

MULTISCALE MODELING OF DIFFUSION PROCESSES IN THE BRAIN

by

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Abstract

This is an abstract text.

To someone

This is a dedication to my cat.

Acknowledgements

I acknowledge my acknowledgements.

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Chapter 1

The beginning is here

Start your chapter by writing something smart. Then go get coffee.

The very first approach was to simply try the problem on a bit. That is to try and substitute some small part of the mesh in a Finite Difference Diffusion solver (Forward Euler scheme) with a stochastic diffusion solver. A random walk method was implemented on part of the mesh to take over the equation-solving. This was done in 1 and 2 spatial dimensions with the aim of finding potential difficulties so that we can further investigate them.

Upon switching length-scales a fundamental question arises almost immediately; what is the continuum limit? In our case this question takes a slightly different, and possibly more answerable form; what is the conversion rate between the continuum model and the microscopic model, and by extension, what does a walker correspond to? The first instinct of this candidate was to just try some conversion rate (say some value corresponds to some number of walkers), and this was implemented in both 1 and 2 dimensions.

Chapter 2

Analysis

In this numerical setup we will potentially introduce several new error sources in addition to the normal errors introduced by numerical solution of any equation. When a part of the solution acquired is replaced by the solution from another model, which in this case is stochastic, we will change the initial condition to the next iteration in time. This might have a number of effects on our final solution. When we solve a differential equation numerically we only get an approximation to the actual solution because we are using approximate derivatives (see figure ??). We can investigate this type of error by doing two simulations of the same problem, but replacing the solution in one of the simulations in some area by the random walk model for one timestep.

```
path = '/home/fredriep/Dropbox/uio/thesis/doc/results/
       experiment_03102013_1218/results/'
import glob

i = []; e = []; s = []

for excl in sorted(glob.glob(path+'Excl*')):
    e.append(np.load(excl))

for incl in sorted(glob.glob(path+'Incl*')):
    i.append(np.load(incl))

for j in xrange(len(i)):
    s.append(np.max(np.abs(i[j]-e[j])))
print s[-1]
```

As we can see this will print the maximum absolute difference between the two solutions. Since we are dealing with a diffusion process we expect that the first timestep should have the largest error and that the error will in time be killed by the diffusion process.

```
...
0.0
0.023472
0.0089632
0.00600192
0.003841664
0.0030460416
0.00225273344
0.001974472704
0.0015961899008
0.00145438941184
0.00126162542592
0.0011517103702
0.00103607220175
0.000954075453981
0.000876626478367
0.000815189817568
```

The error is larger than Δt by a factor of 10 ($\Delta t = 0.002$)... This might be improved by increasing the number of walkers, or finding a better steplength for the walkers as well as improving the walk-algorithm etc..