Compendium Geometry

Conversion between 3D Coordinate Systems

Conversion from cartesian to cylinrical coordinates:

Cartesian [x, y, z] ⟶ Cylindrical [ρ, φ, z']

$$\rho = \sqrt{x^2 + y^2}$$

φ = arctan(y/x) for x > 0
φ = π/2 for x = 0 and y > 0
φ = π + arctan(y/x) for x < 0
φ = 3π/2 for x = 0 and y < 0
z' = z

Conversion from cylindrical to cartesian coordinates:

Cylindrical [ρ, φ, z'] ⟶ Cartesian [x, y, z]

x = ρ . cos φ
y = ρ . sin φ
z = z'

Conversion from spherical to cartesian coordinates:

Spherical [r, θ, φ] ⟶ Cartesian [x, y, z]

x = r . sin θ . cos φ
y = r . sin θ . sin φ
z = r . cos θ

Conversion from cartesian to spherical coordinates:

Cartesian [x, y, z] ⟶ Spherical [r, θ, φ]

$$r = \sqrt{x^2 + y^2 + z^2}$$

$$\cos\varphi = \frac{x}{\sqrt{x^2 + y^2}} \qquad \sin\varphi = \frac{y}{\sqrt{x^2 + y^2}} \qquad \tan\varphi = \frac{y}{x}$$

$$\cos\theta = \frac{z}{r} = \frac{z}{\sqrt{x^2 + y^2 + z^2}}$$

Conversion from spherical to cylindrical coordinates:

Spherical [r, θ, φ] ⟶ Cylindrical [ρ, φ', z']

ρ = r . sin θ
φ' = φ
z' = r . cos θ

Conversion from cylindrical to spherical coordinates:

Cylindrical [ρ, φ, z] ⟶ Spherical [r, θ, φ']

$$r = \sqrt{\rho^2 + z^2}$$

θ = arctan(ρ/z)
φ' = φ