Logical Templates

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The *template language* is a formal language used to generate instances of contradictions and non-contradictions in a natural language. This language is composed of two basic entities: people, $Pe = \{x_1, x_2, ..., x_n\}$ and places, $Pl = \{p_1, p_2, ..., p_m\}$. We also define three binary relations: $V(x, y)$, $x > y$, $x \geq y$. It is a simplistic universe with the intended meaning for binary relations such as "x *has visited* y", "x *is taller than* y" and "x *is as tall as* y", respectively.

For a collection of objects $a_1, \ldots, a_n$, we will say "a^* *is new*" to denote the fact that $a^* \notin \{a_1, \ldots, a_n\}$.

1 Simple Negation

The negation operator $\neg$ corresponds to the word "*not*".

1.1 Contradiction Templates

- $P := V(x_1, p_1), \ldots, V(x_n, p_n)$
  $H := \neg V(x_i, p_i)$

1.2 Non-contradiction Templates

- $P := V(x_1, p_1), \ldots, V(x_n, p_n)$
  $H := \neg V(x_i, p^*)$
  where p^* is new.

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  where x^* is new.

2 Boolean Coordination

The connectives $\wedge$ and $\vee$ correspond to "*and*" and "*or*", respectively.

2.1 Contradiction Templates

- $P := V(x_1, p) \wedge V(x_2, p) \wedge \cdots \wedge V(x_n, p)$
  $H := \neg V(x_i, p)$

- $P := V(x, p_1) \wedge V(x, p_2) \wedge \cdots \wedge V(x, p_n)$
  $H := \neg V(x, p_i)$

2.2 Non-contradiction Templates

- $P := V(x_1, p) \wedge V(x_2, p) \wedge \cdots \wedge V(x_n, p)$
  $H := \neg V(x^*, p^*)$
  where either $x^* = x_i$ and $p^* \neq p$, or x^* is new and $p^* = p$.

- $P := V(x, p_1) \wedge V(x, p_2) \wedge \cdots \wedge V(x, p_n)$
  $H := \neg V(x^*, p^*)$
  where either $x^* \neq x$ and $p^* = p_i$, or $x^* = x$ and p^* is new.

3 Quantification

The quantifiers $\forall$ and $\exists$ should be read as "*for every*" and "*some*", respectively. We will add also quantifiers restricted to the sets Pe and Pl. Hence, $(\forall x \in Pe)$ should be read as "*every person*", and $(\forall x \in Pl)$ should be read as "*every place*". A similar interpretation holds for $\exists$.

3.1 Contradiction Templates

- $P := (\forall x \in Pe)\ V(x, p_1) \wedge \cdots \wedge V(x, p_n)$
  $H := \neg V(x_i, p_i)$

- $P := (\forall x \in Pe)(\forall p \in Pl)\ V(x, p)$
  $H := \neg V(x_i, p_i)$

- $P := (\forall x \in Pe)(\forall y \in Pe)\ V(x, y)$
  $H := \neg V(x_i, y_i)$

- $P := (\forall x \in Pe)(\forall y \in Pe)(\forall p \in Pl)\ V(x, y) \wedge V(x, p)$
  $H := \neg V(x_i, y_i)$ or $H := \neg V(x_i, p_i)$

3.2 Non-contradiction Templates

- $P := (\forall x \in Pe)\ V(x, p_1) \wedge \cdots \wedge V(x, p_n)$
  $H := \neg V(x_i, p^*)$

where p^* is new.

- $P := (\forall x \in Pe)(\forall p \in Pl)\ V(x,p)$
  $H := \neg V(x_i, y_i)$
  where $x_i, y_i \in Pe$.

- $P := (\forall x \in Pe)(\forall y \in Pe)\ V(x,y)$
  $H := \neg V(x_i, p_i)$
  where $p_i \in Pl$.

- $P := (\exists x \in Pe)(\forall y \in Pe)(\forall p \in Pl)\ V(x,y) \wedge V(x,p)$
  $H := \neg V(x_i, y_i)$ or $H := \neg V(x_i, p_i)$

4 Definite Description

Here we will add the operator ι to perform description and the equality relation $=$. Hence, $x = \iota y Q(y)$ is to be read as "*x is the one that has property Q*".

4.1 Contradiction Templates

- $P := x = \iota y (\forall p \in Pl)\ V(y,p)$
  $H := \neg V(x, p_i)$

- $P := x = \iota y (\forall z \in Pe)\ V(y,z)$
  $H := \neg V(x, x_i)$

4.2 Non-contradiction Templates

- $P := x = \iota y (\forall p \in Pl)\ V(y,p)$
  $H := \neg V(x^*, p_i)$
  where $x^* \neq x$.

- $P := x = \iota y (\forall z \in Pe)\ V(y,z)$
  $H := \neg V(x^*, x_i)$
  where $x^* \neq x$.

5 Comparatives

For a set $\{x_1, \ldots, x_n\}$ and a binary relation R, we will use $chain(\{x_1, \ldots, x_n\}, R)$ to denote the facts $x_1 R x_2,\ x_2 R x_3,\ \ldots,\ x_{n-1} R x_n$. We will also use $yR\{x_1, \ldots, x_n\}$ to denote $yRx_1,\ yRx_2,\ \ldots,\ yRx_n$.

5.1 Contradiction Templates

- $P := chain(\{x_1, \ldots, x_n\}, >)$
  $H := x_j > x_i$
  where $i < j$

- $P := chain(\{x_1, \ldots, x_n\}, \geq),\ x_n > y$
  $H := y > x_i$

- $P := xR\{x_1, \ldots, x_n\},\ y \geq x$
  $H := x_i > y$

5.2 Non-contradiction Templates

- $P := chain(\{x_1, \ldots, x_n\}, >)$
  $H := x_j > x_i$
  where $j < i$

- $P := chain(\{x_1, \ldots, x_n\}, \geq),\ x_n > y$
  $H := x_i > y$

- $P := xR\{x_1, \ldots, x_n\},\ y \geq x$
  $H := y > x_i$

6 Counting

We introduce the counting quantifier $\exists_{=n}$ ("*exactly n*"). It will be also restricted to the sets Pe and Pl. For example, $(\exists_{=3} p \in Pl)\ Q$ should be read as "*exists only three places such that Q*").

6.1 Contradiction Templates

- $P := \exists_{=n} a\ V(x,a)$
  $H := V(x,p_1) \wedge \cdots \wedge V(x,p_{n+1})$ or $H := V(x,y_1) \wedge \cdots \wedge V(x,y_{n+1})$

- $P := (\exists_{=n} p \in Pl)(\exists_{=m} y \in Pe)\ V(x,p) \wedge V(x,y)$
  $H := V(x,p_1) \wedge \cdots \wedge V(x,p_{n+1})$ or $H := V(x,y_1) \wedge \cdots \wedge V(x,y_{m+1})$

6.2 Non-contradiction Templates

- $P := \exists_{=n} a\ V(x,a)$
  $H := V(x,p_1) \wedge \cdots \wedge V(x,p_k)$ or $H := V(x,y_1) \wedge \cdots \wedge V(x,y_k)$
  where $k < n$.

- $P := (\exists_{=n} p \in Pl)(\exists_{=m} y \in Pe)\ V(x,p) \wedge V(x,y)$
  $H := V(x,p_1) \wedge \cdots \wedge V(x,p_k)$ or $H := V(x,y_1) \wedge \cdots \wedge V(x,y_k)$
  where $k < n$ (or $k < m$).