RAICES

1. Bisección $\quad P_m = \frac{(a+b)}{2} \quad \log_2\left(\frac{b-a}{e}\right) = m$

2. Punto fijo $\quad g(x) = x - \frac{f(x)}{f'(x)} \quad , P_m = g(P_{m-1})$

   $(f(x) = x - g(x))$

3. Newton Raphson $\quad P_m = P_{m-1} - \frac{f(P_{m-1})}{f'(P_{m-1})}$

4. Newton Raphson modificado $\quad P_{m+1} = P_m - \frac{f(P_m) \cdot f'(P_m)}{f'(P_m)^2 - f(P_m) \cdot f''(P_m)}$

5. secante $\quad p_m = P_{m-1} - \frac{f(P_{m-1}) \cdot (P_{m-1} - P_{m-2})}{f(P_{m-1}) - f(P_{m-2})}$

Interpolacion

1. Lagrange $\quad P(x) = \sum_{k=0}^{n} \left(f(x_k) \left[\prod_{\substack{i=0 \\ i \neq k}}^{n} \frac{(x - x_i)}{(x_k - x_i)} \right] \right)$

2. Newton $\quad P(x) = f(x_0) + DD_1 \cdot (x - x_0) + \dots$

3. hermite

4. Spline
   - $s_j(x_j) = f(x_j) \rightarrow$ reemplazo en x_i
   - igualo borde
   - igualo borde derivadas 1 y 2

   $\}$ en x_1 (el medio)
   - frontera
     - libre $\quad S''(x_0) = S''(x_n)$
     - ligada $\quad S'(x_0) = f'(x_0) \, , \, S'(x_n) = f'(x_n)$
     - natural $\quad S''(x_0) = S''(x_n) = 0$

Sistemas lineales

① Directos $Ax=b$

- cuadrados mínimos

$A^tAx=A^tb$

- exponencial $y=a^{bx}$ → ln y, x
- potencial $y=ax^b$ → ln y, ln(x)
- polinomial $y=a_0+a_1x+\dots+a_mx^n$

- LU

$A=LU$, $L=(\Delta^0)$, $U=({}_0\nabla)$

$LU(x)=b$ → $Ux=y$, $Ly=b$

② iterativos $x^k=Tx^{k-1}+c$, $A=D-L-U$

- jacobi $T_J=D^{-1}(L+U)$ $c_J=D^{-1}b$
- gauss seidel $T_g=(D-L)^{-1}U$ $c_g=(D-L)^{-1}b$
- refinamiento $K(A)=\|A\|\|A^{-1}\|=10^t\cdot\frac{\|y\|_\infty}{\|x\|_\infty}$

$r=b-A\hat{x}$, $A\tilde{y}=r$ ⇒ $x=\hat{x}+\tilde{y}$

$$e=\frac{\|x^n-x^{n-1}\|_\infty}{\|x^n\|_\infty}$$

$\rho(T)=\max\lambda_i<1$ ⇒ sol. única

Sistemas no lineales

- Newton $J(x^{k-1})y^{k-1}=-F(x^{k-1})$

$x^k=x^{k-1}+y^{k-1}$

$G(x)=x-J^{-1}(x)F(x)$

Diferenciación

- numérica

① hacia adelante $f'(x_2)=\frac{f(x_3)-f(x_2)}{h}$ ⇒ $f''(x_2)=\frac{f(x_4)-2f(x_3)+f(x_2)}{h^2}$

② hacia atrás $f'(x_2)=\frac{f(x_2)-f(x_1)}{h}$ ⇒ $f''(x_2)=\frac{f(x_2)-2f(x_1)+f(x_0)}{h^2}$

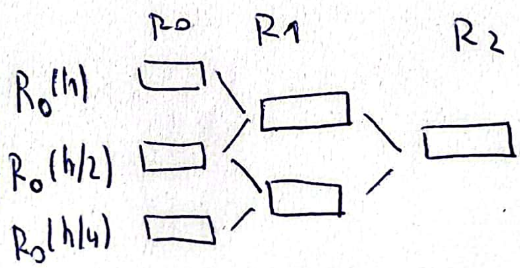
③ centrada $f'(x_2)=\frac{f(x_3)-f(x_1)}{2h}$ ⇒ $f''(x_2)=\frac{f(x_4)-2f(x_2)+f(x_0)}{4h^2}$

- Richardson

$R^0(h)=\frac{f(x+h)-f(x-h)}{2h}$

$R_1(h)=\frac{4\cdot R_0(\frac{h}{2})-R_0(h)}{3}$

$R_2(h)=16\cdot R_1(h/4)-R_1(h/2)$



Integración

- trapecios
$$\int_a^b f(x)\,dx = \frac{h}{2}\left[f(a)+f(b)+2\sum_{k=1}^{m-1} f(x_k)\right] \qquad O(h^2)$$
$$h=\frac{b-a}{N}$$

- simpson 1/3
$$\int_a^b f(x)\,dx = \frac{h}{3}\left[f(a)+f(b)+4\sum f(x_{2k+1})+2\sum f(x_{2k})\right] \qquad O(h^4)$$

- Romberg
$$h_i=\frac{b-a}{2^{i-1}} \qquad N=2^{k-1}-1$$
$$R_{11}=\frac{b-a}{2}$$
$$R_{21}=\frac{1}{2}\cdot\left[R_{11}+h_1\sum_{k=1}^{1} f\left(a+\frac{2k-1}{2}h_1\right)\right]$$
$$R_{31}=\frac{1}{2}\left[R_{21}+h_2\sum_{k=1}^{2} f\left(a+\frac{2k-1}{2}h_2\right)\right]$$

$$\begin{array}{ccc}
R_{i1} & R_{i2} & R_{i3} \\
R_{11} & & \\
 & \boxed{R_{22}} & \\
R_{21} & & \boxed{R_{33}} \\
 & \boxed{R_{32}} & \\
R_{31} & &
\end{array}$$

EDOS

① PVI
$$\begin{cases} y'(t)=f(t,y) \\ y(a)=y_0 \end{cases} \qquad \left|\frac{dy}{dt}\right| < K \rightarrow \text{sol. única}$$

- euler $\quad y_{i+1}=y_i+h f(t_i,y_i)$
- RK $\quad y_{i+1}=y_i+K_2 h \quad , \; K_1=f(t_i,y_i)$
$$K_2=f\left(t_i+\frac{h}{2},\, y_i+\frac{h}{2}\cdot K_1\right)$$

② PVI 2do orden
$$\begin{cases} y''=f(x,y,y') \\ y(x_0)=y_0 \\ y'(x_0)=u_0 \end{cases} \Rightarrow \begin{cases} y'=u \\ u'=f(x,y,u) \\ y(x_0)=y_0 \\ u(x_0)=u_0 \end{cases}$$

- euler
$$\begin{pmatrix} y_{n+1} \\ u_{n+1} \end{pmatrix}=\begin{pmatrix} y_n \\ u_n \end{pmatrix}+h\begin{pmatrix} u_n \\ f(x_n,y_n,u_n) \end{pmatrix}$$

- RK2
$$\begin{pmatrix} y_{n+1} \\ u_{n+1} \end{pmatrix}=\begin{pmatrix} y_n \\ u_n \end{pmatrix}+h\begin{pmatrix} m_2 \\ K_2 \end{pmatrix}, \quad K_1=f(t,u,y)$$
$$K_2=f\left(t+\frac{h}{2},\, u+\frac{h}{2}K_1,\, y+\frac{h}{2}m_1\right)$$
$$m_1=u_n$$
$$m_2=u_n+\frac{h}{2}K_1.$$

③ PVF $\begin{cases} y'' = f(x, y, y') \\ y(a) = \alpha \\ y(b) = \beta \end{cases}$

- diferencias finitas

$$y'' + P(x)y' + Q(x)y = f(x)$$

$$h^2 f_i = \left(1 + \frac{h}{2} P_i\right) y_{i+1} + \left(-2 + h^2 Q_i\right) y_i + \left(1 - \frac{h}{2} P_i\right) y_{i-1}$$

CamScanner