

# Fair, Energy-aware Connectivity in Wireless Sensor Networks

Niels Kasch\*  
Email: nkasch1@umbc.edu

Dave Feltenberger\*  
Email: dfelten1@umbc.edu

Fatih Senel\*  
Email: fsenel1@umbc.edu

\*Department of Computer Science and  
Electrical Engineering  
University of Maryland, Baltimore County  
Baltimore, MD 21250

**Abstract**—The abstract is a substitution for the entire paper (about 250 words).

**Index Terms**—Wireless Sensor Network, Energy Conservation, Fairness, Energy Consumption

## I. INTRODUCTION

Over the past decade, Wireless Sensor Networks (WSN) have been employed in a variety of domains ranging from macroscopic applications such as weather monitoring and traffic control to microscopic applications in medical screening and biomedication. WSNs consist of a collection of independent devices (nodes) that are connected wirelessly in an ad hoc fashion. Individual nodes are equipped with sensors collecting information from the environment. The types of sensors employed by a node depend on the purpose of the WSN and may include active and passive sensors. The size of WSNs (i.e. the number of nodes in the network) also depends on the intended purpose of the network as well as other factors such as the network's resolution in terms of sampling frequency across space and transmission range, to name a few.

As wireless transmission ranges are limited, multi-hop networking plays an integral role in ensuring the connectivity of the network as a whole. Nodes transmit collected information to a centralized collection point, which in turn may distribute instructions or management data to nodes. Therefore, it is essential that segments of the network do not become disconnected due to wireless transmission range limitations. Section II reviews past and current research of connectivity in WSNs.

A second important criterion, which is one of the foci of this paper, concerns the energy efficiency of WSNs. Consider, for example, a remotely located earth quake

sensing station or a patient with implanted heart rate and blood gas sensors. Due to geographical (non-availability of power line infrastructure), economic (prohibitive cost of expanding infrastructure) and/or medical (infeasibility of permanent power connections) reasons, among others, it often is impractical or impossible to power the nodes of a WSN using existing energy grids. For those reasons, nodes are often battery powered, and as such, their lifetime is subject to the life of their batteries. Energy efficiency of nodes is therefore a primary concern for extending the lifetime of individual nodes and the network as a whole.

Furthermore, in this paper we introduce the notion of fairness with regard to power consumption. The aim of fair power consumption is to equalize the power consumption rates of nodes across an entire WSN. Fair power consumption has an immediate impact on economical and practical considerations for WSNs. Fair power consumption enables precise predictability of battery lifetimes of all the nodes in a network. Such predictability can be used to optimize node replacement schedules such that (1) all nodes fail (due to battery exhaustion) simultaneously, (2) groups of nodes fail simultaneously, or (3) nodes fail in a predetermined order. For example, it is desirable to replace implanted biomedical sensor nodes of a patient all at once at the largest possible intervals in order to minimize frequent multiple invasive procedures. Therefore, it is desirable to maximize the fair battery lifetime of all nodes in the WSN.

Wireless radio transmissions drop off at an exponential rate due to ground reflection of radio signals. Since the signal attenuation rate is exponential with distance, it requires a significant amount of energy to transmit

a signal only a small distance further. As such, it is often desirable to introduce relay nodes. Relay nodes differ in that their primary task is not sensing their environment but to ensure connectivity of the network. For the purposes of this paper, relay nodes will also be utilized to approximately equalize power consumption rates. Relay nodes are introduced in areas (as long as resource limitations permit) where power consumption is highest.

In this work, we propose an algorithm with the aim to minimize overall power consumption of WSNs. While overall power consumption has been studied previously, our algorithm considers fairness in power consumption rates in the minimization process. As seen in Section III, our algorithm may introduce additional, limited resources (i.e. nodes) to a fixed network. These resources may be moved to achieve minimal overall power consumption and maximum fairness. Hence, we introduce an approximation algorithm for three dimensions in WSNs: (1) minimal power consumption, (2) maximum fairness in power consumption and (3) minimal additional resources.

In Section II we present previous work relevant to our research followed by a precise problem definition, our approach and resulting algorithm in Section III. Section IV gives a formal analysis of our algorithm. We conclude our discussion in Section V.

## II. RELATED WORK

Construction and design of wireless sensor networks often heavily relies on common graph algorithms such as Prim and Kruskal's *Minimum Spanning Tree (MST)* [1] to ensure connectivity while minimizing the total length of all edges (i.e. wireless connections). The *Steiner Tree* problem extends this notion by introducing additional vertices to a graph in order to reduce the length of the spanning tree. The Steiner Tree problem is of particular interest to WSNs, in that additional vertices called relay nodes are often introduced to ensure connectivity. Estrin et al. [2] focus on connectivity establishment and maintainability using relay nodes in WSNs. They note that WSNs are subject to wireless transmission range constraints which in turn affect the interconnections between nodes. As the energy required to transmit data on wireless links is directly proportional to the distance between source and destination nodes, a significant amount of additional energy is required to establish and maintain a connection with more distant nodes. Hence, the most prominent source of energy consumption in wireless sensor networks is message transmission. Estrin

et al. utilize relay node placement to combat increased energy requirements by placing nodes along wireless links of maximum distance in the network.

Lloyd [3] discusses two strategies for ensuring a WSN is fully connected. The first is a single-tiered node placement strategy in which every node is connected via some path consisting of either sensor nodes or relay nodes. The length between a sensor node and any other node must be  $\leq r$ , while the length between two relay nodes can be of distance  $R$ , where  $R$  is defined as  $R \geq r$ . Node placement is achieved using Steiner Tree nodes. The second strategy discussed is what they call a two-tiered node placement strategy, in which any two nodes of the WSN are fully connected by all relay nodes. That is, between any two sensor nodes in the WSN there exists a path by which every intermediary node is a relay node.

Gao et al. [4] introduce the notion of reducing the radio range of nodes and employing a collaborating scheme between nodes to forward data to a base station or sink in order to conserve energy.

Song et al. [5] address the *energy hole* problem - the problem of depleting the energy reserves of high load nodes faster than non-high load nodes - by proposing an NP hard multi-objective optimization problem (MOP). They propose a centralized algorithm as well as a distributed algorithm for assigning the transmission ranges of sensors in order to maximize the network lifetime. Notice that increasing the network lifetime also increases the lifetime the shortest lived nodes. We differ in our work in that we aim to equalized the power consumption rates of all nodes while simultaneously increasing the network lifetime.

Cheng [6] proposes a relay node placement algorithm using the minimum number of relay nodes, so that the distance between each hop is less than or equal to the common transmission range. This problem is similar to the Steiner Minimum Tree with minimum number of Steiner Points and bounded edge-length (SMT-MSPBEL). They propose a 3-approximation algorithm as well as a 2.5 approximation algorithm. The 2.5 approximation algorithm follows a randomized strategy whose performance is faster than 3-approximation algorithm.

Gandham et al. [7] investigate energy efficiency in wireless sensor networks using multiple base stations. In order to prolong the lifetime of the sensor network, multiple base stations are employed that cover the entire network area. The lifetime of the sensor network is separated into equal periods of time called *rounds* and base stations are relocated at the start of a round. They propose an algorithm for base station placement at

the beginning of each round which maximizes network lifetime.

Tang et al. [8] study relay node placement problem in large scale wireless sensor networks such that sensor nodes are connected to at least one relay node and all relay nodes are connected amongst each other. To provide relay node fault tolerance they define the *2-Connected Relay Node Double Cover* (2CRNDC) problem and present a polynomial time approximation algorithms to solve the 2CRNSC.

Nguyen et al. [9] address the problem of unbalanced energy consumption among sensor nodes. This paper touches on concepts directly related to our work. They notice that unbalanced energy consumption among sensor nodes in home network domains is the result of employing only a single base station to collect sensor data. As sensor nodes further from the base station have to utilize more energy to transmit data across further distances, they propose a sensor network architecture with 2 base stations (and their corresponding communication protocol). While this solution approximately equalizes power consumption in relatively small networks, we are interested in a more general network topology.

### III. PROPOSED APPROACH

#### A. System Model

The model and algorithms described in this paper make the following assumptions:

- Each sensor/node sends an equal amount of data per time unit.
- Each node can receive an arbitrary amount of data per time unit.
- Each node can store an arbitrary amount of data without energy penalties. This assumption enables this model to ignore data transmission bottlenecks.
- The transmission range of each sensor node does not exceed the maximum transmission range  $T$ .
- Only nodes within transmission range of each other are connected (i.e. have an edge between each other in the network).
- A node's energy dissipation per bit transmitted (according to the first order radio model [10]).
- Transmitting a bit over distance  $d$  requires energy  $d^2$ .
- The power consumption rate  $PCR$  for a node  $n$  is defined as  $PCR(n) = \frac{\max_{v \in N} d(v)^2}{\text{unit time}}$ , where  $N$  is the set of neighbors of a node.
- Sensor nodes have a fixed location. Their location may not change during the lifetime of the network.

- Relay nodes are movable. Their location is flexible during the lifetime of the network.
- Relay nodes can be added to the original network.

#### B. Problem Formulation

We formulate the Minimal Fair Energy Consumption with Minimal Additional Resources (MFEC-MAR) problem as follows: Given a set of fixed sensor nodes  $S$ , find a minimal set of relay sensor nodes  $R$  such that the power consumption rate  $PCR$  is equal for each node  $v \in \{S \cup R\}$ , that is  $\forall_{i=0}^{n=|S \cup R|} PCR(v_i) = PCR(v_2) = \dots = PCR(v_n)$ . The induced graph  $G = (S \cup R, E)$  must be connected, where an edge  $(u, v) \in E$  if  $u$  is within transmission range  $T_v$  of  $v$  and  $v$  is within transmission range  $T_u$  of  $u$ . A node  $u$  is within transmission range of  $v$  if  $PCR(u) \leq \text{distance}(u, v)^2$ . Formally, MFEC-MAR is defined as follows:

- **Input:** A set of fixed sensor nodes  $S$  in Euclidean space.
- **Output:** A connected network  $G = (S \cup R, E)$  such that:
  - $R$  is the set of introduced relay nodes such that:
    - \*  $|R|$  is minimal
    - \*  $\forall_{i=0}^{n=|S \cup R|} PCR(v_i) = PCR(v_2) = \dots = PCR(v_n)$
  - $G$  is connected
  - $\sum_{v \in |S \cup R|} PCR(v)$  is minimal

The Minimal Fair Energy Consumption with Minimal Additional Resources Approximation (MFEC-MAR-Approx) problem aims to find an approximate solution to the MFEC-MAR problem. In the latter part of this paper, we will give an algorithm for MFEC-MAR-Approx. The MFEC-MAR-Approx problem is defined as follows:

Given a set of fixed sensor nodes  $S$ , an allowable standard deviation  $\alpha$  and a maximum number of relay nodes  $k$ , find a set of relay sensor nodes  $R$ , where  $|R| \leq k$ , such that the power consumption rates  $PCR$  for all nodes is within  $\alpha$ . The induced graph  $G = (S \cup R, E)$  must be connected, where an edge  $(u, v) \in E$  if  $u$  is within transmission range  $T_v$  of  $v$  and  $v$  is within transmission range  $T_u$  of  $u$ . A node  $u$  is within transmission range of  $v$  if  $PCR(u) \leq \text{distance}(u, v)^2$ . Formally, MFEC-MAR-Approx is defined as follows:

- **Input:** A set of fixed sensor nodes  $S$  in Euclidean space, a standard deviation  $\alpha$  of power consumption rates and a maximum number of relay nodes  $k$ .
- **Output:** A connected network  $G = (S \cup R, E)$  such that:
  - $R$  is the set of introduced relay nodes such that:

- \*  $|R| \leq k$
- \*  $\sigma(PCR) \leq \alpha$
- $G$  is connected

For the remainder of this paper, whenever we refer to fairness of power consumption rates ( $PCR$ ), we denote this fairness measure  $\alpha$  as the maximum allowable standard deviation of power consumption rates.

#### IV. ALGORITHM AND ANALYSIS

##### A. Algorithms for MFEC-MAR (Fairness in WSNs)

In this section we will explain our two step approach to solving the fairness problem in WSNs.

1) *Connecting Nodes:* Given a set  $T$  of  $n$  terminals which are deployed in an Euclidean plane, and a positive constant  $R$ , our aim is to find a Steiner Minimum Tree  $\tau$  with minimum number of Steiner points (SMT-MSP). In 1999, Lin *et al.* [11] showed that SMT-MSP is NP-Hard. In 2008, Cheng *et al.* [6], presented a  $O(n^3)$ -time approximations with performance ratio is at most 3. In this project we implemented Ratio-3 approximation presented in [6], to connect initially disconnected terminals. The idea behind the algorithm is simple: First we assume a fully connected undirected graph  $G = (V, E)$  where  $V$  = set of terminals, and  $(u, v) \in E \forall u, v \in V$  and  $u \neq v$ . Then we sort all  $\frac{n \cdot (n-1)}{2}$  edges. For each subset of 3 terminals  $a, b$  and  $c$  respectively in three connected components, if there exists such a point  $s$  within distance  $R$ , we put a 3-star  $s$  which includes edges  $(s, a)$ ,  $(s, b)$ , and  $(s, c)$ . Then for each edge where  $|e_i| > R$  and  $e_i$  connects two connected component, we move the steinerized  $e_i$  into  $\tau$ . Thus we establish the connectivity of terminals.

##### 1 FairSMT( $G, \alpha, k, R$ )

**Input:**  $G = (T, E)$  such that  $T$  is the set of terminals and  $E$  is the initial set of edges,  $\alpha$  is the target Standard Deviation,  $k$  is the maximum number of relay nodes, and  $R$  is the positive constant which indicates the radio range of a sensor

**Output:**  $G' = (V, E')$  with  $V = T \cup S$  where  $S$  is the set of relay nodes and  $E'$  is the new set of edges

```

2 begin
3    $G' \leftarrow \text{Ratio-3-SMT}(G)$ 
4    $G' \leftarrow \text{MakeFair}(G', \alpha, k)$ 
5   return  $G'$ 
6 end
```

**Algorithm 1:** FairSMT

2) *Optimizing Node Location:* SMT-MSP provides connectivity using minimum number of Steiner points. In some cases, the relay nodes in the resulting topology, may be too close or too far from each other. However this is not desirable, since it violates fairness of power consumption rates.

In this section we explain an extension to the SMT problem to include a fairness measure across the network of nodes. The explanation will include the moving of Steiner Points to optimal locations as well as the introduction of additional resources (nodes) to achieve optimality. The discussion includes an overview of the trade-off between additional resources and fairness and illustrates results of the optimization operations across multiple dimensions (such as minimizing the deployment of additional resources, maximizing fairness while minimizing power consumption).

Initially, our fairness approximation algorithm calculates the power consumption rates of each relay node (PCR) and inserts the nodes to a max heap. The main loop of the MakeFair method iteratively moves and adds relay nodes until the max number relay nodes allowed, is reached. The inner while loop of Algorithm 2 (lines 8-16), invokes MoveRelayNodes-geometric function, which is described in Algorithm 3. MoveRelayNodes-geometric takes each 3-stars as defined in [6], and moves the relay nodes to the center of circle that circumscribes the 3-neighbors. Here, our aim is to provide local fairness. After moving each relay node to the center of 3-star, we extract the node which has the highest PCR, and find its farthest neighbor (see Algorithm 4). Since the longest edge determines the PCR of the node, we add an additional relay node to the middle point the longest edge. Then we add newly deployed node along with the extracted node to the heap. Deploying new node will definitely change PCRs of some set of nodes. We move each relay node to the local optimal positions. In Algorithm 5, it is seen that, the optimal location for a node  $u$  is in between itself and its farthest neighbor. We calculate this location using some trigonometric functions. Then we insert  $u$  to the heap and continue iteration until  $|S| = k$  where  $S$  is the set of relay nodes and  $k$  is the maximum number of relay nodes which can be deployed.

##### B. Theoretical Analysis

In Fairness Approximation we first create a heap and insert the relay nodes to the heap. Assuming that

```

1 MakeFair( $G'$ ,  $\alpha$ ,  $k$ )
  Input:  $G = (V, E)$  with  $V = T \cup S$  where  $T$  is the
    set of terminals and  $S$  is set of nonterminals
    returned by Ratio-3 approximation,  $\alpha$  is the
    target Standard Deviation,  $k$  is the
    maximum number of relay nodes
  Output: ( $G' = (V', E')$ , achieved) where
     $V' = V \cup S'$  where  $S'$  is the set of newly
    added relay nodes and  $E'$  is the new set of
    edges, and achieved is boolean variable
    that indicates if target Standart Deviation
    is achieved or not
2 begin
3   // build a max-heap with the PCR of each
    steiner node
4    $heap \leftarrow Max - Heap(S)$ 
5   while  $|S| < k$  do
6      $\beta \leftarrow STDEVofPCRs()$ 
7      $dec \leftarrow true$ 
8     while  $dec = true$  do
9        $MoveRelayNode - geometric(G)$ 
10      if  $STDEVofPCRs() < \beta$  then
11         $dec \leftarrow true$ 
12         $\beta \leftarrow STDEVofPCRs()$ 
13      else
14         $dec \leftarrow false$ 
15      end
16    end
17     $AddRelayNode(G)$ 
18     $MoveRelayNodes(G)$ 
19  end
20   $G' \leftarrow G$ 
21   $\beta \leftarrow STDEVofPCRs()$ 
22  if  $\beta \leq \alpha$  then
23     $return (G', true)$ 
24  else
25     $return (G', false)$ 
26  end
27 end

```

**Algorithm 2:** Pseudo-code of fairness approximation

FIB-HEAP structure is used to implement the MAX-HEAP. Since amortized cost single insert operation is  $O(1)$ , the cost of inserting  $k$  elements into the heap is  $O(k)$  where  $k$  is maximum number of relay nodes which can be deployed. In the worst case, the outer while loop of Algorithm iterates  $k$  times and inner while loop iterates  $\beta$  times where  $\beta$  is the initial standard deviation of PCRs. The time complexity of

```

1 MoveRelayNodes-geometric( $G$ )
  Input:  $G = (V, E)$  with  $V = T \cup S$  where  $T$  is the
    set of terminals and  $S$  is set of nonterminals
  Output:  $G = (V, E)$  same graph with different
    edge lengths
2 begin
3   for  $s_i \in S$  do
4     if  $s_i$  has 3 neighbors then
5       if neighbors of  $s_i$  is on a circle then
6         move  $s_i$  to the center of the circle
7       end
8     end
9   end
10 end

```

**Algorithm 3:** Detailed description of MoveNodes-geometric function

```

1 AddRelayNodes( $G$ )
  Input:  $G = (V, E)$  with  $V = T \cup S$  where  $T$  is the
    set of terminals and  $S$  is set of nonterminals
  Output:  $G' = (V', E')$  where  $V' = V \cup u$  and  $u$  is
    the newly added relay node and  $E'$  is the
    new set of edges, and achieved is boolean
    variable that indicates if target Standart
    Deviation is achieved or not
2 begin
3    $s \leftarrow heap.extractMax()$ 
4    $t \leftarrow s.getFarthestNeighbor()$ 
5   remove  $(s, t)$  from the steiner tree.
6   deploy new steiner node  $u$ 
7    $p \leftarrow middle\ point(s, t)$ 
8    $u.setCoordinates(p)$ 
9   add  $(s, u)$  and  $(u, t)$  to the tree
10   $heap.insert(u)$ 
11   $heap.insert(s)$ 
12   $heapify()$ 
13 end

```

**Algorithm 4:** Pseudo-code of AddSteinerNodes

MoveRelayNodes-geometric is  $O(k)$ . So the total cost of inner while loop (lines 8-16 of Algorithm 2) is  $O(k\beta)$

In Algorithm 4, we extract the node with highest PCR from the heap, and add a relay node to middle of the longest edge. The cost of extracting from the heap is  $O(\lg k)$ . In Algorithm 5, we move each and every relay node to their local optimal locations. The time complexity of MoveRelayNodes function is  $O(k \lg k)$ .

```

1 MoveRelayNodes( $G$ )
  Input:  $G = (V, E)$  with  $V = T \cup S$  where  $T$  is the
    set of terminals and  $S$  is set of nonterminals
  Output:  $G = (V, E)$  same graph with different
    edge lengths
2 begin
3   while heap is not empty do
4      $s \leftarrow \text{heap.extractMax}()$ 
5      $t \leftarrow s.\text{getFarthestNeighbor}()$ 
6     // calculateLocation finds the best location  $p$ 
      which is in between  $s$  and  $t$ , where  $p$ 
      minimizes the PCR of  $s$ .
7      $p \leftarrow \text{calculateLocation}(s, t)$ 
8      $\text{move}(s, p)$ 
9      $\text{heap.insert}(s)$ 
10     $\text{heapify}()$ 
11  end
12 end

```

**Algorithm 5:** Pseudo-code of MoveRelayNodes

The overall time complexity is  $O(n^3) + O(k)(O(k\beta) + O(\lg k) + O(k \lg k)) = O(n^3) + O(k^2\beta) + O(k^2 \lg k)$

### C. Experimental Analysis

The theoretical analysis will be verified by an experimental analysis.

## V. CONCLUSION

Concluding remarks summarizing our approach and future work will be outlined in this section.

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