$\boxed{1}$

N3 var 5

1. $\lim\limits_{n\to\infty} \frac{8n+5}{2n+7} = 4;$

$\forall \varepsilon>0 \quad \exists n_0 \in N : \forall n \in N,\ n \geq n_0 : \quad \left|\frac{8n+5}{2n+7} - 4\right| < \varepsilon$

$\left|\frac{-23}{2n+7}\right| < \varepsilon; \qquad \frac{23}{2n+7} < \varepsilon; \qquad 2n+7 > \frac{23}{\varepsilon};$

$n > \frac{23-7\varepsilon}{2\varepsilon};$

$n_0 = \left[\frac{23-7\varepsilon}{2\varepsilon}\right] + 1$

2. $\lim\limits_{n\to\infty} (3-2^n) = -\infty$

$\forall M>0 \quad \exists n_0 \in N : \forall n \in N,\ n \geq n_0 : \quad -M \leq 3-2^n;$

$M \geq 2^n - 3; \qquad 2^n > M+3; \qquad n > \log_2(M+3)$

$n_0 = [\log_2(M+3)] + 1;$

3. $\lim\limits_{n\to\infty} \frac{n^2+n}{2n-1} = +\infty$

$\forall M>0 \quad \exists n_0 \in N ; \forall n \in N,\ n \geq n_0 : \quad \frac{n^2+n}{2n-1} > M$

$\frac{n^2+n}{2n-1} > \frac{n^2+n}{2n} > M; \qquad \frac{n+1}{2} > M; \qquad n > 2M-1$

$n_0 = [2M-1] + 1;$

№4 var 5

1. $\lim\limits_{n\to\infty} \dfrac{3n\sqrt[3]{n^4+7n^2+5}-\sqrt{n^3-5}}{\sqrt[4]{n^6-5n^3+4}+3n^2\sqrt[3]{n+1}} \left[\dfrac{\infty}{\infty}\right] = \lim\limits_{n\to\infty} \dfrac{3n\cdot n^{4/3}\sqrt[3]{1+\frac{7}{n^2}+\frac{5}{n^4}}-n^{3/2}\sqrt{1-\frac{5}{n^3}}}{n^{3/2}\sqrt{1-\frac{5}{n^3}+\frac{4}{n^6}}+3n^2\cdot n^{1/3}\sqrt[3]{1+\frac{1}{n}}} =$

$= \dfrac{3}{3} = 1$

2. $\lim\limits_{n\to\infty} \sqrt[n]{\dfrac{3n^2+2n+7}{n-5}} \left[\dfrac{\infty}{\infty}\right] = \left(\lim\limits_{n\to\infty} \dfrac{3n^2+2n+7}{n-5}\right)^{\lim\limits_{n\to\infty}\frac{1}{n}} = \left(\lim\limits_{n\to\infty}\left(\dfrac{3n^2+2n+7}{n-5}\right)\right)^0 =$

$= 1$

3. $\lim\limits_{n\to\infty} \dfrac{4^{1+n}-5^n+n^4}{4^{n-1}-3n^3} \left[\dfrac{\infty}{\infty}\right] = \lim\limits_{n\to\infty} \dfrac{4-\left(\frac{5}{4}\right)^n+\frac{n^4}{4^n}\to 0}{\frac{1}{4}-\frac{3n^3}{4^n}\to 0} =$

$= -\infty$

4. $\lim\limits_{n\to\infty} \dfrac{5(n+1)!-4^{n-4}}{4(n+7)\cdot n!-5^{2n+7}} \left[\dfrac{\infty}{\infty}\right] = \lim\limits_{n\to\infty} \dfrac{n!\left(5(n+1)-\frac{4^{n-4}}{n!}\right)}{n!\left(4(n+7)-\frac{5^{2n+7}}{n!}\right)} =$

$= \lim\limits_{n\to\infty} \dfrac{5(n+1)-\frac{4^{n-4}}{n!}\to 0}{4(n+7)-\frac{5^{2n+7}}{n!}\to 0} = \dfrac{5}{4}$

5. $\lim\limits_{n\to\infty} \left(\dfrac{9n+1}{17n-9}\right)^{5n+1}$; $\lim\limits_{n\to\infty} \dfrac{9n+1}{17n-9} = \dfrac{9}{17} \Rightarrow \lim\limits_{n\to\infty} \left(\dfrac{9n+1}{17n-9}\right)^{5n+1} = 0$

6. $\lim\limits_{n\to\infty} \left(\dfrac{8n+5}{8n-4}\right)^{5n+1} [1^\infty] = \lim\limits_{n\to\infty} \left(1+\dfrac{9}{8n-4}\right)^{5n+1}$;

$n\to\infty$: $\dfrac{9}{8n-4} = \dfrac{1}{y}$; $8n = 9y+4$, $n = \dfrac{9y+4}{8}$; $y\to\infty$

$\lim\limits_{y\to\infty} \left(1+\dfrac{1}{y}\right)^{5\cdot\left(\frac{9y+4}{8}\right)+1} = e^{45}$

7. $$\lim_{n\to\infty} n^{1/3}\left(\sqrt[3]{3n^2+2n+4}-\sqrt[3]{3n^2-3n-4}\right)^{[\infty-\infty]}=$$

$$=\lim_{n\to\infty} n^{1/3}\cdot\frac{3n^2+2n+4-3n^2+3n+4}{\left(\sqrt[3]{3n^2+2n+4}\right)^2+\sqrt[3]{3n^2+2n+4}\cdot\sqrt[3]{3n^2-3n-4}+\left(\sqrt[3]{3n^2-3n-4}\right)^2}=$$

$$=\lim_{n\to\infty}\frac{n^{1/3}(5n+8)}{n^{4/3}\left(\sqrt[3]{3+\frac{2}{n}+\frac{4}{n^2}}\right)^2+n^{2/3}\cdot\sqrt[3]{3+\frac{2}{n}+\frac{4}{n^2}}\cdot n^{2/3}\sqrt[3]{3-\frac{3}{n}-\frac{4}{n^2}}+n^{4/3}\left(\sqrt[3]{3-\frac{3}{n}-\frac{4}{n^2}}\right)^2}=$$

$$=\lim_{n\to\infty}\frac{n^{4/3}\left(5+\frac{8}{n}\right)}{n^{4/3}\left(\left(\sqrt[3]{3+\frac{2}{n}+\frac{4}{n^2}}\right)^2+\sqrt[3]{3+\frac{2}{n}+\frac{4}{n^2}}\sqrt[3]{3-\frac{3}{n}-\frac{4}{n^2}}+\left(\sqrt[3]{3-\frac{3}{n}-\frac{4}{n^2}}\right)^2\right)}=$$

$$=\frac{5}{\left(\sqrt[3]{3}\right)^2+2\left(\sqrt[3]{3}\right)^2+\left(\sqrt[3]{3}\right)^2}=\frac{5}{3\left(\sqrt[3]{3}\right)^2}=\frac{5\cdot\sqrt[3]{3}}{9}$$

8. $$\lim_{n\to\infty}\sqrt[n]{\frac{2n+7}{n-5}}=\left(\lim_{n\to\infty}\left(2+\frac{17}{2n-5}\right)\right)^{\lim\limits_{n\to\infty}\frac{1}{n}}=1$$

9. $$\lim_{n\to\infty}\frac{\sqrt[n]{64}-1}{\sqrt[n]{4}-1}\;\left[\frac{0}{0}\right]=$$

$$\frac{\sqrt[n]{64}-1}{\sqrt[n]{4}-1}\qquad \sqrt[n]{4}=a\qquad \frac{a^3-1}{a-1}=a^2+a+1$$

$$\lim_{n\to\infty}\left(\sqrt[n]{16}+\sqrt[n]{4}+1\right)=1+1+1=3$$

10. $$\lim_{n\to\infty}\frac{2-5+4-7+\ldots+2n-(2n+3)}{3n+2}=\lim_{n\to\infty}\frac{(2+2n)n\cdot\cancel{(-1)}(5+2n+3)n}{2(3n+2)}=$$

$$=\lim_{n\to\infty}\frac{n(2+2n-5-2n-3)}{2(3n+2)}=\lim_{n\to\infty}\frac{-6n}{6n+4}=-1$$

$\boxed{2}$ var 25.

1. $f(x) = \sin(x^2+5x)$; $g(x) = (x^3-25x)$

$$\lim_{x\to 0} \frac{f(x)}{g(x)} = \cancel{\frac{\sin(x^2+5x)}{x^3-25x}} = \lim_{x\to 0} \frac{\sin(x^2+5x)}{x^3-25x} = \lim_{x\to 0} \frac{x^2+5x}{x^3-25x} =$$

$$= \lim_{x\to 0} \frac{1}{x-5} = -\frac{1}{5} \Rightarrow f(x) \sim g(x)$$, как бм одного порядка

2. $$\lim_{x\to 0} \frac{e^{5x}-1}{x^3-27} = \lim_{x\to 0} \frac{5x}{x^3-27} = 0$$

3. $$f(x) = \begin{cases} x, & x < -2 \\ -x+1, & -2 \le x \le 1 \\ x^2-1, & x > 1 \end{cases}$$

~~[illegible]~~ $x=-2$

$$\lim_{x\to -2-0} (x) = -2$$

$$\lim_{x\to -2+0} (-x+1) = 3$$

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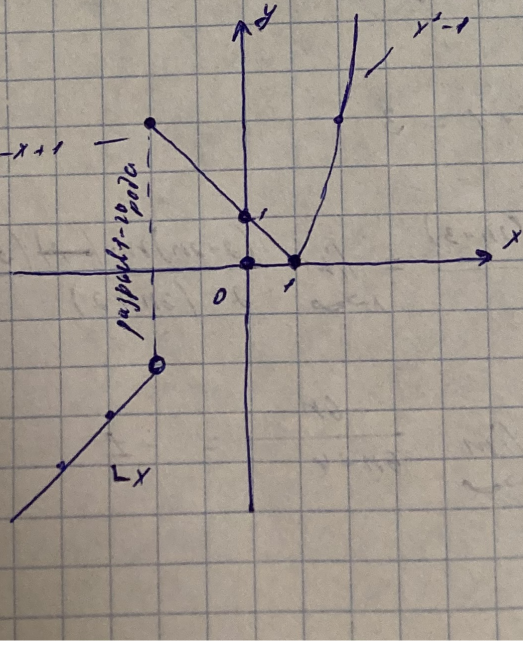
разрыв 1-го рода

$x=1$

$$\lim_{x\to 1-0} (-x+1) = 0$$

$$\lim_{x\to 1+0} (x^2-1) = 0$$

нет разрыва



4. $f(x) = \frac{x-4}{x+3}$ $\quad x_1 = -3; \; x_2 = -2$

$\lim\limits_{x \to -3-0} \left(\frac{x-4}{x+3}\right) = +\infty$; $\lim\limits_{x \to -3+0} \left(\frac{x-4}{x-3}\right) = -\infty$ разрыв 2-го рода

$\lim\limits_{x \to -2-0} \left(\frac{x-4}{x+3}\right) = -6$; $\lim\limits_{x \to -2+0} \left(\frac{x-4}{x+3}\right) = -6$ непрерывна