Constructive Volume Geometry

— A Fundamental Modelling Methodology for Volume Graphics

Professor Min Chen and Professor John V. Tucker
Department of Computer Science, University of Wales Swansea
URL (*including all publications*): http://www.cs.swansea.ac.uk/~cschenm/R25286/

1. Background/Context

In traditional computer graphics, the majority of existing modelling and rendering methods deal with graphics models specified as surfaces or surface-bounded solids. This collection of methods is often referred to as *surface graphics* techniques. The primary deficiencies of surface graphics include its inability to encapsulate the internal description of a model, and the difficulties in modelling and rendering amorphous phenomena.

Having evolved from volume visualisation, *volume graphics* is emerging as an important sub-field of computer graphics. Just as 2D raster graphics superseded vector graphics, volume graphics has the potential to match and overtake surface graphics for the representation, manipulation, rendering and animation of 3D graphics scenes. This EPSRC-funded research project focused on the modelling of complex graphics objects and scenes using volume data types, which is one of the most fundamental aspects of volume graphics.

2. Key Advances and Supporting Methodology

The aim of this proposed project was to conduct a comprehensive research programme on *Constructive Volume Geometry (CVG),* which was a volume-based modelling method proposed and outlined by the investigators. The objectives of the project were:

1. To develop an algebraic theory for volume data types and Constructive Volume Geometry (CVG).
2. To develop advanced techniques for modelling and rendering complex volume graphics scenes, and for facilitating the integration of volume data types and functionalities into main stream graphics systems.
3. To demonstrate the significance of the theory of CVG and CVG-based software technologies through three main applications of volume graphics.

We investigated a broad range of avenues over the course of the project, and made the following key achievements in meeting these objectives.

2.1 CVG Theoretic Framework (Objective 1)

CVG is a theoretical framework for defining high-level algebraic operations on the 3D spatial and 4D spatio-temporal objects of volume graphics. The CVG framework offers a set of precisely defined common concepts and results based the general idea of a CVG data type modelled as an algebra of spatial objects. The basic theoretical framework of the investigators mapped out how to design and specify CVG data types. We have studied the expressive power of CVG:

Can CVG algebras construct all the spatial objects one wants to consider? Continuous spatial objects have good approximation properties and we have found types of operations that provide a sufficient condition *for every continuous spatial object to be approximable by CVG terms* for a wide variety of CVG algebras. Thus, we have proved the wide scope of these CVG data types for programming purposes.

Using CVG data types, can we program everything we want? The fundamental research in this project [11, 14] points to the imperative language constructs needed. Further rtheoretic development has established the wide scope of programming with CVG data types. We have proved this Expressiveness Theorem: *Any computably implementable volume graphics transformation of spatial objects can be programmed in a high level CVG framework.*

In [3], the mathematical analysis of CVG data types and their representation led to a general theory of spatial objects modelled by generalised scalar fields. We defined a spatial object to be a partial map $\mathbf{o}: X \to A$, where X is topological space and A is a data type of attributes modelled by algebras, and considered various classes of general operations to form algebras $O(X, A)$ of spatial objects. This abstraction is perfect for our aim of giving a uniform treatment of continuous and discrete models, and of including advanced attributes (e.g., spectra). Since X, A and $O(X, A)$ can contain continuous data (e.g., real numbers) they are polymorphic data types that are complex semantically.

It was essential to understand the nature of imperative high-level programming with data types like $O(X, A)$. However, this required new theoretical research into imperative programming models and specification techniques for continuous data types in general, since existing knowledge was inadequate to tackle our questions about $O(X, A)$. A full analysis was undertaken for data types with metrics in [11, 14]. The results were surprising: we showed that the data types needed (i) partially defined and continuous operations and (ii) nondeterministic constructs. We proved a completeness theorem that, in a wide variety of cases, means that *while programs + arrays + nondeterministic choice can programme approximations to all computable/ implementable processes on a metric data type.*

The mathematical theory of spatial objects, how to programme with them, and how to use them in CVG programming is the subject of Johnson's PhD thesis. The main above results will be reported in a major journal paper [21]. In the thesis, a detailed investigation is made of the case $CO(X, A)$ of *continuous* spatial objects and their applications in CVG. A general theory of these spatial data types, based on viewing them as topological algebras, has been worked out. Discretisation and rendering methods have been studied using general forms of structural induction, recursion, homomorphism $CO(X, A) \rightarrow CO(Y, A)$ with discretisation $X=\mathbf{R}^3$ and $Y=\mathbf{Z}^3$, and rendering $X=\mathbf{E}^3$ and $Y=\mathbf{Z}^2$. Finally, the main Expressiveness Theorems are proved.

2.2 Spectral Volume Modelling and Rendering (Objectives 1 & 2)

In volume graphics and visualization, the most commonly used object class is the 4-colour model [Porter and Duff, 1984], in which each spatial object is defined with an opacity field and three colour fields: α, R, G, B: $\mathbf{E}^3 \rightarrow [0, 1]$. [Oddy and Willis, 1991] improved the optical accuracy of the this model by introducing a 7-colour model with a β-channel and separate RGB colour properties for pigments and medium filters. The deployment of these models for volume modelling and rendering stumbled upon several problems:

1. The constructive operations defined upon colour fields usually do not satisfy some important physical laws (e.g., colour mixing), or mathematical laws (such as the commutative law).
2. A volume rendering integral based on such a model cannot handle selective absorption in translucent media, and thereby has difficulties in synthesis optically correct images.
3. Scattering and absorption are usually considered as two independent attributes of an object, though in physics, they are closely related to each other.

With this project, we developed a new approach to volume modelling and rendering based on the Kubelka-Munk theory of diffuse reflectance [17]. We showed that not only the Kubelka-Munk theory facilitates a consistent spectral volume rendering integral suitable for both solid objects and amorphous matters in volume data sets, but also provides volume visualization with more accurate optical effects than the traditional volume rendering integral based on RGBα compositing. The novelty and benefits of using Kubelka-Munk theory in volume graphics and volume visualization include:

- A new volume rendering integral that takes into account both scattering and absorption properties of volume objects as well as the sampling density. This integral is superior to the commonly used RGBα compositing which approximates primary scattering by treating voxels as light emitting entities and ignoring all secondary scattering effects.
- Using post-illumination for combining pre-processed reflectance images with different light sources in real time, facilitating interactive visualization.
- The constructive operations based on spectral specification of scattering and absorption, which have better algebraic properties than those operations defined upon the 4-colour model.

2.3 Spatial Transfer Functions (Objectives 1 & 2)

Spatial transfer functions are a class of CVG operations that facilitate the modelling of deformation and animation in volume scene graphs. It utilises the discrete sampling mechanism in direct volume rendering, enabling deformation by transferring a sampling point to a different location in space. In addition, the specification of a deformation action can be defined as a spatial object, in a manner similar to conventional spatial objects. The concept was first proposed in *Dagstuhl Scientific Visualization Seminar 2001* [9], and was realised and demonstrated through several pieces of work developed within this project, including:

- Image-swept volume [1], which allows an image or a video to be swept through multiple trajectories to form a complex volumetric object. For example, one can sweep a small fire video to create large fires of a variety of shapes and effects.
- Volume deformation and animation [7], which gives a comprehensive study on the algebraic properties of spatial transfer functions, and their use in volume visualization, deformation and animation. In particular, for the first time, spatial transfer functions enabled volume animation without the need for creating intermediate volume datasets which usually have excessive space requirements. Through this piece of work, we established a productive collaboration with VIZLAB in Rutgers (USA) and the two groups have continued and extended their collaboration activities since.
- Volume splitting [10, 19], which focuses on a particular use of spatial transfer functions in the context of deformation-centred medical illustration and explosion effects in volume animation.
- Visualization of cardiac anatomy (see 2.7).

The above-mentioned work has also led to a new concept developed recently in conjunction with a different project, in which we have proposed a new concept, namely *spatial displacement functions*, which exhibit more algebraic properties than spatial transfer functions. In addition to hierarchical combination of deformation objects (each of which is the specification of a deformation action) of as in spatial transfer functions, spatial deformation functions enable combinational operations on deformation objects, the application of masking and transfer functions to deformation objects, and discrete specification of deformation [20].

2.4 Translating Scene Description Languages with Semantic Approximation (Objective 2)

During the first year of this project, we investigated several avenues for delivering volume graphics into the hands of developers and users of graphics applications. We considered

1. developing a new volume graphics language based on an existing volume graphics API called *vlib*,

2. incorporating volume graphics features in established surface graphics software, and
3. developing a software tool for translating between volume and surface models.

For the third avenue, we were inspired by the recent developments in translation techniques for XML-based languages, and formulated a novel approach to the translation of graphical scene description languages using independent stylesheets. We were particularly excited by the scientific scope of this approach, and decided to pursue this avenue further after considering carefully the relative merits of each direction (see also Section 3). The success of this approach would fundamentally change the inter-operability of graphical scene description languages, allowing easy integration of volume graphics or other new graphics concepts into main stream graphics systems.

Translation between graphical scene description languages, such VRML and POV-Ray, presents a non-trivial scientific challenge, while constructing and maintaining numerous translation filters is a costly burden to graphics software developers. The complication arises largely from the semantic diversity of different languages and the frequent additions of new graphical features, which hinders the deployment of traditional approaches to automatic translation. The recent advent of XML and XSLT provides a practical means for facilitating direct translation through a generic translator (i.e., XSLT) and mapping specifications (i.e., stylesheets). However, the complexity remains $O(n^2)$ for n languages in terms of stylesheets of XSLT.

In this project, we proposed a new approach to automatic translation among scene description languages [12, 18]. We extended the principal concept of XSLT further by replacing $n(n-1)$ binary mapping stylesheets with n individual stylesheets, one for each scene description language. We named this approach as Independent Stylesheet Language Translation (ISLT). We designed and prototyped a generic translator to translate from one language to another based on their corresponding individual stylesheets. Through several examples of scene translation between X3D and POV-Ray, we demonstrated the main technical features, including the extensibility of independent stylesheets, the knowledge-based semantic approximation, the integration of several software technologies, and the handling of semantic disparity among graphical scene description languages.

2.5 Refraction Rendering in Volume Graphics (Objective 2)

In close association with this project, research was carried out to develop advanced refractive rendering capability for volume scene graphs. Our previous work has shown that one can render CVG scene with reflection and shadows. However, refractive rendering is more susceptible to noise in the data than that do not involve refraction. We developed algorithms for addressing the need for smoothly distributed normals in discretely sampled spatial objects (e.g., volume data sets) in order to synthesise good quality visual representations. We used nonlinear diffusion methods, which were originally developed for image denoising, and developed a volumetric filter for regularised anisotropic nonlinear diffusion (R-ANLD). Our results showed that this approach compared favourably against the Gaussian filter and the linear diffusion filter and the anisotropic nonlinear diffusion (ANLD) filter. This work confirmed that one can render CVG scenes with all conventional effects, including refraction.

2.6 Application 1: Comparative Visualization (Objective 3)

Comparative evaluation of visualization and experiment results is a critical step in computational steering. In this project, we conducted a study, in collaboration with the computational fluid dynamics group in Swansea, on 2D constructive operations for imagery data for visualizing the difference between a visualization of a computer simulation and a photographic image captured from an experiment [2]. We examined eleven image comparison metrics, including three spatial domain, four spatial-frequency domain and four HVS (human-vision system) metrics, and built 2D CVG operators based on these metrics. Among these metrics, a spatial-frequency domain metric called 2nd-order Fourier comparison was proposed specifically for this work. Our study consisted of two stages: base cases and field trials. The former is a general study on a controlled comparison space using purposely selected data, and the latter involves imagery results from computational fluid dynamics and a rheological experiment. This study introduced a methodological framework for analyzing image-level methods used in comparative visualization, and demonstrated the potential use of CVG-based software in 2D, 3D and 4D comparative visualization.

2.7 Application 2: Visualization of Cardiac Anatomy (Objective 3)

The complete virtual engineering of the entire beating heart requires the support of sophisticated visualisation tools that can effectively extract meaningful visual information from a combination of data sets and depict complex spatial, temporal and physical properties encoded in the data sets. Traditional surface graphics techniques have not been able to serve such requirements adequately, largely because of the necessity of transforming field-based data sets (simulated computationally and recorded clinically or experimentally) to surface representations, and because of the information loss during the process. In close collaboration with the Biomedical research group in Leeds, we applied CVG techniques to the visualisation of cardiac anatomy and electrophysiology. Through a serious of experiments, we demonstrated the capability of CVG technique in generating visualisations that can depict multiple structures in a combination of data sets and heterogenous interior structure through effective use of opacity and combinational operators. We applied CVG concepts to tracking and mapping intramural bundles of muscle fibres, and to map voltage isosurfaces of re-entrant activity, and the filaments around which re-entrant waves propagate [5, 8]. We have also employed spatial transfer functions (see also 2.3) for performing digital dissection and `context and focus' visualization of cardiac anatomy [5].

2.8 Application 3: Video Visualization (Objective 3)

Video visualisation is a computation process that extracts meaningful information from original video data sets and conveys extracted information to users in appropriate visual representations. In this project, we developed a novel method for handling large volumes of video data [4, 6]. Building upon the volume visualisation techniques, and using both the concepts of CVG and spatial transfer functions, we can summarise video sequences by considering a video as a 3D volume of imagery data. We can extract and highlight interesting features in the video, and convey such features using one or a few pictures, each of which provides users with a visual representation of complex information and can be used to assist in our decision processes. For example, when a security officer arrives at his/her office in the morning, he/she can be presented with one or a few visualisations for each surveillance camera that has been monitoring a premise during the previous night. From the visualisations, the officer can observe the level and patterns of activities recorded overnight, and decide if any specific section of a particular video needs to be replayed for further investigation.

Through a prototype system V3, we completed a video visualisation pipeline by integrating a range of techniques in volume visualisation, volume graphics and image processing. In particular, we developed stream-based volume rendering techniques for rendering video sequences associated a volume scene graph progressively. We conducted several case studies, including television programmes, and indoor and outdoor video sequences. Our studies have provided the first set of evidence to our hypotheses:

1. Video visualisation is an intuitive and cost-effective means of processing large volumes of video data.
2. Well-constructed visualisations of a video are able to show information that numerical and statistical indicators (and their diagrammatic illustrations) cannot.

As the first piece of research reported in the field of visualisation, it represents a major breakthrough, and opens a new subject area in the field. It has demonstrated the technical feasibility and usefulness of video visualisation in applications such as security surveillance, traffic monitoring, and video segmentation.

3. Project Plan Review

The project started on 1 October 2001. Prior to the starting date, we made two appointments as planned, an RA, Mr. Andrew Winter, and a PhD student, Mr. David Clark. Mr. Winter, who was a Swansea PhD student at that time working on a volume graphics API, *vlib*, would be the most appropriate researcher to take this project further in terms of both objectives 1 and 2 set in the proposal.

However, as Mr. Winter indicated his wish to complete his thesis first, he deferred his starting date to 1 April 2002, while arranging to work (with his existing studentship) on certain aspects of the project that could be integrated into his thesis, including image-swept volumes and spatial transfer functions. On 31 March 2002, Mr. Winter informed the University that he would not take up the RA post as he was offered a position in Qinetiq. Mr. Winter completed his thesis in September 2002, and was awarded a PhD degree in 2003.

After two unsuccessful attempts subsequently to appoint an RA, in July 2002, we sought the approval of EPSRC to use the RA budget for PhD studentships. With the agreement of EPSRC, we appointed two PhD students, Miss Alfie Abul-Rahman and Mr. Kenneth Johnson.

With the departure of an experienced RA, and a project team made up with three PhD students, we made the following changes to the project plan:

- We focused each PhD student on a relatively long term research topic due to their initial learning curve. We set Johnson on Objective 1, Clark on Objective 2, and Abdul-Rahman on the interface between the two.
- We (i.e., investigators) took over the development of deliverables in Objective 3 with the help of collaboration partners and other research students.
- We replaced the original Objective 2 for developing a volume graphics modelling language with a new objective for developing translation tool to facilitate the integration of volume graphics into main stream graphics systems. This change was partially due to the lack of system development capability, among three PhD students, which was necessary for handling both a new modelling language and its underlying API, *vlib*. It was also because of the emergence of an exciting research avenue for automatic translation of graphical scene description languages with semantic approximation (see also 2.4). Looking back, not only we appreciate that we made a sensible decision with the available human resources at that time, we are very pleased with the scientific significance of our work in automatic translation of graphical scene description languages.

We also partially funded another PhD student, Shoukat Islam with the remaining staffing budget. He worked in the area of spatial transfer functions in close collaboration with the VIZLAB in Rutgers.

From October 2002 onwards, the project progressed smoothly and effectively according to the revised plan. In July 2004, EPSRC agreed a six month extension to the project due to the change of Objective 2 and the difficulties surrounding the RA appointment in the first 12 months of the project. This extension is most helpful, enabling some work of the PhD students in this project to be published within the project period.

The success of this project attributes partially to the excellent research partnerships available to this project, including collaboration with:

- Professor J. I. Zucker in Canada [11, 14] and Dr. J. E. Blanck and Dr. V. Stoltenberg-Hansen in Sweden [3];
- Professor D. Silver and her visualization group (VIZLAB) in Rutgers, USA [7, 10, 19, 20];
- Professor A. V. Holden and his biomedical group in Leeds [5, 8];
- Professor M. F. Webster and his computational fluid dynamics group in Swansea [2].

4. Research Impact and Benefits to Society

In 1999 when the first international workshop on volume graphics (VG99) was held in Swansea, the Swansea group is considered to be a young and active group leading in aspects of volume modelling. In VG99, the programme committee proposed nine challenging problems in volume graphics [Chen, Kaufman and Yagel 2000]. The scientific and technical progress in relation to these nice problems was recently reviewed in a panel of VG05 held in Stony Brook, New York. The chair of the Panel, Dr. Klaus Mueller pointed out specifically that the Swansea group has worked on many of these problems, and has made noticeable contributions in many aspects.

Many of these contributions were delivered through this project, including

- New concepts, such as *spatial transfer functions* [7, 9], *independent stylesheet language translation* [12, 18], and spectral volume rendering integral based on the KM-model [17];
- Underlying theoretic developments [3, 11, 14];
- New algorithms and techniques, such as image-swept volumes [1], volume splitting [10, 19], point-based volume modelling [15], refraction rendering [13, 16];
- New applications, such as comparative visualization [2], digital dissection [5, 8] and video visualization [4, 6].

In particular, the development of the *KM-model-based modelling and rendering* [17] has raised great interest in the international community. Although the publication of the work was only announced recently, two groups, Stuttgart and Rutgers have already arranged to collaborate with Swansea to develop further the concept, techniques and applications of spectral volume modelling and rendering.

The development of *independent stylesheet language translation* [12, 18] is expected to have a profound long term impact in computer science. The shift from direct syntactic mapping to semantic approximation may represent a new focus of research in this area, addressing the needs of complex language translation in areas such as computer graphics, multimedia, and financial systems.

The novel approach to *video visualization* timely opened a new subject area in visualization. The work, which is seen as the first benchmark on the subject, has raised interests of many international researchers, some specific government bodies, and several commercial companies.

5. Explanation of Expenditure

	authorised	expenditure
staff	106,605.40	83,250.00
travel	8,404.14	9,814.29
consumables	1,595.73	988.15
exceptional items	8,744.55	28,700.00
PCTF	0.00	0.00
equipment	15,895.00	12,893.47
large c. equipment	0.00	0.00
sub-total	**141,244.82**	**135,645.91**
under-spending		***5,598.91***
indirect costs	49,038.55	38,295.00
TOTAL	**190,283.37**	**173,940.91**
under-spending		***16,342.46***

The major change to the budget was due to the change from the 36 month appointment of an RA to 78 month PhD studentships (30+30+18). Hence part of the staffing budget was reassigned to ‘exceptional items’ to cover tuition fees. Other expenditures were managed as planed. The overall budgetary management throughout the project was cautious, resulting in a reasonable amount of under-spending.

6. Further Research and Dissemination Activities

The further development of the concept of *spatial transfer function* will be focused on a new variation, *spatial displacement function* [20], and its deployment in *volume deformation* and *volume animation*. This development is being undertaken within an EPSRC-funded collaborative project involving Swansea (Professor Chen and Dr. Jones), Bath (Professor Willis) and Rutgers (Professor Silver).

A Royal Society grant for short research visits was recently awarded to Swansea, enabling Professor Chen to work with Professor Thomas Ertl and his visualization group in Stuttgart in July-September in 2005. They have already made an ambitious plan to develop further the techniques for *video visualization* [4, 6] and *comparative visualization* [2]. New topics, including *KM-based volume modelling and rendering* [17] and *point-based volume modelling and rendering* [15] will also likely featured in this visit.

Further work on *point-based volume modelling and rendering* will focus on the handling of very large data sets. A link will University of Utah was recently established, and further collaboration is expected to take place in the next 12 months.

Further work on *KM-based volume modelling and rendering* will focus on a complete pipeline from theory to applications. Preliminary collaboration with Rutgers has started, and collaboration with other international partners is in discussion.

Further work in various areas also continues in Swansea through several new PhD projects, including *independent stylesheet language translation*, and *video visualization*.

While we will focus our dissemination activities on academic publications and collaboration in the international community, we are supporting technology transfer to the industry through the Centre for Communications and Software Technologies at Swansea. Several UK companies have expressed their interests in video visualization and independent stylesheet language translation (for financial data).

References

1. A. S. Winter and M. Chen, **Image-swept volumes**, *Computer Graphics Forum*, 21(3):441-456, 2002.
2. H. Zhou, M. Chen and M. F. Webster, **Comparative evaluation of visualization and experimental results using image comparison metrics**, *Proc. IEEE Visualization 2002*, Boston, MA, 315-322, November 2002.
3. J. E. Blanck, J. V. Tucker, and V. Stoltenberg-Hansen, **Domain representations of partial functions, with applications to spatial objects and constructive volume geometry**, *Theoretical Computer Science*, 284:207-240, 2002.
4. G. W. Daniel and M. Chen, **Video visualization**, *Proc. IEEE Visualization 2003,* 409-416, Seattle, WA, October 2003.
5. M. Chen, R. H. Clayton, A. V. Holden and J.V. Tucker, **Constructive volume geometry applied to visualisation of cardiac anatomy and electrophysiology**, *International Journal of Bifurcation and Chaos*, 13(12) 2003.
6. G. W. Daniel and M. Chen, **Visualising video sequences using direct volume rendering**, *Proc. the 1st International Conference on Vision, Video and Graphics (VVG2003)*, 103-110, Eurographics/ ACM Workshop Series, Bath, July 2003.
7. M. Chen, D. Silver, A. S. Winter, V. Singh and N. Cornea, **Spatial transfer functions — a unified approach to specifying deformation in volume modeling and animation**, *Proc. Volume Graphics 2003*, 35-44, 163, Eurographics/ACM Workshop Series, Tokyo, July 2003.
8. M. Chen, R. H. Clayton, A. V. Holden and J. V. Tucker, **Visualising cardiac anatomy using constructive volume geometry**, *Functional Imaging and Modeling of Heart*, Lecture Notes in Computer Science, 2674:30-38, June 2003.
9. M. Chen, A. S. Winter, D. Rodgman and S. M. F. Treavett, **Enriching volume modelling with scalar fields**, in F. Post, G. M. Nielson and G.-P. Bonneau (eds) *Data Visualization: the State of the Art*, 345-362, Kluwer Academic Publishers, 2003.
10. S. Islam, S. Dipankar, D. Silver and M. Chen, **Spatial and temporal splitting of scalar fields in volume graphics**, in *Proc. IEEE Symposium of Volume Visualization and Graphics*, Austin, October 2004. Invited to submit an extended version to *IEEE Transactions on Visualization and Computer Graphics* (see also 19).
11. J. V. Tucker and J. I. Zucker, **Abstract versus concrete models of computation on partial metric algebras**, *ACM Transactions on Computational Logic*, 5 (4):611-668, 2004.
12. D. P. Clark, M. Chen and J. V. Tucker, **Automatic program translation — the third way**, in *Proc. IEEE Multimedia Software Engineering*, Miami, December 2004.
13. D. Rodgman and M. Chen, **Volume denoising for visualizing refraction**, to appear in G.-P. Bonneau, G. M. Nielson, T. Ertl (eds), *Scientific Visualization, Dagstuhl Seminar 2003*, Springer, 2004.
14. J. V. Tucker and J. I. Zucker, **Computable total functions on metric algebras, universal algebraic specifications and dynamical systems**, *Journal of Algebraic and Logic Programming*, 62:71-10, 2005.
15. M. Chen, **Combining point clouds and volume objects in volume scene graphs**, in *Proc. Volume Graphics 2005*, 127-135, 235, Eurographics/IEEE Workshop Series, Stony Brook, NY, June 2005.
16. D. Rodgman and M. Chen, **Regularised anisotropic nonlinear diffusion for rendering refraction in volume graphics**, to appear in *Proc. IMA Conference on Vision, Video and Graphics* July 2005.
17. A. Abdul-Rahman and M. Chen, **Spectral volume rendering based on the Kubelka-Munk theory**, to appear in *Computer Graphics Forum*, 24(3), 2005, and to be presented in *Eurographics 2005*.

Submitted and in Preparation

18. D. Clark, M. Chen and J. V. Tucker, **Translating scene description languages with semantic approximation**, submitted to *Computer Graphics Forum*.
19. S. Islam, D. Silver and M. Chen, **Volume splitting and its applications**, submitted to *IEEE Transactions on Visualization and Computer Graphics* (see also 10).
20. S. Islam, D. Silver and M. Chen, **Constructive spatial displacement functions**, submitted to *Computer Graphics Forum*.
21. K Johnson, J V Tucker and M Chen, **Spatial data types and the foundations of constructive volume geometry**, in preparation.
22. D. Clark, **Automatic Translation of Graphical Scene Description Languages Using Independent Stylesheets**, PhD thesis, in preparation 2005, (to be submitted in September 2005).
23. K. Johnson, **On Spatial Data Types** (provisional title), PhD Thesis, in preparation, 2006.
24. A. Abdul-Rahman, **Physically-based Volume Modelling and Rendering,** PhD Thesis, in preparation, 2006.