Parallel Computations on Immutable Data Structures

Florian Biermann

fbie@itu.dk

IT University of Copenhagen & UCAS

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- Ropes and their parallel capabilities.

A Sequential Expression

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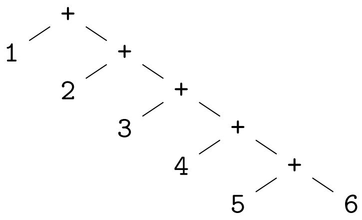
The expression tree:



What Is The Problem?

All operations are performed sequentially, even though it would be perfectly ok to compute

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We can re-formulate the expression and obtain the same result!

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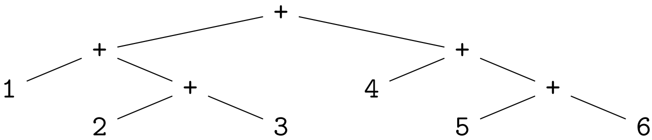

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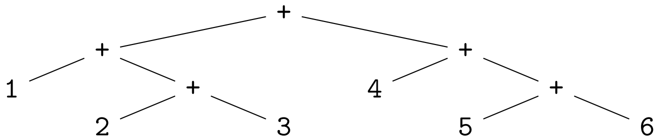

Why is this possible? Because + is an associative operation!

```
(+ x (+ y z)) = (+ (+ x y) z)
```

Generalizing Parallel Expressions

We can do this for any function `f` iff

```
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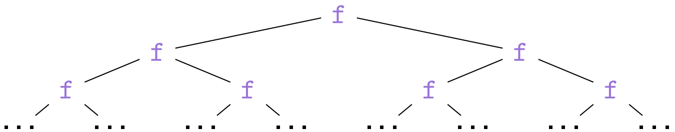

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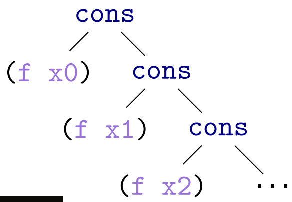
Expression tree of `seq-map`:



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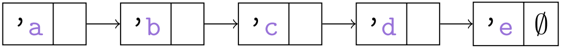

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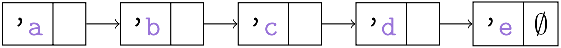

The parallel expressions we have seen so far are *trees*. Maybe we can develop a tree data structure that we can use to implement a parallel version of `map`?

A New Parallel Data Type

We represent the list as a tree instead:

```
(define-type (CatListof A) (U (leaf A)
                              (cat A)))
(struct (A) leaf ([a : A]))
(struct (A) cat  ([l : (CatListof A)]
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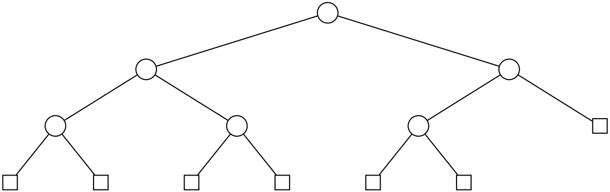

Let’s Implement Map!

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Note: This function is not *truly* parallel yet. We have just created a *possibility* for parallelism.

Parallelism in Racket

In Racket we use a fork/join style model of parallel computations. Fork is called `future` and join is called `touch`:

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(: in-parallel (All (A B C)
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   (let ([fx (future (lambda () (f x)))]
         [gx (g x)])
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Racket has a built-in scheduler for `future`s, so we give a lot of control to the run-time.

```
(: future (All (A) (->(->A) (Futureof A))))
(: touch (All (A) (->(Futureof A) A)))
```

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             [fr (future ;; Map r in parallel.
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         (cat (touch fl) (touch fr)))]))
```

The recursive call is wrapped in a `future`!

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The bad parts:

- A list has $O(n)$ memory overhead, a balanced tree $O(n \log n)$.
- If the tree is not balanced, we lose parallelism – mitigate by balancing algorithm!
- Maybe we spawn **excessively many futures** and put too much strain on the scheduler?

An Improved Variant

If the size of the tree is too small, then the overhead of constructing `future`s dominates the run-time cost. We can handle this problem by using lists as leaves instead of scalar values.

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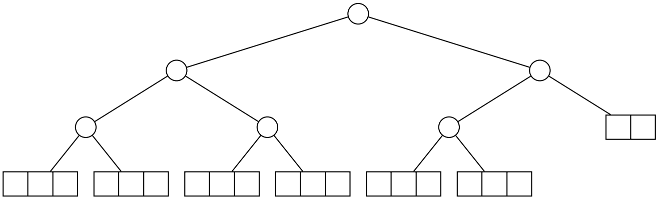
We call such a tree a **Rope**:

```
(define-type (Ropeof A) (U (leaf A)
                           (cat A)))
(struct (A) leaf ([as : (Listof A)]))
(struct (A) cat  ([l  : (Ropeof A)]
                  [r  : (Ropeof A)]))
```

Ropes: Trees With Lists as Leaves

We must define a **maximum size** $s_{\max}$ for leaves. In this example, $s_{\max} = 3$:



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Solution: `letrec`, recursive let-binding:

```
(letrec ([x 1]
         [y (+ 2 x)])
  (* y x))
```

Allows bindings to reference one-another.

De-sugaring Recursive Let

```
(letrec ([x 1]
         [y (+ 2 x)])
  (* y x))
```

is equal to

```
(let ([x 1])
  (let ([y (+ 2 x)])
    (* y x)))
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- Hans-J. Boehm et al.: **Ropes: an Alternative to Strings** (optional)

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