Fan Bu
Duke University - Department of Statistical Science

Application Exercise 4: Hourly rates of manufacturing workers - KEY

Your name: _____

Write your responses in the spaces provided below. WRITE LEGIBLY and SHOW ALL WORK! Concise and coherent are best!

In this activity we'll work with data on average hourly wage for manufacturing workers, in the United States as well as in North Carolina. The data come from the The 2012 Statistical Abstract. Assume that the distributions of the manufacturing wage rates, nationwide and in North Carolina, can be approximated by a normal distribution.

Source: U.S. Bureau of Labor Statistics, Current Employment Statistics, "State and Metro Area Employment, Hours, and Earnings (SAE), March, 2010, http://www.bls.gov/sae/#data.htm.

Part 1: Government data indicates that the average hourly wage for manufacturing workers in the United States is \$18.61, with a standard deviation of \$1.35.

1. What percent of manufacturing workers make more than \$20/hour?

   Given: $X_{US} \sim N(\mu = 18.61, \sigma = 1.35)$

   $$P(X > 20) = P\left(Z > \frac{20 - 18.61}{1.35}\right) = P(Z > 1.02) = 0.154 \rightarrow 15.4\%$$

2. What percent of manufacturing workers make between \$18 - \$20/hour?

   $$P(X_{US} > 18) = P\left(Z > \frac{18 - 18.61}{1.35}\right) = P(Z > -0.45) = 0.674$$
   $$P(18 < X_{US} < 20) = 0.674 - 0.154 = 0.52 \rightarrow 52\%$$

Part 2: Government data also indicates that the average hourly wage for manufacturing workers in North Carolina is \$15.85.

3. An unemployed worker did a job search in North Carolina, and found that 15% of the manufacturing jobs paid more than \$17 per hour. What is the standard deviation of the distribution of hourly wage for manufacturing workers in North Carolina?

   Given: $\mu = 15.85$, $P(X_{NC} > 17) = 0.15$
   This corresponds to a Z score of 1.04 since $P(Z > 1.04) = 0.15$

$$1.04 = \frac{17 - 15.85}{\sigma} \rightarrow \sigma = \frac{17 - 15.85}{1.04} = 1.11$$

4. Suppose that a worker applies for a manufacturing job in North Carolina, and receives the good news that she got the job and that her pay will be at least \$16.50 per hour. She would really like to be able to make at least \$17 per hour. What is the probability that she will get what she wants? Assume that the company she will be working for is a run-of-the-mill manufacturing company in NC, i.e. the distribution of the hourly wages at this company reflects the state distribution. *Hint:* This is a conditional probability.

   Given: $X_{NC} \sim N(\mu = 15.85, \sigma = 1.11)$

$$P(X_{NC} > 17 \mid X_{NC} > 16.50) = \frac{P(X_{NC} > 17 \text{ and } X_{NC} > 16.50)}{P(X_{NC} > 16.50)} = \frac{P(X_{NC} > 17)}{P(X_{NC} > 16.50)}$$
$$= \frac{0.15}{P\left(Z > \frac{16.50 - 15.85}{1.11}\right)} = \frac{0.15}{P(Z > 0.59)} = \frac{0.15}{0.28} = 0.53$$

Part 3: Government data also indicates that the average hourly wage for manufacturing workers in New York is \$18.39, with a standard deviation of \$1.5.

5. Who is doing better within their state: a NC manufacturing worker who makes \$17/hr or a NY manufacturing worker who makes \$19/hr?

   - *Given:* $X_{NY} \sim N(\mu = 18.39, \sigma = 1.5) \rightarrow X_{NY} = \$19 \rightarrow Z = \frac{19-18.39}{1.5} = 0.41$
   - *Given:* $X_{NC} \sim N(\mu = 15.85, \sigma = 1.11) \rightarrow X_{NC} = \$17 \rightarrow Z = 1.04$

   NC manufacturing worker doing better since has a higher Z score.

6. If 34% of NY manufacturing workers make more than \$19/hr, what is the probability that in a random sample of 100 NY manufacturing workers less than 30% make more than \$19/hr.

   $p = 0.34$, $n = 100$
   S/F: checks
   $\mu = 0.34 \times 100 = 34$ *and* $\sigma = \sqrt{100 \times 0.34 \times 0.66} = 4.74$
   $P(K < 30) = P\left(Z < \frac{30-34}{4.74}\right) = P(Z < -0.84) = 0.2$