School on Column Generation: *Applications*

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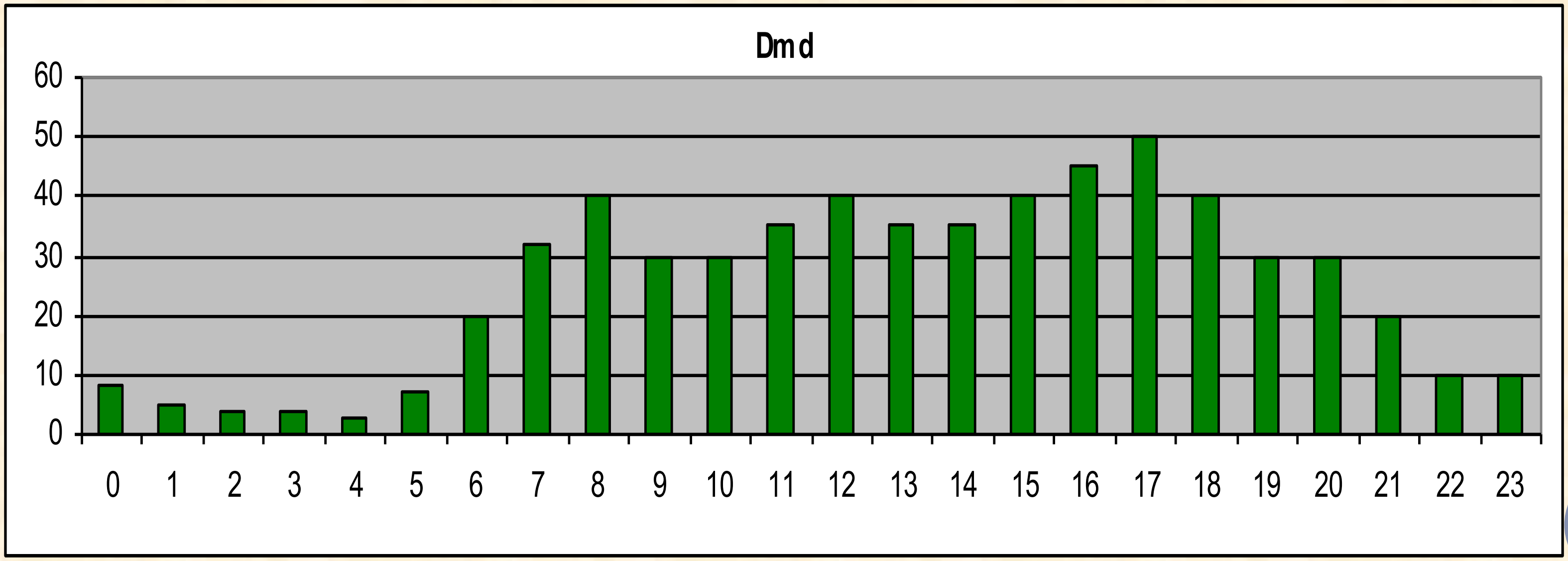
What is the relationship between *La Commission scolaire des Chênes de Drummondville, UPS, Tokyo Metro Co., NAV Canada, Mount Sinai Hospital (Toronto), La Société des Casinos du Québec, Bombardier Flexjet, Autoroutes de France, and Wal-Mart?*

A Simple Scheduling Problem

- Goal : Find work schedules for a set of people.
- Each person works 6 consecutive hours and can start at the beginning of any hour.
- Data : demand curve per hour.



Simple Scheduling Patterns

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	offer		dmd
0	1																			1	1	1	1	1	0	≥	8
1	1	1																			1	1	1	1	0	≥	5
2	1	1	1																			1	1	1	0	≥	4
3	1	1	1	1																			1	1	0	≥	4
4	1	1	1	1	1																			1	0	≥	3
5	1	1	1	1	1	1																			0	≥	7
6		1	1	1	1	1	1																		0	≥	20
7			1	1	1	1	1	1																	0	≥	32
8				1	1	1	1	1	1																0	≥	40
9					1	1	1	1	1	1															0	≥	30
10						1	1	1	1	1	1														0	≥	30
11							1	1	1	1	1	1													0	≥	35
12								1	1	1	1	1	1												0	≥	40
13									1	1	1	1	1	1											0	≥	35
14										1	1	1	1	1	1										0	≥	35
15											1	1	1	1	1	1									0	≥	40
16												1	1	1	1	1	1								0	≥	45
17													1	1	1	1	1	1							0	≥	50
18														1	1	1	1	1	1						0	≥	40
19															1	1	1	1	1	1					0	≥	30
20																1	1	1	1	1	1				0	≥	30
21																	1	1	1	1	1	1			0	≥	20
22																		1	1	1	1	1	1		0	≥	10
23																			1	1	1	1	1	1	0	≥	10

VARIOUS SCHEDULING PROBLEMS

- Minimum number of persons
- Minimum cost

- Split duties : 6 on 7 hrs
  - 3hrs, 1hr lunch, 3hrs
  - Fractional solution
  - Large branch & bound tree (very difficult !)
  - Rounding cut

Split Patterns

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	offer		dmd
0	1																		1	1	1		1	1	8	≥	8
1	1	1																		1	1	1		1	5	≥	5
2	1	1	1																		1	1	1		6	≥	4
3		1	1	1																		1	1	1	17	≥	4
4	1		1	1	1																		1	1	20	≥	3
5	1	1		1	1	1																		1	17	≥	7
6	1	1	1		1	1	1																		23	≥	20
7		1	1	1		1	1	1																	32	≥	32
8			1	1	1		1	1	1																40	≥	40
9				1	1	1		1	1	1															30	≥	30
10					1	1	1		1	1	1														30	≥	30
11						1	1	1		1	1	1													35	≥	35
12							1	1	1		1	1	1												42	≥	40
13								1	1	1		1	1	1											44	≥	35
14									1	1	1		1	1	1										35	≥	35
15										1	1	1		1	1	1									40	≥	40
16											1	1	1		1	1	1								45	≥	45
17												1	1	1		1	1	1							50	≥	50
18													1	1	1		1	1	1						40	≥	40
19														1	1	1		1	1	1					30	≥	30
20															1	1	1		1	1	1				31	≥	30
21																1	1	1		1	1	1			24	≥	20
22																	1	1	1		1	1	1		18	≥	10
23																		1	1	1		1	1	1	10	≥	10

VARIOUS SCHEDULING PROBLEMS

- Split duties : 7 on 8 hrs
  - 3hrs, 1hr lunch, 4hrs or 4hrs, 1hr lunch, 3hrs
  - 48 variables

- Additional am & pm breaks (15 minutes)
  - More constraints

Various Split Patterns

48 Variables

	0a	0b	1a	1b	…	offer		dmd
0	1	1					≥	**8**
1	1	1	1	1			≥	**5**
2	1	1	1	1			≥	**4**
3		1	1	1			≥	**4**
4	1			1			≥	**3**
5	1	1	1				≥	**7**
6	1	1	1	1			≥	**20**
7	1	1	1	1			≥	**32**
8			1	1			≥	**40**
9							≥	**30**
10							≥	**30**
11							≥	**35**
12							≥	**40**
13							≥	**35**
14							≥	**35**
15							≥	**40**
16							≥	**45**
17							≥	**50**
18							≥	**40**
19							≥	**30**
20							≥	**30**
21							≥	**20**
22							≥	**10**
23							≥	**10**

AIRLINE APPLICATIONS

- Replace **time periods to be covered,**
  each by a certain number of people by

  flights to be covered,
  each by a single aircraft

  or
  each by a certain number of pilots
  each by a certain number of flight attendants

  …

AIRLINE APPLICATIONS: AIRCRAFT PATTERNS

Possible aircraft routes

Flights			…			
1	1		…		=	1
2		1			=	1
3	1				=	1
…					=	1
		1			=	1
					=	1
	1	1			=	1
					=	1
		1			=	1
	1	1			=	1
					=	1
	1				=	1
	1				=	1
		1			=	1
		1			=	1
					=	1
	1				=	1
					=	1
					=	1
					=	1
					=	1
					=	1
865					=	1
866			…		=	1

AIRLINE APPLICATIONS: PILOT PATTERNS

Possible pilot schedules

Flights						
1	1		…		≥	1
2		1			≥	1
3	1				≥	1
…					≥	1
		1			≥	1
					≥	1
	1	1			≥	1
					≥	1
		1			≥	1
	1	1			≥	1
					≥	1
	1				≥	1
	1				≥	1
		1			≥	1
		1			≥	1
					≥	1
	1				≥	1
					≥	1
					≥	1
					≥	1
					≥	1
					≥	1
865					≥	1
866					≥	1

AIRLINE APPLICATIONS: FLIGHT ATTENDANT PATTERNS

Possible flight attendant schedules

Flights					
1	1		…	≥	12
2		1		≥	4
3	1			≥	10
…				≥	…
		1		≥	
				≥	
	1	1		≥	
				≥	
		1		≥	
	1	1		≥	
				≥	
	1			≥	
	1			≥	
		1		≥	
		1		≥	
				≥	
	1			≥	
				≥	
				≥	
				≥	
				≥	
				≥	
865				≥	10
866				≥	8

RAIL APPLICATIONS

- Replace **time periods to be covered, each by a certain number of people** by

  trains to be covered,

  each by a certain number of locomotives

Rail Applications: Locomotive patterns

Possible locomotive routes

Trains						
1	1		…		≥	**6**
2		1			≥	**4**
3	1				≥	**5**
…					≥	**…**
		1			≥	
					≥	
	1	1			≥	
					≥	
		1			≥	
	1	1			≥	
					≥	
	1				≥	
	1				≥	
		1			≥	
		1			≥	
					≥	
	1				≥	
					≥	
					≥	
					≥	
					≥	
					≥	
865					≥	**5**
866					≥	**4**

GENERAL STRUCTURE

- Replace **time periods to be covered, each by a certain number of people** by

  tasks to be performed,
  each by a certain number of vehicles or crews

GENERAL STRUCTURE

- Each column provides a feasible pattern, represented by a set of 0/1 values, that is, uncovered and covered tasks.

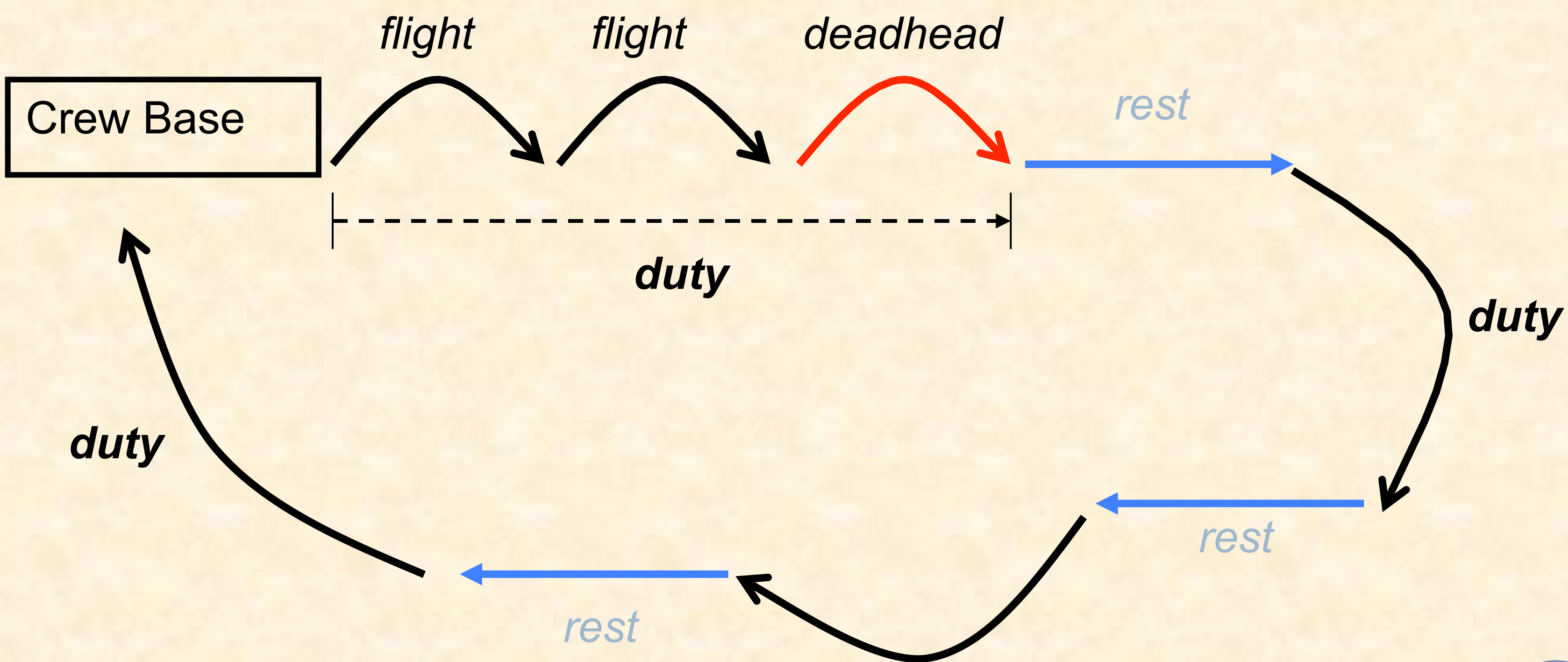
- **Patterns** are given by **paths on time space networks**.

PROBLEM STRUCTURE

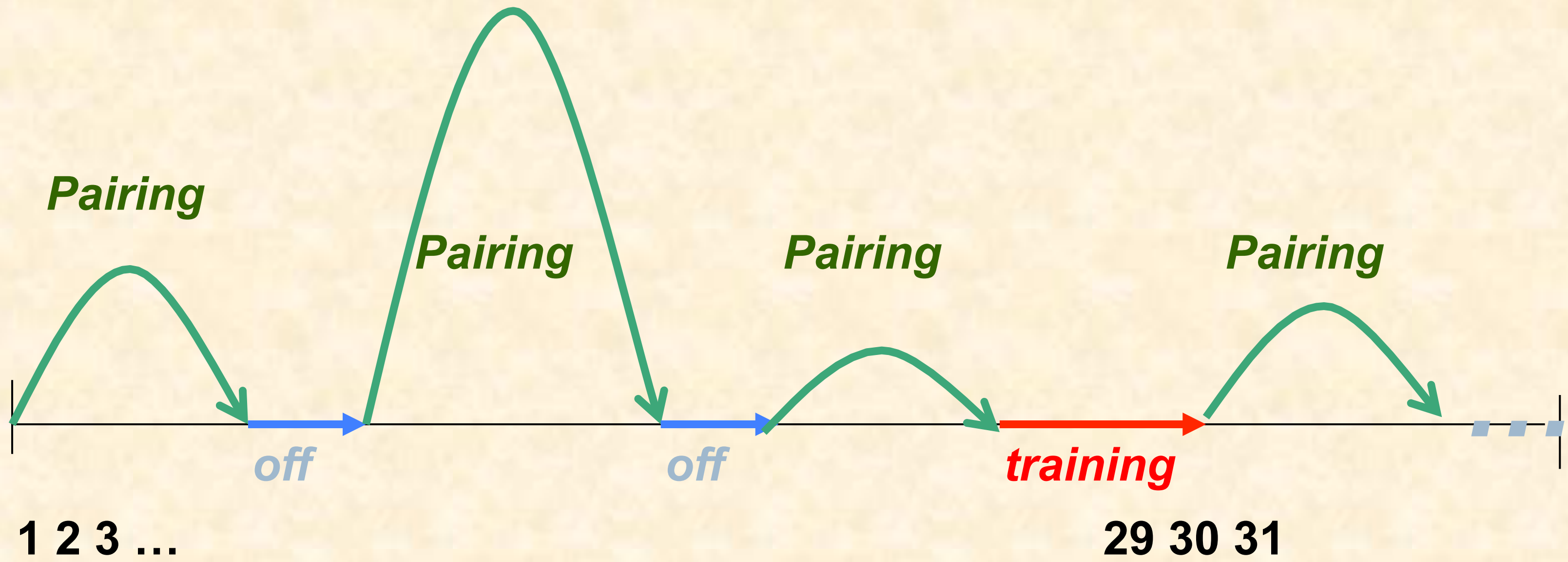
- Time-Space Networks
- Local (*Path)* Restrictions
- Covering of a Set of Tasks
- Schedule Composition

- **Non Linear Cost Functions**

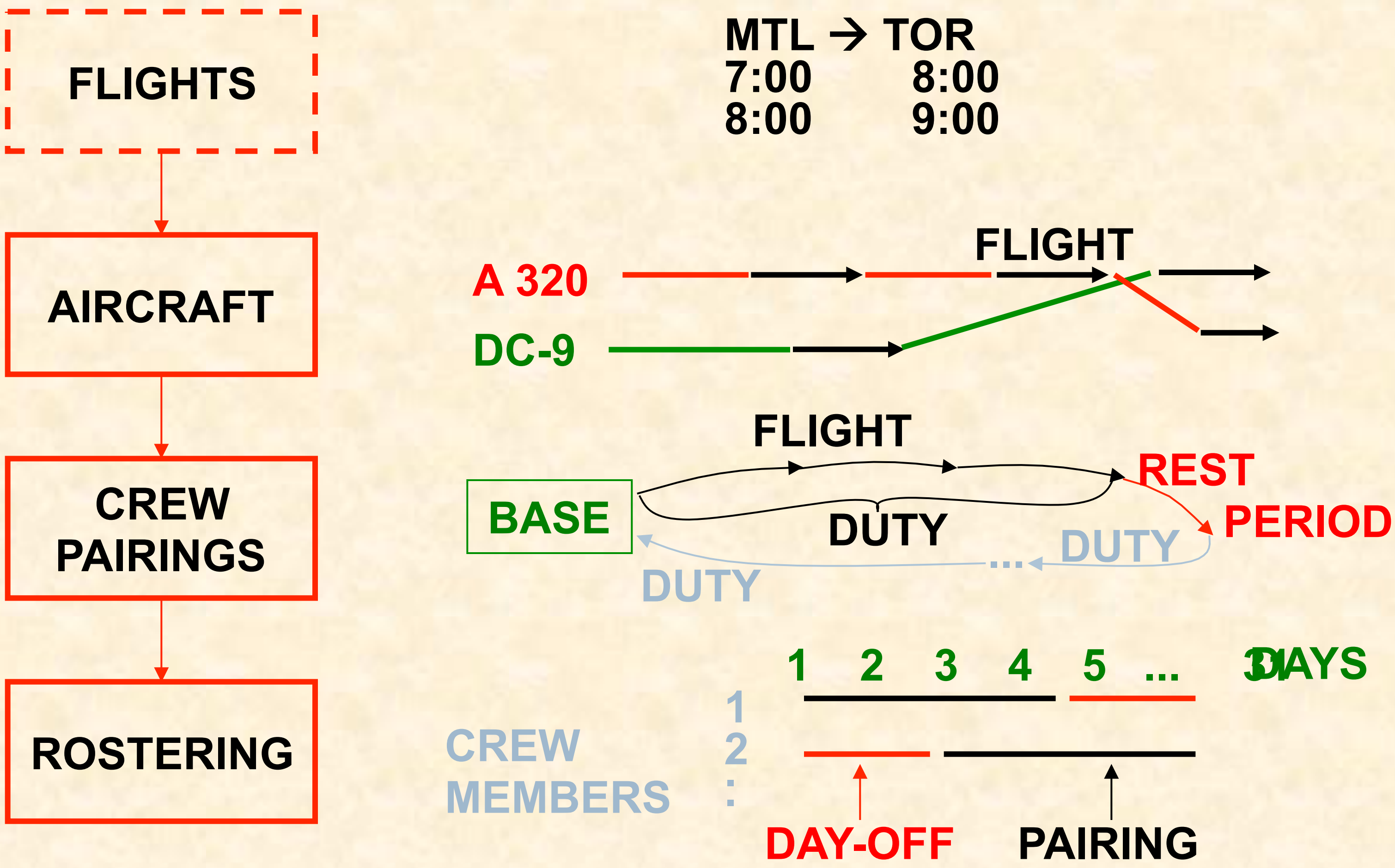
Airline Crew Scheduling: Pilot Pairings



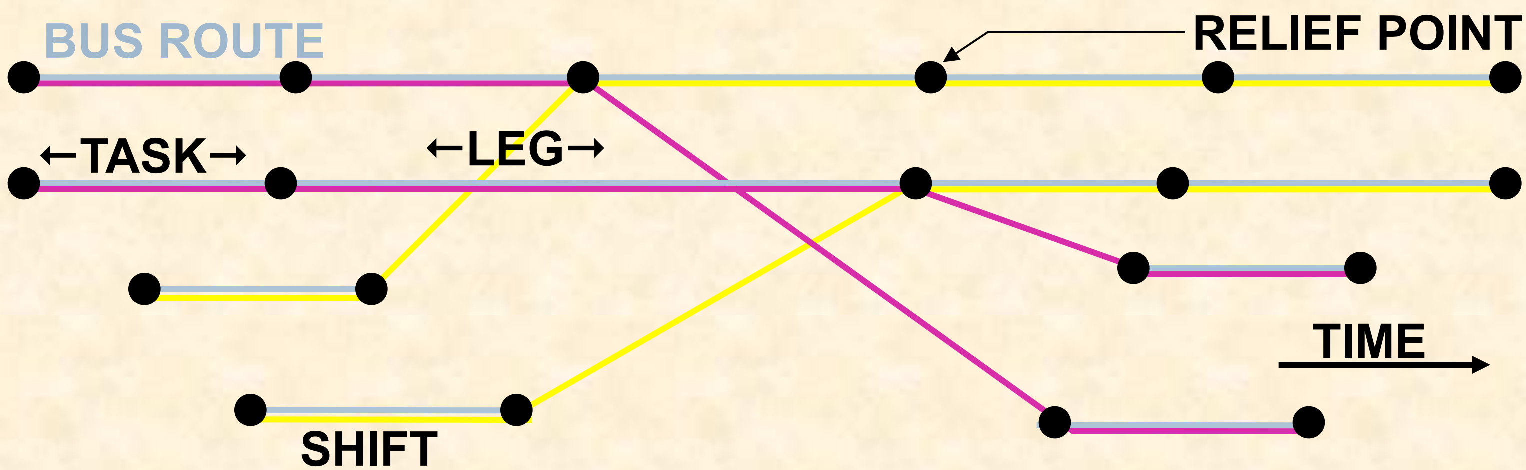
Monthly Schedules: Monthly Rostering



AIRLINE OPTIMIZATION DIVIDED IN 3 STEPS



Bus Driver Scheduling



WORK SHIFT CONSTRAINTS

MAX 8 HOURS

MIN 6 HOURS

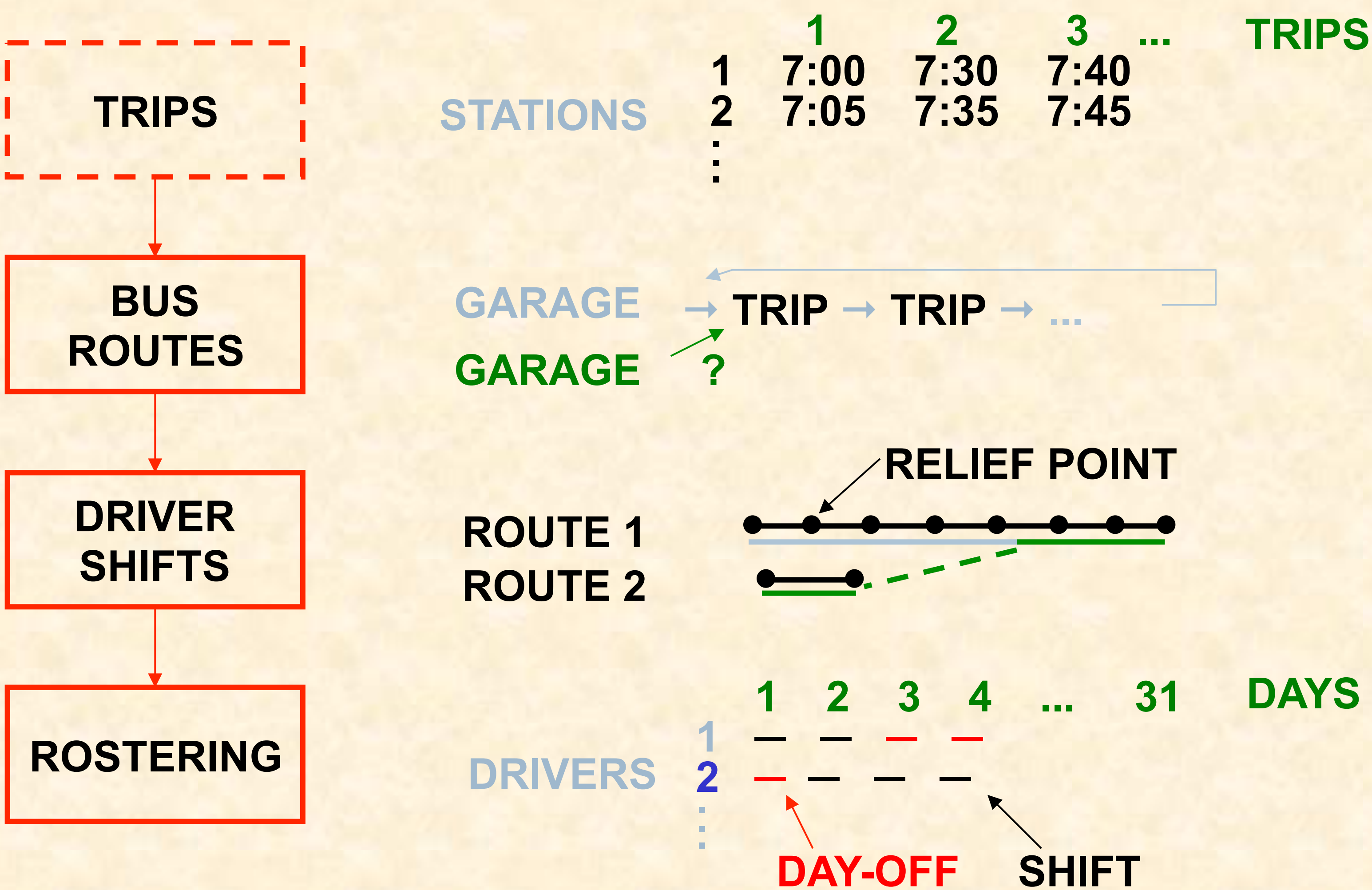
1 HOUR LUNCH TIME

…

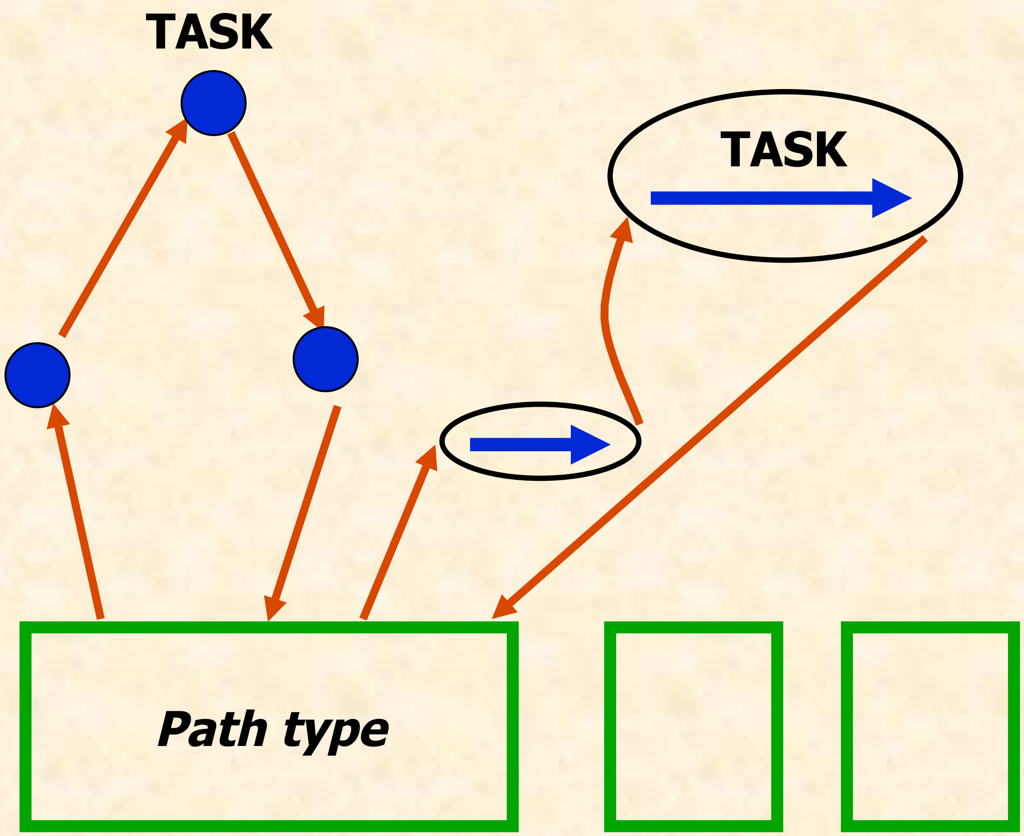
GLOBAL CONSTRAINTS

80% OF SHIFTS ≥ 7 HOURS

BUS DRIVER OPTIMIZATION DIVIDED IN 3 STEPS



A GENERIC PROBLEM



- ***COVER AT MINIMUM COST***
- ***A SET OF TASKS***
- ***WITH FEASIBLE PATHS***

Various Examples

	TASKS	PATHS
URBAN BUS		
BUS ROUTING	BUS TRIPS	ROUTES
DRIVER SCHEDULING	TRIP SEGMENTS	SHIFTS
ROSTERING	SHIFTS	ROSTERS
AIRLINE		
AIRCRAFT ROUTING	FLIGHTS	ROUTES
CREW PAIRING	FLIGHTS	PAIRINGS
MONTHLY BLOCKS	PAIRINGS	BLOCKS
RAIL		
ROUTING of LOCOMOTIVES	TRAINS	ROUTES
PRODUCTION		
JOB-SHOP	OPERATIONS	SEQUENCES ON A MACHINE

Some Elements of Crew Pairing Problems

- Task — *Flights*
- Types — ***# Bases x 7 (implicit pairing duration)***
- Paths — *Pairings*
- Networks
  - Arcs — Duties; Night rests; Connections
  - Resources — Min rest time; Max worked time;…
- Global Constraints — *Max credit/base;*
  Max/Min of pairing types
- **Cost Function**

$$Min \sum_{Pairings} \max\left\{\frac{duration}{3.5}, \sum_{Duties} \max\{4, credits\}\right\}$$

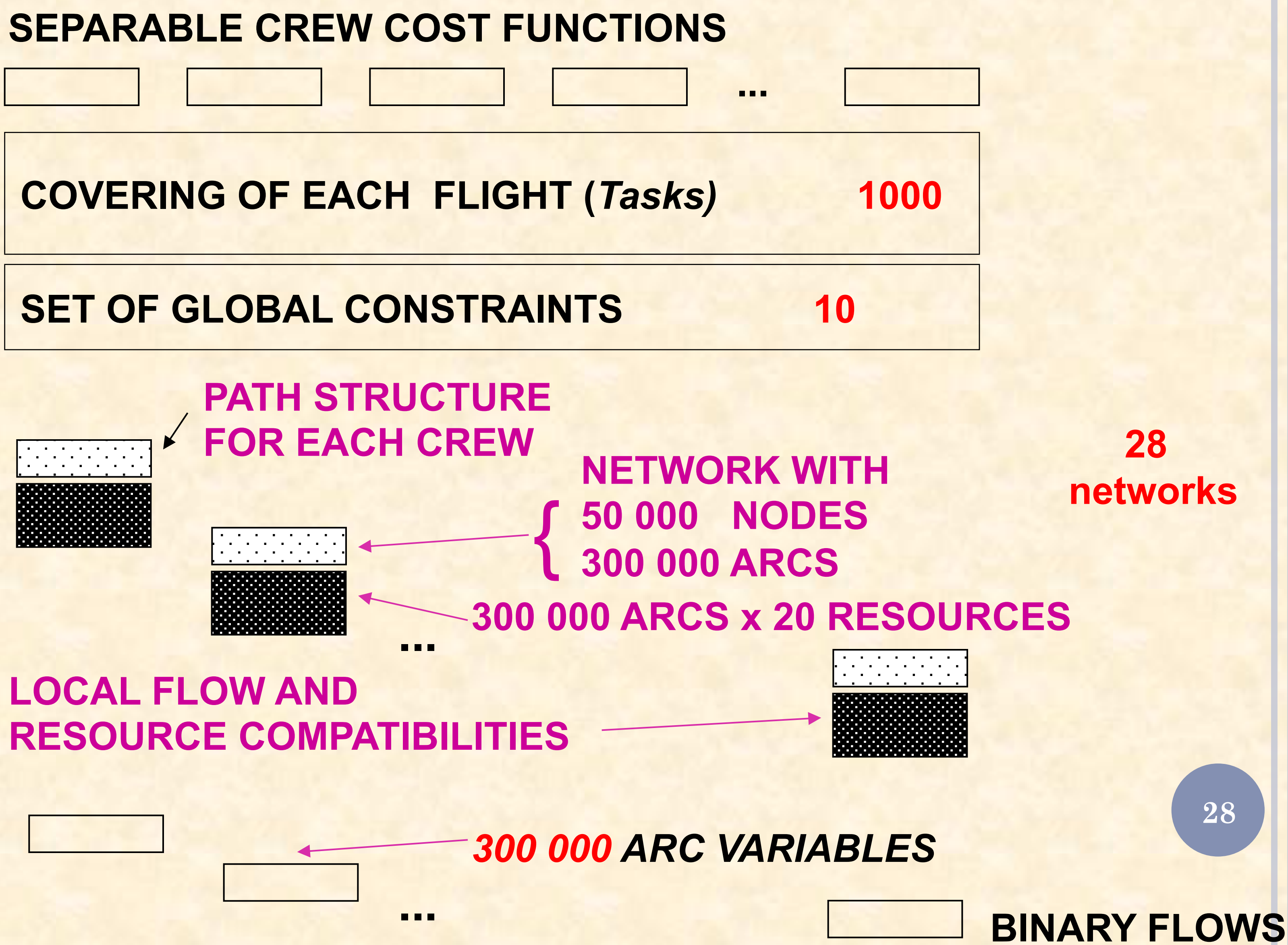
Some Elements of Monthly Rostering Problems

- Task — *Pairing*
- Types — ***# Crew members***
- Paths — *Blocks*
- Networks
  - Arcs — Pairings; Rests; Trainings; …
  - Resources — Max credits; no. days off; Days off patterns;…
- Global Constraints — *Max/Min of block types*
- **Cost Function** — ***Min Uncovered Tasks; Balance workload***

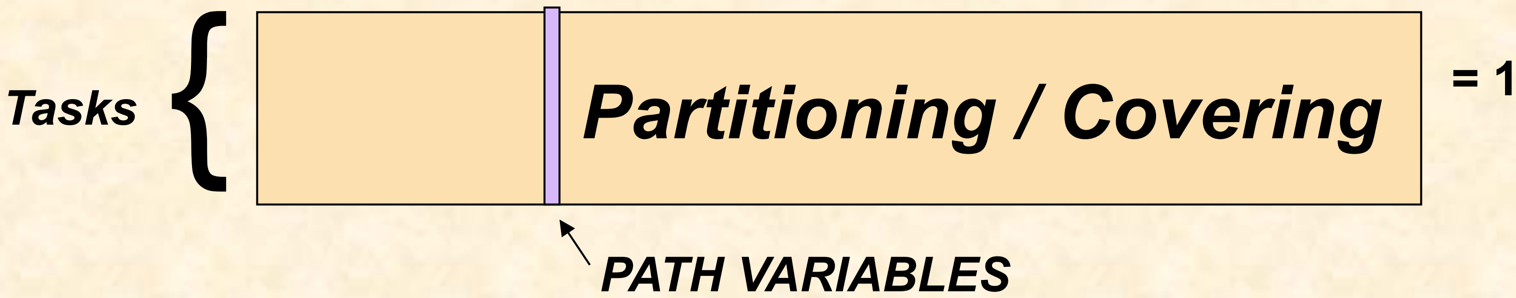
Some Elements of Preferential Bidding Problems

▪ Task	*Pairing*
▪ Types	***# Crew members***
▪ Paths	*Blocks*
▪ Networks	
• Arcs	Pairings; Rests; Trainings; …
• Resource	Max credits; no. days off; Days off patterns8/24 rule;…
▪ Global Constraints	*Max/Min of block types*
▪ **Cost Function**	***Max Satisfaction Scores in Seniority Order***

PROBLEM STRUCTURE
(CREW PAIRING: 1000 FLIGHTS)



DANTZIG-WOLFE REFORMULATION



- ADVANTAGES
  - Simpler and fewer constraints
  - Complex costs can be easily pre-computed

- DIFFICULTIES
  - Astronomical number of path variables !!!

COLUMN GENERATION OF VARIABLES … AS NEEDED

Master Problem

- Tasks Covering
- Global Constraints

Sub-Problems

- Path Structure
- Local Feasibility by Using Resource Variables

SUBPROBLEM: CONSTRAINED SHORTEST PATH

MIN REDUCED COST

$$\min_{\text{PAIRINGS}} \sum \text{MAX}\left(\frac{\text{Pairing Duration}}{3.5}, \sum_{\text{DAYS}} \text{MAX (4, Credits)}\right) - \text{Dual Costs}$$

S.T. - PATH STRUCTURE
- DAY DURATION ≤ 12 HOURS
- WORK TIME / DAY ≤ 8 HOURS
- WORK TIME / PAIRING ≤ MAX
- NIGHT REST ≥ MIN
- ...

10 TO 20 RESOURCES

Linear Master Problem

COSTS

TASKS	39	43	42	40	38	38	…	43	44	34	41	34	33	38	58	41			
1	1						…	1			1				1		…	=	**1**
2		1	1			1	…		1	1		1		1		1	…	=	**1**
3	1	1		1			…	1			1		1		1		…	=	**1**
4		1	1		1	1	…		1	1			1	1	1		…	=	**1**
5			1		1		…	1	1			1				1	…	=	**1**
6			1	1		1	…				1			1	1	1	…	=	**1**
7				1			…	1		1		1					…	=	**1**
8	1	1			1		…		1				1		1	1	…	=	**1**
9					1	1	…	1			1	1		1	1		…	=	**1**
10	1	1		1			…		1	1					1		…	=	**1**
	13	17	15	11	13	14	…										…		**25**
							…	12	18	13	12	9	10	14	24	13	…	≤	**40**
	1	1	1	1	1	1	…										…	≤	**2**
							…	1	1	1	1	1	1	1	1	1	…	≤	**4**

SUCCESSFUL APPLICATIONS

- **Vehicle Routing with Time Windows**
- **Dial-a-Ride for Physically Disabled Persons**
- **Urban Transit Crew Scheduling**
- **Multiple Depot Vehicle Scheduling**
- **Aircraft Routing**
- **Crew Pairing**
- **Crew Rostering (Pilots & Flight Attendants)**
- **Locomotive and Car Assignment**

CREW PAIRING AIR CANADA

FLIGHT ATTENDANTS DC-9 + A 320

5 BASES	*FLIGHTS*	*% FAT*
DAILY	**430**	**0.47**
WEEKLY	**2425**	**1.39**
MONTHLY	**11914**	**2.03**

Air Canada solution vs. Savings : 7.8 % → 2.03 %

TRANSAT, CAN. REGIONAL, NORTHWEST, U.P.S., DELTA, SWISSAIR, FEDEX

Crew Rostering Air France

Flight Attendants

	ORLY	CDG
PAIRINGS	454*7	3000*5
PERSONS	240	840
SAVINGS	7.4%	7.6%

A. C., TRANSAT, CAN. REGIONAL, TWA, DELTA, SWISSAIR, AMERICA WEST

FLEET ASSIGNMENT & AIRCRAFT ROUTING
AIR CANADA

91 AIRCRAFT, 9 TYPES, 33 STATIONS

SAVINGS	3.8 %	8.9 %	13.9 %
TWs	± 10 MIN	± 20 MIN	± 30 MIN

FLEET REDUCTION

WITH TIME WINDOWS ON FLIGHT SCHEDULE

Aircraft Routing & Scheduling
Canadian Army (C-130)

WEST CHALLENGE (19 city-pairs)

- 750 soldiers and equipment
- MAX 65 soldiers per flight

	FLIGHT TIME	NUMBER OF AIRCRAFT
Manual solution	**59 HRS**	**4**
***GENCOL* Optimizer**	**39 HRS**	**3**
SAVINGS	**20 HRS (34 %)**	**1 (33 %)**

SUBWAY DRIVER TOKYO

2000 – 3000 bus segments

One- or two-day shifts

Collective agreement rules

SAVINGS ≈ 15 %

GIRO contract > 1.5M $US (in 1993)

Stockholm, Helsinky, Singapore, Barcelone, New York, Chicago… 250 cities

LOCOMOTIVES CANADIAN NATIONAL
WEEKLY PROBLEM

2000 Trains 26 Locomotive types

Maintenance constraints

Minimal demand for each train : no. of locos & hp

SAVINGS : 100 locos over 1090 (9.17 %)

GENCOL ***R&D***

School Busing

Dial-A-Ride

Bus Drivers

Airline Crew Scheduling

Vehicle Routing

Crew Rostering

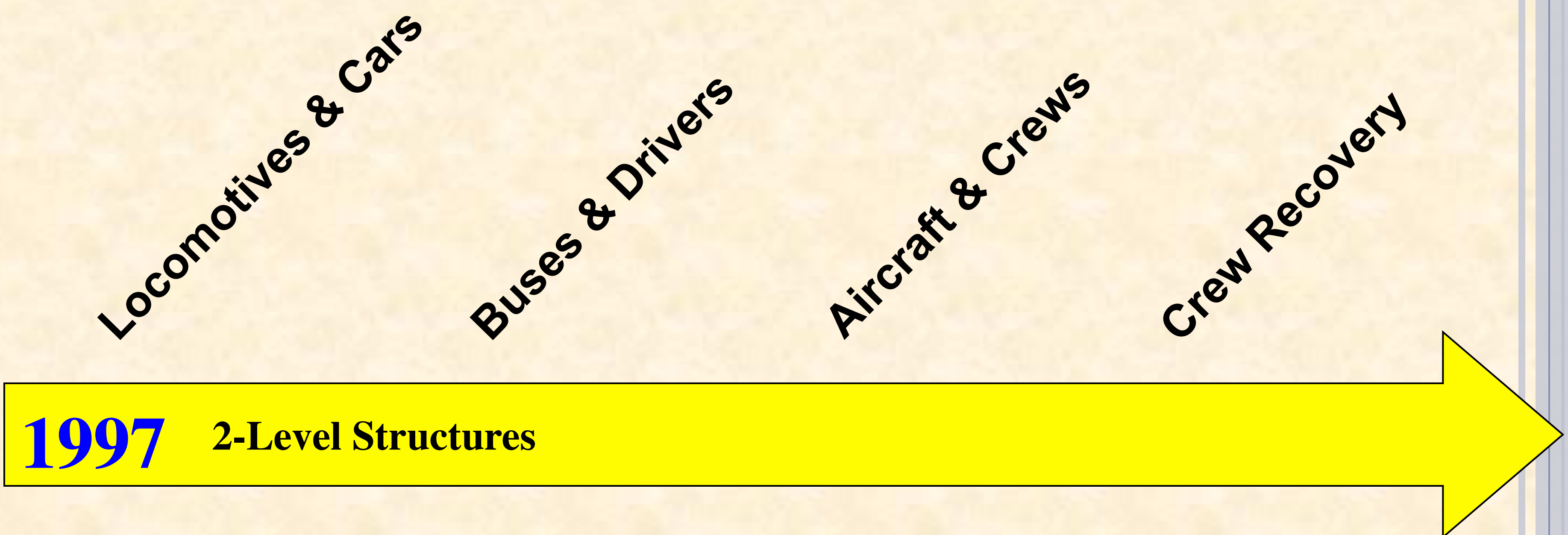
Locomotive Assignment

Fleet Assignment

Preferential Bidding

1981 **Integer Programming Column Generation**

GENCOL R&D


Locomotives & Cars
Buses & Drivers
Aircraft & Crews
Crew Recovery
1997
2-Level Structures

Flexjet Fractional Raised the Bar

Bombardier Flexjet
Aircraft Fractional Ownership Operations

Flight Scheduling
& Fleet Assignment
& Aircraft Routing
& Crew Scheduling

2000 **A 4-Level Integrated Structure**

WHAT IS THE RELATIONSHIP BETWEEN …?

- ***Commission scolaire des Chênes de Drummondville***
- ***UPS***
- ***Tokyo Metro Co.***
- ***NAV Canada***
- ***Mount Sinai Hospital (Toronto)***
- ***Casinos du Québec (Montréal, Charlevoix, Lac-Leamy)***
- ***Bombardier Flexjet***
- ***Autoroutes de France***
- ***Wal-Mart***
- ***…***

VEHICLE ROUTING & CREW SCHEDULING SOLVED BY GENCOL

Two Additional Set Partitioning Applications

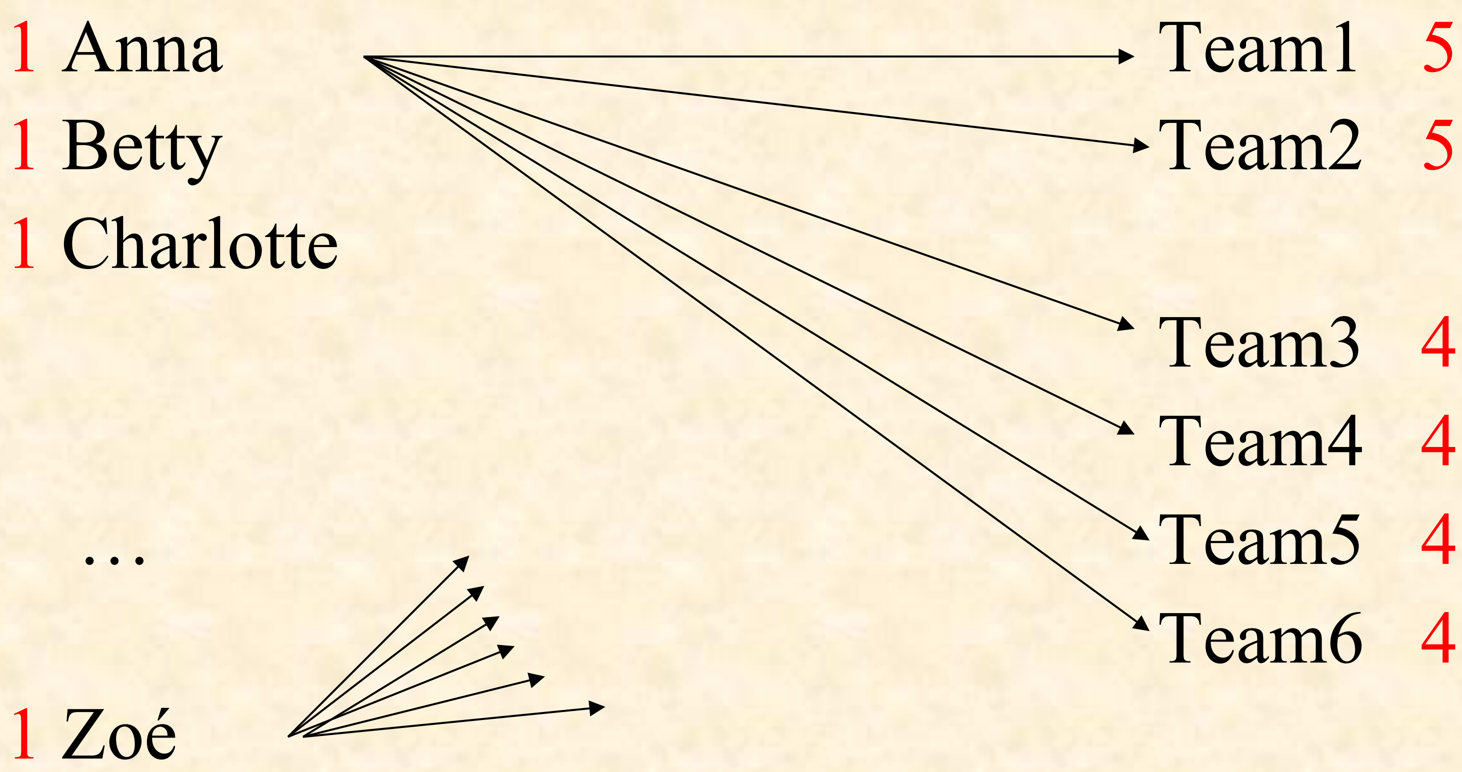
1. MBA Teams

2. A Secret Ballot Problem

1. MBA Teams

- 26 persons to form teams of 4 or 5 persons.
- 4 teams of 4 and 2 teams of 5
- A *Transportation Prob. Constraint Structure*

TRANSPORTATION CONSTRAINTS



MBA Teams: Quadratic objective function

$$\min \sum_{teams} \mathrm{dist}(\mathrm{team}, \mathrm{target})$$

- Target vector
  - Average (proportion) of attributes within the class group
- Team vector
  - Average (proportion) of attributes within the team

- **Attributes**
  - ***Male/Female***
  - ***Scientist***
  - ***Country***
  - ***IQ***
  - ***etc.***

MBA TEAMS

- Some integrality difficulties in solving this *quadratic* transportation problem.
- Moreover, assume 70% males

  => 2.8 in team of 4; 3.5 in team of 5

  2 and 3 are acceptable; 3 and 4 are acceptable

- **Solution procedure**
  - complete enumeration of all acceptable team patterns
  - cost easily computed a priori.

MBA TEAM PATTERNS

Acceptable team patterns of 4 and 5 people

people	cost 1	cost 2	…														
1	1		…													=	1
2																=	1
3	1		1						1							=	1
…			1								1					=	1
			1						1		1					=	1
			1								1					=	1
	1		1						1		1					=	1
					teams of 5											=	1
	1	1											teams of 4			=	1
	1	1														=	1
									1							=	1
																=	1
																=	1
		1														=	1
		1														=	1
																=	1
																=	1
																=	1
																=	1
																=	1
25		1														=	1
26																=	1
Five	1	1	1	1	…	1	1	1								=	2
Four									1	1	1	1	…	1	1	=	4

2. A Secret Ballot Problem: Can you decode the vote?

	p_i	a	b	c	d	e	f
1	**14.3%**						
2	**13.2%**						
3	**12.4%**						
4	**8.4%**						
5	**7.8%**						
6	**6.2%**						
7	**5.7%**						
8	**5.5%**						
9	**4.5%**						
10	**4.2%**						
11	**3.6%**						
12	**3.1%**						
13	**2.7%**						
14	**2.4%**						
15	**1.5%**						
16	**1.4%**						
17	**1.3%**						
18	**1.1%**						
19	**0.4%**						
20	**0.3%**						
	v_j	**35.9%**	**11.1%**	**17.4%**	**17.3%**	**13.8%**	**4.5%**

		a	b	c	d	e	f	
1	14.3%	1						1
2	13.2%	1						1
3	12.4%				1			1
4	8.4%			1				1
5	7.8%					1		1
6	6.2%		1					1
7	5.7%	1						1
8	5.5%			1				1
9	4.5%				1			1
10	4.2%					1		1
11	3.6%		1					1
12	3.1%						1	1
13	2.7%	1						1
14	2.4%			1				1
15	1.5%					1		1
16	1.4%						1	1
17	1.3%		1					1
18	1.1%			1				1
19	0.4%				1			1
20	0.3%					1		1
		35.9%	11.1%	17.4%	17.3%	13.8%	4.5%	

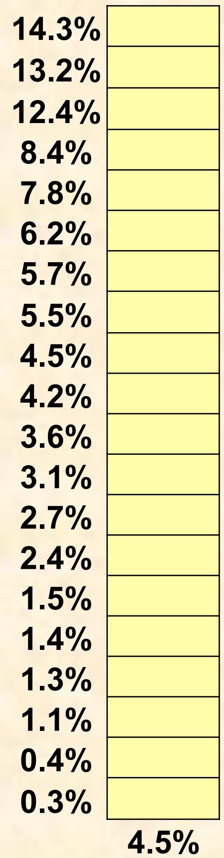
SOLUTION PROCEDURE

- An infinite number of *fractional* solutions
- 4.5% = 0.1*14.3% + 0.23257576*13.2%

  etc.
- A very limited number of *integer* solutions
  - 4.5% : 10 combinations
  - 35.9% : 12 combinations

Full enumeration and Set Partitioning Formulation

WHO CAN HAVE VOTED FOR CANDIDATE *f*?



ONLY
10 PATTERNS FOR CANDIDATE *f*

4.5%		1									
4.2%			1								
3.6%											
3.1%				1	1						
2.7%						1	1	1			
2.4%									1		
1.5%						1				1	
1.4%				1			1		1	1	1
1.3%										1	1
1.1%					1			1			1
0.4%							1	1	1		1
0.3%			1		1	1		1	1	1	1
	4.5%	4.5%	4.5%	4.5%	4.5%	4.5%	4.5%	4.5%	4.5%	4.5%	4.5%

ENUMERATION OF ALL POSSIBLE PATTERNS

14.3%											1	1	1	1	1	1	1	1	1	1	1	1
13.2%											1	1	1	1	1	1	1	1	1	1	1	1
12.4%																						
8.4%											1											
7.8%																						
6.2%												1										
5.7%													1	1	1							
5.5%																1	1	1				
4.5%	1																		1	1	1	1
4.2%		1																				
3.6%																			1			
3.1%			1	1																		
2.7%					1	1	1						1									
2.4%								1						1						1	1	
1.5%					1				1			1				1	1			1		1
1.4%			1			1		1	1	1					1	1		1				
1.3%									1	1					1							1
1.1%				1			1			1							1	1			1	1
0.4%						1	1	1		1		1						1			1	
0.3%		1		1	1		1	1	1	1		1		1			1		1			
	4.5%	4.5%	4.5%	4.5%	4.5%	4.5%	4.5%	4.5%	4.5%	4.5%	35.9%	35.9%	35.9%	35.9%	35.9%	35.9%	35.9%	35.9%	35.9%	35.9%	35.9%	35.9%
	1	1	1	1	1	1	1	1	1	1												
											1	1	1	1	1	1	1	1	1	1	1	1

35.9%	11.1%	17.4%	17.3%	13.8%	4.5%

ENUMERATION OF ALL POSSIBLE PATTERNS

										1	1	1	1	1	1	1	1	1	1	1	1																											=	1
										1	1	1	1	1	1	1	1	1	1	1	1																											=	1
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										1													1	1	1	1	1	1	1	1																		=	1
																														1	1	1	1	1	1	1	1	1	1	1	1	1	1	1			=	1
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		1	1																																				1	1						=	1
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							1						1						1	1					1			1	1							1		1						1			=	1
				1				1			1				1	1			1		1		1						1			1		1							1		1	1	1			=	1
		1			1		1	1	1					1	1		1							1			1			1					1						1	1	1		1			=	1
								1	1					1							1						1	1		1								1		1								=	1
			1			1			1							1	1			1	1	1			1			1	1				1			1		1		1		1		1				=	1
					1	1	1		1		1						1			1				1	1				1				1		1	1						1	1	1	1			=	1
	1		1	1		1	1	1	1		1		1			1		1				1	1		1	1		1		1	1			1		1								1	1			=	1
4.5%	4.5%	4.5%	4.5%	4.5%	4.5%	4.5%	4.5%	4.5%	4.5%	35.9%	35.9%	35.9%	35.9%	35.9%	35.9%	35.9%	35.9%	35.9%	35.9%	35.9%	35.9%	13.8%	13.8%	13.8%	13.8%	13.8%	13.8%	13.8%	13.8%	13.8%	13.8%	13.8%	13.8%	13.8%	13.8%	13.8%	13.8%	13.8%	13.8%	13.8%	13.8%	13.8%	13.8%	13.8%	13.8%	1.9%	0.0%		
1	1	1	1	1	1	1	1	1	1																																							=	1
										1	1	1	1	1	1	1	1	1	1	1	1																											=	1
																					1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1		=	1
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ADDITIONAL APPLICATIONS

1. **Cutting Stock Problem**
2. **Single Depot Vehicle Scheduling Problem**
3. **Multiple Depot Vehicle Scheduling Problem**

CUTTING STOCK PROBLEM

- *Gilmore-Gomory formulation (binary)*
- *Gilmore-Gomory formulation (integer)*
- *Kantorovich formulation*
- *Valério de Carvalho formulation*
- *Dantzig-Wolfe reformulations*

CUTTING STOCK PROBLEM

Given are indentical rolls of length L
and demands d_i for small items i in N of length l_i ($<L$).
Find the minimum number of rolls to satisfy the demand.

P: set of cutting patterns of a roll
a_{ip}: number of times item i appears in pattern p

$$\sum_{i \in I} a_{ip} \leq L, \quad \forall p \in P$$

CUTTING STOCK PROBLEM

λ_p : number of times pattern p is choosen in the solution

$$\min \sum_{p\in P} \lambda_p$$

$$\sum_{p\in P} a_{ip}\lambda_p = d_i \qquad \forall i \in I$$

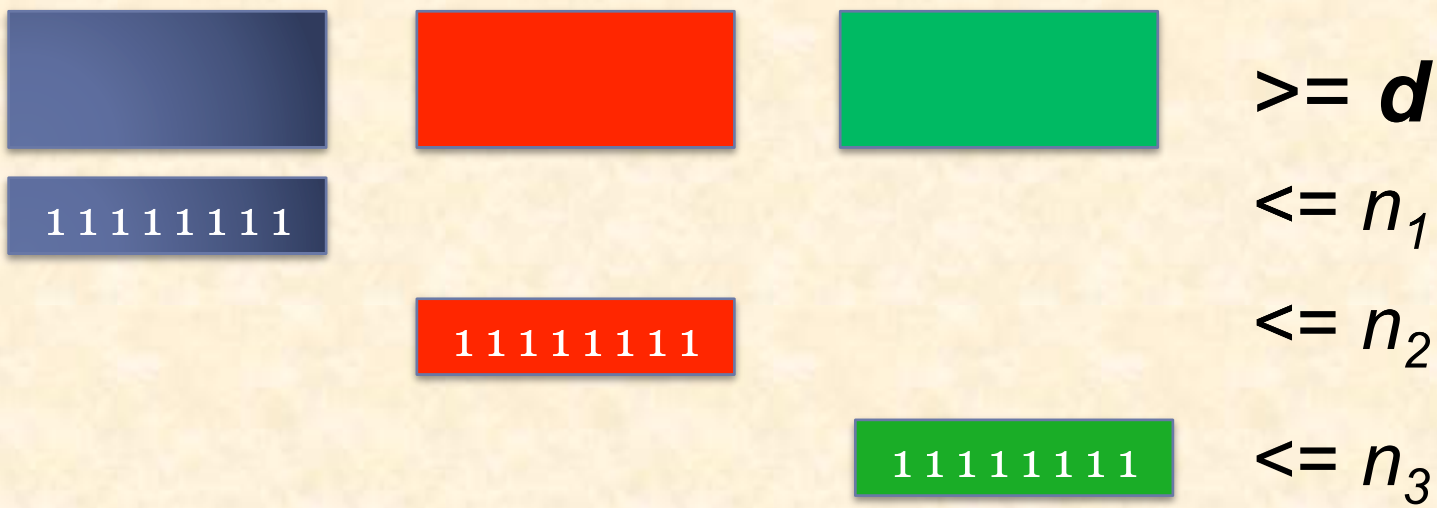
$$\lambda_p \geq 0, \text{integer} \qquad \forall p \in P$$

The linear relaxation is $Z_{LP} = \dfrac{\sum_{i\in I} l_i d_i}{L}$

Lower bound on $Z_{IP} = \lceil Z_{LP} \rceil$

CUTTING STOCK PROBLEM

For non identical rolls, patterns are generated according to the selected roll length.



SINGLE DEPOT VEHICLE SCHEDULING PROBLEM

- ***Network formulation (homogeneous fleet)***
- ***DW with extreme points***
- ***DW with extreme rays and a single extreme point***

- ***Specialized network representations***
  - ***School busing***
  - ***Urban bus routing***
  - ***Periodic aircraft routing***
- ***Homogeneous vs. Heterogeneous fleet***

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MULTIPLE DEPOT VEHICLE SCHEDULING PROBLEM

- ***Multi-commodity network formulation***
- ***Discussion on the objective function***
- ***Block diagonal decomposition***