1. Consider the universal relation R = {a, b, c, d, E} and the set of functional dependencies $FD = \{\{a\} \rightarrow \{b, c\}, \{b\} \rightarrow \{d\}\}$ and MVD = $\{\{b\} \twoheadrightarrow \{c, d\}\}$

a → b, c, d

⇒ b ↠ c

Minimal set

a → b, c, d

b → d

b ↠ c

① 1. (1) Write all the candidate keys for R.

aE

② 2. (6) What schema(s) would be produced by the 4NF decomposition algorithm?

A. R1(a,d), R2(b,c), R3(a,b), R4(a,E)

B. R1(b,d), R2(b,c), R3(a,b), R4(a,E)

C. both options are correct

D. both options are wrong

Start with b → d

R1 {b, d}

R2 {a, b, c, E} ✗

R3 {b, c}

R4 {a, b, E} ✗

R5 {a, b}

R6 {a, E}

Start with b ↠ c

R1(b, c) ✓

R2(a, b, d, E) ✗

R3 (a, b, d) ✗

R4 (a, E) ✓

R5 (a, b) ✓

R6 (b, d) ✓

Start with a → b, c, d

a → bcd

R1 (a, b, c, d) ✗

R2 (a E)

R3 (b, c)

R4 (b, d)

R5 (a, b)

$a \twoheadrightarrow b$
$ab \to E$
$a \to d$
$\Rightarrow a \to E$

$a \twoheadrightarrow b$
$ab \twoheadrightarrow c$
$\Rightarrow a \twoheadrightarrow c$

2. Consider the universal relation R = {a, b, c, d, E} and the set of functional dependencies $FD = \{\{a, b\} \to \{E\}, \{a\} \to \{d\}\}$ and the set of MVDs = $\{\{a\} \twoheadrightarrow \{b\}, \{a, b\} \twoheadrightarrow \{c\}\}$.

$a \twoheadrightarrow c\,d\,E$ $\quad ab \twoheadrightarrow d\,E$
$a \twoheadrightarrow d$
$a \twoheadrightarrow b$
$a \twoheadrightarrow c\,E$

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1. (1) Write all the candidate keys for R.

(abc)

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2. (4) What schema(s) would be produced by the 4NF decomposition algorithm?

A. R1={a,b}, R2={a,d}, R3={a,c,E}

B. R1={a,b},R2={a,E}, R3={a,c}, R4={a,d}

C. R1={a,b},R2={a,d,E}, R3={a,c}

D. R1={a,b,E},R2={a,b}, R3={a,c}, R4={a,d}

$a \twoheadrightarrow b$

a — b1 — c1 d1 E1, c2 d2 E2
a — b2 — c1 d1 E1, c2 d2 E2

$a \to d$

R1 {a, b, c} ×
R2 {a, b, d, E)

R3 (a, b) $\quad$ R4 (a, c)
R5 (a, d, E)

$a \to d, E$
$a \twoheadrightarrow b$
$ab \twoheadrightarrow c$

FD: $ab \to E$

R1 (a, b, E) ×
R2 (a, b, c, d) ×

R3 (a, b) $\quad$ R4 (a, E)
R5 (a, c) $\quad$ R6 (a, d)

Prove by contradiction:
Suppose $a \not\to E$
$\Rightarrow \exists$ $a e_1$ & $a e_2$
$a b c d e_1$; $a b c d e_2$ but $ab \to e$
$\Rightarrow a b_1 c d e_1$, $a b_2 c d e_2$
But $a \twoheadrightarrow b$

a — b_1 — c d e1, c d e2
a — b_2 — c d e1, c d e2

But this contradicts
$ab \to E$.
Hence $a \to E$

$a \twoheadrightarrow b$

$a\ b_1\ c_1\ d_1\ e_1$
$a\ b_2\ c_1\ d_1\ e_1$
$a\ b_1\ c_2\ d_2\ e_2$
$a\ b_2\ c_2\ d_2\ e_2$

$ab \twoheadrightarrow c$

a b_1 — c_1 — $d_1\ e_1$, $d_2\ e_2$
a b_1 — c_2 — $d_1\ e_1$, $d_2\ e_2$

3. Consider the universal relation R = {a, b, c, d, E} and the set of functional dependencies $FD = \{\{a\} \to \{b\}, \{b\} \to \{ce\}\}$ and the set of MVDs = $\{\{b\} \twoheadrightarrow \{d\}$.

$a \longrightarrow b, c, e$ $\quad a \twoheadrightarrow d$

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1. (1) Write all the candidate keys for R.

ad

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2. (3) Decompose the relation R into 4NF relations.

R1 (b, c, e) R2 (b, d) R3 (a, b)

or

R1 (b, c, e) R2 (a, d) R3 (a, b)

Even if you start with A → BCE

R1 (a, b, c, e) ✗ → R3 (b, c, e) R4 (a, b)

R2 (a, d)

A	B	C	D
a1	b1	c1	d1
a1	b2	c1	d1
a1	b2	c2	d2
a1	b1	c2	d2
a2	b3	c1	d2

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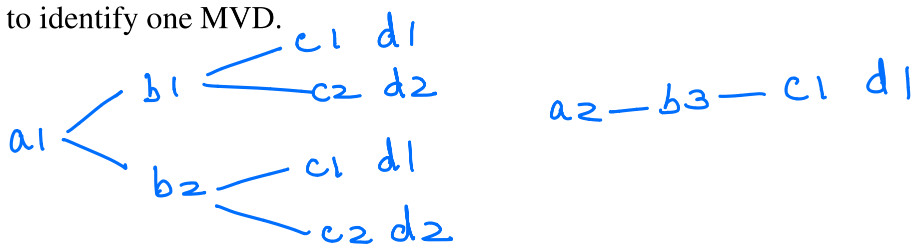
1. (4) Does the above table have a non-trivial MVD that is not a FD? If so , identify the MVD. You just have to identify one MVD.

$A \twoheadrightarrow B$



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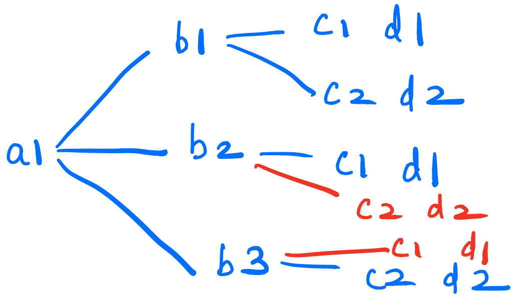
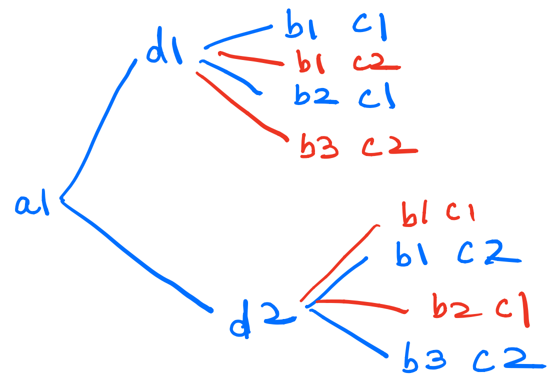
2. (2) Identify one candidate key.

(b, c) (b, d)



4.

A	B	C	D
a1	b1	c1	d1
a1	b1	c2	d2
a1	b2	c1	d1
a1	b3	c2	d2
a2	b2	c1	d1



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1. (3) What functional dependencies would be satisfied by inserting the tuples (a1,b1,c2,d1) and (a1,b1,c1,d2) in the table:

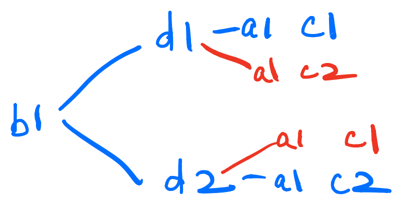
   A. $A \twoheadrightarrow B$

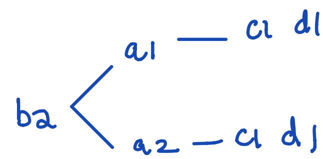
   B. $A \twoheadrightarrow D$

   C. $B \twoheadrightarrow A$ — okay answer

   D. $B \twoheadrightarrow D$

   E. none of the above





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2. (3) What functional dependencies would be satisfied by inserting the tuples (a1,b2,c2,d2) and (a1,b3,c1,d1) in the table:

   A. $A \twoheadrightarrow B$

   B. $A \twoheadrightarrow D$

   C. $B \twoheadrightarrow D$

   D. $D \twoheadrightarrow A$

   E. none of the above

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3. (3) What functional dependencies would be satisfied by inserting the tuples (a1,b1,c1,d2), (a1,b2,c1,d2), (a1,b1,c2,d1) and (a1,b3,c2,d1) in the table:

   A. $A \twoheadrightarrow B$

   B. $A \twoheadrightarrow D$

   C. $D \twoheadrightarrow B$

   D. $B \twoheadrightarrow A$

   E. none of the above

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4. (3) What functional dependencies would be satisfied by inserting the tuple (a2,b1,c1,d1) in the table:

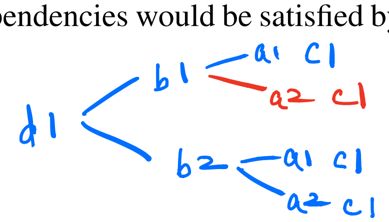
   A. $B \twoheadrightarrow A$

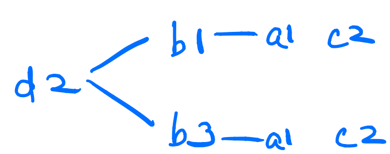
   B. $A \twoheadrightarrow D$

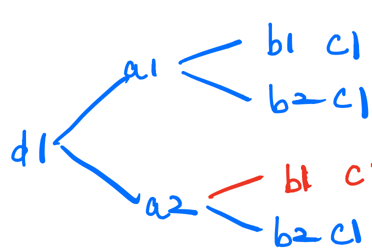
   C. $D \twoheadrightarrow B$

   D. $D \twoheadrightarrow A$

   E. none of the above







5. Consider the relation R = {a, b, ~~c~~, d, E} and the set of functional dependencies $FD = \{\{b\} \rightarrow \{c\}\}$ and the set of MVDs = $\{\{a\} \twoheadrightarrow \{b\}\{c\} \twoheadrightarrow \{d\}\}$.

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1. (1) Write all the candidate keys for R.

~~(a bd)~~ (abd E)

2. (3) Decompose the relation R into 4NF relations.

R1 (b, c) ✓

R2 (a, b, d, E) X

R3 (a, b)

R4 (a, d, E)

$R = R1 \bowtie R3 \bowtie R4$

There may be other 4NFs.

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6. (6) Consider a relation R(A,B,C,D) that satisfies $A \twoheadrightarrow B$ and $A \twoheadrightarrow C$. Prove that $A \twoheadrightarrow BC$ OR present a counter example showing the relationship as false.

Proof:

$A \twoheadrightarrow B \Rightarrow A \twoheadrightarrow CD$ Rule 4

$A \twoheadrightarrow C \Rightarrow A \twoheadrightarrow BD$ Rule 4

Now $A \twoheadrightarrow CD$ and $A \twoheadrightarrow BD$

$\Rightarrow A \twoheadrightarrow D$ (by Rule 5)

$\Rightarrow A \twoheadrightarrow BC$ (by Rule 4)

x —— x —— x

Proof 2

You can also prove by contradiction.

This is a longer proof.

Proof 2 : by contradiction.

Suppose $A \twoheadrightarrow B$ and $A \twoheadrightarrow C$, but $A \not\twoheadrightarrow BC$

Case 1 $\exists$ tuples $(a\ b\ c\ d)$, $(a\ b_1\ c\ d_1)$
but $\nexists$ tuples $(a\ b\ c\ d_1)$ and $(a\ b_1\ c\ d)$. — ①
Since $A \twoheadrightarrow B$, (a, b, c, d) $(a\ b_1\ c\ d_1)$
$\Rightarrow$ $\exists$ tuples (a, b, c, d_1) and $(a\ b_1\ c\ d_1)$
Thus statement ① is false

Case 2 $\exists$ tuples $(a\ b\ c\ d)$, $(a\ b\ c_1\ d_1)$ but
$\nexists$ tuples $(a\ b\ c\ d_1)$ and $(a\ b\ c_1\ d)$ — ②
Since $A \twoheadrightarrow C$, $(a\ b\ c\ d)$, $(a\ b\ c_1\ d_1)$
$\Rightarrow$ $(a\ b\ c_1\ d)$ and $(a\ b\ c\ d_1)$
statement ② is false.

Case 3: $\exists$ $(a\ b\ c\ d)$ $(a\ b_1\ c_1\ d_1)$ but
$\nexists$ $(a\ b\ c\ d_1)$ and $(a\ b_1\ c_1\ d)$ — ③
Since $A \twoheadrightarrow B$ $\exists$ $(a\ b\ c_1\ d_1)$ and $(a\ b_1\ c\ d)$
Since $A \twoheadrightarrow C$ $\exists$ $(a\ b\ c\ d_1)$ and $(a\ b_1\ c_1\ d)$,
Statement ③ is false.

Thus, $A \twoheadrightarrow B$, $A \twoheadrightarrow C$ $\Rightarrow$ $A \twoheadrightarrow BC$