STA101 Problem Set 7 - KEY

Summer I, 2021, Duke University

Exercises from the OpenIntro book - 100pts total

Problems include: Chapter 7 exercises 7.7, 7.16, 7.27, 7.38, 7.46 and 1 additional problem

7.7 (22pts total)

(a) **(2pts (two-sided or one-sided alternate is ok))** $H_0 : \mu = 8$ (New Yorkers sleep 8 hrs per night on average.)
$H_A : \mu \neq 8$ (New Yorkers sleep less or more than 8 hrs per night on average.)

(b) **(2pts for assumptions, 2pts for t-statistic, 2 pts for df)** Before calculating the test statistic we should check that the conditions are satisfied.

1. Independence: The sample is random.
2. Normality: All observations are within three standard deviations of the mean. While this is encouraging, it would be useful to see the raw data. However, for now we will proceed while acknowledging that we are assuming there aren't any clear outliers.

The test statistic and degrees of freedom can be calculated as follows:

$$T = \frac{\overline{x} - \mu_0}{s/\sqrt{n}} = \frac{7.73 - 8}{0.77/\sqrt{25}} = -1.75$$
$$df = 25 - 1 = 24$$

(c) **(2pts for p-value, 4pts for interpretation)** p-value $= 2\mathbb{P}(T_{24} < -1.75) = 0.093$. If in fact the true population mean of the amount New Yorkers sleep per night was 8 hours, the probability of getting a random sample of 25 New Yorkers where the average amount of sleep is 7.73 hours per night or less (or 8.27 hours or more) is 0.093.

(d) **(4pts for "do not reject H_0")** Since p-value > 0.05, do not reject H_0. The data do not provide convincing evidence that New Yorkers sleep more or less than 8 hours per night on average.

(e) **(4pts for "no")** The 90% confidence critical t-value is $t^*_{24} = 1.711$. Using the sample mean and standard error from part b, we can construct the 90% CI: $7.73 \pm (1.711)(0.154) = (7.467, 7, 993)$. Thus, the answer is no because we wouldn't expect 8 hours to be in the interval.

7.16 (16pts total)

(a) **(4pts for "True")** True.

(b) **(4pts for "True")** True.

(c) **(4pts for "True")** True.

(d) **(2pts for "False", 2pts for explanation)** False. We find the difference of each pair of observations, and then we do inference on these differences.

7.27 (22pts total)

(a) **(4pts for reasonable descriptions)** Chicken that were fed linseed on average weigh 218.75 grams while those that were given horsebean weigh on average 160.20 grams. Both distributions are relatively symmetric with no apparent outliers. There is more variability in the weights of chicken that were given linseed.

(b) **(2pts for hypotheses, 2pts for assumptions, 2pts for t-statistic, 2pts for df, 2pts for p-value, 2pts for interpretation)** The hypotheses are $H_0 : \mu_{ls} = \mu_{hb}$ and $H_A : \mu_{ls} \neq \mu_{hb}$. Before calculating the test statistic we should check that the conditions for the sampling distribution of $(\overline{x}_{ls} - \overline{x}_{hb})$ to be nearly normal and the estimate of the standard error to be sufficiently accurate are as follows:

1. Independence: Chickens are randomly assigned to feed groups, and 12 and 10 $<$ 10% of all chickens fed linseed and horsebean, respectively. Therefore we can assume that the weights of chicken fed linseed are independent of each other, as well as the weights of chicken fed horsebean.
2. Normality: The distributions look fairly symmetric, therefore we can assume that the distribution of average differences will be nearly normal.

Since population standard deviations are unknown and samples are small, we calculate a T-score.

$$\begin{aligned}
T &= \frac{(\overline{x}_{ls} - \overline{x}_{hb}) - (\mu_{ls} - \mu_{hb})}{\sqrt{\frac{s_{ls}^2}{n_{ls}} + \frac{s_{hb}^2}{n_{hb}}}} \\
&= \frac{(218.75 - 160.20) - 0}{\sqrt{\frac{52.24^2}{12} + \frac{38.63^2}{10}}} \\
&= 3.02 \\
df &= \min(n_1 - 1, n_2 - 1) \\
&= \min(11, 9) \\
&= 9 \\
\text{p-value} &= \mathbb{P}(|T_9| > 3.02) \\
\implies \text{p-value} &> 0.01 \quad \& \quad < 0.02
\end{aligned}$$

Since p-value < 0.05, we reject H_0. The data provide strong evidence that there is a significant difference between the average weights of chicken that were fed linseed and horsebean.

(c) **(3pts for "Type 1")** Type 1, since we rejected H_0.

(d) **(3pts for "yes")** Yes, since p-value > 0.01, we would fail to reject H_0 and conclude that the data do not provide convincing evidence of a difference between the average weights of chickens that were fed linseed and horsebean.

7.38 (12pts total)

(a) **(4pts, alternate cannot be "all of the means are different")** They hypotheses are:

$$\begin{aligned}
&H_0 : \mu_{\text{lec +disc}} = \mu_{\text{textbook}} = \mu_{\text{text+lec}} = \mu_{\text{comp}} = \mu_{\text{comp+lec}} \\
&H_A : \text{The average score varies across some (or all) groups.}
\end{aligned}$$

(b) **(2pts for df_G, 2pts for df_E)**

$$df_G = 5 - 1 = 4 \quad df_E = 45 - 5 = 40$$

(c) **(4pts for favoring the alternate hypothesis)** Since the p-value is small (assuming $\alpha = 0.05$), the data provide convincing evidence that the average score varies across some (or all) groups.

7.46 (16pts total)

(a) **(2pts for "False", 2pts for reasoning)** False, we conclude that at least one pair of means are different.

(b) **(4pts for "True")** True.

(c) **(2pts for "False", 2pts for reasoning)** False, it is possible to reject the null hypothesis using ANOVA and then to not subsequently identify differences in the pairwise comparisons.

(d) **(2pts for "False", 2pts for reasoning)** False, the Bonferroni correction requires dividing the original by the number of pairwise tests to be conducted, not the number of groups. With 4 groups to start with, $k = 4$, the number of pairwise tests will be $K = \frac{k(k-1)}{2} = \frac{4 \cdot 3}{2} = 6$, and hence the new significance level will be $\alpha^* = \frac{\alpha}{K} = \frac{0.05}{6} = 0.0083$.

Problem 6 (12pts total)

(4pts for identifying the z-score at 80% power, 4pts for setting up the equation to solve for SE, 2pts for computation, 2pts for correct number of plots) Difference we care about: 50. Single tail of 80%: $0.84 \cdot SE$. At the 5% significance level, the rejection region bounds span $\pm 1.96 \cdot SE$

$$\begin{aligned} 50 &= 0.84 \cdot SE + 1.96 \cdot SE = 2.8 \cdot SE \\ SE &= 17.86 = \sqrt{\frac{100^2}{n} + \frac{100^2}{n}} \\ n &= 62.7 \end{aligned}$$

We will need 63 plots of land for each fertilizer.