UTILITY COMPENSATION IN THE PURE THEORY OF INTERNATIONAL TRADE: AN EMPIRICALLY-ORIENTED GENERALISATION

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Abstract

One of the cornerstones of the modern welfare approach to international trade theory is the compensation principle due to Samuelson. The difficulty with the compensation principle is that it pays scant attention to the way in which the equilibrium of the system changes when compensation is actually paid. This paper is concerned with the efficient location of the set of utility-compensated equilibria; that is, the set of equilibria that satisfies the condition that compensation be paid such that no-one is worse off following a particular event or policy change. Using the gains from trade as an example, Dixon's theory of joint maximisation is shown to be an ideal tool, in an empirically-implementable form, for problems of this type.

I INTRODUCTION

One of the cornerstones of the modern welfare approach to international trade theory is the compensation principle. The gains from trade, the existence of an optimal tariff, and the benefits from regional integration are just a few of the many widely accepted propositions that hinge crucially on the assertion that in the movement to such situations, those who gain could compensate those who lose and still be better off. The objective of this paper is to reformulate the concept of a utility-compensated equilibrium in a form that is appropriate for empirical research and to indicate the types of questions that could be examined within this framework.

The framework of the reformulation is Dixon's (1975) theory of joint maximisation. The paper takes the form of an exercise aimed at showing the usefulness of Dixon's model for the analysis of questions involving utility compensation. The exercise is a proof of the gains from trade. The tradition proof due to Samuelson (1939 and 1962), and extended by Kemp (1962) and Kenen (1959), is based on the expansion of the national consumption possibilities frontier under free trade. A free-trade equilibrium is regarded as superior to an autarkic equilibrium if the gainers could compensate the losers and still be better off. The emphasis in the traditional proof is on the word "could" since only minor attention is paid to how the equilibrium and the gains alter when compensation is actually paid. Formal proofs of the gains from trade, allowing for the actual payment of compensation, were provided by Chipman and Moore (1972) and Grandmot and McFadden (1972). As in those papers, the alternative proof presented in this paper has the attractive feature that the payment of compensation is fundamental to the nature of the equilibrium. Unlike those proofs, however, the alternative based on joint maximisation has the advantage that it allows an economic inter-

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pretation of the processes involved; an interpretation that does not get lost in the forest of excessively technical mathematics.

The empirical relevance of the model stems from the dependence of the solution on the distribution of gains and on the fact that models of this type, which require explicit formulation of utility and production functions, have become computationally feasible with the advances made in the past decade in computing general equilibria.

Section II reviews the traditional arguments for utility compensation and the gains from trade. Section III introduces the generalisation in terms of the joint maximisation model and Section IV discusses how this framework can be used as an efficient approach to a number of interesting questions in international trade theory.

II THE COMPENSATION PRINCIPLE AND THE GAINS FROM TRADE

In his classic pair of papers, Samuelson established the gains from free trade using the criterion that a movement from one situation to another is beneficial if those who gain from the move could compensate those who lose and still be better off. The precise meaning of this statement was made clear in the second of the two papers using the concept of the utility possibilities frontier.

Figure I illustrates the small country case. I(a) shows the consumption possibilities frontier under autarky, PQ, and under free trade, EF. Assuming for simplicity that there are only two individuals, A and B, in the domestic economy we can define a utility possibilities frontier associated with each of the production possibilities frontiers. These are shown as pq for autarky, and ef for free trade in Figure I(b). These loci are constructed by first defining the utility frontier associated with each fixed consumption bundle on the consumption frontiers and then forming the envelope of the respective points. The locus ef lies outside pq since EF lies outside PQ.

The autarkic equilibrium is a single point on PQ, such as D, and the free trade equilibrium is a single point on EF, such as V. Similarly, these equilibria correspond to the unique points on pq and ef, d and v respectively. Samuelson noted correctly that d and v are not directly comparable. Redistribution of the fixed bundle of goods at V could well generate a utility locus, such as v'v", that lies below the point d. Samuelson asserted that the potential gain from trade lay in the fact that ef lies nowhere below pq. He argued that if compensation was actually paid, the equilibrium would move from v south-east along ef until it reached the unamiguously preferred zone st.

In fact, as noted by the literature,[1] his proof is a little loose. The problem is that not all points on ef or pq are attainable as competitive equilibria, regardless of the amount of compensation. The limiting case is where the attainable equilibria consist of single points on each frontier. This would occur, for example, if A and B had identical homothetic preferences. In this case, d and v would be directly comparable, since compensation would only involve redistributing the fixed bundles D and V. For given preferences, the attainable utility possibilities frontiers will be determined by the attainable real trade equilibria.

Suppose that by considering all possible income redistributions the range of

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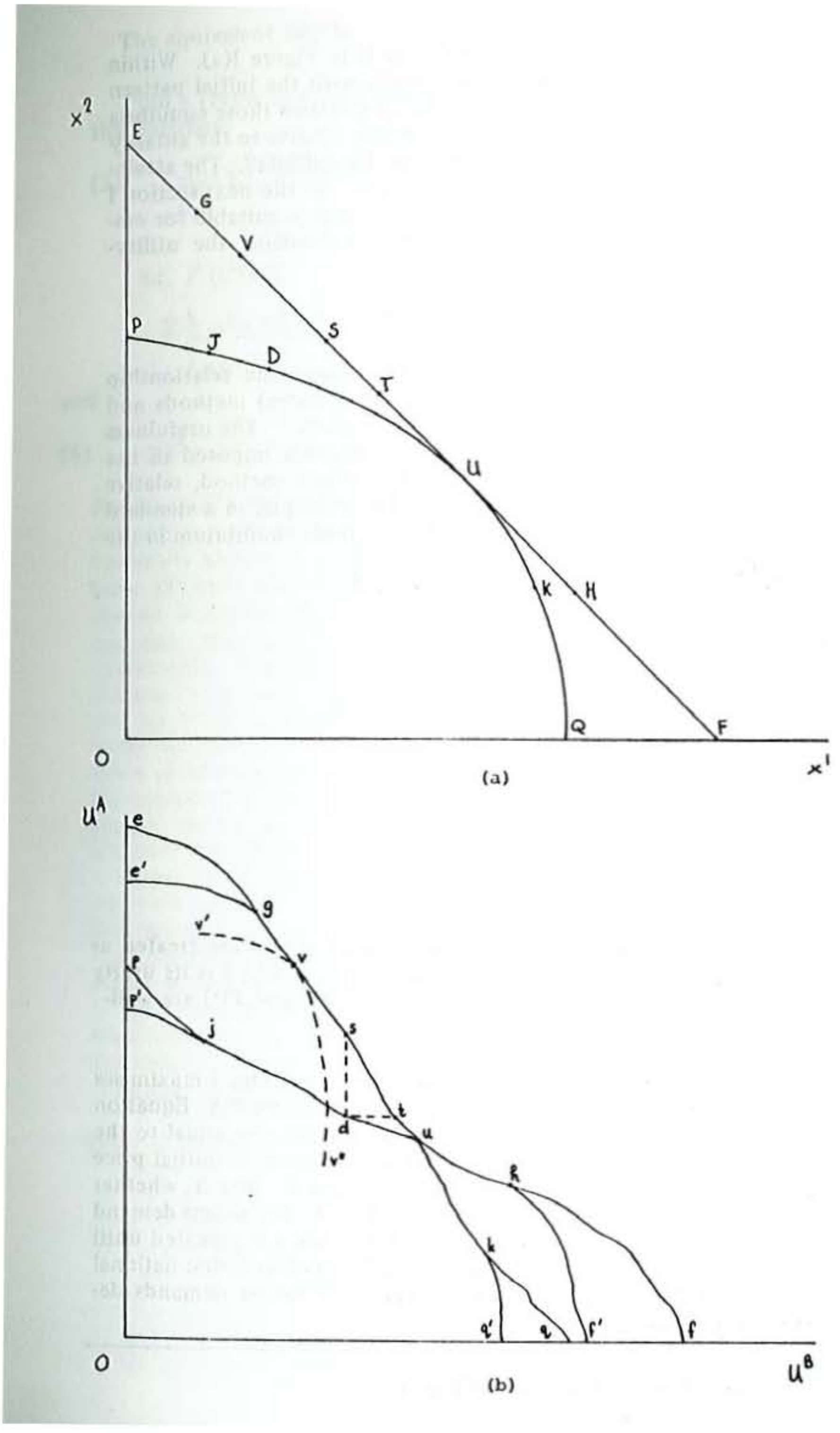
1. See for example the papers by Chipman and Moore, and Grandmont and McFadden, and the treatment of the gains from trade in Chacholiades (1973).


x^2
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(b)

attainable trade equilibria are the points between G and H in Figure I(a). Within this set, V is the equilibrium corresponding to free trade with the initial pattern of factor ownership and no compensation. The region ST contains those equilibria in which compensation is paid such that noone loses utility relative to the autarky point D. I will call the points in ST "utility-compensated equilibria". The attainable utility possibilities frontier corresponding to GH is e'f'. In the next section I will present a general proof of this proposition in a form that is suitable for empirical work and which is efficient for locating and characterising the utility-compensated equilibria.

III A GENERAL PROOF USING JOINT MAXIMISATION

Underlying the theory of joint maximisation is the one-to-one relationship between general equilibria located by price search (or offer curve) methods and those located by searching the utility possibilities frontier directly.[2] The usefulness of this observation is reflected in the nature of the constraints imposed in the joint maximisation problem (JMP) based on the utility search method, relative to those imposed in the offer curve problem (OCP). For example, in a standard trade model, if there are k countries and n goods, the free-trade equilibrium in the OCP is found by solving the problem:

Find a price vector $p' = (p_1, \ldots p_n)$ that satisifes:

$$(1) \quad \max_{x,z} U^i (x^i) \qquad , i = 1, \ldots k$$

$$\text{s.t.} \; p'x^i \leqslant p'z^i \qquad , i = 1, \ldots k$$

$$f^i (z^i) \leqslant 0 \qquad , i = 1, \ldots k$$

$$x^i \geqslant 0 \qquad , i = 1 \ \ldots k$$

and

$$(2) \qquad \Sigma_{i=1}^{k} z^i - \Sigma_{i=1}^{k} x^i \leqslant 0$$

where $z^i = (z_1^i, \ldots, z_n^i)$ is country i's production vector (factors are treated as negative outputs), $x^i = (x_1^i, \ldots x_n^i)$ is its consumption vector, U^i (•) is its utility function, f^i(•) is its production possibilities frontier, and U^i(•) and f^i(•) are well-behaved in terms of the usual concavity conditions.

Equation (1) states that for a given price vector p, each country i maximises its agregate utility index subject to its budget and production constraints. Equation (2) states that the world demands generated by (1) are less than or equal to the generated world supplies. The price search method is to choose an initial price vector, p, solve (1) and then ask whether or not (2) is satisfied; that is, whether all excess demands are zero. If not, p is adjusted according to the excess demand pattern implied by (2) and then (1) is solved again. The process is repeated until (2) is satisfied. The important feature of the price search problem is that national budget constraints are satisfied at all times, while aggregate excess demands determine the adjustment path to equilibrium.

2. This equivalence is established in Dixon (1975) ch. 1.

The equivalent JMP is:

Find a vector of utility weights $\lambda' = (\lambda^1, \ldots \lambda^k)$ where $\Sigma_{i=1}^{k}\lambda^i \equiv 1$ that satisfies:

(3) $$\max_{x,z} \Sigma_{i=1}^{k}\lambda^i U^i(x^i)$$

$$\text{s.t. } f^i(z^i) \leqslant 0 \qquad , i = 1, \ldots k$$

$$\Sigma_{i=1}^{k} x^i \leqslant \Sigma_{i=1}^{k} z^i$$

$$x^i \geqslant 0 \qquad , i = 1, \ldots k$$

and

(4) $$p'z^i - p'x^i = 0$$

Equation (3) states that for a given distribution of utilities, λ, production is optimal and world demands equal world supplies. Equation (4) states that each country's budget is satisfied.[3] The search method for the JMP is to choose a λ, solve (3) then ask whether or not (4) is satisfied. If (4) is not satisfied, then the chosen λ implies an equilibrium with a particular set of inter-country income transfers; that is, $p'z - p'x$ is country i's trade balance. To find the free-trade equilibrium, λ is adjusted in accordance with the implied trade balances until (3) and (4) are simultaneously satisfied. The important feature of the JMP search process is that equality of world supplies and demands is imposed at all stages while the national budgets are not. In a sense, the OCP and JMP are dual to each other in terms of their disequilibrium behaviour. Between the two, suitable tools are provided to analyse disequilibrium situations of either the excess demand of transfer type. As is it the transfer situation that is involved in the compensation problem, the JMP is the relevant tool.

Returning to the multi-dimensional compensation problem, assume that there are k + 1 agents in all; k individuals in the domestic economy and one representing the rest of the world (to be represented by the superscript w). The objective of this section is to show that there exists a trading equilibrium in which every one of the k domestic agents attains a utility level U^i which is greater than or equal to some pre-specified level H^i (here taken to be the autarkic level). This equilibrium can be found by using what Dixon calls the JMP-2. This consists of solving the following modified JMP problem:

Find a vector of minimum utility levels $H' = (H^1, \ldots H^k)$ that satisfies:

(5) $$\max_{x,z} U^w(x^w)$$

$$\text{s.t. } f^w(z^w) \leqslant 0$$

$$f^d(z^d) \leqslant 0$$

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3. The prices for equation (4) are provided by the Lagrange multipliers from the constraint: $\Sigma_{i=1}^{k} x^i = \Sigma_{i=1}^{k} z^i$.

$$U^i(x^i) \geqslant H^i \qquad , i = 1, \ldots k$$

$$\Sigma_{i=1}^{k+1} z^i - \Sigma_{i=1}^{k+1} x^i \geqslant 0$$

$$x^i \geqslant 0 \qquad , i = 1, \ldots k+1$$

and

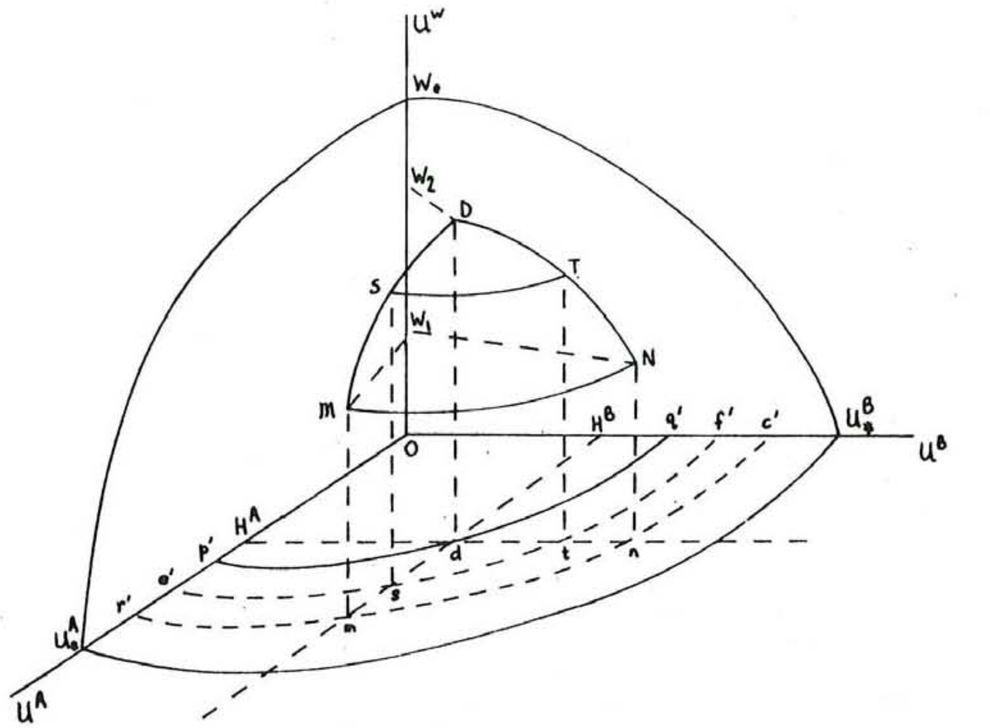
(6) $p'z^W - p'x^W = 0$

Equation (5) states that the rest of the world maximises its utility subject to its production constraint, the home country's production constraint, $(f^d(\cdot))$, the condition that all k consumers in the home economy are no worse off than under autarky, and that world supplies equal world demands. Equation (6) states that inter-country trade must be balanced; that is, transfers within countries are allowed but transfers between countries are not.

An iterative solution to this model consists of choosing an initial set of minimum utility levels H_0^i (equal to the autarkic levels), solving (5) and testing to see if (6) is satisfied. If (6) is positive, then the rest of the world is running a balance of trade surplus. In this case, the utility levels H_0^i can be adjusted upwards according to some pre-determined rule and the model solved again, the process being repeated until the deficit is eliminated. The proof of the gains from trade lies in establishing that a solution to (5) and (6) exists for any adjustment rule that satisfies the condition that H^i is greater than or equal to the autarkic level H_0^i for all i. The proof is mathematically complex and as such is relegated to an appendix. An intuitive proof in the case where k = 2 follows.

In Figure II , the attainable autarkic utility possibilities frontier is p' q' plus the point W_1. Under free trade, with no restrictions on the feasibility of income transfers, the attainable frontier becomes the entire curved surface $U_*^A U_*^B W_*$ (A and B again represent the two individuals in the home country). The JMP-2 problem of equation (5) requires that the rest of the world maximises U^W subject to $U^A \geqslant H^A$ and $U^B \geqslant H^B$. It will do this by setting $U^A = H^A$, $U^B = H^B$, and $U^W = W_2$ (the utility distribution corresponding to D on the utility surface). But at D, equation (6) will not be satisfied since the rest of the world is receiving an implicit income transfer from the home country that enables it to reap all the benefits of trade.

To move towards a solution to (5) that does satisfy (6) requires specification of a redistribution function that allocates utility from the rest of the world back to A and B while satisfying the constraints of the model. This function moves the equilibrium from D to some point in the surface bounded by DSMNT. Only points within this set are feasible distributions of utility for utility-compensated equilibria because D, M and N define the limits within which $U^A \geqslant H^A$, $U^B \geqslant H^B$, and $U^W \geqslant W_1$ (this last constraint is not essential but is imposed on the grounds that if U^W is less than W_1 then it will be unprofitable for the rest of the world to trade with the home economy). Within DSMNT there is a single locus of points, ST, that satisfy equation (6), and the redistribution process will cease once a point on ST is reached. The counterpart of Figure I(b) is shown in the $U^A U^B$ plane. The curve $r'c'$ lies outside $e'f'$ because at all points on $r'c'$ the domestic individuals receive an income transfer from the rest of the world.



IV SOME POSSIBLE APPLICATIONS

The preceding section used the JMP-2 model to prove that a utility-compensated free-trade equilibrium exists. The proof itself is abstract and of small value in its own right. What is of value is the insight that the joint maximisation approach gives us into the nature of the utility-compensated equilibrium and the wealth of empirical opportunities that it opens up. The important message from Figure II is that there is a range of utility-compensated free-trade equilibria, each of which corresponds to a different level (and possibly direction) of compensation payments, and each of which can be located through the JMP-2 algorithm simply by altering the adjustment function used in computing the equilibrium.

The pure theory of trade, since Samuelson's contribution, has tended to focus almost exclusively on uncompensated equilibria. There is a strong case for empirical work to consider more closely the actual utility-compensated equilibria. The joint maximisation approach is an efficient means of mapping out this set and its associated set of transfers. It has two big advantages in empirical work: first, it retains the welfare framework of trade theory and second, it is not restricted to small changes in the way that conventional partial-equilibrium studies are.

The most obvious applications of this framework lie in the area of commercial policy. For example, the traditional argument concerning the existence and properties of an optimal tariff uses the uncompensated equilibrium concept. Using the JMP-2 model in an empirical investigation of the tariff structure it would be possible to map out the set of feasible optimal tariffs. Each optimal tariff will be optimal with respect to a given domestic income distribution, and each will be feasible in the sense of being compatible with a utility-compensated equilibrium (relative to some reference point such as autarky or the historical tariff situation).

The joint maximisation approach is also relevant to preferential trading arrangements. It is well-known from the theory of the second best that a preferential trading area will have some detrimental effects. Some of the members may even face potential losses from joining such an arrangement. By treating the member countries in the way that individuals were treated in the gains from trade exercise and by specifying a tariff structure it would again be possible, with the JMP-2, to locate the set of feasible external tariff structures that are compatible with utility-compensated equilibrium. A subset of these feasible tariffs will be optimal in the sense of maximising the welfare of the trading group for a given allocation of the overall gains. The choice of a unique external tariff structure from within this optimal set would be an extremely useful first step in any evaluation of a proposed or existing discriminatory trading arrangement. It is also possible within this framework to identify the partial equilibrium concepts found in the literature such as trade creation, trade diversion, consumption effects, terms-of-trade effects, and so on.

As a final note in this section something should be said about the actual computational experience with the joint maximisation algorithm. In a recent article Dixon (1978) tested the computational efficiency of his algorithm on a series of examples previously worked by Scarf (1973) and others. These examples involved locating non-compensated equilibria. Dixon found that the joint maximisation methods will be preferred to price search methods when the number of agents, k, is small relative to the number of goods, n. This condition is likely to be met in problems involving utility compensation and reinforces the argument for the efficiency of the joint maximisation approach to these problems.

V CONCLUSION

The modern welfare theory of international trade has relied heavily on the principle of compensation. The analytical models nevertheless deal almost exclusively with uncompensated situations. The main weakness of this analytical approach, particularly with respect to its empirical implications is that the solutions and conclusions of the typical trade models are not always invariant to the pattern of compensation if it is actually paid. In this paper I have attempted to show the relevance of Dixon's theory of joint maximisation to the analysis of questions involving utility compensation. In particular, the joint maximisation model was argued to be an efficient computational framework for locating and evaluation utility-compensated equilibria.

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APPENDIX

The objective of this appendix is to prove that the utility-compensated free-trade equilibrium exists. The proof draws heavily on a similar proof for a closed economy in Chapter 5 of Arrow and Hahn (1971).

Define a utility-compensated free-trade equilibrium as (p^*, u^*, x^*, z^*) such that:

(a) $p^* > 0$

(b) $\Sigma_{i=1}^{k+1} x^{i*} \leqslant \Sigma_{f=1}^{h+1} z^{f*} + \Sigma_{i=1}^{k+1} \bar{x}^{i}$, $i = 1, \ldots k+1, f = 1, \ldots h+1$

(c) z^{f*} maximises $p^{*\prime} z^f$ subject to $z^f \epsilon Z^f$

(d) x^{i*} minimises $p^{*\prime} x^i$ subject to $U^i(x^i) \geqslant u^{i*}$

(e) $p^* x^{w*} = p^{*\prime} \bar{x}^{w} + p^{*\prime} z^{w*}$

(f) $\Sigma_{i=1}^{k} p^{*\prime} x^{i*} = \Sigma_{i=1}^{k} [p^{*\prime} \bar{x}^{i} + \Sigma_{f=1}^{h} d^{if}(p^{*\prime} z^{f*})]$

(g) $\bar{u}^{j*} / \bar{u}^{m*} = a^{jm}$, $j, m \neq w$, $a^{jm} \geqslant 0$

where the following new notation has been introduced: * indicates equilibrium values; $\bar{x}^{i}$ is individual i's initial endowment of commodities (factors are treated as a subset of the n commodities); z^f is the output of firm f (it is assumed that there are h firms in the domestic economy and as with consumers the $h+1^{th}$ firm, labelled w, represents the rest of the world); z^f is the production set for firm f; d^{if} is the fraction of firm f owned by individual i; and, $\bar{\bar{u}}^i = (u^i - H^i)$.

The full set of assumptions needed for the utility-compensated equilibrium to exist are detailed in Arrow and Hahn. The more important of these are that z^f is closed and convex and that $U^i(\cdot)$ is strictly quasi-concave. Subject to these regularity assumptions, the proof proceeds as below.

The equilibrium concept has the conventional demand, supply, and maximisation requirements (a) – (d). The effect of conditions (e) and (f) is to allow transfers between the individuals of the domestic economy but not from one country to the other. Condition (g) allows the incorporation of any domestic redistribution function of the gains from trade that satisfies the condition that no individual is worse off than under autarky.

Let the utility-compensated Pareto-efficient frontier be the set:

$$U' = U(u \mid u^i \geqslant H^i, i, \ldots k+1)$$

where U is the entire Pareto-efficient frontier (U′ corresponds to DSMNT in Figure II of the text). Using the definition of $\bar{\bar{u}}^i$ as $u^i - H^i$, normalise the set U' as:

$$\bar{\bar{U}}' = (\bar{\bar{u}} \mid \bar{\bar{u}}^i \geqslant 0, = 1, \ldots k+1)$$

Define the mapping v from $\bar{\bar{U}}'$ to the k + 1 dimensional simplex S_{k+1} by:

$$v(\bar{\bar{u}}) = \bar{\bar{u}} / \Sigma_{i=1}^{k+1} \bar{\bar{u}}^i$$

This mapping has the properties that $v(u) > 0$, $\Sigma_{i=1}^{k+1} v^i = 1$ and that the inverse $\bar{\bar{u}} = \bar{\bar{u}}(v)$, and therefore $u = u(v)$, are well defined and continuous on S_{k+1} (see Arrow and Hahn, pp. 112-113 for a proof of continuity).

Every point in $\bar{\bar{U}}$, can be supported by a set of non-negative prices, $P(\bar{\bar{u}})$, and a set of u-feasible allocations $Q(\bar{\bar{u}})$; where a u-feasible allocation is a vector $q = (x,z)$ of consumption x^i and production z^f that satisfies (b) – (d). $P(\bar{\bar{u}})$ is defined on the n-dimensional simplex S_n and $Q(\bar{\bar{u}})$ is defined on the set Q of u-feasible allocations.

Given an element (v,p,q) from the set $S_{k+1} \times S_n \times Q$, define individual i's budget surplus as:

$$T^i(p,q) = p'[\dot{x}^i + \Sigma_{f=1}^{h} d^{if}(p'z^f) - x^i]$$

and w's surplus as:

$$T^w(p,q) = p'[\dot{x}^w + p'z^w - x^w]$$

Next, define the adjustment function G(v) such that:

$$G^w(v^w) = \begin{cases} < 0 \text{ if } T^w(p,q) < 0 \\ > 0 \text{ if } T^w(p,q) > 0 \\ = 0 \text{ if } T^w(p,q) = 0 \end{cases}$$

$$G(v^1, \ldots v^k) = \left[\begin{array}{ll} \bar{\bar{u}}^j(v)/\bar{\bar{u}}^m(v) = a^{jm} & \\ \Sigma_{i=1}^{k} G^i(v) > 0 \text{ if } T^W(p,q) < 0 & \\ \quad " \qquad < 0 \text{ if } T^W(p,q) > 0 & \\ \quad " \qquad = 0 \text{ if } T^W(p,q) = 0 & \end{array} \right.$$

then let:

$$V(p,q) = [v + G(v)] / [v + G(v)]'e$$

where $e = (1, \ldots 1)$

The correspondence $V(p,q) \times P[\bar{\bar{u}}(v)] \times Q[\bar{\bar{u}}(v)]$ maps points in the compact convex set $S_{k+1} \times S_n \times Q$ into convex subsets of itself. As $P(u)$ and $Q(u)$ are upper semi-continuous (see Arrow and Hahn, pp. (114-115)) and $V(p,q)$ is continuous, the mapping satisfies the conditions of the Kakutani fixed point theorem. Thus there exists a fixed point (v^*, p^*, q^*) in $S_{k+1} \times S_n \times Q$ such that $(v^*, p^*, q^*) \in V(p,q) \times P[\bar{\bar{u}}(v)] \times Q[\bar{\bar{u}}(v)]$. Since the correspondence is constructed to satisfy (a) – (g), there exists a utility-compensated equilibrium (p^*, u^*, x^*, z^*). It remains only to show that $\bar{\bar{U}}'$ is non-empty. But this must be the case since the consumption possibilities frontier under free trade is expanded and therefore contains the autarky point, and the autarky point is an element of $\bar{\bar{U}}'$.