

# Exercise 3.0 — Pose Graph Optimization and Iterative Closest Point

Artificial Intelligence For Robotics

Timo Hinzmann

Spring Semester 2017

## 1 Pose Graph Optimization (PGO)

In this exercise you will implement a pose graph optimization algorithm to estimate the position of a robot based on wheel odometry and loop closure constraints. The repetition in Sec. 1.1 is based on [1] and [2].

### 1.1 Repetition: PGO

Recall that the edges of a pose graph encode the constraints between two nodes  $i$  and  $j$  given an observation  $z_{ij}$ . Based on the current pose estimates stored in node  $i$  and  $j$  we can formulate the error as:

$$\mathbf{e}_{ij}(\mathbf{x}_i, \mathbf{x}_j) = \mathbf{z}_{ij} - \hat{\mathbf{z}}_{ij} \quad (1)$$

The goal of maximum likelihood pose graph optimization is to minimize the negative log-likelihood given all constraints

$$\mathbf{x}^* = \arg \min_{\mathbf{x}} \sum_{<i,j> \in \mathcal{C}} \mathbf{e}_{ij}^\top \Omega_{ij} \mathbf{e}_{ij} \quad (2)$$

where  $\mathbf{x}^*$  are the optimized poses,  $\mathcal{C}$  is the set of pairwise constraints from node  $i$  to node  $j$  and  $\Omega_{ij}$  is the corresponding measurement information matrix. The error function can be approximated with a first order Taylor expansion given an initial guess  $\check{\mathbf{x}}$  and a perturbation  $\Delta\mathbf{x}$ :

$$\mathbf{e}_{ij}(\check{\mathbf{x}} + \Delta\mathbf{x})^\top \Omega_{ij} \mathbf{e}_{ij}(\check{\mathbf{x}} + \Delta\mathbf{x}) \quad (3)$$

$$\approx (\mathbf{e}_{ij} + \mathbf{J}_{ij} \Delta\mathbf{x})^\top \Omega_{ij} (\mathbf{e}_{ij} + \mathbf{J}_{ij} \Delta\mathbf{x}) \quad (4)$$

$$= \underbrace{\mathbf{e}_{ij}^\top \Omega_{ij} \mathbf{e}_{ij}}_{c_{ij}} + 2 \underbrace{\mathbf{e}_{ij}^\top \Omega_{ij} \mathbf{J}_{ij}}_{\mathbf{b}_{ij}} \Delta\mathbf{x} + \Delta\mathbf{x}^\top \underbrace{\mathbf{J}_{ij}^\top \Omega_{ij} \mathbf{J}_{ij}}_{\mathbf{H}_{ij}} \Delta\mathbf{x} \quad (5)$$

where we defined  $\mathbf{e}_{ij} := \mathbf{e}_{ij}(\check{\mathbf{x}})$ . Minimization of Eq. 5 translates into solving the linear system  $\mathbf{H}\Delta\mathbf{x}^* = -\mathbf{b}$  using sparse Cholesky factorization. The linearized solution is then given as  $\mathbf{x}^* = \check{\mathbf{x}} + \Delta\mathbf{x}^*$ .

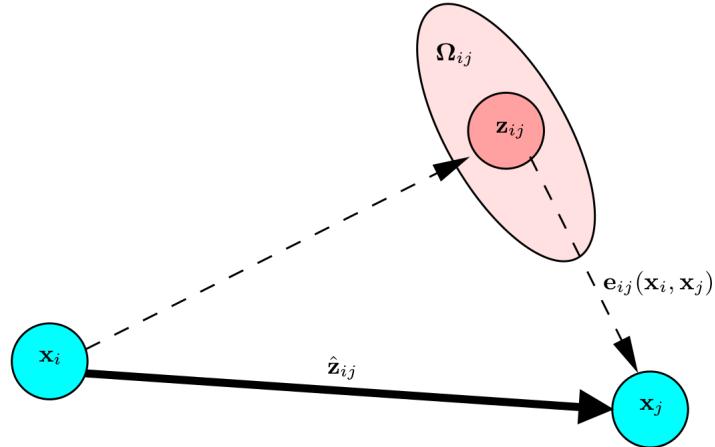


Figure 1: The measurement  $z_{ij}$  and entry of the information matrix  $\Omega_{ij}$  results in a constraint between node  $i$  and  $j$ . The expected observation  $\hat{z}_{ij}$  is computed from the current estimate  $x_i$  and  $x_j$ . [Figure taken from [1]]

## 1.2 Assignments

### 1.2.1 Synthetic Dataset: Robot with odometry and loop closure constraints in 1D

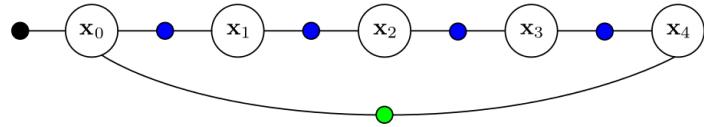


Figure 2: Pose graph in factor graph formulation. The constraints consist of wheel odometry (blue), loop closure (green) and fixed first node (black). [Figure taken from [2]]

For simplicity, we consider the problem in one dimension and estimate only the position in  $x$ -direction, i.e. the state is  $\mathbf{x} := [x]$ .

Tasks: Follow the steps described in [1], Algorithm 1 and fill in the sections marked with `#TODO` in `pgo_1D.py`. In particular:

1. Compute the residual  $\mathbf{e}_{ij}$  as well as Jacobians  $\mathbf{A}_{ij}$  and  $\mathbf{B}_{ij}$  for
  - (a) Wheel odometry: Returns the measured distance  $u_{ij}$  from node  $i$  to node  $j$ .
  - (b) Loop closure: The loop closure constraint can be interpreted as an odometry constraint from node  $i$  to node  $j$  with  $u_{ij} = 0$ .
  - (c) Fixation of first node: We fix the first pose to the origin which is (arbitrarily) selected as starting point.

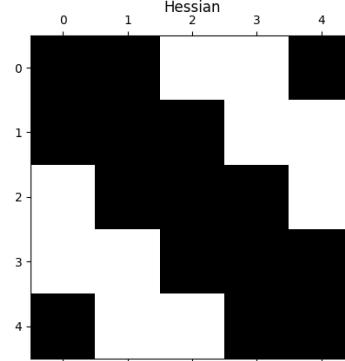
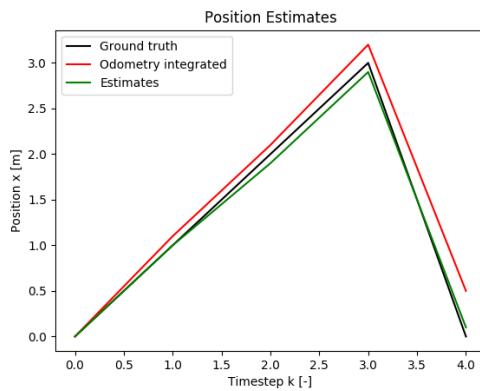


Figure 3: Sample position output: Ground truth, integrated odometry, fused integrated odometry and loop closure constraints.

Figure 4: Hessian showing the odometry and loop closure constraints. Black squares are non-zero matrix entries.

2. Set the corresponding values of the measurement information matrix:
  - Wheel odometry:  $\Omega_{ij} = 100$
  - Loop closure:  $\Omega_{ij} = 100$
  - Fixation of first node:  $\Omega_{ij} = 1000$
3. Implement the Gauss-Newton based pose graph optimization which computes  $\mathbf{H}$ ,  $\mathbf{b}$  and solves for  $\Delta\mathbf{x}$  using sparse Cholesky factorization. The positions estimated by your pose graph optimization algorithm are automatically stored to `results_1D.txt`.

### 1.2.2 Synthetic Dataset: Robot with odometry and loop closure constraints in 2D

In this assignment we augment the problem from the 1D to the 2D case. The state of the robot is now  $\mathbf{x} := [x \ y \ \theta]$ .

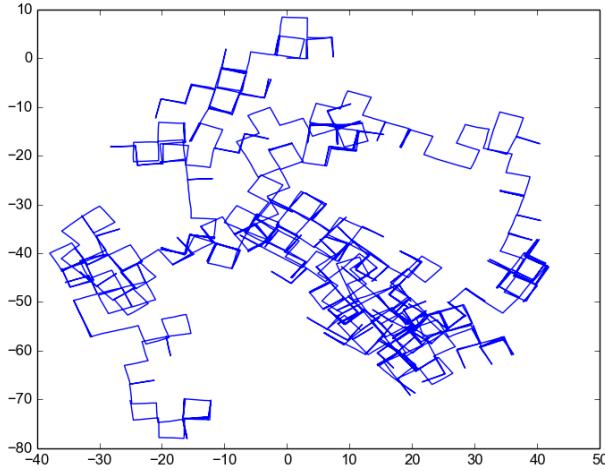


Figure 5: Initial guess for  $x$  and  $y$  computed from odometry.

The input data is given as follows:

- `vertices.dat`:  
`vertex_id x y theta`
- `edges.dat`:  
`vertex_id_from vertex_id_to dx dy dth 011 012 022 033 013 023`
- `loop_closures.dat`:  
`vertex_id_from vertex_id_to dx dy dth 011 012 022 033 013 023`

where  $dx, dy, dth$  is the odometry measurement between two vertices and  $\Omega_{ij}$  is the entry of the information matrix defined as

$$\boldsymbol{\Omega} = \begin{bmatrix} \Omega_{11} & \Omega_{12} & \Omega_{13} \\ \Omega_{12} & \Omega_{22} & \Omega_{23} \\ \Omega_{13} & \Omega_{23} & \Omega_{33} \end{bmatrix} \quad (6)$$

The edge and loop closure constraints are obtained from wheel and laser odometry as well as laser scan matching respectively. Fig. 5 shows the initial guess for  $x$  and  $y$  computed from the odometry.

Tasks: Follow the steps described in [1], Algorithm 1 and fill in the sections marked with `#TODO` in `pgo_2D.py`. The optimized states  $\hat{\mathbf{x}} := [\hat{x} \ \hat{y} \ \hat{\theta}]$  are automatically saved to `results_2D.txt`. Feel free to reuse the code from Sec. 1.2.1.

## 2 Pointcloud Registration

In the first assignment you will implement the point-to-point iterative closest point (ICP) algorithm to register a synthetic source pointcloud to a target pointcloud. In the second assignment you will implement a pointcloud registration algorithm of your choice to register a real-world vision pointcloud to a laser pointcloud. The repetition about ICP in Sec. 2.1 is based on [3].

### 2.1 Repetition: ICP

The goal of ICP is to find the transformation matrix  $\mathbf{T}$  that best transforms all points from source to target pointcloud. Several ICP variants exist - such as point-to-point, point-to-plane and generalized ICP. In the first assignment, we focus on point-to-point ICP and follow the algorithm as described in Algorithm 2.

---

#### Algorithm 2 Point-to-point ICP

**input:** Two pointclouds: source  $\mathbf{p}_s$ , target  $\mathbf{p}_t$ ; an initial transformation  $\hat{\mathbf{T}}$   
**output:** The optimized transformation  $\hat{\mathbf{T}}$  which aligns source and target pointcloud.  
 $\mathbf{p}_t^c$ : Nearest neighbors in target cloud for every point in source cloud.

---

```

1: function COMPUTEBESTTRANSFORMATION( $\mathbf{p}_s, \mathbf{p}_t$ )
2:    $\bar{\mathbf{p}}_s \leftarrow \text{computeCentroid}(\mathbf{p}_s)$ 
3:    $\bar{\mathbf{p}}_t \leftarrow \text{computeCentroid}(\mathbf{p}_t)$ 
4:    $[\mathbf{U} \quad \mathbf{D} \quad \mathbf{V}^\top] \leftarrow \text{SVD}(\sum_i (\mathbf{p}_{s,i} - \bar{\mathbf{p}}_s)(\mathbf{p}_{t,i} - \bar{\mathbf{p}}_t)^\top)$ 
5:    $\hat{\mathbf{R}} = \mathbf{U}^\top \mathbf{V}$ 
6:    $\hat{\mathbf{t}} = \bar{\mathbf{p}}_t - \hat{\mathbf{R}}\bar{\mathbf{p}}_s$ 
7:    $\hat{\mathbf{T}} \leftarrow \text{createTransformationMatrix}(\hat{\mathbf{t}}, \hat{\mathbf{R}})$ 
8: end function

9: function ICP( $\mathbf{p}_t, \mathbf{p}_s, \hat{\mathbf{T}}$ )
10:    $\mathbf{p}_s^{orig} \leftarrow \mathbf{p}_s$                                  $\triangleright$  Save original source pointcloud for line 16.
11:    $\mathbf{p}_s \leftarrow \text{transformCloud}(\mathbf{p}_s, \hat{\mathbf{T}})$ 
12:   while not converged do
13:      $\text{rejectPointsTooFarAway}()$                        $\triangleright$  [Optional]
14:      $\mathbf{p}_t^c \leftarrow \text{getNearestNeighbors}(\mathbf{p}_s, \mathbf{p}_t)$ 
15:      $\hat{\mathbf{T}} \leftarrow \text{computeBestTransformation}(\mathbf{p}_s, \mathbf{p}_t^c)$ 
16:      $\mathbf{p}_s \leftarrow \text{transformCloud}(\mathbf{p}_s, \hat{\mathbf{T}})$ 
17:   end while
18:    $\hat{\mathbf{T}} \leftarrow \text{computeBestTransformation}(\mathbf{p}_s^{orig}, \mathbf{p}_s)$ 
19: end function

```

---

## 2.2 Assignments

### 2.2.1 Synthetic Dataset

In Fig. 6 you can see the (misaligned) source pointcloud (red) and the target pointcloud (green). The goal is to estimate the transformation  $\mathbf{T}$  that best aligns the source to the target pointcloud as shown in Fig. 7 . Note that these are the same pointclouds, generated by applying a transformation  $\mathbf{T}$ , i.e. there exist unique point-to-point correspondences which is an underlying assumption of ICP.

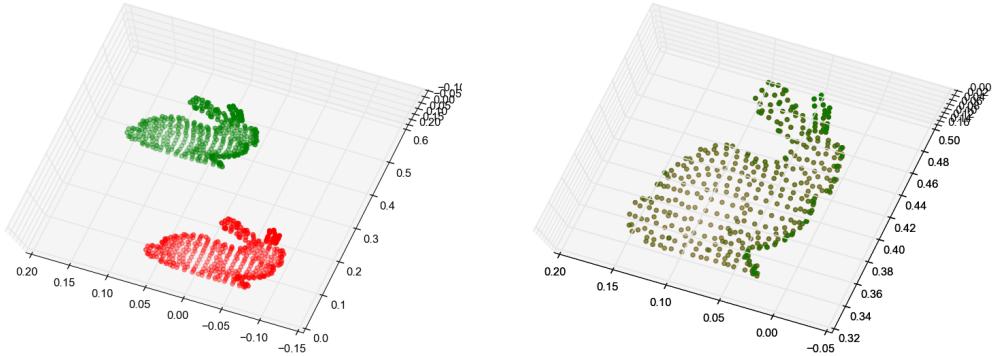


Figure 6: Misaligned pointclouds: source (red) and target (green).      Figure 7: Aligned pointclouds: source (red) and target (green).

Task: Follow the steps described in Algorithm 2 and modify the sections marked with `#TODO` in `results_alignment_synthetic.py`. The program will automatically save the final transformation  $\mathbf{T}$  which will be used for grading.

### 2.2.2 Real-World Dataset: Vision-laser alignment

In this assignment you will have to register a pointcloud obtained from a camera to a laser pointcloud (cf. Fig. 8 and 9). Notice that the difference to Sec. 2.2.1 is that, in general, unique point-to-point matches do not exist. Your task is to implement a robust pointcloud registration algorithm of your choice that aligns the vision to the laser pointcloud by modifying the



Figure 8: Platforms used for point-cloud generation: Fixed-wing (Vision) and helicopter (Laser) UAV. [Figure taken from [4]]

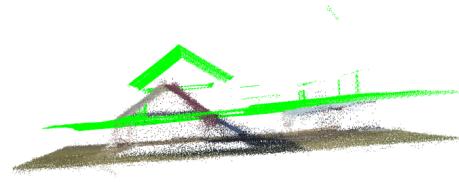


Figure 9: Misaligned source (vision, colored) and target (laser, green) pointcloud. [Figure taken from [4]]

```
#TODOs in alignment_real_world.py. The final transformation T is stored in results_alignment_real_world.txt. Feel free to reuse the code from Sec. 2.2.1.
```

### 3 Submission

To hand in the exercise you have to zip the whole folder (3\_0\_pgo\_icp) with your code and result files and upload it on moodle. The zip file you upload should have the name *aifr\_lastname\_ex3* (replace *lastname* with your last name). The python scripts are written in a way, such that it always creates a **results\*.txt** or **results\*.pkl** file with the data that is used for evaluation. **Please make sure that your python scripts are executable.**

### References

- [1] G. Grisetti, R. Kuemmerle, C. Stachniss, *et al.*, “A tutorial on graph-based SLAM,” *Intelligent Transportation Systems Magazine, IEEE*, vol. 2, no. 4, pp. 31–43, 2010.
- [2] N. Sünderhauf, *Robust Optimization for Simultaneous Localization and Mapping*. PhD thesis, 2012.

- [3] C. Cadena, I. Gilitschenski, and R. Siegwart, “Artifical intelligence for robotics: Pose graph optimization & iterative closest points.” Lecture Notes, Lecture 7.
- [4] T. Hinzmann, T. Stastny, G. Conte, *et al.*, “Collaborative 3D Reconstruction using Heterogeneous UAVs: System and Experiments,” in *Experimental Robotics - The 15th International Symposium on Experimental Robotics, ISER 2016, October 3-6, 2016, Tokyo, Japan*, 2016.