Visualizing demographic evolution using geographically inconsistent census data

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Abstract—We propose a visual analytics system that enables the exploration evolutionary patterns in geographically inconsistent data, removing the need to harmonize it into the same geographical regions, a time consuming and error-prone process that is currently used in virtually all longitudinal analysis of geographical data. This work also includes incremental developments in the representation, clustering, and visual exploration of geographical data. We illustrate it considering census data, where it allows an easier understanding of the demographic groups present in a city and their evolution over time. We present the feedback of experts in urban sciences and sociology, along with illustrative scenarios in the USA and Canada on the decennial censuses between 1970 and 2010.

1 Introduction

URBAN sciences are blooming thanks to a renewed interest in understanding and improving the urban environment. Visual analytics is following this trend, fueled by new public datasets that encompass progressively more of our daily lives [1]. There is no shortage of methods to explore mobility patterns [2], social media [1], traffic [3], and so on, providing experts, planners, policy makers, and the general population with deeper insights about their cities.

These new datasets usually contain GPS coordinates for the records, leading to *point-based* data. Combined with the corresponding timestamps, this data is easily suitable for longitudinal analysis. But most demographic datasets are *region-based*, where the measurements are associated with pre-defined regions, not only for an additional level of privacy protection, but because some measurements only make sense over a defined area. Census data is a classic example of this format, with datasets available from 1790 onwards for the US [4]. Despite this unmatched temporal availability, longitudinal analyses of census data are often restricted in time, especially when smaller tabulation areas are considered, such as census tracts (CT) or dissemination areas, which evolve to reflect changes in population density, leading to geographic inconsistencies across time, and the traditional time-series based approach is no longer viable.

However, these analyses are necessary to understand the urban environment. Indeed, two different regions can have similar average income for a given year, while one is experiencing a process of economic improvement and the other one impoverishment. Quite obviously, a single snapshot cannot be used to identify gentrification, migration, education changes, or any of the relevant processes that happen over time.

To overcome these inconsistencies, the traditional approach is the *geographical harmonization* of the data, the interpolation of the measurements into a common set of regions [5], [6], [7], so that each variable can be represented using time-series. This is laborious work that inevitably introduces some amount of error [8], even when additional data is provided [9]. Nevertheless, this step is considered mandatory in the current literature: *"(...) tract-by-tract comparison is not possible unless data from 2000 is interpolated to 2010 boundaries (...)"* [10].

The main contribution of this application paper is an visualization-based alternative to the geographical harmonization, a combination of established graph based processing and information visualization techniques allowing tract-by-tract comparison, the identification and visualization of patterns of demographic evolution without geographic harmonization, effectively removing one of the most challenging problems in longitudinal demographic analysis. We also include illustrative scenarios and our prototype is available at http://uoft.me/piccard, including more than fourty regions in the US and Canada.

2 Related Work

Since our problem emcompasses several fields, we divided this section into specific subproblems: *longitudinal demographic studies*, describing the traditional approach to perform longitudinal studies; *data representation*, exploring how evolving geographic data can be represented for processing; *Data clustering*, briefly reviewing existing clustering methods; and *cluster characterization*, exploring how the clusters can be visually summarized.

2.1 Longitudinal demographic studies

Census data is used not only to discover demographic patterns [11], but to correlate demographic charateristics to other measurements [12]. However, longitudinal studies are rare: *"(...) One of the most challenging and fascinating areas in spatial statistics is the synthesis of spatial data collected at different spatial scales(...)"* [13].

While CT level data is readily available for the US since 1910 [4], most studies consider the period between 1970 and 2010, using pre-harmonized data [4], [5]. Despite the inherent errors [6], [8], this dataset became the standard

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Manuscript received MMMM DD, YYYY; revised MMMM DD, YYYY.

source for longitudinal demographic data, with similar efforts appearing in other countries [7], [14], [15]. This result was significant for the field, but it also restricts the useable data, since new datasets need to be similarly processed.

Another option considers the use of grid data [10], [16], where small rectangular areas are used, in an approach similar to satellite imagery. Beyond the increased spatial accuracy, this approach does not require complex harmonizations when new data is considered. However, demographic data is usually not available in this format, especially from older sources, and the conversion from tabulation areas can introduce significant errors.

In the proposed methodology, we avoid the harmonization by considering each measurement using its actual geographic region. It does not require the regions to be consistent across time because they are already represented as different entities.

2.2 Data representation

Most data is represented in tabular form, where the rows and columns have coherent definitions. For example, consider a table with rain measurements over time, with the rows representing different locations and the columns different times. This representation can also be interpreted as a collection of time-series, one for each location. Geographic data followed this format, only including an additional field that describes the associated geographic area. Following the example, the data would now represent the amount of rain for a given region and time. As long each region remains the same, the data is coherent and can be interpreted again as a collection of time-series.

In the proposed method, we remove the requirement for consistency in the measurement regions by leveraging a graph-based representation, where each region in time corresponds to a different node. Instead of a collection of time-series, the data is represented as a dynamic graph. Graph based representation of geographic information is fairly well explored in the literature, as a basis for topological methods for event detection [17], leveraging signal processing on graphs [18], [19] to find patterns and outliers [20], [21], [22]. Graphs are well suited to represent trajectories as well [2], [3], [23], allowing the use of graph visualization methods [24], [25].

Graphs were used to represent census data for clustering purposes before [21], [26], but these works did not explore temporal evolution, where graphs are particular powerful as they allow a natural representation of inconsistent regions, with both spatial and temporal connections. Note that there are other possible representations that have similar properties, but we adopted graphs to allow the use of the existing literature and methods.

2.3 Data clustering

Data clustering is one of the elementary processes for data analysis, simplifying the data into a smaller number of homogeneous sets that can be interpreted in the same way. While there is no shortage of contributions for this problem [27], most applications still rely on k-means [28], [29] and, to a lesser extent, Self Organizing Maps [30], [31].

However, a method for geographic data analysis should not ignore the geographic component of the data. One straightforward option, for agglomerative methods [32], is to consider only nearby clusters for merging [33], which can also be done for k-means [34]. Alternatively, the spatial distance could be directly added to the inter-cluster metric [33] via a mixing parameter, which adds flexibility to the method, but introduces the problem of finding the correct application-dependent values.

Indeed, one crucial step in most clustering algorithms is the definition of the number of clusters. We sidestep this problem by considering hierarchical methods [35], where the result is not a partition of the data, but a tree of partitions. This approach is interesting for interactive methods, because it allows the user to change the number of displayed clusters with minimal processing. Since our data is represented as a graph, one option would be the watershed cuts algorithm [36], inspired by the well known image processing segmentation and equally prone to oversegmentation. Considering that the processing time is also a relevant factor, we opted for an heuristic variation of the maximum weighted matching algorithm called *sorted maximal matching* [37], which merges clusters based on the weights of the edges between pairs of clusters.

2.4 Cluster characterization

Visually representing evolving spatial data is a challenging old problem [38], [39], [40], [41]. Most geographic data is naturally bidimensional and maps work well in this case [41], [42], but the temporal dimension cannot be so naturally represented. One straightforward option is to leverage tridimensional plots [43], [44], but this can lead to visual obstructions or scaling problems unless a tridimensional display device is used. Animation can also be explored in some specific cases [45], but it is not a general approach. Glyphs can also be used [46], [47], but this may lead to cluttering when many small regions are present. A simpler, well adopted, option is to display a map that corresponds to a subset of the temporal information, allowing the user to change the time with an associated control [1], [17], [20], [22]. Small multiples can be used [2], but only when there are few temporal snapshots. However, none of these options is suitable to represent many variables at the same time.

Using data clustering, we can represent the region's cluster instead of all the its variables [2], [20], [22]. While this simplifies the geographic portion of the visualization, it introduces the problem of how to summarize the contents of each cluster. One traditional approach is to use parallel coordinates plot [48], [49], [50], [51], but these they can get cluttered representing similar clusters over several variables. Further, for demographic applications, the clusters are usually strongly characterized by a small subset of values [29], [30]. Therefore, in the proposed method, we identify the variables that are most relevant to the characterization of each cluster. The distribution of values on that variable is then represented using a boxplot, a well known statistical plot displaying basic properties of the distributions.

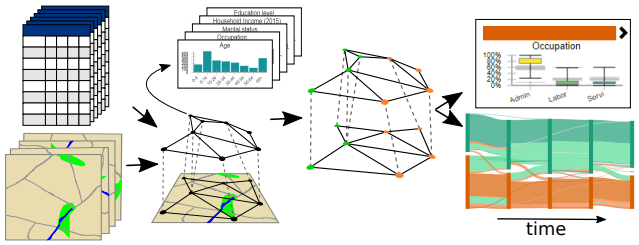

Fig. 1. Overview of the proposed method. A graph is generated combining the original census data, encoding the changing geographical information. The graph is partitioned into an hierarchy [37]. The characteristics and evolution of the clusters are then visually represented.

3 Visualizing the Demographic Spatio-Temporal Evolution

Beyond the objective of allowing the study of inconsistent data, our method includes incremental developments in most steps of the analysis, from data representation to the visualization method for the clusters. Figure 1 presents an overview of the processing steps of the proposed method.

3.1 Census methodology and data representation

Census data is disseminated in a tabulated form for aggregation areas: whole country, state/province, metropolitan region, and so on. To provide as much detail as possible, we focus on the smallest region with available data: *census tracts* (CT). They are usually defined to maintain the anonymity of the population, leading to a population count in the order of thousands in densely populated areas. Physical barriers are usually adopted as borders, so these regions can change because of new roads, construction or removal of high density buildings, and so on. Some census entities also consider demographic characteristics, aiming to establish the CTs as a cohesive unit. Therefore, CTs are the least geographically stable tabulation area.

Each CT is associated with a series of variables, with counts derived from the census questionnaires, covering several aspects of the demographic characteristics of the population. Some questions allow for multiple choices or open answers, that are then tabulated into the most frequent categories. Since the census is often used to direct government initiatives, which variables are measured/disseminated is dependent on administrative interests, the general understanding of the population, and current customs. For instance, income is disseminated with a finer tabulation in the lower portion than on the higher.

To match these variables over time and allow for direct comparison across different census years, we aggregated similar ones (e.g. White, Black, Asian, Other) into *aspects* (e.g. Race), encoding the distribution of that facet of the population. In this convention, we refer to the composing parts of an aspect as a *part* or the traditional *variable*. Internally, the aspects are represented using normalized histograms. This normalized representation is crucial for the comparison between inconsistent regions.

In our graph based representation, each CT of each census year is represented as a node, and edges are placed between nodes if the corresponding CTs share geographic borders in the same year. Further, edges are placed between nodes if the corresponding CTs belong to sequential years and there is geographical overlap between them. This approach leads to a single graph representing the whole spatio-temporal space of the data. Our objective then becomes to identify partitions of this graph such that the nodes of each partition are more similar between themselves than to the other nodes. This representation is not the only option, nor unique, but it allows the use of existing graph-based methods for the other steps.

3.2 Geographic content clustering

To partition the graph we must first establish a distance function between the nodes, measuring the data similarity. This similarity is then associated with the edges, leading to a weighted dynamic graph. Every node has a collection of histograms, each representing the distribution of certain aspect in the population.

Let $G = (V, E)$ be a graph, where $V = \{v_1, v_2, \ldots, v_n\}$ is the set of nodes and $E = \{(v_i, v_j), i \neq j \text{ and } i, j \in [1, n]\}$ is the set of edges. A function H associates each node to a set of K histograms. We define the distance D between two nodes v_i and v_j as:

$$D(v_i, v_j) = \sum_{k \in [1,K]} w_k \, d(H_k(v_i), H_k(v_j)) \qquad (1)$$

where d is a distance metric between histograms and w is a sequence of non-negative weights associated with each aspect, $\sum_{k \in [1,K]} w_k = 1$. While any histogram metric can be used, we adopted a euclidean distance between the vectors, because it led to reasonable results with reduced computational cost. Therefore the distance between two nodes is defined as the weighted average distance between its associated histograms, where the weights can be adjusted by the user.

Once the distances are associated to the edges, we use watershed cuts [36] to create an initial clustering, which is then refined into a hierarchy using the Sorted Maximal Matching (SMM) [37] with median linkage. The initial watershed step is performed to create an initial clustering and reduce the running time of the SMM. For completeness, we briefly review this method, but we refer the reader to the original paper [37] for more details, including a complete performance evaluation using several metrics.

We included two application-specific parameters: the maximum number of clusters to be shown and a distance threshold. Contrarily to the original SMM, which merges all clusters in all steps, we only merge two clusters where the distance is above the threshold after we reach the maximum number of displayed clusters. Without this restriction, significantly different clusters would be merged early, leading to increased intra cluster variance and the disappearance of small outlier regions. Further, after the maximum number of clusters is reached, we create one step of the hierarchy for each merge, leading to a binary partition tree. In this structure we can directly access a result with an arbitrary number of clusters.

Each resulting cluster is contiguous in the graph. This means that two similar, but non-contiguous, sets of CTs will

be classified into two different clusters, which can be counterintuitive. To overcome this issue, we *augment* the graph with two new edges per node from a nearest neighbors graph [52] using only the distances between the histograms. These edges connect nodes with similar content, if they are not already connected, providing a path for the algorithm to group similar nodes. Theoretically, adding more of these content based edges could be used to decrease the impact of the spatio-temporal edges, controling the balance between content and topology in the result. In practice, the effect is dependent on the data itself, and the results are not consistent, or predictable, across different cities. We fixed it at two edges because it was the lowest number that empircally led to consistent clusters, but we believe that this idea warrants further investigation, as an alternative to mixing parameters in the distance metric [33].

3.3 Cluster characterization and variable relevance

The composition of each cluster is determined by simple statistical measures, considering each aspect separately. We compute the minimum, maximum, median, 25%, and 75% quantiles for each part of each aspect for all clusters in the hierarchy. While interpreting these values is more complex than interpreting just the average, they provide far more information about the underlying distribution.

We also use these statistical measurements to discover what characterizes each cluster, that is, what makes it different from the others. We define the *relevance* of a part of an aspect based on the distance between the interquantile ranges (IQR) of the clusters in the same hierarchical level. If the IQRs overlap for all clusters, that variable is not relevant to the characterization of the cluster, but if the IQRs are distant, it means that this specific range of values is something that only occurs in this cluster.

3.4 Clusters and trajectories

While the partition of the data into different clusters helps the user to understand what demographic groups exist and where they are, we are also interested in the changes in those groups. To this end, we introduce the concept of *trajectories*, which are composed by regions that are classified into the same sequence of clusters over the considered period. This enables direct access to regions that evolved in the same manner. While the interface provides access to data by census tract, the trajectories are the main unit of exploration in this work.

3.5 Colors

As illustrated by Figure 3 and further explored in the next subsection, our proposed interface heavily relies on color to express cluster-related information. We adopted this convention because colors can be used in all our visual tools in a coherent manner. However, this also introduced significant challenges. The first is the limit on the number of clusters that can be visually represented. We limited the number of clusters to eight because this was the largest number of colors that we could reliably use, derived from the 8-class Dark2 set from ColorBrewer [53].

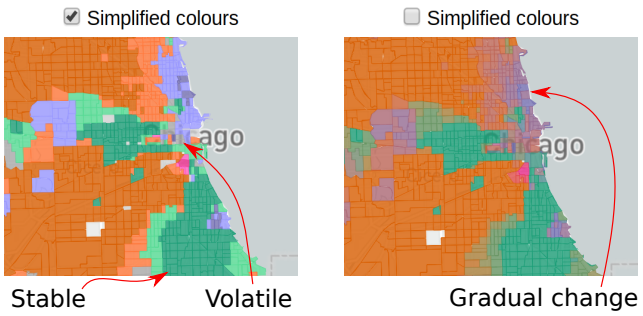

Fig. 2. Different color schemes for Chicago with four clusters. Left: simplified, right: average color.

While we can reasonably limit the number of clusters, there are far more possible trajectories. And the color associated with each trajectory should bear some resemblance to the clusters included in it. Therefore, we were left with a conundrum: *Should we associate each trajectory with a unique color, which the user probably cannot distinguish, or should we use a reduced set of colors and associate the same color to different trajectories?* Since there are advantages and disadvantages for each of those options, we adopted both. The user can control which color policy is used via a checkbox in the configuration panel, on the top left of the interface.

By default, the interface adopts a simplified color scheme, where a trajectory is painted in the same color of a cluster if the regions were associated to that cluster for *all* times; in a slightly less saturated version of that color if the regions were associated to that cluster for *the simple majority* of the time, and gray otherwise. In this mode, the colors will mostly represent stability, immediately identifying the regions that were consistently associated with each cluster. It also easily identifies volatile regions, painted gray.

When this simplified color scheme is disabled, each trajectory will be painted using the average of the colors of the involved clusters, in the LAB color space. In this mode, the map becomes more similar to a heatmap, where stronger presence of a color indicates more temporal affinity to the cluster. Volatile regions will also tend to be displayed in gray, as the average of three or more colors.

While both approaches will use more than eight colors, in practice this is not as significant because most cities can be explained using less than eight clusters. In fact, articles in the literature usually employ from two to five, which are fairly stable across time. For the more dynamic scenarios, user interaction can be used to alleviate the shortcomings of both approaches.

3.6 User interface

The initial interface is illustrated in Figure 3. Since demographic data can be nuanced, with intricate interconnections, we decided against presenting the interface using synthetic data, considering instead data from the Chicago region between 1970 and 2010, using previous published studies as corroboration. This region is known for its entrenched racial divide and the emergence of a *'young urban'* population with a higher education level [29], [30]. More details about this scenario are presented in Section 4.

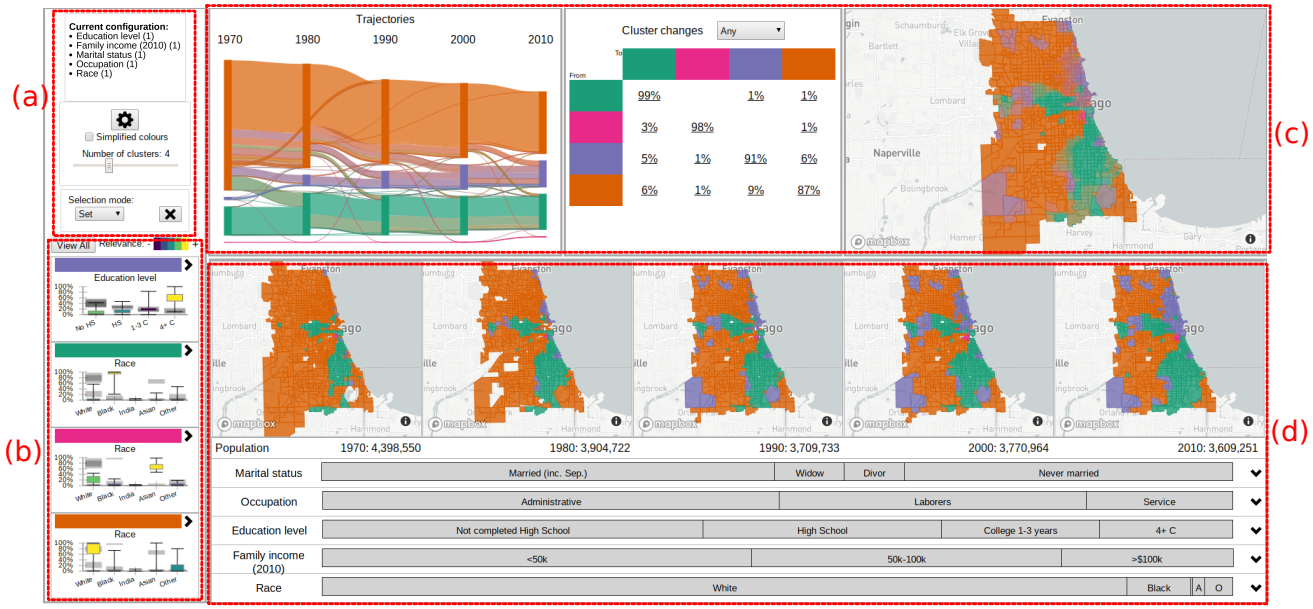

Fig. 3. Initial interface of our method showing the demographic evolution of Chicago. **(a)**: Configuration panel with the current clustering parameters and controls. **(b)**: Cluster overview illustrating the most relevant aspect for each cluster. **(c)**: Trajectories overview and the general evolution of the population, geographical information, and how it changed. **(d)**: Details of the selected trajectories, including precise geographic locations, population numbers, and the composition of the aspects.

The configuration panel, on top left in Figure 3, displays which aspects were considered and their weights (following Equation 1). It also includes other configuration options that can be altered without re-processing the data, such as the number of clusters and the color option. The gear button allows access to the other configuration options that do require further processing, such as changing location, aspects, and weights. This panel also includes the configuration of the selection mode for the trajectories, which allows the user to set, add, or remove the next selected trajectories to the current selection. This feature enables the analysis of complex sets of trajectories.

The cluster overview panel, on the bottom left in Figure 3, displays a brief summary of each cluster, based on the distance between the IQRs, as detailed in Section 3.3. The *View all* button opens a new panel where all aspects are represented, while the chevron at the side of the color lets the user expand each cluster separately. While the standard approach to represent cluster characteristics is to use parallel coordinates [50], [51], this representation occupies screen space proportional to the number of variables and can get cluttered with a higher number of clusters, or when the clusters are not well defined for multiple variables. To save space and leverage the familiarity scientists have with statistical tools, we opted to use boxplots to properly convey the distribution of each variable in the current cluster. However, a simple boxplot would not include information about the other clusters, forcing the user to mentally compare them to find what is relevant.

We adopt an *enhanced* version of the traditional boxplot, which includes the minimum, maximum, 25% and 75% quantiles for the current cluster, but also the IQRs for the other clusters, in slightly larger and faded black rectangles. We also color the current IQR according to its relevance. While there might be some degree of similarity between the color schema for relevance and for trajectory identification, none of the experts consulted reported confusion. These simple changes allow the user to easily understand the composition of the cluster and how it relates to the others. Violin plots [54] could also be used, but with increased potential for cluttering the representation of the other clusters, hindering the comparisons.

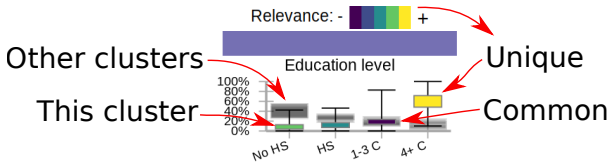

Fig. 4. Enhanced boxplot of the clusters' characteristics allows a quick comparison to the other clusters.

For instance, the boxplot that summarizes the purple cluster, detailed on Figure 4, illustrates that this cluster is best defined by the proportion of the population with four or more years of college. The user can quickly see that this is relevant because the corresponding IQR is colored with the highest relevance present in the legend. It is also clear that, while this cluster includes CTs that have between 10% to 90% of people in this variable, approximately, half of them have about 60% of the population with four or more years of college. Since all the other IQRs are well separated, this is a defining characteristic of this cluster. By clicking on the colored bar above the boxplot, the user can select all trajectories that contain this cluster at any point in time.

The other clusters identified on Figure 3 correspond to higher concentration of people that identify as Black in the green cluster, people that identify as "Asian, Hawaiian, other pacific islander" in the pink cluster, and people that identify as White in the orange cluster. From these plots,

it is clear that the city is indeed racially divided [29], with several CTs that are almost exclusively occupied by people of the same racial category.

The trajectories overview aims to convey basic information about the trajectories present in the data, where they are, and what changes are involved. This is done using three sub panels. The first, on the left, contains a Sankey diagram illustrating the evolution of the clusters over time. The widths are proportional to the population involved, the colors follow a policy detailed in Section 3.5. A stacked graph could also be used to represent the proportions of each cluster [20] with less clutter, since the transitions between clusters would not be represented. However, this is only viable if more temporal steps are available, making the plot smoother. Another option to remove clutter is to remove portions of this plot when trajectories are selected, but this would change the layout and compromise the user's mental map.

In our example in Figure 3, we can see that the total population of Chicago is decreasing. Additionally, the orange and green clusters contain most of the population and are fairly stable over time. The pink cluster is small and mostly stable. The purple cluster is increasing, mostly by incorporating areas that were previously orange. Since the purple group corresponds to the emergent 'young urban' group, this corroborates the findings of Delmelle [29], [30]. This diagram can also be used to select specific trajectories, by clicking on the bands, or all trajectories that contain a specific cluster at a specific time, by clicking on the rectangles.

The next sub panel, in the top middle of Figure 3, is a transition matrix between the clusters. It indicates the percentage of the population whose area changed in each pattern. This kind of table can be found in the related literature [29], so it is familiar to the advanced users, and it not only informs the proportional changes, but allows the selection of the corresponding trajectories for further analysis.

Contrary to the trajectories plot, this representation is more Markovian, where only the current and next state are considered. This panel also enables easier access to trajectories with specific changes, by clicking on the corresponding percentage values. The combo box allows the user to refine the transitions, from 'Any', which includes all transitions between years, to specific transitions, to changes from the first year to the last year. In this example, approximately 99% of the population in areas classified as green were also in areas classified as green in the next year, while 1% changed to purple at some point and another 1% to orange. The total is over 100% due to rounding errors. Regions changed from the orange to the green cluster for 6% of its population, 1% to pink, and 9% to purple. This further corroborates the fact that most of the growth of the purple cluster came from the orange cluster. Additionally, the lack of transitions is also relevant, for instance, no CT changed from majority of Black population (green) to Asian population (pink), and no CT with significant Asian population had significant increase in education levels (purple).

The last sub panel, in the top right of Figure 3, is a map of the region under analysis, summarizing the evolution over time. The objective of this panel is to provide an overview of the general position of the trajectories and clusters. The colors are derived from the clusters involved in each trajectory as detailed in Section 3.5, which are consistent across the linked views.

The bottom part of the interface contains the details for the selected trajectories. If no trajectory is selected, then the details for the whole city are displayed, as illustrated in Figure 3. This panel contains two main regions: the small multiple maps, depicting the clusters at each year, and the stacked bar plots that summarize the overall composition of these regions. Some finer localization information is lost using small multiples, such as small border changes, but that information is available at the larger map. All the maps are linked with syncronized navigation, and the use of small multiples allows the exploration of each temporal census individually, and its comparison to the others, with minimal interaction.

In this example, the maps show the transition from orange to green and purple in several regions over time. Clicking on a region in these maps will bring up a new panel with the original census data of this specific region. The actual population numbers are below the maps, and they confirm that the total population is indeed decreasing.

Each aspect is represented by a stacked bar plot, where the width of each rectangle corresponds to the average percentage of that variable. We chose stacked bar plots to represent the composition of the regions because they can accurately and succinctly inform the proportions of each aspect, without any interaction. In this case, about half of the people in Chicago in the considered period are married, and the percentage that are Widowers or Divorced is roughly similar. About half of the population work in Administrative jobs, a third never completed high-school, approximately half have gross family income below 50,000USD per year. The vast majority identify as white. Placing the mouse over one of the bars will open a small panel with the temporal evolution of that specific variable, and clicking on the chevron on the right side expands the corresponding aspect, showing details of the temporal evolution of each variable and also the corresponding IQRs for the whole city.

4 Illustrative Scenarios

We used decennial census data from the United States [4] and Canada[1], tabulated by CTs, from 1970 to 2010. The prototype allows access to 40 regions, 28 in the US and 12 in Canada. Due to the high number of CTs, New York City was split into its boroughs.

We used five aspects for the USA: Education level, Family income, Marital status, Occupation, and Race; and seven for Canada: Age, Education level, Home language, Household Income, Marital status, Occupation, Place of birth, and Religion. While our method does not require geographic harmonization, it requires matching variables over time. The supplementary material contains the details of which census columns were used for each aspect. Income is slightly inaccurate, even though we did correct for inflation. We grouped the original ranges into three larger ranges, but they do not match precisely.

1. http://datacentre.chass.utoronto.ca/census/

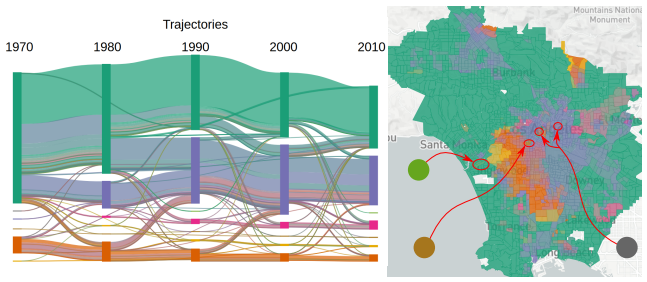

Fig. 5. Result for Los Angeles with 8 clusters, including three small and ephemeral clusters. Cluster characterization is displayed in Figure 6.

4.1 Los Angeles

We selected a region around the metropolitan area of Los Angeles (LA). This region follows urban density, not administrative boundaries. The summary of the results using all aspects with equal weights and eight clusters is illustrated in Figure 5, while the statistical description of the clusters is illustrated in Figure 6, where the most relevant aspect of each cluster is highlighted. From the trajectories plot, we can see that there is a large but shrinking cluster, depicted in green, one increasing cluster in purple, an almost constant orange cluster, a smaller but increasing pink cluster, and three other small clusters. The corresponding map illustrates where these clusters are located, and that they are somewhat geographically stable, with some movement between the green, orange, and purple clusters.

From Figure 6, we can see that the green cluster is characterized by a high percentage of White population, low percentage of population in the lowest income bracket, mostly administrative occupations, and about 30% of the population with four or more years of college. The orange cluster is characterized by a high percentage of Black population, with few people in the highest range of income and education. The purple cluster corresponds to a high concentration of "Other" in race, which includes Hispanic for this dataset, high concentration of Laborers, and low education and income. The pink cluster contains a high percentage of Asian population and about 30% of the population with four or more years of college. The light green cluster contains very few people in the lower income bracket, mostly White population, with the highest percentage of population with four or more years of college, working administrative jobs, and a high concentration of singles. The yellow cluster represents Black population, with higher level of education and income, mostly working administrative jobs. The brown cluster represent a majority of single population, working administrative jobs with mostly low income. The dark gray cluster is characterized by all its population in the lowest income bracket, low education level, with a majority of White population.

Further, the larger clusters, green, orange, and purple, present a significant intra-cluster variance in most variables, as indicated by the maximum and minimum whiskers of the boxplots. For instance, while fifty percent of the CTs in the green cluster have between 20% and 40% of people in the lowest income bracket, that cluster also includes CTs where none and all the population belongs to that bracket. This might indicate that this cluster could be further divided, and currently represents different groups of people that are not different enough to be separated into clusters in this level of the hierarchy.

While the larger clusters are expected, the light green, brown, and dark gray clusters are more surprising. These clusters are small and ephemeral, including only a few CTs, and are peculiarly different enough to be separated into their own clusters, at this level of the hierarchy. While these small clusters might be interesting enough to warrant further study, they might suppress larger, but more subtle, clusters from appearing in this interface, due to the limited number of clusters.

The orange area in the map in Figure 5 presents movement, indicated by the presence of green and purple tones mixed with the orange, which may warrant further exploration. By clicking on the orange bar above the boxplot, we select all trajectories that contain the orange cluster. The corresponding details are illustrated in Figure 7. This shows a location change, where the orange cluster is progressively replaced by the purple cluster on its east side, and in turn expanding to the west. Interestingly, the population increased, the racial profile changed, but the distribution of income was reasonably stable, with a higher amount of the population in the lowest income range and very few people in the highest income range. Indeed, the income difference is significant when compared to the city-wide distribution.

A portion of this region is classified into the green cluster in 2010, indicating a majority white population. To further understand that change, we clear the current selection, and select all regions that changed from orange in 1970 to green in 2010, using the transition matrix. A portion of the resulting region, near the Florence-Graham region, is depicted in Figure 8, along with the temporal evolution of Race. Despite this difference, the other aspects are similar to the ones from the region in Figure 7, with slightly lower income and education profiles. While the racial aspect changed considerably, the economic and educational aspects stayed the same.

4.2 Toronto

We considered a region that is approximately the administrative border of the current city of Toronto and all seven available aspects. The results are summarized in Figure 9, considering eight clusters.

The population with low percentage of University degrees is represented in orange, mostly anglophone population in green, Asian immigrants in yellow, high percentage of income in the highest bracket in purple, high percentage of Jewish people in light green and brown, high percentage of Eastern Non-Christian religion in pink, and high concentration of single people in dark gray.

From the trajectories plot, we can see that Toronto is more dynamic than both Chicago and LA, with one cluster constantly shrinking, and others growing more prominent. In the 1970s, the city was divided into four clusters: low number of university degrees, Jewish population, majority anglophones, and high income, with the latter being rather small. There is some variance in the other characteristics of the orange cluster, but none as significant as the education level, especially when compared to the other years. While

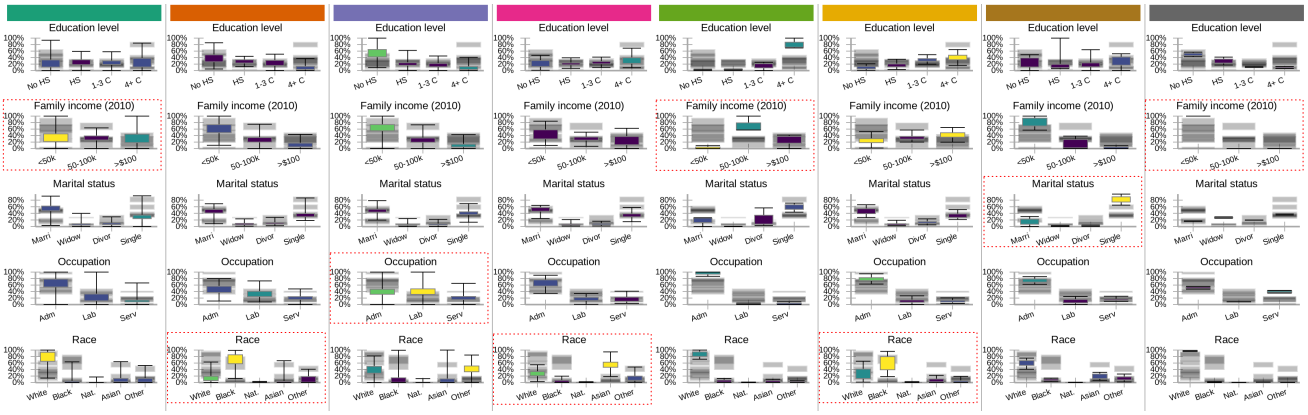

Fig. 6. Full characterization of the eight clusters found for LA. The red rectangles indicate the most relevant aspects for each cluster.

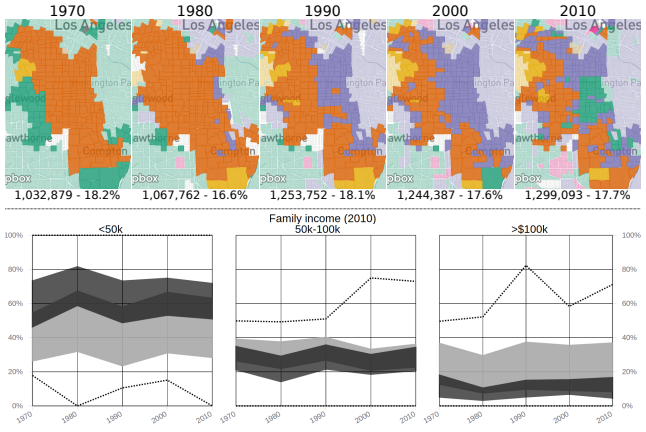

Fig. 7. Top: Geographic changes in the majority Black population cluster (orange) and Laborers cluster (green). Bottom: Income evolution for this region (black) and the whole city (gray).

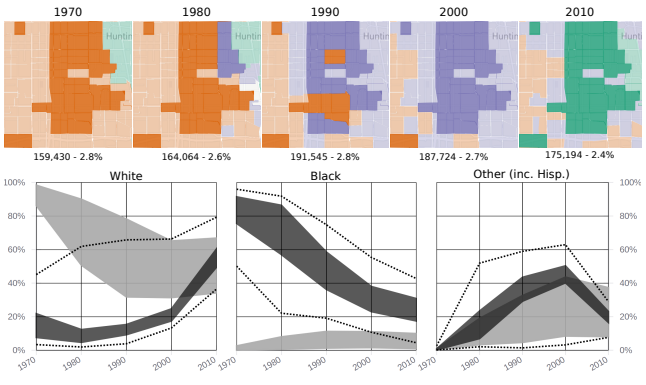

Fig. 8. Details for a volatile region contained in the area of Figure 7. This region went from Black to Hispanic to White.

the orange cluster persists until 2011, the other clusters present have similar education profiles to one another, leading the clustering method to use other aspects to characterize them.

Toronto's dynamism also appears in that we see the influx of Asian population starting in the 1980s and building thereafter, as represented by the yellow and pink clusters. While both include a high percentage of people born in Asia, the pink region is more defined by religion, with low

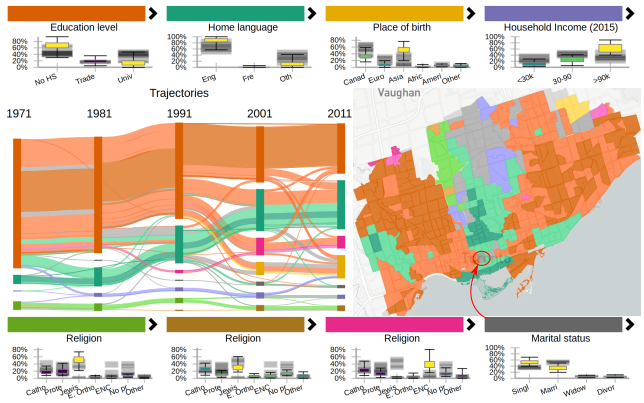

Fig. 9. Results for Toronto, with eight clusters.

percentage of university degrees, and contains the lowest percentage of people in the highest income bracket for these clusters; the yellow is less defined by religion, and has higher education and income, geographically corresponding to the Markham region, know for its Chinese population. A similar division also happens for the two Jewish clusters, where the light green cluster has lower education and income levels than the brown cluster. The purple cluster of high income is somewhat stable. This cluster includes until 2011 the Bridle Path neighborhood, known for its wealthy population. In 2011, however, it was classified into the yellow cluster that includes Asian immigration, since about 35% of the population for this CT were born in Asia. It maintained its income profile, with 85% with 90k CAD or more.

The grey regions of the map are another indication of Toronto's volatility over this period. While there is a cluster that is painted dark grey, annotated by a red circle in Figure 9, the light gray regions on the north part of the city indicate volatility. Here the same region was classified into three or more clusters for this five census period. Using the 'Add' mode for the trajectory selection, we select these volatile regions. A subset of the details is illustrated in Figure 10.

These regions account for about 5% of Toronto's population. The whole region was classified into the orange cluster in 1971 (low level of university degrees). By 1991,

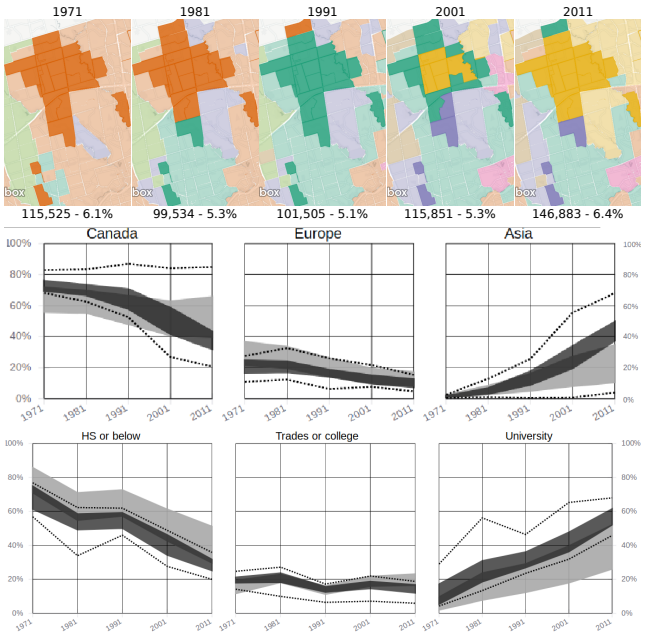

Fig. 10. Details for some regions of Toronto that were classified into 3 or more clusters over time.

most of the region was classified into the green cluster, representing anglophone population, mostly Canadian born, with a higher level of education. As the corresponding plot indicates, this trend in increasing education is city-wide, but this region has people with better education than most.

In 2001, the purple cluster of high income annexes neighboring parts of the volatile region, and the Asian born population increases sharply, as illustrated by the appearance of the yellow cluster. This cluster, once again, indicates well educated, higher income, and about 30%-50% Asian born population. By 2011, the yellow cluster increased considerably, annexing parts of the high income purple cluster, including the neighboring Bridle Path area.

While Toronto is more dynamic than Los Angeles, possibly due to size differences, the volatile regions shown in Figure 10 did not change as quickly or dramatically as the ones shown in Figure 8, which involved twice as many people. We found this trend to be related to the countries themselves, Canadian cities have larger areas undergoing slow, gradual changes, whereas American cities have more general stability, but quicker changes in smaller scales. The supplementary material contains brief summaries of all the regions accessible in this prototype.

5 Expert Feedback

To assess the proposed method, we contacted academic and industry experts in sociology and urban sciences. The supplementary material contains all our communication with five experts, identified from A to E.

Their response was positive, mentioning that the prototype allows them to analyze census data without the additional work of obtaining and cleaning the data (A, B, E), and it allows the inclusion of geographic visual analysis tools in their research process (D). It enables the users to tell different stories about neighborhoods/cities and their changes (A), visualize the relationship between key urban variables over time (D), offering a quick way to identify particular neighborhoods that one may be interested in studying more in depth around a particular issue or efficiently understanding the context of an area (E). Indeed, the experts identified gentrification processes in Manhattan (B) and Dallas (E), reinforced a hypothesis for occupational clustering (D), and highlighted how the method can be used to compare neighborhoods and cities (A). These remarks attest that we satisfied the design requirements regarding the existing demographic groups and their changes.

While the interface was "easy to navigate" (B), it was also considered "overwhelming" (A), "intimidating" (E), and "tricky to interpret" (C), possible side-effects of our effort to increase representational accuracy, where we avoided using simplified labels. Specifically, while identifying clusters by their most relevant variables was welcome, the overlap of information from different clusters in the boxplot was "a bit confusing" (C). While the map of trajectories was mentioned as a "good summary map", how it related to the clustering method was unclear (C). While we provide different options on how the colors are used on this map, both are sub optimal, reliably representing several distinct entities using colors is still an open problem in visualization.

The experts also mentioned the poor responsiveness of the method when changes in the clustering parameters required server-side processing (B,D). Indeed, the current implementation can take a few minutes to cluster regions with high number of CTs, like Los Angeles or Brooklyn.

Most of the experts demonstrated interest in using our method in their research (A, B, D, E), aiming to use the census data as a backdrop for other datasets, providing demographic context. They also mentioned the need to export subsets of data, plots, and maps to be used in reports and publications (C, D, E).

More importantly, while these experts were aware that our method does not perform geographic normalization, none of them mention it. We did not specifically ask if this difference led to unexpected results, but rather inquired if they found interesting insights, leading them to consider the interface, its options, and results as a whole. Therefore the method was neutrally compared against other, traditionally normalized results. We interpret the fact that most of them were interested in the next steps as an affirmation of the accuracy of the method.

6 Discussion and Limitations

The objective of this work was to leverage a graph based data representation with visual tools to allow for the exploration of geographically inconsistent census data. While we successfully replicated and corroborated results from the literature, this method still has significant limitations.

Removing the geographical normalization/interpolation step greatly reduces the amount of work necessary, but the method still requires consistent variables across the years. Matching the fields of the public census can be trivial for some aspects, like Age, but challenging for others, like Income. The divulged income ranges vary over time, the actual value changes due to inflation and other factors, and so on. Moreover, some fields were not considered in earlier

censuses, such as Race in Canada, or Hispanic population in the USA, hampering its use when they are available. We matched some aspects, but a deeper demographic analysis would greatly benefit from all available information.

Another limitation is the lack of control on how much the geographical information will impact the clustering result. While the adopted method met our needs for this work, a configurable control would add another dimension to the exploration, allowing for more intra-cluster variance to obtain more 'compact' clusters. We explored changing the number of content based augmented edges, but this proved to be unreliable and hard to interpret. The *ClustGeo* method [33] can be a viable option for this, allowing a graph based input and a hierarchical output, combined using a single mixing parameter. Alternatively, one could cluster the changes [55] instead of the stable states.

There are also technological limitations, such as memory use on the visualization client. To allow for changes on the CTs over the years, we use a geographic file that contains all possible intersections, which can grow rather large if the original city was expansive and contained several CTs, like NYC or LA. However, the most significant technological limitation relates to parameters that are not immediately interactive, such as the clustering configuration. Since the clustering is computationally expensive and performed on the server, which allows for cached results, some changes can take a few minutes to be considered, removing any possibility of a continuous exploration.

Indeed, the cognitive load on the user is already significant, as we compromised simplicity for accuracy. While other works labelled the clusters, as 'young urban', 'struggling', and so on [29], [30], we show the statistical characteristics of the clusters, which are harder to interpret, as the data may have subtle nuances that labels would otherwise hide. This also led to a crowded interface, mitigated somewhat the use of pop-up panels and collapsible sections. For some cities, especially if they are small and stable, the panels can appear redundant, but each provide a different way to interact with the information that can ease the exploration process for larger and dynamic cities.

7 Conclusion

Our objective was to allow for the exploration of census data without geographical harmonization, an approach that is usually not considered. Our method was able to corroborate previous findings from the specialized literature, with an increased level of detail due to our data representation and visualization choices. The feedback from experts was remarkably positive, most of them were able to extract insight from the prototype and demonstrated interest in using it on their research efforts.

While we focused on census data in this work, the proposed method can be used with most similarly area-based datasets. Further, while there is interest in the use of census data, it is mostly in combination with other, larger and more dynamic, datasets. We postulate that, while census data is interesting, the amount of effort required to obtain and prepare it often discourages its use as complementary information, reinforcing our hypothesis that there is need for methods that are able to deal with geographically inconsistent data.

"(...) tract-by-tract comparison is not possible unless data from 2000 is interpolated to 2010 boundaries (...)" [10]

Acknowledgments

This research was supported by a University of Toronto Connaught Global Challenge grant and is part of the Urban Genome Project.

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