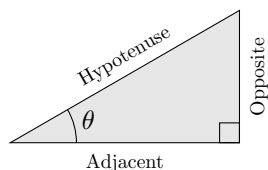
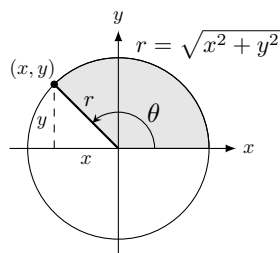


TRIGONOMETRY

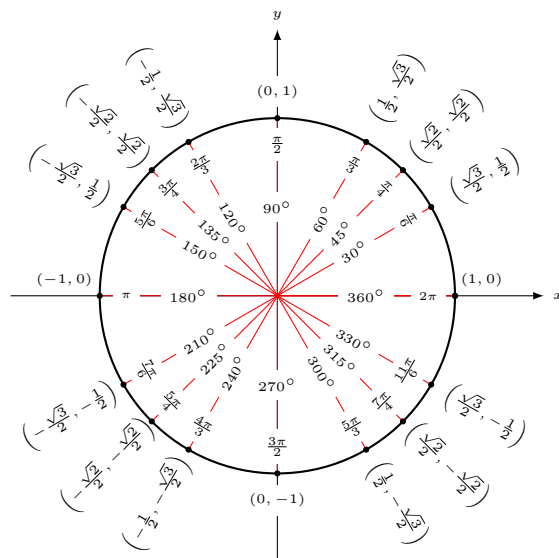
Definition of the Six Trigonometric Functions



$$\begin{aligned}\sin \theta &= \frac{\text{opp}}{\text{hyp}} & \csc \theta &= \frac{\text{hyp}}{\text{opp}} \\ \cos \theta &= \frac{\text{adj}}{\text{hyp}} & \sec \theta &= \frac{\text{hyp}}{\text{adj}} \\ \tan \theta &= \frac{\text{opp}}{\text{adj}} & \cot \theta &= \frac{\text{adj}}{\text{opp}}\end{aligned}$$



$$\begin{aligned}\sin \theta &= \frac{y}{r} & \csc \theta &= \frac{r}{y} \\ \cos \theta &= \frac{x}{r} & \sec \theta &= \frac{r}{x} \\ \tan \theta &= \frac{y}{x} & \cot \theta &= \frac{x}{y}\end{aligned}$$



Reciprocal Identities

$$\begin{aligned}\sin \theta &= \frac{1}{\csc \theta} & \cos \theta &= \frac{1}{\sec \theta} & \tan \theta &= \frac{1}{\cot \theta} \\ \csc \theta &= \frac{1}{\sin \theta} & \sec \theta &= \frac{1}{\cos \theta} & \cot \theta &= \frac{1}{\tan \theta}\end{aligned}$$

Tangent and Cotangent Identities

$$\tan \theta = \frac{\sin \theta}{\cos \theta} \quad \cot \theta = \frac{\cos \theta}{\sin \theta}$$

Pythagorean Identities

$$\begin{aligned}\sin^2 \theta + \cos^2 \theta &= 1 \\ 1 + \tan^2 \theta &= \sec^2 \theta \quad 1 + \cot^2 \theta = \csc^2 \theta\end{aligned}$$

Cofunction Identities

$$\begin{aligned}\sin \left(\frac{\pi}{2} - \theta \right) &= \cos \theta & \cos \left(\frac{\pi}{2} - \theta \right) &= \sin \theta \\ \csc \left(\frac{\pi}{2} - \theta \right) &= \sec \theta & \tan \left(\frac{\pi}{2} - \theta \right) &= \cot \theta \\ \sec \left(\frac{\pi}{2} - \theta \right) &= \csc \theta & \cot \left(\frac{\pi}{2} - \theta \right) &= \tan \theta\end{aligned}$$

Reduction Formulas

$$\begin{aligned}\sin(-\theta) &= -\sin \theta & \cos(-\theta) &= \cos \theta \\ \csc(-\theta) &= -\csc \theta & \tan(-\theta) &= -\tan \theta \\ \sec(-\theta) &= \sec \theta & \cot(-\theta) &= -\cot \theta\end{aligned}$$

Sum and Difference Formulas

$$\begin{aligned}\sin(\theta \pm \phi) &= \sin \theta \cos \phi \pm \cos \theta \sin \phi \\ \cos(\theta \pm \phi) &= \cos \theta \cos \phi \mp \sin \theta \sin \phi \\ \tan(\theta \pm \phi) &= \frac{\tan \theta \pm \tan \phi}{1 \mp \tan \theta \tan \phi}\end{aligned}$$

Double-Angle Formulas

$$\begin{aligned}\sin 2\theta &= 2 \sin \theta \cos \theta \\ \cos 2\theta &= \cos^2 \theta - \sin^2 \theta = 2 \cos^2 \theta - 1 = 1 - 2 \sin^2 \theta \\ \tan 2\theta &= \frac{2 \tan \theta}{1 - \tan^2 \theta}\end{aligned}$$

Power-Reducing Formulas

$$\begin{aligned}\sin^2 \theta &= \frac{1}{2}(1 - \cos 2\theta) \\ \cos^2 \theta &= \frac{1}{2}(1 + \cos 2\theta) \\ \tan^2 \theta &= \frac{1 - \cos 2\theta}{1 + \cos 2\theta}\end{aligned}$$

Sum-to-Product Formula

$$\begin{aligned}\sin \theta + \sin \phi &= 2 \sin \left(\frac{\theta + \phi}{2} \right) \cos \left(\frac{\theta - \phi}{2} \right) \\ \sin \theta - \sin \phi &= 2 \cos \left(\frac{\theta + \phi}{2} \right) \sin \left(\frac{\theta - \phi}{2} \right) \\ \cos \theta + \cos \phi &= 2 \cos \left(\frac{\theta + \phi}{2} \right) \cos \left(\frac{\theta - \phi}{2} \right) \\ \cos \theta - \cos \phi &= -2 \sin \left(\frac{\theta + \phi}{2} \right) \sin \left(\frac{\theta - \phi}{2} \right)\end{aligned}$$

Product-to-Sum Formulas

$$\begin{aligned}\sin \theta \sin \phi &= \frac{1}{2} [\cos(\theta - \phi) - \cos(\theta + \phi)] \\ \cos \theta \cos \phi &= \frac{1}{2} [\cos(\theta - \phi) + \cos(\theta + \phi)] \\ \sin \theta \cos \phi &= \frac{1}{2} [\sin(\theta + \phi) + \sin(\theta - \phi)] \\ \cos \theta \sin \phi &= \frac{1}{2} [\sin(\theta + \phi) - \sin(\theta - \phi)]\end{aligned}$$

DERIVATIVES AND INTEGRALS

Basic Differentiation Rules

- | | | |
|---|---|--|
| 1. $\frac{d}{dx}[cu] = cu'$ | 13. $\frac{d}{dx}[\sin u] = (\cos u)u'$ | 25. $\frac{d}{dx}[\sinh u] = (\cosh u)u'$ |
| 2. $\frac{d}{dx}[u \pm v] = u' \pm v'$ | 14. $\frac{d}{dx}[\cos u] = -(\sin u)u'$ | 26. $\frac{d}{dx}[\cosh u] = (\sinh u)u'$ |
| 3. $\frac{d}{dx}[uv] = uv' + u'v$ | 15. $\frac{d}{dx}[\tan u] = (\sec^2 u)u'$ | 27. $\frac{d}{dx}[\tanh u] = (\operatorname{sech}^2 u)u'$ |
| 4. $\frac{d}{dx}\left[\frac{u}{v}\right] = \frac{u'v - uv'}{v^2}$ | 16. $\frac{d}{dx}[\cot u] = -(\csc^2 u)u'$ | 28. $\frac{d}{dx}[\coth] = -(\operatorname{csch}^2 u)u'$ |
| 5. $\frac{d}{dx}[c] = 0$ | 17. $\frac{d}{dx}[\sec u] = (\sec u \tan u)u'$ | 29. $\frac{d}{dx}[\operatorname{sech} u] = -(\operatorname{sech} u \tanh u)u'$ |
| 6. $\frac{d}{dx}[u^n] = nu^{n-1}u'$ | 18. $\frac{d}{dx}[\csc u] = -(\csc u \cot u)u'$ | 30. $\frac{d}{dx}[\operatorname{csch} u] = -(\operatorname{csch} u \coth u)u'$ |
| 7. $\frac{d}{dx}[x] = 1$ | 19. $\frac{d}{dx}[\arcsin u] = \frac{u'}{\sqrt{1-u^2}}$ | 31. $\frac{d}{dx}[\sinh^{-1} u] = \frac{u'}{\sqrt{u^2+1}}$ |
| 8. $\frac{d}{dx}[u] = \frac{u}{ u }(u'), \quad u \neq 0$ | 20. $\frac{d}{dx}[\arccos u] = \frac{-u'}{\sqrt{1-u^2}}$ | 32. $\frac{d}{dx}[\cosh^{-1} u] = \frac{u'}{\sqrt{u^2-1}}$ |
| 9. $\frac{d}{dx}[\ln u] = \frac{u'}{u}$ | 21. $\frac{d}{dx}[\arctan u] = \frac{u'}{1+u^2}$ | 33. $\frac{d}{dx}[\tanh^{-1} u] = \frac{u'}{1-u^2}$ |
| 10. $\frac{d}{dx}[e^u] = e^u u'$ | 22. $\frac{d}{dx}[\operatorname{arccot} u] = \frac{-u'}{1+u^2}$ | 34. $\frac{d}{dx}[\coth^{-1} u] = \frac{u'}{1-u^2}$ |
| 11. $\frac{d}{dx}[\log_a u] = \frac{u'}{(\ln a)u}$ | 23. $\frac{d}{dx}[\operatorname{arcsec} u] = \frac{u'}{ u \sqrt{u^2-1}}$ | 35. $\frac{d}{dx}[\operatorname{sech}^{-1} u] = \frac{-u'}{u\sqrt{1-u^2}}$ |
| 12. $\frac{d}{dx}[a^u] = (\ln a)a^u u'$ | 24. $\frac{d}{dx}[\operatorname{arccsc} u] = \frac{-u'}{ u \sqrt{u^2-1}}$ | 36. $\frac{d}{dx}[\operatorname{csch}^{-1} u] = \frac{-u'}{ u \sqrt{1+u^2}}$ |

Basic Integration Formulas

- | | |
|--|---|
| 1. $\int kf(u) du = k \int f(u) du$ | 10. $\int \sec u du = \ln \sec u + \tan u + C$ |
| 2. $\int [f(u) \pm g(u)] du = \int f(u) du \pm \int g(u) du$ | 11. $\int \csc u du = -\ln \csc u + \cot u + C$ |
| 3. $\int du = u + C$ | 12. $\int \sec^2 u du = \tan u + C$ |
| 4. $\int a^u du = \left(\frac{1}{\ln a}\right)a^u + C$ | 13. $\int \csc^2 u du = -\cot u + C$ |
| 5. $\int e^u du = e^u + C$ | 14. $\int \sec u \tan u du = \sec u + C$ |
| 6. $\int \sin u du = -\cos u + C$ | 15. $\int \csc u \cot u du = -\csc u + C$ |
| 7. $\int \cos u du = \sin u + C$ | 16. $\int \frac{du}{\sqrt{a^2 - u^2}} = \arcsin\left(\frac{u}{a}\right) + C$ |
| 8. $\int \tan u du = -\ln \cos u + C$ | 17. $\int \frac{du}{a^2 + u^2} = \frac{1}{a} \arctan\left(\frac{u}{a}\right) + C$ |
| 9. $\int \cot u du = \ln \sin u + C$ | 18. $\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a} \operatorname{arcsec}\left(\frac{ u }{a}\right) + C$ |