# Identifying Causal Effects in Experiments with Social Interactions and Non-compliance

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### Empirical Example with Potential for Indirect Treatment Effects

Crepon et al. (2013; QJE)

- Large-scale job-seeker assistance program in France.
- Randomized offers of intensive job placement services.

#### Displacement Effects of Labor Market Policies

"Job seekers who benefit from counseling may be more likely to get a job, but at the expense of other unemployed workers with whom they compete in the labor market.

This may be particularly true in the short run, during which vacancies do not adjust: the unemployed who do not benefit from the program could be partially crowded out."

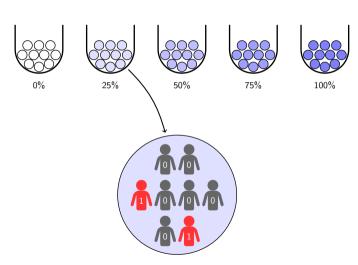
# Studying Social Interactions Without Network Data

#### Partial Interference

Spillovers within but not between groups.

#### Randomized Saturation

Two-stage experimental design.



# This Paper: Non-compliance in Randomized Saturation Experiments

#### Identification

Beyond ITT: Average direct/indirect causal effects under 1-sided non-compliance.

#### Estimation

Simple, asymptotically normal estimator under large/many-group asymptotics.

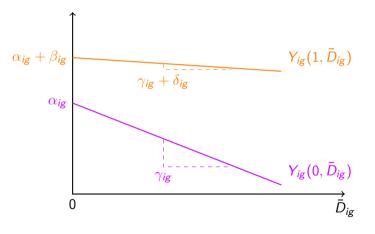
### **Application**

French labor market experiment: Crepon et al. (2013; QJE)

# Overview of Assumptions

- (i) Experimental Design: Randomized Saturation ✓
- (ii) Exclusion Restriction for  $(Z_{ig}, S_g)$
- (iii) Treatment Take-up:  $\mathbf{1}(\mathsf{Take}\ \mathsf{Treatment}) = \mathbf{1}(\mathsf{Offered}) imes \mathbf{1}(\mathsf{Complier})$
- (iv) Potential Outcomes: Correlated Random Coefficients Model

$$Y_{ig}(D_{ig}, \bar{D}_{ig}) = \alpha_{ig} + \beta_{ig}D_{ig} + \gamma_{ig}\bar{D}_{ig} + \delta_{ig}D_{ig}\bar{D}_{ig}$$



#### Indirect Effects

 $Y_{ig}(1, \bar{D}_{ig})$  Treated:  $\gamma_{ig} + \delta_{ig}$ 

Untreated:  $\gamma_{\it ig}$ 

#### Direct Effects

$$eta_{ig} + \delta_{ig}ar{D}_{ig}$$

# Näive IV Does Not Identify The Effects of Interest

#### Unoffered Individuals

$$Y_{ig} = \alpha_{ig} + \beta_{ig} \overline{D_{ig}} + \gamma_{ig} \overline{D}_{ig} + \underline{\delta}_{ig} \overline{D_{ig}}$$

$$= \mathbb{E}[\alpha_{ig}] + \mathbb{E}[\gamma_{ig}] \overline{D}_{ig} + (\alpha_{ig} - \mathbb{E}[\alpha_{ig}]) + (\gamma_{ig} - \mathbb{E}[\gamma_{ig}]) \overline{D}_{ig}$$

$$= \alpha + \gamma \overline{D}_{ig} + \varepsilon_{ig}$$

#### **IV** Estimand

$$\gamma_{IV} = \frac{\mathsf{Cov}(Y_{ig}, S_g)}{\mathsf{Cov}(\bar{D}_{ig}, S_g)} = \gamma + \frac{\mathsf{Cov}(\varepsilon_{ig}, S_g)}{\mathsf{Cov}(\bar{D}_{ig}, S_g)} = \ldots = \gamma + \frac{\mathsf{Cov}(\gamma_{ig}, \bar{C}_{ig})}{\mathbb{E}(\bar{C}_{ig})}$$

# Identification – Unoffered Individuals: $Y_{ig}(0, \bar{D}_{ig}) = \alpha_{ig} + \gamma_{ig}\bar{D}_{ig}$

#### **Theorem**

 $(Z_{ig}, S_g, \bar{D}_{ig})$  are independent of the unobserved heterogeneity given  $(\bar{C}_{ig}, N_g)$ .

$$\mathbb{E}\left\{\begin{bmatrix}1\\\bar{D}_{ig}\end{bmatrix}Y_{ig}\middle|\left(Z_{ig}=0,\bar{C}_{ig},N_{g}\right)\right\} = \mathbb{E}\left\{\begin{bmatrix}1&\bar{D}_{ig}\\\bar{D}_{ig}&\bar{D}_{ig}^{2}\end{bmatrix}\begin{bmatrix}\alpha_{ig}\\\gamma_{ig}\end{bmatrix}\middle|\left(Z_{ig}=0,\bar{C}_{ig},N_{g}\right)\right\}$$

$$= \mathbb{E}\left\{\begin{bmatrix}1&\bar{D}_{ig}\\\bar{D}_{ig}&\bar{D}_{ig}^{2}\end{bmatrix}\middle|\left(Z_{ig}=0,\bar{C}_{ig},N_{g}\right)\right\}\mathbb{E}\left\{\begin{bmatrix}\alpha_{ig}\\\gamma_{ig}\end{bmatrix}\middle|\left(\bar{C}_{ig},N_{g}\right)\right\}$$

#### "Localized" Average Coefficients

$$\mathbb{E}\left\{egin{aligned} \left[lpha_{i\mathsf{g}}
ight]\left(ar{C}_{i\mathsf{g}}, \mathsf{N}_{\mathsf{g}}
ight)
ight\} &= \mathbb{E}\left\{egin{bmatrix} 1 & ar{D}_{i\mathsf{g}} \ ar{D}_{i\mathsf{g}} & ar{D}_{i\mathsf{g}}^2 \end{bmatrix} \middle| \left(\mathsf{Z}_{i\mathsf{g}}=0, ar{C}_{i\mathsf{g}}, \mathsf{N}_{\mathsf{g}}
ight)
ight\}^{-1} \mathbb{E}\left\{egin{bmatrix} 1 \ ar{D}_{i\mathsf{g}} \end{bmatrix} \mathsf{Y}_{i\mathsf{g}} \middle| \left(\mathsf{Z}_{i\mathsf{g}}=0, ar{C}_{i\mathsf{g}}, \mathsf{N}_{\mathsf{g}}
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# Identification – Unoffered Individuals: $Y_{ig}(0, \bar{D}_{ig}) = \alpha_{ig} + \gamma_{ig}\bar{D}_{ig}$

Iterated Expectations over  $(\bar{C}_{ig}, N_g)$ 

$$\mathbb{E}egin{bmatrix} lpha_{ig} \ \gamma_{ig} \end{bmatrix} = \mathbb{E}\left\{ egin{bmatrix} \left[\mathbf{Q}_0(ar{C}_{ig}, m{N}_g)
ight]^{-1} egin{bmatrix} 1 \ ar{D}_{ig} \end{bmatrix} Y_{ig} \ m{Z}_{ig} = 0 
ight\}, & \mathbf{Q}_0(ar{C}_{ig}, m{N}_g) \equiv \mathbb{E}\left\{ egin{bmatrix} 1 & ar{D}_{ig} \ ar{D}_{ig} & ar{D}_{ig}^2 \end{bmatrix} \ m{Z}_{ig} = 0, ar{C}_{ig}, m{N}_g \end{pmatrix} 
ight\}$$

#### $\mathbf{Q}_0$ is *Known*

Distribution of  $\bar{D}_{ig}|(\bar{C}_{ig},N_g)$  determined by experimental design.

#### Feasible Estimation

Need  $\widehat{C}_{ig} \rightarrow_p \overline{C}_{ig}$ . Hence: large/many–group asymptotics.

# We Identify these Average Causal Effects:

#### Spillover

 $ar{D}_{ig} 
ightarrow Y_{ig}$  for the population, holding  $D_{ig} = 0$ .

#### Treatment on the Treated

 $D_{ig} 
ightarrow Y_{ig}$  for compliers as a function of  $\bar{D}_{ig}$ .

#### Spillover on the Treated

 $ar{D}_{ig} 
ightarrow Y_{ig}$  for compliers holding  $D_{ig} = 0$  or  $D_{ig} = 1$ .

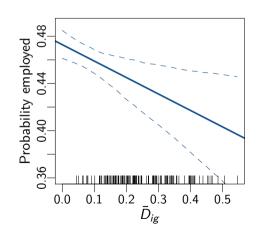
#### Spillover on the Untreated

 $ar{D}_{ig} 
ightarrow Y_{ig}$  for never-takers holding  $D_{ig} = 0$ .

# Average Spillover to Long-term Employment: $Y_{ig}(0, \bar{D}_{ig}) = \alpha_{ig} + \gamma_{ig}\bar{D}_{ig}$

Data from Crepon et al. (2013; QJE)

	$\mathbb{E}(lpha_{\mathit{ig}})$	$\mathbb{E}(\gamma_{ig})$
Our estimator	0.47	-0.14
	(0.01)	(0.07)
Naïve IV	0.47	-0.06
	(0.01)	(0.06)



#### Conclusion

#### Identification

Go beyond ITTs to identify average direct and indirect effects in randomized saturation experiments with 1-sided non-compliance.

#### Estimation

Simple asymptotically normal estimator under large/many-group asymptotics.

#### **Application**

Substantial labor market spillovers in Crepon et al. (2013; QJE) experiment.

# Appendix - Notation

### Sample Size and Indexing

- ightharpoonup Groups:  $g = 1, \dots, G$
- ▶ Individuals in g:  $i = 1, ..., N_g$

#### **Observables**

- $ightharpoonup Z_{ig} = \text{binary treatment offer to } (i,g)$
- $ightharpoonup D_{ig} = \text{binary treatment take-up of } (i,g)$
- $ightharpoonup Y_{ig} = \text{outcome of } (i,g)$
- $ightharpoonup S_g = \text{saturation of group } g$
- $ightharpoonup ar{D}_{ig} = ext{take-up fraction in } g ext{ excluding } (i,g)$

# Appendix - Correlated Random Coefficients Model

$$Y_{ig}(\mathbf{\mathcal{D}}) = Y_{ig}(\mathbf{\mathcal{D}}_g) = Y_{ig}(D_{ig}, \bar{D}_{ig}) = \mathbf{f}(\bar{D}_{ig})' \left[ (1 - D_{ig})\boldsymbol{\theta}_{ig} + D_{ig}\boldsymbol{\psi}_{ig} \right]$$

- ightharpoonup f is a vector of known functions, bounded on [0,1]
- $lackbox{ } heta_{ig}$  and  $\psi_{ig}$  are RVs that may be dependent on  $(D_{ig}, ar{D}_{ig})$ .

#### This Talk

$$Y_{ig}(D_{ig}, \bar{D}_{ig}) = \alpha_{ig} + \beta_{ig}D_{ig} + \gamma_{ig}\bar{D}_{ig} + \delta_{ig}D_{ig}\bar{D}_{ig}$$

# Appendix – Assumptions on Treatment Take-up

#### One-sided Non-compliance

Only those offered treatment can take it up.

Individualistic Offer Response (IOR)

$$D_{ig}(\boldsymbol{Z}) = D_{ig}(\boldsymbol{Z}_g) = D_{ig}(Z_{ig}, \bar{Z}_{ig}) = D_{ig}(Z_{ig})$$

#### Notation

 $C_{ig} = 1$  iff (i, g) is a complier;  $\bar{C}_{ig} \equiv$  share of compliers among (i, g)'s neighbors.

$$(\mathsf{IOR}) + (\mathsf{1}\text{-Sided}) \Rightarrow D_{ig} = C_{ig} \times Z_{ig}$$

# Appendix - Exclusion Restriction

#### Notation

- $ightharpoonup \mathbf{B}_g = \text{random coefficients for everyone in group } g.$
- $ightharpoonup oldsymbol{\mathcal{C}}_g = ext{complier indicators for everyone in group } g$
- $ig> m{Z}_g = ext{treatment offers for everyone in group } g$

#### **Exclusion Restriction**

- (i)  $S_g \perp \!\!\! \perp (\boldsymbol{C}_g, \mathbf{B}_g, N_g)$
- (ii)  $Z_g \perp \!\!\! \perp (C_g, B_g) | (S_g, N_g)$

#### Theorem 1

Under the randomized saturation design, IOR assumption, and exclusion restriction,  $(Z_{ig}, \bar{D}_{ig}, S_g) \perp \!\!\! \perp (\mathbf{B}_{ig}, C_{ig}) | (\bar{C}_{ig}, N_g).$ 

# Appendix - Testable Implications of IOR in Crepon et al. (2013; QJE)

#### Recall: IOR Assumption

Person (i,g)'s take-up  $D_{ig}$  depends only on her own treatment offer  $Z_{ig}$ .

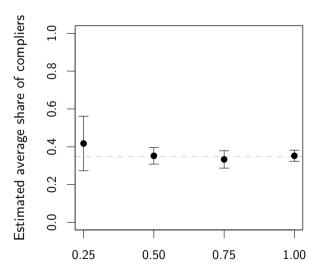
#### A "Regression-based" Test

 $\mathbb{E}[D_{ig}|Z_{ig}=1,S_g=s]$  should not depend on s.

#### **Implementation**

- ightharpoonup Regress  $D_{ig}$  on an intercept and saturation dummies for those offered treatment.
- Test the joint null that the coefficients on all saturation dummies are zero.
- ► (Equivalently test that share of compliers is the same across saturation "bins")
- ► Crepon Experiment: p-value of 0.62

# Appendix – Testable Implications of IOR



# Appendix - Identification: Rank Condition

$$Y_{ig}(D_{ig}, \bar{D}_{ig}) = \alpha_{ig} + \beta_{ig}D_{ig} + \gamma_{ig}\bar{D}_{ig} + \delta_{ig}D_{ig}\bar{D}_{ig}$$

#### **Notation**

$$\mathbf{Q}_z(ar{c},n) \equiv \mathbb{E} \left\{ egin{bmatrix} 1 & ar{D}_{ig} \ ar{D}_{ig} & ar{D}_{ig}^2 \end{bmatrix} \middle| (Z_{ig} = z, ar{C}_{ig} = ar{c}, N_g = n) 
ight\}$$

#### Rank Condition

 $\mathbf{Q}_0(\bar{c},n)$  and  $\mathbf{Q}_1(\bar{c},n)$  are invertible for all  $(\bar{c},n)$  in the support of  $(\bar{C}_{ig},N_g)$ .

#### Bernoulli Offers

$$\mathbf{Q}_0(\bar{c},n) = \frac{1}{\mathbb{E}(1-S_g)} \begin{bmatrix} \mathbb{E}\left\{1-S_g\right\} & \bar{c} \mathbb{E}\left\{S_g(1-S_g)\right\} \\ \bar{c} \mathbb{E}\left\{S_g(1-S_g)\right\} & \bar{c}^2 \mathbb{E}\left\{S_g^2(1-S_g)\right\} + \frac{\bar{c}}{n-1} \mathbb{E}\left\{S_g(1-S_g)^2\right\} \end{bmatrix}$$