Journal of Natural Gas Science and Engineering 24 (2015) 365–378

Contents lists available at ScienceDirect

Journal of Natural Gas Science and Engineering

journal homepage: www.elsevier.com/locate/jngse

Invited review

Gas storage valuation and optimization

Cyriel de Jong

KYOS Energy Consulting BV, Nieuwe Gracht 49, 2011 ND Haarlem, The Netherlands

ARTICLE INFO

Article history:
Received 16 October 2014
Received in revised form
23 March 2015
Accepted 24 March 2015
Available online 11 April 2015

Keywords:
Gas storage
Hedging
Valuation
Stochastic optimization

ABSTRACT

In this paper we review the pricing and optimization of natural gas storage in competitive natural gas markets. Over the past decade valuation approaches have been suggested. Of those approaches, the most general ones are based on Monte Carlo price simulations, allowing the evaluation of different market trading strategies and different assumptions about the underlying price process. In a simulation exercise we first demonstrate that the impact of parameter (e.g. volatility) uncertainty on storage value is relatively limited. Inevitably, different market parameters lead to different storage values, but the trading strategy is relatively robust for a reasonably wide range of market parameters. Parameter uncertainty is also evaluated in a large-scale backtest of different storage trading strategies. The backtest of three different virtual gas storage types in the UK market provides a unique insight in how spot optimization combined with forward hedging would have fared over the past 17 years. On average, the estimated storage value is realized with a combination of spot optimization and dynamic delta hedging in the forward market. Dynamic intrinsic hedging of the spot exposures works relatively well too, but less so than delta hedging.

1. Gas storage assets role in the gas value chain

Storage plays a vital role in competitive natural-gas markets, because the average variability in the consumption of natural gas is much greater than the average variability in production. Demand is not only fluctuating, but often also at a considerable distance from the production sources. In both North-America (US + Canada) and Europe (OECD countries) natural gas storage capacity measured by working volume is around 18% of total consumption (IEA, 2012). Flexibility in the gas supply is also provided by production variations, pipeline and LNG transportation, but gas storage takes a large share of flexibility in many demand areas. For example, the US natural gas production has sharply increased due to the shale gas revolution in the past 5–10 years. The locations of production were not always well connected to the traditional demand areas, which has boosted investments in both transportation and storage (IHS, 2013).

For both the optimal planning of investments and the operation of existing facilities, it is necessary to fully understand the value drivers of natural gas storage. This article provides a review of the gas storage valuation literature and the relevant issues concerning different methodologies. This is combined with a backtest analysis, which provides very insightful results about the actual ability to monetize the storage value in trading markets.

The insights from this article can be applied to both physical storage assets and to storage services. In any case, a gas storage (asset or product) has three main operating characteristics: working gas volume, withdrawal rate and injection rate. The working gas volume is the capacity which can be actively used in cycling the gas through the storage in several days, weeks or months. Another part of the storage volume, the cushion gas volume, is needed to maintain enough pressure, but is not used operationally; it may be a big portion of the initial investment though. The withdrawal or send-out rate defines the volume which can be withdrawn, often expressed per day or hour. It may be volume-dependent with lower rates when there is relatively little gas in the storage asset. Likewise, the send-out rate, the third primary storage parameter is often decreasing when the storage is almost full. Other important storage parameters are the variable costs for injection and withdrawal, the maintenance and support costs for operating the facility, and of course the location of the storage.

Natural gas storage assets have been constructed in different geological structures, which are often categorized in the following

E-mail address: dejong@kyos.com.

http://dx.doi.org/10.1016/j.jngse.2015.03.029

three groups: empty oil and gas fields, salt caverns and aquifers. Empty oil and gas fields generally provide the largest working volume, i.e. the volume which can effectively be used for making cycles of injection and withdrawal in a year. Albeit being large in working volume, the deliverability, measured by the withdrawal or send-out rate, is often comparatively low. This means that former oil and gas fields are mostly used for providing seasonal flexibility, having the ability to cycle once or at most twice a year. Aquifers and salt caverns tend to have smaller working volumes but (much) higher deliverability. The right mix of seasonal and high-cycle storage assets, in combination with other sources of supply flexibility, is important for demand areas to absorb fluctuations in natural gas demand in a variety of market conditions. The larger and slower storage assets help to bridge the demand differences between seasons, for example fill up in summer and deliver in winter (heating). In turn, high-cycle storage assets, often in combination with LNG storage tanks, provide quick short-term security of supply, needed during a series of cold winter days (heating) or hot summer days (air-conditioning creating high power demand).

In liberalized markets, the natural-gas storage service is unbundled from the production, sales and transportation services. This means that storage is offered as a distinct, separately charged service under different regulations of third-party access. When there is a sufficiently liquid market for spot and forward or futures trading, market players can adjust their trading and operating decisions to the price signals. This allows them to benefit from price spreads and price movements (volatility).

Market players tend to own or contract natural gas storage flexibility primarily for managing the fluctuations in their own portfolio. In areas with limited or no trading possibilities, storage capacity is inefficiently used, because every player has to secure sufficient flexibility in his own portfolio. Thanks to liberalization, market players may not have to find the perfect balancing within their own portfolio. They may be net short or long flexibility and make their ultimate operational decisions based on a combination of internal flexibility sources and market prices. This leads to a more efficient use of available capacities for the market as a whole. The ongoing liberalization process in the European markets, and the improved decision making processes within the energy companies, is therefore one of the explanations of the lower price volatility and lower winter–summer spreads, especially on the continental markets. This has gone hand in hand with larger trade volumes and lower profitability of storage assets. From the viewpoint of system security of supply this may be a dangerous equilibrium: under normal market conditions the available storage capacity is efficiently used and available at relatively low cost. However, over longer horizons and in unusual market conditions there may be a shortage of storage flexibility in the system. This is a general policy maker's concern in liberalized markets which require long-term investments. The perfect policy mechanism for dealing with such potential under investments does not exist and individual countries have adopted a variety of approaches, ranging from holding 'strategic' reserves to investment subsidies or obligations on supply companies to contract a minimum level of storage capacity. All such mechanisms may increase security of supply, but introduce other market inefficiencies, both in the operational use of capacities and in new investments (Chaton et al., 2008; Redpoint Energy, 2013).

In a situation of liquid gas markets, the value of gas storage can be primarily derived from market prices. It depends much less so on the individual portfolio of single players, since all players have the ability to trade in the common marketplace. The valuation approaches which we discuss in the next section all take a market oriented approach, and all rely on a specific underlying financial economic model. An important element in those models is the dynamics of the price process. We provide an analysis of model risk and parameter uncertainty in Section 3. The final main section discusses the backtest. The backtest is essentially a very practical review of storage valuation in which all the topics of previous sections are reviewed from a practical pricing and optimization perspective. In Section 5 we conclude.

2. Valuation approaches to gas storage

There are basically four valuation approaches to natural gas storage: intrinsic, rolling intrinsic, basket of spreads and spot trading (see e.g. Boogert and de Jong, 2011). We discuss each approach individually, starting with the intrinsic calculation. It takes the current forward curve, calculates the optimal trades in the forward market and the corresponding cash-flows. The search for the optimal trades in the forward market includes all trades whose flows can be backed by the storage. This is the asset-backed trading principle. When the storage has volume-dependent injection or withdrawal rates, there may actually be no trades which can be exactly absorbed by the storage and at the same time use the storage capacities fully. In such situations it is common practice to calculate the optimal trades on a daily basis and then to spread the volumes over the products which are actually traded, such as months, quarters and seasons. In general, the intrinsic value, if it can be traded in the market, provides an immediate value and forms the lower bound to what can be actually achieved. The optimization for the intrinsic calculation can be based on linear-programming (possibly with integers) or on dynamic programming.

The rolling intrinsic approach is very similar to the intrinsic, but also considers profits of rebalancing the portfolio over time. At every rehedge date a new intrinsic optimization is executed, but including an initial position. This initial position is taken from the intrinsic optimization and any subsequent rebalancing trades. The rolling intrinsic trading strategy is relatively popular among traders, because it is a safe strategy (the profit cannot go below intrinsic) which can be easily explained to others. In order to judge the potential future value of the rolling intrinsic approach, a representative set of potential future market price developments has to be simulated. For each simulation and each rehedge date the rolling intrinsic approach requires a separate intrinsic optimization, which may make it somewhat slow to calculate. The estimated future roll profits, averaged over the simulations, largely depend on the methodology to describe the forward price dynamics. Roll trades are only profitable if the portfolio can be rebalanced, which is typically when a forward spread changes sign. Such spread sign reversals tend to happen in the shorter end of the forward curve. In any case, a fair rolling intrinsic valuation depends heavily on a realistic price process. Hence, a multi-factor price model, with multiple stochastic factors, is needed. The first (known) description of rolling intrinsic for gas storage is in Gray and Khandelwal (2004). Another article describing this approach is from Bjerksund et al. (2011). It should be noted though that they overestimate the benefits of this approach, mainly because they ignore the requirement that forward contracts should be actually traded.

The basket of spreads approach treats a gas storage as a set of time spread options. As a simple example, suppose four quarters ahead can be traded and the intrinsic strategy is to buy the Jul–Sep forward and sell the same volume in the Jan–Mar forward of the following year. Then at any future date until end of June, whenever the Oct–Dec forward becomes cheaper than the Jul–Sep forward, a trader can roll the two products (sell Jul–Sep, then buy Oct–Dec). This is also the essence of the rolling intrinsic strategy. Whereas the rolling intrinsic strategy evaluates many potential scenarios to find out if a roll (and other rolls) are profitable, the basket of spreads values the optionality of a roll directly. Under certain assumptions similar to the ones underlying the Black–Scholes formula the spread options can be valued with the Margrabe's formula or variations of it (Margrabe, 1978; Kirk, 1995; Li et al., 2008; Lo, 2014). The basket of spreads approach (Eydeland and Wolyniec, 2003; Manoliu, 2004; Lai et al., 2010) can be regarded as a simplification of the rolling intrinsic. It has a number of important restrictions, in particular that the storage asset should be described as a set of non-path-dependent options. In our simple example, there are potentially more options to be exercised, but the potential exercise of one option may hinder (or allow) the exercise of another option. As a result, only few spread options can be considered and the approach undervalues the true storage value.

Eventually, traders can take operating decisions on at least a daily basis. In the presence of spot trading markets, this is why the true storage value should include a spot trading component. Within the spot trading valuation approaches, three solution approaches can be distinguished: tree building, stochastic control and least-squares Monte Carlo. Each has its own advantages and disadvantages, but essentially rely on the same principles. That is, all approaches model the dynamics of the underlying gas prices and find the optimal spot trading actions while taking into account future optionality. In general, the real option principle for storage assets is to find the right balance between immediate cash-flows and the creation of flexibility for higher future cash-flows. A simple example may help to clarify the main concept. Suppose that we have a gas storage with 100 therm working volume and 1 therm withdrawal rate per day; it is filled with 1 therm and the forward curve is flat at a price of 50 p/th. When the spot price is 50.5 p/th, an intrinsic strategy will advise to withdraw spot and sell at the forward price, thus locking in 0.5 pence profit (ignoring costs and discounting issues). However, releasing the gas today means the lost optionality to inject the next day and later (unless we inject again). Therefore, depending on price volatility, it may be optimal to keep the single therm in store, thereby creating more flexibility for the future.

In a tree based approach (Manoliu, 2004; Felix and Weber, 2012; Parsons, 2013) the spot price is described by upward and downward movements, each of which has a (possibly time-varying) magnitude and probability. In order to capture the mean-reversion and possibly other spot price dynamics, the tree should be constructed differently than in a standard binomial recombining tree used for many stock options (Cox et al., 1979). To capture the mean-reversion, there are typically three possible movements from a node to another node (a node is a price on a specific day) required. In each node of the price tree, the expected continuation value is stored for a discrete set of possible gas storage volumes. Then at any node a backward procedure determines which is the optimal storage action (injection, withdrawal, no action) which yields the highest sum of expected continuation value and immediate cash-flow. This backward procedure can be relatively fast. The main drawback is the limited flexibility to model prices in a tree.

Another optimization approach to the spot trading decisions can be grouped under the name stochastic control. This type of approach is described in Thompson et al. (2009), Carmona and Ludkovski (2010), and Chen and Forsyth (2010). For example, Chen and Forsyth solve Hamilton–Jacobi–Bellman equation defining the stochastic control problem of the gas storage. The stochastic control approach is rather elegant, but does not (yet) seem to be widely adapted.

In contrast, the least-squares Monte Carlo (LSMC) approach has become a popular approach in the field of derivatives, with gas storage as a special application. It is actually rather similar to the tree based approaches, with the advantage that there is complete flexibility in defining the gas price model. As long as the gas price model can be discretized and able to generate Monte Carlo simulations, the LSMC can be implemented. The LSMC approach has been popularized by Longstaff and Schwartz (2001), who actually give the main credits of the method to Carriere (1996). These early papers describe the valuation of American (financial) options with typically a single exercise. De Jong and Walet (2003) were the first to show how the LSMC can be applied to the problem of gas storage valuation. This short article is followed by more extensive mathematical descriptions and quantitative analysis in Boogert and de Jong (2008, 2011), as well as Schlüter and Davison (2010). The methodology bears great similarity to the tree building approach, with the main difference that the expected continuation values are not derived from nodes in the price tree. Instead, the storage LSMC performs a linear regression on each day, bundling the information from the cross-section of the simulated paths. Similar to the tree building approach the calculation of expected continuation values is executed on a discrete grid of volume points. To generalize the method and allow for movements to any possible volume level, Boogert and de Jong (2008) propose a linear interpolation scheme. In Boogert and de Jong (2011) it is shown that the scheme works well in combination with a multi-factor model for the price process. All of the spot trading approaches can essentially be combined with a multi-factor price process, but the flexibility of Least-squares Monte Carlo makes this especially easy. The paper also shows the quick convergence and calculation times of just a few seconds on ordinary computers, providing an extra explanation for the popularity of this approach.

3. The impact of model risk

The gas storage valuation literature has mainly focused on valuation algorithms of gas storage. In practical applications, however, the selection of the gas price model and the estimation of parameters poses a significant challenge. As De Jong and Walet (2003) already pointed out: changes in volatility and mean-reversion have a great impact on the estimated storage value.

As an example of model risk analysis, Boogert and de Jong (2011) use a single-factor model in the backward optimization and a multi-factor model for the forward simulation. It is demonstrated that the suboptimal price process in the backward loop leads to a significant value reduction in the forward loop. Other articles addressing the issue of model risk are Secomandi et al. (2010) and Henaff et al. (2013).

Model risk relates to uncertainty in the true parameters of the model and uncertainty in the model specification. The analysis of Boogert and de Jong (2011) addresses the second type of model risk. However, because the single factor model is nested in the multi-factor model, one could also label it an extreme case of parameter uncertainty: some parameters are incorrectly estimated to be zero in the backward loop. Our analysis focuses on the first type of model risk, in a less extreme and hence more realistic setting than the aforementioned paper. More precisely, we set up a scheme of parameter re-estimation which leads to a realistic distribution of parameters around the 'true' parameter set. Throughout this section we apply the Least Squares Monte Carlo (LSMC) method developed by Boogert and de Jong (2008, 2011), and using the KYOS commercial software product KyStore for all calculations. For more details about the model equations and parameters we refer the interested reader to both papers.

In general, parameter uncertainty (PU) can lead to increasing uncertainty in the distribution of gas storage values. Potentially, it can also make the trading strategy suboptimal and thereby reduce the expected value of the gas storage. Our findings point out that parameter uncertainty hardly affects the expected value of gas storage though. Nevertheless, it systematically increases the higher moments of the gas storage distribution. The former result may initially be counter intuitive, but asserts the robustness of LSMC in gas storage valuation, while the latter result shows gas storage distribution behavior under parameter uncertainty.

3.1. Methodology

The LSMC method computes the value of gas storage in two well-defined steps. The first step is the computation of an optimal decision rule by using a dynamic programming approach on simulated gas price paths. This is the backward loop and leads to regression parameters which define the expected storage value as a function of gas prices and inventory level. The second step is the application of the optimal decision rule on new simulated paths of the commodity at hand, called forward loop. The latter step generates the distribution of gas storage values.

Parameter uncertainty can be introduced in this procedure by assuming that the parameters used to generate Monte Carlo gas price paths in the forward loop differ from those used in the backward loop. As a result, the decision rule calculated in the backward loop will no longer be optimal in the forward loop due to biased simulated paths in the latter step. Assume a general gas price stochastic process $P(\cdot)$, which can depend on some known or noisy parameters values θ or $\widetilde{\theta}$, respectively. We can generate N_s paths with $P(\theta)$, and compute the decision rule in the backward loop. After that, new paths have to be generated with $P(\widetilde{\theta})$ and then used in the value distribution computation in the forward loop.

We thought about three different ways to generate $\widetilde{\theta}$. The first one is to re-estimate θ from each simulated path generated in the backward loop. This yields N_s estimates of θ, i.e. $\widetilde{\varphi}_{ES} = \{\widetilde{\theta}_{ES,1}, \ldots, \widetilde{\theta}_{ES,N_s}\}$. The second one is to draw randomly N_s parameters form the estimator's distribution of θ, which yields $\widetilde{\varphi}_{DI} = \{\widetilde{\theta}_{DI,1}, \ldots, \widetilde{\theta}_{DI,N_s}\}$. The third one is to draw randomly N_s parameters form a uniform distribution with domain in a chosen interval around θ, which generates $\widetilde{\varphi}_{UN} = \{\widetilde{\theta}_{UN,1}, \ldots, \widetilde{\theta}_{UN,N_s}\}$. Note that $E[\widetilde{\varphi}_i] = \theta,\ \forall_i\ i \in [ES, DI, UN]$, and that each element in $\widetilde{\varphi}_i$ is used to generate one new simulated path of the stochastic process in the forward loop. Hence, $\widetilde{\varphi}_i$ is used to generate N_s new paths for the forward loop calculation. The following analysis focuses on the re-estimation methodology, because it does not assume a specific distribution for the parameters, so is more general. We also evaluated the other two methodologies and verified that they lead to similar results.

The stochastic process adopted in the subsequent analysis is a generalization of the one-factor Schwartz model, Schwartz (1997). We define the logarithm evolution of the spot gas price as follows (Boogert and de Jong, 2008):

$$\mathrm{d}\ln(P(t)) = k\left[\ln(F(t)) - \ln(P(t)) - \frac{\sigma^2}{2k}\right]dt + \sigma\, dW(t)$$

where $P(t)$ is the spot gas price, $F(t)$ is the forward gas price, $dW(t)$ is a Brownian motion, k and σ are the mean reversion and diffusion parameters, respectively. Accordingly, $\theta = [k, \sigma]'$ and $\widetilde{\theta}_{ES,j} = [\widetilde{k}_{ES,j}, \widetilde{\sigma}_{ES,j}]'$, $j \in [1, \ldots, N_s]$, which are estimated with ordinary least squares (OLS).

Boogert and de Jong (2011) describe how the forward price dynamics can be modeled, thus creating a multi-factor specification. They introduce a long-term volatility which shifts the whole curve up or down, and a volatility of the winter–summer spread, which creates a widening or narrowing of the difference on the forward price spread between winter and summer. In the next section we set the forward volatilities to zero, so the focus is on the uncertainty in the spot parameters only. In the chapter discussing the backtest, the full multi-factor model is used.

3.2. Results

The storage contract under analysis is for 1 year (1 April 2011–1 April 2012), with a working volume of 50,000 MWh, a daily injection rate of 834 MWh and withdrawal rate of 1667 MWh. This corresponds to 60 days injection and 30 days withdrawal and can be characterized as a medium-cycle storage. The initial forward curve used for the valuation has an average price level of 27.10 €/MWh and a difference between winter (Q1-2012) and summer (Q3-2011) of 2.01 €/MWh The number of simulations is 10,000 and the assumed known parameters are k = 15% per day and σ = 30% per year.

The analysis is applied to a spot storage valuation with and without static delta hedge. The delta hedge strategy is defined as a hedge position taken on top of the spot strategy at the beginning of the contract period and kept unchanged. We report four different cases: the first one is the case with no PU and no delta hedge (No PU), the second one is with PU but no delta hedge (Yes PU), the third one has no PU but with delta hedge (No PU H), the forth one with PU and with delta hedge (Yes PU H).

Table 1 shows the first four gas storage distribution moments in the four different cases. The expected values with and without hedge are the same, because the hedge has an expected profit of 0. However, the standard deviations of the hedged results are smaller because the hedge reduces the price exposure. When parameter uncertainty is taken into account, the expected storage value slightly increases. Nonetheless, this expected value increment is only 0.13% of the storage value, which is basically negligible. Such a result is counter intuitive at first sight: we would have expected a lower value of the storage due to a slightly suboptimal trading strategy. The explanation of this

Table 1
Gas storage moments. This table shows the first four moments of gas storage distribution for four different situations. The first one is the case with no PU (No PU), the second one is with PU (Yes PU), the third one does not have PU but it has a static delta hedge strategy (No PU H) and lastly a scenario with PU and with static delta hedge strategy (Yes PU H).

Moment	No PU	Yes PU	No PU H	Yes PU H
Mean	251,270	251,597	251,270	251,597
Standard deviation	24,610	26,295	15,415	17,470
Skewness	−0.06862	0.11751	0.28298	0.55022
Kurtosis	3.29066	3.50519	3.49258	3.87109

phenomenon resides in the shape of gas storage value surface as a function of k and σ. The convex dependence of storage value on spot volatility (σ) means that different levels of σ lead to a higher storage value on average. More generally, the expected value of a convex function is greater than the value function evaluated at the expectation of its domain (the range of noisy parameters in our case). Internal tests show such a feature in different settings too. It is possible to separate the PU impact from the convexity effect if wanted. However, this would be a mere mathematical exercise and the main result would not change, i.e. the expectation change is minimal. This leads us to state that the LSMC method is fairly robust in computing the expected value even if parameter uncertainty is taken into account.

The insertion of parameter uncertainty substantially increases standard deviation, skewness and kurtosis. The standard deviation increases by 1685 (no hedge) and 2055 (hedge). With a hedge the increase is percentage wise even larger: it is +6.85% (no hedge) and 13.33% (hedge). So, parameter uncertainty leads to uncertainty in the distribution of storage value, and this uncertainty cannot be hedged; in our example the hedge becomes even slightly less effective. The introduction of PU leads to a positively skewed distribution in the case without hedge, while it makes the distribution even more positively skewed in the hedge case. The gas storage distribution always becomes more leptokurtic when PU is taken into account (note: a value of 3 implies no excess kurtosis relative to the normal distribution).

These features can also be seen in Fig. 1. The graph shows the non-parametric Gaussian kernel distribution estimates of the four different cases. The graph points out the presence of a larger right tail due to parameter uncertainty. In the author's opinion, this can be deriving from those scenarios when $\tilde{k}$ is lower than k, increasing the value of this particular storage, and those situations when $\tilde{\sigma}$ is higher than σ, thus increasing the value of the storage. The asymmetric impact on the tails can derive from the convexity effect.

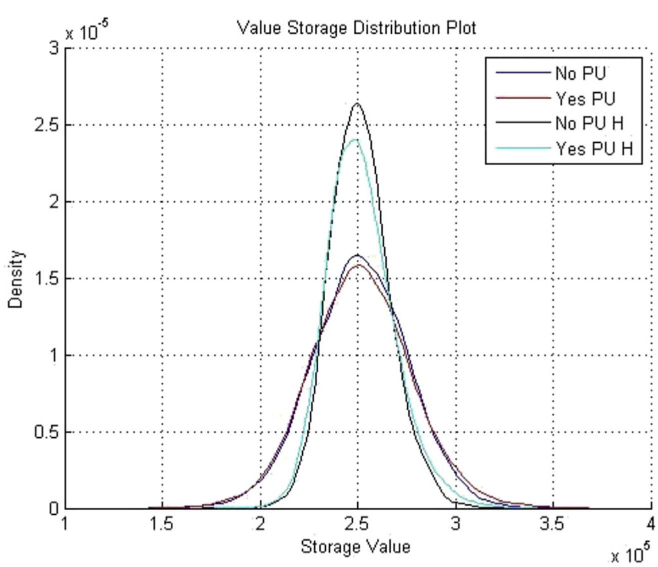

Fig. 1. Gas storage vale distribution. This graph shows the value distribution of gas storage in four different scenarios. The first one is the case with no parameter uncertainty (No PU), the second one is with parameter uncertainty (Yes PU), the third one is with no PU but with a static hedge strategy (No PU H) and the fourth one is with PU and with a static hedge strategy (Yes PU H).

3.3. Conclusion model risk

Several internal tests with different settings, such as slower and faster storage assets as well as different parameters values, show similar features and patterns in the behavior of gas storage distribution. To conclude, we can state that the expected value of gas storage, assuming a realistic distribution around the true model, is hardly affected by model risk. However, model risk makes the value uncertainty, the higher moments of the distribution, larger.

4. Backtesting gas storage valuation approaches

4.1. Purpose of a backtest

Large investments are made in gas storage facilities in order to balance supply and demand and to make profits in the trading market. Whereas the ultimate use of the storage is to balance physical gas of various users, the majority of storage valuation approaches assumes a market trading strategy. This is for a good reason, because it leads to a consistent valuation framework, more or less available to everyone in the marketplace, and independent from company-specific portfolios.

Different approaches have been suggested in the literature to value gas storage assets against market prices and assuming different trading approaches. The approaches can be evaluated, for example, based on the underlying economic model and ease of implementation. Another form of evaluation is to assess their actual ability to support valuation and trading decisions. For this second type of evaluation, a backtest is well suited. Backtesting can be generally applied to testing a trading strategy or the outcome of an analytical model if it had been employed in the past.

A backtest does not only provide an assessment of a particular storage valuation methodology, it also provides wider insight in storage trading strategies, the effectiveness of hedging, and the impact of uncertainty in estimating market parameters. Finally, the backtest provides interesting insight in the historical development of gas market dynamics (seasonal spreads and volatilities) and their impact on gas storage values over time. For these many reasons, this section provides an extensive review of a gas storage backtest. The primary methodology under consideration is the Least-squares Monte Carlo approach, but this is combined with various forward trading strategies, rolling intrinsic and delta hedging.

Pricing models for financial assets try to estimate the fair value of the asset in future market conditions. In the case of gas storage assets, the pricing is derived from the current forward curve, an expected future evolution of market prices, and an optimal trading strategy in those market conditions. The first two elements, the forward curve and the price dynamics, are inputs to a model which derives the optimal trading strategy. Such a pricing exercise can be carried out on day t for a future horizon until day T. Ideally in a backtest, the expected value on day t is exactly realized over the storage period [t, T]. For example, if we value a one year storage product on 1 April 2013, we hope to realize exactly that value by trading according to the storage model's strategy between 1 April 2013 and 1 April 2014.

Apart from a flaw in the underlying economic model, there are several reasons for a discrepancy between realized and expected value. First of all, the initial valuation gives the value that can be realized *on average*. In individual scenarios the realized value may be quite different. This is mainly caused by price (level) risk, part of which may be successfully hedged in the market, but another part not. Secondly, the price *dynamics* may be different than expected. In particular, the price model itself may be correctly describing the gas price dynamics, but the parameter estimates are imperfect. For example, if the storage contract was priced with an assumed 50% spot volatility and the realized volatility is just 25%, we can probably not make as much money with the storage as expected. Finally, the price model itself may be imperfect. For example, if the price model assumes a static forward curve (single-factor model), whereas actually the curve changes shape over time (multi-factor model), the valuation will be inaccurate. All points are relevant and inevitably present in any analysis: a price model is always imperfectly describing reality (it is a model) and parameters of a model are unknown and only estimated, typically using a set of quite recent historical price data.

4.2. Backtest design and trading strategies

The gas storage valuation literature has paid limited attention to backtesting. One exception is the paper by Parsons (2013). The analysis shows how a tree based approach has been virtually implemented in the US natural gas market (Henry Hub) in the period 1999 to 2006. Our backtest implementation is similar, but with a couple of differences. First, our spot trading strategy is based on the least-squares Monte Carlo approach instead of the tree approach. In principle, under the same price assumptions both approaches should lead to the same operating decisions, so this difference can be neglected. Secondly, we have a larger dataset with 17 years and 3 storage types, giving more confidence that the results can be generalized to other years and markets. Finally, we include and analyze a wider variety of forward hedging strategies.

The backtest has been implemented as follows, for a total of 17 storage years in the UK NBP gas market:

1. At the beginning of each period (always starting on 1 April) a gas storage asset is valued for the upcoming storage year. This valuation is based on the forward price curve at the initial date and based on a set of market parameters (volatility, etc.).
2. Then every day in this storage year, an injection or withdrawal is made (hypothetically), and the gas bought or sold at spot prices as they were historically. This requires a new storage calculation every day in the storage year, based on the updated market price information (spot and forward prices). The storage calculation is a complete optimization and valuation for the remaining time in the storage year, but we only use the information which tells us how to trade in the spot market on that day. The recommended trades are 'virtually' carried out in the market and the recommended injection or withdrawal changes the gas storage volume for the next day.
   - A small example, focusing on the spot trading decisions, may be helpful: suppose that it is 10 April 2013 and the storage contains 50,000 therm of working gas, based on our actions from 1 to 9 April. The storage model makes a valuation over the period 10 April 2013 to 1 April 2014, based on the forward market prices of 10 April 2013. As part of the output, the model advises to withdraw 10,000 therm of the working volume when the spot price is above 40 p/th. When the spot price is 42 p/th, we withdraw the 10,000 therm and collect 4200 GBP, which is added to our cash balance (being positive or negative). If there are storage withdrawal costs, those are deducted from the cash balance. The next day, a new storage calculation is carried out with a starting volume of 40,000 therm. If we continue like this, then the model will lead us to a zero end volume on 1 April 2014. Along the way, the gas volume in store has gone up and down, and the accumulated cash balance has hopefully become positive at the end.
   - The example can be extended with forward or futures trading on 1 April 2013, the beginning of the storage period. We call this a static hedge, because it is not (yet) adapted during the year. Suppose that the storage exposure is hedged by buying 10 mln therm in Q3-13 at a price of 40 p/th and simultaneously selling 10 mln therm in Q1-14 at a price of 50 p/th. For ease of calculation, this is best regarded as a separate position which does not affect the spot trading actions. At the end of the storage period, the hedge position has its own profit and loss (P&L). For the long position (Q3-13) it equals the average spot price in Q3-13 minus the forward price at initiation. For the short position, the calculation is similar, but with a minus sign. The hedge P&L is added to the spot trading cash balance. Typically, when the realized spot prices were relatively low in Q1-14 compared to Q3-13, the spot trading cash balance will be low, which is compensated for by a positive P&L from the hedge trades. In general, the forward hedge counterbalances part of the variations in the spot cash balance.
   - The example can further be extended with a dynamic strategy of forward or futures trading. Suppose that the storage calculation on 1 May 2013 reveals that the Q3-13 position of $+10$ mln therm should be sold, and moved to a Q4-13 position, which trades 1 p/th lower. Ignoring trading costs, this leads to an immediate cash income of 100,000 GBP (10 mln pence). Furthermore, at the end of the storage year, the financial P&L should be calculated on the Q4-13 position, not the Q3-14 position (which we sold). This dynamic rehedging can be carried out in certain time intervals, such as once a month.
3. At the end of the storage year, we compare the realized value (the cash balance) from step 2 with the estimated value from step 1. This is repeated for several storage years. For example, if we observe that the realized values are consistently lower than the anticipated values, then that is an indication that the pricing model is too optimistic, i.e. positively biased.

We consider three different storage assets, or actually small bundles: a very fast (high-cycle), a relatively fast (mid-cycle) and a rather slow (seasonal) bundle. All have a working volume of 100 therms, but different daily injection and withdrawal rates:

- Slow: injection = 1 therm/day, withdrawal = 2 therm/day (150 days cycling)
- Fast: injection = 2 therm/day, withdrawal = 4 therm/day (75 days cycling)
- Very fast: injection = 5 therm/day, withdrawal = 5 therm/day (40 days cycling)

We assume there are no injection or withdrawal costs, and no discounting is applied. In addition, in all calculations we assume no trading costs (no bid-ask spread). These assumptions make the results easier to interpret and compare, but of course lead to some overvaluation in both the estimated values and the realized values.

At the beginning of the storage year, the storage bundles are valued using three methodologies:

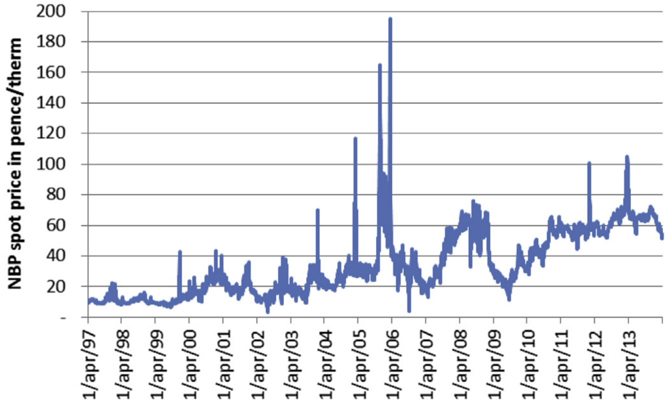

Fig. 2. NBP spot price development from 1 April 1997 to 1 April 2014.

- Monthly intrinsic, so the maximum value which can be locked in at current monthly forward prices. For example, with the very fast storage, which needs less than a month to inject and a month to withdraw, the monthly intrinsic strategy buys 100 therm of the cheapest forward month and sells 100 therm of the most expensive forward month. The fast storage strategy to buy one month extra to spread the injections (e.g. 60 in the cheapest and 40 in the second cheapest). The slow storage needs 4 months to inject and 2 months to withdraw.
- Monthly rolling intrinsic, with monthly rehedging. So, after the monthly intrinsic value has been locked in, every month the intrinsic value is calculated anew, and the portfolio is rebalanced if that is profitable. The rolling intrinsic value equals the intrinsic value at the start date plus the additional income from the roll trades during the year. In order to estimate the potential future value of the roll trades the model simulates forward and spot prices using a multi-factor model described in Boogert and De Jong (2011). The average across the simulations is the estimated value.
- Spot trading, based on the Least-squares Monte Carlo methodology. This requires Monte Carlo simulations of spot and forward prices as well, and we use the same multi-factor simulations as for rolling intrinsic.

The pure spot trading strategy can lead to very variables results. For example, when the storage is being filled during the summer, and the prices go down afterwards, the gas has to be sold at a loss in the autumn or winter. In practice, no sensible trader will just trade in the spot market and be fully exposed to spot market conditions. Therefore, the spot trading strategy is evaluated in combination with four different hedging strategies. The static intrinsic and the static delta hedge are traded at the beginning of the storage year and not rebalanced afterwards. The dynamic intrinsic and delta hedges have the same initial forward positions as their static counterparts, but are rebalanced once a month.

4.3. Historical price data for the backtest

For the generality of the backtest, we selected a gas market with a relatively long price history, which is the UK market (NBP). The dataset covers 17 storage years, with the first being 1 April 1997 to 1 April 1998 and the last period being 1 April 2013 to 1 April 2014. Fig. 2 displays the development of the spot price over this time period. It can be seen that the gas price has mostly trended upwards, with a temporary drop in 2007 and 2009 (mainly due to the financial crisis and US shale gas development).

A careful inspection of the graph also reveals that there is seasonality in the gas prices, with higher winter than summer prices. Fig. 3, displaying the average realized spot prices, shows this more clearly. It can be seen that the average spot prices were actually highest in December and January, and lowest in June and August. This is the typical winter–summer pattern in western Europe, and an important source of income for gas storage operators. Comparing the realized spot prices with the average front-month prices of the preceding month yields an interesting extra insight: for the winter months December to February the front-month forward price was on average 1.5 to 2.5 p/th higher than the realized spot prices. This may be considered a risk premium in the winter forward prices, though the result is not statistically significant at the 5% level and two years are mainly responsible for this difference: 2006 and 2007. Nevertheless, even before having the results of the backtest, it shows that forward hedging is important in order to stabilize (or increase) income. For the winter months, selling gas forward would have been more profitable than selling spot, at least on average. As shown in Fig. 3, selling the January gas at 1

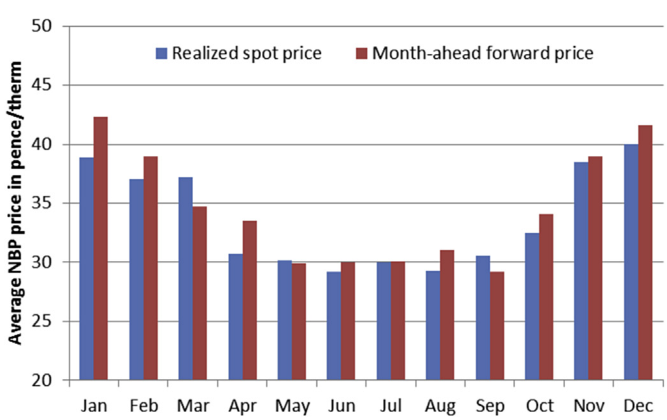

Fig. 3. NBP seasonal pattern from 1 April 1997 to 31 March 2014. For both time-series the prices have been averaged per month of delivery. For the front-month data, this means that the trading days were in the month prior to delivery.

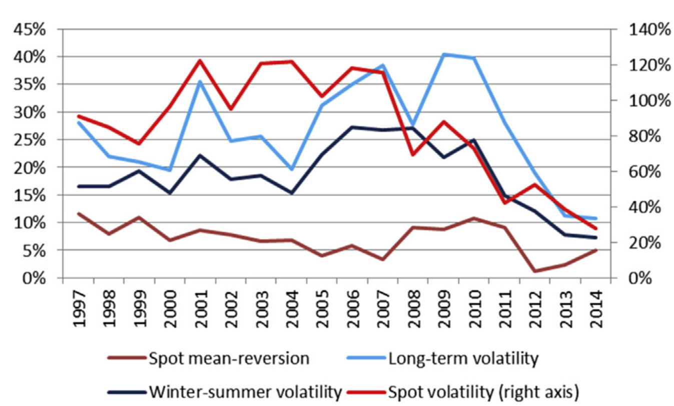

Fig. 4. Parameter estimates of the price simulation model using UK NBP market price data. Estimates are from a rolling window of 1 years, always with 1 April as the end date.

month ahead forward prices (i.e. in December) instead of at January spot prices, gave an average benefit of 3.42 p/th. In a separate calculation not visible in the graph, we verified that having sold 2 months prior to delivery the benefit would have been over 6 p/th, and having sold between 3 and 6 months ahead the benefit would have been even higher at 9 p/th. We will come back to these results when discussing the backtest results. But before that, the market parameters are explained in more detail.

Both the rolling intrinsic and the spot trading values rely on Monte Carlo price simulations. This creates a need for estimating the model parameters. The multi-factor model used for the backtest is the same as in Section 3.1. It contains four parameters: the long-term forward volatility (σ_L), the winter–summer forward volatility (σ_{WS}), the short-term (spot) volatility (σ) and the spot mean-reversion rates (k). All of the parameters can be estimated with ordinary least-squares on the basis of historical price data. We choose one year of history to match with the length of the storage year, but with two different selections:

- Out-of-sample: the sampling period precedes the storage year. This would be the set of information that you have at the date of valuation.
- In-sample: the sampling period equals the storage year. At the date of valuation this can be considered perfect foresight regarding the future price dynamics. However, comparing the out-of-sample estimated storage values with the in-sample based estimated allows us to evaluate the impact of parameter uncertainty on storage value. For example, if the estimated storage value is 40% lower than the realized value, it is interesting to know if that could be due to the parameters estimates being far off or the model not performing well.

The development of the parameter estimates is displayed in Fig. 4. It provides an interesting insight in the change of the UK gas market over the years. Most notable is the sharp decline in volatility in the last years, from 2009 onwards. One of the primary explanations is the increased diversity in supply sources, with more interconnections between European markets, European markets with Russia and from further abroad in the form of LNG. Another explanation is the lower demand for natural gas from power stations due to renewable production growth, cheaper coal (and CO_2 emission) prices; the power sector is one of the most volatile demand sectors.

The parameter estimates are used to make an initial valuation at the start of the storage year. During the storage year, when the storage trading actions are executed according to the storage model, the same model parameters are used. This ensures consistency between valuation and execution of the underlying trading model.

4.4. Initial valuation results

The estimated storage value has varied largely over time, as shown in Tables 2–5. The estimated storage value in the period 2005–2009 was several times higher than in the other years, both the intrinsic, rolling intrinsic and spot trading value. The primary explanation is the size of the winter–summer forward spread in the market: it was between 5 and 10 p/th initially, then rising up to 50 p/th in April 2006, then fell back to about 10–15 p/th in recent years. This dependence of storage value on winter–summer spread is most explicit for the intrinsic value. The rolling intrinsic value and, especially, the spot trading value also depend on the market price volatility. Nevertheless, commercial storage bundles, especially seasonal bundles, are regularly priced as a multiple of the forward winter–summer spread. For example, a particular storage bundle may be offered at a price equal to 1.3 times the spread between the Q1 (Jan–Mar) and Q3 (Jul–Sep) product of particular delivery years.

The estimated extrinsic value is, as expected, larger for the spot trading strategy than the rolling intrinsic, and primarily dependent on the spot volatility. That is why the winter–summer multiple pricing is sometimes conditional on the spot volatility being in a certain range.

The impact of spot volatility on storage value is very clear when we run a storage valuation and increase the spot volatility. In the backtest results, however, it appeared much more difficult to quantitatively explain the variations of the estimated storage value by the individual market parameters. There is actually a complex interdependence and multi-collinearity between forward spreads, spot and forward volatilities and the mean-reversion rate, which make it hard to attribute value changes to individual parameters.

The average multiple of estimated spot trading value over intrinsic value (i.e. the extrinsic value multiple) is 1.14, 1.25 and 1.29 for the monthly rolling intrinsic strategy, going from the slow storage to the very fast storage. The estimated spot trading value is a much higher multiple of the intrinsic value, especially for the fast storage. The numbers are 1.47, 1.85 and 2.15, again from slow to very fast storage.

4.5. Backtest realized values

This brings us to the primary question of the backtest: how well could the estimated value be realized? Or stated more formally: are the storage valuation methodologies efficient and unbiased estimates of the true storage value? Especially for the spot trading strategy, in private conversations market players have sometimes expressed their skepticism about the ability to attain the high extrinsic value. Tables 2–5 show all valuation results. We will discuss various interesting insights starting with the in-sample results.

On average, the realized rolling intrinsic value is rather well in line with its prediction. Over all the years and the three storage types, the realized value is 1900 pence versus a prediction of 1964 pence. Measuring the difference year by year and then taking the average, there is actually a plus of 2.9% in realized value relative to the prediction. While the prediction is on average quite right, there are rather good and rather bad individual years. Remember that the rolling intrinsic value (estimated and realized) cannot be below the intrinsic value, so a bad year should be seen in comparison to the expectation. Its standard deviation is 27.2%, with outliers below −20% and above +50%.

In the analysis of the realized spot valuation results, the focus is again on the difference with the estimated spot trading value. At first sight, the spot trading realization, without any forward hedging, seems very low. In certain cases, the spot trading would even have resulted in a net loss. That may not be too surprising, but the average performance is also really bad. The overall average realized value (over 17 years and 3 storage bundles) is 1492 pence. This is not only below the rolling intrinsic value, but even below the intrinsic value. After more careful analysis, there are good reasons for this discrepancy and the spot valuation with least-squares Monte Carlo is actually very accurate as long as we include a forward hedging strategy in the realization. First of all, forward hedging stabilizes the income from an otherwise speculative strategy. Secondly, forward hedging allows a trader to capture the risk premium in (winter) forward prices. Essentially, at the time of valuation (1 April) and also in the months

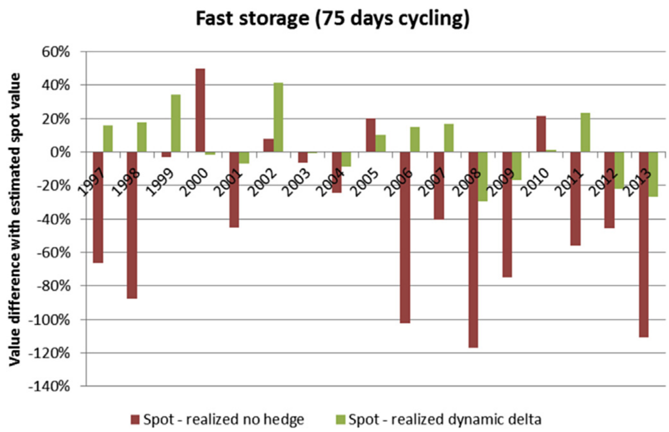

Fig. 5. Percentage difference between the realized value and the estimated spot trading value for each of the 17 storage years.

up to the start of the winter, the forward prices for the winter months were considerably higher than the realized spot prices. For example, the risk premium for the February product, comparing the forward value on 1 April of the previous year with the realized spot price of the subsequent February, is 8.33 p/th. For the July forward, the premium is 0.83 p/th, meaning that the assumed spread between February and July was on average 7.5 p/th lower in the spot market than expected[1] at the date of valuation. By setting up an initial hedge, the risk premium is largely locked in. This is visible in the results of the spot trading strategy with the static intrinsic and the static delta hedge. The average deviation relative to the estimated spot value, across 17 years and 3 storage bundles, is −38.2% without any hedge. With the static intrinsic hedge, the under-performance in the realizations is reduced to −21.4%. With the static delta hedge, this is further reduced to −14.5%. The reason for the better performance of static delta hedge, compared to static intrinsic hedge, can be explained by the sizes of the hedges. One of the characteristics of a gas storage delta hedge is that a slightly larger *volume* is sold forward than bought forward so as to match the *values* of the long and short exposures (and the value of the storage). Over the 17 years, selling relatively larger winter volumes forward paid off and lifted the static delta hedge performance.

For a seasonal storage, a static hedge is quite effective. It takes out most of the uncertainty. In the NBP backtest, the under-performance of the static delta hedge for the slow storage is relatively small: −12%. Nevertheless, it is better to adjust this hedge over time, in response to changing market conditions. The two dynamic forward hedging strategies (intrinsic and delta) assume adjustments once a month. The spot trading strategy with dynamic delta hedge even has a small over-performance in realized value over the expected value (+3.5% average per year and per storage). It also performs better than a dynamic intrinsic hedge (−14.5%). The dynamic delta hedge is also very effective in reducing the uncertainty. Without any forward hedge, the percentage difference between realized and expected value was 51%. With a dynamic delta hedge this is reduced to 19.5%. Fig. 5 shows the evolution of the performance of the fast (mid-cylce) storage, without hedge and with the dynamic delta hedge. Results for the slow and very fast storage show a similar pattern, though with either more variation (very fast storage) or less variation (slow storage).

Finally, the backtest results shed some light on the impact of parameter uncertainty. Remember that the in-sample parameters are estimated on price data from within the storage year, whereas the out-of-sample parameters are from the storage year before. In practice, the estimates are likely to be based on the price dynamics of the past year, so equal to the out-of-sample. The realized values of the spot with dynamic delta hedge strategy are just above the estimated value in both samples. It is only marginally higher (+3.5% above expectation) with in-sample parameters than with out-of-sample parameters (+1.1%). However, the standard deviation, measured over the percentage differences between realized and expected values, is considerably lower with the in-sample parameters (19.5% versus 27.3%). This shows the risk of estimating the storage value when the actual market parameters are not known. It introduces around 8 percentage point extra standard deviation around the estimated value. At the same time, the moderate drop in average realized value seems to indicate that the storage trading strategy is only marginally suboptimal with the 'inaccurate' market parameters. Both results confirm our findings of Section 3, based on simulations, about parameter uncertainty.

5. Conclusions and further research

The valuation and optimal operation of gas storage assets in liberalized gas markets relies on a model, a gas forward curve and market parameter assumptions. So far, the literature has mainly focused on the modeling aspect. On the one hand, our results indicate that different forward curves and market parameters, such as winter–summer spread in the forward curve and volatility of the spot prices, inevitably lead to different estimated and realized storage values. On the other hand, our results indicate that an incorrect estimate of the market parameters, within a reasonable range of imprecision, has a limited negative impact on the optimality of the trading strategy. This confirms earlier results of Parsons (2013).

Most value can be generated with a gas storage if the daily injections and withdrawals are adjusted to spot prices, for example based on the daily trading signals of the least-squares Monte Carlo method. However, realized values can fluctuate dramatically from year to year. Part of the fluctuations have to be accepted and are due to fundamental changes in the supply-demand balance resulting in different levels of price seasonality and price volatility. Another part of the fluctuations are more short-lived and can be effectively hedged in the forward market. The backtest demonstrates how bad a trading performance can be without any forward hedging and how accurate valuations become with a dynamically adjusted forward hedge. Among the evaluated hedging strategies, the delta hedging performs best.

The backtest analysis mimics actual market trading opportunities relatively closely. Still, there are differences with actual markets. For example, in practice traders face limitations and uncertainty in liquidity and transaction costs, traders can trade on different times of the day and on different platforms, and so on. Some of these factors may be included in future research.

Acknowledgments

The author wishes to thank Adriano Tosifor for his contribution to the section about model risk, and to Emilian Enev and Marcus Nossman for their contribution to the storage model development and data analysis.

[1] Assuming no risk premium in forward prices, the forward price equals the expected future spot price.

Appendix. Detailed backtest results

Table 2a–2c
Estimated and realized gas storage values in pence, with in-sample parameter estimates, for 3 storage types.

Slow storage		Intrinsic	Rolling - expected	Rolling - realized	Spot - expected	Spot - realized			
Begin	End	monthly	monthly	monthly		no hedge	dynamic delta	dynamic intrinsic	static intrinsic
Apr-97	Mar-98	544	591	550	816	236	893	584	556
Apr-98	Mar-99	691	723	693	909	82	1053	887	908
Apr-99	Mar-00	445	520	786	770	657	911	877	684
Apr-00	Mar-01	374	446	505	854	1641	922	1050	743
Apr-01	Mar-02	409	516	523	1024	775	1017	744	702
Apr-02	Mar-03	912	1065	1085	1538	1531	1949	1608	1415
Apr-03	Mar-04	964	1141	1066	1683	1540	1530	1032	1300
Apr-04	Mar-05	1292	1559	1320	2007	2130	2409	2057	2138
Apr-05	Mar-06	3008	3393	3016	4303	5670	4549	4099	4944
Apr-06	Mar-07	4946	5873	5324	7079	-895	7702	5508	5166
Apr-07	Mar-08	3212	3331	3266	3788	2414	4170	3962	3255
Apr-08	Mar-09	1831	2159	2032	3505	-638	2735	2462	2104
Apr-09	Mar-10	2808	2859	3004	3443	964	2887	2730	2573
Apr-10	Mar-11	1211	1228	1467	1494	1935	1476	1633	1796
Apr-11	Mar-12	1220	1777	2018	2035	456	2182	1948	1215
Apr-12	Mar-13	1658	1997	1685	2297	1347	1743	1195	1969
Apr-13	Mar-14	905	1057	1624	1369	-208	1187	921	783
Average result		**1555**	**1779**	**1763**	**2289**	**1155**	**2313**	**1959**	**1897**

Fast storage		Intrinsic	Rolling - expected	Rolling - realized	Spot - expected	Spot - realized			
Begin	End	monthly	monthly	monthly	no hedge	no hedge	dynamic delta	dynamic intrinsic	static intrinsic
Apr-97	Mar-98	544	646	587	1107	373	1280	795	521
Apr-98	Mar-99	694	759	713	1159	146	1363	1109	1054
Apr-99	Mar-00	450	590	1044	1062	1029	1428	1233	1051
Apr-00	Mar-01	377	522	562	1312	1968	1292	1673	969
Apr-01	Mar-02	422	636	488	1599	879	1491	1042	409
Apr-02	Mar-03	926	1224	1117	2125	2287	3003	2392	2188
Apr-03	Mar-04	1004	1324	1041	2356	2202	2334	1336	1869
Apr-04	Mar-05	1330	1815	1416	2584	1960	2363	1945	2528
Apr-05	Mar-06	3076	3803	3140	5599	6711	6166	4868	5858
Apr-06	Mar-07	5051	6653	5919	8659	-185	9964	5794	5327
Apr-07	Mar-08	3315	3531	3349	4511	2692	5273	4674	3627
Apr-08	Mar-09	1844	2479	2091	5152	-878	3628	2767	1996
Apr-09	Mar-10	2958	3046	3520	4276	1085	3561	3475	2645
Apr-10	Mar-11	1266	1303	1596	1861	2257	1878	2045	2248
Apr-11	Mar-12	1320	2102	2414	2458	1088	3028	2596	2271
Apr-12	Mar-13	1709	2281	1759	2783	1512	2169	1424	2300
Apr-13	Mar-14	932	1232	2006	1842	-201	1348	1077	474
Average result		**1601**	**1997**	**1927**	**2967**	**1466**	**3033**	**2367**	**2196**

Very fast storage		Intrinsic	Rolling - expected	Rolling - realized	Spot - expected	Spot - realized			
Begin	End	monthly	monthly	monthly	no hedge	no hedge	dynamic delta	dynamic intrinsic	static intrinsic
Apr-97	Mar-98	544	673	561	1372	534	1572	933	669
Apr-98	Mar-99	697	780	737	1418	170	1567	1220	1116
Apr-99	Mar-00	452	623	1179	1305	1324	1822	1516	1326
Apr-00	Mar-01	380	564	648	1733	2660	1774	2442	1687
Apr-01	Mar-02	431	708	548	2119	1423	2241	1693	812
Apr-02	Mar-03	943	1315	1198	2603	2782	3564	2821	2700
Apr-03	Mar-04	1048	1433	1104	2908	2691	2892	1857	2398
Apr-04	Mar-05	1373	1960	1459	2969	1411	2079	1599	2133
Apr-05	Mar-06	3143	4039	3225	6642	8031	7441	5855	7109
Apr-06	Mar-07	5099	7001	6053	9619	437	11559	6414	5737
Apr-07	Mar-08	3405	3652	3411	5206	3532	6439	5526	4537
Apr-08	Mar-09	1860	2650	2213	6720	-241	4422	3221	3535
Apr-09	Mar-10	3076	3186	3707	5164	1174	4037	3785	2685
Apr-10	Mar-11	1290	1347	1638	2245	2641	2499	2676	2750
Apr-11	Mar-12	1387	2260	2491	2681	1208	3415	2851	2475
Apr-12	Mar-13	1768	2428	1807	3097	1651	2556	1609	2518
Apr-13	Mar-14	969	1341	2173	2226	116	1635	1385	632
Average result		**1639**	**2115**	**2009**	**3531**	**1856**	**3618**	**2788**	**2636**

Table 3a–3c
Estimated and realized gas storage values in pence, with out-of-sample parameter estimates, for 3 storage types.

Slow storage		Intrinsic	Rolling - expected	Rolling - realized	Spot - expected	Spot - realized				
Begin	End	monthly	monthly	monthly		no hedge	dynamic delta	dynamic intrinsic	static intrinsic	static delta
Apr-97	Mar-98	544	576	550	820	228	882	580	549	710
Apr-98	Mar-99	691	741	693	949	83	1039	888	909	823
Apr-99	Mar-00	445	473	786	671	656	922	878	683	740
Apr-00	Mar-01	374	457	505	746	1617	938	985	719	722
Apr-01	Mar-02	409	540	523	1227	626	960	654	552	1018
Apr-02	Mar-03	912	999	1085	1378	1531	1920	1599	1415	1678
Apr-03	Mar-04	964	1142	1066	1677	1523	1519	1027	1283	1475
Apr-04	Mar-05	1292	1494	1320	2152	1755	2014	1813	1763	1587
Apr-05	Mar-06	3008	3440	3016	4091	5923	4978	4356	5197	5471
Apr-06	Mar-07	4946	5575	5324	7159	-930	7626	5489	5132	6243
Apr-07	Mar-08	3212	3704	3266	4278	2570	4017	4000	3410	2955
Apr-08	Mar-09	1831	2101	2032	3120	-502	2692	2452	2240	2690
Apr-09	Mar-10	2808	2907	3004	3645	941	2984	2759	2550	2956
Apr-10	Mar-11	1211	1251	1467	1794	2028	1193	1557	1888	1128
Apr-11	Mar-12	1220	1367	2018	1993	437	2164	1993	1196	1437
Apr-12	Mar-13	1658	2146	1685	2382	1341	1751	1187	1963	1437
Apr-13	Mar-14	905	1273	1624	1589	-202	1148	930	789	873
Average result		**1555**	**1776**	**1763**	**2334**	**1155**	**2279**	**1950**	**1896**	**1997**

Fast storage		Intrinsic	Rolling - expected	Rolling - realized	Spot - expected	Spot - realized				
Begin	End	monthly	monthly	monthly	no hedge	no hedge	dynamic delta	dynamic intrinsic	static intrinsic	static delta
Apr-97	Mar-98	544	615	587	1141	371	1267	793	519	905
Apr-98	Mar-99	694	798	713	1223	162	1357	1123	1070	946
Apr-99	Mar-00	450	505	1044	913	1033	1452	1242	1056	1184
Apr-00	Mar-01	377	539	562	1088	1895	1373	1600	895	864
Apr-01	Mar-02	422	679	488	1980	744	1466	1006	273	1428
Apr-02	Mar-03	926	1105	1117	1858	2279	2966	2372	2180	2646
Apr-03	Mar-04	1004	1327	1041	2340	2196	2333	1338	1863	2267
Apr-04	Mar-05	1330	1721	1416	2968	1927	2252	2009	2494	1835
Apr-05	Mar-06	3076	3860	3140	5090	6695	6134	4549	5841	6971
Apr-06	Mar-07	5051	6229	5919	9163	-309	10049	5714	5204	8051
Apr-07	Mar-08	3315	4139	3349	5172	2856	5241	4855	3791	3351
Apr-08	Mar-09	1844	2353	2091	4431	-518	3686	2837	2355	3223
Apr-09	Mar-10	2958	3133	3520	4633	1094	3670	3466	2655	3520
Apr-10	Mar-11	1266	1348	1596	2465	2648	1621	2186	2638	1263
Apr-11	Mar-12	1320	1603	2414	2864	1194	3238	2806	2377	2462
Apr-12	Mar-13	1709	2429	1759	2785	1504	2183	1417	2292	1475
Apr-13	Mar-14	932	1545	2006	2029	-234	1277	1062	441	1056
Average result		**1601**	**1996**	**1927**	**3067**	**1502**	**3033**	**2375**	**2232**	**2556**

Very fast storage		Intrinsic	Rolling - expected	Rolling - realized	Spot - expected	Spot - realized				
Begin	End	monthly	monthly	monthly	no hedge	no hedge	dynamic delta	dynamic intrinsic	static intrinsic	static delta
Apr-97	Mar-98	544	636	561	1480	540	1580	937	675	1105
Apr-98	Mar-99	697	828	737	1473	169	1535	1218	1115	991
Apr-99	Mar-00	452	522	1179	1165	1309	1810	1480	1311	1513
Apr-00	Mar-01	380	584	648	1375	2672	2078	2500	1699	1494
Apr-01	Mar-02	431	765	548	2678	1282	2108	1523	671	2089
Apr-02	Mar-03	943	1167	1198	2290	2713	3485	2732	2631	3233
Apr-03	Mar-04	1048	1435	1104	2882	2691	2891	1857	2398	2804
Apr-04	Mar-05	1373	1853	1459	3635	1431	2055	1793	2152	1466
Apr-05	Mar-06	3143	4105	3225	5806	7801	7566	5692	6879	8554
Apr-06	Mar-07	5099	6511	6053	10695	546	11979	6425	5846	9636
Apr-07	Mar-08	3405	4328	3411	5735	3555	6338	5577	4560	4306
Apr-08	Mar-09	1860	2478	2213	5713	-316	4337	3205	3460	3880
Apr-09	Mar-10	3076	3295	3707	5609	1131	4123	3737	2642	3923
Apr-10	Mar-11	1290	1406	1638	3192	2755	2055	2719	2863	967
Apr-11	Mar-12	1387	1736	2491	3719	1419	3801	3192	2686	2954
Apr-12	Mar-13	1768	2564	1807	3003	1608	2564	1569	2475	1791
Apr-13	Mar-14	969	1682	2173	2303	95	1644	1400	611	1457
Average result		**1639**	**2111**	**2009**	**3691**	**1847**	**3644**	**2798**	**2628**	**3068**

Table 4a–c
Percentage deviation of realized gas storage values relative to estimated spot trading value, with parameter estimates, for 3 storage types.

Slow storage		Percentage deviation				
Begin	End	no hedge	dynamic delta	dynamic intrinsic	static intrinsic	static delta
Apr-97	Mar-98	-71%	9%	-28%	-32%	-13%
Apr-98	Mar-99	-91%	16%	-2%	0%	-7%
Apr-99	Mar-00	-15%	18%	14%	-11%	-3%
Apr-00	Mar-01	92%	8%	23%	-13%	-18%
Apr-01	Mar-02	-24%	-1%	-27%	-31%	13%
Apr-02	Mar-03	0%	27%	5%	-8%	10%
Apr-03	Mar-04	-8%	-9%	-39%	-23%	-11%
Apr-04	Mar-05	6%	20%	2%	6%	-1%
Apr-05	Mar-06	32%	6%	-5%	15%	18%
Apr-06	Mar-07	-113%	9%	-22%	-27%	-12%
Apr-07	Mar-08	-36%	10%	5%	-14%	-21%
Apr-08	Mar-09	-118%	-22%	-30%	-40%	-28%
Apr-09	Mar-10	-72%	-16%	-21%	-25%	-16%
Apr-10	Mar-11	30%	-1%	9%	20%	-21%
Apr-11	Mar-12	-78%	7%	-4%	-40%	-31%
Apr-12	Mar-13	-41%	-24%	-48%	-14%	-39%
Apr-13	Mar-14	-115%	-13%	-33%	-43%	-30%
Average result		**-37%**	**3%**	**-12%**	**-16%**	**-12%**
Standard dev.		**57%**	**15%**	**20%**	**18%**	**16%**

Fast storage		Percentage deviation				
Begin	End	no hedge	dynamic delta	dynamic intrinsic	static intrinsic	static delta
Apr-97	Mar-98	-66%	16%	-28%	-53%	-18%
Apr-98	Mar-99	-87%	18%	-4%	-9%	-18%
Apr-99	Mar-00	-3%	34%	16%	-1%	12%
Apr-00	Mar-01	50%	-2%	27%	-26%	-37%
Apr-01	Mar-02	-45%	-7%	-35%	-74%	-4%
Apr-02	Mar-03	8%	41%	13%	3%	26%
Apr-03	Mar-04	-7%	-1%	-43%	-21%	-3%
Apr-04	Mar-05	-24%	-9%	-25%	-2%	-26%
Apr-05	Mar-06	20%	10%	-13%	5%	21%
Apr-06	Mar-07	-102%	15%	-33%	-38%	-9%
Apr-07	Mar-08	-40%	17%	4%	-20%	-24%
Apr-08	Mar-09	-117%	-30%	-46%	-61%	-44%
Apr-09	Mar-10	-75%	-17%	-19%	-38%	-21%
Apr-10	Mar-11	21%	1%	10%	21%	-37%
Apr-11	Mar-12	-56%	23%	6%	-8%	-6%
Apr-12	Mar-13	-46%	-22%	-49%	-17%	-47%
Apr-13	Mar-14	-111%	-27%	-42%	-74%	-39%
Average result		**-40%**	**4%**	**-15%**	**-24%**	**-16%**
Standard dev.		**48%**	**20%**	**24%**	**26%**	**22%**

Very fast storage		Percentage deviation				
Begin	End	no hedge	dynamic delta	dynamic intrinsic	static intrinsic	static delta
Apr-97	Mar-98	-61%	15%	-32%	-51%	-21%
Apr-98	Mar-99	-88%	11%	-14%	-21%	-30%
Apr-99	Mar-00	1%	40%	16%	2%	17%
Apr-00	Mar-01	53%	2%	41%	-3%	-25%
Apr-01	Mar-02	-33%	6%	-20%	-62%	3%
Apr-02	Mar-03	7%	37%	8%	4%	28%
Apr-03	Mar-04	-7%	-1%	-36%	-18%	-3%
Apr-04	Mar-05	-52%	-30%	-46%	-28%	-47%
Apr-05	Mar-06	21%	12%	-12%	7%	28%
Apr-06	Mar-07	-95%	20%	-33%	-40%	-7%
Apr-07	Mar-08	-32%	24%	6%	-13%	-14%
Apr-08	Mar-09	-104%	-34%	-52%	-47%	-41%
Apr-09	Mar-10	-77%	-22%	-27%	-48%	-26%
Apr-10	Mar-11	18%	11%	19%	22%	-41%
Apr-11	Mar-12	-55%	27%	6%	-8%	-2%
Apr-12	Mar-13	-47%	-17%	-48%	-19%	-41%
Apr-13	Mar-14	-95%	-27%	-38%	-72%	-33%
Average result		**-38%**	**4%**	**-15%**	**-23%**	**-15%**
Standard dev.		**46%**	**22%**	**28%**	**24%**	**24%**

Table 5a–c
Percentage deviation of realized gas storage values relative to estimated spot trading value, with out-of-sample parameter estimates, for 3 storage types.

Slow storage		Percentage deviation				
Begin	End	no hedge	dynamic delta	dynamic intrinsic	static intrinsic	static delta
Apr-97	Mar-98	-72%	8%	-29%	-33%	-13%
Apr-98	Mar-99	-91%	9%	-6%	-4%	-13%
Apr-99	Mar-00	-2%	37%	31%	2%	10%
Apr-00	Mar-01	117%	26%	32%	-4%	-3%
Apr-01	Mar-02	-49%	-22%	-47%	-55%	-17%
Apr-02	Mar-03	11%	39%	16%	3%	22%
Apr-03	Mar-04	-9%	-9%	-39%	-23%	-12%
Apr-04	Mar-05	-18%	-6%	-16%	-18%	-26%
Apr-05	Mar-06	45%	22%	6%	27%	34%
Apr-06	Mar-07	-113%	7%	-23%	-28%	-13%
Apr-07	Mar-08	-40%	-6%	-7%	-20%	-31%
Apr-08	Mar-09	-116%	-14%	-21%	-28%	-14%
Apr-09	Mar-10	-74%	-18%	-24%	-30%	-19%
Apr-10	Mar-11	13%	-34%	-13%	5%	-37%
Apr-11	Mar-12	-78%	9%	0%	-40%	-28%
Apr-12	Mar-13	-44%	-27%	-50%	-18%	-40%
Apr-13	Mar-14	-113%	-28%	-41%	-50%	-45%
Average result		**-37%**	**0%**	**-14%**	**-19%**	**-14%**
Standard dev.		**61%**	**22%**	**25%**	**20%**	**20%**

Fast storage		Percentage deviation				
Begin	End	no hedge	dynamic delta	dynamic intrinsic	static intrinsic	static delta
Apr-97	Mar-98	-68%	11%	-30%	-54%	-21%
Apr-98	Mar-99	-87%	11%	-8%	-13%	-23%
Apr-99	Mar-00	13%	59%	36%	16%	30%
Apr-00	Mar-01	74%	26%	47%	-18%	-21%
Apr-01	Mar-02	-62%	-26%	-49%	-86%	-28%
Apr-02	Mar-03	23%	60%	28%	17%	42%
Apr-03	Mar-04	-6%	0%	-43%	-20%	-3%
Apr-04	Mar-05	-35%	-24%	-32%	-16%	-38%
Apr-05	Mar-06	32%	21%	-11%	15%	37%
Apr-06	Mar-07	-103%	10%	-38%	-43%	-12%
Apr-07	Mar-08	-45%	1%	-6%	-27%	-35%
Apr-08	Mar-09	-112%	-17%	-36%	-47%	-27%
Apr-09	Mar-10	-76%	-21%	-25%	-43%	-24%
Apr-10	Mar-11	7%	-34%	-11%	7%	-49%
Apr-11	Mar-12	-58%	13%	-2%	-17%	-14%
Apr-12	Mar-13	-46%	-22%	-49%	-18%	-47%
Apr-13	Mar-14	-112%	-37%	-48%	-78%	-48%
Average result		**-39%**	**2%**	**-16%**	**-25%**	**-16%**
Standard dev.		**53%**	**28%**	**30%**	**28%**	**28%**

Very fast storage		Percentage deviation				
Begin	End	no hedge	dynamic delta	dynamic intrinsic	static intrinsic	static delta
Apr-97	Mar-98	-64%	7%	-37%	-54%	-25%
Apr-98	Mar-99	-89%	4%	-17%	-24%	-33%
Apr-99	Mar-00	12%	55%	27%	13%	30%
Apr-00	Mar-01	94%	51%	82%	24%	9%
Apr-01	Mar-02	-52%	-21%	-43%	-75%	-22%
Apr-02	Mar-03	19%	52%	19%	15%	41%
Apr-03	Mar-04	-7%	0%	-36%	-17%	-3%
Apr-04	Mar-05	-61%	-43%	-51%	-41%	-60%
Apr-05	Mar-06	34%	30%	-2%	18%	47%
Apr-06	Mar-07	-95%	12%	-40%	-45%	-10%
Apr-07	Mar-08	-38%	11%	-3%	-20%	-25%
Apr-08	Mar-09	-106%	-24%	-44%	-39%	-32%
Apr-09	Mar-10	-80%	-27%	-33%	-53%	-30%
Apr-10	Mar-11	-14%	-36%	-15%	-10%	-70%
Apr-11	Mar-12	-62%	2%	-14%	-28%	-21%
Apr-12	Mar-13	-46%	-15%	-48%	-18%	-40%
Apr-13	Mar-14	-96%	-29%	-39%	-73%	-37%
Average result		**-38%**	**2%**	**-17%**	**-25%**	**-16%**
Standard dev.		**54%**	**31%**	**35%**	**29%**	**33%**

References

Bjerksund, P., Stensland, G., Vagstad, F., 2011. Gas storage valuation: price modelling v. optimization methods. Energy J. Int. Assoc. Energy Econ. 32 (1), 203–228 (earlier version published in 2008).

Boogert, A., de Jong, C., 2008. Gas storage valuation using a Monte Carlo method. J. Deriv. 15 (3), 81–91.

Boogert, A., de Jong, C., 2011. Gas storage valuation using a multi-factor process. J. Energy Mark. 4 (4), 1–24.

Carmona, R., Ludkovski, M., 2010. Valuation of energy storage: an optimal switching approach. Quant. Financ. 10 (4), 359–374.

Carriere, J., 1996. Valuation of the early-exercise price for options using simulations and nonparametric regression. Insur. Math. Econ. 19, 19–30.

Chaton, C., Creti, A., Villeneuve, B., 2008. Storage and Security of Supply in the Medium Run. MPRA Paper No. 11986.

Chen, C. and P. Forsyth, „Implications of a regime-switching model on natural gas storage valuation and optimal operation“, Quant. Financ., Vol. 10, No. 2, pp. 159–176

Cox, J., Ross, S., Rubinstein, A., 1979. Option pricing: a simplified approach. J. Financ. Econ. 7 (3), 229–263.

De Jong, C., Walet, K., 2003. To Store or Not to Store. Energy Price Risk Management (now Energy Risk), October, pp. 8–11.

Eydeland, A., Wolyniec, K. (Eds.), 2003. Energy and Power Risk Management. John Wiley & Sons.

Felix, B., Weber, C., 2012. Gas storage valuation applying numerically constructed recombining trees. Eur. J. Oper. Res. 216 (1), 178–187.

Gray, J., Khandelwal, P., 2004. Towards a realistic gas storage model. Commodities. Now, June.

Henaff, P., Laachir, I., Russo, F., 2013. Gas Storage Valuation and Hedging: a Quantification of the Model Risk (working paper).

IHS Global, 2013 (December). Oil and Natural Gas Transportation and Storage Infrastructure: Status, Trends and Economic Benefits. Report for the American Petroleum Institute. http://www.api.org/~/media/Files/Policy/SOAE-2014/API-Infrastructure-Investment-Study.pdf.

International Energy Association, 2012. Natural Gas Information 2012. http://www.iea.org/media/training/presentations/statisticsmarch/NaturalGasInformation.pdf.

Kirk, E., 1995. In: Kaminski, Vince (Ed.), Correlation in the Energy Markets“, Book Chapter in Managing Energy Price Risk. Risk Publications (London).

Lai, G., Margot, F., Secomandi, N., 2010. An approximate dynamic programming approach to benchmark practice-based heuristics for natural gas storage valuation. Oper. Res. 58, 564–582.

Li, M., Deng, S.-J., Zhoc, J., 2008. Closed-form approximations for spread option prices and Greeks. J. Deriv. 15 (3), 58–80. Spring.
Lo, C.-F., 2014. A simple generalisation of Kirk's approximation for multi-asset spread options by the Lie-Trotter operator splitting method. J. Math. Financ. (4), 178–187.
Longstaff, F., Schwartz, E., 2001. Valuing American options by simulation: a simple least-squares approach. Rev. Financ. Stud. 14 (1), 113–147.
Manoliu, M., 2004. Storage options valuation using multilevel trees and calendar spreads. Int. J. Theor. Appl. Financ. 7 (4), 425–464.
Margrabe, W., 1978. The value of an option to exchange one asset for another. J. Financ. 33 (1), 177–186.
Parsons, C., 2013. Quantifying natural gas storage optionality: a two-factor tree model. J. Energy Mark. 6 (1).
Redpoint Energy, 2013 (July). The Impact of Gas Market Interventions on Energy Security - a Report for DECC report commissioned by the UK energy regulator DECC. https://www.gov.uk/government/uploads/system/uploads/attachment_data/file/236757/DECC_FI_Final_report_09072013.pdf.
Schlüter, S., Davison, M., 2010. Pricing an European Gas Storage Facility Using a Continuous-time Spot Price Model with GARCH Diffusion. IWQW discussion paper series, No. 2.
Schwartz, E., 1997. The stochastic behavior of commodity prices: implications for valuation and hedging. J. Financ. 52 (3), 923–973.
Secomandi, N., Lai, G., Margot, F., Scheller-Wolf, A., Seppi, D., 2010. The Effect of Model Error on the Valuation and Hedging of Natural Gas Storage (Tepper working paper).
Thompson, M., Davison, M., Rasmussen, H., 2009. Natural gas storage valuation and optimization: a real options application. Nav. Res. Logist. 56 (3), 226–238 (earlier published as working paper in 2003).