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Abstract

$$V = \{v_1, v_2, v_3, v_4\}$$
$$E = \{e_1, e_2, e_3\}$$
$$G = \{V, E\}$$

$$D = \begin{bmatrix} 3 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & & \\ 1 & & 0 & \\ 1 & & & 0 \end{bmatrix}$$

$$L = D - A = \begin{bmatrix} 3 & -1 & -1 & -1 \\ -1 & 1 & & \\ -1 & & 1 & \\ -1 & & & 1 \end{bmatrix}$$

$$|\lambda I - L| = 0$$
$$L\overrightarrow{u} = \lambda \overrightarrow{u}$$

$$L = U\Lambda U^T = \sum_1^N \lambda_k \overrightarrow{u_k} \cdot \overrightarrow{u_k}^T$$

$$\forall \overrightarrow{s} \in R^N$$
$$p_k = |\overrightarrow{s}| \cos\theta_k = \frac{\overrightarrow{u_k}^T \cdot \overrightarrow{s}}{|\overrightarrow{u_k}^T|} = \overrightarrow{u_k}^T \cdot \overrightarrow{s}$$

$$\overrightarrow{p} = U^T \overrightarrow{s}$$
$$\overrightarrow{s} = U \overrightarrow{p}$$

$$TV\left(\overrightarrow{s}\right) = \overrightarrow{s}^T L \overrightarrow{s} = \overrightarrow{s}^T U\Lambda U^T \overrightarrow{s}$$

$$= \left(U^T \overrightarrow{s}\right)^T \Lambda \left(U^T \overrightarrow{s}\right) = \overrightarrow{p}^T \Lambda \overrightarrow{p} = \sum_1^N p_k^2 \lambda_k \geq 0$$

$$E\left(\overrightarrow{s}\right)=|\overrightarrow{s}|^2=\left(U\overrightarrow{p}\right)^T\cdot\left(U\overrightarrow{p}\right)=\overrightarrow{p}^T\overrightarrow{p}=\sum_1^N p_k^2$$

$$\overrightarrow{s_{in}}\left(\sum_1^N p_k\overrightarrow{u_k}\right)\to H\to\overrightarrow{s_{out}}\left(\sum_1^N p_k^{'}\overrightarrow{u_k}\right)$$

$$\overrightarrow{s_{in}}\left(\sum_1^N p_k\overrightarrow{u_k}\right)\to H\left(filter\right)\to\overrightarrow{s_{out}}\left(\sum_1^N h\left(\lambda_k\right)p_k\overrightarrow{u_k}\right)$$

$$\overrightarrow{s_{out}}=H\overrightarrow{s_{in}}=\sum_1^N h\left(\lambda_k\right)p_k\overrightarrow{u_k}$$

$$=\begin{bmatrix}u_1 & u_2 & ... & u_n\end{bmatrix}\begin{bmatrix}h\left(\lambda_1\right)p_1\\ h\left(\lambda_2\right)p_2\\ ...\\ h\left(\lambda_n\right)p_n\end{bmatrix}$$

$$=U\begin{bmatrix}h\left(\lambda_1\right) & & & \\ & h\left(\lambda_2\right) & & \\ & & ... & \\ & & & h\left(\lambda_n\right)\end{bmatrix}\begin{bmatrix}p_1\\ p_2\\ ...\\ p_n\end{bmatrix}$$

$$=U\begin{bmatrix}h\left(\lambda_1\right) & & & \\ & h\left(\lambda_2\right) & & \\ & & ... & \\ & & & h\left(\lambda_n\right)\end{bmatrix}U^T\overrightarrow{s_{in}}$$

$$H=U\begin{bmatrix}h\left(\lambda_1\right) & & & \\ & h\left(\lambda_2\right) & & \\ & & ... & \\ & & & h\left(\lambda_n\right)\end{bmatrix}U^T=U\Lambda_h U^T$$

$$\Lambda_h=\lim_{K\to\infty}\sum_0^K h_k\Lambda^k$$

$$K\ll N$$

$$H=U\left(h_0\Lambda^0+h_1\Lambda^1+h_2\Lambda^2+...+h_K\Lambda^K\right)U^T$$

$$=U\left(h_0\Lambda^0\right)U^T+U\left(h_1\Lambda^1\right)U^T+U\left(h_2\Lambda^2\right)U^T+...+U\left(h_K\Lambda^K\right)U^T$$

$$=h_0L^0+h_1L^1+h_2L^2+...+h_KL^K=\sum_0^K h_kL^k$$

$$K=1$$
$$H=h_0L^0+h_1L^1$$

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