Random walks in cones

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Outline

- Definition
  Let be i.i.d. random variables on . Let , . is a random walk.
  Fix a point , is a random walk starting at .
- The cone
  Consider .
- Exit time
  Let and let be a mapping from to . is a stopping time if the event is in .
  For , we study the stopping time .

- Definition
  Let be mappings on such that
  - (Independent increments) If then are independent.
  - (Normally distributed) For all , .
  - (Continuous path) With probability 1, the mapping is continuous.

  One can also view a Brownian motion as a mapping from to -the space of continuous functions on .
- Exit time
  The corresponding exit time is
- The Donsker's theorem

The Donsker's theorem

Let be i.i.d. random variables from a probability space to whose mean is and variance . Set and

For any , we denote by the random variable with values in defined by

where . Then the law of converges weakly to that of a Brownian motion having a.e.

- Previous approach
  The Wiener Hopf factorization approach
- New approach
  Constructing a harmonic function for random walk and coupling with Brownian motion

The approach for Brownian motion

- Consider harmonic function on , vanishing on the boundary

- Results

Proposition

There exists a constant such that for every and ,

The approach for Brownian motion

- Results

Proposition

There exists a constant such that for every sequence converging to 0,

The approach for random walk

- Construct a harmonic function for the random walk
- Coupling with Brownian motions

A function defined on is harmonic before for the random walk if for every in , one has

Define

Properties

There exists an such that for every , the function is well defined, finite, strictly positive inside the cone and harmonic for the random walk and

The Sakhanenko's theorem

Assume that there exists an such that . Let . There exists a Brownian motion such that

where .

This provides a good control for the convergence in the invariant principle.

Asymptotics for exit time

Proposition

For all

Integral limit theorem

Proposition

For any , the distribution for weakly converges to the distribution with the density .

Thank you for your attention