Edge capacity derivation for MAP inference with min-cut

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$$\lambda_t = \log\left(\frac{P\left(y \mid x=1\right)}{P\left(y \mid x=0\right)}\right)$$

Using Bayes' rule we have

$$\text{posterior odds} = \text{likelihood ratio} \cdot \text{prior odds}$$
$$\frac{P\left(x=1 \mid y\right)}{P\left(x=0 \mid y\right)} = \frac{P\left(y \mid x=1\right)}{P\left(y \mid x=0\right)} \cdot \frac{P\left(x=1\right)}{P\left(x=0\right)}$$

So

$$\lambda_t = \log\left(\frac{P\left(x=1 \mid y\right)}{P\left(x=0 \mid y\right)} \cdot \frac{P\left(x=0\right)}{P\left(x=1\right)}\right)$$

We are currently assuming that the prior odds are fifty-fifty, so we get

$$\begin{aligned}\lambda_t &= \log\left(\frac{P\left(x=1 \mid y\right)}{P\left(x=0 \mid y\right)}\right)\\ &= \log\left(P\left(x=1 \mid y\right)\right) - \log\left(P\left(x=0 \mid y\right)\right)\end{aligned}$$

We are modeling $P\left(x=1 \mid y\right)$ is modeled as $\sigma\left(w^\top h\left(y\right)\right)$, so we derive that

$$\lambda_t = \log\left(\sigma\left(w^\top h\left(y\right)\right)\right) - \log\left(1 - \sigma\left(w^\top h\left(y\right)\right)\right)$$