FD Rules

1) Splitting Rule for FDs:

$A_1 A_2 \cdots A_n \longrightarrow B_1 B_2 \cdots B_m$

$\Leftrightarrow$

$A_1 A_2 \cdots A_n \longrightarrow B_1$

$A_1 A_2 \cdots A_n \longrightarrow B_2$ valid

$\vdots$

$A_1 A_2 \cdots A_n \longrightarrow B_m$

$A_i \longrightarrow B_1 B_2 \cdots B_n$ not necessarily valid

$A_i \longrightarrow B_j$

Example: Fname, Lname $\longrightarrow$ SSN, Address

Fname, Lname $\longrightarrow$ SSN

Fname, Lname $\longrightarrow$ Address

Fname $\longrightarrow$ SSN ✗

2) Combining Rule of FDs

Fname, Lname $\longrightarrow$ SSN R1 (Fname, Lname, SSN)

Fname, Lname $\longrightarrow$ Address R2 (Fname, Lname, Addr)

Combine into a single table R (Fname, Lname, SSN, Addr)

3) Key, Superkey Rule of FDs.

$A_1 A_2 \cdots A_n$ is a key for a relation $R(A_1, A_2, \cdots, A_n, B_1, B_2, \cdots B_m)$ if

(I) $A_1 A_2 \cdots A_n \longrightarrow B_1 B_2 \cdots B_m$

(II) no proper subset of $\{A_1, A_2, \cdots, A_n\}$ is a FD of R

↓ minimal

If more than 1 key, pick one key as ~~primary key~~; others are <u>candidate keys</u>.

R (SSN, Fname, Lname, Addr, Phone, Dno)

SSN ⟶ Fname, Lname, Addr, Phone, Dno

Fname, Lname ⟶ SSN, Addr, Phone, Dno

(SSN) (Fname, Lname) are keys

Superkey : not minimal

e.g., SSN, Addr ⟶ Fname, Lname, Phone, Dno

Fname, Lname, Phone ⟶ SSN, Dno, Addr

<u>Defn:</u> A set of attributes that contains a key is a superkey.

$A_1 A_2 A_3 \cdots A_n \longrightarrow B_1 B_2 \cdots B_m$

$A_1 A_2 A_3 \cdots A_n$ $B_1 B_2 \cdots B_m$: Superkey

↓ prime attributes A_i

↓ non-prime attributes B_i

Example: R (a, b, c, d)

a ⟶ b c d

↓ prime ↓ non prime

Superkey: ab, abc, abcd, ad ....

4) Transitive Rule of FDs

a ⟶ b b ⟶ c ⟹ a ⟶ c

A	B	C
a1	b1	c1
a2	b2	c2
a3	b1	c1
a4	b2	c2

Example: R(a, b, c, d) Find FDs Find key : B

A	B	C	D
a1	b1	c1	d1
a1	b2	c1	d1
a2	b3	c2	d1
a2	b4	c3	d1

B ⟶ A, C, D

C ⟶ D C ⟶ A C ⟶ A, D

A ⟶ D

Question: How to find key given a relation R with all attributes and the FDs.

FD1: SSN ⟶ name, mgr, dno

FD2: dno ⟶ dname, mgr

FD3 : pno → pname, Plocation, dno

FD4 : SSN, pno → hrs

R : (SSN, name, mgr, dno, dname, pno, pname, Plocation, hrs)

Closure of attributes

$(SSN)^+$ = { SSN, name, mgr, dno, dname }
FD1 FD2

$(Pno)^+$ = { Pno, Pname, Plocation, dno, dname, mgr }
FD3 FD2

$(SSN, Pno)^+$ = { SSN, pno, hrs, Pname, Plocation, dno, name, mgr, dname }
FD4 FD3

Key of R is (SSN, pno)

Example : R(a, b, c, d, E) Find key

d → b

ab → c

aE → d

$(d)^+ = \{d, b\}$

$(ab)^+ = \{a, b, c\}$

$(aE)^+ = \{a, E, d, b, c\}$

Example: $R(a, b, c, d, e, f)$

$ab \rightarrow c$

$bc \rightarrow ad$

$d \rightarrow e$

$cf \rightarrow b$

Find keys.

$(ab)^+ = \{a, b, c, d, e\}$

$(bc)^+ = \{b, c, a, d, e\}$

$(cf)^+ = \{c, f, b, a, d, e\}$ cf is key

$(abf)^+ = \{a, b, f, c, d, e\}$

candidate keys (cf) (abf)

$(bcf)^+ = \{a, b, c, d, e, f\}$

↳ superkey