① а) $\lim\limits_{x\to1}\left(\frac{3}{1-\sqrt{x}}-\frac{2}{1-\sqrt[3]{x}}\right)=\lim\limits_{x\to1}\left(\frac{3-3\sqrt[3]{x}-2+2\sqrt{x}}{(1-\sqrt{x})(1-\sqrt[3]{x})}\right)=$

$=\lim\limits_{x\to1}\left(\frac{1+2\sqrt{x}-3\sqrt[3]{x}}{(1-\sqrt{x})(1-\sqrt[3]{x})}\right)\boxed{t=\sqrt[6]{x}}=\lim\limits_{x\to1}\frac{1+2t^3-3t^2}{(1-t^3)(1-t^2)}=\lim\limits_{x\to1}\frac{2t^3-2t^2-t^2+t-t+1}{(t-1)(t^2+t+1)(t-1)(t+1)}=$

$=\lim\limits_{x\to1}\frac{2t^2(t-1)-t(t-1)-(t-1)}{(t-1)(t^2+t+1)(t-1)(t+1)}=\lim\limits_{x\to1}\frac{2t^2-t-1}{(t^2+t+1)(t-1)(t+1)}=\lim\limits_{x\to1}\frac{(2t+1)(t-1)}{(t^2+t+1)(t-1)(t+1)}=$

$=\lim\limits_{x\to1}\frac{2\sqrt[6]{x}+1}{(\sqrt[3]{x}+\sqrt[6]{x}+1)(\sqrt[6]{x}+1)}=\frac{3}{3\cdot2}=\frac{1}{2}$

б) $\lim\limits_{x\to0}\frac{\sqrt[4]{\cos4x}-1}{\sin^2 8x}=\lim\limits_{x\to0}\frac{\sqrt{\cos4x}-1}{(\sqrt[4]{\cos4x}+1)\sin^2 8x}=\lim\limits_{x\to0}\frac{\cos4x-1}{(\sqrt[4]{\cos x}+1)(\sqrt{\cos x}+1)(\sin^2 8x)}=$

$=\lim\limits_{x\to0}\frac{-\left(\frac{(4x)^2}{2}\right)}{(\sqrt[4]{\cos x}+1)(\sqrt{\cos x}+1)\,64x}=\lim\limits_{x\to0}\frac{-8x^2}{(\sqrt[4]{\cos x}+1)(\sqrt{\cos x}+1)\,64x}=\frac{-8}{2\cdot2\cdot64}=-\frac{1}{32}$

в) $\lim\limits_{x\to\frac{\pi}{2}}\frac{e^{\sin2x}-e^{\operatorname{tg}2x}}{\ln\frac{2}{\pi}}=\lim\limits_{x\to\frac{\pi}{2}}\frac{(e^{\sin2x}-1)-(e^{\operatorname{tg}2x}-1)}{\ln\frac{2}{\pi}}=$

$=\lim\limits_{x\to\frac{\pi}{2}}\frac{\sin2x-\operatorname{tg}2x}{\ln\frac{2}{\pi}}=\lim\limits_{x\to\frac{\pi}{2}}\frac{2x-2x}{\ln\frac{2}{\pi}}=0$

② $f(x)=\sqrt[3]{x^2}-2\sqrt[3]{x}+1;\quad x_0=1$

$\lim\limits_{x\to1}\left(\sqrt[3]{x^2}-2\sqrt[3]{x}+1\right)=0$ – бесконечно малая величина.

$\lim\limits_{x\to1}\frac{\sqrt[3]{x^2}-2\sqrt[3]{x}+1}{C(x-1)^k}=1;\quad y=\sqrt[3]{x}$

$\lim\limits_{x\to1}\frac{y^2+2y+1}{C(y^3-1)^k}=1;\quad \lim\limits_{y\to1}\frac{(y-1)^2}{C(y^3-1)^k}=1;\quad \lim\limits_{y\to1}\frac{(y-1)^2}{C(y^3-1)^{k-2}(y-1)^2(y^2+y+1)^2}=$

$=\lim\limits_{y\to1}\frac{1}{C(y^3-1)^{k-2}(y^2+y+1)^2};\quad k=2,\quad C=\frac{1}{9}$

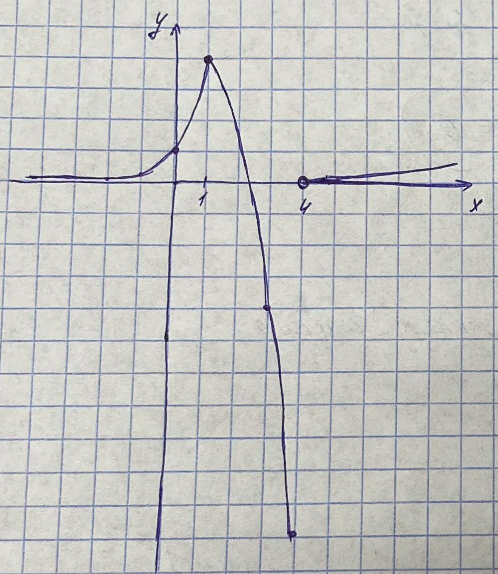
③ $f(x) = \begin{cases} 4^x, & x < 1 \\ 5 - x^2, & 1 \le x \le 4 \\ \lg(x-4), & x > 4. \end{cases}$

$x_0 = 4$

$$\left.\begin{aligned} &\lim_{x \to 1-0} (4^x) = 4 \\ &\lim_{x \to 1+0} (5 - x^2) = 4 \end{aligned}\right]$$ нет разрыва

$x_1 = 4$

$$\left.\begin{aligned} &\lim_{x \to 4-0} (5 - x^2) = -11 \\ &\lim_{x \to 4+0} (\lg(x-4)) = -\infty \end{aligned}\right]$$ $f(x)$ терпит разрыв 2-го рода; бесконечный разрыв в т. $x_1 = 4$



④ $\lim\limits_{x \to \infty} \left(\frac{x^2+5}{x^2+3} \right)^{4x^2} = \lim\limits_{x \to \infty} \left(1 + \frac{2}{x^2+3} \right)^{4x^2}$

$x \to \infty$: $\frac{2}{x^2+3} = \frac{1}{y}$; $x^2 + 3 = 2y$

$x^2 = 2y - 3$; $y \to \infty$

$\lim\limits_{y \to \infty} \left(1 + \frac{1}{y} \right)^{4(2y-3)} = e^8$

(5) $f(x) = 2^{\frac{x}{x^2-1}}$

$x_0 = 1$:

$$\lim_{x \to 1-0} \left(2^{\frac{x}{x^2-1}}\right) = 0$$

$$\lim_{x \to 1+0} \left(2^{\frac{x}{x^2-1}}\right) = +\infty$$

$f(x)$ терпит бесконечный разрыв 2-го рода в т. $x_0 = 1$

$x_1 = -1$:

$$\lim_{x \to -1-0} \left(2^{\frac{x}{x^2-1}}\right) = 0$$

$$\lim_{x \to -1+0} \left(2^{\frac{x}{x^2-1}}\right) = +\infty$$

$f(x)$ терпит бесконечный разрыв 2-го рода в т. $x_1 = -1$