

We wish to implement the formula

$$(1) \quad D \equiv \frac{e^{-rT} \Delta^2}{(2\pi)^2} \sum_{k_1, k_2=1}^N e^{i(\Delta(k_1 - N/2 - 1) + i\epsilon_1, \Delta(k_1 - N/2 - 1) + i\epsilon_2) \cdot (X_1, X_2)} F(k_1, k_2),$$

where

$$F(k_1, k_2) = \Phi(\Delta(k_1 - N/2 - 1) + i\epsilon_1, \Delta(k_1 - N/2 - 1) + i\epsilon_2) \\ \times \widehat{P}(\Delta(k_1 - N/2 - 1) + i\epsilon_1, \Delta(k_1 - N/2 - 1) + i\epsilon_2),$$

Φ is the characteristic function and \widehat{P} is the Fourier transform of the payoff (and this can be the payoff for spread options or basket options). Now by setting

$$X_1 = X_{01} + \lambda(\ell - 1), \quad X_2 = X_{02} + \lambda(m - 1), \quad \ell, m = 1, \dots, N$$

and

$$\lambda = \frac{2\pi}{N\Delta},$$

we rewrite (1) as

$$(2) \quad \frac{e^{-rT} \Delta^2}{(2\pi)^2} e^{V_1(\ell) + V_2(m)} \sum_{k_1, k_2=1}^N e^{i \frac{2\pi}{N} [(k_1 - 1)(\ell - 1) + (k_2 - 1)(m - 1)]} e^{i\Delta(k_1 X_{01} + k_2 X_{02})} F(k_1, k_2),$$

with

$$V_1(\ell) = (-\epsilon_1 - i\Delta(N/2 + 1)) \left(X_{01} + \frac{2\pi}{N\Delta}(\ell - 1) \right) + i \frac{2\pi}{N}(\ell - 1),$$

and

$$V_2(m) = (-\epsilon_2 - i\Delta(N/2 + 1)) \left(X_{02} + \frac{2\pi}{N\Delta}(m - 1) \right) + i \frac{2\pi}{N}(m - 1).$$

Note that inside the summation we have the two-dimensional discrete Fourier transform of the matrix

$$A_{(k_1 k_2)} \equiv e^{i\Delta(k_1 X_{01} + k_2 X_{02})} F(k_1, k_2);$$

its Fast Fourier transform will produce a matrix

$$B_{\ell m} = \{b_{\ell m}\};$$

now the matrix

$$C = \{c_{\ell m}\} = \left\{ \frac{e^{-rT} \Delta^2}{(2\pi)^2} e^{V_1(\ell) + V_2(m)} b_{\ell m} \right\}$$

will give the call option price for each choice of share values

$$(X_\ell, X_m) = (X_{01} + \lambda(\ell - 1), X_{02} + \lambda(m - 1)).$$

Let me know if something is not clear.