Захаров Сергей

1) $\hat{A}, \hat{B}$ эрмитовы, $\hat{L}$ - произвольный

а) $\int \varphi^* \hat{L}\hat{L}^+ \psi\, dq = \int (\hat{L}^+\varphi)^* \hat{L}^+ \psi\, dq = \int (\hat{L}\hat{L}^+\varphi)^* \psi\, dq \Rightarrow \langle \varphi | \hat{L}\hat{L}^+ | \psi \rangle = \langle \hat{L}\hat{L}^+ \varphi | \psi \rangle$ ч.т.д.

$\int \varphi^* \hat{L}^+\hat{L} \psi\, dq = \int (\hat{L}\varphi)^* \hat{L}\psi\, dq = \int (\hat{L}^+\hat{L}\varphi)^* \psi\, dq$ ч.т.д.

б) $\langle \varphi | \hat{L} + \hat{L}^+ | \psi \rangle = \langle \varphi | \hat{L} | \psi \rangle + \langle \varphi | \hat{L}^+ | \psi \rangle = \langle \varphi | \hat{L}^+ | \psi \rangle + \langle \varphi | \hat{L} | \psi \rangle = \langle \varphi | \hat{L}^+ + \hat{L} | \psi \rangle$ ч.т.д.

По условию $\hat{L}$ и значит $\hat{L}^+$ линейны => перестановки легальны

в) $\langle \varphi | i(\hat{L} - \hat{L}^+) | \psi \rangle = \langle \varphi | i\hat{L} | \psi \rangle - \langle \varphi | i\hat{L}^+ | \psi \rangle = \langle -i\hat{L}^+\varphi | \psi \rangle + \langle i\hat{L}\varphi | \psi \rangle = \langle i(-\hat{L}^+ + \hat{L})\varphi | \psi \rangle$ ч.т.д.

эрмитовы по усл.

г) $\langle \varphi | \hat{A}\hat{B} + \hat{B}\hat{A} | \psi \rangle = \langle \varphi | \hat{A}\hat{B} | \psi \rangle + \langle \varphi | \hat{B}\hat{A} | \psi \rangle = \langle \hat{A}\varphi | \hat{B}\psi \rangle + \langle \hat{B}\varphi | \hat{A}\psi \rangle = \langle \hat{B}\hat{A}\varphi | \psi \rangle + \langle \hat{A}\hat{B}\varphi | \psi \rangle = \langle (\hat{B}\hat{A} + \hat{A}\hat{B})\varphi | \psi \rangle$ ч.т.д.

д) $\langle \varphi | i(\hat{A}\hat{B} - \hat{B}\hat{A}) | \psi \rangle = \langle \varphi | i\hat{A}\hat{B} | \psi \rangle - \langle \varphi | i\hat{B}\hat{A} | \psi \rangle = \langle -i\hat{A}\varphi | \hat{B}\psi \rangle + \langle i\hat{B}\varphi | \hat{A}\psi \rangle = \langle \hat{B}(-i)\hat{A}\varphi | \psi \rangle + \langle \hat{A}\, i\hat{B}\varphi | \psi \rangle =$

$= \langle -i\hat{B}\hat{A}\varphi | \psi \rangle + \langle i\hat{A}\hat{B}\varphi | \psi \rangle = \langle i(\hat{A}\hat{B} - \hat{B}\hat{A})\varphi | \psi \rangle$ ч.т.д. Числа с операторами коммутируют

2) $\langle \hat{L}\hat{L}^+ \rangle = \int \psi^* \hat{L}\hat{L}^+ \psi\, dq = \int (\hat{L}^+\psi)^* \hat{L}^+\psi\, dq = \langle \hat{L}^+\psi | \hat{L}^+\psi \rangle$ а это норма $\psi' \geq 0$

$\langle \hat{L}^+\hat{L} \rangle = \int \psi^* \hat{L}^+\hat{L} \psi\, dq = \int (\hat{L}\psi)^* \hat{L}\psi\, dq = \langle \hat{L}\psi | \hat{L}\psi \rangle$ тоже норма ≥ 0 ч.т.д.

3) $[\hat{A}, \hat{B}\hat{C}] = \hat{A}\hat{B}\hat{C} - \hat{B}\hat{C}\hat{A}$ осталось это посчитать

Посмотрим $[\hat{A}, \hat{B}] = \hat{A}\hat{B} - \hat{B}\hat{A}$, умножим справа на $\hat{C}$ чтобы получить это: $[\hat{A}, \hat{B}]\hat{C} = \hat{A}\hat{B}\hat{C} - \hat{B}\hat{A}\hat{C}$

Аналогично $[\hat{A}, \hat{C}] = \hat{A}\hat{C} - \hat{C}\hat{A}$ $|\cdot$ слева на $\hat{B}$: $\hat{B}[\hat{A}, \hat{C}] = \hat{B}\hat{A}\hat{C} - \hat{B}\hat{C}\hat{A}$, сумма убьёт ненужный член $\Big| \Rightarrow$

$\Rightarrow \boxed{[\hat{A}, \hat{B}\hat{C}] = [\hat{A}, \hat{B}]\hat{C} + \hat{B}[\hat{A}, \hat{C}]}$

4) а) $[\hat{p}_i, \hat{p}_j]\psi = (\hat{p}_i\hat{p}_j\psi - \hat{p}_j\hat{p}_i\psi) = \hbar^2\left(-\frac{\partial}{\partial x_i}\left(\frac{\partial}{\partial x_j}\psi\right) + \hbar^2 \frac{\partial}{\partial x_j}\left(\frac{\partial}{\partial x_i}\psi\right)\right) = 0$ коммутатор 0

б) $[\hat{p}_i, \hat{r}_j]\psi = (\hat{p}_i\hat{r}_j\psi - \hat{r}_j\hat{p}_i\psi) = -i\hbar\left(\frac{\partial}{\partial x_i}(r_j\psi) - r_j\frac{\partial}{\partial x_i}\psi\right) = -\delta_{ij}\cdot i\hbar\psi$ коммутатор $-i\hbar\delta_{ij}$

Если $i=j$: $-i\hbar\left(\frac{\partial}{\partial x_i}(r_i\psi) - r_i\frac{\partial}{\partial x_i}\psi\right) = -i\hbar\left(\psi + r_i\cancel{\frac{\partial\psi}{\partial x_i}} - r_i\cancel{\frac{\partial\psi}{\partial x_i}}\right) = -i\hbar\psi$

Если $i \neq j$: $-i\hbar\left(\cancelto{0}{\frac{\partial r_j}{\partial x_i}}\cdot\psi + \cancel{\frac{\partial\psi}{\partial x_i}}r_j - r_j\cancel{\frac{\partial\psi}{\partial x_i}}\right) = 0$

4) в) $[\hat{r}_i, \hat{r}_j] * \Psi = \hat{r}_i(\hat{r}_j \Psi) - \hat{r}_j(\hat{r}_i \Psi) = \hat{r}_i \hat{r}_j \Psi - \hat{r}_i \hat{r}_j \Psi = 0$ Коммутатор 0

Коммутатор

г) $[\hat{p}_x, f(x)]\Psi = \hat{p}_x(f(x)\Psi) - f(x)\hat{p}_x\Psi = -i\hbar\left(\frac{\partial}{\partial x}(f(x)*\Psi) - f(x)\frac{\partial}{\partial x}\Psi\right) = -i\hbar\left(\Psi\frac{\partial f(x)}{\partial x} + f(x)\frac{\partial\Psi}{\partial x} - f(x)\frac{\partial\Psi}{\partial x}\right) = \boxed{-i\hbar\frac{\partial f(x)}{\partial x}}\Psi$

д) $[\hat{p}_x^2, f(x)]\Psi = \hat{p}_x^2(f(x)\Psi) - f(x)\hat{p}_x^2\Psi = -\hbar^2\left(\frac{\partial^2}{\partial x^2}(f(x)\Psi) - f(x)\frac{\partial^2}{\partial x^2}\Psi\right) = \cancel{\hbar^2\left(\Psi\frac{\partial^2}{\partial x^2}f(x) + f(x)\frac{\partial^2}{\partial x^2}\Psi - f(x)\frac{\partial^2}{\partial x^2}\Psi\right)} =$

$= -\hbar^2\left(\frac{\partial}{\partial x}\left(\Psi\frac{\partial f(x)}{\partial x} + f(x)\frac{\partial\Psi}{\partial x}\right) - f(x)\frac{\partial^2}{\partial x^2}\Psi\right) = -\hbar^2\left(\frac{\partial\Psi}{\partial x}\frac{\partial f(x)}{\partial x} + \Psi\frac{\partial^2 f(x)}{\partial x^2} + \frac{\partial f(x)}{\partial x}\frac{\partial\Psi}{\partial x} + \cancel{f(x)\frac{\partial^2\Psi}{\partial x^2}} - \cancel{f(x)\frac{\partial^2\Psi}{\partial x^2}}\right) =$

$= -\hbar^2\left(\Psi\frac{\partial^2 f(x)}{\partial x^2} + 2\frac{\partial\Psi}{\partial x}\frac{\partial f(x)}{\partial x}\right)$ Коммутатор $-\hbar^2\left(\frac{\partial^2 f(x)}{\partial x^2} + 2\frac{\partial f(x)}{\partial x}\frac{\partial}{\partial x}\right)$

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