1 Formulaire : trigonométrie circulaire et hyperbolique

Propriétés trigonométriques : remplacer cos par ch et sin par $\mathrm{i}\cdot\mathrm{sh}$.

$$\cos^2 x + \sin^2 x = 1$$

$$\mathrm{ch}^2 x - \mathrm{sh}^2 x = 1$$

$$\begin{aligned}\cos(a+b) &= \cos a\cdot\cos b - \sin a\cdot\sin b\\ \sin(a+b) &= \sin a\cdot\cos b + \sin b\cdot\cos a\\ \tan(a+b) &= \frac{\tan a+\tan b}{1-\tan a\cdot\tan b}\end{aligned}$$

$$\begin{aligned}\mathrm{ch}(a+b) &= \mathrm{ch}\,a\cdot\mathrm{ch}\,b + \mathrm{sh}\,a\cdot\mathrm{sh}\,b\\ \mathrm{sh}(a+b) &= \mathrm{sh}\,a\cdot\mathrm{ch}\,b + \mathrm{sh}\,b\cdot\mathrm{ch}\,a\\ \mathrm{th}(a+b) &= \frac{\mathrm{th}\,a+\mathrm{th}\,b}{1+\mathrm{th}\,a\cdot\mathrm{th}\,b}\end{aligned}$$

$$\begin{aligned}\cos(a-b) &= \cos a\cdot\cos b + \sin a\cdot\sin b\\ \sin(a-b) &= \sin a\cdot\cos b - \sin b\cdot\cos a\\ \tan(a-b) &= \frac{\tan a-\tan b}{1+\tan a\cdot\tan b}\end{aligned}$$

$$\begin{aligned}\mathrm{ch}(a-b) &= \mathrm{ch}\,a\cdot\mathrm{ch}\,b - \mathrm{sh}\,a\cdot\mathrm{sh}\,b\\ \mathrm{sh}(a-b) &= \mathrm{sh}\,a\cdot\mathrm{ch}\,b - \mathrm{sh}\,b\cdot\mathrm{ch}\,a\\ \mathrm{th}(a-b) &= \frac{\mathrm{th}\,a-\mathrm{th}\,b}{1-\mathrm{th}\,a\cdot\mathrm{th}\,b}\end{aligned}$$

$$\begin{aligned}\cos 2a &= 2\cos^2 a - 1\\ &= 1-2\sin^2 a\\ &= \cos^2 a - \sin^2 a\end{aligned}$$

$$\sin 2a = 2\sin a\cdot\cos a$$

$$\tan 2a = \frac{2\tan a}{1-\tan^2 a}$$

$$\begin{aligned}\mathrm{ch}\,2a &= 2\,\mathrm{ch}^2 a - 1\\ &= 1+2\,\mathrm{sh}^2 a\\ &= \mathrm{ch}^2 a + \mathrm{sh}^2 a\end{aligned}$$

$$\mathrm{sh}\,2a = 2\,\mathrm{sh}\,a\cdot\mathrm{ch}\,a$$

$$\mathrm{th}\,2a = \frac{2\,\mathrm{th}\,a}{1+\mathrm{th}^2 a}$$

$$\begin{aligned}\cos a\cdot\cos b &= \frac{1}{2}\big[\cos(a+b)+\cos(a-b)\big]\\ \sin a\cdot\sin b &= \frac{1}{2}\big[\cos(a-b)-\cos(a+b)\big]\\ \sin a\cdot\cos b &= \frac{1}{2}\big[\sin(a+b)+\sin(a-b)\big]\end{aligned}$$

$$\begin{aligned}\mathrm{ch}\,a\cdot\mathrm{ch}\,b &= \frac{1}{2}\big[\mathrm{ch}(a+b)+\mathrm{ch}(a-b)\big]\\ \mathrm{sh}\,a\cdot\mathrm{sh}\,b &= \frac{1}{2}\big[\mathrm{ch}(a+b)-\mathrm{ch}(a-b)\big]\\ \mathrm{sh}\,a\cdot\mathrm{ch}\,b &= \frac{1}{2}\big[\mathrm{sh}(a+b)+\mathrm{sh}(a-b)\big]\end{aligned}$$

$$\begin{aligned}\cos p+\cos q &= 2\cos\frac{p+q}{2}\cdot\cos\frac{p-q}{2}\\ \cos p-\cos q &= -2\sin\frac{p+q}{2}\cdot\sin\frac{p-q}{2}\\ \sin p+\sin q &= 2\sin\frac{p+q}{2}\cdot\cos\frac{p-q}{2}\\ \sin p-\sin q &= 2\sin\frac{p-q}{2}\cdot\cos\frac{p+q}{2}\end{aligned}$$

$$\begin{aligned}\mathrm{ch}\,p+\mathrm{ch}\,q &= 2\,\mathrm{ch}\frac{p+q}{2}\cdot\mathrm{ch}\frac{p-q}{2}\\ \mathrm{ch}\,p-\mathrm{ch}\,q &= 2\,\mathrm{sh}\frac{p+q}{2}\cdot\mathrm{sh}\frac{p-q}{2}\\ \mathrm{sh}\,p+\mathrm{sh}\,q &= 2\,\mathrm{sh}\frac{p+q}{2}\cdot\mathrm{ch}\frac{p-q}{2}\\ \mathrm{sh}\,p-\mathrm{sh}\,q &= 2\,\mathrm{sh}\frac{p-q}{2}\cdot\mathrm{ch}\frac{p+q}{2}\end{aligned}$$

Avec $t=\tan\frac{x}{2}$ on a

$$\begin{cases}\cos x &= \frac{1-t^2}{1+t^2}\\ \sin x &= \frac{2t}{1+t^2}\\ \tan x &= \frac{2t}{1-t^2}\end{cases}$$

Avec $t=\mathrm{th}\,\frac{x}{2}$ on a

$$\begin{cases}\mathrm{ch}\,x &= \frac{1+t^2}{1-t^2}\\ \mathrm{sh}\,x &= \frac{2t}{1-t^2}\\ \mathrm{th}\,x &= \frac{2t}{1+t^2}\end{cases}$$

Dérivées : la multiplication par i n'est plus valable

$$\begin{aligned}\cos' x &= -\sin x\\ \sin' x &= \cos x\\ \tan' x &= 1+\tan^2 x = \frac{1}{\cos^2 x}\end{aligned}$$

$$\begin{aligned}\mathrm{ch}' x &= \mathrm{sh}\,x\\ \mathrm{sh}' x &= \mathrm{ch}\,x\\ \mathrm{th}' x &= 1-\mathrm{th}^2 x = \frac{1}{\mathrm{ch}^2 x}\end{aligned}$$

$$\begin{aligned}\arccos' x &= \frac{-1}{\sqrt{1-x^2}} \quad (|x|<1)\\ \arcsin' x &= \frac{1}{\sqrt{1-x^2}} \quad (|x|<1)\\ \arctan' x &= \frac{1}{1+x^2}\end{aligned}$$

$$\begin{aligned}\mathrm{Argch}' x &= \frac{1}{\sqrt{x^2-1}} \quad (x>1)\\ \mathrm{Argsh}' x &= \frac{1}{\sqrt{x^2+1}}\\ \mathrm{Argth}' x &= \frac{1}{1-x^2} \quad (|x|<1)\end{aligned}$$