Example : R (a, b, c, d)

A	B	C	D
1	10	100	x
1	20	100	y
1	20	100	x
1	10	100	y
2	10	200	x
2	30	300	x
2	30	200	x
2	10	300	x

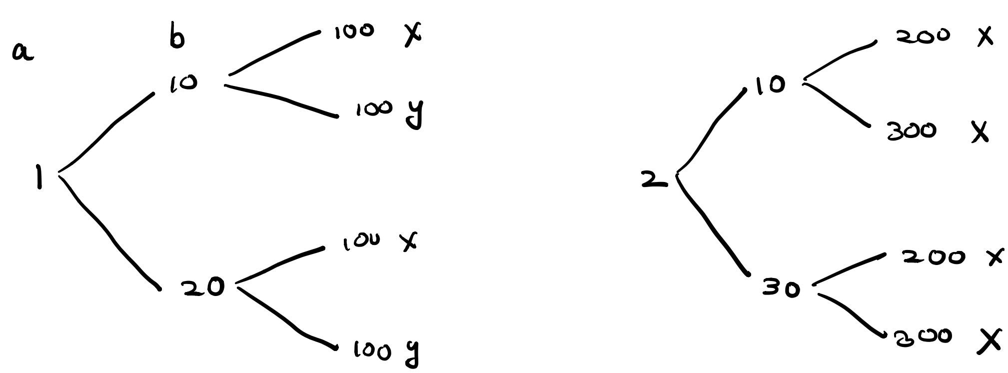
① $a \twoheadrightarrow b$? ✓

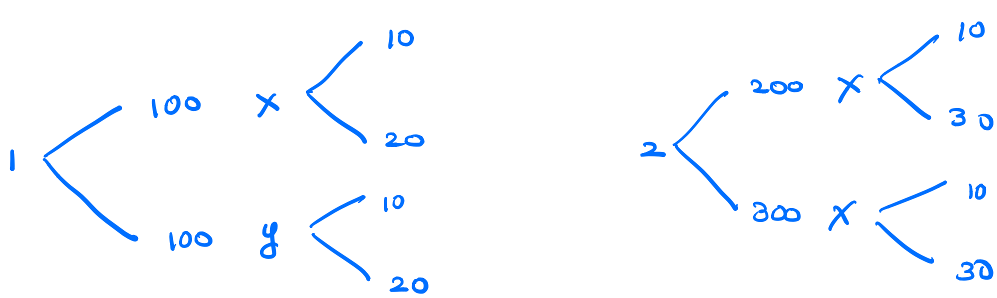
② $a \twoheadrightarrow cd$? ✓

③ $a \twoheadrightarrow c$? ✓

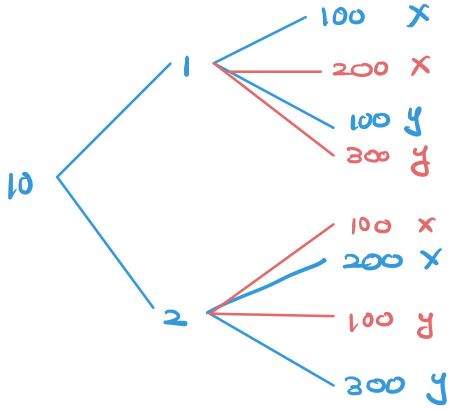
④ $a \twoheadrightarrow d$? ✓

⑤ $b \twoheadrightarrow a$?



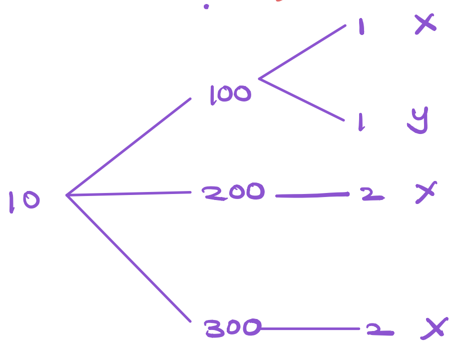


$b \twoheadrightarrow a$? ✗



A	B	C	D
1	10	100	x
1	20	100	y
1	20	100	x
1	10	100	y
2	10	200	x
2	30	300	x
2	30	200	x
2	10	300	x

$b \twoheadrightarrow c$? ✗



$c \twoheadrightarrow a$?

$c \twoheadrightarrow d$?

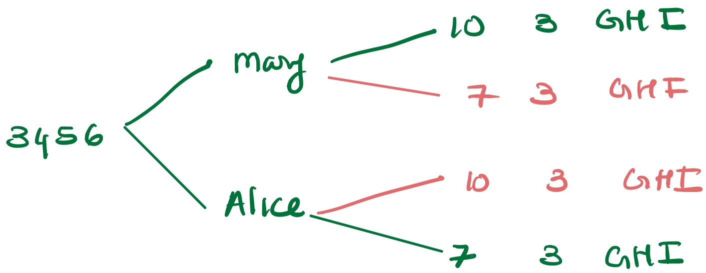
Example: Emp # Dependent, DepAge, Proj#, Pname

Emp#	Dep	Age	Proj#	Pname
1234	Bob	2	1	ABC
1234	Bob	2	2	DEF
3456	Mary	10	3	GHI
3456	Alice	7	3	GHI

Emp # $\twoheadrightarrow$ Dep, Age

$Emp\# \twoheadrightarrow Prog\#, Pname$

$Emp\# \twoheadrightarrow Dep$? ✗



Property: $A \twoheadrightarrow BC \nRightarrow A \twoheadrightarrow B, \; A \twoheadrightarrow C$

Trivial MVDs

$\bar{A}$: $A_1 A_2 \dots$

$\bar{B}$: $B_1 B_2 \dots$

1) If $\bar{B} \subseteq \bar{A}$ then $\bar{A} \twoheadrightarrow \bar{B}$

e.g. $AB \twoheadrightarrow A$; $AB \twoheadrightarrow B$

2) If $R = \bar{A} \cup \bar{B}$ then $\bar{A} \twoheadrightarrow \bar{B}$ and $\bar{B} \twoheadrightarrow \bar{A}$

e.g. $R(a, b, c)$

$a \rightarrow bc$

$ab \rightarrow c$

$bc \rightarrow a$

$b \rightarrow ac$

$c \rightarrow ab$

$\vdots$

Rules :

1) If $R = (\bar{A}, \bar{B}, \bar{C})$ and $\bar{A} \twoheadrightarrow \bar{B} \Rightarrow \bar{A} \twoheadrightarrow \bar{C}$

$A \twoheadrightarrow B \mid C$

2) If $A \rightarrow B$ then $A \twoheadrightarrow B$

3) Splitting rule does not apply.

if $A \twoheadrightarrow BC \not\Rightarrow A \twoheadrightarrow B, \; A \twoheadrightarrow C$

4) Transitive Rule

$A \twoheadrightarrow B \quad B \twoheadrightarrow C \Rightarrow A \twoheadrightarrow C$

5) $R(A, B, C, D)$

if $A \twoheadrightarrow B$ then $A \twoheadrightarrow CD$

if $A \twoheadrightarrow CD$ then $A \twoheadrightarrow B$

6) if $A \twoheadrightarrow BC$ and $A \twoheadrightarrow CD$

then $A \twoheadrightarrow C$

7) if $A \twoheadrightarrow B, \; A \twoheadrightarrow C$ then $A \twoheadrightarrow BC$

8) FD: $AB \rightarrow C$

MVD: $ABC \twoheadrightarrow D$

$\Rightarrow AB \twoheadrightarrow D$

can be derived from 1 – 6

Question: if $A \twoheadrightarrow C$ and $B \twoheadrightarrow C$ does $AB \twoheadrightarrow C$?

Defn: A relation R is in 4NF if whenever $\bar{A} \twoheadrightarrow \bar{B}$ is a nontrivial MVD, then $\bar{A}$ is a key / superkey.
(i.e., R is in 4NF if it has no non-trivial MVDs)

e.g., R(a, b, c, d) R is not in 4NF since b is not key

4NF Decomposition Algorithm

Input: R, FDs, MVDs

① Find keys for R

② loop until 4NF:

* select FD: $A \rightarrow B$ or nontrivial MVD: $A \twoheadrightarrow B$ where A is not key / superkey

* decompose R into
  R1 (A, B) if $A \rightarrow B$ key (A)
  or
  R1 (A, B) if MVD $A \twoheadrightarrow B$ key (A B)
  and
  R2 (R − B)

Find FDs and MVDs, keys for R1 & R2

Example: Student (id, name, course#, major)

$id \rightarrow name$

$id \twoheadrightarrow course\#$

Key (id, course#, major)

Start with FD: $id \rightarrow name$ that violates

R1 (<u>id</u>, name) ✓ 4NF

R2 (<u>id</u>, <u>course#</u>, <u>major</u>) ✗ 4NF

nontrivial

R3 (<u>id</u>, <u>course#</u>) trivial MVD ✓ 4NF

R4 (<u>id</u>, <u>major</u>) trivial MVD ✓ 4NF

Student = R1 ⋈ R3 ⋈ R4   R1, R3, R4 are in 4NF

Example: