# 1 Formules de développements limités

Développements limités usuels (au voisinage de 0)

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\begin{aligned}
e^x &= 1+\frac{x}{1!}+\frac{x^2}{2!}+\cdots+\frac{x^n}{n!} \quad + \quad o(x^n)=\sum_{k=0}^{n}\frac{x^k}{k!} \quad + \quad o(x^n)\\
\cos x &= 1-\frac{x^2}{2!}+\frac{x^4}{4!}-\cdots+(-1)^n\cdot\frac{x^{2n}}{(2n)!} \quad + \quad o(x^{2n+1})=\sum_{k=0}^{n}(-1)^k\frac{x^{2k}}{(2k)!} \quad + \quad o(x^{2n+1})\\
\sin x &= x-\frac{x^3}{3!}+\frac{x^5}{5!}-\cdots+(-1)^n\cdot\frac{x^{2n+1}}{(2n+1)!} \quad + \quad o(x^{2n+2})=\sum_{k=0}^{n}(-1)^k\frac{x^{2k+1}}{(2k+1)!} \quad + \quad o(x^{2n+2})\\
\tan x &= x+\frac{x^3}{3}+\frac{2}{15}x^5+\frac{17}{315}x^7 \quad + \quad o(x^8)\\
\mathrm{ch}\, x &= 1+\frac{x^2}{2!}+\frac{x^4}{4!}+\cdots+\frac{x^{2n}}{(2n)!} \quad + \quad o(x^{2n+1})=\sum_{k=0}^{n}\frac{x^{2k}}{(2k)!} \quad + \quad o(x^{2n+1})\\
\mathrm{sh}\, x &= x+\frac{x^3}{3!}+\frac{x^5}{5!}+\cdots+\frac{x^{2n+1}}{(2n+1)!} \quad + \quad o(x^{2n+2})=\sum_{k=0}^{n}\frac{x^{2k+1}}{(2k+1)!} \quad + \quad o(x^{2n+2})\\
\mathrm{th}\, x &= x-\frac{x^3}{3}+\frac{2}{15}x^5-\frac{17}{315}x^7 \quad + \quad o(x^8)
\end{aligned}
$$

$$
\begin{aligned}
\ln(1+x) &= x-\frac{x^2}{2}+\frac{x^3}{3}-\cdots+(-1)^{n-1}\cdot\frac{x^n}{n} \quad + \quad o(x^n)=\sum_{k=1}^{n}(-1)^{k+1}\frac{x^k}{k} \quad + \quad o(x^n)\\
(1+x)^\alpha &= 1+\alpha x+\frac{\alpha(\alpha-1)}{2!}x^2+\cdots+\frac{\alpha(\alpha-1)\cdots(\alpha-n+1)}{n!}x^n \quad + \quad o(x^n)\\
&= \sum_{k=0}^{n}\binom{\alpha}{k}x^k \quad + \quad o(x^n)\\
\frac{1}{1+x} &= 1-x+x^2-\cdots+(-1)^n x^n \quad + \quad o(x^n)=\sum_{k=0}^{n}(-1)^k x^k \quad + \quad o(x^n)\\
\frac{1}{1-x} &= 1+x+x^2+\cdots+x^n \quad + \quad o(x^n)=\sum_{k=0}^{n}x^k \quad + \quad o(x^n)\\
\sqrt{1+x} &= 1+\frac{x}{2}-\frac{1}{8}x^2-\cdots+(-1)^{n-1}\cdot\frac{1\cdot1\cdot3\cdot5\cdots(2n-3)}{2^n n!}x^n \quad + \quad o(x^n)\\
\frac{1}{\sqrt{1+x}} &= 1-\frac{x}{2}+\frac{3}{8}x^2-\cdots+(-1)^n\cdot\frac{1\cdot3\cdot5\cdots(2n-1)}{2^n n!}x^n \quad + \quad o(x^n)\\
\arccos x &= \frac{\pi}{2}-x-\frac{1}{2}\frac{x^3}{3}-\frac{1\cdot3}{2\cdot4}\frac{x^5}{5}-\cdots-\frac{1\cdot3\cdot5\cdots(2n-1)}{2\cdot4\cdot6\cdots(2n)}\frac{x^{2n+1}}{2n+1} \quad + \quad o(x^{2n+2})\\
\arcsin x &= x+\frac{1}{2}\frac{x^3}{3}+\frac{1\cdot3}{2\cdot4}\frac{x^5}{5}+\cdots+\frac{1\cdot3\cdot5\cdots(2n-1)}{2\cdot4\cdot6\cdots(2n)}\frac{x^{2n+1}}{2n+1} \quad + \quad o(x^{2n+2})\\
\arctan x &= x-\frac{x^3}{3}+\frac{x^5}{5}+\cdots+(-1)^n\cdot\frac{x^{2n+1}}{2n+1} \quad + \quad o(x^{2n+2})
\end{aligned}
$$