Modelling Human Induced Allee Effects on Biological Invasion in a Two Patch Model

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Abstract

Biological Invasion is a process wherein the natural environment is negatively altered by the importation and spread of non-native species. Invasions can have far-reaching ecological and economic consequences, as non-native species can out-compete and displace the native species, resulting in disruptions to natural food webs and local biodiversity. Harnessing the Allee effect, (negative density dependence) has been explored as a method of managing biological invasion. By artificially lowering the Allee threshold, and therefore increasing the chances of reversing the positive growth rate of the invaders, biological management may be possible through one-time extinction events. In this model, human-induced Allee effects are investigated as a measure to reduce the persistence of invaders in a two-patch model. The basis of this model includes invaders originating in one patch and migrating to the second patch by human or other means. We hope to answer the question of how we can use human induced allee thresold in patch 1 to manage the spread of invasive species in patch 2.

1 Introduction

Biological invasion

Biological invasion, also known as species invasion, is a process wherein the natural environment is negatively altered by the importation (establishment and spread) of non-native species [2]. This phenomenon includes animals, plants, and microorganisms such as bacteria, and it can lead to challenges in ecological, economic, management, and conservation efforts [4]. Some of the causes of this are globalization, habitat alteration, climate change, unintentional transport, ballast water, and others [4, 3]. Allee effects are phenomena in which species (individuals or populations) have lower fitness and survival at low population densities. This can make it difficult for them to establish themselves in new environments. Essentially, some species encounter challenges such as mate finding, cooperation, or predator avoidance when their numbers are scarce. Allee effects play an important role in population dynamics and the success of biological invasions [5, 1]. Allee effects certainly affect the spread of invasive species when they are trying to establish themselves in some places. Therefore, reducing the population density of invasive species below a critical Allee threshold can be a successful approach in the management of them. Some of these strategies are culling, disruption of successful mating, and augmentation of natural enemy populations [2].

Culling methods: Depending on the taxa being addressed they encompass eliminating host material that is infested, decontaminating vehicles that transport invasive species, manual extraction, herbicides use, traps, aerial poisoning, etc [2].

Disruption of successful mating: An example in the management of insect populations is the use of synthetic pheromones to hinder males from finding females. Another method involves the release of sterile males or the sterilization of individuals of one gender, as observed in specific mammal and avian species [2].

Augmentation of natural enemy populations: Releasing specialized natural enemies shortly after a successful species invasion holds the potential to impede or halt further expansion and even lead to extinction. However, this needs the presence of an Allee threshold within the invading species and a high rate of natural enemy dispersal relative to their prey. One example is the introduction of the parasitoid Compsilura concinnata, which resulted in the decline and geographical reduction of the brown-tail moth population in eastern North America [2].

2 Methods

2.1 The Model

The Allee two-patch migration model begins with two patches connected by migration. On each patch, there is a modified logistic growth ODE with the Allee effect. The Allee effect on patch one is the combined effect of the species' natural Allee effect with human-induced Allee effects. These human-induced Allee effects come from either reproduction interruptions or scavenging interruptions. K (carrying capacity) is kept the same between the two patches. Finally, a migration term is added, but for simplicity migration symmetric between patch 1 and patch 2.

$$\frac{dx}{dt} = r\left(\frac{x(t)}{a_1} - 1\right)\left(1 - \frac{x(t)}{k}\right)(x(t)) - m_{12}x(t) + m_{21}y(t) \tag{1}$$
$$\frac{dy}{dt} = r\left(\frac{y(t)}{a_2} - 1\right)\left(1 - \frac{y(t)}{k}\right)(y(t)) + m_{12}y(t) - m_{21}y(t) \tag{2}$$

Variable	Description
r	Growth rate of new individuals per day
a_1	Allee effect in first population due to humans
a_2	Natural Allee effect
k	Carrying capacity
m_{12}	migration from patch 1 to patch 2
m_{21}	migration from patch 1 to patch 2
m	migration from patch 1 to patch 2 or patch 2 to patch 1

Table 1: Descriptions of the variables used in the two patch model, (1) and (2), depicting rate of movement of invasive species from lake 1 to lake 2. In the scaled equation, m is scaled based on r and k.

2.2 Non-dimensional Equation with Symmetric Migration

$$\frac{dx}{dt} = \left(\frac{x(t)}{a_1} - 1\right)(1 - x(t))(x(t)) - m(y(t) - x(t)) \tag{3}$$
$$\frac{dy}{dt} = \left(\frac{y(t)}{a_2} - 1\right)(1 - y(t))(y(t)) + m(y(t) - x(t)) \tag{4}$$

2.3 Equilibrium points

The equilibrium points acquired for this two patch model are found by setting each equation to 0 and solve for x and y respectively. We found one equilibrium point for this particular model, which is $(0, 0)$. These can now be used in the Jacobian to assess stability.

2.4 The Jacobian

We can substitute the equilibrium values in the Jacobian. For the first equilibrium value we get

$$J(0,0) = \begin{pmatrix} -m_{12} + r & 0 \\ m_{12} & r \end{pmatrix}$$

$$\begin{pmatrix} -m_{12} + r & 0 \\ m_{12} & \frac{r(a_2 - k)}{k} \end{pmatrix}$$

3 Results

3.0.1 Test Cases

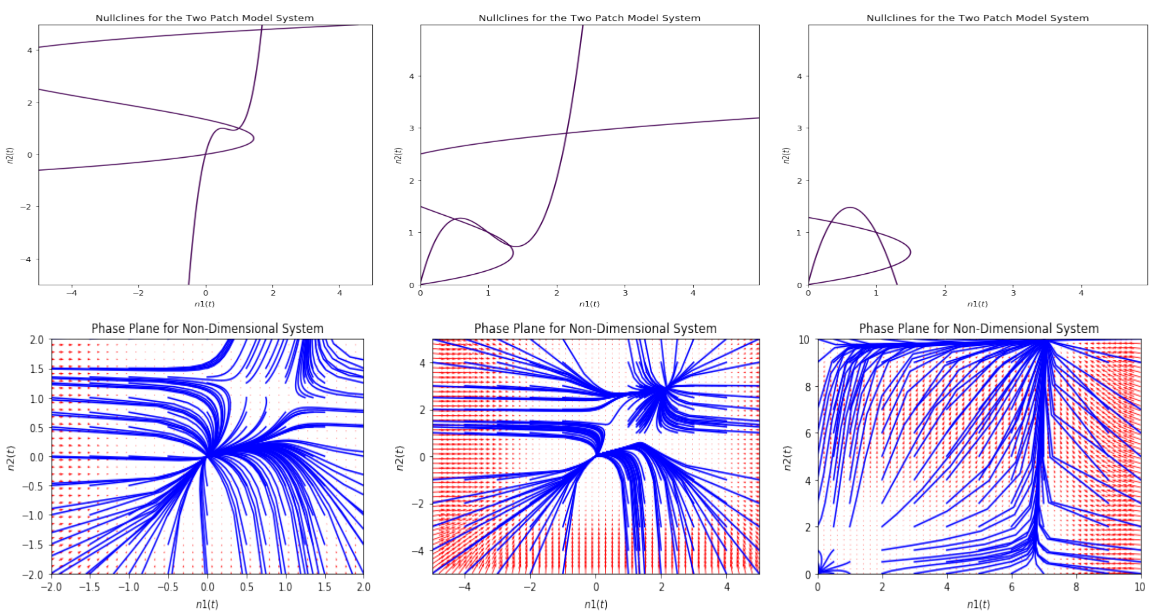

Figure 1: Figure 1: Phase Plane and Nullcline for three test cases. (Left) a1 = 1, a2 = 5, m = 0.25 (Middle) a1 = 2, a2 = 3, m = 0.25 (Right) a1 = 7, a2 = 10, m = 0.25

4 Discussion

This project focuses on changes in population over time with a spatially implicit model incorporating the Allee effect and Logistic model. It is a two-patch model depicting the rate of growth of invasive species from one Patch to the next. The goal of the code provided is to give an insight into the stability of the system and to show classic species spread. We first start by solving the two-patch model which includes symmetric migration seen in equations (1) and (2). Due to the complexity of the system, non-dimensionalization was necessary to reduce the number of parameters. In equations (1) and (2) we see there are 6 parameters, $(r, k, a_1, a_2, m_{12}, m_{21})$, but after non-dimensionalization, we see the number of parameters is reduced down to 3, (a_1, a_2, m). This facilitates the equilibria solving algorithms.

We then evaluate the system's equilibrium points, which we initially found to be a singular equilibria at $(0, 0)$. This was unexpected; we anticipated at least two equilibrium values. In order to find other equilibria, which may have not been captured the first time, we do a simple substitution to reduce the system of equations to one differential equation. Doing it this way gives us a 9^{th} order polynomial for which we need to find the zeros, however the run time was extremely lengthy and we were not able to get the roots of the polynomial this way. Instead, using user-inputted parameters, that may be chosen to b species-specific, we used a root-finding algorithm. Only real values were kept, while imaginary roots were discarded. The Jacobian was found for each real positive root, which was then analyzed for stability.

Since we chose to keep the system non-species-specific there were no universal parameters for the system. Due to this, we constructed several phase planes and nullclines by inputting different values for the 3 parameters with a constant migration rate of 25 %. The migration rate was chosen to be constant as traffic between the two patches can be made standardized. We can see Figure 1 shows the equilibrium points change as the Allee effects differ.

To answer our question - "How can use human-induced Allee thresholds in Patch 1 manage the spread of invasive species in Patch 2" it is necessary to obtain parameter values of the species of interest, along with the correct initial conditions and be able to use numerical simulations. Further research would include conducting experiments or simulate data to get those parameters. Numerical simulations should be conducted for a range of initial conditions and Allee Effects to see whether managing the population in the first Patch through Allee Effects is sufficient to guarantee extinction in the second Patch. Another interesting question arises when the system begins at a non-equilibrium state- the population is at carrying capacity in Patch 1, but at zero in Patch 2, which may simulate when one migration between patches is closed, but becomes open at t=0. Further extending this project may also include visualizing the spread of n-discrete patches. Lastly, adding a stochastic harvesting event will change the dynamics of this problem further the research question. These ideas all provide fascinating extensions to this two-patch problem.

5 Conclusion

The two-patch system proposed models the general effect of invasive species between the two Patches. The user is able to provide different values for the parameters and explore the effects they have on the system's stability, nullclines and eventual outcomes. Any further analysis would require exact values in order to answer the question of how humans can control the spread of invasive species between the two patches.

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