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GAINS FROM TRADE WITH AND WITHOUT LUMP-SUM COMPENSATION

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It is shown by example that there are situations in which free trade is Pareto-superior to autarky if and only if the compensation of losers is effected by lump-sum transfers and that there are situations in which free trade is Pareto-superior to autarky if and only if compensation is not lump-sum.

1. Introduction

According to the traditional gains-from-trade theorem, for any country entering trade there always exists a system of post-trade lump-sum compensating payments such that after compensation each member of the country is not worse off than under autarky. Recently, however, Dixit and Norman (1980) have considered the possibility of achieving trade gains with compensation effected by means of non-lump-sum taxes and subsidies. On the assumption that the authorities are restricted to the 'taxation of goods and factors', they argue that 'even this more limited set of instruments suffices to make free trade Pareto superior to autarky' and claim that '[t]his set of results [their finding combined with the traditional doctrine] constitutes a powerful argument in favour of trade...' (p. 76). In particular, they appear to suggest that if free trade is strictly gainful with lump-sum compensation then it is strictly gainful when compensation is effected by carefully chosen (non-lump-sum) taxes on goods and services. The suggestion is an interesting one for, if valid, it would imply that any internal misallocation generated by (carefully chosen) non-lump-sum taxes is always sufficiently offset by an improvement in the terms of trade. However, the

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Dixit–Norman analysis is quite informal; no proofs are provided. Indeed, it is not even clear just how broad is the class of economies they have in mind.

In the present paper we consider the following questions: Are there situations in which trade is strictly gainful with lump-sum compensation but not with any scheme of non-lump-sum compensation? Are there situations in which trade is strictly gainful with non-lump-sum compensation but not with any scheme of lump-sum compensation? To each question we give an affirmative answer. Thus, let us define a *situation* as an ordered pair $s=(\mathscr{E}, e^a)$, where $\mathscr{E}$ is a vector of characteristics of the world economy and e^a is an autarkic equilibrium state of that economy. In the space of all situations let A be the class for which trade is strictly gainful for a particular country under some scheme of internal lump-sum compensation, and let B be the class for which trade is strictly gainful for that country under some scheme of non-lump-sum compensation. Then it is our finding that neither class is contained in the other: there exists a situation s' which belongs to A but not to B, and there exists a situation s'' which belongs to B but not to A. The proof is by example; of three examples offered in section 2, the first pair establishes the existence of s' and the third establishes the existence of s''. None of the examples contains any out-of-the-ordinary (non-Arrow–Debreu) features.

Finally, in section 3 we offer a general comment on the work of Dixit and Norman. It is there argued that any useful proposition concerning the gains from trade, be it with lump-sum or with non-lump-sum compensation, must specify a scheme of compensation (as a function of s) and assert the existence of a compensated trading equilibrium under that scheme; and it is noted that Dixit and Norman have failed to provide such a proposition.

2. Three examples

The first two examples show that there are circumstances in which trade is strictly gainful if and only if compensation is lump-sum.

Example 1. There are three goods. In the country under observation (the 'home' country) each of the three goods may be consumed; and the first two may be combined to produce the third. In the home country there are six households with the common utility index

$$u^i = 2\sqrt{2}x_{i1}^{1/4}x_{i2}^{1/4}x_{i3}^{1/2},$$

where x_{ik} is the consumption of good k by household i. The commodity endowments of the six households are

$$\omega_1=\omega_4=(1,0,0), \qquad \omega_2=\omega_5=(0,1,0), \qquad \omega_3=\omega_6=(0,0,1).$$

Moreover, households 4, 5 and 6 share equally in the ownership of a single firm with the constant-returns production function

$$y_3=(-y_1)^{1/2}(-y_2)^{1/2},$$

where y_k is the net output of good k and inputs are treated as negative outputs. The autarkic prices (normalized so that $p_1+p_2+p_3=1$) are $p^a=(\frac{1}{3},\frac{1}{3},\frac{1}{3})$, the autarkic production vector is $y^a=(-\frac{1}{2},-\frac{1}{2},1)$, and the autarkic consumption vector is $x_i^a=(\frac{1}{4},\frac{1}{4},\frac{1}{2})$, all i.

Since households 1–3 own no shares in the firm, their indirect utility functions are

$$v_1=p_1^{3/4}/[p_2^{1/4}(1-p_1-p_2)^{1/2}],$$

$$v_2=p_2^{3/4}/[p_1^{1/4}(1-p_1-p_2)^{1/2}],$$

$$v_3=(1-p_1-p_2)^{1/2}/(p_1^{1/4}p_2^{1/4}),$$

where p_3 is written as $1-p_1-p_2$. It is easy to verify that the autarkic utility $v_i^a=1$, $i=1,2,3$. Fig. 1 displays the set $\mathscr{P}_i(v_i^a)$ of prices (p_1,p_2) such that household i, $i=1,2,3$, is not better off than under autarky. Neither the three price sets nor their complements have an element in common other than p^a. Thus, whatever the world price vector (and, therefore, whether or not trade with lump-sum compensation is strictly gainful), it is impossible to find a set of domestic prices such that all individuals are better off under autarky. □

Example 1 demonstrates that there can be no general gains-from-trade proposition without lump-sum compensation. However, the example may strike some trade theorists as unconventional in that there is no good for which the firm appears alone on one side of the market. Our second example therefore is taken from the familiar class of two-by-two economies.

Example 2. Two tradeable consumption goods 1 and 2 are produced by two inelastically-supplied primary or non-produced factors (goods 3 and 4) in a Heckscher–Leontief technology:

$$\begin{bmatrix}\frac{4}{5} & \frac{1}{5}\\ \frac{1}{5} & \frac{4}{5}\end{bmatrix}\begin{bmatrix}y_1\\ y_2\end{bmatrix}\leqq\begin{bmatrix}-y_3\\ -y_4\end{bmatrix}.$$

There are two households, with utility indices

$$u_1=(3/4^{1/3})x_{11}^{2/3}x_{12}^{1/3}, \qquad u_2=(3/4^{1/3})x_{21}^{1/3}x_{22}^{2/3}.$$

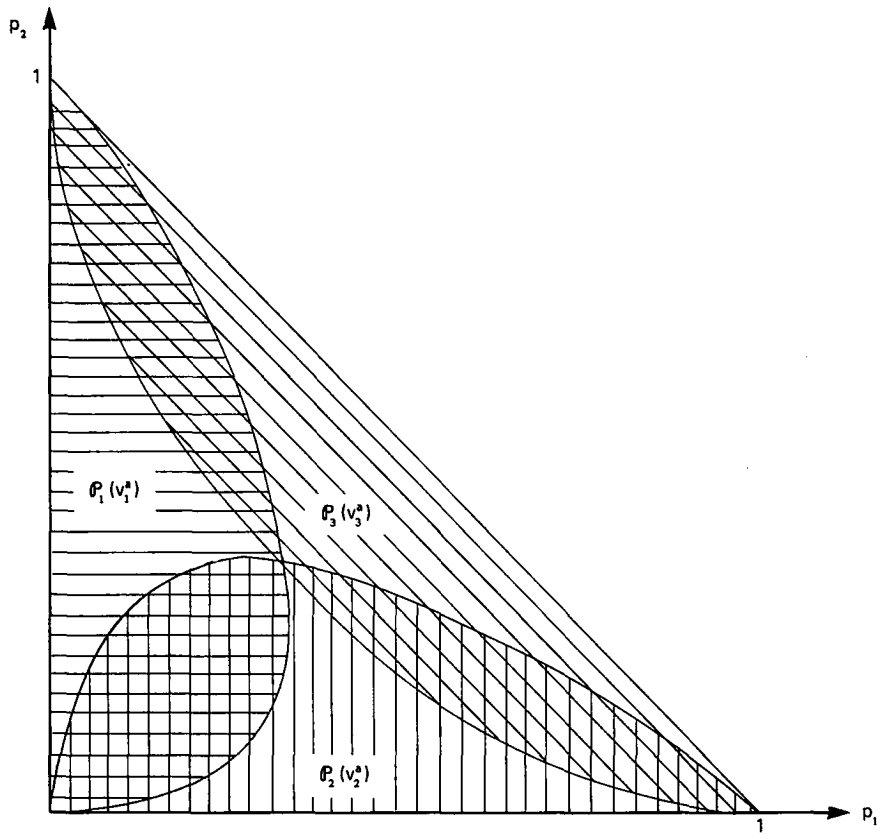

Fig. 1

Thus the consumption of household i is 'intensive' in commodity i, $i=1,2$. The community has an endowment of primary factors but no endowment of producible commodities; moreover, the endowment is shared equally by the two households:

$$\omega_1=(0,0,\tfrac{1}{2},\tfrac{1}{2})=\omega_2.$$

It follows that households supply primary factors and demand produced consumption goods. Firms, on the other hand, demand primary factors and supply produced consumption goods. Thus, firms and households are on opposite sides of each market. In addition, the two households share ownership of the firms equally. However, under constant returns to scale and freedom of entry, profits are zero in equilibrium, so this assumption plays no role.

Let us normalize domestic prices so that $p_1+p_2=1$, write $p_1=p$ and $p_2=1-p$, and denote by w_i the income of household i. Then, in autarkic equilibrium,

$$p^a=\tfrac{1}{2}=1-p^a, \qquad w_1^a=\tfrac{1}{2}=w_2^a,$$

$$x_{11}^a=\tfrac{2}{3}, \qquad x_{12}^a=\tfrac{1}{3}, \qquad x_{21}^a=\tfrac{1}{3}, \qquad x_{22}^a=\tfrac{2}{3},$$

$$y_1^a=1=y_2^a,$$

$$u_1^a=1=u_2^a.$$

Now let the economy be opened to trade; and suppose that it is small in relation to the rest of the world, so that world prices for the two tradeables are given numbers. Indicating world prices by primes, we have $p_1'=kp'$ and $p_2'=k(1-p')$ for some $k>0$. Let $\frac{1}{3}<p'<\frac{1}{2}$. Then $\frac{1}{2}\leqq p'/(1-p')<1$ so that, if firms face world prices, production remains at the autarkic point; but, since $p'/(1-p')\neq 1$, international trade is possible.

Consider fig. 2. In the absence of compensation, the post-trade budget line for each household is SS', which passes through the common per capita output point P and has a slope of $p'/(1-p')<1$. It is easy to see that hosuehold 1 gains from trade and that household 2 loses. It is easy to see also that if lump-sum compensation were feasible then trade would be strictly gainful; for a post-trade transfer of $PQ=RP$ of the output bundle from household 1 to household 2 would leave both households better off than under autarky.

Suppose, alternatively, that only non-discriminatory non-lump-sum taxes are allowed. We notice at once that, since the two households are equally endowed and since primary factors of production enter neither of the utility functions, taxes on the sale or initial holdings of factors can do nothing to redistribute income; such taxes are equivalent to poll taxes and can be useful only in achieving balance in the government's tax–subsidy budget. Ruling out import and export taxes on the ground that they distort production, we are left with taxes on consumption as the effective devices for redistributing real income.

Now in any tax-ridden trading equilibrium, since the two household endowments are equal, $w_1=w_2$. Moreover, from the log-linearity of the utility functions, in equilibrium

$$px_{11}=\tfrac{2}{3}w_1 \quad (1-p)x_{12}=\tfrac{1}{3}w_1,$$

$$px_{21}=\tfrac{1}{3}w_2 \quad (1-p)x_{22}=\tfrac{2}{3}w_2.$$

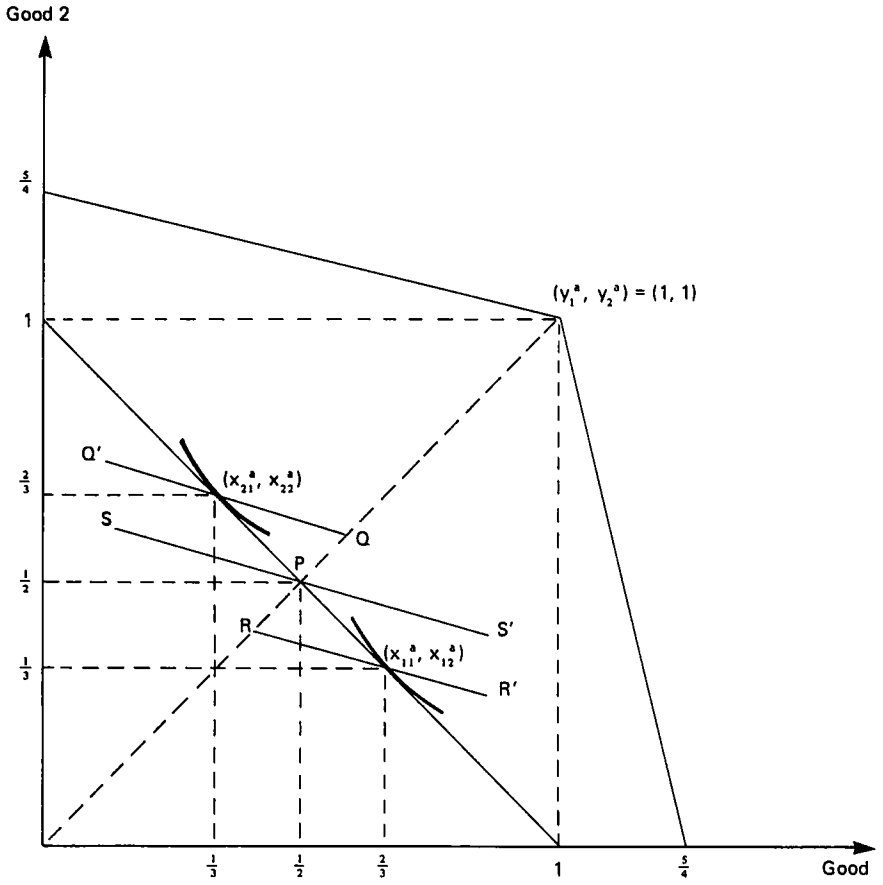

Fig. 2

Hence,

$$p(x_{11}+x_{21})=(1-p)(x_{12}+x_{22}).$$

On the other hand, since international payments balance,

$$p'(x_{11}+x_{21})+(1-p')(x_{12}+x_{22})=p'\cdot 1+(1-p')\cdot 1=1.$$

Hence, solving the last four equations,

$$x_{11}=2x_{21}=2(1-p)/(3D)$$

$$2x_{12}=x_{22}=2p/(3D),$$

where $D \equiv p + p' - 2pp'$; and

$$u_1 = p^{1/3}(1-p)^{2/3}/D,$$

$$u_2 = p^{2/3}(1-p)^{1/3}/D.$$

It is easy to verify that (i) u_1 and u_2 are zero at $p=0$ and $p=1$, (ii) u_1 is increasing up to $p=p'/(2-p')<p'$ and thereafter decreasing, (iii) u_2 is increasing up to $p=2p'/(1+p')>p'$ and thereafter decreasing, and (iv) $u_1 = u_2 = 1$ at $p=\frac{1}{2}$. Moreover, by assumption, $\frac{1}{3}<p'<\frac{2}{3}$; hence (v) $2p'/(1+p')>\frac{1}{2}>0$ and $p'/(2-p')<\frac{1}{2}<1$. It follows from (ii), (iv) and (v) that if $\frac{1}{2} \leqq p \leqq 1$, then $u_1 \leqq 1$, the autarkic level of utility; and it follows from (iii), (iv) and (v) that if $0 \leqq p \leqq \frac{1}{2}$, then $u_2 \leqq 1$. Hence, not both households can be better off than under autarky. □

In example 2 it was assumed that coefficients of production are rigidly fixed. However, that extreme assumption was made only for convenience. All that is needed is that autarkic output have multiple supporting price lines. As Kemp et al. (1985) have shown, the production frontier can contain sharp points even when the underlying production functions are smooth.

Remark 1 about examples 1 and 2. In the construction of examples 1 and 2 it was assumed that the government runs a balanced tax–subsidy budget. However, it will be obvious that our conclusion, that non-lump-sum compensation cannot always be substituted for lump-sum compensation, holds a fortiori if there is a surplus which is spent on goods which are then removed from the system.

Remark 2 about examples 1 and 2. It is well known from the recent literature on tax reform that if and only if there is a single or composite good such that all consumers are on the same side of the market, then there is a direction of change of consumer prices such that, if the change is sufficiently small, all individuals are left better off. [See, in particular, Weymark (1979, theorem 1), which refines Diamond and Mirrlees (1971).] It is easy to verify that the condition is violated by example 1. However, example 1 provides more than a simple illustration of Weymark's theorem, for the example is global in scope, whereas the theorem applies only to infinitesimal price changes. In example 2, on the other hand, the condition of Weymark's theorem is satisfied but there are no *feasible* Pareto-improving price changes.

Our third example shows that there are circumstances in which trade is strictly gainful if and only if compensation is non-lump-sum.

Example 3. In each of two countries there is a single non-tradeable factor of production, labour, which produces two tradeable consumption-goods in a constant-returns no-joint-products technology. The technology differs from country to country; in particular, each country has a comparative advantage in producing some commodity. Moreover, at home all individuals are alike both in their labour endowments and in their preferences. For concreteness it is assumed that preferences can be summarized by a Mill–Cobb–Douglas utility function. Finally, the home country is larger than the foreign country, in the limited sense that the tax-free world trading equilibrium lies on the linear segment of the home offer curve.

For the home country, trade without taxes means only a change in production; the consumption and well-being of each individual is unchanged. Suppose, however, that the home country imposes a tax on the consumption of the imported commodity and a subsidy on the consumption of the exported commodity, with the rates of tax and subsidy nicely calculated to equate home consumer prices to foreign autarkic prices and to balance the government's tax–subsidy budget. Then trade takes place at *foreign* autarkic prices, home production shifts marginally, if at all, and each home individual is better off than before trade. Diagrammatically, the home offer curve is so distorted by the tax that it passes through the straight segment of the foreign offer curve. □

3. Comment on Dixit and Norman

Examples 1 and 2 show conclusively that it is not generally the case that trade with suitable non-lump-sum compensation is strictly gainful whenever trade with lump-sum compensation is strictly gainful. This does not mean that valid and policy-relevant propositions about the gains from trade with non-lump-sum compensation are beyond our reach. However, such propositions must state sufficient conditions [restrictions on $s=(\mathscr{E}, e^a)$] for the existence of equilibrium with strictly gainful trade with specified schemes of compensation (vectors of taxes which depend on s); thus any useful gains-from-trade theorem under non-lump-sum compensation is an existence theorem.[1] Dixit and Norman have failed to carefully specify either the class of economies or the redistributive scheme and therefore cannot even approach the all-important question of existence.

That question presents technical difficulties.[2] More important, for present

[1]It will be recalled that the propositions of Grandmont and McFadden (1972) and of Kemp and Wan (1972) were of precisely this kind.

[2]Dixit and Norman (1980) appear to believe that the question of existence with non-lump-sum compensation contains no novelties. 'Having seen the manner in which the existence of equilibrium can be checked when treating lump-sum transfers, we will not repeat the *same* argument' (p. 79, italics added). As the recent contributions of Sontheimer (1971), Shoven (1974), Mantel (1975) and Shafer and Sonnenschein (1976) suggest, questions of existence cannot be simply brushed aside in this manner.

purposes, is the problem of finding a suitable scheme of compensation. Consider, by way of illustration, the two-step procedure which Dixit and Norman seem to have in mind. At the first step one searches for an 'interim equilibrium' in which each household faces the same prices and consumes the same quantities as under autarky, each firm achieves maximum profit at equilibrium world prices and the government applies its tax surplus to stockpile all l goods, spending the same amount on each good. At the next step, one reduces the tax surplus by $1/l$ in some non-discriminatory manner (e.g. by a negative poll tax). The households then buy back the stockpiled goods at autarkic prices. The remaining stockpiles may be disposed of physically. If no good is inferior, this procedure works; that is, it yields a trading equilibrium which is Pareto-superior to autarky. (Of course, the procedure is very wasteful.) Otherwise, households buying less inferior goods, may increase spending on one good by more than the entire tax surplus. With households' preferences unrevealed, no stockpile is adequate in all events. Try now a one-step procedure without disposal. Again, it may break down as the following example demonstrates.

Example 4. In a world of three goods, tradeable consumption goods are produced under constant returns by the non-tradeable primary good 3. Thus $y_1+y_2 \leqq -y_3$, where y_k is the net output of the kth good. There is a single type of household or, equivalently, a single price-taking household; the household owns both the production sector and the endowment vector $\omega=(5.3, 19, 1)$. The Engel curve defined by the autarkic prices $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$ is described by the equation

$$x_1 = 5 + 50(x_2 - 5)/[25 + (x_2 - 5)^2].$$

In fig. 3(a), the supply set $OABCD$ contains those pairs of goods 1 and 2 which can be represented as the sum of the endowment vector (5.3, 19) and the feasible output vector (y_1, y_2), where $y_1 + y_2 \leqq 1$. The Engel curve is labelled $OEFCG$. In the companion fig. 3(b), the offer curve of the 'home' country consists of three branches: both (i) the curve $B'C'G'$ and (ii) the curve $E'F'$ indicate possible trades when output is at point B in fig. 3(a). Branch (iii), the linear segment $C''B'$, summarizes the home country's capacity to trade when it produces at point C. We now superimpose the offer curve of the world, $\alpha B'\gamma$; that curve exhibits no peculiarity at all. Since the home country would not trade at any price ratio $p_1/p_2 < 1$, point B' is an equilibrium (without trade); so are points H' and I', which correspond to points H and I of fig. 3(a). Clearly, trade does not improve welfare in any of these three cases; at H and I it is welfare-reducing. □

The purpose of this exercise is not simply to reinforce example 2, by showing again that non-lump-sum compensation cannot always be substituted for lump-sum compensation. Rather, its purpose is to reveal the possibility that an apparently innocuous scheme of compensation may yield an 'inappropriate' equilibrium in which everyone loses from trade, and this in spite of the presence of a production gain (at equilibrium world prices, post-trade output at B dominates pre-trade output at C), in spite of the fact that producers and consumers are on opposite sides of each market, and in the absence of any need for redistribution (all individuals are identical in

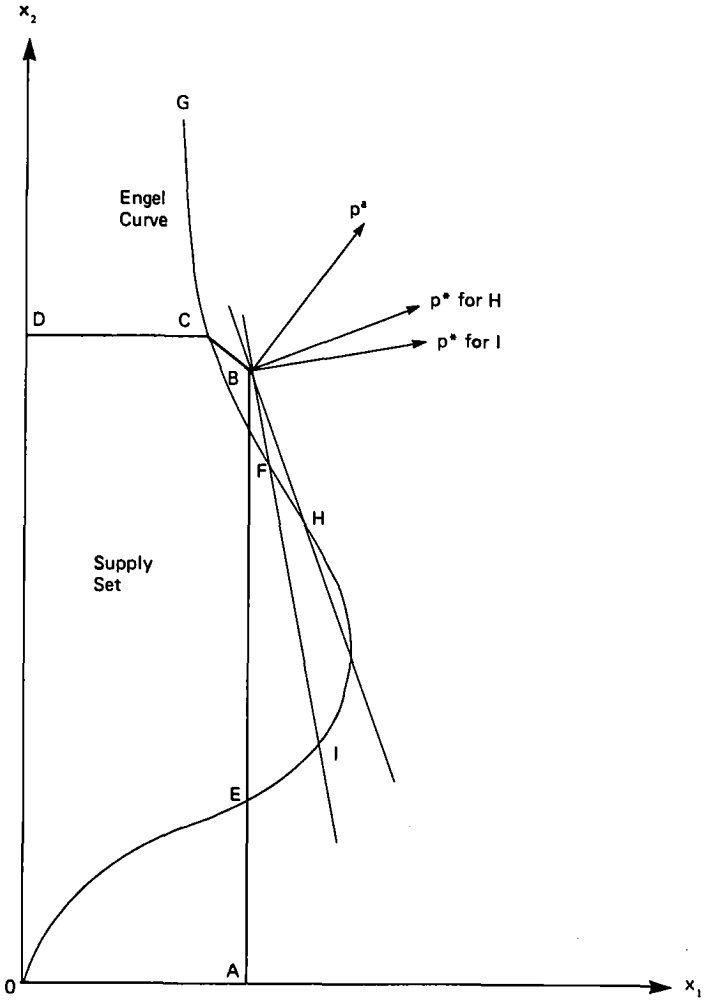

Fig. 3a

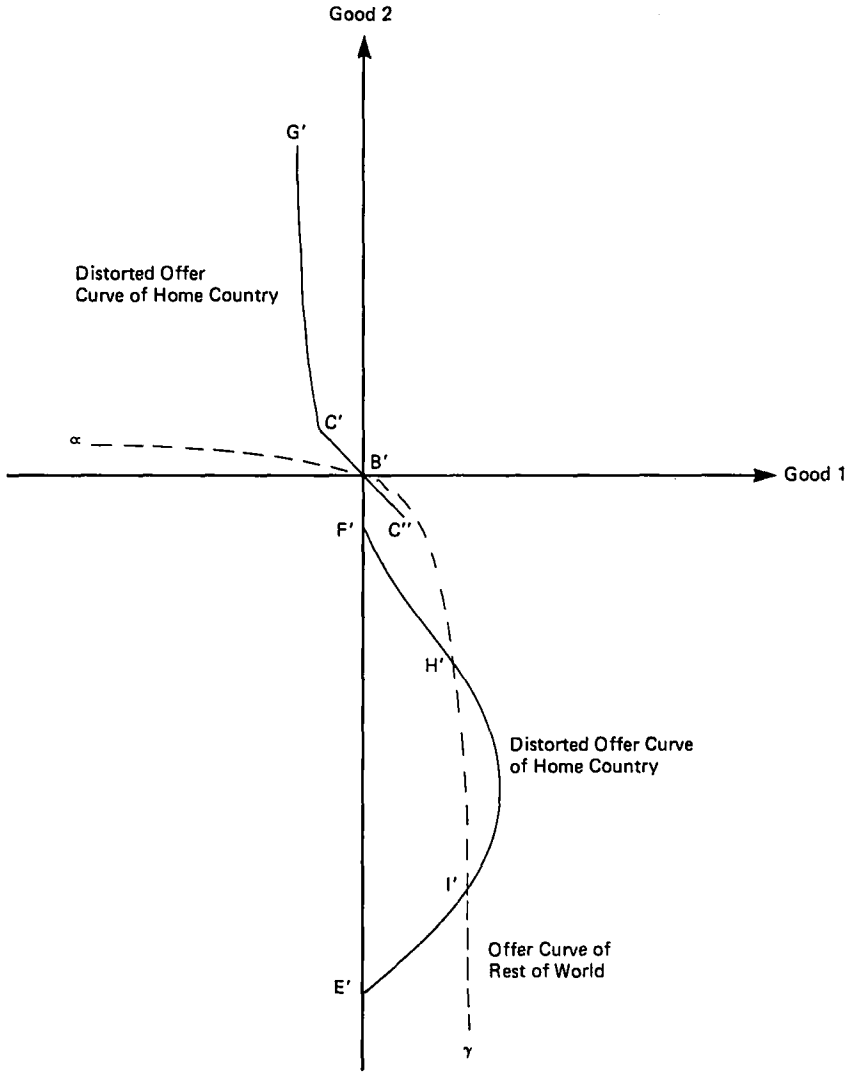

Fig. 3b

preferences and endowments). Of course there may well be some other non-lump-sum scheme which, *for this example*, guarantees that everyone gains from trade. But that is beside the point. What we wish to emphasize is the need to specify an operational rule which assigns to any situation $s=(\mathscr{E}, e^a)$ a policy vector $t=f(s)$ of tax rates, etc. so that each equilibrium e^* in the class of policy-assisted equilibria $E^*(s,t)=E^*(s,f(s))$ is Pareto-not-worse than autarky for all citizens.

There remains for comment the suggestion of Dixit and Norman that their proposition, if valid, would constitute a 'powerful argument in favour of trade'. Why this should be so is not made clear. The only hint we have is their remark that non-lump-sum redistributive instruments are 'weaker' than lump-sum transfers (p. 80). If by this it is meant that the proper deployment of non-lump-sum instruments presupposes less information, or less costly information, than does the proper deployment of lump-sum instruments, then we must ask for a demonstration. In this connection it is worth recalling that the Grandmont–McFadden (1972) and Grinols (1981) schemes of *lump-sum* compensation require knowledge of autarkic equilibrium quantities only.

That completes our discussion of the Dixit–Norman proposition concerning the gains from trade. In concluding we merely note that their related proposition concerning the formation of customs unions (pp. 192–194) is vulnerable to similar comment; see also Kemp and Wan (1986).

References

Diamond, P.A. and J.A. Mirrilees, 1971, Optimal taxation and public production I: Tax rules, American Economic Review 70, 8–27.

Dixit, A.K. and V. Norman, 1980, Theory of international trade (James Nisbet and Company, Welwyn, Herts.).

Grandmont, J.M. and D. McFadden, 1972, A technical note on classical gains from trade, Journal of International Economics 2, 109–125.

Grinols, E.L., 1981, An extension of the Kemp–Wan theorem on the formation of customs unions, Journal of International Economics 11, 259–266.

Kemp, M.C. and H.Y. Wan, Jr., 1972, The gains from free trade, International Economic Review 13, 509–522.

Kemp, M.C. and H.Y. Wan, Jr., 1986, The comparison of second-best equilibria: The case of customs unions, Zeitschrift für Nationalökonomie, to appear.

Kemp, M.C., N.V. Long and M. Tawada, 1985, Sharp points in production surfaces, Oxford Economic Papers, to appear.

Mantel, R.R., 1975, General equilibrium and optimal taxes, Journal of Mathematical Economics 11, 187–200.

Shafer, W. and H. Sonnenschein, 1976, Equilibrium with externalities, commodity taxation and lumpsum transfers, International Economic Review 17, 601–611.

Shoven, J.B., 1974, General equilibrium with taxes, Journal of Economic Theory 8, 1–25.

Sontheimer, K.C., 1971, The existence of international trade equilibrium with trade tax-subsidy distortions, Econometrica 39, 1015–1035

Weymark, J.A., 1979, A reconciliation of recent results in optimal taxation theory, Journal of Public Economics 12, 171–189.