this simulation and previously mentioned ascent guidance parameters. It consists of a PI-controller that attempts to cancel out any vertical velocity.

The conditions for entering terminal guidance is:

- $h > g_{ap} - 55$
- OR:
- $h > 100$ and $|\vec{v} \cdot \hat{r}| < 50$

Within terminal guidance, θ is controlled with the regulator

$$\theta = 5 \cdot e(t) + 5 \int_{t_{triggered}}^{t} e(t')dt'$$

where $t_{triggered}$ is the time at which terminal guidance was triggered, and

$$e(t) = \vec{v} \cdot \hat{r}$$

Furthermore, throttle is controlled by

$$\eta = \begin{cases} 0 & : \quad r_{pe} > g_{pe} - 2km, \quad g_{ap} - 2km < r_{ap} < g_{ap} + 3km \\ 0 & : \quad r_{pe} > g_{pe} - 2km, |\vec{v} \cdot \hat{r}| < 15km \\ 0.05 & : \quad r_{pe} > g_{pe} - 10km \\ 0.25 & : \quad r_{pe} > 0km \\ 0.5 & : \quad r_{pe} > -100km \\ 1 & : \quad otherwise \end{cases}$$

If $\eta = 0$, the simulation ends in orbit.

5 Integration

6 Optimization of parameters

7 Results

7.1 Orbit insertion accuracy

		Achieved	
Surface motion	Goal altitude	Perigee	Apogee
0 m/s	170 km	168.21 km	169.76 km
460 m/s	170 km	168.17 km	169.49 km
0 m/s	220 km	218.87 km	219.39 km
460 m/s	220 km	218.93 km	223.90 km
0 m/s	275 km	274.85 km	274.88 km
460 m/s	275 km	273.93 km	274.38 km

7.2 Propellant consumption

		Propellant	
Surface motion	Goal altitude	Consumed	Remaining
0 m/s	170 km	511909 kg	6490 kg
460 m/s	170 km	510103 kg	8297 kg
0 m/s	220 km	516129 kg	2271 kg
460 m/s	220 km	512725 kg	5675 kg
0 m/s	275 km	516911 kg	1489 kg
460 m/s	275 km	515460 kg	2940 kg

7.3 Equatorial vs polar

	Propellant	
Goal altitude	Consumed	Remaining
170 km	99.64%	127.8 %
220 km	99.34%	250.0 %
275 km	99.72%	197.4 %

8 Analysis

By the results achieved, it's clear that orbital launch from the equator is better than launch from the poles. One must be careful just looking at the results however, as one might say that it is not by a huge margin that equatorial launch is better, but that's where the tyranny of the rocket equation comes into play. If our payload goal is to place 10000 kg into a 220 km orbit, then reducing the fuel consumption by 3500 kg allows a much smaller first stage, as the rocket equation tends toward exponential changes.

It is however interesting that the fuel consumption is the most equal for a target orbit of 170 km. Without knowing for certain what the result should be, the way that the trajectory is created and optimized will play a role. The *very* simple guidance algorithm is definitely just scraping the surface of what is possible.

A Raw data

A.1 170 km orbit

	Goal		Achieved	
Surface motion	Perigee	Apogee	Perigee	Apogee
0 m/s	170 km	170 km	168.21 km	169.76 km
460 m/s	170 km	170 km	168.17 km	169.49 km

		Propellant	
Surface motion	Payload	Remaining	Consumed
0 m/s	10000 kg	6490.91 kg	511909 kg
460 m/s	10000 kg	8297.18 kg	510103 kg
		127.82%	99.64%

A.2 220 km orbit

	Goal		Achieved	
Surface motion	Perigee	Apogee	Perigee	Apogee
0 m/s	220 km	220 km	218.87 km	219.39 km
460 m/s	220 km	220 km	218.93 km	223.90 km

		Propellant	
Surface motion	Payload	Remaining	Consumed
0 m/s	10000 kg	2271.10 kg	516129 kg
460 m/s	10000 kg	5675.03 kg	512725 kg
		250%	99.34%

A.3 275 km orbit

	Goal		Achieved	
Surface motion	Perigee	Apogee	Perigee	Apogee
0 m/s	275 km	275 km	274.85 km	274.88 km
460 m/s	275 km	275 km	273.93 km	274.38 km

		Propellant	
Surface motion	Payload	Remaining	Consumed
0 m/s	10000 kg	1489.10 kg	516911 kg
460 m/s	10000 kg	2939.79 kg	515460 kg
		197.4%	99.72%