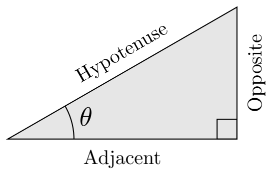
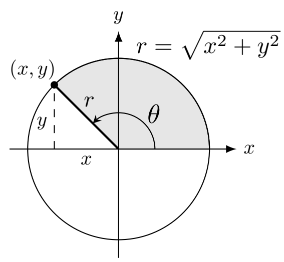
TRIGONOMETRY

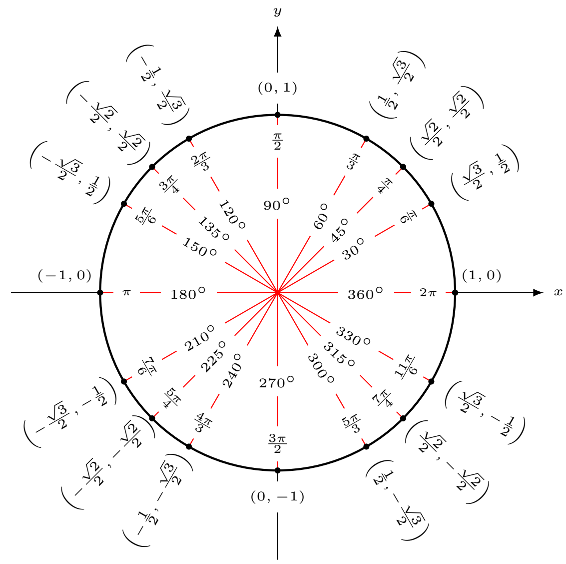
Definition of the Six Trigonometric Functions



$$\sin\theta = \frac{\text{opp}}{\text{hyp}} \qquad \csc\theta = \frac{\text{hyp}}{\text{opp}}$$
$$\cos\theta = \frac{\text{adj}}{\text{hyp}} \qquad \sec\theta = \frac{\text{hyp}}{\text{adj}}$$
$$\tan\theta = \frac{\text{opp}}{\text{adj}} \qquad \cot\theta = \frac{\text{adj}}{\text{opp}}$$



$$\sin\theta = \frac{y}{r} \qquad \csc\theta = \frac{r}{y}$$
$$\cos\theta = \frac{x}{r} \qquad \sec\theta = \frac{r}{x}$$
$$\tan\theta = \frac{y}{x} \qquad \cot\theta = \frac{x}{y}$$



Reciprocal Identities

$$\sin\theta = \frac{1}{\csc\theta} \qquad \cos\theta = \frac{1}{\sec\theta} \qquad \tan\theta = \frac{1}{\cot\theta}$$
$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

Tangent and Cotangent Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1$$
$$1 + \tan^2\theta = \sec^2\theta \qquad 1 + \cot^2\theta = \csc^2\theta$$

Cofunction Identities

$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta \qquad \cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$$
$$\csc\left(\frac{\pi}{2} - \theta\right) = \sec\theta \qquad \tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta$$
$$\sec\left(\frac{\pi}{2} - \theta\right) = \csc\theta \qquad \cot\left(\frac{\pi}{2} - \theta\right) = \tan\theta$$

Reduction Formulas

$$\sin(-\theta) = -\sin\theta \qquad \cos(-\theta) = \cos\theta$$
$$\csc(-\theta) = -\csc\theta \qquad \tan(-\theta) = -\tan\theta$$
$$\sec(-\theta) = \sec\theta \qquad \cot(-\theta) = -\cot\theta$$

Sum and Difference Formulas

$$\sin(\theta \pm \phi) = \sin\theta\cos\phi \pm \cos\theta\sin\phi$$
$$\cos(\theta \pm \phi) = \cos\theta\cos\phi \mp \sin\theta\sin\phi$$
$$\tan(\theta \pm \phi) = \frac{\tan\theta \pm \tan\phi}{1 \mp \tan\theta\tan\phi}$$

Double-Angle Formulas

$$\sin 2\theta = 2\sin\theta\cos\theta$$
$$\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$$
$$\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$$

Power-Reducing Formulas

$$\sin^2\theta = \frac{1}{2}(1 - \cos 2\theta)$$
$$\cos^2\theta = \frac{1}{2}(1 + \cos 2\theta)$$
$$\tan^2\theta = \frac{1 - \cos 2\theta}{1 + \cos 2\theta}$$

Sum-to-Product Formula

$$\sin\theta + \sin\phi = 2\sin\left(\frac{\theta + \phi}{2}\right)\cos\left(\frac{\theta - \phi}{2}\right)$$
$$\sin\theta - \sin\phi = 2\cos\left(\frac{\theta + \phi}{2}\right)\sin\left(\frac{\theta - \phi}{2}\right)$$
$$\cos\theta + \cos\phi = 2\cos\left(\frac{\theta + \phi}{2}\right)\cos\left(\frac{\theta - \phi}{2}\right)$$
$$\cos\theta - \cos\phi = -2\sin\left(\frac{\theta + \phi}{2}\right)\sin\left(\frac{\theta - \phi}{2}\right)$$

Product-to-Sum Formulas

$$\sin\theta\sin\phi = \frac{1}{2}\left[\cos(\theta - \phi) - \cos(\theta + \phi)\right]$$
$$\cos\theta\cos\phi = \frac{1}{2}\left[\cos(\theta - \phi) + \cos(\theta + \phi)\right]$$
$$\sin\theta\cos\phi = \frac{1}{2}\left[\sin(\theta + \phi) + \sin(\theta - \phi)\right]$$
$$\cos\theta\sin\phi = \frac{1}{2}\left[\sin(\theta + \phi) - \sin(\theta - \phi)\right]$$

DERIVATIVES AND INTEGRALS

Basic Differentiation Rules

1. $\frac{d}{dx}[cu] = cu'$
2. $\frac{d}{dx}[u \pm v] = u' \pm v'$
3. $\frac{d}{dx}[uv] = uv' + u'v$
4. $\frac{d}{dx}\left[\frac{u}{v}\right] = \frac{u'v - uv'}{v^2}$
5. $\frac{d}{dx}[c] = 0$
6. $\frac{d}{dx}[u^n] = nu^{n-1}u'$
7. $\frac{d}{dx}[x] = 1$
8. $\frac{d}{dx}[|u|] = \frac{u}{|u|}(u'), \quad u \neq 0$
9. $\frac{d}{dx}[\ln u] = \frac{u'}{u}$
10. $\frac{d}{dx}[e^u] = e^u u'$
11. $\frac{d}{dx}[\log_a u] = \frac{u'}{(\ln a)u}$
12. $\frac{d}{dx}[a^u] = (\ln a)a^u u'$
13. $\frac{d}{dx}[\sin u] = (\cos u)u'$
14. $\frac{d}{dx}[\cos u] = -(\sin u)u'$
15. $\frac{d}{dx}[\tan u] = (\sec^2 u)u'$
16. $\frac{d}{dx}[\cot u] = -(\csc^2 u)u'$
17. $\frac{d}{dx}[\sec u] = (\sec u\ \tan u)u'$
18. $\frac{d}{dx}[\csc u] = -(\csc u\ \cot u)u'$
19. $\frac{d}{dx}[\arcsin u] = \frac{u'}{\sqrt{1-u^2}}$
20. $\frac{d}{dx}[\arccos u] = \frac{-u'}{\sqrt{1-u^2}}$
21. $\frac{d}{dx}[\arctan u] = \frac{u'}{1+u^2}$
22. $\frac{d}{dx}[\text{arccot}\, u] = \frac{-u'}{1+u^2}$
23. $\frac{d}{dx}[\text{arcsec}\, u] = \frac{u'}{|u|\sqrt{u^2-1}}$
24. $\frac{d}{dx}[\text{arccsc}\, u] = \frac{-u'}{|u|\sqrt{u^2-1}}$
25. $\frac{d}{dx}[\sinh u] = (\cosh u)u'$
26. $\frac{d}{dx}[\cosh u] = (\sinh u)u'$
27. $\frac{d}{dx}[\tanh u] = (\text{sech}^2\, u)u'$
28. $\frac{d}{dx}[\coth] = -(\text{csch}^2\, u)u'$
29. $\frac{d}{dx}[\text{sech}\, u] = -(\text{sech}\, u\ \tanh u)u'$
30. $\frac{d}{dx}[\text{csch}\, u] = -(\text{csch}\, u\ \coth u)u'$
31. $\frac{d}{dx}\left[\sinh^{-1} u\right] = \frac{u'}{\sqrt{u^2+1}}$
32. $\frac{d}{dx}\left[\cosh^{-1} u\right] = \frac{u'}{\sqrt{u^2-1}}$
33. $\frac{d}{dx}\left[\tanh^{-1} u\right] = \frac{u'}{1-u^2}$
34. $\frac{d}{dx}\left[\coth^{-1} u\right] = \frac{u'}{1-u^2}$
35. $\frac{d}{dx}\left[\text{sech}^{-1}\, u\right] = \frac{-u'}{u\sqrt{1-u^2}}$
36. $\frac{d}{dx}\left[\text{csch}^{-1}\, u\right] = \frac{-u'}{|u|\sqrt{1+u^2}}$

Basic Integration Formulas

1. $\int kf(u)\,du = k\int f(u)\,du$
2. $\int [f(u) \pm g(u)]\,du = \int f(u)\,du \pm \int g(u)\,du$
3. $\int du = u + C$
4. $\int a^u\,du = \left(\frac{1}{\ln a}\right)a^u + C$
5. $\int e^u\,du = e^u + C$
6. $\int \sin u\,du = -\cos u + C$
7. $\int \cos u\,du = \sin u + C$
8. $\int \tan u\,du = -\ln|\cos u| + C$
9. $\int \cot u\,du = \ln|\sin u| + C$
10. $\int \sec u\,du = \ln|\sec u + \tan u| + C$
11. $\int \csc u\,du = -\ln|\csc u + \cot u| + C$
12. $\int \sec^2\ du = \tan u + C$
13. $\int \csc^2 u\,du = -\cot u + C$
14. $\int \sec u \tan u\,du = \sec u + C$
15. $\int \csc u \cot u\,du = -\csc u + C$
16. $\int \frac{du}{\sqrt{a^2-u^2}} = \arcsin\left(\frac{u}{a}\right) + C$
17. $\int \frac{du}{a^2+u^2} = \frac{1}{a}\arctan\left(\frac{u}{a}\right) + C$
18. $\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\,\text{arcsec}\left(\frac{|u|}{a}\right) + C$