Parallel Computations on Immutable Data Structures

Florian Biermann
`fbie@itu.dk`

IT University of Copenhagen & UCAS

2016-06-02

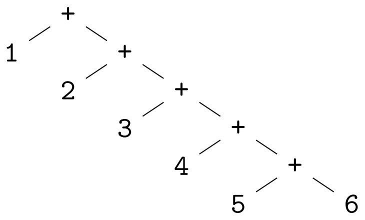
A Sequential Expression

$\sum_{i=1}^{6} i$ = `(+ 1 (+ 2 (+ 3 (+ 4 (+ 5 6)))))`

A Sequential Expression

$\sum_{i=1}^{6} i$ = `(+ 1 (+ 2 (+ 3 (+ 4 (+ 5 6)))))`

The expression tree:



What Is The Problem?

All operations are performed sequentially, even though it would be perfectly ok to compute

```
(+ 1 (+ 2 3))
```

and

```
(+ 4 (+ 5 6))
```

in parallel. The result would be the same.

What Is The Problem?

All operations are performed sequentially, even though it would be perfectly ok to compute

```
(+ 1 (+ 2 3))
```

and

```
(+ 4 (+ 5 6))
```

in parallel. The result would be the same.

We can re-formulate the expression and obtain the same result!

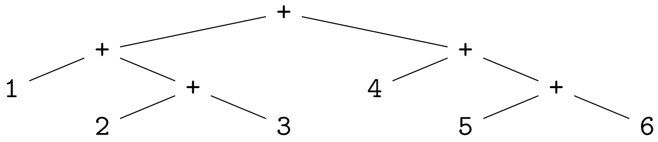
A Parallel Expression

$\sum_{i=1}^{6} i$ `= (+ (+ 1 (+ 2 3)) (+ 4 (+ 5 6)))`

A Parallel Expression

$\sum_{i=1}^{6} i$ = `(+ (+ 1 (+ 2 3)) (+ 4 (+ 5 6)))`

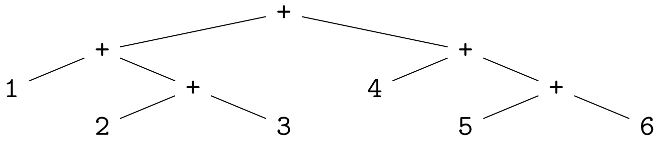
The expression tree:



A Parallel Expression

$$\sum_{i=1}^{6} i \quad = \quad \texttt{(+ (+ 1 (+ 2 3)) (+ 4 (+ 5 6)))}$$

The expression tree:



Why is this possible? Because + is an associative operation!

$$\texttt{(+ x (+ y z))} = \texttt{(+ (+ x y) z)}$$

Generalizing Parallel Expressions

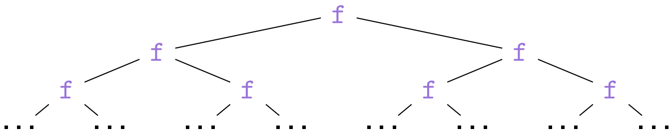
We can do this for any function `f` iff

```
(f a (f b c)) = (f (f a b) c)
```

Generalizing Parallel Expressions

We can do this for any function `f` iff

```
(f a (f b c)) = (f (f a b) c)
```



Another Problem

Mapping a function `f` over a `cons` list.

```
(: seq-map (All (A B)
    (-> (-> A B) (Listof A) (Listof B))))
```

Another Problem

Mapping a function `f` over a `cons` list.

```
(: seq-map (All (A B)
    (-> (-> A B) (Listof A) (Listof B))))

(define (seq-map f xs)
  (match xs
    ['() '()]
    [(cons x xs) (cons (f x) (seq-map f xs))]))
```

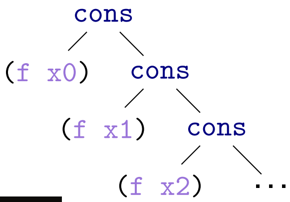
Another Problem

Mapping a function `f` over a `cons` list.

```
(: seq-map (All (A B)
     (-> (-> A B) (Listof A) (Listof B))))

(define (seq-map f xs)
  (match xs
    ['() '()]
    [(cons x xs) (cons (f x) (seq-map f xs))]))
```

Expression tree of `seq-map`:

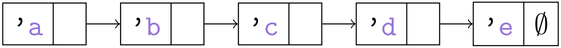


What Is The Problem?

`seq-map` is not the problem. It is the only way to handle a `cons` list. **The real problem is the single-linked list!** It is inherently sequential:

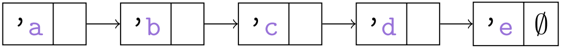
What Is The Problem?

`seq-map` is not the problem. It is the only way to handle a `cons` list. **The real problem is the single-linked list!** It is inherently sequential:



What Is The Problem?

`seq-map` is not the problem. It is the only way to handle a `cons` list. **The real problem is the single-linked list!** It is inherently sequential:



The parallel expressions we have seen so far are *trees*. Maybe we can develop a tree data structure that we can use to implement a parallel version of `map`?

A New Parallel Data Type

We represent the list as a tree instead:

```
(define-type (CatListof A) (U (leaf A)
                              (cat A)))
(struct (A) leaf ([a : A]))
(struct (A) cat  ([l : (CatListof A)]
                  [r : (CatListof A)]))
```

A New Parallel Data Type

We represent the list as a tree instead:

```
(define-type (CatListof A) (U (leaf A)
                              (cat A)))
(struct (A) leaf ([a : A]))
(struct (A) cat  ([l : (CatListof A)]
                  [r : (CatListof A)]))
```

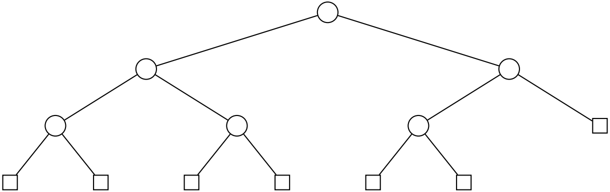
`(leaf a)` produces a singleton `CatList`.
`(cat l r)` concatenates two `CatList` instances.

A New Parallel Data Type

We represent the list as a tree instead:

```
(define-type (CatListof A) (U (leaf A)
                               (cat A)))
(struct (A) leaf ([a : A]))
(struct (A) cat  ([l : (CatListof A)]
                  [r : (CatListof A)]))
```

(`leaf a`) produces a singleton `CatList`.
(`cat l r`) concatenates two `CatList` instances.



Let's Implement Map!

```
(: par-map (All (A B)
  (-> (-> A B) (CatListof A) (CatListof B))))
```

Let's Implement Map!

```
(: par-map (All (A B)
  (-> (-> A B) (CatListof A) (CatListof B))))

(define (par-map f xs)
  (match xs
    [(leaf x) (leaf (f x))]
    [(cat l r) (cat (par-map f l)
                    (par-map f r))]))
```

Let's Implement Map!

```
(: par-map (All (A B)
  (-> (-> A B) (CatListof A) (CatListof B))))

(define (par-map f xs)
  (match xs
    [(leaf x) (leaf (f x))]
    [(cat l r) (cat (par-map f l)
                    (par-map f r))]))
```

Note: This function is not *truly* parallel yet. We have just created a *possibility* for parallelism.

Parallelism in Racket

In Racket we use a fork/join style model of parallel computations. Fork is called `future` and join is called `touch`:

```
(: in-parallel (All (A B C)
  (-> (-> A B) (-> A C) A (Pairof B C))))
```

Parallelism in Racket

In Racket we use a fork/join style model of parallel computations. Fork is called `future` and join is called `touch`:

```
(: in-parallel (All (A B C)
   (-> (-> A B) (-> A C) A (Pairof B C))))

(define (in-parallel f g x)
   (let ([fx (future (lambda () (f x)))]
         [gx (g x)])
     (cons (touch fx) gx)))
```

Parallelism in Racket

In Racket we use a fork/join style model of parallel computations. Fork is called `future` and join is called `touch`:

```
(: in-parallel (All (A B C)
  (-> (-> A B) (-> A C) A (Pairof B C))))

(define (in-parallel f g x)
  (let ([fx (future (lambda () (f x)))]
        [gx (g x)])
    (cons (touch fx) gx)))
```

Racket has a built-in scheduler for `future`s, so we give a lot of control to the run-time.

```
(: future (All (A) (->(->A) (Futureof A))))
(: touch (All (A) (->(Futureof A) A)))
```

Parallelizing Map, For Real

```
(: par-map (All (A B)
  (-> (-> A B) (CatListof A) (CatListof B))))
```

Parallelizing Map, For Real

```
(: par-map (All (A B)
  (-> (-> A B) (CatListof A) (CatListof B))))

(define (par-map f xs)
  (match xs
    [(leaf x) (leaf (f x))]
```

Parallelizing Map, For Real

```
(: par-map (All (A B)
  (-> (-> A B) (CatListof A) (CatListof B))))

(define (par-map f xs)
  (match xs
    [(leaf x) (leaf (f x))]
    [(cat l r)
       (let ([fl (future ;; Map l in parallel.
                          (lambda () (par-map f l)))]
```

Parallelizing Map, For Real

```
(: par-map (All (A B)
  (-> (-> A B) (CatListof A) (CatListof B))))

(define (par-map f xs)
  (match xs
    [(leaf x) (leaf (f x))]
    [(cat l r)
       (let ([fl (future ;; Map l in parallel.
                         (lambda () (par-map f l)))]
             [fr (future ;; Map r in parallel.
                         (lambda () (par-map f r)))])
```

Parallelizing Map, For Real

```
(: par-map (All (A B)
  (-> (-> A B) (CatListof A) (CatListof B))))

(define (par-map f xs)
  (match xs
    [(leaf x) (leaf (f x))]
    [(cat l r)
       (let ([fl (future ;; Map l in parallel.
                         (lambda () (par-map f l)))]
             [fr (future ;; Map r in parallel.
                         (lambda () (par-map f r)))])
         (cat (touch fl) (touch fr)))]))
```

The recursive call is wrapped in a `future`!

Pro and Contra

The good parts:

Pro and Contra

The good parts:

- The build-in scheduler decides on the number of threads to use.

Pro and Contra

The good parts:

- The build-in scheduler decides on the number of threads to use.
- A thread that is blocked by `touch` is de-scheduled until the `touch`ed `future` is done, no locking, spinning etc.

Pro and Contra

The good parts:

- The build-in scheduler decides on the number of threads to use.
- A thread that is blocked by `touch` is de-scheduled until the `touch`ed `future` is done, no locking, spinning etc.
- As a result, we do not have to manually decide **how** to split up work across hardware threads.

Pro and Contra

The good parts:

- The build-in scheduler decides on the number of threads to use.
- A thread that is blocked by `touch` is de-scheduled until the `touch`ed `future` is done, no locking, spinning etc.
- As a result, we do not have to manually decide **how** to split up work across hardware threads.

The bad parts:

Pro and Contra

The good parts:

- The build-in scheduler decides on the number of threads to use.
- A thread that is blocked by `touch` is de-scheduled until the `touch`ed `future` is done, no locking, spinning etc.
- As a result, we do not have to manually decide **how** to split up work across hardware threads.

The bad parts:

- A list has $O(n)$ memory overhead, a balanced tree $O(n \log n)$.

Pro and Contra

The good parts:

- The build-in scheduler decides on the number of threads to use.
- A thread that is blocked by `touch` is de-scheduled until the `touch`ed `future` is done, no locking, spinning etc.
- As a result, we do not have to manually decide **how** to split up work across hardware threads.

The bad parts:

- A list has $O(n)$ memory overhead, a balanced tree $O(n \log n)$.
- If the tree is not balanced, we lose parallelism – mitigate by balancing algorithm!

Pro and Contra

The good parts:

- The build-in scheduler decides on the number of threads to use.
- A thread that is blocked by `touch` is de-scheduled until the `touch`ed `future` is done, no locking, spinning etc.
- As a result, we do not have to manually decide **how** to split up work across hardware threads.

The bad parts:

- A list has $O(n)$ memory overhead, a balanced tree $O(n \log n)$.
- If the tree is not balanced, we lose parallelism – mitigate by balancing algorithm!
- Maybe we spawn **excessively many futures** and put too much strain on the scheduler?

An Improved Variant

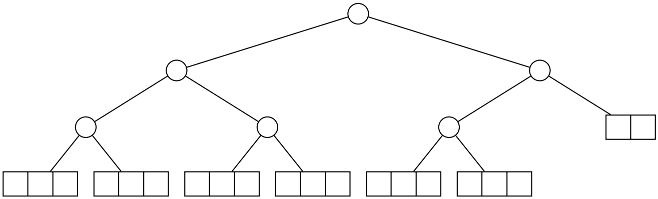
If the size of the tree is too small, then the overhead of constructing `future`s dominates the run-time cost. We can handle this problem by using lists as leaves instead of scalar values.

We call such a tree a **Rope**:

```
(define-type (Ropeof A) (U (leaf A)
                           (cat A)))
(struct (A) leaf ([as : (Listof A)]))
(struct (A) cat  ([l  : (Ropeof A)]
                  [r  : (Ropeof A)]))
```

Ropes: Trees With Lists as Leaves

We must define a **maximum size** $s_{\max}$ for leaves. In this example, $s_{\max} = 3$:



Live Coding Session

Parallel higher-order functions on ropes.

Live Coding Session

Parallel higher-order functions on ropes.

- I code on the big screen and show you how to do functional programming in Racket.

Live Coding Session

Parallel higher-order functions on ropes.

- I code on the big screen and show you how to do functional programming in Racket.
- You help with ideas and suggestions and experiment yourselves!

Live Coding Session

Parallel higher-order functions on ropes.

- I code on the big screen and show you how to do functional programming in Racket.
- You help with ideas and suggestions and experiment yourselves!
- All code we write will be available on
  - github.com/fbie/parallel-functional-programming

Live Coding Session

Parallel higher-order functions on ropes.

- I code on the big screen and show you how to do functional programming in Racket.
- You help with ideas and suggestions and experiment yourselves!
- All code we write will be available on
  - `github.com/fbie/parallel-functional-programming`
- Want to start now and code along?
  - Download the code from `2/ropes-lecture.rkt`

Live Coding Session

Parallel higher-order functions on ropes.

- I code on the big screen and show you how to do functional programming in Racket.
- You help with ideas and suggestions and experiment yourselves!
- All code we write will be available on
  - `github.com/fbie/parallel-functional-programming`
- Want to start now and code along?
  - Download the code from `2/ropes-lecture.rkt`
- You can download Racket from
  - `racket-lang.org`

New Syntax: Recursive Let

Standard `let`:

```
(let ([x 1]
      [y (+ 2 x)])
  (* y x))
```

New Syntax: Recursive Let

Standard `let`:

```
(let ([x 1]
      [y (+ 2 x)])
  (* y x))
```

```
Undefined identifier:  x
```

New Syntax: Recursive Let

Standard `let`:

```
(let ([x 1]
      [y (+ 2 x)])
  (* y x))
```

```
Undefined identifier:  x
```

Solution: `letrec`, recursive let-binding:

```
(letrec ([x 1]
         [y (+ 2 x)])
  (* y x))
```

Allows bindings to reference one-another.

De-sugaring Recursive Let

```
(letrec ([x 1]
         [y (+ 2 x)])
  (* y x))
```

is equal to

```
(let ([x 1])
  (let ([y (+ 2 x)])
    (* y x)))
```