1. Notation

$$b := \text{number of components in the basket} \tag{1}$$
$$\boldsymbol{X} := (X_0, \ldots, X_{b-1}) \tag{2}$$
$$\boldsymbol{e} := (e_0, \ldots, e_{b-1}) \tag{3}$$
$$N := \text{a positive integer} \tag{4}$$
$$K = \{(k_0, \ldots, k_{b-1}) : k_j \in \{0, \ldots, N-1\}\} \tag{5}$$
$$\boldsymbol{k} \in K \tag{6}$$
$$\boldsymbol{I}(\alpha) := (\alpha, \ldots, \alpha) \qquad (b \text{ times}) \tag{7}$$
$$\boldsymbol{u}(\boldsymbol{k}) := \Delta(\boldsymbol{k} - \boldsymbol{I}(N/2)) + i\boldsymbol{e} \tag{8}$$
$$\Phi := \text{characteristic function of spread/basket payoff} \tag{9}$$
$$\widehat{P} := \text{Fourier transform of spread/basket payoff} \tag{10}$$
$$F(\boldsymbol{k}) = \Phi(\boldsymbol{u}(\boldsymbol{k}))\widehat{P}(\boldsymbol{u}(\boldsymbol{k})) \tag{11}$$
$$\Delta := \text{some real number, maybe } 1/4 \tag{12}$$
$$\lambda = \frac{2\pi}{N\Delta} \tag{13}$$
$$\boldsymbol{X}(\boldsymbol{k}) = \boldsymbol{X} + \lambda\boldsymbol{k} \tag{14}$$
$$\boldsymbol{f} = -\boldsymbol{e}\Delta - \boldsymbol{I}\left(i\frac{\Delta N}{2}\right) \tag{15}$$
$$\boldsymbol{\ell} \in K \tag{16}$$
$$\boldsymbol{v}(\boldsymbol{\ell}) = \boldsymbol{X} + \frac{2\pi}{N\Delta}\boldsymbol{\ell} \tag{17}$$
$$V(\boldsymbol{\ell}) = \boldsymbol{f} \cdot \boldsymbol{v}(\boldsymbol{\ell}) \tag{18}$$

2. Idea

Consider the following: whilst the function Φ is identical for both spread and basket options, we have

$$\widehat{P}(\xi) = \frac{\prod_{j=1}^{b} \Gamma(-i\xi_j)}{\Gamma(-i(\sum_{j=1}^{b} \xi_j) + 2)}, \qquad \text{basket} \tag{19}$$
$$\widehat{P}(\xi) = \frac{\Gamma(i(\xi_1 + \xi_2) - 1)\Gamma(-i\xi_2)}{\Gamma(i\xi_1 + 1)}, \qquad \text{spread} \tag{20}$$
$$D := \frac{e^{-rT}\Delta^b}{(2\pi)^b} \sum_{\boldsymbol{k}\in K} e^{i(\Delta(\boldsymbol{k}+i\boldsymbol{e})\cdot\boldsymbol{X} - \Delta N\boldsymbol{X}/2)} F(\boldsymbol{k}) \tag{21}$$
$$= \frac{e^{-rT}\Delta^b}{(2\pi)^b} e^{V(\boldsymbol{\ell})} \sum_{\boldsymbol{k}\in K} e^{i\frac{2\pi}{N}(\boldsymbol{k}\cdot\boldsymbol{\ell})} e^{i\Delta(\boldsymbol{k}\cdot\boldsymbol{X})} F(\boldsymbol{k}) \tag{22}$$
$$= \frac{e^{-rT}\Delta^b}{(2\pi)^b} e^{V(\boldsymbol{\ell})} \sum_{\boldsymbol{k}\in K} e^{i\frac{2\pi}{N}(\boldsymbol{k}\cdot\boldsymbol{\ell})} A_{\boldsymbol{k}} \tag{23}$$

with

$$A_{\boldsymbol{k}} := e^{i\Delta(\boldsymbol{k}\cdot\boldsymbol{X})} F(\boldsymbol{k}) \tag{24}$$

$$A_{\boldsymbol{k}} \xrightarrow{FFT} B_{\boldsymbol{\ell}} \tag{25}$$

and the call option price for the share values $\boldsymbol{X(\ell)}$ is

$$C_{\boldsymbol{\ell}} = \frac{e^{-rT}\Delta^b}{(2\pi)^b} e^{V(\boldsymbol{\ell})} B_{\boldsymbol{\ell}} \tag{26}$$

3. Implementation

- The set K is created by TupleFactory::generate_tuples_mod_N().
- We have $\#K = N^b$.
- All b-vectors are created using the LinAlgSys class.
- Currently the entire implementation is in Basket::basketpricingFFT()