

1. Consider the universal relation $R = \{a, b, c, d, e\}$ and the set of functional dependencies $FD = \{\{ab\} \rightarrow \{cde\}, \{cd\} \rightarrow \{abe\}, \{e\} \rightarrow \{d\}\}$.

Give a 3NF decomposition. You should underline the keys or write down all the keys for each of the 3NF relations. Show all work.

ANSWER:

The keys are (ab) , (cd) , (ce) .

The relation R is in 3NF; in the case of each FD, the LHS is key or the RHS is a prime attribute.

2. What is the highest normal form of the relation $R(a, b, c, d, e, f)$ given the following FDs: $ab \rightarrow cdef$; $c \rightarrow d$.

ANSWER: 2NF.

key (ab)

3. Consider the relation $R(a, b, c, d, e, f, g, h)$ and suppose we have the functional dependencies $b \rightarrow e$; $a \rightarrow b$; $de \rightarrow f$; $eg \rightarrow h$; $ch \rightarrow a$; $bd \rightarrow c$.

$$ch \rightarrow a \rightarrow b \Rightarrow ch \rightarrow b$$

1. (2) Which of the following FDs is also guaranteed to be satisfied by R ?

A. $ade \rightarrow ch$

B. $bdg \rightarrow ae$

C. $bg \rightarrow ae$

D. $gh \rightarrow bf$

$$(ade)^+ = (a, d, e, b, f, c)$$

$$(bdg)^+ = (b, d, g, e, c, h, a)$$

$$(bg)^+ = (b, g, e, f, h)$$

$$(gh)^+ = (g, h)$$

4. Consider the relation $R(a, b, c, d, e)$ and suppose we have the functional dependencies $ab \rightarrow c; ae \rightarrow d; d \rightarrow be$.
 $ae \rightarrow bd$ key (ae) (ad)

1. Check all 2NF decompositions of R .

A. $R_1(b, d, e)$ $R_2(a, c, d)$

B. $R_1(b, d)$ $R_2(a, c, d, e)$

C. $R_1(b, d)$ $R_2(a, b, c, e)$

D. $R_1(b, d, e)$ $R_2(a, b, c)$

$R \neq R_1 \cup R_2$

$d \rightarrow be$

but e is prime attribute

5. Consider the relation $R(a, b, c, d, e)$ and the set of FDs $ab \twoheadrightarrow c; ae \twoheadrightarrow d; d \twoheadrightarrow b$. Which of the following FDs does not follow from the above set? Check one answer.

1. A. $ad \twoheadrightarrow b$

B. $ae \twoheadrightarrow bc$

C. $ad \twoheadrightarrow c$

D. $be \twoheadrightarrow d$

$(ad)^+ = (a, d, b, c)$

$(ae)^+ = (a, e, d, b, c)$

$(be)^+ = (b, e)$

6. Assume that the relation $R(\underline{a}, \underline{b}, c, d, e)$ is in 3NF. Which of the following FDs must be false? Check all correct answers.

1. A. $ac \twoheadrightarrow e$

B. $cd \twoheadrightarrow b$ — prime

C. $abcd \twoheadrightarrow e$

D. $d \twoheadrightarrow c$

key —

7. Consider the relation $R(a, b, c, d, e)$ and the set of FDs: $ab \twoheadrightarrow de; b \twoheadrightarrow c; cd \twoheadrightarrow e$. Give a 2NF decomposition of R that is not in 3NF. You should underline the keys or write down all the keys for each of the 2NF relations. Also explain why the relation is not in 3NF.

ANSWER

Key of R is (ab) .

$R_1(b, c)$

$R_2(a, b, d, e)$

R_1 and R_2 in 2NF. R_2 is not in 3NF since $bd \twoheadrightarrow e$ where bd is not key and e is not a prime attribute.

8. Reconsider the above question: Give a 3NF decomposition of R . Show all work. You should underline or write down all the keys for each of the 3NF relations.

ANSWER

Key for R is (ab) .

$R_1(b, c)$

$R_2(a, b, d, e)$

R_1 in 3NF but not R_2 since $bd \twoheadrightarrow e$.

$R_3(b, d, e)$

$R_4(a, b, d)$

$R = R_1 \text{ join } R_3 \text{ join } R_4$.

$$(ab)^+ = \{a, b, c, d, e, f\}$$

$$\left. \begin{array}{l} b \rightarrow e \\ e \rightarrow c \end{array} \right\} \Rightarrow b \rightarrow c$$

9. Consider the relation $R(a, b, c, d, e, f)$ and the set of FDs: $ab \twoheadrightarrow cd$; $e \twoheadrightarrow c$; $b \twoheadrightarrow ef$. Which of the following are 3NF decompositions of R ? Check all the correct answers.

key (ab)

1. A. $R(a, b, c, d, e, f)$

B. $R_1(a, b, c, d)$ $R_2(e, f, b)$ $R_3(c, e)$ ✗

C. $R_1(a, b, c, d)$ $R_2(e, f, b)$ $R_3(f, c)$ ✗

D. $R_1(a, b, d)$ $R_2(e, c)$ $R_3(e, f, b)$

$e \rightarrow c$ $R_1(\underline{e}, c)$

$R_2(\underline{a}, \underline{b}, d, e, f)$ ✗

$R_3(\underline{b}, e, f)$

$R_4(\underline{a}, \underline{b}, d)$

10. Consider the relation $R(a, b, c, d, e)$ and the set of FDs: $a \twoheadrightarrow c$; $d \twoheadrightarrow ce$. Check all the BCNF decompositions of R .

key (abd)

1. A. $R(a, b, c, d, e)$

B. $R_1(a, c)$ $R_2(d, c, e)$ $R_3(a, b, d)$ $R \neq R_1 \cup R_2 \cup R_3$

C. $R_1(d, c, e)$ $R_2(a, b, d)$

D. $R_1(a, c)$ $R_2(d, e)$ $R_3(a, b, d)$

$R_1(\underline{a}, c)$ ✗

$R_2(\underline{a}, \underline{b}, \underline{d}, e)$ ✗

$R_3(\underline{d}, e)$ $R_4(a, b, d)$

11. Consider the relation $R(B, O, I, S, Q, D)$ and the set of FDs: $I \twoheadrightarrow B$; $IS \twoheadrightarrow Q$; $B \twoheadrightarrow O$; $S \twoheadrightarrow D$. Check all the BCNF decompositions of R .

1. A. $R_1(I, B)$ $R_2(S, D)$ $R_3(I, O)$ $R_4(I, S, Q)$

B. $R_1(B, O)$ $R_2(I, B)$ $R_3(S, D)$ $R_4(I, S, Q)$

C. $R_1(\underline{I}, B, O)$ $R_2(S, D)$ $R_3(I, S, Q)$ ✗

D. $R_1(B, O)$ $R_2(I, B)$ $R_3(I, S, Q, D)$ ✗

12. Reconsider your answer(s) to the previous question. List the BCNF decompositions that preserve all the FDs.

Option A: last FD: $B \rightarrow O$

ANSWER

Option B: $R1(B, O) R2(I, B) R3(S, D), R4(I, S, Q)$ preserves all the FD.

13. Consider the relation $R(a, b, c, \cancel{d}, \cancel{e}, \cancel{f}, \cancel{g}, \cancel{h})$ and the set of FDs: $ab \twoheadrightarrow e; c \twoheadrightarrow d; f \twoheadrightarrow gh; b \twoheadrightarrow f$. If R is not in 2NF, decompose into 2NF relations that are not in 3NF. Write down the key of the original relation R and of the 2NF relations. Show all work.

ANSWER $R1(\underline{a}, \underline{b}, e)$

key $(abc)^+ = (a, b, c, e, d, f, g, h)$

$R2(\underline{c}, d)$

$R3(\underline{b}, f, g, h)$

$R4(\underline{a}, \underline{b}, \underline{c})$

Not in 3NF since $R3(\underline{b}, \underline{f}, \underline{g}, \underline{h})$

14. Reconsider the above question. If R is not in 3NF, decompose into 3NF relations. Write down the key of the original relation R and of the 3NF relations. Show all work.

ANSWER

$R1(\underline{a}, \underline{b}, e)$

$R2(\underline{c}, d)$

$R3(f, g, h)$

$R5(\underline{b}, \underline{f})$

$R6(a, b, c)$

$(ab)^+ = \{a, b, c, d, e\}$ $(ad)^+ = \{a, d, b, e, c\}$ $(ac)^+ = \{a, c, d, b, e\}$

- ✓ 15. Consider the relation $R(a, b, \cancel{c}, \cancel{d}, \cancel{e})$ and the set of FDs: $ab \twoheadrightarrow c; c \twoheadrightarrow d; d \twoheadrightarrow be$. Check all BCNF decompositions of R .

key $(ab) (ac) (ad)$

1. A. $R1(\underline{c}, d) R2(b, \underline{c}, e) R3(\underline{a}, \underline{c})$

B. $R1(\underline{c}, d) R2(b, \underline{d}, e) R3(\underline{a}, \underline{c})$

C. $R1(a, b, c) R2(b, d, e) R3(c, d)$

D. $R1(a, b, c) R2(c, e) R3(c, d)$

key $(ab) (ac)$

$d \twoheadrightarrow b, e$
 $R1(\underline{d}, b, e)$ ✓

$R2(\underline{a}, \underline{d}, \underline{c})$ ✗

$R3(\underline{c}, d)$
 $R4(\underline{a}, \underline{c})$

$c \twoheadrightarrow d; d \twoheadrightarrow be \Rightarrow c \twoheadrightarrow be$

cannot be (\underline{c}, d, b, e)

✓ $R1(\underline{c}, d)$

$R2(a, b, c, e)$ ✗

$R3(\underline{b}, \underline{c}, e)$

$R4(\underline{a}, \underline{c})$

16. Suppose relation $R(a, b, c)$ currently has only the tuple $(0,0,0)$, and it must always satisfy the functional dependencies $a \twoheadrightarrow b$, $b \twoheadrightarrow c$. Which of the following tuples may be inserted into R legally?

1. A. $(0, 2, 1)$
 B. $(1, 1, 0)$
 C. $(0, 1, 1)$
 D. $(2, 0, 1)$

17. Let relation $R(A,B,C,D,E)$ satisfy the following functional dependencies: $ab \twoheadrightarrow c$, $bc \twoheadrightarrow d$, $cd \twoheadrightarrow e$, $de \twoheadrightarrow a$, $ae \twoheadrightarrow b$. Which of the following FDs is also guaranteed to be satisfied by R ? Check one answer.

1. A. $bc \twoheadrightarrow a$ $(bc)^+ = (b, c, d, e, a)$
 B. $bd \twoheadrightarrow a$
 C. $b \twoheadrightarrow a$
 D. $ac \twoheadrightarrow b$

18. Consider the relation $R(a, b, c, d, e)$ and the set of FDs: $a \twoheadrightarrow b$, $d \twoheadrightarrow e$, $ad \twoheadrightarrow c$, $c \twoheadrightarrow d$. Check one BCNF decomposition of R .

- best answer
 1. A. $R_1(a, c) R_2(d, e) R_3(a, b) R_4(c, d)$
 B. $R_1(a, b) R_2(a, c) R_3(a, d), R_4(c, e)$
 C. $R_1(a, b, c) R_2(c, d) R_3(c, e)$
 D. $R_1(a, c) R_2(a, c, e) R_3(a, b) R_4(c, d)$

$$c \rightarrow de \quad d \rightarrow e$$

$$(ad)^+ = (a, b, d, c, e)$$

$$(ac)^+ = (a, b, c, d, e)$$

$$R_1(a, b)$$

$$R_2(a, c, d, e) \times$$

$$R_3(c, d)$$

$$R_4(a, c, e) \times$$

$$R_5(c, e)$$

$$R_6(a, c)$$

19. Consider the relation $R(a, b, c, d, e)$ with the following FDs: $b \twoheadrightarrow e$, $be \twoheadrightarrow d$, $cd \twoheadrightarrow a$, $d \twoheadrightarrow e$. List all the keys for R . Give a BCNF decomposition. You should underline the keys or write down all the keys for each of the BCNF relations. Show all work.

ANSWER:

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$$FD: b \rightarrow ed$$

$$R_1(\underline{b}, e, d) \times$$

$$R_2(\underline{b}, a, c)$$

$$R_1'(\underline{b}, d)$$

$$R_1''(\underline{d}, e)$$

$$R = R_1' \bowtie R_1'' \bowtie R_2$$

Key = bc

R1(d, e) R3(c, d, a) R5(b, d) R6(b, c)

R1(d, e)
R2(a, b, c, d)
↑ ↓ ↓

R3(c, d, a)

R4(b, c, d) X
↑

R5(b, d)
R6(b, c)

There are other decompositions

20. Consider relation R(a, b, c, d) with functional dependencies:

$a \twoheadrightarrow b, c \twoheadrightarrow d, ad \twoheadrightarrow c, bc \twoheadrightarrow a$

Suppose we decompose R into BCNF. Which of the following schemas could not be in the result of the decomposition? Check one answer.

1. A. R1(a, b, c)

B. R1(a, c)

C. R1(c, d)

D. R1(a, d)

key(a c) (ad) (bc)

R1(d, e)
R2(a, b, c, d) X
↑

R3(b, d)

R4(a, b, c)

another one