STA101 Problem Set 3 - KEY

Summer I, 2021, Duke University

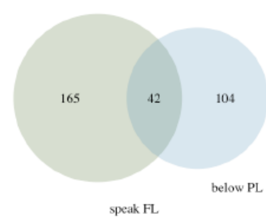
Exercises from the OpenIntro book - 100pts total

Problems include: 3.8, 3.32, and 4 additional problems

3.8 (22pts total)

(a) **(2pts for "no")** No, there are people who are both living below the poverty line and speak a language other than English at home.

(b) **(4pts)** The Venn diagram is shown below.



(c) **(4pts)** Each person living below the poverty line either speaks only English at home or doesn't. Since 14.6% of Americans live below the poverty line and 4.2% speak a language other than English at home, the other 10.4% only speak English at home.

(d) **(4pts)** Using the General Addition Rule:

$$\begin{aligned}\mathbb{P}(\text{below PL or speak FL}) &= \mathbb{P}(\text{below PL}) + \mathbb{P}(\text{speak FL}) - \mathbb{P}(\text{both})\\ &= 0.146 + 0.207 - 0.042\\ &= 0.311\end{aligned}$$

(e) **(4pts)** $\mathbb{P}(\text{neither below PL nor speak FL}) = 1 - \mathbb{P}(\text{below PL or speak FL}) = 1 - 0.311 = 0.689$

(f) **(2pts for "dependent", 2pts for explanation/justification/computation)** Two approaches:

- Using the multiplication rule: $\mathbb{P}(\text{below PL}) \times \mathbb{P}(\text{speak FL}) = 0.146 \times 0.207 = 0.030$, which does not equal $\mathbb{P}(\text{below PL and speak FL}) = 0.042$. Therefore, the events are dependent.
- Using Bayes' theorem: if the two events are independent, then $\mathbb{P}(\text{below PL} \mid \text{speak FL}) = \mathbb{P}(\text{below PL})$. Using Bayes' theorem,

$$\begin{aligned}\mathbb{P}(\text{below PL} \mid \text{speak FL}) &= \frac{\mathbb{P}(\text{below PL and speak FL})}{\mathbb{P}(\text{speak FL})}\\ &= \frac{0.042}{0.207}\\ &\approx 0.203\end{aligned}$$

Since this probability is different than $\mathbb{P}(\text{below PL}) = 0.146$, we determine that the two events are dependent.

3.32 (16pts total)

(a) **(2pts for each of number, J/Q/K, Ace, and Ace of clubs, and 2pts for correct expectation (10pts for part a))** The probability model for the amount Andy can profit at this game and the calculation of the expected profits is as follows:

Event	X	$\mathbb{P}(X)$	$X \cdot \mathbb{P}(X)$
Number	-2	$\frac{36}{52} = 0.6923$	$-2 \times \frac{36}{52} = -1.38$
J, Q, K	1	$\frac{12}{52} = 0.2308$	$1 \times \frac{12}{52} = 0.23$
Ace	3	$\frac{3}{52} = 0.0577$	$3 \times \frac{3}{52} = 0.17$
Ace of clubs	23	$\frac{1}{52} = 0.0192$	$23 \times \frac{1}{52} = 0.44$
			$\mathbb{E}(X) = -0.54$

(b) **(6pts for "no")** No, he is expected to lose money on average.

Problem 3 (12pts total)

In the following table, the row/column labels denote the possible outcomes of a dice role, and the grid values represent their sum.

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

To find the probability that the sum of two twice is equal to a number x, we find the number of times x appears in the table above and divide by the total possible number of outcomes (36).

(a) **(4pts)** $\mathbb{P}(\text{sum is 2}) = \mathbb{P}(\text{die 1 } = 1 \text{ and die 2 } = 1) = \frac{1}{36}$

(b) **(4pts)** $\mathbb{P}(\text{dice sum to 7}) = \frac{6}{36} = \frac{1}{6}$.

(c) **(4pts)** $\mathbb{P}(\text{dice sum to 13}) = \frac{0}{36} = 0$. Note that for this to be possible, one die would have to land on at least a 7, which is impossible for a pair of fair six-sided dice.

Problem 4 (22pts total)

(a) **(4pts)** $\mathbb{P}(\text{man or partner has brown eyes}) = \frac{55+54-23}{204} = 0.42$

(b) **(4pts)** $\mathbb{P}(\text{partner with brown eyes} \mid \text{man with blue eyes}) = \frac{23}{114} \approx 0.20$

(c) **(4pts for each probability)** $\mathbb{P}(\text{partner with brown eyes} \mid \text{man with brown eyes}) = \frac{23}{54} \approx 0.43$
$\mathbb{P}(\text{partner with brown eyes} \mid \text{man with green eyes}) = \frac{9}{36} = 0.25$

(d) **(6pts for "dependent")** It is much more likely for a man with brown eyes to have a partner with brown eyes than a man with another eye color to have a partner with brown eyes. Therefore it appears that eye colors of males and their partners are not independent.

Problem 5 (8pts total)

(6pts for computing correct probability for any order of draws, 2pts for recognizing that the 3 probabilities of the 3 possible orders should be added up.) Let us denote the colors of the balls by R, O, B, and G, and let us use the notation $\mathbb{P}(X \mid Y)$ to mean "the probability we picked a ball with color X given we have already picked a ball with color Y. Note that there are 3 distinct ways to get 1 red ball and 2 blue balls: (1) get aa red ball on the first draw (RBB), (2) get a red ball on the second draw (BRB), and (3) get a red ball on the third draw (BBR). Then we have

$$\begin{aligned}
\mathbb{P}(1R \text{ and } 2B) &= \mathbb{P}(RBB) + \mathbb{P}(BRB) + \mathbb{P}(BBR) \\
&= \mathbb{P}(R)\mathbb{P}(B \mid R)\mathbb{P}(B \mid R, B) + \mathbb{P}(B)\mathbb{P}(R \mid B)\mathbb{P}(B \mid R, B) + \mathbb{P}(B)\mathbb{P}(B \mid B)\mathbb{P}(R \mid B, B) \\
&= \frac{6}{20} \cdot \frac{7}{19} \cdot \frac{6}{18} + \frac{7}{20} \cdot \frac{6}{19} \cdot \frac{6}{18} + \frac{7}{20} \cdot \frac{7}{19} \cdot \frac{6}{18} \\
&\approx 0.037 + 0.037 + 0.037 \\
&= 0.11
\end{aligned}$$

Problem 6 (20pts total)

(a) **(4pts for expectation, 4pts for standard deviation)** The amount of ice cream served by 1 box and 4 scoops can be represented by $X + Y_1 + Y_2 + Y_3 + Y_4$.

$$\mathbb{E}[X + Y_1 + Y_2 + Y_3 + Y_4] = \mathbb{E}[X] + 4\mathbb{E}[Y] = (48) + 4(2) = 56.$$

$$\begin{aligned}
SD(X + Y_1 + Y_2 + Y_3 + Y_4) &= \sqrt{\text{Var}(X + Y_1 + Y_2 + Y_3 + Y_4)} = \sqrt{\text{Var}(X) + 4\text{Var}(Y)} \\
&= \sqrt{(2.25) + 4(0.09)} \approx 1.62
\end{aligned}$$

(b) **(4pts for expectation, 4pts for standard deviation)**

$$\mathbb{E}[X - Y_1 - Y_2] = \mathbb{E}[X] - \mathbb{E}[Y_1] - \mathbb{E}[Y_2] = 48 - 2 - 2 = 44.$$

$$SD(X - Y_1 - Y_2) = \sqrt{\text{Var}(X) + \text{Var}(Y_1) + \text{Var}(Y_2)} = \sqrt{(2.29) + 2(0.09)} = 1.57$$

(c) **(4pts)** Initially we do not know exactly how much ice cream is in the box. Then we scoop out an unknown amount. We should now be even more unsure about the amount of ice cream that is left in the box.