Jurgen A. Doornik

An Object-oriented Matrix Programming Language

Ox™ 7

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Preface

My interest in computer languages was awakened by reading a short book on compiler building written by Niklaus Wirth (see Wirth, 1987). The first opportunity to dabble in this field was the design and implementation of the algebra language in PcGive 7 (also used in later versions). The result was a tiny vector language, not very efficient, but it worked. My ambition was to write a more powerful language, to leverage the growing body of computational code that I was developing. The next attempt took a few weeks at the end of 1992. It did not lead to a useful program, but the experience helped in the third and serious attempt: Ox. That project was started in April 1994, just after completing version 8 of PcGive. The aim was to use it for the simulations required for my doctoral thesis. Having done most of my programming of recent years in C, I was unhappy with the syntax of the matrix languages I tried. By November I had a preliminary version. It had a database and PcFiml class, and I could use it for my simulations. The Ox library gradually expanded, but my thesis had a higher priority. In the summer of 1995, the number of Ox users tripled: both Neil Shephard and Richard Spady started to use it. Their wishes and comments helped push Ox towards its current form, including Unix versions, support for DLLs, more graphics and many types of random number generators.

The origin of the name Ox is a bit vague. It is the first and last letter of Object-oriented matrix. Initially I was comparing the program to an ox: a solid work animal but quite slow. Since then, however, Ox has become a lot faster, to the point where it is even beating some native C and Fortran programs. Alternatively, the name can be interpreted as a tribute to Oxford and its University.

Of course, there is still much to be added to Ox, and development will continue. OxEdit supplies the integrated environment for development, but a visual debugger remains on the wish-list.

Please keep sending your suggestions for improvements. Questions regarding Ox and Ox packages should primarily be addressed to the ox-users discussion list. You can contact me if you need Ox on a platform which is currently not supported. My work page at `www.doornik.com` as well as `www.oxmetrics.net` are regularly updated with pointers to relevant Ox information.

Clearly, I wish to thank Neil Shephard and Richard Spady for adopting Ox early on, and their many comments and suggestions. Also to their students, who were encouraged to use Ox and gave feedback. By now, many more people have downloaded

Ox, and given it a try, among these Francisco Cribari-Neto deserves special thanks. I thank David Hendry for continuing support for this project, and also wish to thank Maureen Baker, Christopher Baum, Charles Bos, Peter Boswijk, Max Bruche, James Davidson, Ola Elerian, Richard Gascoigne, Frank Gerhard, Siem Jan Koopman, Hans-Martin Krolzig, Michal Kurcewicz, Ulrich Küsters, Richard Lewney, Sébastien Laurent, James MacKinnon, Aurora Manrique, Michael Massmann, Sophocles Mavroeidis, Steve Moyle, Bent Nielsen, Marius Ooms, Mike Orszag, Felix Ritchie, Pieter Jelle van der Sluis, Ana Timberlake, Giovanni Urga. And, of course, thanks to all those people who have sent me email messages saying how much they appreciate Ox (I like those!).

As the proverbial last but not least, I wish to thank Kate Doornik: without her support and company I would not have managed.

Oxford, May 2012

I wish you enjoyable and productive use of

Ox

Part I

Introduction to Ox

Chapter 1

Summary information

1.1 What is Ox?

Ox is an object-oriented matrix programming language with a comprehensive mathematical and statistical function library. Matrices can be used directly in expressions, for example to multiply two matrices, or to invert a matrix. The major features of Ox are its speed, extensive library, and well-designed syntax, which leads to programs which are easier to maintain.

1.2 Availability

The full Windows version of Ox, called **Ox Professional** is available from Timberlake Consultants. Timberlake can be found on the internet at `www.timberlake.co.uk` and `www.timberlake-consultancy.com`, or contacted via telephone in the UK on: +44 (0)20 8697 3377 , and in the US on: +1 908 686 1251.

The Windows command-line, and Unix versions of Ox can be downloaded from: `www.doornik.com`. These are called **Ox Console**.

Please check the `ox\doc\readme.ox` file before installation (and for Unix installations: `ox/doc/readunix.txt`).

1.3 Ox version

This documentation refers to version 7.10. Check the web addresses given in §1.8 for changes which were made after publication of this book.

1.4 Learning Ox

The best place to learn Ox is Doornik and Ooms (2006), which gives an introduction to the Ox language, complemented with econometric and statistical examples, as well as

Table 1.1 Ox executables

Platform	Name	Console	DLL	Debug	Graphs	
					Save	Show
Windows/Linux/OS X	oxl	yes	yes	no	yes	no
Windows	oxli	yes	yes	yes	yes	no
Windows/Linux/OS X	OxRun	no	yes	yes	yes	yes

many exercises (the tutorial files are installed in `ox\tutorial`). Also see the *Getting started* section in the on-line help system. Chapter 3 gives a shorter introduction.

1.5 Ox platforms

Ox is currently available on Windows, Linux for PC, and OS X. Table 1.1 lists the current set of Ox executables. Console indicates whether the compiler is launched from the command line, or using an interactive program. The first yes/no under graphics indicates whether graphs can be created and saved to disk, the second whether graphs can be displayed on screen by Ox.

Under Linux the programs are run through scripts: `oxl` for 32-bit and `oxl64` for 64-bit Linux.

Ox can be run in four ways:

1. from the console (command line) using `oxl` (bin64/oxl in 64-bit Windows; oxl64 in 64-bit Linux).
   Graphs cannot be displayed.
2. from `OxEdit` (the ‘running person’ icon) using `oxl`
   Graphs cannot be displayed.
3. from `OxMetrics` (either using OxRun or the ‘running person’ icon)
   Graphs are shown in OxMetrics. This requires Ox Professional.
4. from the console (command line) using `oxli`
   This supports command line debugging and interactive use. Graphs cannot be displayed. This requires Ox Professional.

1.6 Ox supported data formats

Ox can read (and write) the following data files directly into a matrix:

- .mat (ASCII matrix file),
- .dat (ASCII data file with load information),
- .in7 (PcGive 7 data file, with data in .bn7 file),
- .xlsx (Excel 2007 or newer workbook files),
- .csv (comma separated data file),
- .xls (Excel old-format spreadsheet files),
- .dht (Gauss data file, with data in .dat file),

- .fmt (Gauss matrix file),
- .dta (Stata data file, version 4–6 and 11).

In addition, there are text and low-level functions for reading and writing binary files.

1.7 Extending Ox

Ox can be extended on all platforms. The Ox Developer's manual documentation provides examples of what you can do:

- Develop an OxPack compatible interactive package when deriving from the Modelbase class.
- Make extensions to Ox in e.g. C/C++ or Fortran, and put that in a DLL; such functions are then callable from Ox code.
- Use Ox as a mathematics library (e.g. if you are programming in C/C++ but do not want to program in Ox; or to call functions such as Choleski decomposition or a random number generator in your Ox extension DLL).
- Write an interface wrapper around Ox code. Examples using Visual Basic and Visual C++ are given.

Using *OxRun*, Ox can use *OxMetrics* as a front-end, which holds databases, and receives text and graphical output from Ox (and also other modules such as PcGive, STAMP, PcNaive, etc.).

1.8 World Wide Web

Check `www.oxmetrics.net` or `www.doornik.com` for information on bugs, bug fixes, new features, benchmarks and other information relevant to Ox.

1.9 Online documentation

The Ox help system is implemented as a set of HTML pages which can be read with an internet browser. Open `\ox\docs\index.html` in your browser to start help.

1.10 Ox-users discussion list

The ox-users discussion group is an email-based forum to discuss any problems related to Ox programming, and share code and programming solutions. Consult the online help for information on joining the list.

1.11 Installation

Installation can be done as follows:

(1) For Windows use the provided setup program, which will do the complete installation. The settings of §1.12 should have been made automatically.
(2) For OS X use the provided setup programs, which will do the complete installation.
(3) Under Linux: use the RPM version for automated installation.

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1.12 Completing the basic installation

No further action is required, unless you wish to run the command line version of Ox (that is: `oxl`) from anywhere in your Command prompt window. In that case you need to add the `ox\bin` folder to your PATH statement (`ox\bin64` for the 64-bit version) . For example, assuming you, installed on the C drive, add:

```
C:\Program Files\OxMetrics7\ox\bin
```

to your PATH statement in the the control panel.

For Unix see §1.15.

1.13 Directory structure

`ox/bin`	– executables and DLLs
`ox/bin64`	– executables and DLLs for 64-bit version
`ox/data`	– default data directory
`ox/dev`	– examples on how to extend Ox
`ox/doc`	– documentation (start `index.html`)
`ox/include`	– Ox header files
`ox/lib`	– useful additional source code files
`ox/packages`	– Ox extension packages
`ox/samples`	– Ox samples directory with code for Ch. 2
`ox/src`	– Ox code for .oxo files in `ox/include`
`ox/tutorial`	– Tutorial files accompanying Doornik and Ooms (2006)

`ox/samples/bench`	– benchmark samples
`ox/samples/classes`	– Line and Angle classes from §13.5.6
`ox/samples/database`	– database class examples
`ox/samples/graphics`	– graphics examples
`ox/samples/inout`	– input/output examples
`ox/samples/lib`	– examples for source code files in `ox/lib`
`ox/samples/maximize`	– function maximization and differentiation
`ox/samples/oxpack`	– application illustrating OxPack dialogs
`ox/samples/pcfiml`	– PcFiml examples
`ox/samples/ranapp`	– C++ wrapper around Ox code*
`ox/samples/simulation`	– Simulator class examples
`ox/samples/virtual`	– Demonstrates virtual class member functions

The main executable files are in `ox/bin/` or `ox/bin64/`:

`oxl.exe`	– standard Ox compiler (Windows)
`oxl`	– standard Ox compiler (Unix)
`oxli.exe`	– debug/interactive Ox compiler (Windows)
`oxrun.exe`	– for using Ox with *OxMetrics* (Windows)
`oxpack.exe`	– for interactive use of many packages with *OxMetrics* (Windows)

1.14 OX7PATH

Ox 7.x uses the `OX7PATH` environment variable if this has been set. Under Windows there is no need to set this variables anymore, because the system will use a default (determined from the location of oxwin.dll). If you installed to `c:\ox` it defaults to:

```
c:\ox\include;c:\ox
```

Under Unix, the scripts that run the program set the `OX7PATH` variable.

Ox will read the `OX7PATH` environment variable on all platforms. If you do set it, you must include the default paths. For example, when Ox is run from the network (`X:\apps\ox` for example), and you wish to also use packages installed on the local harddisk at `c:\ox`, you could set:

```
set OX7PATH=X:\apps\ox\include;X:\apps\ox;c:\ox
```

Ox 6.x used the `OX6PATH` environment variable, allowing Ox version 6 and 7 installations to exist side by side.

1.15 Ox for Unix

The directory structure under Unix is identical to that under Windows. Setting environment variables, and resolving the dynamic link library works a bit differently. Here are some notes on Unix installation:

(0) Dynamic link libraries have the `.so` extension under Unix, except for HPUX, which uses `.sl`.

(1) Dynamic linking:
For Ox version 5.1, the naming scheme is as follows:

`libox.so.5.10.0`	is the (dynamically linked) Ox library,
`oxl`	is the executable which links to it.

Normally, step (3) and (4) below would be added to a startup script.

(2) version numbers:
In (e.g.) `oxl.so.5.10.0` the 5 is the major, and 10.0 the minor version number. The executable only looks for a file matching the major version number (`oxl.so.5`). A symbolic link is required to resolve the search:

```
rm libox.so.5
ln -s libox.so.5.00.0 libox.so.5
```

This numbering system is not used for HPUX and AIX. Under HPUX the DLL is called `oxl.sl`, and no symbolic link is required. Under AIX the shared library file is called `libox.o`.

(3) library search paths:
`LD_LIBRARY_PATH` is used to search for the library file when the file is in a non-standard location. This must be set to the directory where `oxl.so.5` is, unless it has been moved to standard directory which is searched by default. (HPUX uses `SHLIB_PATH`, and AIX uses `LIBPATH`, and OS-X uses DYLD_LIBRARY_PATH or DYLD_FALLBACK_LIBRARY_PATH.)

(4) `OX7PATH`
set the OX7PATH variable to allow Ox to find include files, e.g. to
`"$HOME/ox/include:$HOME/ox"`

Note that setting environment variables is shell specific. For example for bash shells you can use:

- to show contents of `LD_LIBRARY_PATH` environment variable:
  `echo $LD_LIBRARY_PATH`
- to set to a directory, e.g. to:
  `LD_LIBRARY_PATH="$HOME/ox/bin"; export LD_LIBRARY_PATH`
- to append a directory (this is one command):
  ```
  LD_LIBRARY_PATH="$LD_LIBRARY_PATH:$HOME/dir";
      export LD_LIBRARY_PATH
  ```
- to set the OX7PATH variable:
  `OX7PATH="$HOME/ox/include:$HOME/ox"; export OX7PATH`

Whereas other shells may use setenv/printenv:

- to show contents of `LD_LIBRARY_PATH` environment variable:
  `printenv $LD_LIBRARY_PATH`
- to set to a directory, e.g. to:
  `setenv LD_LIBRARY_PATH "$HOME/ox/bin"`
- to set the OX7PATH variable:
  `setenv OX7PATH "$HOME/ox/include:$HOME/ox"`

(5) File mode:
On some platforms it might be necessary to set the file mode to executable, for example:

```
chmod +x oxl
```

(6) `bin` directory
On most systems, when in the bin directory, it is necessary to run Ox as `./oxl` instead of just `oxl`.

(6) developing dynamic link libraries
When developing DLLs for Ox, the `ldd` command can sometimes help with finding out why a DLL does not link.

(7) The threes example shows how to create and call a dynamic link library. Run `make -f threes.mak` to compile `threes.so` (the header file `oxexport.h` and dependencies must be in the search path). On some platforms there may be unresolved messages from the linker, which may be ignored. Then run `oxl threes` to see if it works. The dynamic linker must be able to find `threes.so`, also (7).

(8) When adding a DLL which is to be used from Ox, Ox will try to locate it in the following way:
1. Try directly
2. Search along `OX7PATH` paths
3. Search along `OX7PATH` relative to `#import` statements which have a path component.
4. Try package/dll_name directory (but only if no path is used in the extern statement).

For example, when the declaration is:

```
        extern "arfima,FnFracSigma" fracsigma(....);
```

and arfima.oxo has:

```
        #import <modelbase>
        #import <lib/testres>
```

and somewhere else is:

```
        #import <packages/arfima/arfima>
```

moreover, the `OX7PATH` is set to `$HOME/ox/include:$HOME/ox`
Then Ox will try:

```
    1.  ./arfima.so
    2a. $HOME/ox/include/arfima.so
     b. $HOME/ox/arfima.so
    3a. $HOME/ox/include/lib/arfima.so
     b. $HOME/ox/lib/arfima.so
     c. $HOME/ox/include/packages/arfima/arfima.so
     d. $HOME/ox/packages/arfima/arfima.so
    4a. $HOME/ox/include/packages/arfima/arfima.so
     b. $HOME/ox/packages/arfima/arfima.so
```

So, when Ox is loading a library through the extern statement, the `LD_LIBRARY_PATH` is not used. Also note that under Windows, the operating system automatically searches along the path statement, which does not happen under Unix. (Of course, Unix is also case-sensitive, except for OS X).
DLLs for different platforms can be kept separate by using _64 and platform-specific suffix. For example, Ox will try first one of:

`arfima.dll`	Windows 32-bit
`arfima_64.dll`	Windows 64-bit
`arfima.so`	Linux 32-bit
`arfima_64.so`	Linux 64-bit
`arfima_osx.so`	OS X 32-bit
`arfima_sparc.so`	Solaris on Sparc, 32-bit
`arfima_sunx86.so`	Solaris on x86, 32-bit
`arfima_sunx86_64.so`	Solaris on x86, 64-bit

using the search method described above. If that fails, the search is done with just arfima.

Chapter 2

Getting started with Ox

2.1 Introduction

Ox is an object-oriented matrix language with a comprehensive mathematical and statistical function library. Matrices can be used directly in expressions, for example to multiply two matrices, or to invert a matrix. The basic syntax elements of Ox are similar to the C, C++ and Java languages (however, knowledge if these languages is not a prerequisite for using Ox). This similarity is most clear in syntax items such as loops, functions, arrays and classes. A major difference is that Ox variables have no explicit type, and that special support for matrices is available.

The advantages of object-oriented programming are that it potentially improves the clarity and maintainability of the code, as well as reducing coding effort through inheritance. Several useful classes are provided with Ox.

This chapter will introduce a first Ox program, and discuss the various ways in which the program can be executed. The next chapter will then give a brief overview of the language elements.

2.2 A first Ox program

As a first example of an Ox program consider the following Ox code:

samples/myfirst.ox

```
#include <oxstd.oxh>// include the Ox standard library header

main()                // function main is the starting point
{
    decl m1, m2;         // declare two variables, m1 and m2

    m1 = unit(3);    // assign to m1 a 3 x 3 identity matrix
    m1[0][0] = 2;               // set top-left element to 2
    m2 = <0,0,0;1,1,1>;  // m2 is a 2 x 3 matrix, the first
               // row consists of zeros, the second of ones

    println("two matrices", m1, m2);  // print the matrices
}
```

The program is in `ox\samples\myfirst.ox`; running this program should produce the following result:

```
two matrices
       2.0000      0.00000      0.00000
      0.00000       1.0000      0.00000
      0.00000      0.00000       1.0000

      0.00000      0.00000      0.00000
       1.0000       1.0000       1.0000
```

An Ox program consists of one or more *source code* files. All source files have the `.ox` extension. *Header files* are used to communicate declarations from one source file to another. Header files have the `.oxh` extension.[1]

The next section explains how to run the Ox program on your system. First we consider the `myfirst.ox` program in more detail:

- The first line includes the `oxstd.oxh` file into the source code (literally: the contents of the file are inserted at that point). This file contains the function declarations of the standard library, so that the function calls can be checked for number of arguments. The file name is between `< >`, indicating that the header file came with the Ox program.
- The function `main` is the starting point, and each program is only allowed one such function. Even though `main` has no arguments, it still requires `()`.
- Variables may be declared with the `decl` statement, and have no type until the program is actually run.
- `unit` is a standard library function, which creates an identity matrix; here it is called with argument `3`. The result is assigned to the variable `m1`. The type of `m1` has become *matrix*, and until a reassignment is made (or it goes out of scope), `m1` will keep its type and value.
- Note that *indexing starts at zero*, so the top-left element is `m1[0][0]`: row 0, column 0. The first index is the row index: `m1[1][2]` is row 1, column 2. Ox has this convention in common with many other programming languages (but it could be changed, see §13.9.5).
- `<0,0,0;1,1,1>` is a *matrix constant*. Elements are listed by row, whereby rows are separated by a semicolon, and elements within a row by a colon. This value is stored in `m2`, which is now also of type matrix.
- `println` is a library function, which can print any type of variable or constant to the standard output screen. It can take any number of arguments. Here it has three: a *string constant* and two variables (which both happen to be matrices).

An important advantage of Ox is that we can directly work with matrices, and do not have to worry about memory allocation and deallocation. Low-level languages may be faster, although we have encountered several cases in which Ox performed better than a comparable C program. Ox code has a much closer correspondence to mathematical expressions used on paper.

[1]Previous versions of Ox used `.h` instead. This still works as well.

2.3 Running the first Ox program

2.3.1 Ox Professional under Windows

Load the `myfirst.ox` program in OxMetrics and click on Run (the running person icon on the toolbar).

Or right-click on myfirst.ox in the workspace window after it has een loaded into OxMetrics, and select Run Ox.

2.3.2 Ox Console under Windows

Load the `myfirst.ox` program in OxEdit and click on Run (the running person icon on the toolbar).

If Ox Console (or Ox Professional) has been installed correctly, the Ox program can also be run from a command window (Command prompt or MS-DOS prompt under Windows) by typing (this assumes Ox is installed in `Program Files\OxMetrics7\ox` on the current drive):

```
cd "\Program Files\OxMetrics7\ox\samples"
```

Followed by

```
oxl myfirst
```

There is no need to add the `.ox` extension. If `oxl` cannot be found, you have to add the path to the executable file, which is in `ox\bin`:

```
..\bin\oxl myfirst
```

Having to add the path to `oxl.exe` everytime is a nuisance, and there are several alternatives which are more convenient:

- Add the `ox\bin` folder to the environment `PATH`. In the default installation this is `C:\Program Files\ox\bin`.
- Use **OxEdit** to run your Ox programs.
- Ox Professional users can run their programs (with graphics) from **OxMetrics**.

In the remainder, we refer to the MS-DOS window as the console window, and to `oxl.exe` as the console version of the Ox compiler.

If you do not get the output listed in the previous section check the installation notes in Chapter 1.11.

2.3.3 Ox Professional under Linux and OS X

Load the `myfirst.ox` program in OxMetrics and click on Run (the running person icon on the toolbar).

Or right-click on myfirst.ox in the workspace window after it has een loaded into OxMetrics, and select Run Ox.

2.3.4 Ox Console under Linux and OS X

Load the `myfirst.ox` program in OxEdit and click on Run (the running person icon on the toolbar).

If Ox has been installed correctly, the Ox program can also be run from a terminal window by typing (this assumes Ox is installed in `/ox` on the current drive, which is unlikely to be the correct path):

```
cd /ox/samples
```

Followed by

```
oxl myfirst
```

There is no need to add the `.ox` extension. Currently, there is only the console version under Unix.

If your output is:

```
myfirst.ox (1): 'oxstd.oxh' include file not found
myfirst.ox (7): 'unit' undeclared identifier
myfirst.ox (12): 'print' undeclared identifier
```

Then the header file was not found, and the `OX7PATH` environment variable is not set, or set wrongly. (Note that the environment variable is specific to the major version of Ox, e.g. `OX5PATH` was used by version 5.)

2.4 Online help

A large part of this book is part of the online help system. The Ox help system is implemented as a set of HTML pages. To start the help open `\ox\docs\index.html` in your browser.

OxMetrics conveniently lists the Ox Help index in the Help pane in the workspace. When editing an Ox file, press F1 for context-sensitive help for the word under the text caret.

2.5 Using file names in Ox

If you specify full path names of files in a string constant, you must either use one forward slash, or two backslashes: `"./data.mat"` or `".\\data.mat"`. Ox will interpret one backslash in a string as an escape sequence (as in `"\n"`, see §13.3.2.2); a single backslash will only work if it does not happen to form an escape sequence. Also note that the Windows and Unix versions of Ox can handle long file names, and that Unix treats file names in a case sensitive manner.

2.6 Ox file extensions

Table 2.1 summarizes file types (by extension) used in Ox.

Table 2.1 Ox extensions

extension	description
`.csv`	comma separated spread sheet text file,
`.dat`	ASCII data file with load information,
`.dht`	Gauss data file (with corresponding .dat file)
`.dll`	Dynamic link library (Windows)
`.eps`	Encapsulated PostScript file
`.fmt`	Gauss matrix file
`.gwg`	*OxMetrics* graphics file
`.h`	Ox header file (up to version 6)
`.in7/.bn7`	PcGive 7 data file (with corresponding .bn7 file)
`.mat`	ASCII matrix file
`.ox`	Ox source code file
`.oxh`	Ox header file (version 7 onwards)
`.oxo`	compiled Ox code (object file)
`.pdf`	PDF file
`.ps`	PostScript file
`.so`	Dynamic link library (Unix)
`.xls`	Excel old-format spreadsheet file
`.xlsx`	Excel 2007 workbook file

2.7 More on running Ox programs

2.7.1 Windows: OxMetrics and OxRun

OxRun is a small Windows front end to Ox. It offers the same services as the command-line compilers, but in the form of a dialog. *OxRun* can be started from *OxMetrics*, and text and graphics output from the Ox program will appear in *OxMetrics*. Alternatively, an Ox program can be loaded into OxMetrics, and then run directly by clicking on the Run icon.

OxRun and *OxMetrics* are part of Ox Professional, and cannot be downloaded. More information is in the introduction to Ox, see Doornik and Ooms (2006).

2.7.2 Windows: command-line compiler

The Ox command-line compiler under Windows is called `oxl`; starting it without arguments produces a list of options. Dynamic link libraries (DLL) are supported. For debugging and interactive mode, use `oxli.exe` instead of `oxl.exe`.

2.7.3 Unix compiler

The Ox compiler under Unix (including Linux) is also called `oxl`. DLLs are supported on all Unix platforms. Under most Unix systems a DLL has the `.so` extension. Debugging and interactive mode are activated as `oxl -d` and `oxl -i` respectively.

2.7.4 Running programs with graphics

Many types of graphs are readily produced in Ox, such as graphs over time of several variables, cross-plots, histograms, correlograms, etc. Several examples are in Ch. 10. There is also a GnuPlot package for Ox.

A graph can be saved in various formats: PDF (`.pdf`, encapsulated PostScript (`.eps`), PostScript (`.ps`), and OxMetrics graphics file (`.gwg`). When using *OxMetrics*, graphs can also be saved in Windows Metafile format (`.wmf`), and copied to the clipboard for pasting into wordprocessors.

Although creating and saving graphs will work on any system supported by Ox, it is only possible to see the result on screen under Windows. This requires Ox Professional, running the Ox program within *OxMetrics*. Then both text and graphical output from the Ox program will appear in *OxMetrics*. There, text and graphs can be edited further, or copied to the clipboard for pasting into other programs.

2.7.5 Compilation into `.oxo` file

The `-c` switch compiles the Ox source code into an object file (.oxo file). Such files are binary, and cross-platform compatible. This means that you can create an .oxo file under Windows, then copy it to the Sun (using binary transfer), and use it directly. Thus it provides a way to distribute modules without the source code.

2.7.6 The debugger

Ox has debug facilities, which can be useful to locate bugs in your programs. A debug session is started with the `-d` switch (use `oxli.exe` under Windows). More information is in the *Introduction to Ox*, see Doornik and Ooms (2006, Appendix 1).

2.7.7 OxEdit

OxEdit is a powerful text editor, and a very useful program in its own right. Ox can be installed within OxEdit, and program output captured in an OxEdit window, see `www.doornik.com` and the introduction to Ox: Doornik and Ooms (2006).

2.7.8 Windows context menu

Once Ox Professional has been installed there are a couple of convenient shortcuts in the Explorer window:

- (double) clicking on an Ox file will run the file using OxRun,
- right click on an Ox file, and choose:
  - run to run the Ox file with OxRun,
  - open to open the Ox file for editing.

2.8 Command line arguments

Arguments before the Ox filename are passed to the compiler, those after to the running program. So in

```
    oxl -DMYTEXT1 prog.ox -DMYTEXT2
```

the string `"-DMYTEXT2"` is not handled by the compiler, but available to the `prog.ox` program when using the `arglist` function. If you just type `oxl` you will get a list of command line options.

2.8.1 General switches

-c Create an object (`.oxo`) file, there is no linking or running of the file. An `.oxo` file is a binary file which holds compiled Ox code. It can be linked in later (see §3.9).

-cl Create object (.oxo) file after linking in dependencies (Ox Professional only).

-d Run program in debug mode (with -c: inserts debug information in compiled file).

-Dtoken Define tokens, e.g. -DOPTION1+OPTION2 corresponds to the preprocessor statements

```
#define OPTION1
#define OPTION2
```

-g The source code is an OxGauss file

-lfilelist Link object file, e.g. -lfile1+file2+file3, which links in the named files (the `.oxo` extension is assumed). If the file cannot be found as specified, the linker will search along the include path.

-i Run Ox in interactive mode (at start up the file called `ox_init.ox` is run automatically).

-ipath Appends path in front of the current include path. Initially, the include path is that specified in the `OX7PATH` environment variable (under Windows when `OX7PATH` is not set, the default is obtained from the location of the binary file); use this switch to prepend directories for searching. Use a semicolon to separate directories. The include path is used to search for files included in search code and link files.

-v# Set verbosity level (-v1 or -v2).
Use -v1 to print information on loaded files and required imports as well as canonical and parallel loops. Use -v2 to also print which include files are opened.

-w0 Switches off parse warnings. Currently, the parser warns for

- `isolated ; is empty statement`
  This refers to expressions such as `if (i == 10);` where the semicolon terminates the expression. The warning is also issued for `;` after `for` and `while` statements.
- `assignment in test expression`
  This refers to expressions such as `if (i = 10)` where an assignment is made inside a test expression. The warning is also issued for assignments in `for`, `while`, and `do while` statements.

-x Clears the current include path. Use this prior to the `-i` switch if you do not wish to search in the directories specified by the `OX7PATH` environment variables.

2.8.2 Optimization switches

-od Switch all code optimizations off. By default this is on. Usually, there is no reason to switch it off, other than to check for speed differences.

-on Switch line numbering off. Use this switch to prevent the emission of line numbers into the compiled code. This makes error messages less helpful; moreover, the speed improvement is negligable.

2.8.3 Run-time switches

-r- Do not run code. The code will be compiled and linked. Could be useful to only do a syntax check.

-rp# Set number of parallel threads (Ox Professional only). The default is the number of processor cores; use `-rp1` to force one thread only (serial code).

-s Sets the set symbol table and stack size. The default is `-s3000,1000`. Setting larger sizes is only required when large programs run out of symbol table or stack space.

2.9 Extending Ox

Ox is an open system to which you can add functions written in other languages. It is also possible to control Ox from another programming environment such as Visual C++ or Visual Basic.

Extending Ox requires an understanding of the innards of Ox, a decent knowledge of C, as well as the right tools. You also need a version of Ox with developer support. In addition, extending Ox is simpler on some platforms than others. Thus, it is unavoidable that writing Ox extensions is somewhat more complex than writing plain Ox code. However, there could be reasons for extending Ox, e.g. when you need the speed of raw C code (but make sure that the function takes up a significant part of the time it takes to run the program and that it actually will be a lot faster in C than in Ox!), when code is already available in e.g. Fortran, or to add a user-friendly interface.

The documentation on extending Ox is provided as part of the Ox Appendices, which is a separate PDF file. Example code on creating extension dynamic-link libraries are also provided.

Chapter 3

Introduction to the Ox language

The previous chapter introduced the first Ox program. We saw that a program always includes header files to define the standard library functions, and that it must have a `main` function, which is where program control starts. We also saw that the body of the function is enclosed in curly braces. This chapter will give a brief overview of the important elements of the Ox language. A more formal description of the Ox syntax is in Ch. 13. That chapter also has many more examples.

A much more extensive introduction is available, see Doornik and Ooms (2006) and the on-line help system. It is strongly recommended that this is used to learn more about the Ox language. The book contains econometric and statistical examples, and provides tutorial programs as well as many exercises.

3.1 Variables, types and scope

Variables are declared using the `decl` keyword. Unlike C, variables are *implicitly* typed. This means that variables do not have a type when they are declared, but get a type when values are assigned to them. So a variable can change type during its lifetime. The most important implicit types are *int* for an integer value, *double* for a real number, *string* for a text string and *matrix* for a matrix (two-dimensional array) of real numbers. The next Ox program illustrates implicit declaration and scope:

```
#include <oxstd.oxh>

main()
{
    decl i, d, m, s;

    i = 1;         // assign integer to i --> i is of type int
    d = 1.0;        // assign real number to d --> d is double
    s = "some text";   // assign string to s --> s is string
    m = zeros(3,3);   // assign to m a 3 x 3 matrix of zeros
                            // --> m is of type matrix
    print("i=", i, " d=", d, " s=", s, "\nm=", m);
}
```

This prints (`\n` is the newline character):

```
i=1 d=1 s=some text
m=
      0.00000      0.00000      0.00000
      0.00000      0.00000      0.00000
      0.00000      0.00000      0.00000
```

The *scope* of a variable refers to the parts of the program which can see the variable. This could be different from its lifetime: a variable can be ‘alive’ but not ‘seen’. If a variable is declared outside any function, its scope is the remainder of the source file. It is possible to export such variables to other source files, as we shall see shortly.

Variables declared inside a function have scope until the closing brace of the level at which it is declared. The following example illustrates:

```
#include <oxstd.oxh>

decl mX;                                // external variable
main()
{
    decl i = 0;                            // local variable
    {
       decl i = 1, j = 0;                             // new i
       mX = ones(3,3);
       print("i=", i, " j=", j);         // prints: i=1 j=0
    }             // brace end: local i and j cease to exist
    print("\ni=", i);        // revert to old i, prints: i=0
}
```

The variable `mX` (here we use *Hungarian notation*, see §3.11), can be seen everywhere in the `main` function. To make sure that it can never be seen in other source files, prefix it with the word `static`. It is good programming practice to use `static` in such cases, because it is very useful to know that it is not used in any other files (we may than rename it, e.g., without any unexpected side effects). An example will be given in `myfunc.ox` on page 30.

3.2 Indexing matrices

Indexing starts at zero, so `m[0][0]` is the first element of the matrix `m`. It is easy to select individual elements or a subset of the matrix. Here are some examples:

```
#include <oxstd.oxh>

main()
{
    decl m = <0, 1, 2; 3, 4, 5; 6, 7, 8>;

    println("m = ", m);
    println("element 1,0: ", m[1][0]);
    println("second row: ",  m[1][]);
    println("second column: ", m[][1]);
    println("without 1st row/3rd col: ", m[1:][:1]);
    println("indexed as a vector ", m[2:3]);
}
```

Which prints as output:

```
m =
      0.00000       1.0000       2.0000
       3.0000       4.0000       5.0000
       6.0000       7.0000       8.0000
element 1,0: 3
second row:
       3.0000       4.0000       5.0000
second column:
       1.0000
       4.0000
       7.0000
without 1st row/3rd col:
       3.0000       4.0000
       6.0000       7.0000
Warning: indexed a matrix as a vector
indexed as a vector
       2.0000
       3.0000
```

These expressions may also be used in assignments, for example:

```
m[1:][:1] = 10;
m[0][1:2] = m[0][0:1];
```

3.3 Functions and function arguments

We have already used various functions from the standard library (such as `print`, `ones` and `zeros`), and written various `main` functions). Indeed, an Ox program is primarily a collection of functions. It is important to know that all function arguments are *passed by value*. This means that the function gets a copy which it can change without changing the original. For example:

```
#include <oxstd.oxh>

func(mA)
{
    mA = zeros(1,2);
    print("ma in func()", mA);
}

main()
{
    decl ma;

    ma = ones(1,2);
    print("ma before func()", ma);
    func(ma);
    print("ma after func()", ma);
}
```

which prints:

```
ma before func()
       1.0000       1.0000
```

```
ma in func()
      0.00000      0.00000
ma after func()
       1.0000       1.0000
```

If the function argument is not changed by the function, it is good programming style to prefix it with the `const` keyword, as in:

```
func(const mA)
{
    print("ma in func()", mA);
}
```

Of course it is possible to return changed values from the function. If there is only one return value, this is most simply done by using the `return` statement:

```
#include <oxstd.oxh>

func(const r, const c)
{
    return rann(r, c);      // return r x c matrix of random
}                            // numbers from standard normal
main()
{
    print("return value from func():", func(1,2) );
}
```

Another way is to pass a *reference* to the variable, rather than the variable itself, as for example in:

```
#include <oxstd.oxh>

func(const pmA)
{
    pmA[0] = zeros(1,2);
    print("ma in func()", pmA[0]);
}
main()
{
    decl ma;

    ma = ones(1,2);
    print("ma before func()", ma);
    func(&ma);
    print("ma after func()", ma);
}
```

which prints:

```
ma before func()
       1.0000       1.0000
ma in func()
      0.00000      0.00000
ma after func()
      0.00000      0.00000
```

Now the change to `ma` is permanent. The argument to the function was the address of `ma`, and `func` received that address as a reference. Now we can modify the contents of the reference by assigning a value to `pmA[0]`. When `func` has finished, `ma` has been changed permanently. Note that we gave the argument a `const` qualification. This was possible because we did not change `pmA` itself, but what it referred to.

3.4 The `for` and `while` loops

Since Ox is a matrix language, there is much less need for loop statements than in C or C++. Indeed, because Ox is compiled and then interpreted, there is a speed penalty for using loop statements when they are not necessary.

The `for`, `while` and `do while` loops have the same syntax as in C. The `for` loop consists of three parts, an initialization part, a termination check, and an incrementation part. The `while` loops only have a termination check.

```
#include <oxstd.oxh>

main()
{
    decl i, d;

    for (i = 0; i < 5; ++i)
    {
        d = i * 0.01;
        println(d);
    }
}
```

which prints (`println` is like `print`, but ensures that the next output will be starting on a new line):

```
0
0.01
0.02
0.03
0.04
```

This could also be written, less elegantly, using `while` as follows:

```
#include <oxstd.oxh>

main()
{
    decl i, d;

    i = 0;
    while (i < 5)
    {
        d = i * 0.01;
        println(d);
        ++i;
    }
}
```

It is not uncommon to have more than one loop counter in the `for` statement, as the following code snippet illustrates:

```
decl i, j;

for (i = 0, j = 10; i < 5 && j > 0; ++i, --j)
    println(i * j);
```

The `&&` is *logical-and*, whereas `||` is *logical-or*. The `++i` statement is called (prefix) incrementation, and means 'add one to `i`'. Similarly, `--j` subtracts one from `j`. There is a difference between prefix and postfix incrementation (decrementation). For example, the second line in

```
i = 3;
j = ++i;
```

means: add one to `i`, and assign the result to `j`, which will get the value 4. But

```
i = 3;
j = i++;
```

means: leave the value of `i` on the stack for assignment, then afterwards increment `i`. So `j` will get the value 3. In the incrementation part of the `for` loop it does not matter whether you use the prefix or postfix form.

3.5 The `foreach` loop

The `foreach` loop is a convenient way to loop over the elements of an array or matrix, without the need to 'count' the number of elements:

```
#include <oxstd.oxh>
main()
{
    decl as = {"AA", "BB"};

    foreach (decl s in as)
    {
        print(" ", s);
    }
}
```

which prints `AA BB`. Similarly, `foreach (xi in mx)` loops over each element in `mx`. The element variable (`xi` here), must be a local variable, while the collection (`mx`) can be any pre-existing variable:

```
#include <oxstd.oxh>
main()
{
    decl xi, mx = <1,2,3;4,5,6;7,8,9>;

    foreach (xi in mx)
    {
        print(xi);
    }
}
```

The elements are accessed element-by-element, ordered by row, so this prints: `123456789`. Sometimes it is useful to access the matrix by entire rows or columns. Or to have access to the iterator. Both are possible:

```
#include <oxstd.oxh>
main()
{
    decl xi, mx = <1,2;3,4>, i, j, vx = vec(mx);

    foreach (xi in mx[i][j])
    {
        println("element ", i, ",", j, ": ", xi);
    }
    foreach (xi in mx[i][])
    {
        println("row ", i, ": ", xi);
    }
    foreach (xi in mx[][j])
    {
        println("column ", j, ": ", xi);
    }
    foreach (xi in vx[i])
    {
        println("vector element ", i, ": ", xi);
    }
}
```

This prints:

```
element 0,0: 1
element 0,1: 2
element 1,0: 3
element 1,1: 4
row 0:
       1.0000       2.0000
row 1:
       3.0000       4.0000
column 0:
       1.0000
       3.0000
column 1:
       2.0000
       4.0000
vector element 0: 1
vector element 1: 3
vector element 2: 2
vector element 3: 4
```

Note that changing the element does not change the matrix, and that the dimension of the matrix os not allowed to change during the loop.

3.6 The `if` statement

The `if` statement allows for conditional program flow. In the following example we draw a uniform random number. Such a random number is always between zero and one. The `ranu` returns a matrix, unless we ask it to generate just one number. Then it returns a double, as is the case here.

```
decl d = ranu(1,1);

if (d < 0.5)
    println("less than 0.5");
else if (d < 0.75)
    println("less than 0.75");
else
    println("greater than 0.75");
```

Again, braces are used to group multiple statements together. They should also be used when nesting `if` statements, to avoid confusion about which `else` belongs to which `if`.

```
decl d1 = ranu(1,1), d2 = ranu(1,1);

if (d1 < 0.5)
{   println("d1 is less than 0.5");
}
else
{   if (d2 < 0.75)
        println("d1 >= 0.5 and d2 < 0.75");
    else
        println("d1 >= 0.5 and d2 <= 0.75");
}
```

The `if` part is executed if the expression evaluates to a non-zero value (*true*). The `else` part otherwise, i.e. when the expression evaluates to zero (*false*: either an integer 0, or a double 0.0). Some care is required when using matrices in `if` statements. A matrix expression is a true statement if all elements are true (non-zero). Even if only one element is zero, the matrix expression is false, so

```
#include <oxstd.oxh>

main()
{
    if (ones(2,2))  print("yes");
    else            print("no");
    if (unit(2))    print("yes");
    else            print("no");
    if (zeros(2,2)) print("yes");
    else            print("no");
}
```

prints: `yesnono`.

There are two forms of relational operators. There is `<  <=  >  >=  ==  !=` meaning 'less', 'less than or equal', 'greater', 'greater than or equal', 'is equal' and 'is not equal'. These always produce the integer value 1 (true) or 0 (false). If any of the arguments is a matrix, the result is only true if it is true for each element:

```
#include <oxstd.oxh>

main()
{
    if (ones(2,2) == 1)  print("yes");   // true for each
    else                 print("no");          // element
    if (unit(2) == 1)    print("yes");//not true for each
```

```
        else                  print("no");          // element
        if (zeros(2,2) == 1) print("yes");//not true for each
        else                  print("no");          // element
    }
```

prints: `yesnono`.

The second form are the dot-relational operators `.<` `.<=` `.>` `.>=` `.==` `.!=` meaning 'dot less', 'dot less than or equal', 'dot greater', 'dot greater than or equal', 'is dot equal' and 'is not dot equal'. If any of the arguments is a matrix, the result is a matrix of zeros and ones, with each element indicating the relevant result.

The `any` library function returns `1` (true) if *any* element of the matrix is non-zero, so that `yesyesno` will be printed by:

```
    #include <oxstd.oxh>

    main()
    {
        if (any(ones(2,2)))  print("yes");
        else                 print("no");
        if (any(unit(2)))    print("yes");
        else                 print("no");
        if (any(zeros(2,2))) print("yes");
        else                 print("no");
    }
```

To conclude: you can test whether all elements of a matrix `m` are equal to one (say) by writing: `if (m == 1)`. To test whether any element is equal to one: `if (any(m .== 1))`. The expression `if (m != 1)`, on the other hand, is only true if none of the elements is equal to one. So, use `if (!(m == 1))` to test whether it is true that not all elements are equal to one.

3.7 Operations and matrix programming

To a large extent, the same operators are available in Ox as in C or C++. Some of the additional operators are power (`^`), horizontal concatenation (`~`), vertical concatenation (`|`) and the Kronecker product (`**`). One important distinction is that the operators are also available for matrices, so that, for example, two matrices can be added up directly. For some operators, such as multiplication, there is a distinction between the dot operators (e.g. `.*` is element by element multiplication and `*` is matrix multiplication if both arguments are matrices). Not available in Ox are the bitwise operators, instead you need to use the library functions `binand` and `binor`.

Because Ox is implicitly typed, the resulting type of the expression will depend on the types of the variables in the expression. When a mixture of types is involved, the result is promoted upwards in the order integer, double, matrix. So in an expression consisting if an integer and a double, the integer will be promoted to a double. An expression of only integers yields an integer. However, there are two important exceptions to this rule:

1. integer division is done in floating point and yields a double. *This is an important difference with C, where integer division is truncated to an integer.*

2. power expressions involving integers which yield a result too large to be expressed as an integer give a double result.

To illustrate, we write the Fahrenheit to Celsius example of Kernighan and Ritchie (1988) in Ox:

```
#include <oxstd.oxh>

const decl LOWER = 0;
const decl UPPER = 100;
const decl STEP  = 20;
main()
{
    decl fahr;

    for (fahr = LOWER; fahr <= UPPER; fahr += STEP)
        print("%3d", fahr, " ",
              "%6.1f", (5.0/9.0) * (fahr-32), "\n");
}
```

which prints:

```
  0  -17.8
 20   -6.7
 40    4.4
 60   15.6
 80   26.7
100   37.8
```

In C we have to write `5.0/9.0`, because `5/9` evaluates to zero. In Ox both expressions are evaluated in floating point arithmetic.

In general we get more more efficient code by vectorizing each program as much as possible:

```
#include <oxstd.oxh>

const decl LOWER = 0;
const decl UPPER = 100;
const decl STEP  = 20;
main()
{
    decl fahr = range(LOWER, UPPER, STEP)';
    print("%6.1f",  fahr ~ (5.0/9.0) * (fahr-32) );
}
```

- As in the first version of the program, we declare three constants which define the Fahrenheit part of the table.
- The `range()` function creates a $1 \times n$ matrix with the values LOWER, LOWER+STEP, LOWER + 2STEP, . . . , UPPER.
- The transpose operator `'` changes this into an $n \times 1$ matrix.
- The conversion to Celsius in the print statement works on the matrix as a whole: multiplication of a matrix by a scalar is equivalent to multiplication by the scalar of each element of the matrix.
- The `~` operator concatenates the two column vectors into an $n \times 2$ matrix.

- Finally, the `print` function is different from the `printf` in C. In Ox each variable to print is simply specified sequentially. It is possible, as done here with `"%6.1f"`, to insert formatting strings for the next variable.

  The program prints a table similar to the earlier output:

```
  0.0  -17.8
 20.0   -6.7
 40.0    4.4
 60.0   15.6
 80.0   26.7
100.0   37.8
```

3.8 Arrays

The Ox syntax allows for arrays, so you may use, for example, an array of strings (often useful), an array of matrices, or even an array of an array of matrices (etc.). The following program gives an example.

```
#include <oxstd.oxh>

const decl MX_R = 2;
const decl MX_C = 2;
main()
{
    decl i, asc, asr, m;

    asr = new array[MX_R];
    asc = new array[MX_C];

    for (i = 0; i < MX_R; ++i)
        asr[i] = sprint("row ", i);
    for (i = 0; i < MX_C; ++i)
        asc[i] = sprint("col ", i);

    m = ranu(MX_R, MX_C);
    print("%r", asr, "%c", asc, m);
}
```

which prints

```
                    col 0        col 1
row 0            0.020192      0.68617
row 1             0.15174      0.74598
```

- The `new` operator declares a new object. That could be a class object, as discussed in the next chapter, a matrix, a string, or, as used here, an array. The argument in square brackets is the size of the array. (When creating a matrix in this way, note that a matrix is always two-dimensional, and needs two arguments, as in: `m = new matrix[2][2]`.)
- The `sprint` functions return a string, which is stored in the arrays.
- In `print()`, we use `"%r"` followed by an array of strings to specify row labels for the subsequent matrix. Column labels use `"%c"`.

3.9 Multiple files: using `#include` and `#import`

The source code of larger projects will often be spread over several source files. Usually the `.ox` file containing the `main` function is only a few tens of lines. We have already seen that information about other source files is passed on through included header files. However, to run the entire program, the code of those files needs to be linked together as well. Ox offers various ways of doing this. As an example, consider a mini-project consisting of two files: a source code file and a header file. The third file will contain the `main` function.

samples/myfunc.ox

```
#include <oxstd.oxh>

static decl s_iCalls = 0;  // counter, initialize to 0

MyFunction(const ma)
{
    ++s_iCalls;             // increment calls counter
    println("MyFunction has been called ", s_iCalls,
          " times and prints:", ma);
}
```

samples/myfunc.h

```
    MyFunction(const ma);
```

The header file `myfunc.h` *declares* the `MyFunction` function, so that it can be used in other Ox files. Note that the declaration ends in a semicolon. The source code file contains the *definition* of the function, which is the actual code of the function. The header of the definition does not end in a semicolon, but is followed by the opening brace of the body of the function. The `s_iCalls` variable is declared outside any function, making it an *external* variable. Here we also use the `static` *type specifier*, which restricts the scope of the variable to the `myfunc.ox` file: `s_iCalls` is invisible anywhere else (and other files may contain their own `s_iCalls` variable). Variables declared inside a block of curly braces have a more limited lifetime. Their life starts when they are declared, and finishes at the closing brace (matching the brace level of declaration).

It is also possible to share variables between various source files, although there can be only one declaration (physical allocation) of the shared variable. The following modifications would do that for the `myfunc.ox` program:
(1) delete the `static` keyword from the declaration,
(2) add to `myfunc.h` the line (renaming `s_iCalls` to `g_iCalls`):

```
    extern decl g_iCalls;
```

Any code which includes `myfunc.h` can now reference or change the `g_iCalls` variable.

3.9.1 Including the code into the main file

The first way of combining the mini project with the `main` function is to `#include` the actual code. In that case the `myfunc.h` header file is not needed:

samples/mymaina.ox
```
#include <oxstd.oxh>
#include "myfunc.ox"

main()
{
    MyFunction("one");
}
```

The result will be just one code file, and `mymaina.ox` can be run as `oxl mymaina`.

3.9.2 Importing the code into the main file

The drawback of the previous method of including source code using `#include`, is that it can only be done once. That is not a problem in this short program, but is difficult to ensure if a library is used at many points in a large project. The `#import` command solves this problem.

samples/mymainc.ox
```
#include <oxstd.oxh>
#import "myfunc"

main()
{
    MyFunction("one");
}
```

Again, `mymainc.ox` can be run as `oxl mymainc`.

There is no extension in the argument to `#import`. The effect is as an `#include "myfunc.h"` statement followed by marking `myfunc.ox` for linking.[1] The actual linking only happens when the file is run, and the same `#import` statement may occur multiple times (as well as in compiled files). So even when the same file is imported many times, it will only be linked once.

3.9.3 Importing Ox packages

If `myfunc.ox` would require the maximization package, it could have at the top:

```
    #include <oxstd.oxh>
    #import <maximize>
```

[1] `#import` will actually try to find the `.oxo` file first. If that is not found, it will search for the `.ox` file. If neither is found, the program cannot run. More detail is in §13.9.3.

Partial paths can be used. Searching is relative to the OX7PATH environment variable. For example, if the Arfima package is in its default location of ox/packages/arfima, we would use:

```
#import <packages/arfima/arfima>
```

The distinction between angular brackets and double quotes in the include and import statements is discussed in §13.9.1. Roughly, the <> form should be used for files which are part of the Ox system, and the double quotes for your own files, which will not be in the Ox tree.

3.9.4 Separate compilation

Ox source code files can be compiled into Ox object files. These files have the .oxo extension, and are binary. The format is identical across operating systems, but since they are binary, transfer from one platform to another has to be done in binary mode.

To compile myfunc.ox into an Ox object file use the -c switch:

```
oxl -c myfunc
```

This creates myfunc.oxo (the .oxo extension is automatically appended). Remember that myfunc.oxo must be recreated every time myfunc.ox changes.

Now, when rerunning mymainc.ox, it will automatically use the .oxo instead of the .ox file.

Compiled Ox files can be useful for very large files (although even then compilation will be very fast), or if you do not wish to distribute the source files. They are inconvenient when the code is still under development.

3.10 Object-oriented programming

Object-oriented programming involves the grouping together of functions and variables in convenient building blocks. These blocks can then be used directly, or as starting point for a more specialized implementation. A major advantage of object-oriented programming is that it avoids the use of global variables, thus making the code more re-entrant: several instances will not conflict wiith each other.

The object-oriented features in Ox are not as sophisticated as in some low-level languages. However, this avoids the complexity of a language such as C++, while still providing most of the benefits.

Ox allows you to completely ignore the object-oriented features. However, you will then not be able to use the preprogrammed classes for data management and simulation. It is especially in the latter task that we found a considerable reduction in the required programming effort after writing the base class.

The *class* is the main vehicle for object-oriented programming. A class is nothing more than a group of variables (the data) and functions (the actions) packaged together. This makes it a supercharged struct (or record in Pascal terminology). Inheritance allows for a new class to add data and functions to the base class, or even redefine functionality of the base class.

In Ox, the default is that all data members of the class are protected (only visible to class members), and all function members are public. Like C++, Ox has the `virtual` keyword to define functions which can be replaced by the derived class. Classes are used by dynamically creating objects of that class. No static objects exist in Ox. When an object is created, the *constructor* function is called, when the object is deleted, the *destructor* function is called. More information on object-oriented programming is given in §13.5.6. Examples based on the preprogrammed classes are in Ch. 12.

3.11 Style and Hungarian notation

The readability and maintainability of a program is considerably enhanced when using a consistent style and notation, together with proper indentation and documentation. Style is a personal matter; this section describes the one I have adopted.

In my code, I always indent by one tab (four spaces) at the next level of control (i.e. after each opening brace), jumping back on the closing brace.

Table 3.1 Hungarian notation prefixes

prefix	type	example
i	integer	iX
c	count of	cX
b	boolean (f is also used)	bX
fl	integer flag	flX
d	double	dX
m	matrix	mX
v	vector ($1 \times n$ or $n \times 1$ matrix)	vX
s	string	sX
as	array of strings	asX
am	array of matrices	amX
a	reference in function argument	amX
m_	class member variable	m_mX
s_	static external variable (file scope)	s_mX
g_	external variable with global scope	g_mX
fn	function reference	fnX

I have found Hungarian notation especially useful (see e.g. Petzold, 1992, Ch. 1). Hungarian notation involves the decoration of variable names. There are two elements to Hungarian notation: prefixing of variable names to indicate type (Table 3.1), and using case to indicate scope (Table 3.2, remember that Ox is case sensitive).

As an example consider:

```
#include <oxstd.oxh>

const decl MX_R = 2;                              /* a constant */
decl g_mX;                                    /* exported matrix */
```

Table 3.2 Hungarian notation, case sensitivity

function	all lowercase
function (exported)	first letter uppercase
`static` external variable	type in lowercase, next letter uppercase (perhaps prefixed with `s_`)
exported external variable	as above, but prefixed with g_
function argument	type in lowercase, next letter uppercase
local variables	all lowercase
constants	all uppercase

```
static decl s_iCount;       /* static external variable */

static func1(const pdX)/* argument is pointer to double */
{
}
                                   /* exported function */
Func2(const mX, const asX, const cT, const cX)
{
    decl i, m;
}
```

`Func2` expects a `cT` $\times$ `cX` matrix, and corresponding array of `cX` variable names. The `c` prefix is used for the number of elements in a matrix or string. Note however, that it is not necessary in Ox to pass dimensions separately. You can ask `mX` and `asX` what dimensions they have:

```
Func2(const mX, const asX)
{
    decl i, m, ct, cx;
    cx = columns(mX);
    ct = rows(mX);
    if (cx != sizeof(asX))
        print("error: dimensions don't match");
}
```

3.12 Optimizing for speed

Ox is very fast: current benchmarks suggest that it is faster than most (if not all) other commonly used matrix language interpreters. A program can never be fast enough though, and here are some tips to achieve even higher speed:

- Use matrices as much as you can, avoiding loops and matrix indexing.
- Use built-in functions where possible.
- When optimizing a program with loops, it usually only pays to optimize the inner most loop. One option is to move loop invariants to a variable outside the loop.
- Avoid using 'hat' matrices, i.e. avoid using outer products over large dimensions when not necessary.

- Note that matrices are stored by row (the C and C++ default, but transposed from the Fortran default), so it could sometimes be faster to transpose matrices (i.e. have data variables in rows instead of columns).
- If necessary, you can link in C or Fortran code, see the separate Ox Developer's manual.

3.13 OxGauss

Ox has the capability of running a wide range of Gauss (GAUSS is a trademark of Aptech Systems, Inc., Maple Valley, WA, USA) programs. Gauss code can be called from Ox programs, or run on its own. The formal syntax of OxGauss is described in the Ox Developer's manual, which also lists some of the limitations of OxGauss, and gives a function summary. The remainder of this chapter gives some examples on its use.

3.13.1 Running OxGauss programs from the command line

As an example we consider a small project, consisting of a code file that contains a procedure and an external variable, together with a code file that includes the former and calls the function. We shall use `.src` or `.oxgauss` extension for the OxGauss programs.

samples/oxgauss/gaussfunc.src
```
declare matrix _g_base = 1;

proc(0)=gaussfunc(a,b);
    "calling gaussfunc";
    retp(a+_g_base*eye(b));
endp;
```

samples/oxgauss/gausscall.src
```
#include gaussfunc.src;

_g_base = 20;
z = gaussfunc(10,2);
"result from gaussfunc" z;
```

To run this program on the command line, enter

```
oxl -g gausscall.src
```

Which produces the output:

```
calling gaussfunc
result from gaussfunc
       30.000000        10.000000
       10.000000        30.000000
```

If there are problems at this stage, we suggest to start by reading the first chapter of the Introduction to Ox.

3.13.2 Running OxGauss programs from OxMetrics

Using Ox Professional, the OxGauss program can be loaded into OxMetrics. The syntax highlighting makes understanding the program easier.

Click on Run (the running person) to execute the program. This runs the program using the *OxGauss* application, with the output in a window entitled 'OxGauss Session'. OxMetrics will treat the file as an OxGauss file if it has the `.src`, `.g` or `.oxgauss` extension. If not, the file can still be run by launching *OxGauss* from the OxMetrics workspace window.

3.13.3 Calling OxGauss from Ox

The main objective of creating OxGauss was to allow Gauss code to be called from Ox. This helps in the transition to Ox, and increases the amount of code that is available to users of Ox.

The main point to note is that the *OxGauss code lives inside the* `gauss` *namespace*. In this way, the Ox and OxGauss code can never conflict.

Returning to the earlier example, the first requirement is to make an Ox header file for `gaussfunc.src`. This must declare the external variables and procedures explicitly in the `gauss` namespace:

samples/oxgauss/gaussfunc.h

```
    namespace gauss
    {
        extern decl _g_base;
        gaussfunc(const a, const b);
    }
```

Next, the OxGauss code must be imported into the Ox program. The `#import` command has been extended to recognize OxGauss imports by prefixing the file name with `gauss::`, as in the following program:

samples/oxgauss/gausscall.ox

```
    #include <oxstd.oxh>
    #import "gauss::gaussfunc"
    main()
    {
        gauss::_g_base = 20;
        decl z = gauss::gaussfunc(10,2);
        println("result from gaussfunc", z);
    }
```

When the OxGauss functions or variables are accessed, they must also be prefixed with the namespace identifier `gauss::`. The output is:

```
calling gaussfunc
result from gaussfunc
       30.000       10.000
       10.000       30.000
```

Chapter 4

Parallel programming in Ox

4.1 Introduction

Ox provides two ways to implement the parallel computation: using threads or using processes. In the multi-threaded framework used by `parallel for`, memory is shared, and any potential race condition must be prevented by forcing such operations to be serial. The `parallel for` loop is only available in Ox Professional. Ox Console executes such loops serially.

The alternative approach is to use multiple processes, as in OxMPI, In that case, each process operates in complete isolation, and shared operations require a synchronization (communication) between processes. The multi-processing case is harder to program, but the memory separation provides a neater framework.

The multi-threaded approach is readily available through just two keywords in Ox: `parallel` and `serial`. However, it is the user's responsibility to avoid race conditions: the compiler cannot detect it. Making mistakes can result in wrong outcomes, or even a crash.

Only certain loops can be made parallel: a minimum requirement is that they are *canonical*.

4.2 Canonical `for` and `foreach` loops

A `for` loop is *canonical* if:

1. the iterator is a local variable,
2. the iterator is an integer,
3. the iterator is not changed in the loop body,
4. the iterator is incremented (or decremented) by an integer constant,
5. the upperbound can be computed before the loop starts,
   In particular, it is either the value of a variable, or `sizer`, `sizec`, `sizerc`, `sizeof`, `rows`, `columns` of a variable.
6. the upperbound is fixed while the loop executes,

7. the loop body is a closed statement list.
   This means that there is no `return` statement in the loop, `break` out of the loop, or `goto` in or out of the loop.

Except for the last condition, all are automatically satisfied by a `foreach` loop. Here are some examples:

```
decl i, j, crep = 10, x = zeros(5, 5), i0 = 2;

for (i = 0; i < crep; i += 2)       // canonical
{
    println("i=", i);
}
for (i = int(i0); i < sizer(x); ++i)// canonical
{                                   // int() forces integer
    println("i=", i);
}
for (i = 0; i < crep; i++)          // canonical
{
    if (i == 1)
        continue;                   // continue is allowed
    println("i=", i);
}

for (i = i0; i < crep; i++)         // not canonical
{                                   // type of i0 unknown
    println("i=", i);               // at compilation time
}
for (i = 0; i < crep; i++)          // not canonical
{                                   // i modified in body
    if (i == 1)
        ++i;
    println("i=", i);
}
for (i = 0; i < crep; i++)          // not canonical
{                                   // body not closed
    if (i == 1)
        break;
    println("i=", i);
}
```

Ox can determine whether a `for` or `foreach` loop is canonical, and use compiled code for the iteration aspect, which is more efficient. If you use the `-v` command line switch, a message will indicate if a loop was optimized this way.

4.3 Parallel `for` and `foreach` loops

A canonical `for` or `foreach` loop can be run in parallel if there is no dependency between iterations, i.e. if the ordering of the iterations does not matter. This requirement is the responsibility of the author, and not verified by Ox. A `for` loop can be labelled as `parallel`:

```
    parallel for (i = 0; i < crep; i += 2)
    {
        println("i=", i);
    }
```

resulting in parallel execution if supported by the run-time system. A `foreach` loop can also be labelled as `parallel`.

The iterations of a loop rarely operate in complete isolation: usually there is an accumulation of results. This can still be run in parallel, provided that the order in which it is done does not matter.[1] A prime example is the loop of a Monte Carlo simulation.

4.3.1 Local variables

Local variables are arguments and variables that are declared inside a function — those within the current scope are declared higher-up inside the current block.

When Ox starts running code in parallel, n threads are created. Each thread has its own space for local variables (called a *stack*). Initially these stacks are the same as that of the main thread (integers and doubles are copied, the remainder are references to the value in the main thread). When assignment is made, it is to the thread-specific version, and, as the threads proceed in parallel, the local variables will become different in each thread. When the parallel section is finished, only the local variables in the main thread survive, the others are removed. This is useful because it separates local variables, allowing function calls to be executed in parallel — provided those functions are re-entrant, i.e. do not use external variables.

So local variables are thread-safe by design. But, as a consequence, they cannot be used for reduction operations such as accumulating a sum: each thread has it own version, and the final value is the sum of the part executed by the main thread only. A local variable that is used for a reduction must be labelled as `serial`, which is the responsibility of the programmer, see §4.4.

4.3.2 Global variables

Variables that are declared outside any function or not replicated to each thread: there is just one version in the program. Writing to a global variable inside parallel code introduces a so-called race condition: if two iterations try to do an update simultaneously they will overlap, and we cannot be sure of the precise outcome (but it will probably be wrong). Similarly, an overlapping write and read is a race condition. But overlapping read operations are always safe. So if a variable is initialized before the loop starts, and doesn't change, then there is no problem.

Race conditions can be difficult to notice: the answer may appear to be random, or even be correct most of the time. They can also result in a crash when memory allocation and deallocation overlap (e.g. changing dimensions of a matrix while trying to read it at the same time).

[1] Rounding error may still accumulate differently.

Good rules of thumb are:

1. never write to a global variable unless it is `serial` and a safe operation,
2. never read *and* write to a global variable in the same parallel block.

4.3.3 Member variables of objects

If an object is created within a thread (as a local variable in the parallel loop), its members are unique to a thread (except the static ones), and safe to use. Remember that Ox does not have garbage collection for class objects: each `new` must be matched by a `delete` within the block.

If an object is shared between threads, the member variables behave like global variables, and similar care is required. Also see the discussion at the end of §4.10.

4.4 Serial variables

Ox variables can be declared as `serial`. In that case only one thread at a time is able to modify the variable, provided one of the following compound assignment operations is used: `*=` `/=` `+=` `-=` `~=` `|=` `.*=` `./=` `++` `--`. When one thread updates, any other thread trying to do the same will be forced to wait.

The following code illustrates the use of `serial`:

```
decl i, j, crep = 10;

decl sum1 = 0;
parallel for (i = 0; i < crep; ++i)
{
    sum1 += 1;
}
println("sum1=", sum1);

serial decl sum2 = 0;
parallel for (i = 0; i < crep; ++i)
{
    sum2 += 1;
}
println("sum2=", sum2);
```

prints

```
sum1=3
sum2=10
```

The precise value of `sum1` depends on the amount of work allocated to the main thread. However, it is clearly incorrect.

The value of `sum2` is correct though: only one thread at a time was allowed to update, so, while one was doing this, the others had to wait. The price we pay for this is slower code.

Simple assignment (=) is not affected by declaring a variable as `serial`. The reason is that simple assignment in a parallel loop is really only useful for a thread-specific local variable. The exception is assigning a value to an iteration-specific location in a matrix or array. Such updating of matrix elements is safe, provided the matrix is pre-allocated, and each iteration updates a different element. So no `serial` is required in this code:

```
decl sum3 = zeros(1, crep);
parallel for (i = 0; i < crep; ++i)
{
    sum3[i] = 1;
}
println("sum3=", sumr(sum3));
```

4.5 Serial functions

Functions calls can be problematic if the functions are not re-entrant, which is usually caused by the fact that some internal state is maintained in a global variable. The solution is again to disallow overlapping calls to such functions.[2] For this reason all Ox functions that do file or text input/output are serial (such as `println`, `sprint`, `loadmat`, `savemat`, etc.). Calls to graphics functions are also serial.

Note that functions written in Ox code cannot be labelled as serial, but calls to functions inside dynamic-link libraries can.

Compare the following two parallel loops:

```
parallel for (i = 0; i < crep; ++i)
{
    println("i=", i);
}
parallel for (i = 0; i < crep; ++i)
{
    print("i=");
    println(i);
}
```

In the first version, the call to `println` is serial, so only executed in one thread, and therefore the output appears together (although the order is mixed up). In the second version, on a quad core computer, four of the first `print` statements are executed serially, but before the second call, resulting in the (truncated) output on the right:

```
--- println("i=", i); ---      --- print("i="); println(i); ---
        i=0                              i=i=i=i=0
        i=3                              8
        i=6                              3
        i=8                              6
        i=1                              i=i=i=i=1
        i=4                              9
```

4.6 Serial sections

Sections of code may need to be executed together serially. This can be achieved by creating a `serial` block. For example, to keep the print statements together:

[2]If possible it would be better to remove the dependency on the global variable entirely.

```
parallel for (i = 0; i < crep; ++i)
{
    // lengthy computation running in parallel
    // ....
    serial
    {
        print("i=");
        println(i);
    }
}
```

Parallel computations are not nested: if a parallel loop contains another parallel loop, the latter is not executed in parallel. Relatedly, any parallel loops inside a serial section will not be executed in parallel, as e.g. in:

```
serial
{
    parallel for (i = 0; i < crep; ++i)
    {
    }
}
```

4.7 Random number generation

An important case of a non-reentrant function is typical random number generation: the seed is maintained in the background, and is updated every time a ‘random’ draw is made. It would be possible to force random number generation to be serial, but that would incur a significant speed penalty. Moreover, because the ordering of the loops could be different every time the program is run, different outcomes will attain. The solution that Ox adopts is to give each iteration the same initial seed, but colour it with the iterator.

initial state: Starting seed is s_0.

before loop: `ranloopseed(0, -1)` is called (internally):
set the loop seed to the initial seed: $s^{-1} = s_0$.

iteration i: `ranloopseed(i, 0)` is called just before entering the next iteration:
set the seed to s^i, which is s^{-1} coloured with the loop counter i.

after loop: `ranloopseed(0, 1)` to return to normal random number behaviour:
reset the seed to s_0, then advance the seed by one step to s_1 (but only if random numbers were generated).
It is necessary to advance the seed when random numbers are used: otherwise two subsequent `parallel for` loops would use the same random number stream.

This approach is only adopted for the outermost parallel loops, even when made sequential through embedding in a `serial` block. As a result, the outcome of a simulation experiment using a parallel loop is the same regardless of the number of threads (even when using a single thread, when Ox is run with the `-rp1` commandline argument).

This loop colouring is also adopted in OxMPI (in that case it has to be implemented explicitly by calls to `ranloopseed`), so that OxMPI and `parallel for` outcomes can be the same.

4.8 Monte Carlo example

.... *samples/simulation/parallel_mc.ox*

```
#include <oxstd.oxh>

Run(cRep, vBeta, dRho_dgp, cT)
{
    decl beta_dgp = vec(vBeta);         // DGP parameters
    decl cn = sizerc(beta_dgp);
    decl asx = new array[cn], k;

    foreach (decl asx_k in asx[k])      // asx_k not used
    {                                   // create names
        asx[k] = sprint("x", k + 1);
    }

    ranseed(-1);

    decl i, mcoefs = zeros(cn, cRep), time = timer();
    serial decl sumcoefs = 0;

    parallel for (i = 0; i < cRep; i++)
    {
        decl vcoefs;
        decl X = rann(cT, cn);
        decl y = X * beta_dgp + rann(cT, 1);

        olsc(y, X, &vcoefs);            // do the regression

        mcoefs[][i] = vcoefs;           // no need for serial
        sumcoefs += vcoefs;             // must be serial
    }
    println("%r", asx, beta_dgp ~ meanr(mcoefs) ~ sumcoefs / cRep);
    println("M=", cRep, " overall time:", timespan(time));
}
main()
{
    decl cm = 1000000;

    Run(cm, zeros(1, 5), 0.9, 100);

    serial
    {
        Run(cm, zeros(1, 5), 0.9, 100);
    }
}
```
....

The output of this program is (the first three lines were added because we used the -v command-line switch):

```
Opening source file: parallel_mc.ox
parallel_mc.ox (12): Remark: canonical loop has been compiled
parallel_mc.ox (29): Remark: canonical loop is parallel
```

```
x1          0.00000  8.6793e-005  8.6793e-005
x2          0.00000   0.00017182   0.00017182
x3          0.00000   0.00015306   0.00015306
x4          0.00000  6.3777e-005  6.3777e-005
x5          0.00000 -3.2096e-005 -3.2096e-005
M=1000000 overall time: 5.08

x1          0.00000  8.6793e-005  8.6793e-005
x2          0.00000   0.00017182   0.00017182
x3          0.00000   0.00015306   0.00015306
x4          0.00000  6.3777e-005  6.3777e-005
x5          0.00000 -3.2096e-005 -3.2096e-005
```

The timings depend on the hardware; in this case a quad core computer was used. The parallel run is 3.3 times faster, and the results are the same in both cases, because both experiments start from the Ox default initial seed.

Removing the `parallel` keyword gives different results as the colouring of the seed by the iteration counter is omitted:

```
x1          0.00000 -6.4666e-005 -6.4666e-005
x2          0.00000  2.1149e-005  2.1149e-005
x3          0.00000 -2.4469e-005 -2.4469e-005
x4          0.00000 -6.3575e-005 -6.3575e-005
x5          0.00000   0.00013504   0.00013504
M=1000000 overall time:16.52
```

4.9 Monte Carlo example using OxMPI

Using OxMPI requires installing the package, as well as the MPI run-time. When this is done successfully, the `Loop::RunEx` function can be used to run the experiment on multiple processes. This in turns requires that the body is expressed as a function, and that other shared information is communicated through shared variables:

. . . . *packages/oxmpi/parallel_mc_oxmpi.ox*

```
#include <oxstd.oxh>
#import <packages/oxmpi/loop>

static decl s_vBeta_dgp, s_cT;

Run_i(const i)
{
    decl vcoefs;
    decl X = rann(s_cT, sizerc(s_vBeta_dgp));
    decl y = X * s_vBeta_dgp + rann(s_cT, 1);

    olsc(y, X, &vcoefs);            // do the regression
    return vcoefs;
}

Run(cRep, vBeta, dRho_dgp, cT)
{
    decl beta_dgp = vec(vBeta);         // DGP parameters
    decl cn = sizerc(beta_dgp);
    decl asx = new array[cn], k;
    foreach (decl asx_k in asx[k])      // asx_k not used
    {                                   // create names
        asx[k] = sprint("x", k + 1);
    }
    ranseed(-1);

    decl i, mcoefs = zeros(cn, cRep), time = timer();
    decl creject;

    s_vBeta_dgp = beta_dgp;
    s_cT = cT;

    mcoefs = Loop::RunEx(0, Run_i, cRep, 0, &creject);

    println("%r", asx, beta_dgp ~ meanr(mcoefs));
    println("M=", cRep, " overall time:", timespan(time));
}
main()
{
    decl cm = 1000000;

    Run(cm, zeros(1, 5), 0.9, 100);
}
```
....

In this example, the overhead of OxMPI is somewhat larger: we increase the number of replications tenfold to measure the difference:

```
x1          0.00000  3.6753e-005
x2          0.00000   0.00024034
x3          0.00000   0.00014868
x4          0.00000  7.3615e-006
x5          0.00000  2.8682e-005
M=1000000 overall time: 5.82
```

The multithreaded version was somewhat faster:

```
x1          0.00000  3.6753e-005  3.6753e-005
x2          0.00000   0.00024034   0.00024034
x3          0.00000   0.00014868   0.00014868
x4          0.00000  7.3615e-006  7.3615e-006
x5          0.00000  2.8682e-005  2.8682e-005
M=1000000 overall time: 5.05
```

The difference between the two approaches tends be small when each iteration is more demanding and the start-up cost ignored. The MPI version can be run on multiple machines.

4.10 Monte Carlo example using the Simulator class

The Simulator class is the Ox 7 successor to the Simulation class (which is still available, but cannot be used in parallel). The main changes are:

1. The class is split in SimulatorBase and Simulator to facilitate different top layers.

2. There is now a `Prepare` function that is called prior to the parallel loop.
3. There are no `GetCoefficients`, `GetPvalues` and `GetTestStatistics` functions, because these were non-reentrant, and therefore prevented parallel use. Instead, the `Generate` function returns an array with four values: integer (1 for success), coefficients, pvalues, and tests.
4. IsTwoSided is replaced by SetTwoSided which should be called as part of the DGP initialization.

The next listing rewrites the experiment using the Simulator class.

....*packages/oxmpi/parallel_mcsim.ox*

```
#include <oxstd.oxh>

#ifdef OX_MPI
  #import <packages/oxmpi/simulator>             // MPI version
#else
  #import <simulator>        // import default simulation class
#endif

class MySim : Simulator         // inherit from simulation class
{
    MySim(cRep, vBeta, cT);                           // constructor
    ~MySim();                                          // destructor
    Generate(const iRep, const cT, const mxT);

    decl m_vBeta_dgp;
};
MySim::MySim(cRep, vBeta, cT)
{
    m_vBeta_dgp = vec(vBeta);
    decl cn = sizerc(m_vBeta_dgp);
    decl asx = new array[cn], k;

    foreach (decl asx_k in asx[k])             // asx_k not used
    {                                             // create names
        asx[k] = sprint("x", k + 1);
    }

    Simulator(cT, cT, cRep, FALSE, -1, <>, m_vBeta_dgp);
    SetCoefNames(asx);
}
MySim::~MySim()
{
}
MySim::Generate(const iRep, const cT, const mxT)
{
    decl vcoefs;
    decl X = rann(cT, sizerc(m_vBeta_dgp));
    decl y = X * m_vBeta_dgp + rann(cT, 1);

    olsc(y, X, &vcoefs);                      // do the regression

return {1, vcoefs, <>, <>};    // 1 indicates success, 0 failure
}
```

```
main()
{
    decl cm = 1000000, time;
    decl exp = new MySim(cm, zeros(1, 5), 100);

    time = timer();
    exp.Simulate();
    println("time=", timespan(time));

    time = timer();
    exp.Simulate_serial();
    println("time=", timespan(time));

    delete exp;
}
```
....

I had to change my programming style to allow Monte Carlo experiments to be run in parallel. In Ox version 6, I would use a dgp and a model member that would hold the data for that replication. Because there was only a single object shared between iteration, as e.g.:

```
    SimArTest1::Generate(const iRep, const cT, const mxT)
    {
        // ...
        y = m_dgp.Generate(mxT);          // generate the data

        m_sys.Renew(y[][0], "YA", 0);     // store in database
        m_sys.Renew(y[][1], "YB", 0);
        m_sys.Renew(y[][2], "YC", 0);
        // ...
        return TRUE;
    }
```

the code could not be used in parallel.

Now I clone the model object, so that a loop specific version is modified. This cloned object receives the data:

```
    SimArTest1::Generate(const iRep, const cT, const mxT)
    {
        decl model = clone(m_sys), tests;

        // generate and store data
        m_dgp.StoreInDatabase(m_dgp.GenerateTo(mxT), model,
            model.GetVarIndex("YA"), model.GetVarIndex("ZA"), -1, 0);

        // ...
        delete model;
        return {1, <>, tests[1][], tests[0][]};
    }
```

A call to `clone` implies that a new object is allocated, so the matching `delete` is required to prevent a memory leak.

Here are a few more examples of code that works in parallel:

```
    fn_ok(const arg)
```

```
{
    decl xi;
    serial decl x = 0;
    decl mx = zeros(crep, 1);
    parallel for (i = 0; i < crep; ++i)
    {
        xi = arg + i;   // thread-specific storage
        x += xi;        // serial update
        mx[i] += xi;    // iteration-specific storage
    }
}
```

And code that fails:

```
fn_errors(const arg)
{
    serial decl xi;
    decl x = zeros(arg);
    decl mx = zeros(crep, 1);
    parallel for (i = 1; i < crep; ++i)
    {
        xi = arg + i;   // error: should be thread-specific storage
        x += xi;        // error: thread-specific update
        mx[i - 1] += xi;// error: depends on other iteration:
                        //        i-1 may not have been done yet
    }
}
```

It is a good idea to run your parallel Ox code first (with fewer iterations) with `-rp1` (one thread) and without (all threads). Because random number behaviour is the same in both cases (but different when the parallel keyword is omitted from the loop), the outcomes should be the same.

Chapter 5

How to …

How to compute/get/achieve:

- bootstrap a data set, see under: 'take a random sample …'.
- censored random variates, for example, a random normal censored at a and b (don't forget any dots; the `setbounds()` function can also be used):
```
x = rann(1000,1);
y = x .< a .? a .: x .> b .? b .: x;
y = setbounds(x, a, b);
```
- check if all elements in a matrix are equal to a value, 1 say:
```
if (x == 1)
```
- check if no element in a matrix is equal to a value, 1 say:
```
if (x != 1)
```
- check if any element in a matrix is not equal to a value, 0 say:
```
if ( !(x == 0) )
if ( max(x .== 0) )
```
- check if any element in a matrix is equal to a value, 1 say:
```
if ( any(x .== 1) )
```
- check if two matrices, x and y, are equal to each other:
```
if (x == y)
```
- check if two matrices, x and y, have any elements in common:
```
if ( any(x .== y) )
```
- concatenation of columns in a loop (inserting columns of zeros):
```
m = <>;
for (i = 0; i < columns(mx); ++i)
{
    m ~= mx[][i];
    m ~= 0;
}
```
Such concatenation can be relatively slow if `columns(m)` is large. An alternative is to pre-allocate the destination matrix:
```
m = zeros(rows(mx), 2 * columns(mx));
for (i = 0; i < columns(mx); i += 2)
{
    m[][i] = mx[][i];
}
```

- concatenation of rows in a loop
  ```
  decl m = <>;
  for (i = 0; i < rows(mx); ++i)
      m |= mx[i][];
  ```
  Again, pre-allocation is more efficient.
- correlation matrix out of a variance matrix:
  ```
  decl sdi = 1 ./ sqrt(diagonal(mvar));
  corrm = sdi .* mvar .* sdi';
  ```
- create a tridiagonal matrix, symmetric, $n \times n$:
  ```
  a * unit(n) + b * lag0(unit(n), 1) + b * lag0(unit(n), -1);
  ```
- delete rows with certain values:
  ```
  deleter(mx, value);
  ```
  or use:
  ```
  mx[ vecindex( !sumr(mx .== value) )][];
  ```
- element-by-element maximum (dot-maximum) (or minimum, etc.) of two matrices, or of a matrix and a number:
  ```
  x = max(a, b);
  x = max(a, 3);
  ```
- factorial: see under the `loggamma()` and `gammafact()` library functions, e.g. for $x!$:
  ```
  fact = exp(loggamma(x + 1));
  fact = gammafact(x + 1);
  ```
- gamma function: see under the `loggamma()` and `gammafact()` library functions; for the incomplete gamma function, see under `gammafunc()`.
- index of the maximum value in each column
  ```
  maxindc = maxcindex(x);
  ```
  Another possibility:
  ```
  maxindc = limits(x)[2][];
  ```
- maximum of each column:
  ```
  maxc = maxc(x);
  ```
- median of each data column:
  ```
  quantilec(x);
  ```
- mode of a data column:
  ```
  max(x[][0]);
  ```
- $N[\boldsymbol{\mu}, \boldsymbol{\Sigma}]$ random numbers
  ```
  chol_t = choleski(mSigma)';        // use P'
  eps = rann(ct, cn) * chol_t + mMu;
  ```
- $N[\mu, \sigma^2]$ quantiles
  ```
  z = quann(p);
  x = z .* sqrt(sigma2) + mu;
  ```
- Numerical variance

  Following maximum likelihood estimation, compute the second derivative matrix Q using `Num2Derivative`. Then $-Q^{-1}$ is an estimate of the parameter variance matrix.
- π (this requires `#include <oxfloat.oxh>`):
  ```
  pi = M_PI;
  ```
- (homogeneous) Poisson process, simulate first n arrival times:
  ```
  cumulate(ranexp(n, 1, 1.0));
  ```

- (homogeneous) Poisson process with rate μ, simulate times of events up to time t:
```
t * ranuorder(ranpoisson(1, 1, mu * t));
```
- quadratic form:
```
mom = x'x;
mom = outer(x', <>);
```
- replace values exceeding a certain value:
```
x = y .> 3 .? y .: 3;
x = y .> z .? y .: z;
```
- select rows with certain values:
```
selectr(mx, value);
```
  or use:
```
mx[ vecindex( sumr(mx .== value) )][];
```
- skewness and kurtosis:
```
mxs = standardize(mx);
n = columns(mx);
skew = sumr(mxs .^ 3) / n;
kurt = sumr(mxs .^ 4) / n;
```
  or use the `moments` library function.
- sorted column index of a matrix x sorted by the first column (column zero):
```
sortindex = sortcindex(x[][0]);
// or use:
//sortindex =
//    sortbyc(x ~ range(0, rows(x)-1)', 0)[][columns(x)];

// Now sortindex can be used to sort
// another matrix y conformably:
z = y[sortindex][];
```
- sequence from a to b of $n+1$ equally spaced points (see under the `range` library function for more information):
```
step = (b - a) / n;
seq = range(0, n) *step + a;
```
- substitute certain values only, say change all the 3's to 1 in a matrix x:
```
x = x .== 3 .? 1 .: x;
```
- take a random sample of size n with replacement from the rows of a matrix x:
```
y = x[ ranu(1,n) * rows(x) ][];
```
- take a random sample of size n without replacement from the rows of a matrix x (this requires `oxprob.oxh`):
```
y = x[ ransubsample(n, rows(x) - 1) ][];
```
  Or using `ranindex` which returns unsorted random indices:
```
y = x[ ranindex(n, rows(x)) ][];
```
- trim the matrix x by deleting the first top and the last bot rows:
```
trim = x[top:rows(x)-bot-1][];
```
- truncated random variates (i.e. random numbers from truncated distributions, see Devroye, 1986, p.39), with the distribution F truncated on the left at a, and on the right at b:
$$F^{-1}\left\{F(a) + u \times [F(b) - F(a)]\right\},$$
  where u is a uniform random number. In Ox code, for a random normal, truncated at a and b:

```
pa = probn(a);       // Pr{value <= a}
     // pa = 0 for no  left truncation
pb = probn(b);       // Pr{value <= b}
     // pb = 1 for no right truncation
y = quann( pa + ranu(1000,1) * (pb - pa) );
```

- two-sided critical values from a t(k) distribution:

```
pvalue = 2 * tailt(fabs(x), k);
```

- unsorting a matrix which is to be sorted by a column i.

```
sorted = sortbyc(x ~ range(0, rows(x)-1)', i)
unsorted = sortbyc(sorted, columns(sorted) - 1);
```

- $y \log y$:

```
y .* log(y .> 0 .? y .: 1);
```

Chapter 6

Numerical accuracy

Any computer program that performs numerical calculations is faced with the problem of (loss of) numerical accuracy. It seems a somewhat neglected area in econometric computations, which to some extent could be owing to a perception that the gradual and steady increase in computational power went hand in hand with improvements in accuracy. This, however, is not the case. At the level of software interaction with hardware, the major (and virtually the only) change has been the shift from single precision (4-byte) floating point computation to double precision (8-byte). Not many modern regression packages have problems with the Longley (1967) data set, which severely tests single precision implementations. Of course, there has been a gradual improvement in the understanding of numerical stability of various methods, but this must be offset against the increasing complexity of the calculations involved.

Loss of numerical accuracy is not a problem, provided we know when it occurs and to what extent. Computations are done with finite precision, so it will always be possible to design a problem with analytical solution which fails numerically. Unfortunately, most calculations are too complex to precisely understand to what extent accuracy is lost. So it is important to implement the most accurate methods, and increase understanding of the methods used. The nature of economic data will force us to throw away many correct digits, but only at the end of the computations.

Real numbers are represented as *floating point* numbers, consisting of a sign, a mantissa, and an exponent. A finite number of bytes is used to store a floating point number, so only a finite set can be represented on the computer. The main storage size in Ox is 8 bytes, which gives about 15 to 16 significant digits. Two sources of error result. The first is the *representation error*: most numbers can only be approximated on a computer. The second is *rounding error*. Consider the *machine precision* ϵ_m: this is the smallest number that can be added to one such that the result is different from one:

$$\epsilon_m = \operatorname{argmin}_\epsilon (1 + \epsilon \neq 1) .$$

So an extreme example of rounding error would be $(1 + \epsilon_m/10) - 1$, where the answer would be 0, rather than $\epsilon_m/10$. In Ox: $\epsilon_m \approx 2.2 \times 10^{-16}$.

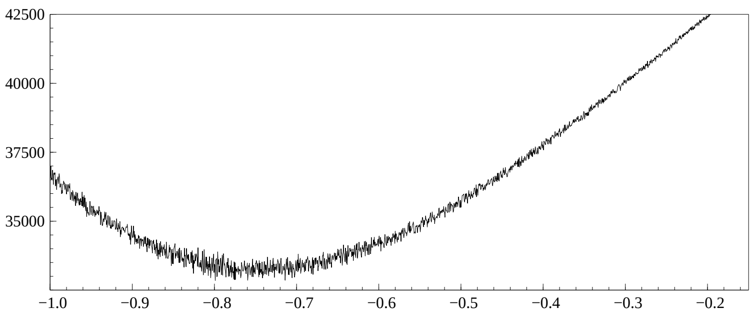


Figure 6.1 AR(1) grid of Longley data

Due to the accumulation of rounding errors, it is possible that mathematically equivalent formulae can have very different numerical behaviour. For example, computing $\mathsf{V}[x]$ as $\frac{1}{T}\sum x_i^2 - \bar{x}^2$ is much less stable than $\frac{1}{T}\sum (x_i - \bar{x})^2$. In the first case, we potentially subtract two quite similar numbers, resulting in cancellation of significant digits (it is even possible to get a negative number). A similar cancellation could occur in the computation of inner products (a very common operation, as it is part of matrix multiplication).

The Windows version of Ox accumulates inner products in extended 10-byte reals, leading to a higher accuracy. In general, one can expect small difference in the results from computations between versions of Ox. Often these are unnoticeable in the accuracy used for printing. The following code example can show the difference between 8 and 10-byte accumulation:

```
#include <oxstd.oxh>
#include <oxfloat.oxh>

main()
{
    decl x, y;

    x = <DBL_MAX; DBL_MAX; DBL_MAX-1;
         DBL_MAX; DBL_MAX>;
    y = <10; 10; 1; -10; -10>;

    print("%20.16g", x'y);
}
```

When using extended precision for inner products, it prints the value for `DBL_MAX` (see Ch. 9) else it prints infinity. When the computations work, it also shows that `DBL_MAX - 1` equals `DBL_MAX`.

An interesting example of harmless numerical inaccuracies is in the case of a grid plot of an autoregressive parameter based on the concentrated likelihood function of an AR(k) model. Rounding errors make the likelihood function appear non-smooth (not differentiable). This tends to occur in models with many lags of the dependent variable

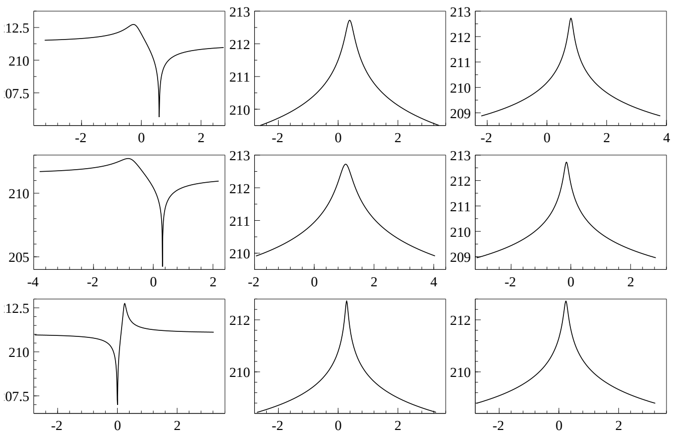


Figure 6.2 Likelihood grid of Klein model I

and a high autoregressive order. It also occurs in an AR(1) model of the Longley data set, see Fig. 6.1, which is a grid of 2000 steps between -1 and 0, done in PcGive (ignoring the warning that numerical accuracy is endangered).

It is important to distinguish numerical accuracy from other problems that may occur. Multicollinearity, for example, is first and foremost a statistical problem. A certain parametrization of a model might make the estimates of one or more parameters statistically imprecise (cf. the concept of 'micronumerosity' playfully introduced by Goldberger in Kiefer, 1989). This imprecision could be changed (or moved) by altering the specification of the model. Multicollinearity could induce numerical instability, leading to loss of significant digits in some or all results.

Another example is the determination of the optimum of a non-linear function that is not concave. Here it is possible to end up in a local optimum. This is clearly not a problem of numerical stability, but inherent to non-linear optimization techniques. A good example is provided by Klein model I. Figure 6.2 provides a grid plot of the FIML likelihood function for each parameter, centred around the maximum found with the 2SLS estimates as a starting point. These grids are of a different type from the AR grid in Fig. 6.1. In the former, all parameters but one are kept fixed, whereas the AR grid graphs the concentrated likelihood. In the case of one autoregressive parameter, the actual optimum may be read off the graph, as is the case in the AR grid plot above.

A matrix frequently used to show the limitations of numerical techniques is the Hilbert matrix. A Hilbert matrix of dimension n, H_n, has elements

$$h_{ij} = (i + j + 1)^{-1}, \quad i, j = 0, 1, \ldots, n - 1.$$

For example:

$$H_4 = \begin{pmatrix} 1/1 & 1/2 & 1/3 & 1/4 \\ 1/2 & 1/3 & 1/4 & 1/5 \\ 1/3 & 1/4 & 1/5 & 1/6 \\ 1/4 & 1/5 & 1/6 & 1/7 \end{pmatrix}$$

This matrix is very ill-conditioned, and many computations involving H_n break down even for n as small as 10. The inverse and determinant of H_n are known analytically. Barnett (1990) gives the determinant:

$$\det(H_n) = \prod_{k=0}^{n-1} \frac{(n-k-1)!(n-k-1)!k!}{(n+k)!}.$$

We could use the `loggamma` function to compute the (reciprocal of the) determinant: $\log\Gamma(z+1) = \log(z!)$. Then we can make a stable computation up to $n = 23$. At $n = 24$ the `exp` function overflows. Restricting ourselves to the logarithm of the determinant, we can go quite a bit further. The `determinant` function breaks down much earlier: at $n = 4$ we have about 13 significant digits correct, at $n = 10$ just 5. At $n = 11$, the function reports that the matrix is singular. If we scale the matrix to keep the determinant under control, we get the message that the determinant is unreliable at $n = 11$, which is borne out by only two correct digits. Beyond that, there is no correct answer from the `determinant` function; using the `logdet` function does not help.

To conclude this chapter, we show that using floating point for computations which should result in integers could lead to unexpected results. Most of the time conversion to an integer works, but not always. The following code has been especially written to show that:

```
#include <oxstd.oxh>

intfuzzy(const d)
{
    return  d > 0 ? int(d * (1 + fuzziness(0)/2))
                  : int(d * (1 - fuzziness(0)/2));
}
main()
{
    decl i, j;

    for (i = 322, j = 122; i < 327; ++i, ++j)
        println("%20.16f", (i*0.1 - 20) * 10, " ",
             "%5d", int( ((i*0.1) - 20) * 10) - j,
             "%5d", int( ((i*0.1) - 20) * 10),
             "%5d", int(floor( ((i*0.1) - 20) * 10)),
             "%5d", int(ceil( ((i*0.1) - 20) * 10)));

    for (i = 322, j = 122; i < 327; ++i, ++j)
        println("%20.16f", (i*0.1 - 20) * 10, " ",
             "%20.16f", (i*0.1 - 20) * 10 * (1+fuzziness(0)/2),
             "%5d", intfuzzy( ((i*0.1) - 20) * 10) - j);
}
```

which has output on Windows (note that there could be minor differences on other platforms):

```
122.0000000000000300      0  122  122  123
123.0000000000000400      0  123  123  124
123.9999999999999900     -1  123  123  124
125.0000000000000000      0  125  125  125
126.0000000000000100      0  126  126  127

122.0000000000000300 122.0000000000061200     0
123.0000000000000400 123.0000000000061800     0
123.9999999999999900 124.0000000000061800     0
125.0000000000000000 125.0000000000062400     0
126.0000000000000100 126.0000000000063100     0
```

The last two zeros in the floating point print-out are beyond the precision, so can be ignored. We see however, that even then the results are not exact: in general most integers cannot be represented exactly in floating point notation (this is the representation error mentioned earlier). Once we start computing, these inexactitudes propagate. Conversion to an integer involves truncation, hence we find 123 for the third value, and not 124 as expected. This also affects the `floor` and `ceil` function. (Another example of this effect is discussed under the `range` library function.)

There is a potential solution, as shown in the code. Add a little bit to positive numbers (subtract for negative numbers), where the little bit is a fraction determined by the current fuzziness value. This is implemented in the `truncf` library function. Alternatively, one could round to the nearest integer, using the `round` function.

Note that, when indexing a matrix by another matrix, a problem like this could occur when the indices are the result from computation, rather than direct storage. Internally, the indices are converted to integers by truncation, so you could decide to round first. When using random indices (e.g. in bootstrapping), such rounding will produce indices out of range, and truncation is precisely what is required.

Part II

Function and Language Reference

Chapter 7

Function summary

This chapter lists all library functions by category, and gives a brief description. More detailed descriptions with examples follow in Chapters 8–12.

date and time functions

`date`	returns a string with the current date
`dayofcalendar`	translates a date in the day of the calendar
`dayofeaster`	finds the calendar date of Easter
`dayofmonth`	finds the *n*-th weekday in the month
`dayofweek`	translates a date in the day of the week
`time`	returns a string with the current time
`timeofday`	translates the time component of a calendar index
`timer`	returns an integer representing the current time
`timespan`	returns the lapsed time
`timestr`	returns a string from no of seconds since 1 Jan 1970
`timing`	returns date/time as no of seconds since 1 Jan 1970
`today`	returns current date/time as no of seconds since 1 Jan 1970

general functions

`any`	returns TRUE if any element is TRUE
`arglist`	get the argument list specified on the command line
`binand`	binary *and* operation
`bincomp`	binary bit-wise complement operation
`binor`	binary *or* operation
`columns`	get number of columns of argument (0 for int,double)
`countc`	count elements in columns in specified intervals
`countr`	count elements in rows in specified intervals
`discretize`	count elements in columns in regularly-spaced intervals
`fuzziness`	set fuzziness parameter
`isdotfeq`	tests for dot fuzzy equality
`isdotinf`	returns boolean matrix from test for infinity

`isdotmissing`	returns boolean matrix, 1 for missing values (.NaN,$\pm$.Inf)
`isdotnan`	returns boolean matrix from test for .NaN
`iseq`	tests for equality with fuzziness 0
`isfeq`	tests for fuzzy equality
`ismissing`	tests for the presence of a missing value (.NaN,+.Inf,-.Inf)
`isnan`	tests for the presence of .NaN
`limits`	maximum/maximum values in matrix plus location
`max`	maximum value in arguments
`maxc`	maximum value of each column
`maxcindex`	row index of the maximum value of each column
`maxr`	maximum value of each row
`min`	minimum value in arguments
`minc`	minimum value of each column
`mincindex`	row index of the minimum value of each column
`minr`	minimum value of each row
`prodc`	compute column products
`prodr`	compute row products
`rows`	get number of rows of argument (0 for int,double)
`sizec`	get number of columns of argument (1 for int,double)
`sizeof`	same as `rows`
`sizer`	get number of rows of argument (1 for int,double)
`sizerc`	get total number of elements of argument (1 for int,double)
`sumc`	compute column sums
`sumr`	compute row sums
`sumsqrc`	compute column sum of squares
`sumsqrr`	compute row sum of squares
`va_arglist`	needed to access arguments in a variable argument list
graphics functions	
`CloseDrawWindow`	close the drawing window
`Draw`	draw a matrix against an x-axis
`DrawAcf`	draw an ACF (correlogram) and/or PACF
`DrawAdjust`	adjust most recent draw object
`DrawAxis`	draw an axis
`DrawAxisAuto`	draw an automatic axis
`DrawBoxPlot`	draw a box plot
`DrawCorrelogram`	draw a correlogram
`DrawDensity`	draw a histogram and/or density
`DrawHistogram`	draw a histogram from vector of heights
`DrawLegend`	draw the legend
`DrawLine`	draw a line
`DrawMatrix`	draw a matrix against an x-axis
`DrawPLine`	draw a line (pixel coordinates)
`DrawPSymbol`	draw a symbol (pixel coordinates)
`DrawPText`	draw text (pixel coordinates)
`DrawQQ`	draw a QQ plot

`DrawSpectrum`	draw a spectral density
`DrawSymbol`	draw a symbol
`DrawT`	draw a matrix against time
`DrawText`	draw text
`DrawTitle`	set the title text
`DrawTMatrix`	draw a matrix against time
`DrawX`	cross plot of a matrix against a vector
`DrawXMatrix`	cross plot of a matrix against a vector
`DrawXYZ`	draw 3-dimensional graph
`DrawZ`	add error bar/band/fan/Z variable
`SaveDrawWindow`	save the drawing to a file
`SetDraw`	set drawing defaults
`SetDrawWindow`	set the name of the drawing window
`SetTextWindow`	set the name of the text window
`ShowDrawWindow`	show the drawing window

input/output

`eprint`	print to stderr
`fclose`	close a file
`feof`	tests for end of file
`flush`	flushes the file buffer
`fopen`	open a file
`format`	set default print format
`fprint`	print to a file
`fprintln`	as `printf`, ensures the next output will be on a new line
`fread`	read data in binary format from a file
`fremove`	removes a file
`fscan`	read from a file
`fseek`	gets or repositions the file pointer
`fsize`	get the file size in bytes
`ftime`	get the file's modification time
`fwrite`	write data in binary format from a file
`loadmat`	load a matrix
`loadsheet`	load an entire sheet from a spread sheet file as an array
`print`	print to stdout
`println`	as `print`, ensures the next output will be on a new line
`savemat`	save a matrix
`scan`	read from the console
`sprint`	print to a string
`sprintbuffer`	resize the sprint buffer
`sscan`	read from a string

is type functions

`classname`	returns the class name of a class object
`clone`	returns the clone of a class object
`isarray`	tests if argument is an array

`isclass`	tests if argument is a class object
`isdouble`	tests if argument is a double
`isfile`	tests if argument is a file
`isfunction`	tests if argument is a function
`isint`	tests if argument is an integer
`ismatrix`	tests if argument is a matrix
`ismember`	tests if a class object has a specified member
`isstring`	tests if argument is a string

mathematical functions

`bessel`	bessel functions of order 0 and 1
`betafunc`	incomplete beta integral
`binomial`	binomial coefficient
`cabs`	complex absolute value
`cdiv`	complex division
`ceil`	ceiling
`cerf`	complex error function
`cexp`	complex exponent
`clog`	complex logarithm
`cmul`	complex multiplication
`csqrt`	complex square root
`dawson`	Dawson integral
`dfft`	discrete Fourier transform
`erf`	error function
`exp`	exponent
`expint`	exponential integral Ei
`fabs`	absolute value
`factorial`	factorial
`fft`	fast Fourier transform, pads to power of two
`fft1d`	fast Fourier transform, any sample size
`floor`	floor
`fmod`	floating point remainder
`gammafact`	gamma function (related to factorial)
`gammafunc`	incomplete gamma function
`idiv`	integer division
`imod`	integer remainder
`log`	natural logarithm
`log10`	base-10 logarithm
`loggamma`	logarithm of gamma function
`polygamma`	derivatives of loggamma function
`pow`	dot-power (alternative to `.^`)
`round`	rounds to nearest integer
`sqr`	square
`sqrt`	square root
`trunc`	truncate towards zero
`truncf`	fuzzy truncation towards zero

matrix creation

`constant`	create a matrix and fill with a value
`diag`	create matrix with specified vector on diagonal
`nans`	create a matrix of .NaN
`ones`	create a matrix of ones
`range`	create a matrix consisting of a range of numbers (trend)
`toeplitz`	create a symmetric Toeplitz matrix
`unit`	create an identity matrix
`zeros`	create a matrix of zeros

matrix decomposition

`choleski`	Choleski decomposition of symmetric positive definite matrix
`decldl`	square root free Choleski decomposition of sym.pd. matrix
`decldlband`	Choleski decomposition of sym.pd. band matrix
`declu`	LU decomposition
`decqr`	QR decomposition
`decqrmul`	applies Q from the QR decomposition to compute $Q'Y$
`decqrupdate`	update a QR decomposition via Givens rotations
`decschur`	real Schur decomposition
`decschurgen`	real generalized Schur decomposition
`decsvd`	singular value decomposition
`eigen`	eigenvalues of matrix
`eigensym`	eigenvalues of symmetric matrix
`eigensymgen`	solves generalized symmetric eigen problem
`polydiv`	divides two polynomials
`polyeval`	evaluates a polynomial
`polymake`	gets polynomial coefficients from the (inverse) roots
`polymul`	multiplies two polynomials
`polyroots`	computes the (inverse) roots of a polynomial
`solveldl`	solves AX=B when A is decomposed with `decldl`
`solveldlband`	solves AX=B when A is decomposed with `decldlband`
`solvelu`	solves AX=B when A is decomposed with `declu`
`solvetoeplitz`	solves AX=B when A is symmetric Toeplitz

matrix functions

`determinant`	returns the determinant of a matrix
`diagcat`	concatenates two matrices long the diagonal
`diagonalize`	set off-diagonal elements to zero
`invert`	invert a matrix
`inverteps`	sets inversion/rank epsilon
`invertgen`	(generalized) inversion
`invertsym`	invert a symmetric matrix
`logdet`	returns the log and sign of the determinant
`norm`	returns the norm of a matrix
`nullspace`	returns the null space of a matrix
`outer`	XSX', or diagonal(XSX') or $\sum x_i x_i'$

`rank`	returns the rank of a matrix
`trace`	returns the trace of a matrix

matrix modification/selection/reordering

`aggregatec`	aggregates the columns of a matrix by taking sums of groups
`aggregater`	aggregates the rows of a matrix by taking sums of groups
`deletec`	deletes columns with specific values (or missing values)
`deleteifc`	deletes columns according to boolean matrix
`deleteifr`	deletes rows according to boolean matrix
`deleter`	deletes rows with specific values (or missing values)
`diagonal`	extract diagonal from a matrix
`dropc`	deletes specified columns
`dropr`	deletes specified rows
`exclusion`	return sorted unique elements which are not in a 2nd matrix
`find`	row indices of elements of one vector in another
`insertc`	inserts columns of zeros
`insertr`	inserts rows of zeros
`intersection`	return sorted unique intersection of two matrices
`lower`	return the lower diagonal of a matrix
`reflect`	reflect a matrix
`reshape`	reshape a matrix by row
`reversec`	reverse column elements
`reverser`	reverse row elements
`selectc`	selects columns with specific values (or missing values)
`selectifc`	selects columns according to boolean matrix
`selectifr`	selects rows according to boolean matrix
`selectr`	selects rows with specific values (or missing values)
`selectrc`	selects elements from specified rows and columns
`setbounds`	set the lower and upper bounds of a matrix
`setdiagonal`	set the diagonal of a matrix
`setlower`	set the lower diagonal of a matrix
`setupper`	set the upper diagonal of a matrix
`shape`	reshape a matrix by column
`sortbyc`	sort one column, and remaining columns accordingly
`sortbyr`	sort one row, and remaining rows accordingly
`sortc`	sort columns of a matrix, or an array of strings
`sortcindex`	sorted index from applying `sortc`
`sortr`	sort rows of a matrix
`submat`	extract a submatrix
`thinc`	thin the columns of a matrix
`thinr`	thin the rows of a matrix
`union`	return the sorted unique elements of two matrices
`unique`	return the sorted unique elements of a matrix
`unvech`	undoes `vech`
`upper`	return the upper diagonal of a matrix
`vec`	vectorize the columns of a matrix

`vech`	vectorize the lower diagonal only
`vecindex`	row indices of non-zero elements of the vec of a matrix
`vecr`	vectorize the rows of a matrix
`vecrindex`	row indices of non-zero elements of the vecr of a matrix

maximization, differentiation (Maximization package, requires `maximize.oxh`**)**

`GetMaxControl`	get maximum no of iterations and print control
`GetMaxControlEps`	get convergence tolerances
`MaxBFGS`	maximize a function using BFGS
`MaxControl`	set maximum no of iterations and print control
`MaxControlEps`	set convergence tolerances
`MaxConvergenceMsg`	get convergence message
`MaxNewton`	maximize a function using Newton's method
`MaxSimplex`	maximize a function using the simplex method
`MaxSQP`	maximize a function under nonlinear constraints
`MaxSQPF`	as MaxSQP, using feasible iterates
`Num1Derivative`	numerical computation of 1st derivative
`Num2Derivative`	numerical computation of 2nd derivative
`NumJacobian`	numerical computation of Jacobian matrix
`SolveNLE`	solves systems of nonlinear equations
`SolveQP`	solves quadratic programming problem

probability

`denschi`	χ^2 density
`densf`	F density
`densn`	standard normal density
`denst`	Student t density
`probchi`	χ^2 distribution function (also non-central)
`probf`	F-distribution function
`probn`	standard normal distribution function
`probt`	Student t-distribution function
`quanchi`	χ^2 distribution quantiles
`quanf`	F-distribution quantiles
`quann`	standard normal quantiles
`quant`	Student t-distribution quantiles
`tailchi`	χ^2 distribution tail probabilities
`tailf`	F-distribution tail probabilities
`tailn`	standard normal tail probabilities
`tailt`	Student t-distribution tail probabilities

probability (Probability package, requires `oxprob.oxh`)

`densbeta`	$B(a, b)$ density
`densbinomial`	Binomial density
`denscauchy`	Cauchy density
`densexp`	Exponential density
`densextremevalue`	Extreme value density

`densgamma`	Gamma density
`densgeometric`	Geometric density
`densgh`	Generalized Hyperbolic density
`densgig`	Generalized Inverse Gaussion density
`denshypergeometric`	Hypergeometric density
`densinvgaussian`	Inverse Gaussian density
`denskernel`	kernel densities
`denslogarithmic`	logarithmic density
`denslogistic`	logistic density
`denslogn`	lognormal density
`densmises`	von Mises density
`densnegbin`	Negative Binomial density
`denspareto`	Pareto density
`denspoisson`	Poisson density
`densweibull`	Weibull density
`probbeta`	$B(a,b)$ cumulative distribution function
`probbinomial`	Binomial cumulative distribution function
`probbvn`	bivariate normal cumulative distribution function
`probcauchy`	Cauchy cumulative distribution function
`probexp`	exponential cumulative distribution function
`probextremevalue`	extreme value cumulative distribution function
`probgamma`	Gamma cumulative distribution function
`probgeometric`	Geometric cumulative distribution function
`probhypergeometric`	Hypergeometric cumulative distribution function
`probinvgaussian`	Inverse Gaussian cumulative distribution function
`problogarithmic`	logarithmic cumulative distribution function
`problogistic`	logistic cumulative distribution function
`problogn`	lognormal cumulative distribution function
`probmises`	von Mises cumulative distribution function
`probmvn`	multivariate normal cdf (up to trivariate)
`probnegbin`	Negative Binomial cumulative distribution function
`probpareto`	Pareto cumulative distribution function
`probpoisson`	cumulative Poisson cumulative distribution function
`probweibull`	Weibull cumulative distribution function
`quanbeta`	$B(a,b)$ quantiles
`quanbinomial`	Binomial quantiles
`quancauchy`	Cauchy quantiles
`quanexp`	exponential quantiles
`quanextremevalue`	extreme value quantiles
`quangamma`	Gamma quantiles
`quangeometric`	Geometric quantiles
`quanhypergeometric`	Hypergeometric quantiles
`quaninvgaussian`	Inverse Gaussian quantiles
`quanlogarithmic`	logarithmic quantiles
`quanlogistic`	logistic quantiles
`quanlogn`	lognormal quantiles

`quanmises`	von Mises quantiles
`quannegbin`	Negative Binomial quantiles
`quanpareto`	Pareto quantiles
`quanpoisson`	Poisson quantiles
`quanweibull`	Weibull quantiles

random numbers

`ranloopseed`	used in parallel loops
`rann`	standard normal distributed random numbers
`ranseed`	set and get seed; choose unform random number generator
`ranu`	uniform $[0, 1]$ distributed random numbers

random numbers (Probability package, requires `oxprob.oxh`)

`ranbeta`	$B(a, b)$ distributed random numbers
`ranbinomial`	binomially distributed random numbers
`ranbrownianmotion`	realizations from a Brownian motion
`rancauchy`	Cauchy random numbers
`ranchi`	χ^2 distributed random numbers
`randirichlet`	Dirichlet$(\alpha_1, \dots, \alpha_{c+1})$ random numbers
`ranexp`	$\exp(\lambda)$ distributed random numbers
`ranextremevalue`	extreme value random numbers
`ranf`	F-distributed random numbers
`rangamma`	gamma-distributed random numbers
`rangeometric`	Geometric random numbers
`rangh`	Generalized Hyperbolic random numbers
`rangig`	Generalized Inverse Gaussion random numbers
`ranhypergeometric`	Hypergeometric random numbers
`ranindex`	draw a random index without replacement
`raninvgaussian`	inverse Gaussian-distributed random numbers
`ranlogarithmic`	logarithmic distributed random numbers
`ranlogistic`	logistic distributed random numbers
`ranlogn`	log normal distributed random numbers
`ranmises`	von Mises distributed random numbers
`ranmultinomial`	multinomial distributed random numbers
`rannegbin`	negative binomial distributed random numbers
`ranpareto`	Pareto random numbers
`ranpoisson`	poisson distributed random numbers
`ranpoissonprocess`	realizations from a poisson process
`ranshuffle`	samples from a vector without replacement
`ranstable`	stable-distributed random numbers
`ransubsample`	samples from a set of integers without replacement
`rant`	Student t-distributed random numbers
`ranuorder`	uniform order statistics
`ranweibull`	Weibull random numbers
`ranwishart`	Wishart$(1, \mathbf{I}_r)$ distributed random drawing

statistics

`correlation`	correlation matrix of matrix (data in columns)
`meanc`	compute column means
`meanr`	compute row means
`moments`	compute column moment ratios (skewness,kurtosis, etc.)
`ols2c`	OLS based on normal equations (data in columns)
`ols2r`	OLS based on normal equations (data in rows)
`olsc`	OLS based on orthogonal decomposition (data in columns)
`olsr`	OLS based on orthogonal decomposition (data in rows)
`quantilec`	quantiles of a matrix (data in columns)
`quantiler`	quantiles of a matrix (data in rows)
`spline`	natural cubic spline smoother (data in columns)
`standardize`	standardize a matrix (data in columns)
`varc`	compute column variances
`variance`	variance matrix of matrix (data in columns)
`varr`	compute row variances

string functions

`find`	finds a string/character in a string or array of strings
`replace`	replace string(s) in a string or array of strings
`strfind`	finds a string/character in an array of strings/string
`strfindr`	finds last occurrance
`strifind`	case insensitive version of `strfind`
`strifindr`	case insensitive version of `strfindr`
`strlwr`	convert a string to lower case
`strtrim`	removes leading and trailing white space
`strupr`	convert a string to upper case

system functions

`chdir`	change directory
`exit`	exits Ox
`getcwd`	get current working directory
`getenv`	get the value of an environment variable
`getfiles`	get list of files matching the specified mask
`getfolders`	get list of folders matching the specified mask
`oxfilename`	returns the name of the Ox file it is called from
`oxprintlevel`	global control of printing
`oxrunerror`	raises a run-time error
`oxversion`	returns the Ox version
`oxwarning`	controls run-time warnings
`systemcall`	make an operating system call

time series (data in columns)

`acf`	autocorrelation function of matrix
`cumprod`	cumulate autoregressive product
`cumsum`	cumulate autoregressive sum

`cumulate`	cumulate (vector) autoregressive process
`diff0`	ith difference, $(1 - L^i)y$
`findsample`	determines the selected sample
`lag0`	ith lag
`periodogram`	periodogram, smoothed periodogram (spectral density)

time series (Arma package, requires `arma.oxh`)

`arma0`	residuals of an ARMA(p, q) filter
`armaforc`	forecasts from an ARMA(p, q) process
`armagen`	fitted values of an ARMA(p, q) process
`armavar`	autocovariances of an ARMA(p, q) process
`diffpow`	dth fractional difference, $(1 - L)^d y$
`modelforc`	forecasts of a dynamic model
`pacf`	partial autocorrelation function of matrix or applies Choleski factor of a Toeplitz matrix

trigonometric functions

`acos`	arccosine
`asin`	arcsine
`atan`	arctangent
`atan2`	arctangent of y/x
`cos`	cosine
`cosh`	cosine hyperbolicus
`sin`	sine
`sinh`	sine hyperbolicus
`tan`	tangent
`tanh`	tangent hyperbolicus

standard classes

Database	Data loading, saving; model selection
Modelbase	Model formulation and estimation, interactive facilities
PcFiml	OLS, VAR, cointegration, simultaneous equations
PcFimlDgp	General reduced form dynamic model DGP
PcNaiveDgp	DGP with up to two lags, may be equilibrium correction
RanMC	Error generation for Monte Carlo experiments
Sample	Basic sample: year (period)
Simulator	Monte Carlo experimentation

`ox/lib/` **code snippets** (examples in `ox/samples/lib/`)

acffft.ox	compute the ACF using the FFT
coigamma.ox	asymptotic distribution of I(1) and I(2) tests
densest.ox	density estimation
hacest.ox	heteroscedasticity and autocorrelation consistent covariance
hpfilter.ox	compute the Hodrick-Prescott filter
longrun.ox	dynamic analysis of dynamic systems
normtest.ox	Normality test

quantile.ox	compute quantiles given a density and cdf
spline3w.ox	computes a cubic spline weight matrix
probimhof.ox	Imhof procedure for cdf of the ratio of quadratic form
ranktest.ox	tests the rank of a matrix
testres.ox	residual-based tests (ARCH, Normality, Portmanteau)

Chapter 8

Function reference

Ox has implicit typing, so function declarations contain no type information. However, at run time, type information is known and checked for validity. The following argument types are distinguished in the function summary (the conversion rules are described in §13.8.2.3):

argument type	legal actual argument	conversion inside function to
int	int, double, 1×1 matrix	int
double	int, double, 1×1 matrix	double
matrix	int, double, matrix	matrix
arithmetic type	int, double, matrix	int $\to$ double
any type	any type	none
string	string	none
array	array	none
address	address	none

All functions documented in this chapter require the `oxstd.oxh` header file, which must be included by writing

```
    #include <oxstd.oxh>
```

at the top of your source code. A few functions need an additional header file, which is indicated explicitly.

Some functions have a variable argument list. An example is the `fread` function. This function is documented as:

```
fread(const file, const am, ...);
fread(const file, const am, const type, const r, const c);
```

which means that the following calls are allowed:

```
fread(file, am);
fread(file, am, type);
fread(file, am, type, r);
fread(file, am, type, r, c);
```

The function documentation will indicate what the default values are when arguments are omitted.

acf

```
acf(const ma, const ilag);
```

`ma`	in:	$T \times n$ matrix
`ilag`	in:	int, the highest lag

Return value

Returns a $(\texttt{ilag} + 1) \times n$ matrix with the autocorrelation function of the columns of `ma` up to lag `ilag`. Returns 0 if `ilag` ≤ 0. If any variance is $\leq 10^{-20}$, then the corresponding autocorrelations are set to 0.

Description

Computes the autocorrelation functions of the columns of a $T \times n$ matrix $A = (a_0, a_1, \ldots, a_{n-1})$. The autocorrelation function of a T-vector $x = (x_0 \cdots x_{T-1})'$ up to lag k is defined as $\mathsf{r} = (\hat{\mathsf{r}}_0 \cdots \hat{\mathsf{r}}_k)'$:

$$\hat{\mathsf{r}}_j = \frac{\sum_{t=j}^{T-1} (x_t - \bar{x})(x_{t-j} - \bar{x})}{\sum_{t=0}^{T-1} (x_t - \bar{x})^2}, \tag{8.1}$$

with the mean defined in the standard way as:

$$\bar{x} = \tfrac{1}{T} \sum_{t=0}^{T-1} x_t.$$

Note that $\hat{\mathsf{r}}_0 = 1$. The approximate standard error for $\hat{\mathsf{r}}_j$ is $1/\sqrt{T}$.

See also

`DrawCorrelogram`, `pacf`, `lib/AcfFft.ox`

Example

The example computes a correlogram twice, once using the library function, and once ‘manually’ (in the matrix `macf`).

```
#include <oxstd.oxh>
main()
{
    decl i, m1 = rann(200,2), m1m, macf, ilag = 5;

    macf = new matrix[ilag + 1][2];
    m1m = m1 - meanc(m1);          // in deviation from mean

    for (i = 0; i <= ilag; ++i)
        macf[i][] = diagonal(m1m'lag0(m1m, i));
    macf = macf ./ macf[0][];           // scale by variance

    print( acf(m1, ilag) ~ macf);
}
```

produces

```
       1.0000        1.0000        1.0000        1.0000
   -0.0021973     -0.046870    -0.0021973     -0.046870
    -0.041011     -0.051470     -0.041011     -0.051470
    -0.050879     -0.039346     -0.050879     -0.039346
     0.056525     -0.093980      0.056525     -0.093980
     0.021034       0.12671      0.021034       0.12671
```

acos

`acos(const ma);`

`ma`	in:	arithmetic type

Return value

Returns the arccosine of `ma`, of double or matrix type.

See also

`asin`, `atan`, `cos`, `cosh`, `sin`, `sinh`, `tan`, `tanh`

Example

```
#include <oxstd.oxh>
main()
{
    print( acos(<0,1>) );
    print( asin(<0,1>) );
    print( atan(<0,1>) );
    print( cos(<0,1>) );
    print( cosh(<0,1>) );
    print( sin(<0,1>) );
    print( sinh(<0,1>) );
    print( tan(<0,1>) );
    print( tanh(<0,1>) );
}
```

produces

```
      1.5708      0.00000
     0.00000       1.5708
     0.00000      0.78540
      1.0000      0.54030
      1.0000       1.5431
     0.00000      0.84147
     0.00000       1.1752
     0.00000       1.5574
     0.00000      0.76159
```

aggregatec, aggregater

```
aggregatec(const ma, const istep);
aggregater(const ma, const istep);
```

`ma`	in:	$m \times n$ matrix A
`istep`	in:	int, size of groups, s

Return value

The `aggregatec` function returns a ceil$(m/s) \times n$ matrix where each group of s observations in every column is replaced by the sum.

The `aggregater` function returns a $m \times$ ceil(n/s) matrix where each group of s observations in every row is replaced by the sum.

See also

`thinc`, `thinr`

Example

```
#include <oxstd.oxh>
main()
{
    decl x = ones(20,1) ~ range(1,20)';

    println(aggregatec(x, 5));
    println(aggregatec(x, 6));
    println(aggregater(x', 5));
    println(aggregater(x', 6));
}
```

produces

```
      5.0000       15.000
      5.0000       40.000
      5.0000       65.000
      5.0000       90.000

      6.0000       21.000
      6.0000       57.000
      6.0000       93.000
      2.0000       39.000

      5.0000       5.0000       5.0000       5.0000
      15.000       40.000       65.000       90.000

      6.0000       6.0000       6.0000       2.0000
      21.000       57.000       93.000       39.000
```

any

`any(const ma);`

`ma`	in:	arithmetic type

Return value

Returns TRUE if any element of `ma` is TRUE, of integer type.

Description

If any element is non-zero, the return value is 1. This is in contrast with the `if` statement, which evaluates to TRUE if *all* elements are TRUE.

See also

§13.8.9

Example

```
#include <oxstd.oxh>
main()
{
    decl m1 = unit(2), m2 = zeros(2,2);

    if (m1 == 0)         print ("TRUE ");
    else                 print ("FALSE ");
    if (any(m1 .== 0))   print ("TRUE ");
    else                 print ("FALSE ");
    if (!(m1 == 0))      print ("TRUE ");
    else                 print ("FALSE ");
    if (any(m1 .!= 0))   print ("TRUE ");
    else                 print ("FALSE ");

    if (m2 == 0)         print ("TRUE ");
    else                 print ("FALSE ");
    if (any(m2 .== 0))   print ("TRUE ");
    else                 print ("FALSE ");
    if (m2 != 0)         print ("TRUE ");
    else                 print ("FALSE ");
    if (any(m2 .!= 0))   print ("TRUE ");
    else                 print ("FALSE ");
}
```

produces: `FALSE TRUE TRUE TRUE TRUE TRUE FALSE FALSE`

arglist

`arglist();`

Return value

Returns an array of strings holding the command line arguments passed to the Ox program. The first entry is the name of the program.

Example

Running the following `arglist.ox` program:

```
#include <oxstd.oxh>
main()
{
    decl args = arglist(), s, i, j;

    for (i = 0; i < sizeof(args); ++i)
    {
        sscan(args[i], "%d", &j);
        println("argument ", i, ": ", args[i], " integer value:", j);
    }
}
```

as `oxl arglist.ox aa 12` (the arguments before `arglist.ox` are passed to `oxl`, those after to `arglist.ox`), produces:

```
    argument 0: arglist.ox integer value:0
    argument 1: aa integer value:0
    argument 2: 12 integer value:12
```

See also

`va_arglist` (for variable number of function arguments)

array

`array(const ma);`

`ma`	in:	any type

Return value

Casts the argument to an array, unless it already is an array.

Example

The `array` cast can be useful when an array indexation must remain an array. For example, a single index on an array of strings returns a string, whereas a multiple index returns an array of strings:

```
#include <oxstd.oxh>
main()
{
    decl as = {"ax", "bx", "cx"};

    print("single index is string: ", as[0],
        "\nmultiple index is array of strings:", as[0:1],
        "keep single index as array:", array(as[0]) );
}
```

which produces:

```
single index is string: ax
multiple index is array of strings:
[0] = ax
[1] = bx
keep single index as array:
[0] = ax
```

asin

```
asin(const ma);
```

`ma`	in:	arithmetic type

Return value

Returns the arcsine of `ma`, of double or matrix type.

See also

`acos` (for examples), `atan`, `cos`, `cosh`, `sin`, `sinh`, `tan`, `tanh`

atan, atan2

```
atan(const ma);
atan2(const my, const mx);
```

`ma`	in:	arithmetic type
`my`	in:	arithmetic type
`mx`	in:	arithmetic type

Return value

The `atan` function returns the arctangent of `ma`, of double or matrix type, between $-\pi/2$ and $\pi/2$.

The `atan2` function returns the arctangent of `my ./ mx`, between $-\pi$ and π. The return type is double if both `my` and `mx` are int or double. If `my` or `mx` is a matrix, the return type is a matrix of the same size.

See also

`acos` (for examples), `asin`, `cos`, `cosh`, `sin`, `sinh`, `tan`, `tanh`

bessel

```
bessel(const mx, const type, const n01);
bessel(const mx, const type, const nu);
```

`mx`	in:	x, arithmetic type, points at which to evaluate
`type`	in:	character, type of Bessel function: `'J'`, `'Y'`, `'I'`, `'K'` or string: `"IE"`, `"KE"`, for scaled Bessel functions
`n01`	in:	0 or 1: order of Bessel function
`nu`	in:	double, fractional order of Bessel function

Return value

Returns a $m \times n$ matrix with the requested Bessel function, or a double when `x` is scalar. The following are available: $J_0(x)$, $Y_0(x)$, $J_1(x)$, $Y_1(x)$, and the modified Bessel functions $I_0(x)$, $K_0(x)$, $I_1(x)$, $K_1(x)$. Similarly, the fractional Bessel functions $J_\nu(x)$, $Y_\nu(x)$, $I_\nu(x)$, $K_\nu(x)$. The modified Bessel functions are also available in scaled form: $e^{-x}I_\nu(x)$ and $e^{x}K_\nu(x)$.

The result is accurate to about 15 digits.

Description

The implementation is based on the code by W. Fullerton (Los Alamos scientific lab), as available in the FN library of netlib. The fractional Bessel functions are based on the Fortran code in Netlib by W.J. Cody.

betafunc

```
betafunc(const mx, const ma, const mb);
```

`mx`	in:	x, arithmetic type
`ma`	in:	a, arithmetic type
`mb`	in:	b, arithmetic type

Return value

Returns the incomplete beta integral $B_x(a, b)$. Returns 0 if $a \le 0$, $b \le 0$ or $x \le 0$. The accuracy is to about 10 digits.
The return type is derived as follows:

returns	`mx`	`ma,mb`
$m \times n$ matrix	$m \times n$ matrix	scalar
$m \times n$ matrix	scalar	$m \times n$ matrix
$m \times n$ matrix	$m \times n$ matrix	$m \times n$ matrix
double	scalar	scalar

Description

The incomplete beta integral is defined as:

$$B_x(a, b) = \int_0^x t^{a-1} (1-t)^{b-1} \, \mathrm{d}t, \quad a > 0, b > 0.$$

Note that the complete beta integral is:

$$B(a, b) = B_1(a, b) = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}.$$

Using the `loggamma` function, $B(a, b)$ can be computed as:

```
exp(loggamma(a) + loggamma(b) - loggamma(a+b))
```

which avoids overflow in the gamma function.
Also note that `betafunc` computes the incomplete beta integral, and *not* $I_x(a, b) = B_x(a, b)/B(a, b)$. $I_x(a, b)$ corresponds to the beta distribution, and can be computed with `probbeta`.
The approximation is based on the continued fraction representation given in Press, Flannery, Teukolsky, and Vetterling (1988, §6.3).

See also

`gammafunc`, `probbeta`, `probf`, `tailf`

binand, bincomp, binor, binpop, binxor

```
binand(const ia, const ib, ...);
bincomp(const ia);
binor(const ia, const ib, ...);
binpop(const ia, ...);
binxor(const ia, const ib, ...);
binvec(const ia);
```

`ia`	in:	int or matrix of integers
`ib`	in:	int or matrix of integers
`...`	in:	optional additional arguments, if present, all arguments must be integers

Return value

`binand` returns the result from *and*-ing all arguments (the `&` operator in C/C++).
`bincomp` returns the binary (bit-wise) complement of the argument (the `~` operator in C/C++).
`binor` returns the result from *or*-ing all arguments (the `|` operator in C/C++).
`binpop` returns the number of non-zero bits (popcount, Hamming weight) in the argument. If there is more than one argument, these are *xor*ed together first (the Hamming distance).
`binvec` returns the $n \times 32$ vector of zeros and ones, with each row the bitwise representation of the integer(s) in the argument. The least significant bit comes first, so 6 is returned as 0,1,1 followed by 29 zeros.
`binoxr` returns the result from *xor*-ing all arguments (the `^` operator in C/C++).

Example

```
#include <oxstd.oxh>
main()
{
    print( binand(1,2,4), " ", binor(1,2,4) );
}
```

produces: `0 7`

binomial

```
binomial(const n, const k);
```

`n`	in:	arithmetic type
`k`	in:	arithmetic type

Return value

Returns the binomial function at the rounded value of each element, of double or matrix type.
For negative integers, the function returns .NaN.

Description

Computes the binomial coefficient:

$$\begin{pmatrix} n \\ k \end{pmatrix} = \frac{n!}{(n-k)!k!}.$$

When $\max(n-k,k) \geq 50$ the computation uses the `loggamma` function:

$$\left(\begin{array}{c} n \\ k \end{array} \right) = \exp\left(\log\Gamma(n+1) - \log\Gamma(n-k+1) - \log\Gamma(k+1)\right).$$

which has about 13 significant digits.

See also

`factorial`, `gammafunc`, `loggamma`

cabs, cdiv, cerf, cexp, clog, cmul, csqrt

```
cabs(const ma);
cdiv(const ma, const mb);
cerf(const ma);
cexp(const ma);
clog(const ma);
cmul(const ma, const mb);
csqrt(const ma);
```

`ma`, `mb`	in:	$2 \times n$ matrix (first row is real part, second row imaginary part), or $1 \times n$ matrix (real part only)

Return value

`cabs` returns a $1 \times n$ matrix with absolute value of the vector of complex numbers.
`cdiv` returns a $2 \times n$ matrix with the result of the division of the vectors of complex numbers. If both `ma` and `mb` have no imaginary part, the return value is $1 \times n$.
`cerf` returns a $2 \times n$ matrix with the result of the complex error function of the vector of (complex) numbers.
`cexp` returns a $2 \times n$ matrix with the result of the complex exponential of the vector of (complex) numbers.
`clog` returns a $2 \times n$ matrix with the result of the complex logarithm of the vector of (complex) numbers. This is the principal branch.
`cmul` returns a $2 \times n$ matrix with the result of the multiplication of the two vectors of complex numbers. If both `ma` and `mb` have no imaginary part, the return value will be $1 \times n$.
`csqrt` returns a $2 \times n$ matrix with the square root of the vector of complex numbers.

Description

Using subscript r for the real part of a, b and subscript i for the imaginary part:
`cabs`: modulus of complex number: $|a| = (a_r^2 + a_i^2)^{1/2}$.
`cmul`: complex multiplication: $ab = (a_r + ia_i)(b_r + ib_i)$.
`cdiv`: complex division: $a/b = (a_r + ia_i)/(b_r + ib_i)$.
`csqrt`: square root of complex number: $a^{1/2} = (a_r + ia_i)^{1/2}$.
`cexp`: complex exponential: $\exp(a) = \exp(a_r)(\cos(a_i) + i\sin(a_i))$.
`clog`: complex logarithm: $\log(a) = \log(|a|) + i\arctan(a_i/a_r)$.
complex conjugate: $(a_r - ia_i)$.

The complex logarithm is a multivalued function, and the `clog` function takes the principal branch, see Olver, Lozier, Boisvert, and Clark (2010, § 4.2):

$$\log(z) = \log|z| + i\text{ph}z, \quad \pi < \text{ph}z, \pi,$$

extended by the cut

$$\log(x + i0) = \log|x| + i\pi \text{ and } \log(x - i0) = \log|x| - i\pi \text{ both for } -\infty < x < 0.$$

This cut along the negative real axis ($x \pm i0$ for $x < 0$) causes some properties not to hold when the line is crossed, e.g.:

$$z_1 = -2 + 1i, z_2 = -1 + 2i, \text{ then } z_1 z_2 = 0 - 5i,$$

and

$$\log(z_1) + \log(z_2) = \log(5) + i\pi 3/2, \text{ but } \log(z_1 z_2) = \log(5) + i\pi 1/2.$$

The identity $\log(z_1 z_2) = \log(z_1) + \log(z_2)$ holds when $|\text{ph} z_1 + \text{ph} z_2| \leq \pi$, or if we were to use the multivalued version of the complex logarithm.
The complex exponential is single valued with period $2\pi i$: $\exp(z + 2\pi i) = \exp(z)$. So in the above example we will find that:$\exp[\log(z_1 z_2)]$ is equal to $\exp[\log(z_1) + log(z_2)]$.

Example

```
#include <oxstd.oxh>
main()
{
    decl v = <1, -1, -2>, rv = csqrt(v);
    rv[0][1] = 1;/* change to a more interesting value */

    print(v, rv, cabs(rv), cdiv(rv, rv), cmul(rv, rv),
        cmul(rv, cdiv(ones(1,3), rv)) );
    print(cexp(clog(rv)) );
}
```

produces

```
       1.0000      -1.0000      -2.0000

       1.0000       1.0000      0.00000
      0.00000       1.0000       1.4142

       1.0000       1.4142       1.4142

       1.0000       1.0000       1.0000
      0.00000      0.00000      0.00000

       1.0000      0.00000      -2.0000
      0.00000       2.0000      0.00000

       1.0000       1.0000       1.0000
      0.00000      0.00000      0.00000

       1.0000       1.0000  8.6593e-017
      0.00000       1.0000       1.4142
```

In the second example the complex functions are used to check if the computed roots of a polynomial indeed correspond to zeros of the polynomial:

```
#include <oxstd.oxh>
main()
{
    decl v1 = <-1, 1.2274, -0.017197, -0.28369, -0.01028>, roots, cr;

    polyroots(v1, &roots);

    cr = columns(roots);
    print("roots", roots,
          "inverse roots", cdiv(ones(1,cr), roots) );

    decl x1, x2, x3, x4, check;
    x1 = roots;
    x2 = cmul(x1, x1);                  /* roots ^ 2 */
    x3 = cmul(x2, x1);                  /* roots ^ 3 */
    x4 = cmul(x2, x2);                  /* roots ^ 4 */
    check = v1[0][4] * (ones(1,cr) | zeros(1,cr)) +
            v1[0][3] * x1 + v1[0][2] * x2 +
            v1[0][1] * x3 + v1[0][0] * x4;

    print("check (near-zeros could be different "
          "with other Ox versions):", check);
}
```

which produces:

```
roots
      0.82865      0.82865     -0.39337    -0.036535
      0.16923     -0.16923      0.00000      0.00000
inverse roots
       1.1585       1.1585      -2.5422      -27.371
     -0.23659      0.23659      0.00000      0.00000
check (near-zeros could be different with other Ox versions):
      0.00000      0.00000 -1.7000e-016 -8.4441e-018
 -2.2204e-016  2.2204e-016      0.00000      0.00000
```

The final example considers the complex logarithm:

```
#include <oxstd.oxh>
main()
{
    decl z, z1, z2, zm;
    z = -1|-0.0; println("%c", {"z","clog(z)"}, z ~ clog(z));
    z = -1|0;    println(z ~ clog(z));
    z =  1|-0.0; println(z ~ clog(z));
    z =  1|0;    println(z ~ clog(z));

    z1=-2|1; z2 = -1|2;
    zm = cmul(z1, z2);
    println("%c", {"clog(z_1z_2)","clog(z_1)+clog(z_2)"},
        clog(zm) ~ clog(z1)+clog(z2));
    println("ph(z_1)=", atan2(z1[1], z1[0]),
            " ph(z_2)=", atan2(z2[1], z2[0]));
}
```

which produces:

```
            z      clog(z)
      -1.0000      0.00000
     -0.00000      -3.1416

      -1.0000      0.00000
      0.00000       3.1416

       1.0000      0.00000
     -0.00000     -0.00000

       1.0000      0.00000
      0.00000      0.00000

 clog(z_1z_2)clog(z_1)+clog(z_2)
       1.6094       1.6094
      -1.5708       4.7124
ph(z_1)=2.67795 ph(z_2)=2.03444
```

ceil

`ceil(const ma);`

`ma`	in:	arithmetic type

Return value

Returns the ceiling of each element of `ma`, of double or matrix type. The ceiling is the smallest integer larger than or equal to the argument

See also

`floor`, `round`, `trunc`

Example

```
#include <oxstd.oxh>
main()
{
    print( ceil(<-1.8, -1.2, 1.2, 1.8>) );
    print( floor(<-1.8, -1.2, 1.2, 1.8>) );
    print( round(<-1.8, -1.2, 1.2, 1.8>) );
    print( trunc(<-1.8, -1.2, 1.2, 1.8>) );

    print( int(-1.8), " ", int(-1.2), " ",
           int(1.2), " ", int(1.8) );
}
```

produces

```
      -1.0000      -1.0000       2.0000       2.0000
      -2.0000      -2.0000       1.0000       1.0000
      -2.0000      -1.0000       1.0000       2.0000
      -1.0000      -1.0000       1.0000       1.0000
-1 -1 1 1
```

chdir

`chdir(const s);`

`s`	in:	new directory

Return value

Returns 1 if successful, 0 otherwise.

Description

Changes the current directory.

Windows specific: if the string starts with a drive letter followed by a semicolon, the current drive is also changed. For example, use `chdir("c:")` to change to the C drive.

See also

`getcwd`, `getfiles` (for example), `systemcall`

choleski

`choleski(const ma);`

`ma`	in:	symmetric, positive definite $m \times m$ matrix A

Return value

Returns the Choleski decomposition P of a symmetric positive definite matrix A: $A = PP'$; P is lower triangular (has zeros above the diagonal).

Returns 0 if the decomposition failed.

Error and warning messages

choleski(): decomposition failed (this implies a negative definite or numerically singular matrix A).

See also

`decldl`, `invertsym`, `solvelu`, `RanMC::Choleski`

Example

The example also shows how `solvelu` may be used to obtain P^{-1}.

```
#include <oxstd.oxh>
main()
{
    decl mp;

    mp = choleski(<4,1;1,3>);
    print(mp, mp*mp');

    print(1/mp ~ solvelu(mp, 0, 0, unit(2)) );
}
```

produces

```
     2.0000      0.00000
   0.500000       1.6583

     4.0000       1.0000
     1.0000       3.0000

     0.50000      0.00000      0.50000      0.00000
    -0.15076      0.60302     -0.15076      0.60302
```

classname

```
classname(const obj);
```

`obj`	in:	object of a class

Return value

Returns a string with the class name of the object (or 0 if the argument is not an object).

See also

`isclass`

clone

```
clone(const obj);
clone(const obj, const iDeep=1);
```

`obj`	in:	object of a class
`iDeep`	in:	int, 1 (deep copy, the default), 0: shallow copy

Return value

Returns a clone of the object.

Description

The clone is an exact copy that must be removed with a call to `delete`.

When writing `a = new Database(); b = a;` both a and b refer to the same object, and only one can be deleted.

Writing `a = new Database(); b = clone(a);` both a and b refer different objects, which happen to hold the same values. Both a and b should be deleted when done.

The default is to make a deep copy: all members that are objects are also cloned (and members of members, etc.). A shallow copy only clones the members that are objects, but not members of members.

columns

`columns(const ma);`

`ma`	in:	any type

Return value

Returns an integer value with the number of columns in the argument `ma`:

type	returns
$m \times n$ matrix	n
string	number of characters in the string
array	number of elements in the array
file	number of columns in the file (only if opened with f format, see `fopen`)
other	0

See also

`rows`, `sizec`, `sizeof`, `sizer`, `sizerc`

Example

```
#include <oxstd.oxh>
main()
{
    println(columns(<0,1;1,2;3,4>), " ", columns("taylor"));
    println(   rows(<0,1;1,2;3,4>), " ",    rows("taylor"));
    println( sizerc(<0,1;1,2;3,4>), " ",  sizeof("taylor"));
}
```

produces

```
2 6
3 6
6 6
```

constant

`constant(const dval, const r, const c);`
`constant(const dval, const ma);`

`dval`	in:	double
`r`	in:	int
`c`	in:	int
`ma`	in:	matrix

Return value

`constant(dval,r,c)` returns an `r` by `c` matrix filled with `dval`.

`constant(dval,ma)` returns a matrix of the same dimension as `ma`, filled with `dval`.

See also

`ones`, `unit`, `zeros`

Example

```
#include <oxstd.oxh>
main()
{
    print( constant(1.5, 2, 2) );
}
```

produces

```
     1.5000       1.5000
     1.5000       1.5000
```

correlation

`correlation(const ma);`

`ma`	in:	$T \times n$ matrix A

Return value

Returns a $n \times n$ matrix holding the correlation matrix of `ma`. If any variance is $\leq 10^{-20}$, then the corresponding row and column of the correlation matrix are set to 0.

Description

Computes the correlation matrix $R = (r_{ij})$ of a $T \times n$ matrix $A = (a_{tj})$:

$$\bar{a}_j = \frac{1}{T} \sum_{t=0}^{T-1} a_{tj}$$

$$\hat{\sigma}_j^2 = \frac{1}{T} \sum_{t=0}^{T-1} (a_{tj} - \bar{a}_j)^2$$

$$r_{ij} = \frac{1}{T\hat{\sigma}_i\hat{\sigma}_j} \sum_{t=0}^{T-1} (a_{ti} - \bar{a}_i)(a_{tj} - \bar{a}_j)$$

Note that $r_{ii} = 1$.

See also

`acf`, `meanc`, `meanr`, `standardize`, `varc`, `varr`, `variance`

Example

```
#include <oxstd.oxh>
main()
{
    decl m1 = rann(100,2), m2;

    m2 = standardize(m1);
    print( correlation(m1), m2'm2/rows(m2) );
}
```

produces

```
       1.0000    -0.039218
    -0.039218       1.0000

       1.0000    -0.039218
    -0.039218       1.0000
```

cos, cosh

`cos(const ma);`
`cosh(const ma);`

`ma`	in:	arithmetic type

Return value

`cos` returns the cosine of `ma`, of double or matrix type.
`cosh` returns the cosine hyperbolicus of `ma`, of double or matrix type.

See also

`acos` (for examples), `asin`, `atan`, `cosh`, `sin`, `sinh`, `tan`, `tanh`

countc

`countc(const ma, const va);`

`ma`	in:	$m \times n$ matrix
`va`	in:	$1 \times q$ or $q \times 1$ matrix

Return value

Returns a matrix `r` which counts of the number of elements in each column of `ma` which is between the corresponding values in `va`:

`r[0][0]` = # elements in column 0 of `ma` $\leq$ `va[0]`
`r[1][0]` = # elements in column 0 of `ma` $>$ `va[0]` and $\leq$ `va[1]`
`r[2][0]` = # elements in column 0 of `ma` $>$ `va[1]` and $\leq$ `va[2]`
`r[q][0]` = # elements in column 0 of `ma` $>$ `va[q-1]`
...
`r[0][1]` = # elements in column 1 of `ma` $\leq$ `va[0]`
`r[1][1]` = # elements in column 1 of `ma` $>$ `va[0]` and $\leq$ `va[1]`
`r[2][1]` = # elements in column 1 of `ma` $>$ `va[1]` and $\leq$ `va[2]`
`r[q][1]` = # elements in column 1 of `ma` $>$ `va[q-1]`
...

If `ma` is $m \times n$, and `va` is $1 \times q$ (or $q \times 1$) the returned matrix is $(q + 1) \times n$ (any remaining columns of `va` are ignored). If the values in `va` are not ordered, the return value is filled with missing values.

Description

Counts the number of elements in each column which is in a supplied interval.

See also

`countr`

Example

```
#include <oxstd.oxh>
main()
{
    print( countc(<0:3;1:4;2:5>, <2,4>) );
    print( countr(<0:3;1:4;2:5>, <2>) );
}
```

produces

```
      3.0000       2.0000       1.0000      0.00000
     0.00000       1.0000       2.0000       2.0000
     0.00000      0.00000      0.00000       1.0000

      3.0000       1.0000
      2.0000       2.0000
      1.0000       3.0000
```

countr

```
countr(const ma, const va);
```

`ma`	in:	$m \times n$ matrix
`va`	in:	$1 \times q$ or $q \times 1$ matrix

Return value

Returns a matrix `r` which counts of the number of elements in each row of `ma` which is between the corresponding values in `va`:

`r[0][0]` = # elements in row 0 of `ma` $\leq$ `va[0]`
`r[0][1]` = # elements in row 0 of `ma` $>$ `va[0]` and $\leq$ `va[1]`
`r[0][2]` = # elements in row 0 of `ma` $>$ `va[1]` and $\leq$ `va[2]`
`r[0][q]` = # elements in row 0 of `ma` $>$ `va[q-1]`
...
`r[1][0]` = # elements in row 1 of `ma` $\leq$ `va[0]`
`r[1][1]` = # elements in row 1 of `ma` $>$ `va[0]` and $\leq$ `va[1]`
`r[1][2]` = # elements in row 1 of `ma` $>$ `va[1]` and $\leq$ `va[2]`
`r[1][q]` = # elements in row 1 of `ma` $>$ `va[q-1]`
...

If `ma` is $m \times n$, and `va` is $1 \times q$ (or $q \times 1$) the returned matrix is $m \times (q + 1)$ (any remaining columns of `va` are ignored). If the values in `va` are not ordered, the return value is filled with missing values.

Description

Counts the number of elements in each row which is in a supplied interval.

See also

`countc` (for an example)

cumprod

```
cumprod(const mfac);
cumprod(const mfac, const cp);
cumprod(const mfac, const cp, const mz);
```

`mfac`	in:	$T \times n$ or $1 \times n$ matrix of multiplication factors S
`cp`	in:	int: autoregressive order p (optional argument; default is 1)
`mz`	in:	(optional argument) $T \times n$ or $1 \times n$ matrix of known components Z (optional argument; default is 0)

Return value

Returns a $T \times n$ matrix with the cumulated autoregressive product. The first p rows of the return value will be identical to the sum of those in `mz` and `mfac`; the recursion will be applied from the pth term onward. If either `mz` or `mfac` is $1 \times n$, the same values are used for every t.

Description

For a column $(z_0, \ldots, z_{T-1})'$ of known values X, and multiplication factors $(s_0, \ldots s_{T-1})'$ the `cumprod` function computes:

$$\begin{array}{ll} a_t = z_t + s_t, & t = 0, \ldots, p-1, \\ a_t = z_t + s_t(a_{t-1} \times \ldots \times a_{t-p}) & t = p, \ldots, T-1. \end{array}$$

See also

`cumsum` (for an example), `cumulate`

cumsum

```
cumsum(const mx, const vp);
cumsum(const mx, const vp, const mstart);
```

`mx`	in:	$T \times n$ matrix of known component X
`vp`	in:	$1 \times p$ or $n \times p$ or $T \times p$ matrix with autoregressive coefficients $\phi_1, \phi_2, \ldots, \phi_p$
`mstart`	in:	(optional argument) $s \times n$ matrix of starting values S, $s \geq p$; default is `mx`

Return value

Returns a $T \times n$ matrix with the cumulated autoregressive sum. The first p rows of the return value will be identical to those of `mstart`; the recursion will be applied from the pth term onward.

If `vp` is $1 \times p$, the same coefficients are applied to each column.

If `vp` is $n \times p$, each row will have coefficients specific to each column of the recursive series.

Finally, if `vp` is $T \times p$, the same coefficients are applied to each column, but the coefficients are specific to each row (time-varying coefficients).

Description

For a column $(x_0, \ldots, x_{T-1})'$ of known values X, and starting values $(s_0, \ldots s_{p-1})'$ the `cumsum` function computes:

$$\begin{array}{ll} a_t = s_t, & t = 0, \ldots, p-1, \\ a_t = x_t + \phi_1 a_{t-1} + \ldots + \phi_p a_{t-p}, & t = p, \ldots, T-1. \end{array}$$

When ϕ is $n \times p$, the AR coefficients are different for each data column, for $j = 0, \ldots, n-1$:

$$\begin{array}{ll} a_{t,j} = s_{t,j}, & t = 0, \ldots, p-1, \\ a_{t,j} = x_{t,j} + \phi_{j,1} a_{t-1} + \ldots + \phi_{t,p} a_{j-p}, & t = p, \ldots, T-1. \end{array}$$

When ϕ is $T \times p$ (and $T \neq n$), the AR coefficients are time-varying:

$$\begin{array}{ll} a_t = s_t, & t = 0, \ldots, p-1, \\ a_t = x_t + \phi_{t,1} a_{t-1} + \ldots + \phi_{t,p} a_{t-p}, & t = p, \ldots, T-1. \end{array}$$

See also

`cumprod`, `cumulate`

Example

```
#include <oxstd.oxh>
main()
{
    decl mx = ones(5,1);
    print( mx ~ cumsum(mx, <0.5>)
              ~ cumsum(mx, <1, 0.5>, <1;2>)
              ~ cumprod(mx * 2)
              ~ cumprod(mx * 2, 2) );

    print(cumsum(mx, <0.5;0.5;0.5;1;1>)' );
}
```

produces

```
      1.0000      1.0000      1.0000      2.0000      2.0000
      1.0000      1.5000      2.0000      4.0000      2.0000
      1.0000      1.7500      3.5000      8.0000      8.0000
      1.0000      1.8750      5.5000      16.000      32.000
      1.0000      1.9375      8.2500      32.000      512.00

      1.0000      1.5000      1.7500      2.7500      3.7500
```

cumulate

```
cumulate(const ma);
cumulate(const ma, const m1, ...);
cumulate(const ma, const am);
```

`ma`	in:	$T \times n$ matrix A
`m1`	in:	$n \times n$ matrix, coefficients of first lags (optional argument)
...	in:	$n \times n$ matrix, coefficients of lags 2, ...
`am`	in:	array of length k with $n \times n$ matrices of coefficients

Return value

Returns a $T \times n$ matrix. The simplest version returns a matrix which holds the cumulated (integrated) columns of `ma`.

The second form cumulates (integrates) the (vector) autoregressive process with current values `ma` using the specified coefficient matrices. The function has a variable number of arguments, and the number of arguments determines the autoregressive order (minimum 2 arguments, which is an AR(1) process). Note that `cumulate(m)` corresponds to `cumulate(m,unit(columns(m)))`.

Description

The version with one arguments cumulates the columns of its argument.

For the form with additional arguments, assume that `ma` and k coefficient matrices have been supplied ($k \geq 1$: at least two arguments) and write $A_0^{T-1} = A =$ `ma`, $M_1 =$ `m1`, $\ldots M_k$. Also define A_{-i}^{T-1-i} as the ith lag, whereby each column is lagged: each column of A is shifted down, and missing values are replaced by zeros, so that e.g. A_{-1}^{T-2} = `lag0(ma, 1)`. The `cumulate` function returns:

$$A_0^{T-1} + A_{-1}^{T-2}M_1 + A_{-2}^{T-3}M_2 + \cdots A_{-k}^{T-1-k}M_k,$$

which has the same dimensions as `ma`.

The univariate case is easier to explain. For example, with three arguments, $(a_0, \ldots, a_{T-1})'$, β_0 and β_1, this function computes y_t:

$$\begin{aligned} y_0 &= a_0, \\ y_1 &= a_1 + \beta_0 y_0, \\ y_t &= a_t + \beta_0 y_{t-1} + \beta_1 y_{t-2}, \quad t = 2, \ldots, T-1. \end{aligned}$$

See also

`cumsum`, `lag0`

Example

```
#include <oxstd.oxh>
main()
{
    print( ones(5,1) ~ cumulate(ones(5,1))
           ~ cumulate(ones(5,1), <0.5>)
           ~ cumulate(ones(5,1), <1>, <0.5>)
           ~ cumulate(ones(5,1), {<1>, <0.5>}) );
}
```

produces

```
      1.0000       1.0000       1.0000       1.0000       1.0000
      1.0000       2.0000       1.5000       2.0000       2.0000
      1.0000       3.0000       1.7500       3.5000       3.5000
      1.0000       4.0000       1.8750       5.5000       5.5000
      1.0000       5.0000       1.9375       8.2500       8.2500
```

date

`date();`

Return value

A string holding the current date.

See also

`time`

Example

```
#include <oxstd.oxh>
main()
{
    println("\ntime=", time(), " date=", date());
}
```

prints the current time and date.

dawson

`dawson(const ma);`

`ma`	in:	arithmetic type

Return value

Returns the Dawson integral

$$F(x) = e^{-x^2} \int_0^x e^{t^2}\, dt$$

of each element of `ma`, of double or matrix type.

Description

The function is based on the Fortran code in Netlib by W.J. Cody.

See also

`erf`

dayofcalendar, dayofeaster, dayofmonth, dayofweek

```
dayofcalendar();
dayofcalendar(const index);
dayofcalendar(const year, const month, const day);
dayofeaster(const year);
dayofmonth(const year, const month, const dayofweek, const nth);
dayofweek(const index);
dayofweek(const year, const month, const day);
```

`index`	in:	in: arithmetic type, calendar index of a certain date, as returned by `dayofcalendar(year, month, day)`
`year`	in:	arithmetic type
`month`	in:	arithmetic type, January=1, etc.
`day`	in:	arithmetic type
`dayofweek`	in:	arithmetic type, day of the week (Sunday = 1, Monday = 2, . . .)
`nth`	in:	arithmetic type, $>$ 0: n-th from start of month, $<$ 0: n-th from end of month

Return value

The `dayofcalendar` function without any arguments returns the calendar index (Julian day number) of today.

The `dayofcalendar` function with three arguments returns the calendar index of the specified date (this is the Julian day number, see below). If all arguments are an integer, the return value will be an integer.

The `dayofcalendar` function with one argument takes a calendar index (or vector of indices), as returned by `dayofcalendar(year, month, day)` as argument, returning a $n \times 3$ matrix with the triplet year, month, day in each row (n is the number of elements in the input).

The `dayofeaster` function returns the calendar index of Easter.

The `dayofmonth` function returns the calendar index of the n-th day of the week in the specified month (n-th from last for a negative value). For example `dayofmonth(2005, 5, 2, -1)` returns the index of the last Monday in May 2005.

The `dayofweek` function with three arguments returns the day of the week (Sunday = 1, Monday = 2, . . .). If all arguments are an integer, the return value will be an integer.

The `dayofweek` function with one argument takes a calendar index (r vector of) as argument, returning the day of the week (Sunday = 1, Monday = 2, . . .).

Description

The calendar[1] index is the Julian day number, and the `dayof...` functions convert from or to the index. For example, Julian day 2453402 corresponds to 2005-01-31. An optional fractional part specifies the fraction of the day: 2453402.75 corresponds to 2005-01-01T18:00. If the day number is zero, it is interpreted as a time only, so

[1]The calendar is Gregorian from 15 October 1582 onwards, and Julian before (so there is no year 0: year -1 precedes year 1; day 0 is on Julian date 1 January -4713).

0.75 is just 18:00 (6 PM).[2]

Use `dayofcalendar(year, month, day) - dayofcalendar(year, 1, 1) + 1` to compute the day in the year. Similarly, the function can be used to compute the number of days between two dates.

The `"%C"` print format is available to print a calendar index.

See also

`print`, `timeofday`, `timing`

Example

```
#include <oxstd.oxh>
main()
{
    println("1-Jan-2000 was weekday ",
        dayofweek(2000, 1, 1), " (7 is Saturday)");
    println("1-Jan-2000 was yearday ",
        dayofcalendar(2000, 1, 1));
    println("2000 had ", dayofcalendar(2001, 1, 1)
        - dayofcalendar(2000, 1, 1), " days");
    println("2001 had ", dayofcalendar(2002, 1, 1)
        - dayofcalendar(2001, 1, 1), " days");

    println("%c", {"Easter Sunday", "Last Wed in May"},
        "%17C", dayofeaster(range(2005, 2010)')
                ~ dayofmonth(range(2005, 2010)', 5, 2, -1));

    println("today ", "%C", dayofcalendar());
}
```

produces

```
1-Jan-2000 was weekday 7 (7 is Saturday)
1-Jan-2000 was yearday 2451545
2000 had 366 days
2001 had 365 days

    Easter Sunday  Last Wed in May
       2005-03-27       2005-05-30
       2006-04-16       2006-05-29
       2007-04-08       2007-05-28
       2008-03-23       2008-05-26
       2009-04-12       2009-05-25
       2010-04-04       2010-05-31

today 2012-11-28
```

[2]This is similar to how Excel stores date and time. The main difference is that Excel uses 1=1900-01-01 (wrongly treating 1900 as a leap year).

decldl

`decldl(const ma, const aml, const amd);`

`ma`	in:	symmetric, positive definite $m \times m$ matrix A
`aml`	in:	address of variable
	out:	$m \times m$ lower diagonal matrix L, $LDL' = A$
`amd`	in:	address of variable
	out:	$1 \times m$ matrix with reciprocals of D

Return value

Returns the result of the Choleski decomposition:

1 no error;

0 the Choleski decomposition failed: the matrix is negative definite or the matrix is (numerically) singular.

Description

Computes the square root free Choleski decomposition of a symmetric positive definite matrix A stored in argument `ma`: $A = LDL'$. L has zeros above the diagonal and ones on the diagonal.

Note that the *reciprocals* of D are stored in `amd`.

Error and warning messages

decldl(): decomposition failed (the matrix is numerically singular or negative definite)

See also

`choleski`, `decldlband`, `solveldl`

Example

```
#include <oxstd.oxh>
main()
{
    decl ma = <4,1;1,3>, md, ml, mi;

    print("result = ", decldl(ma, &ml, &md));
    print(" L =", ml, "D =", md);
    print(ml*diag(1 ./ md)*ml');

    mi = solveldl(ml, md, unit(2));
    print(mi*ma);
}
```

Note that `diag(1 ./ md)` and `diag(1./md)` are not the same. The program produces (the final matrix could have values of around 1e-16 instead of 0):

```
result = 1 L =
       1.0000      0.00000
      0.25000       1.0000
D =
      0.25000      0.36364

       4.0000       1.0000
       1.0000       3.0000

       1.0000      0.00000
      0.00000       1.0000
```

decldlband

decldlband(const ma, const aml, const amd);

ma	in:	$p \times m$ vector specifying the A^b matrix
aml	in:	address of variable
	out:	holds $p \times m$ lower diagonal matrix L
amd	in:	address of variable
	out:	$1 \times m$ matrix with reciprocals of D

Return value

Returns the result of the Choleski decomposition:

1 no error;

0 the Choleski decomposition failed: the matrix is negative definite or the matrix is (numerically) singular.

Description

Computes the square root free Choleski decomposition of a symmetric positive definite band matrix A stored in argument ma: $A = LDL'$. L has zeros above the diagonal and ones on the diagonal. Note that the reciprocals of D are stored.

If $A = (a_{ij}), i, j = 0, \dots m-1$ is the underlying $m \times m$ symmetric positive definite band matrix, with bandwidth p, so that $a_{ij} = 0$ for $|i-j| > p$, then the input matrix ma $= A^b$ is formed as:

$$\begin{pmatrix} 0 & \cdots & \cdots & 0 & a_{0,p-1} & \cdots & a_{m-p,m-1} \\ \vdots & & & & & & \vdots \\ 0 & a_{0,1} & a_{1,2} & \cdots & \cdots & \cdots & a_{m-2,m-1} \\ a_{0,0} & \cdots & \cdots & \cdots & \cdots & \cdots & a_{m-1,m-1} \end{pmatrix}$$

The example below also shows how to create A^b out of A and vice versa.

Error and warning messages

decldlband(): decomposition failed (the matrix is numerically singular or negative definite)

See also

diagonal, solveldlband, solvetoeplitz

Example

```
#include <oxstd.oxh>
main()
{
    decl i, j, k, m, mab, ma, ml, md, ct = 5, cb = 2;

    ma = toeplitz(<5,4,3>, ct);   // create test matrix ma
    for (i = 0; i < ct; ++i)
        ma[i][i] += i;

    mab = diagonal(ma, cb);  // create band matrix version
    print("original matrix", ma, "band version", mab);

    if (decldlband(mab, &ml, &md))  // decompose and solve
        print("solved:", solveldlband(ml, md, <1;2;3;4;5>)' );

    // undo banded storage:store L in lower diagonal of ma
    for (i = 0, m = -cb; i < ct; ++i, m++)
        for (j = max(0,m), k = j - m; j < i; ++j, ++k)
            ma[i][j] =  ml[k][i];

    print("band L=", ml, "L:U=", ma);
}
```

produces

```
original matrix
       5.0000       4.0000       3.0000      0.00000      0.00000
       4.0000       6.0000       4.0000       3.0000      0.00000
       3.0000       4.0000       7.0000       4.0000       3.0000
      0.00000       3.0000       4.0000       8.0000       4.0000
      0.00000      0.00000       3.0000       4.0000       9.0000
band version
      0.00000      0.00000       3.0000       3.0000       3.0000
      0.00000       4.0000       4.0000       4.0000       4.0000
       5.0000       6.0000       7.0000       8.0000       9.0000
solved:
     0.012378      0.26172    -0.036251      0.17507      0.48983
band L=
      0.00000      0.00000      0.60000       1.0714      0.70000
      0.00000      0.80000      0.57143      0.53333      0.67290
       1.0000       1.0000       1.0000       1.0000       1.0000
L:U=
       5.0000       4.0000       3.0000      0.00000      0.00000
      0.80000       6.0000       4.0000       3.0000      0.00000
      0.60000      0.57143       7.0000       4.0000       3.0000
      0.00000       1.0714      0.53333       8.0000       4.0000
      0.00000      0.00000      0.70000      0.67290       9.0000
```

declu

`declu(const ma, const aml, const amu, const amp);`

`ma`	in:	square $m \times m$ matrix A
`aml`	in:	address of variable
	out:	$m \times m$ matrix lower diagonal matrix L, has ones on the diagonal
`amu`	in:	address of variable
	out:	$m \times m$ matrix upper diagonal matrix U, $LU = PA$
`amp`	in:	address of variable
	out:	$2 \times m$ matrix, the first row holds the permutation matrix P', $A = (LU)[P'][\,]$, the second row holds the interchange permutations

Return value

Returns the result of the LU decomposition:

1 no error;

2 the decomposition could be unreliable;

0 the LU decomposition failed: the matrix is (numerically) singular.

Description

Computes the LU decomposition of a matrix A as: $PA = LU$ by Gaussian elimination (using accumulation of inner-products) with partial pivoting, as described, e.g. in Wilkinson (1965, §4.39) (also see Golub and Van Loan, 1989 §3.4 for an analysis). *Note that L has ones on the diagonal.*

The permutation matrix P' is stored as a vector of row indices so that $A = (LU)[P'][\,]$ (see the example below). The actual permutation matrix $P' = P^{-1}$ can be created as `pt = (unit(rows(ma)))[vp][]` where `ma` is the original matrix, and `vp` holds the row indices as returned by `declu` (in the first row of `amp`, the last argument). P can be computed as `vp[][vp]`. The second row of `amp` holds the interchange permutations p, such that rows $p[0][i]$ and i are swapped.

Error and warning messages

declu(): decomposition failed (the matrix is numerically singular)

See also

`determinant`, `invert`, `solvelu`

Example

```
#include <oxstd.oxh>
main()
{
    decl ma, ml, mu, vp, mx;

    ma = <3,17,10;2,4,-2;6,18,-12>;
    declu(ma, &ml, &mu, &vp);
    print( (ml*mu)[ vp[0][] ][], (unit(rows(ma)))[ vp[0][] ][] );

    mx = solvelu(ml, mu, vp, ma);
    print(mx);
}
```

produces (note that the last matrix is the identity matrix: whether it has zeros, or nearly zeros, could dependent on which Ox version was used):

```
   3.0000      17.000      10.000
   2.0000      4.0000     -2.0000
   6.0000      18.000     -12.000

  0.00000      1.0000     0.00000
  0.00000     0.00000      1.0000
   1.0000     0.00000     0.00000

   1.0000     0.00000     0.00000
  0.00000      1.0000     0.00000
  0.00000     0.00000      1.0000
```

decqr

`decqr(const ma, const amq, const amr, const amp);`

`ma`	in:	$m \times n$ matrix A
`amq`	in:	address of variable
	out:	$n \times m$ matrix upper diagonal matrix H', has ones on the diagonal
`amr`	in:	address of variable
	out:	$n \times n$ matrix upper diagonal matrix R_1
`amp`	in:	address of variable (use 0 as argument to avoid pivoting; note that pivoting is recommended)
	out:	$2 \times n$ matrix, the first row holds the permutation matrix P', the second row holds the interchange permutations

Return value

Returns the result of the QR decomposition:

0:	out of memory,
1:	success,
2:	ratio of diagonal elements of $A'A$ is large, rescaling is advised, (ratio of smallest to largest $\leq \epsilon_{inv}$)
-1:	$(A'A)$ is (numerically) singular ($|R_{ii}| \leq \epsilon_{inv} \left[\max_j (A'A)_{jj}\right]^{1/2}$),
-2:	combines 2 and -1.

The inversion epsilon, ϵ_{inv}, is set by the `inverteps` function.

Description

Computes the QR decomposition of a matrix A as: $AP = QR$ based on Householder transformations with column pivoting, as described, e.g. in Golub and Van Loan (1989, §5.4). A is $m \times n$, Q is an $m \times m$ orthogonal matrix, and R is an $m \times n$ upper diagonal matrix. Note that this function does *not* return Q and R. Instead it returns R_1, which is the $\min(n, m) \times n$ upper block of R (the rest of R is zero). Q' is returned as an $\min(n, m) \times m$ matrix H' which stores the Householder vectors. H is lower diagonal with ones on the diagonal. H will have columns of zeros if A is reduced rank (in that case pivoting is essential).

The `decqrmul` function uses H' to compute $Q'Y$.

The permutation matrix P' is stored in the same way as for `declu`.

See also

`decqrmul` (for another example), `decqrupdate`, `inverteps`, `olsc`, `solvelu`

Example

```
#include <oxstd.oxh>
main()
{
    decl ma, mht, mr, mp, vp;
    ma = <2,1,4;5,1,7;8,1,9;11,1,12>;

    decqr(ma, &mht, &mr, &mp);
    vp = mp[0][];
    print("A=", ma, "A\'A", ma'ma,
        "R\'R (ignoring pivoting)", mr'mr,
        "R\'R (after undoing pivoting)", (mr'mr)[vp][vp]);
    println("Note that mp[0][] contains P':", vp);
    println("The pivots on A (where AP=QR) are:",
        sortcindex(vp') ');
}
A=
       2.0000       1.0000       4.0000
       5.0000       1.0000       7.0000
       8.0000       1.0000       9.0000
       11.000       1.0000       12.000
A'A
       214.00       26.000       247.00
       26.000       4.0000       32.000
       247.00       32.000       290.00
R'R (ignoring pivoting)
       290.00       247.00       32.000
       247.00       214.00       26.000
       32.000       26.000       4.0000
R'R (after undoing pivoting)
       214.00       26.000       247.00
       26.000       4.0000       32.000
       247.00       32.000       290.00
Note that mp[0][] contains P':
       1.0000       2.0000      0.00000
The pivots on A (where AP=QR) are:
       2.0000      0.00000       1.0000
```

decqrmul

```
decqrmul(const mht, const my);
decqrmul(const mht);
```

`mht`	in:	$n \times m$ matrix H' from `decqr`
`my`	in:	$m \times p$ matrix Y

Return value

Returns $Q'Y$, where Q is the orthogonal matrix derived from the QR decomposition. The version with one argument returns the $m \times m$ matrix Q'.

Description

The `decqr` composition returns Q in the form of householder vectors H'. This function may be used to obtain $Q'Y$ or Q' (the latter can be costly as it requires an $m \times m$ identity matrix). To compute QY, reverse the elements in each column of H': `decqrmul(reversec(mht), my)`.

See also

`decqr`, `olsc`, `solvelu`

Example

The example shows how to obtain Q', reconstructs the original matrix, and implements regression using the QR decomposition (note that `olsc` is also QR based). Because the input matrix is singular, the solution is not unique. Different versions of Ox may find different solutions depending on differences in accumulation of rounding errors.

```
#include <oxstd.oxh>
main()
{
    decl iret, ma, maa, mht, mr, mp, mq, mb, vy;

    ma = <1,2,3;1,5,6;1,8,9;1,11,12>;

    iret = decqr(ma, &mht, &mr, &mp);
    if (iret < 0)
        println("Input matrix is singular");
    print("H\'=", mht', "R1=", mr, "pivots", mp);

    mq = decqrmul(mht);
    maa = mq' * (mr | <0,0,0>);
    print("Q\'=", mq', "ma (pivoted)=", maa,
        "ma=", maa[][mp[0][]]);

    vy = <2;1;2;4>;
    olsc(vy, ma, &mb);
    print("regression coefficients (transposed)", mb');

    decl rank = sumr(fabs(diagonal(mr)) .> 1e-14);
    println("rank=", rank);
    mr[rank:][] = 0;

    mb = solvelu(0, mr, 0, decqrmul(mht, vy)[:2][]);
    print("from QR", mb', "in correct order", mb[mp[0][]][]');
}
```

```
Input matrix is singular
```

```
H'=
       1.0000     0.00000     0.00000
      0.30877      1.0000     0.00000
      0.46316    -0.32710      1.0000
      0.61755    -0.78925     0.46524
R1=
      -16.432     -1.8257     -14.606
      0.00000    -0.81650     0.81650
      0.00000     0.00000  2.7577e-015
pivots
       1.0000      2.0000     0.00000
       2.0000      2.0000      2.0000
Q'=
     -0.18257    -0.81650     0.54384    0.065078
     -0.36515    -0.40825    -0.77363     0.31859
     -0.54772  5.5511e-017  -0.084268    -0.83241
     -0.73030     0.40825     0.31406     0.44874
ma (pivoted)=
       3.0000      1.0000      2.0000
       6.0000      1.0000      5.0000
       9.0000      1.0000      8.0000
       12.000      1.0000      11.000
ma=
       1.0000      2.0000      3.0000
       1.0000      5.0000      6.0000
       1.0000      8.0000      9.0000
       1.0000      11.000      12.000
regression coefficients (transposed)
      0.50000     0.00000     0.23333
rank=
       2.0000
from QR
      0.23333     0.50000     0.00000
in correct order
      0.50000     0.00000     0.23333
```

decqrupdate

```
decqrupdate(const amq, const amr, const i1, const i2);
decqrupdate(const amq, const amr, const i1);
```

`amq`	in:	address of $m \times m$ matrix Q
	out:	updated matrix Q
`amr`	in:	address of $m \times n$ matrix R
	out:	updated matrix R

No return value.

Description

Updates the QR decomposition using Givens rotations.

The version with only the `i1` argument zeroes the subdiagonal elements from subdiagonal `i1` to the diagonal (i.e. subdiagonal 0). It is assumed that subdiagonals below `i1` are already zero.

The version with both the `i1` and `i2` arguments zeroes the subdiagonal from column `i1` to column `i2`. It is assumed that columns before `i1` are already zero below the diagonal.

Both `decqrupdate(&q, &a, 0, columns(r));` and `decqrupdate(&q, &a, rows(r));` compute a complete QR decomposition (like `decqr`, although `decqr` does not compute Q explicitly). However, the `decqrupdate` function is primarily intended to update a QR factorization.

See also

`decqr`, `decqrmul`

Example

The example shows first how the QR decomposition of an upper Hessenberg matrix (a matrix with zeros below the subdiagonal) can be computed, and then updates after appending a column to a lower triangular matrix.

```
#include <oxstd.oxh>
main()
{
    decl ma, maa, mht, mr, mp, mq, mb, vy;

    ma = <1,2,3,4,5;1,5,6,7,8;0,1,8,9,10;0,0,1,11,12>;

    println("Upper Hessenberg matrix A", ma);
    mr = ma;
    mq = unit(sizer(ma));
    decqrupdate(&mq, &mr, 1);
    println("triangular R:", mr);
    println("original:", mq*mr);

    mr[1:][0] = 1;
    mq = unit(sizer(ma));
    println("Column 0 changed:", mr);
    decqrupdate(&mq, &mr, 0, sizer(mr));
    println("Made triangular:", mr);
    println("original:", mq*mr);
}
```

```
Upper Hessenberg matrix A
       1.0000       2.0000       3.0000       4.0000       5.0000
```

```
      1.0000       5.0000       6.0000       7.0000       8.0000
     0.00000       1.0000       8.0000       9.0000       10.000
     0.00000      0.00000       1.0000       11.000       12.000
triangular R:
      1.4142       4.9497       6.3640       7.7782       9.1924
     0.00000       2.3452       5.3300       5.7564       6.1828
     0.00000      0.00000       6.4102       8.8637       9.9131
     0.00000      0.00000      0.00000       9.7365       10.583
original:
      1.0000       2.0000       3.0000       4.0000       5.0000
      1.0000       5.0000       6.0000       7.0000       8.0000
     0.00000       1.0000       8.0000       9.0000       10.000
     0.00000      0.00000       1.0000       11.000       12.000
Column 0 changed:
      1.4142       4.9497       6.3640       7.7782       9.1924
      1.0000       2.3452       5.3300       5.7564       6.1828
      1.0000      0.00000       6.4102       8.8637       9.9131
      1.0000      0.00000      0.00000       9.7365       10.583
Made triangular:
      2.2361       4.1793       9.2753       15.812       17.745
     0.00000       3.5403       1.4789      -3.9779      -4.0002
     0.00000      0.00000       4.6671     -0.80946     -0.78739
     0.00000      0.00000      0.00000     -0.70954      -1.2216
original:
      1.4142       4.9497       6.3640       7.7782       9.1924
      1.0000       2.3452       5.3300       5.7564       6.1828
      1.0000  3.8760e-016       6.4102       8.8637       9.9131
      1.0000  3.8760e-016  8.6736e-018       9.7365       10.583
```

decschur, decschurgen

```
decschur(const ma, const amval, const ams, ...);
decschur(const ma, const amval, const ams, const amv,
    const dselmin, const dselmax);
decschurgen(const ma, const mb, const amalpha, const ambeta,
    const ams, const amt, ...);
decschurgen(const ma, const mb, const amalpha, const ambeta,
    const ams, const amt, const amvl, const amvr, const dselmin,
    const dselmax);
```

`ma`	in:	$m \times m$ matrix A
`amval`	in:	address of variable
	out:	complex eigenvalues: $2 \times m$ matrix with eigenvalues of A first row is real part, second row imaginary part only real eigenvalues: $1 \times m$ matrix The eigenvalues are not ordered unless `dselmin` and `dselmax` are specified.
`ams`	in:	address of variable
	out:	upper quasi-triangular Schur form S, such that $A = VSV'$
`amv`	in:	(optional) address of variable
	out:	orthogonal matrix V with Schur vectors: $A = VSV'$
`dselmin`	in:	(optional) double, min absolute eigenvalue to move forward
`dselmax`	in:	(optional) double, max absolute eigenvalue to move forward
`ma`	in:	$m \times m$ matrix A
`mb`	in:	$m \times m$ matrix B for generalized Schur decomposition
`amalpha`	in:	address of variable
	out:	complex values: $2 \times m$ matrix with α first row is real part, second row imaginary part only real αs: $1 \times m$ matrix The generalized eigenvalues are $(\alpha_r[j] + i\alpha_i[j])/\beta[j],\ j = 0, \ldots, m-1$. The generalized eigenvalues are not ordered unless `dselmin` and `dselmax` are specified.
`ambeta`	in:	address of variable
	out:	$1 \times m$ matrix with β
`ams`	in:	address of variable
	out:	upper quasi-triangular Schur form S, with $A = V_l S V_r'$
`amt`	in:	address of variable
	out:	upper-triangular Schur form T, such that $B = V_l T V_r'$
`amvl`	in:	(optional) address of variable
	out:	orthogonal matrix V_l with left Schur vectors
`amvr`	in:	(optional) address of variable
	out:	orthogonal matrix V_r with right Schur vectors
`dselmin`	in:	(optional) double, minimum absolute generalized eigenvalue to include move forward
`dselmax`	in:	(optional) double, maximum absolute generalized eigenvalue to include move forward

Return value

Returns the result of the Schur decomposition:

0	no error;
1	maximum no of iterations reached;
-1	ill conditioning prevented ordering;
-2	rounding errors in ordering affected complex eigenvalues.

Description

The `decschur` function computes the Schur decomposition of a real matrix A:

$$A = VSV',$$

where V is orthogonal, and S upper quasi-triangular, with 2×2 blocks on the diagonal corresponding to complex eigenvalues.

The `decschurgen` function computes the generalized Schur decomposition of two real matrices A, B:

$$A = V_l S V_r', \quad B = V_l T V_r',$$

where V is orthogonal, and S upper quasi-triangular, with 2×2 blocks on the diagonal corresponding to complex eigenvalues. T is an upper-triangular matrix. The generalized eigenvalues are $\alpha[i]/\beta[i]$, where α may be complex and β is real.

The Schur decomposition can be ordered if the `dselmin` and `dselmax` arguments are specified. Any (generalized) eigenvalues that are $\geq$ `dselmin` and $\leq$ `dselmax` in absolute value, are selected for reordering, and moved top left. Note the reordering may affect complex eigenvalue when the matrices are ill-conditioned.

Sources: these routines are based on LAPACK 3.0 (see LAPACK, 1999).

Error and warning messages

decschur(): maximum no. of iterations reached

decschurgen(): maximum no. of iterations reached

Example

```
#include <oxstd.oxh>

main()
{
    decl a, b, ev, t, s, v, i, alpha, beta, vl, vr;
    a = rann(4,4);  b = rann(4,4);
    print("a", a);

    i = decschur(a, &ev, &s);
    print("eigenvalues", ev);
    print("s", s);

    i = decschur(a, &ev, &s, &v);
    print("v*s*v'", v*s*v');

    i = decschur(a, &ev, &s, &v, 0, 1);
    print("cabs(eigenvalues) between 0 and 1 first, S=", s);

    i = decschurgen(a, b, &alpha, &beta, &s, &t, &vl, &vr);
    print("b", b);
    println("decschurgen i=", i);
    print("alpha", alpha);
    print("beta", beta);
    // print("s", s, "vl*s*vr'", vl*s*vr');
    print("t", t, "vl*t*vr'", vl*t*vr');

    decschurgen(a, unit(rows(a)), &alpha, &beta, &s, &t, &vl, &vr,0,1);
    println("selecting gen. eigenvalues between 0 and 1 first");
    print("generalized eigenvalues", alpha ./ beta);
}
```

produces

```
a
      0.22489       1.7400     -0.20426     -0.91760
     -0.67417     -0.34353      0.22335     -0.14139
     -0.18338      0.68035     0.090558     -0.83328
      0.81350       1.1174      0.31499     -0.50031
eigenvalues
     -0.25959     -0.25959   -0.0046060   -0.0046060
       1.3775      -1.3775      0.32694     -0.32694
s
     -0.25959      -2.1654      -1.2665     -0.37296
      0.87631     -0.25959     -0.51481      0.18777
      0.00000      0.00000   -0.0046060      0.16910
      0.00000      0.00000     -0.63214   -0.0046060
v*s*v'
      0.22489       1.7400     -0.20426     -0.91760
     -0.67417     -0.34353      0.22335     -0.14139
     -0.18338      0.68035     0.090558     -0.83328
      0.81350       1.1174      0.31499     -0.50031
cabs(eigenvalues) between 0 and 1 first, S=
   -0.0046060     -0.20780      0.49340      0.64443
      0.51441   -0.0046060      0.66321      0.24688
      0.00000      0.00000     -0.25959      0.78487
      0.00000      0.00000      -2.4177     -0.25959
b
      -1.6268      0.61943      -1.4574      -1.8035
       2.0016      0.57912     -0.70797      0.59336
     -0.58939       1.4674    -0.020230      0.73706
       1.4795     -0.26881       1.2282       1.5784
decschurgen i=0
alpha
       1.9293      0.70758     -0.68938     -0.22323
beta
     0.089639       3.2454       2.0066       1.7759
t
     0.089639      0.68167     -0.46602     -0.52514
      0.00000       3.2454       1.6897     -0.89339
      0.00000      0.00000       2.0066     -0.75847
      0.00000      0.00000      0.00000       1.7759
vl*t*vr'
      -1.6268      0.61943      -1.4574      -1.8035
       2.0016      0.57912     -0.70797      0.59336
     -0.58939       1.4674    -0.020230      0.73706
       1.4795     -0.26881       1.2282       1.5784
selecting gen. eigenvalues between 0 and 1 first
generalized eigenvalues
   -0.0046060   -0.0046060     -0.25959     -0.25959
      0.32694     -0.32694       1.3775      -1.3775
```

decsvd

```
decsvd(const ma);
decsvd(const ma, const amu, const amw);
decsvd(const ma, const amu, const amw, const amv);
```

`ma`	in:	$m \times n$ matrix A
`amu`	in:	address of variable
	out:	$m \times n$ matrix U, $U'U = I_n$
`amw`	in:	address of variable
	out:	$1 \times n$ matrix with diagonal of W
`amv`	in:	(optional argument) address of variable
	out:	if not 0 on input: $n \times n$ matrix V, $UWV' = A$, $V'V = I_n$

Return value

Returns the result of the singular value decomposition:

- one argument: returns a $1 \times \min(m, n)$ matrix with the singular values, or 0 if the decomposition failed.
- two or more arguments: an integer indicating the result from the decomposition:
  0 — no error;
  k — if the k-th singular value (with index $k - 1$) failed after 50 iterations.

Note that the singular values are in *decreasing order*, with the columns of U, V sorted accordingly.

Description

Decomposes a $m \times n$ matrix A, rank$(A) = r > 0$, into $A = UWV'$:
U is $m \times n$ and $U'U = I_n$, V is $n \times n$ and $V'V = I_n$
W is $n \times n$ and diagonal, with non-negative singular values on the diagonal.
The rank of A is the number of non-zero diagonal elements of W.

Error and warning messages

decsvd(): decomposition failed

See also

§13.8.5.1, §13.8.5

Example

```
#include <oxstd.oxh>
main()
{
    decl x=<1,2;3,4;5,6>, mu, mv, mw;
    print("singular values: ", decsvd(x));
    print("result = ", decsvd(x, &mu, &mw, &mv));
    print(" A =", mu * diag(mw) * mv');
    decsvd(x', &mu, &mw, &mv);
    print(" A =", mu * diag(mw) * mv');
}
```

produces

```
singular values:
       9.5255      0.51430
result = 0 A =
       1.0000       2.0000
       3.0000       4.0000
       5.0000       6.0000
 A =
       1.0000       3.0000       5.0000
       2.0000       4.0000       6.0000
```

deletec, deleter, deleteifc, deleteifr

```
deletec(const ma);
deletec(const ma, const mval);
deleter(const ma);
deleter(const ma, const mval);
deleteifc(const ma, const mifc);
deleteifr(const ma, const mifr);
```

`ma`	in:	$m \times n$ matrix to delete from
`mval`	in:	$p \times q$ matrix with values to use for deletion
`mifc`	in:	$p \times n$ boolean matrix specifying columns to delete
`mifr`	in:	$m \times q$ boolean matrix specifying rows to delete

Return value

All functions return an empty matrix (<>) if the result is empty.
The `deletec` function with one argument returns an $m \times s$ matrix, deleting columns from `ma` which have a missing value (NaN: not a number).
The `deleter` function with one argument returns an $s \times n$ matrix, deleting rows from `ma` which have a missing value (NaN: not a number).

The remaining forms have no special treatment of missing values.

The `deleter` function with two arguments returns an $s \times n$ matrix, deleting the rows from `ma` which have at least one element equal to an element in `mval`.
The `deletec` function with two arguments returns an $m \times s$ matrix, deleting the columns from `ma` which have at least one element equal to an element in `mval`.
The `deleteifc`/`deleteifr` functions can be used to delete rows or columns based on a logical expression: all rows (columns) which have a zero in the corresponding row (column) are kept, the remainder is dropped.
The `deleteifc` function returns an $m \times s$ matrix, deleting only those columns from `ma` which have at least one non-zero element in the corresponding column of `mifc`.
The `deleteifr` function returns an $s \times n$ matrix, deleting only those rows from `ma` which have at least one non-zero element in the corresponding row of `mifr`.

See also

`dropc/r`, `selectc/r`, `selectrc`, `selectifc/r`, `isdotnan`, `vecindex`

Example

```
#include <oxstd.h>
main()
{
    decl m = <.,1,2,3;4:7;8,9,.,11>;
    print(m, "Rows with .NaN deleted", deleter(m));
    print("%r", {"deleter","deleteifr"},
        deleter(m, <1,.NaN>) | deleteifr(m, m .< 6 .|| m .>= 14));
}
```

produces:

```
          .NaN       1.0000       2.0000       3.0000
        4.0000       5.0000       6.0000       7.0000
        8.0000       9.0000         .NaN       11.000
Rows with .NaN deleted
        4.0000       5.0000       6.0000       7.0000
deleter              4.0000       5.0000       6.0000       7.0000
deleteifr            8.0000       9.0000         .NaN       11.000
```

denschi, densf, densn, denst

```
denschi(const ma, const df);
densf(const ma, const df1, const df2);
densn(const ma);
denst(const ma, const df);
```

`ma`	in:	arithmetic type
`df`	in:	arithmetic type, degrees of freedom
`df1`	in:	arithmetic type, degrees of freedom in the numerator
`df2`	in:	arithmetic type, degrees of freedom in the denominator

Return value

Returns the requested density at `ma` (the returned densities are positive):

`denschi`	$\chi^2(df)$ density
`densf`	$\mathsf{F}(df1, df2)$ density
`densn`	standard normal density
`denst`	student-t(df) density

The return type is derived as follows:

returns	`ma`	degrees of freedom arguments
$m \times n$ matrix	$m \times n$ matrix	scalar
$m \times n$ matrix	scalar	$m \times n$ matrix
$m \times n$ matrix	$m \times n$ matrix	$m \times n$ matrix
double	scalar	scalar

See also

`prob...`, `quan...`, `tail...`, Probability package (§11.3)

determinant

`determinant(const ma);`

`ma`	in:	$m \times m$ matrix

Return value

Returns the determinant of `ma`. Return type is double.

Description

Computes the determinant of a matrix. The determinant is obtained from the LU decomposition of the matrix (see `declu`). Use `invert` if both the inverse and determinant are required. Note that for ill-conditioned or large matrices, the determinant could be a very large or very small number.

Error and warning messages

determinant(): overflow (determinant set to `DBL_MAX_E_EXP`)
determinant(): underflow (determinant set to 0)
determinant(): matrix is singular (determinant set to 0)
determinant(): unreliable (warns that the result may be unreliable)

See also

`declu`, `invert`, `logdet`

Example

```
#include <oxstd.oxh>
main()
{
    print( determinant(<2,1;1,4>) );
}
```

produces: 7

dfft

```
dfft(const ma);
dfft(const ma, const inverse);
```

`ma`	in:	$2 \times n$ matrix (first row is real part, second row imaginary part), or $1 \times n$ matrix (real part only, imaginary part is zero)
`inverse`	in	(optional argument), int: 1: do inverse discrete FT 2: do inverse discrete real FT

Return value

If only one argument is used, the return value is a $2 \times n$ matrix which holds the Fourier transform.

If `inverse` equals 1, the return value is a $2 \times n$ matrix which holds the inverse Fourier transform.

If `inverse` equals 2, the return value is a $1 \times n$ matrix which holds the inverse real Fourier transform.

Description

Performs an (inverse) discrete Fourier transform. Computing the discrete Fourier transform is of order n^2, whereas the FFT is of order $n \log_2(n)$, so much faster for large sample sizes.

If the input has no complex part, in the absence of the `inverse` argument, a real FT is performed.

See also

`fft1d`

Example

```
#include <oxstd.oxh>
main()
{
    print( "dfft", dfft(<1,0,1>), "fft1d", fft1d(<1,0,1>),
        "inverse dfft(dfft))", dfft(dfft(<1,0,1>), 2) );
}
```

produces

```
dfft
       2.0000      0.50000      0.50000
      0.00000      0.86603     -0.86603
fft1d
       2.0000      0.50000      0.50000
      0.00000      0.86603     -0.86603
inverse dfft(dfft))
       1.0000 -1.4599e-016       1.0000
```

diag

`diag(const ma);`

`ma`	in:	double, or $m \times 1$ or $1 \times m$ matrix

Return value

Returns a $m \times m$ matrix with `ma` on the diagonal.

See also

`diagonal`, `diagonalize`, `toeplitz`

Example

```
#include <oxstd.oxh>
main()
{
    print( diag(<1,1>), diag(<1;1>) );
}
```

produces

```
       1.0000      0.00000
     0.00000       1.0000

       1.0000      0.00000
     0.00000       1.0000
```

diagcat

`diagcat(const ma, const mb);`

`ma`	in:	$m \times n$ matrix
`mb`	in:	$p \times q$ matrix

Return value

Returns a $(m+p) \times (n+q)$ matrix with `mb` concatenated to `mb` along the diagonal; the off-diagonal blocks are set to zero.

Example

```
#include <oxstd.oxh>
main()
{
    print( diagcat(<2,2>, unit(2)) );
}
```

produces

```
         2.0000       2.0000      0.00000      0.00000
       0.00000      0.00000       1.0000      0.00000
       0.00000      0.00000      0.00000       1.0000
```

diagonal

```
diagonal(const ma);
diagonal(const ma, const upr);
diagonal(const ma, const upr, const lwr);
```

`ma`	in:	arithmetic type
`upr`	in:	(optional argument), int: upper bandwidth (≥ 0, default 0)
`lwr`	in:	(optional argument), int: lower bandwidth (≤ 0, default 0)

Return value

The version with one argument returns a matrix with the diagonal from the specified matrix in the first row. *Note that the diagonal is returned as a row vector, not a column.* If `ma` is $m \times n$, the returned matrix is $1 \times \min(m, n)$ (exception: 0×0 when $m = 0$); if `ma` is scalar, the returned matrix is 1×1.

The version with more than one argument extracts the matrix in band format. If $A = (a_{ij})$ is $m \times n$ input matrix, then the output matrix `ma` $= A^b$ is formed as:

$$\begin{pmatrix} \vdots & & & \\ 0 & 0 & a_{2,3} & \cdots \\ 0 & a_{0,1} & a_{1,2} & \cdots \\ a_{0,0} & a_{1,1} & a_{2,2} & \cdots \\ a_{1,0} & a_{2,1} & a_{3,2} & \cdots \\ \vdots & & & 0 \end{pmatrix}$$

The diagonal is returned with `diagonal(., 0, 0)`.

See also

`decldlband` (for another example), `diag`, `diagonalize`, `setdiagonal`

Example

```
#include <oxstd.oxh>
main()
{
    decl x = <1:5;11:15;21:25>;
    print( "%6.0f", diagonal(x) );
    print( "%6.0f", diagonal(x, 1, -1) );
}
```

produces

```
     1    12    23

     0     2    13    24
     1    12    23     0
    11    22     0     0
```

diagonalize

`diagonalize(const ma);`

`ma`	in:	arithmetic type

Return value

Returns a matrix with the diagonal of `ma` on its diagonal, and zeros in off-diagonal elements. If `ma` is $m \times n$, the returned matrix is $m \times n$; if `ma` is scalar, the returned matrix is 1×1.

See also

`diag`, `diagonal`, `setdiagonal`

Example

```
#include <oxstd.oxh>
main()
{
    print( diagonalize( constant(2, 3, 4) ) );
}
```

produces

```
     2.0000      0.00000      0.00000      0.00000
    0.00000       2.0000      0.00000      0.00000
    0.00000      0.00000       2.0000      0.00000
```

diff, diff0

```
diff(const ma);
diff(const ma, const ilag);
diff(const ma, const ilag, const dmisval);
diff0(const ma);
diff0(const ma, const ilag);
diff0(const ma, const ilag, const dmisval);
```

`ma`	in:	$T \times n$ matrix A
`ilag`	in:	int, lag length of difference (1 for first difference)
`dmisval`	in:	(optional argument) double, value to set missing observations to (default is 0 for `diff0`, `.NaN` for `diff`)

Return value

The `diff` function returns a $T \times n$ matrix with the `ilag`th difference of the specified matrix, whereby missing values are replaced by `.NaN`. E.g. the result matrix `r` using second differences (`ilag` = 2) is:

```
r[0][0] = .NaN                r[0][1] = .NaN                ...
r[1][0] = .NaN                r[1][1] = .NaN                ...
r[2][0] = ma[2][0]-ma[0][0]   r[2][1] = ma[2][1]-ma[0][1]   ...
r[3][0] = ma[3][0]-ma[1][0]   r[3][1] = ma[3][1]-ma[1][1]   ...
r[4][0] = ma[4][0]-ma[2][0]   r[4][1] = ma[4][1]-ma[2][1]   ...
...
```

The result has the same dimensions as `ma`.

The `diff0` function is the same, but using zero for the missing value (by default).

Description

Differences the specified matrix, missing values are replaced by zero (unless a missing value is specified as the third argument). Using the lag operator L, for a column $a = (a_0, \ldots, a_{T-1})'$ of A, this function computes $(1 - L^d)a$. For $d = 1$, this is: $(0, a_1 - a_0, \ldots, a_{T-1} - a_{T-2})'$. The value of d must be integer, but may be negative (a forward difference). Note that $(1 - L^0)a = 0$.

See also

`lag`, `lag0`

Example

```
#include <oxstd.oxh>
main()
{
    print( diff0(<1:5>',2) );
}
```

produces

```
    0.00000
    0.00000
     2.0000
     2.0000
     2.0000
```

discretize

```
discretize(const vx, const dmin, const dmax,
    const icount, const ioption);
```

`mx`	in:	$1 \times T$ data vector
`dmin`	in:	double, first point a
`dmax`	in:	double, last point b
`icount`	in:	int, number of points M
`ioption`	in:	int, 0: raw discretization; 1: weighted discretization

Return value

Returns a $1 \times M$ matrix with the discretized data.

Description

Define a horizontal axis $a, a+\delta, a+2\delta, \ldots, b$, where $\delta = (b-a)/(M-1)$. The return value is the observation count, where each data value is assigned to the nearest point on the horizontal axis (this is raw discretization). Points outside the interval $[a-\delta/2, b+\delta/2)$ are ignored. The sum of the return value corresponds to the number of data points actually used.

In weighted discretization, an observation which falls between two points is distributed to both points, with weight proportional to the distance.

See also

`countc`, `lib/DensEst.ox` (for an application)

Example

In this example, the three intervals are $[-3,-1)$, $[-1,1)$ and $[1,3)$. So the last observation of x will be ignored. The raw discretization simply counts the numbers in each interval, giving the first line of output. The weighted version looks at the distance to the points $-2, 0, 2$ (also printed as the last output line): -3 is to the left of the minimum, so fully assigned to the first interval. Apart from -1, all observations are exactly on a point, so fully assigned; -1 falls halfway between -2 and 0, so half is assigned to the first interval, and half to the second (if the value would have been -1.5, 0.75 would go to the first interval, 0.25 to the second.

```
#include <oxstd.oxh>
main()
{
    decl a = -2, b = 2, m = 3, t;
    decl x = <-3,-2,-1,0,2,3>;

    t = a + (b - a) * range(0, m - 1) / (m - 1);
    print( discretize(x, a, b, m, 0)
        | discretize(x, a, b, m, 1) | t);
}
```

produces

```
       2.0000       2.0000       1.0000
       2.5000       1.5000       1.0000
      -2.0000      0.00000       2.0000
```

double

```
double(const ma);
```

`ma`	in:	arithmetic type

Return value

Casts the argument to a double:

input	returns
integer	converted to a double
double	unchanged
matrix	element 0,0
string	see §13.8.2.3 (also see the example under `fread`)
other types	error

See also

`int`, `matrix`, `string`, §13.8.2.3

dropc, dropr

```
dropc(const ma, const midxc);
dropr(const ma, const midxr);
dropr(const aa, const midxr);
```

`ma`	in:	$m \times n$ matrix to delete from
`aa`	in:	m array to delete from
`midxc`	in:	scalar or $p \times q$ matrix with the indices of columns to delete
`midxr`	in:	scalar or $p \times q$ matrix specifying the index of rows to delete

Return value

The `dropc` function returns a copy of `ma` with the specified columns deleted.
The `dropr` function returns a copy of the input matrix with the specified rows deleted; `dropr` also works for arrays.

All functions return an empty matrix (`<>`) if all rows or columns are deleted (or empty array for arrays).

See also

`deleteifc`, `deleteifr`, `insertc`, `insertr`, `vecindex`

Example

```
#include <oxstd.oxh>
main()
{
    decl x = <1,2,3;4,5,6>;
    print( dropc(x, 1), dropr(x, 1) );
    print( insertc(x, 0, 1) );

    decl a = {{"A","B"},{1,2},{<1>,<2>}};
    println("dropr(a, <1,2>)", dropr(a, <1,2>));
    println("a[0]=", a[0], "dropr(a[0], <1>)", dropr(a[0], <1>));
    println("insertr(a[0], 0, 2)", insertr(a[0], 0, 2));
}
```

produces

```
       1.0000       3.0000
       4.0000       6.0000

       1.0000       2.0000       3.0000

      0.00000       1.0000       2.0000       3.0000
      0.00000       4.0000       5.0000       6.0000
dropr(a, <1,2>)
[0][0] = A
[0][1] = B
a[0]=
[0] = A
[1] = B
dropr(a[0], <1>)
[0] = A
insertr(a[0], 0, 2)
[0] = .Null
[1] = .Null
[2] = A
[3] = B
```

eigen, eigensym

```
eigen(const ma, const amval);
eigen(const ma, const amval, const amvec);
eigensym(const ms, const amsval);
eigensym(const ms, const amsval, const amsvec);
```

`ma`	in:	$m \times m$ matrix A
`amval`	in:	address of variable
	out:	complex eigenvalues: $2 \times m$ matrix with eigenvalues of A first row is real part, second row imaginary part only real eigenvalues: $1 \times m$ *The eigenvalues are not sorted.*
`amvec`	in:	address of variable
	out:	complex eigenvectors: $2m \times m$ matrix with eigenvectors of A in columns top $m \times m$ block is real part, bottom $m \times m$ block is imaginary part only real eigenvalues: $m \times m$ matrix with eigenvectors in columns (the vectors are scaled by the largest absolute element in the vector)
`ms`	in:	symmetric $m \times m$ matrix A^s
`amsval`	in:	address of variable
	out:	$1 \times m$ matrix with eigenvalues of A^s, sorted in decreasing order
`amsvec`	in:	address of variable
	out:	$m \times m$ matrix with eigenvectors of A^s in columns

Return value

Returns the result of the eigenvalue decomposition:

0 no error;

1 maximum no of iterations (50) reached.

Description

Computes the eigenvalues of a real matrix and a symmetric real matrix. The `eigensym` function delivers the eigenvalues sorted, with the *largest first*. If eigenvectors are requested, these are in corresponding order.

The `eigen` function uses the balanced form of the matrix. (`eigensym`: if the matrix has elements of widely varying order of magnitude, the smaller elements should be in the bottom right hand corner.)

Sources: these routines are based on algorithms by J.H. Wilkinson and colleagues in Numerische Mathematik (Martin, Reinsch, and Wilkinson, 1968, Martin and Wilkinson, 1968b, Martin and Wilkinson, 1968a, Parlett and Reinsch, 1969, Peters and Wilkinson, 1970, Dubrulle, 1970). From Ox version 3.2 onwards, the non-symmetric eigenvalue code is based on LAPACK 3.0 (see LAPACK, 1999).

Error and warning messages

eigen(): maximum no. of iterations reached

eigensym(): maximum no. of iterations reached

Example

```
#include <oxstd.oxh>
main()
{
    decl meval, mevec;

    print("result=", eigensym(<2,1;1,3>, &meval, &mevec));
    print(" eigenvalues:", meval, "eigenvectors:", mevec);

    print("result=", eigen(<2,1;-3,1>, &meval));
    print(" eigenvalues:", "%r",
        {"real", "imaginary"}, meval);
}
```

produces

```
result=0 eigenvalues:
     3.6180       1.3820
eigenvectors:
   -0.52573      0.85065
   -0.85065     -0.52573
result=0 eigenvalues:
real                1.5000       1.5000
imaginary           1.6583      -1.6583
```

eigensymgen

`eigensymgen(const ma, const mb, const amval,const amvec);`

`ma`	in:	symmetric $m \times m$ matrix A
`mb`	in:	symmetric positive definite $m \times m$ matrix B
`amval`	in:	address of variable
	out:	$1 \times m$ matrix with sorted (generalized) eigenvalues of A
`amvec`	in:	address of variable
	out:	$n \times m$ matrix (generalized) eigenvectors of A in columns

Return value

Solves the general eigenproblem $Ax = \lambda Bx.$ returning the result of the eigenvalue decomposition:

0	no error;
1	maximum no of iterations (50) reached.
–1	Choleski decomposition failed.

Description

Solves the general eigenproblem $Ax = \lambda Bx$, where A and B are symmetric, B is also positive definite. The problem is transformed in standard eigenproblem by decomposing $B = CC' = LDL'$ and solving $Py = \lambda y$, where $y = C'x$, $P = C^{-1}AC'^{-1}$

Error and warning messages

eigensymgen(): matrices not conformant
eigensymgen(): maximum no. of iterations reached
eigensymgen(): decomposition failed (Choleski decomposition)

See also

`decldl`, `eigensym`

Example

```
#include <oxstd.oxh>
main()
{
    decl meval, mevec;

    print("result = ",
        eigensymgen(<2,1;1,3>,<1,0;0,1>, &meval, &mevec));
    print(" generalized eigenvectors:", mevec);
}
```

produces

```
result = 0 generalized eigenvectors:
     -0.52573      0.85065
     -0.85065     -0.52573
```

eprint

`eprint(const a, ...);`

`a`	in:	any type
`...`	in:	any type

Return value

Returns the number of arguments supplied to the function.

Description

Prints to stderr. See `print` for a further description.

See also

`fprint`, `print`, `sprint`

Example

```
#include <oxstd.oxh>
main()
{
    eprint( "\nerror message\n" );
}
```

prints `error message` to the console (even when the output is redirected to a file).

erf

`erf(const ma);`

`ma`	in:	arithmetic type

Return value

Returns the error function of each element of `ma`, of double or matrix type.

Description

The error function is related to the normal CDF as follows:

$$\mathrm{erf}(x) = 2\Phi(x\sqrt{2}) - 1.$$

See also

`cerf`, `probn`

exclusion

```
exclusion(const ma, const mb);
exclusion(const ma, const mb, const amidx);
```

`ma`	in:	matrix
`mb`	in:	matrix
`amidx`	in:	address of matrix
	out:	$2 \times c$ matrix, first row is index of exclusion in `ma`, second row is index in `mb`.

Return value

Returns the sorted unique elements of `ma` which are not in `mb` as a row vector. Returns an empty matrix if the result is empty. Missing values are skipped.

See also

`intersection`, `union`, `unique`

Example

```
#include <oxstd.oxh>
main()
{
    decl x = <-1,1,.,-2,-2,.,4>, y = <3,3,.,-2,1>;
    format("%5.1g");
    print("exclusion", exclusion(x, y) );
    print("intersection", intersection(x, y) );
    print("union", union(x, y) );
    print("union using unique", unique(x ~ y) );
}
```

produces

```
exclusion
   -1    4
intersection
   -2    1
union
   -2   -1    1    3    4
union using unique
   -2   -1    1    3    4
```

exit

```
exit(const iexit);
```

`iexit`	in:	integer, exit code

No return value.

Description

Exits the Ox run-time environment. The specified exit code is ignored.

exp

`exp(const ma);`

`ma`	in:	arithmetic type

Return value

Returns the exponent of each element of `ma`, of double or matrix type.

See also

`log`

Example

```
#include <oxstd.oxh>
main()
{
    print( exp(<0,1>) );
}
```

produces

```
      1.0000       2.7183
```

expint

`expint(const ma);`

`ma`	in:	arithmetic type

Return value

Returns the exponential integral $\mathrm{Ei}(x)$ of each element of `ma`, of double or matrix type.

Description

Note that $E_1(x) = -\mathrm{Ei}(-x)$. The function is based on the Fortran code in Netlib by W.J. Cody.

fabs

`fabs(const ma);`

`ma`	in:	int, double, matrix

Return value

Returns the absolute value of each element of `ma`, of the same type as `ma`.

Example

```
#include <oxstd.oxh>
main()
{
    print( fabs(<-1.1,1.1>) );
}
```

produces

```
      1.1000       1.1000
```

factorial

`factorial(const ma);`

`ma`	in:	arithmetic type

Return value

Returns the factorial function at the rounded value of each element of `ma`, of double or matrix type.

For negative integers, the function returns .NaN.

Description

Computes the factorial:

$$n! = n \times (n-1) \times (n-2) \ldots 2 \times 1.$$

The gamma function is related to the factorial for integer arguments: if n is integer then $\Gamma(n+1) = n!$.

Often a ratio of factorials functions is needed. Note that the factorial can overflow rapidly. However, often there is an offsetting factor in the denominator/numerator, and it is advised to use the `loggamma` or `binomial` function instead in that case.

Computation is based on the `gammafunc` function.

See also

`binomial`, `gammafunc`

fclose

`fclose(const file);`

`file`	in:	an open file which is to be closed

Return value

Returns 0.

Description

Closes the specified file, which was previously opened by a call to `fopen`. All open files are automatically closed when the program exits. On some operating systems, there is a limit on the number of open files.

Use `fclose("l")` to close the log file.

See also

`fopen`, `fprint` (for an example)

feof, fflush

`feof(const file);`
`fflush(const file);`

`file`	in:	an open file

Return value

The `feof` function checks for end of file; returns 0 if not at end of file, a non-zero value otherwise. `fflush` flushes the file buffer.

fft, fft1d

```
fft(const ma);
fft(const ma, const inverse);
fft1d(const ma);
fft1d(const ma, const inverse);
```

`ma`	in:	$2 \times n$ matrix (first row is real part, second row imaginary part), or $1 \times n$ matrix (real part only, imaginary part is zero)
`inverse`	in	(optional argument), int: 1: do inverse FFT 2: do inverse real FFT

Return value

If only one argument is used, the return value is a $2 \times s$ matrix which holds the Fourier transform.

If `inverse` equals 1, the return value is a $2 \times s$ matrix which holds the inverse Fourier transform.

If `inverse` equals 2, the return value is a $1 \times s$ matrix which holds the inverse real Fourier transform.

For `fft1d`, $s = n$, so it returns the same number of columns as the input.

But `fft` pads with zeros until a power of two is reached: s is the smallest power of 2 which is $\geq n$.

Description

Performs an (inverse) fast Fourier transform. The code is based on FFTE 2.0 by Daisuke Takahashi, see `www.ffte.jp`. FFTE provides Discrete Fourier Transforms of sequences of length $2^p 3^q 5^r$, which has been extended to work for all sample size. If the input has no complex part, in the absence of the `inverse` argument, a real FFT is performed.

See also

for some applications: `lib/AcfFft.ox`, `lib/DensEst.ox`

Example

```
#include <oxstd.oxh>
main()
{
    print( fft(<1,0,1>), fft(fft(<1,0,1>), 2) );
}
```

produces

```
       2.0000      0.00000       2.0000      0.00000
      0.00000     -0.00000      0.00000      0.00000

       1.0000      0.00000       1.0000     -0.00000
```

find

```
find(const where, const what);
find(const where, const what, const mode);
```

`where`	in:	object to search in
`what`	in:	what to search for
`mode`	in	(optional argument), string controlling search method: ”i”, ”ir”, ”r” or nothing

Return value

`where`	`what`	return type
m-vector	c-vector	$c \times 1$ matrix with indices of occurrence (or -1 if not found)
array of strings	array of c string	$c \times 1$ matrix with indices of occurrence (or -1 if not found)
array of strings	string	int: index of occurrence of string `what`, or -1 if not found
string	string	int: index of occurrence of substring `what`, or -1 if not found
string	$r \times c$ matrix with character values	$rc \times 1$ matrix with indices of occurrence (-1 if not found)
string	character	int: index of occurrence of character `what`, or -1 if not found

Description

When strings are involved, this function is identical to the `strfind` family, except that the return value is a column vector. The default is case-sensitive forward search; use mode ”i” to ignore case, ”r” to search in reverse order, ”ir” to do both.

When the two arguments are a vector (either column or row), `find` returns the location of the `what` elements in `where` (the numbers have to match exactly, or both be a missing value). The return value is a row vector with the same number of elements as `what`. Only the first instance will be located if there are multiple occurences in `where`; use `vecindex` to find all occurences of a certain value.

See also

`replace`, `strfind`, `vecindex`

Example

```
#include <oxstd.oxh>
main()
{
    decl x1 = <4;0;3> ~ <0;4;1>, xm = <4;3;2;1;0> ~ <.;.;2;3;.>, x2;

    x2 = xm;
    println("before, x1=", x1, "x2=", x2);
    println("vecindex - sorted indices (column):",
        vecindex(x2[][0], x1[][0]));

    x2 = xm;
    println("find returns the locations (column):",
        find(x2[][0], x1[][0]));

    // no find failures:
```

```
    decl sel = find(x2[][0], x1[][0]);
    x2[sel][1] = x1[][1];
    println("can be used to insert the missing values in x2:", x2);

    // not all found:
    x2 = xm;
    sel = find(x2[][0], x1[][0] | 5);
    decl selfound = vecindex(sel .>= 0);
    x2[sel[selfound]][1] = x1[selfound][1];
    println("redo, but now with a value that is not found:", x2);
}
```

produces:

```
before, x1=
       4.0000      0.00000
      0.00000       4.0000
       3.0000       1.0000
x2=
       4.0000         .NaN
       3.0000         .NaN
       2.0000       2.0000
       1.0000       3.0000
      0.00000         .NaN
vecindex - sorted indices (column):
      0.00000
       1.0000
       4.0000
find returns the locations (column):
       0.0000
       4.0000
       1.0000
can be used to insert the missing values in x2:
       4.0000      0.00000
       3.0000       1.0000
       2.0000       2.0000
       1.0000       3.0000
      0.00000       4.0000
redo, but now with a value that is not found:
       4.0000      0.00000
       3.0000       1.0000
       2.0000       2.0000
       1.0000       3.0000
      0.00000       4.0000
```

findsample

```
findsample(const mdata, const vvarsel, const vlagsel,
    const it1, const it2, const imode, const ait1, const ait2);
```

`mdata`	in:	$T \times n$ data matrix
`vvarsel`	in:	p-dimensional selection vector with indices in `mdata` or empty matrix to use whole `mdata` as selection
`vlagsel`	in:	p-dimensional vector with lag lengths for selection or empty matrix to use no lags
`it1`	in:	int, first observation index to consider (≥ 0)
`it2`	in:	int, last observation index to consider (can use -1 for $T-1$)
`mode`	in:	int, sample selection mode `SAM_ALLVALID`: all observations must be valid `SAM_ENDSVALID`: only the first and last observation must be wholly valid (there may be missing observations in between) `SAM_ANYVALID`: first and last obs. must have some valid data
`ait1`	in:	address of variable
	out:	the first observation index
`ait2`	in:	address of variable
	out:	the last observation index

Return value

The number of observation in the selected sample.

Example

```
#include <oxstd.oxh>
main()
{
    decl x = range(0,5)' ~ range(10,15)', t1, t2;
    x[2][1] = x[5][1] = .NaN;  x[4][] = .NaN;

    println(x);
    findsample(x, <>, <>, 0, -1, SAM_ALLVALID, &t1, &t2);
    println("SAM_ALLVALID: t1=", t1, " t2=", t2);
    findsample(x, <>, <>, 0, -1, SAM_ENDSVALID, &t1, &t2);
    println("SAM_ENDSVALID:t1=", t1, " t2=", t2);
    findsample(x, <>, <>, 0, -1, SAM_ANYVALID, &t1, &t2);
    println("SAM_ANYVALID: t1=", t1, " t2=", t2);

    findsample(x, <0,0>, <0,1>, 0, -1, SAM_ALLVALID, &t1, &t2);
    println("SAM_ALLVALID: t1=", t1, " t2=", t2, " column 0,lags 0-1");
}
```

```
      0.00000       10.000
       1.0000       11.000
       2.0000         .NaN
       3.0000       13.000
         .NaN         .NaN
       5.0000         .NaN
SAM_ALLVALID:  t1=0 t2=1
SAM_ENDSVALID: t1=0 t2=3
SAM_ANYVALID:  t1=0 t2=5
SAM_ALLVALID:  t1=1 t2=3 column 0, lags 0-1
```

floor

`floor(const ma);`

`ma`	in:	arithmetic type

Return value

Returns the floor of each element of `ma`, of double or matrix type. The floor is the largest integer less than or equal to the argument.

See also

`ceil` (for an example), `round`, `trunc`

fmod

`fmod(const ma, const mb);`

`ma`	in:	arithmetic type
`mb`	in:	arithmetic type

Return value

Returns the floating point remainder of `ma / mb`. The sign of the result is that of `ma`. The return type is double if both `ma` and `mb` are int or double. If `ma` is a matrix, the return type is a matrix of the same size, holding the floating point remainders `ma`$[i][j]/$`mb`$[i][j]$, etc. The return type is derived as follows:

returns	`ma`	`mb`
$m \times n$ matrix	$m \times n$ matrix	scalar
$m \times n$ matrix	scalar	$m \times n$ matrix
$m \times n$ matrix	$m \times n$ matrix	$m \times n$ matrix
double:	scalar	scalar

See also

`imod`

Example

```
#include <oxstd.oxh>
main()
{
    print( fmod(3,2), " ", fmod(-3,2), " ",
           fmod(3,-2), " ", fmod(-3,-2) );
}
```

produces: `1 -1 1 -1`

fopen

`fopen(const filename);`
`fopen(const filename, const smode);`

`filename`	in:	name of file to open
`smode`	in:	text with open mode

Return value

Returns the opened file if successful, otherwise the value 0. Use `isfile` to test if `fopen` was successful.

Description

The `smode` argument can be:

`"w"`	open for writing (the default when omitting this argument);
`"r"`	open for reading;
`"a"`	open for appending;
`"r+"`	open for reading and writing (update);
`"l"`	open a log file for writing (use `"la"` to append).

In addition, the following letters can be used in the `smode` argument:

b	Opens the file in binary mode (Windows only). Binary leaves a `\r\n` as `\r\n`, whereas non-binary translates `\r\n` to `\n` on output (and vice versa on output). On Windows, it is customary to open text files without the b, and binary files (when using fread and fwrite) with the b.
e	Forces the file reading and writing (using fread and fwrite) to be in little-endian mode. This allows Ox on Unix (not Linux on Intel) to handle files which use the MS-DOS byte ordering (which is little-endian).
E	Forces the file reading and writing (using fread and fwrite) to be in big-endian mode. This allows Ox on Windows/MS-DOS to handle files which use the Unix (not Linux on Intel) byte ordering (which is big-endian).
F	Use for a v96 `.fmt` file, which stores a matrix. Reading and writing can then be done by blocks of rows. When writing, the file must be explicitly closed with `fclose`. Also used for a v96 `.dat` file, which stores variable names and binary data. When writing, the first `fwrite` should be an array of strings, which also determines the number of variables. When reading, use `fread(file, &as, 's')` to read the variable names; this sets the file pointer to the first row.
f	Same as `'F'`, but only for extended v89 `.fmt` files.
v	Print a message if opening the file failed.
V	Print a message if opening the file failed, then generate a run-time error.

When using `"r+"`, it is necessary to use `fseek` or `fflush` when switching from reading to writing. To send the output from all `print` and `println` statements to a file (in addition to the screen), use `fopen(`*filename*`,"l")`.

Finally, it is possible to *read from a zip file*, by specifying the archive name, followed by two slashes, followed by the name in the archive. For example

```
    file = fopen("mydata/data.zip//data/data.in7");
```

opens the file `data/data.in7` in the zip archive `mydata/data.zip`. Ox will search for `data.zip` in the standard way. It is not possible to write to a zip file.

See also

`fclose`, `fflush`, `fprint` (example), `fread`, `fscan`, `fseek`, `fwrite`, `isfile`

format

`format(const sfmt);`

`sfmt`	in:	string: new default format for double or int
		int: new line length for matrix printing

No return value.

Description

Use this function to specify the default format for double and int types. The function automatically recognizes whether the format string is for int or double (otherwise it is ignored). The specified double format will also be used for printing matrices. See under the `print` function for a complete description of the formatting strings.

Use an integer argument to set the line length for matrix printing (default is 80, the maximum is 1024).

The default format strings are:

int	`"%d"`
double	`"%5g"`
matrix	each element `"%#13.5g"`, 6 elements on a line (depending on the line length).

Notes:

- The `print` function allows setting of format for the next argument only.
- Be careful with the `%f` format. For example, when printing 1e-300, the output field will need 302 characters.
- By default, integers and doubles are printed without a leading space. To use a space as separator: `format(" %d");`. Specifying a wider field can also insert extra spaces: `format("%6d");`. For a double, you could set the field to `"%#13.5g"`. Because at most 7 characters are needed on top of the 5 for significant digits, this format will always have at least one space.
- When a matrix is printed, no extra space is inserted between elements. So, make sure that the field width is at least one character larger than the maximum number of printed characters (as is the case for `"%#13.5g"` and `"%13.5g"`).

See also

`fprint`, `print`, `sprint`

fprint, fprintln

```
fprint(const file, const a, ...);
fprintln(const file, const a, ...);
```

`file`	in:	file which is open for writing
`a`	in:	any type
`...`	in:	any type

Return value

Returns the number of arguments supplied to the function.

Description

Prints to the specified file. See `print` for a further description. `fprintln` is as `fprint` but ensures the next output will be on a new line.

See also

`fclose`, `fopen`, `print`

Example

```
#include <oxstd.oxh>
main()
{
    decl file = fopen("test.tmp", "w");

    if (isfile(file))
    {
        fprintln(file, "some text" );

        fclose(file);
    }
}
```

produces a file test.tmp with the specified text.

fread

```
fread(const file, const am, ...);
fread(const file, const am, const type, const r, const c);
```

`file`	in:	file which is open for writing
`am`	in:	address, address for storing read item
`type`	in:	(optional argument), type of object to read, see below
`r`	in:	(optional argument), number of rows to read; default is 1 if argument is omitted
`c`	in:	(optional argument), number of columns to read; default is 1 if argument is omitted, unless file is opened with f, in which case the number of columns is read from the file

Return value

Returns an integer:

-1	nothing read, because end-of-file was reached;
0	nothing read, unknown error;
> 0	object read, return value is size which was actually read:

`type`	data type read	return value
'i', 'd'	integer	1
'e', 'f'	double	1 (r and c omitted, or both equal to 1)
'e', 'f'	matrix	$r \times c$
'e', 'f'	matrix	r (number of complete rows read; file opened with f in format)
'c'	integer	1 (if $r = 1$: just one byte read)
'c'	string	r (if $r > 1$: r bytes read)
's'	string	string length
'4'	float	1 (r and c omitted, or both equal to 1)
'4'	float matrix	$r \times c$

When reading a matrix, for example as `fread(file,&x,'f',r,c)`, the size of `x` will always be `r` by `c`. If less than `rc` elements could be read, the matrix is padded with zeros. If no elements could be read at all, because the end of the file was reached, the return value is –1.

The `'4'` format reads 4-byte real values ('float'), these are not written by Ox, but may be needed to read externally created files.

The `'s'` type reads a string up to (and including) the first null character or the end of file.

Description

Reads binary data from the specified file. The byte ordering is the platform specific ordering, unless the f format was used (also see `fopen` and `fwrite`).

See also

`fclose`, `fopen`, `fscan`, `fseek`, `fwrite` (for example using f format)

Example

A number of input/output examples is in the `samples/inout` directory. Below is `samples/inout/inout7.ox`. The programs `inout10` and `inout11` show how data can be read and written in blocks when the file is not a `.fmt` file.

This example writes a matrix as a .fmt file using savemat. Then the matrix is written using fread, in such a way that the same format is used.
Note that under Windows and MS-DOS these files are identical, but that on some platforms (such as the Sun) the files differ: iotest7.ox is little endian, but reading here assumes the platform ordering (which is big endian on a Sun).

```
#include <oxstd.oxh>
main()
{
    decl file, x;
    decl s, r, c, rc8;

    x = rann(2,3);
    x[0][] = double("tinker");
    savemat("iotest7.fmt", x);
                // open mode: read, binary, little-endian
    file = fopen("iotest7.fmt", "rbe");
    fread(file, &s, 'c', 4);

    if (s == "\xDD\xEE\x86")
        println("signature OK");
    else
    {   println("signature NOT OK!");
        exit(1);
    }

    fread(file, &r, 'i');
    fread(file, &c, 'i');
    println("rows=", r, " columns=", c);

    fread(file, &rc8, 'i');
    fread(file, &x, 'f', r, c);
    println("-1 indicates eof: ", fread(file, &s, 'c', 1));
    if (feof(file))
        println("Was indeed end of file.");

    fclose(file);

    println(string(x[0][0]), x[1:][]);
}
```

produces:

```
signature OK
rows=2 columns=3
-1 indicates eof: -1
Was indeed end of file.
tinker
     -0.91760     -0.67417     -0.34353
```

fremove

`fremove(const filename);`

`filename`	in:	name of file to remove

Return value

Returns 1 if the file was removed successfully, 0 otherwise.

fscan

`fscan(const file, const a, ...);`

`file`	in:	file which is open for writing
`a`	in:	any type
`...`	in:	any type

Return value

Returns the number of arguments successfully scanned and assigned, or -1 when the end of the file was encountered and nothing was read.

Description

Reads text from a file. The arguments are a list of scanning strings and the addresses of variables.

A scanning string consists of text, optionally with a format specifier which starts with a % symbol. The string is truncated after the format. Any text which precedes the format, is skipped in the file. A space character will skip any white space in the file.

If the scanning string holds a format (and assignment is not suppressed in the format), the string must be followed by the address of a variable.

The format specification is similar to that for the scanf function of the C language:

%[`*` or `#`][*width*]*type*

The *width* argument specifies the width of the input field. A `*` suppresses assignment. A `#` can only be used with `m` and `M`.

Notes:

- The `"%m"` and `"%M"` formats can be used to read a matrix from a file. They first read the number of rows and columns, and then the matrix row by row; this corresponds to the format used by `loadmat`.
  No dimensions are read by `"%#m"` and `"%#M"`, in that case the scanning string has to be followed by two integers indicating the number of rows and columns to be read. For `fscan` the two integers can be -1. In that case all numbers are read and returned as a column vector.
- The `"%z"` format reads a whole line up to `\n`, the `\n` (and `\r`) are removed from the return value. The line can be up to 2048 characters long (or whatever buffer size is set with `sprintbuffer`). If the line in the file is too long, the remainder is skipped.
- When scanning a string, the maximum string length which can be read is 2048. The `sprintbuffer` function can be used to enlarge the buffer size.
- The `"%t"` and `"%T"` formats can be used to read a token, using a simplified syntax that is similar to Ox code. Five types of tokens are distinguished:

Table 8.1 Formatting types for scanning

double *type*:	
`e`,`f`,`g`	field is scanned as a double value
`le`,`lf`,`lg`	field is scanned as a double value
`C`	field is scanned as a calendar double value
integer *type*:	
`d`	signed decimal notation,
`i`	signed decimal notation,
`o`	unsigned octal notation,
`x`	unsigned hexadecimal notation,
`u`	unsigned decimal notation,
`c`	(no width) scan a single character (i.e. one byte),
string *type*:	
`s`	scan a string up to the next white space,
`z`	scan a whole line,
`c`	(width $>$ 1) scan a number of characters,
matrix *type*:	
`m`,`M`	scan a matrix row by row,
token *type*:	
`t`	scan a token, returning the value,
`T`	scan a token, returning a triplet.
any type:	
`v`	scan an Ox constant.

`SCAN_EOF`	End of the file or text.
`SCAN_IDENTIFIER`	An identifier.
`SCAN_LITERAL`	A literal integer, double or string.
`SCAN_SYMBOL`	A symbol.
`SCAN_SPACE`	White space.

The `"%t"` version returns the value that was read, while `"%T"` returns an array with three elements: the value, the actual text that was read and the token type (`SCAN_`...).

Note that a negative number is read as two tokens: a minus symbol and the value. Space is returned as a token. To skip leading spaces use `" %t"` and `" %T"`. Note that `fscan` may reach the end (i.e. return -1) before it gets to process the token, so the return value of `fscan` should be taken into account.

The token format can be useful when a simple parser is required, or to read strings that are not delimited by white space. An example using `sscan` is given below.

- The `"%C"` format is used to scan a date/time field written in ISO format: *yyyy-mm-dd*, *hh:mm::ss.hh*, or *yyyy-mm-ddThh:mm::ss.hh*. Examples are `1999-03-31`, `13:10` (a 24-hour clock is used, seconds and hundreds are optional) and `1999-3-31T13:10`.

  Years with week number are also recognised, e.g. `1976-W3` returns the calendar index for the Monday of week 3 in 1976. (Week 1 is the first week that contains the first Thursday; or equivalently, the week that contains 4 January.)

- The `"%v"` format reads a variable that has been written in the format of an Ox constant. It is especially useful to read a variable that consist of a derived types, such as an array or a class object, but also for a matrix. When scanning a class object, the variable must already have the type of that class (using `new`), because the scan functions cannot create the object themselves. An example is given in `ox/samples/inout/percent_v.ox` and under the `print` function.

See also

`fprint`, `fread`, `print`, `scan`, `sscan` (for another example)

Example

The example (`samples/inout/iotest2.ox`) writes a file, and reads it twice. The first time, the string read is `tinker123`, but then reading gets stuck, because the word tailor can not be read is an integer, double or matrix. Failure to read the matrix dimension generates an error message.
The second time, the file is read properly.

```
#include <oxstd.oxh>
main()
{
    decl file;

    file = fopen("iotest2.txt", "w");
    fprint(file, "tinker123\ntailor456.78\n 2 2 1 0 0 1\n");
    fclose(file);

    decl c = -2, s, i = 0, d = 0, m = 0;

    file = fopen("iotest2.txt");
    println("Next statement will print message: "
          "\"load matrix: no matrix elements\"");

    c = fscan(file, "%s", &s,       // stops after &s
        "%d", &i, "%f", &d, "%m", &m);
    fclose(file);

    print("\nitems read=", c, " s=", s, " int=", i,
        " dbl=", d, " mat=", m);

    file = fopen("iotest2.txt");
    c = fscan(file, "tinker%d", &i, " tailor%f", &d, "%m", &m);
    fclose(file);

    print("\nitems read=", c, " int=", i, " dbl=", d,
        " mat=", m);

    // token example:
    decl str = "GMM(\"a\", 1.5, -3);";
    decl func, arg0, arg1, arg2, arg3;

    println("\ntoken string: ", str);
    sscan(str, "%t", &func, "( %t", &arg0, ", %f", &arg1,
        ", %d", &arg2);
    println("scanned using \"%t\": ", func, " ", arg0,
        " ", arg1, " ", arg2);

    sscan(str, "%T", &func, "(%T", &arg0, ", %T", &arg1,
        ", %T", &arg2, "%T", &arg3);
    println("scanned using \"%T\":", func, arg0, arg1, arg2, arg3);
}
```

produces

```
Next statement will print message: "load matrix: no matrix elements"
load matrix: no matrix elements

items read=1 s=tinker123 int=0 dbl=0 mat=0
items read=3 int=123 dbl=456.78 mat=
       1.0000      0.00000
      0.00000       1.0000

token string: GMM("a", 1.5, -3);
scanned using "%t": GMM a 1.5 -3
scanned using "%T":
[0] = GMM
[1] = GMM
[2] = 0

[0] = a
[1] = "a"
[2] = 1

[0] = 1.5
[1] = 1.5
[2] = 1

[0] = -
[1] = -
[2] = 2

[0] = 3
[1] = 3
[2] = 1
```

fseek

```
fseek(const file);
fseek(const file, const type);
fseek(const file, const type, const r);
```

`file`	in:	file which is open for writing
`type`	in:	(optional argument), type of object use in seeking, see below
`r`	in:	(optional argument), number of rows to move; default is 1 if argument is omitted

Return value

The first form, with only the `file` argument, tells the current position in the file as an offset from that start of the file (as the standard C function `ftell`). The return vale is an integer, except in 64-bit Ox when the file is larger than 2GB: in that case it is a double.

The second and third form return 0 if the seek was successful, else a non-zero number,

Description

Repositions the file pointer to a new position specified from the start of a file. The `type` argument is interpreted as follows:

type	seek data type	byte equivalent
'i', 'd'	integer	$4r$
'e', 'f'	double	$8r$
'e', 'f'	matrix rows	$16 + 8rc$ (file opened with f in format)
'c'	character	r

So when a file is opened as `"rbf"`, `fseek(file,'f',r)` moves the file pointer to row `r` in the `.fmt` file.

To position the file pointer at the end specify -1 for the third argument. This can be used to determine the length of a file, as the following example shows:

```
    fseek(file, 'c', -1);           // move to end
    length = fseek(file);           // get byte position at end
    ::fseek(file, 'c', 0);          // move to beginning
    fread(file, &s, 'c', length);   // read the whole file into s
```

See also

`fclose`, `fopen`

Example

This example (`samples/inout/iotest9.ox`) reads and writes to a matrix opened with the f format. In that case, the number of columns applies to the whole file, and seeking is by row. Once the file file holds data, each subsequent write must match the number of columns already in the file.

```
#include <oxstd.oxh>
main()
{
    decl file, x, i;

    file = fopen("iotest9.fmt", "wbf");  // write

    fwrite(file, ones(1, 4));
    fwrite(file, 1 + ones(1, 4));
    fwrite(file, zeros(27, 4));

    fclose(file);

    file = fopen("iotest9.fmt", "abf");  // append
    println("file is ", rows(file), " by ", columns(file));

    fwrite(file, 2 + ones(1, 4));
    fclose(file);

    file = fopen("iotest9.fmt", "rbf");    // read
    println("file is ", rows(file), " by ", columns(file));

    fseek(file, 'f', 1);             // second row
    fread(file, &x, 'f', 1);            // read it
    print("row of twos:", x);

    fseek(file, 'f', rows(file)-1);  // second row
    fread(file, &x, 'f', 1);   // read it
    print("row of threes:", x);
}
```

produces:

```
file is 29 by 4
file is 30 by 4
row of twos:
       2.0000       2.0000       2.0000       2.0000
row of threes:
       3.0000       3.0000       3.0000       3.0000
```

fsize, ftime

```
fsize(const file);
ftime(const file);
```

`file`	in:	an open file

Return value

`fsize` returns the size of the file in bytes (an integer, or a double if the file is larger than 2GB).
`ftime` returns the modification date and time of the file (a double).

fwrite

```
fwrite(const file, const a);
```

`file`	in:	file which is open for writing
`a`	in:	int, double, matrix or string

Return value

Returns 0 if failed to write, or the number of items written to the file:

input	return value (integer)
integer	1,
double	1,
$m \times n$ matrix	number of elements written (normally $m \times n$),
$m \times n$ matrix	opened with f format: no of rows written (normally m),
string	number of characters written.

Description

Writes binary data to the specified file. The byte ordering is the platform specific ordering, unless the f format was used (also see `fopen`), in which case writing is to a `.fmt` file in little-endian mode (also see `savemat`). When data is written to a `.fmt` file, the number of columns must match that already in the file (use `columns(file)` to ask for the number of columns in the file).

See also

`fclose`, `fopen`, `fread`, `fseek` (for example using f format)

Example

A number of input/output examples is in the `samples/inout` directory. Below is `samples/inout/inout6.ox`, which saves a matrix as a `.ftm` file using `savemat`. Then the matrix is written using `fwrite`, in such a way that the same format is used. See under `fread` for a read example.
Note that under Windows and MS-DOS these files are identical, but that on some platforms (such as the Sun) the files differ: `iotest6a.fmt` is little endian, but `iotest6b.fmt` big endian. So on a Sun, using `loadmat` on `iotest6b.fmt` fails to read the matrix correctly.
The example also shows how a short string can be stores in matrix, and retrieved from it.

```
#include <oxstd.oxh>
main()
{
    decl file, x = rann(2,3);
    x[0][] = double("tinker");
    savemat("iotest6a.fmt", x);
                             // force little-endian mode
    file = fopen("iotest6b.fmt", "wbe");
//  two ways if writing signature, first:
//  decl s = new string[4];      // need four bytes
//  s[0:2] = "\xDD\xEE\x86";     // signature is DDEE8600
//  fwrite(file, s);
//
//  and second way:
    fprint(file, "%c", 0xdd, "%c", 0xee, "%c", 0x86, "%c", 0x00);

    fwrite(file, rows(x));
    fwrite(file, columns(x));
    fwrite(file, rows(x) * columns(x) * 8);
    fwrite(file, x);
    fclose(file);

    decl y = loadmat("iotest6b.fmt");

    println(string(x[0][0]), string(y[0][0]), x[1][1]-y[1][1]);
}
```

produces: `tinkertinker0`

fuzziness

`fuzziness(const deps);`

`deps`	in:	double, 0 or new fuzziness value

Return value

Sets and returns the new fuzziness parameter if `deps` > 0. If `deps` ≤ 0, no new fuzziness value is set, but the current one is returned. The default fuzziness is 10^{-13}.

See also

`isfeq`

gammafact

`gammafact(const ma);`

`ma`	in:	arithmetic type

Return value

Returns the complete gamma function at the value of each element of `ma`, of double or matrix type.

For argument zero, or a negative integer, the function returns .NaN.

Description

Computes the gamma function at the argument:

$$\Gamma(a) = \int_0^\infty x^{a-1} e^{-x} \mathrm{d}x.$$

Note that:

$$a\Gamma(a) = \Gamma(a+1).$$

The gamma function is related to the factorial for integer arguments: if $a = i$ is integer then $\Gamma(i+1) = i!$.

Often a ratio of gamma functions is needed. Since the Gamma function can overflow quite rapidly, it is advised to use the `loggamma` function instead.

The function is accurate to about 14 to 15 significant digits (a table is used to look up integer values up to 13). The approximation uses a series expansion of the reciprocal for arguments $\leq$ 13 (see Abramowitz and Stegun, 1984, §6.1.34). Otherwise the exponential of the loggamma is used. For negative arguments the following relation is used:

$$\Gamma(a) = -\frac{\pi}{\sin(\pi a) a \Gamma(-a)}.$$

See also

`factorial`, `gammafunc`, `loggamma`, `polygamma`

gammafunc

`gammafunc(const dx, const dr);`

`mx`	in:	x, arithmetic type
`mr`	in:	r, arithmetic type

Return value

Returns the incomplete gamma function $G_x(r)$. Returns 0 if $r \leq 0$ or $x \leq 0$. The accuracy is to about 10 digits.

The return type is derived as follows:

returns	`mx`	`mr`
$m \times n$ matrix	$m \times n$ matrix	scalar
$m \times n$ matrix	scalar	$m \times n$ matrix
$m \times n$ matrix	$m \times n$ matrix	$m \times n$ matrix
double	scalar	scalar

Description

The incomplete gamma function is defined as:

$$G_x(r) = \int_0^t \frac{1}{\Gamma(r)} x^{r-1} e^{-t} \mathrm{d}t, \quad t > 0, r > 0.$$

Source: `gammafunc` uses Applied Statistics algorithm AS 239 (Shea, 1988).

See also

`betafunc`, `loggamma`, `probgamma`

Example

```
#include <oxstd.oxh>
#include <oxprob.oxh>       // required for probgamma
main()
{
    print(probgamma(5.99, 1, 0.5), " ",
          gammafunc(5.99 * 0.5, 1), "\n");
    print(probgamma(5.99, 0.5, 1), " ",
          gammafunc(5.99, 0.5) );
}
```

produces

```
0.949963 0.949963
0.999462 0.999462
```

getcwd

`getcwd();`

Return value

Returns the current directory. *Windows specific*: returns the current directory on the current drive. Use `chdir` to change the current drive.

See also

`chdir`, `getfiles` (for example), `systemcall`

getenv

`getenv(const senv);`

`senv`	in:	string

Return value

Returns a string with the value of the environment variable, or an empty string if the environment variable is undefined.

See also

`systemcall`

getfiles, getfolders

`getfiles(const sfilemask);`
`getfolders(const sfilemask);`

`sfilemask`	in:	string, mask for files, may have a path or wild cards

Return value

`getfiles` returns an array of strings with file names matching the specified mask. `getfolders` returns an array of strings with folder names matching the specified mask.

See also

`chdir`, `getcwd`

Example

```
#include <oxstd.oxh>
main()
{
    println("Current folder = ", getcwd());
    chdir("D:\\OxMetrics6\\ox\\include");
    //default:chdir("C:\\Program Files\\OxMetrics6\\ox\\include");
    println("Current folder = ", getcwd());
    println("Files in folder: ", getfiles("*.ox"));
}
```

produces

```
Current folder = D:\Waste
Current folder = D:\OxMetrics6\ox\include
Files in folder:
[0] = g2ox.ox
[1] = oxgauss.ox
[2] = ox_init.ox
```

hyper_2F1

```
hyper_2F1(const a, const b, const c, const z);
```

`a,b,c`	in:	arithmetic type, parameters of ${}_2F_1$
`z`	in:	arithmetic type, argument of ${}_2F_1$

Return value

Returns the value of the Gauss Hypergeometric function ${}_2F_1$ with real parameters a, b, c and real argument z.

Description

The Gauss hypergeometric function is defined as:

$$ {}_2F_1(a,b;c;z) = \sum_{i=0}^{\infty} \frac{(a)_i(b)_i}{(c)_i i!} z^i \equiv F(z) \tag{8.2}$$

for argument $|z| < 1$ and parameters $a, b \in \mathbb{C}$, $c \in \mathbb{C}\backslash\{\nvdash, -\nVdash, -\nvDash, ...\}$. The hypergeometric function is extended to $z \in \mathbb{C}$ by analytic continuation. The function does not exist when $c = 0$ or for c a negative integer (unless a or b is a smaller negative integer or zero). The principal branch corresponds to $|\mathrm{ph}(1-z)| \leq \pi$. The notation in (8.2) uses the Pochhammer symbol:

$$(a)_i = a(a+1)(a+2)...(a+i-1), \quad (a)_0 = 1.$$

The Pochammer symbol can be expressed as a ratio of gamma functions:

$$(a)_i = \frac{\Gamma(a+i)}{\Gamma(a)}.$$

provided a is not zero or a negative integer.
On the unit circle $|z| = 1$, the Gauss hypergeometric function converges absolutely when $\mathcal{R}(c-a-b) > 0$, converges conditionally when $-1 < \mathcal{R}(c-a-b) \leq 0$ (except for $z = 1$), and diverges otherwise. Abramowitz and Stegun (1970, Ch. 15) and Olde Daalhuis (2010) provide more information.

idiv, imod

```
idiv(const ia, const ib);
imod(const ia, const ib);
```

`ia`	in:	arithmetic type
`ib`	in:	arithmetic type

Return value

The `imod` function returns the integer remainder of `int(ia) / int(ib)`. The sign of the result is that of `ia`.

The `idiv` function returns the result of the integer division `int(ia) / int(ib)`.

The return type is a matrix of integer values if either arguments is a matrix, else it is a scalar int.

See also

`fmod`

Example

```
#include <oxstd.oxh>
main()
{
    print( idiv(3,2), " ", idiv(-4,2), " ",
           idiv(3,-2), " ", idiv(-4,-2), " ");
    print( imod(3,2), " ", imod(-3,2), " ",
           imod(3,-2), " ", imod(-3,-2) );
}
```

produces: `1 -2 -1 2 1 -1 1 -1`

insertc, insertr

```
insertc(const ma, const c, const cadd);
insertr(const ma, const r, const radd);
insertr(const aa, const r, const radd);
```

`ma`	in:	$m \times n$ matrix to insert into
`aa`	in:	m array to insert into
`c`	in:	scalar, column index of insertion
`cadd`	in:	scalar, number of columns of zeros to add
`r`	in:	scalar, row index of insertion
`radd`	in:	scalar, number of rows of zeros to add

Return value

The `insertc` function returns a copy of the input matrix with the specified columns of zeros inserted.

The `insertr` function returns a copy of the input matrix with the specified rows of zeros inserted; `insertr` also works for arrays.

See also

`dropc` (for an example), `dropr`

int

```
int(const ma);
```

`ma`	in:	arithmetic type

Return value

Casts the argument to an integer:

input	returns
integer	unchanged
double	rounded towards zero
matrix	element 0,0 rounded towards zero
string	element 0
other types	error

See also

`ceil` (for an example), `double` , `matrix` , `trunc`, §13.8.2.3

intersection

```
intersection(const ma, const mb);
intersection(const ma, const mb, const amidx);
```

`ma`	in:	matrix
`mb`	in:	matrix
`amidx`	in:	address of matrix
	out:	$2 \times c$ matrix, the first row is the index of the common elements in `vecr(ma)`, the second row is the index in `vecr(mb)`. The order of the indices correspond to the order of the return value.

Return value

Returns the sorted unique elements of `ma` which are also in `mb` as a $1 \times c$ vector, where c is the number of elements `ma` and `mb` have in common. Returns an empty matrix if the result is empty. Missing values are skipped.

See also

`exclusion` (for an example), `union`, `unique`

invert

`invert(const ma);`
`invert(const ma, const alogdet, const asign);`

`ma`	in:	$m \times m$ real matrix A
`alogdet`	in:	(optional argument) address of variable
	out:	double, *logarithm of the absolute value* of the determinant
`asign`	in:	(optional argument) address of variable
	out:	int, the sign of the determinant of A; 0: singular; $-1, -2$: negative determinant; $+1, +2$: positive determinant; $-2, +2$: result is unreliable

Return value

Returns the inverse of A, or the value 0 if the decomposition failed.

Description

Inverts the matrix A using the LU decomposition (see under `declu`). The exponent of the log-absolute-determinant can only be computed for values $\leq$ `DBL_MAX_E_EXP` and $\geq$ `DBL_MIN_E_EXP` (see Ch. 9). Note that `1 / ma` also returns the inverse (if `ma` is square, `invert` is tried, if that fails, or the matrix is not square, the generalized inverse is used), see §13.8.5 and `invertgen`.

Error and warning messages

invert(): decomposition failed (the matrix is numerically singular)

See also

`declu`, `invertgen`, `invertsym`, `logdet`

Example

```
#include <oxstd.oxh>
main()
{
    decl mp = <4,1;1,3>;
    print(invert(mp)*mp ~ invertsym(mp)*mp);
}
```

produces (note that the both matrices are the identity matrix: whether it has zeros, or nearly zeros, could dependent on which Ox version was used):

```
      1.0000      0.00000       1.0000      0.00000
     0.00000       1.0000      0.00000       1.0000
```

inverteps

`inverteps(const dEps);`

`dEps`	in:	sets the inversion epsilon ϵ_{inv} to `dEps` if `dEps` > 0, to the default if `dEps` < 0; leaves the value unchanged if zero

Return value

Returns the inversion epsilon (the new value if `dEps != 0`).

Description

The following functions return singular status if the pivoting element is less than or equal to ϵ_{inv}: `decldl`, `declu`, `decldlband`, `invert`, `invertsym`, `orthmgs`. Less than $10\epsilon_{inv}$ is used by `olsc` and `olsr`.

A singular value is considered zero when less than $||A||_\infty 10\epsilon_{inv}$ in `rank`, `nullspace`, and when using the generalized inverse.

The default value for ϵ_{inv} is $1000 \times$ `DBL_EPSILON`.

invertgen

```
invertgen(const ma);
invertgen(const ma, const mode);
```

`ma`	in:	$m \times n$ matrix A
`mode`	in:	int, mode of inversion (optional argument, default is 0)

Return value

Returns the (generalized) inverse of A, or the value 0 if the decomposition failed.

Description

mode	description	A
0	generalized inverse using SVD	
1	gen. symmetric p.s.d. inverse using SVD	$m = n$, symmetric p.s.d.
2,20	first try `invert` then mode 0	$m = n$
3,30	first try `invertsym` then mode 1	$m = n$, symmetric p.s.d.
4,40	use `olsc` (QR dec.) for inverse of $A'A$	$m \geq n$
$\geq$10	print warning if matrix is singular	

0. When `mode` equals 0, or the mode argument is omitted, `invertgen` defaults to the generalized inverse (see §13.8.5.1) when only one argument is used.
1. When `mode` equals 1, the matrix must be symmetric positive semi-definite. The generalized inverse can use the fact that $\mathbf{U} = \mathbf{V}$ in the singular value decomposition. *Do not use this mode for negative definite matrices.*
2. Mode 2 first tries the normal inversion routine (`invert`), and then, if the matrix is singular, uses the generalized inverse. This mode is the same as using `1 / x`.
3. Mode 3 first tries the normal inversion routine (`invertsym`), and then, if the matrix is singular, uses the generalized inverse (as mode 1). *Do not use this mode for negative definite matrices.*
4. Mode 4 uses the QR decomposition, and the inverse is the same as obtained from using `olsc`. This is a different type of generalized inverse, so that, when the matrix is singular a different value is obtained then from the other modes.

If the matrix is full rank, the generalized inverse equals the normal inverse (for modes 1,3 this also requires symmetry and positive definiteness).
When the mode argument is multiplied by ten, a warning is printed if the matrix is singular (or negative definite for mode 30), but the return value is not affected.

Error and warning messages

invertgen: invert failed, proceeding with generalized inverse (mode 20
invertgen: invertsym failed, proceeding with generalized p.s.d. inverse (mode 30)
invertgen: matrix has reduced rank (mode 40)
invertgen: decomposition failed (some other problem)

See also

`invert`, `invertsym`

Example

```
#include <oxstd.oxh>
main()
{
    decl x, xx;
    x = rann(20,2);
    x = x ~ x[][0];
    xx = x'x;

    println("\nAA^A=A:");
    print(xx * invertgen(xx, 30) * xx - xx);
    print(xx * invertgen(x, 40) * xx - xx);

    println("These generalized inverses are different:");
    print("Choleski failed, so use SVD", invertgen(xx, 3));
    print("Using QR", invertgen(x, 4));
}
```

produces (note that the exact value of the zeros can depend on the computer platform and the version of Ox):

```
AA^A=A:
Warning: invertgen: invertsym failed, proceeding with
generalized p.s.d. inverse
invertgen.ox (10): main

 -1.4211e-014 -4.4409e-016 -1.4211e-014
 -2.2204e-016  1.4211e-014 -2.2204e-016
 -1.4211e-014 -4.4409e-016 -1.4211e-014
Warning: invertgen: matrix has reduced rank
invertgen.ox (11): main

 -7.1054e-015 -6.6613e-016 -7.1054e-015
 -8.8818e-016  3.5527e-015 -8.8818e-016
 -7.1054e-015 -6.6613e-016 -7.1054e-015
These generalized inverses are different:
Choleski failed, so use SVD
     0.014260   -0.0023020     0.014260
   -0.0023020     0.049276   -0.0023020
     0.014260   -0.0023020     0.014260
Using QR
     0.057041   -0.0046039      0.00000
   -0.0046039     0.049276      0.00000
      0.00000      0.00000      0.00000
```

invertsym

```
invertsym(const ma);
invertsym(const ma, const alogdet);
```

`ma`	in:	symmetric, positive definite $m \times m$ matrix A
`alogdet`	in:	(optional argument) address of variable
	out:	double, the *logarithm* of the determinant of A

Return value

Returns the inverse of A, or the value 0 if the decomposition failed.

Description

Inverts the symmetric positive definite matrix A using the Choleski decomposition (see under `decldl`). The exponent of the log-determinant can only be computed for values $\leq$ `DBL_MAX_E_EXP` and $\geq$ `DBL_MIN_E_EXP` (see Ch. 9).

Error and warning messages

invertsym(): decomposition failed (the matrix is numerically singular or negative definite)

See also

`decldl`, `invert` (for an example), `invertgen`

isarray, isclass, isdouble, isfile, isfunction, isint, ismatrix, ismember, isstring

```
isarray(const a);
isclass(const a);
isclass(const a, const sclass);
isdouble(const a);
isfile(const a);
isfunction(const a);
isint(const a);
ismatrix(const a);
ismember(const a, const smember);
isstring(const a);
```

`a`	in:	any type
`sclass`	in:	string, class name
`smember`	in:	string, member name

Return value

Returns TRUE (i.e. the value 1) if the argument is of the correct type, FALSE (0) otherwise.

`isclass(a, "class")` returns `TRUE` if a is an object of type `"class"`, or derived from `"class"`.

`ismember` returns 1 if `a` is an object of a class and has a function member `"smember"`; 2 if `"smember"` is a data member and 0 otherwise.

See also

`classname`

isdotfeq, isfeq

```
isdotfeq(const ma, const mb);
isfeq(const ma, const mb);
```

`ma`	in:	arithmetic type
`mb`	in:	arithmetic type

Return value

`isfeq` always returns an integer: it returns 1 if the argument `ma` is fuzzy equal to `mb`, 0 otherwise. When strings are compared, the comparison is case insensitive.

`iseq` is as `isfeq`, but using fuzziness of zero. When strings are compared, the comparison is case sensitive.

`isdotfeq` returns a matrix if either argument is a matrix; the matrix consists of 0's and 1's: 1 if the comparison holds, 0 otherwise. If both arguments are scalar, `isdotfeq` is equal to `isfeq`.

In both cases the current fuzziness value is used.

See also

`fuzziness`

Example

```
#include <oxstd.oxh>
main()
{
    decl m1 = <1+1e-17,1-1e-17;1+1e-17,1-1e-17 >;
    decl m2 = <1+1e-17,1-1e-10;1+1e-17,1-1e-17 >;

    print( "m1 is ", isfeq(m1,1) ? "" : "*** not *** ",
        "fuzzy equal to 1\n");
    print( "m2 is ", isfeq(m2,1) ? "" : "*** not *** ",
        "fuzzy equal to 1\n");
    print(isdotfeq(m1,1));
}
```

produces

```
m1 is fuzzy equal to 1
m2 is *** not *** fuzzy equal to 1
       1.0000       1.0000
       1.0000       1.0000
```

isdotinf

```
isdotinf(const ma);
```

`ma`	in:	arithmetic type

Return value

Returns a matrix of the same dimensions if the input is a matrix; the returned matrix consists of 0's and 1's: 1 if the element is +/- infinity, 0 otherwise. If the arguments is a double, `isdotinf` returns 1 if the double is +/- infinity.

See also

`isdotmissing`, `isdotnan`

isdotmissing, isdotnan, ismissing, isnan

```
isdotmissing(const ma);
isdotnan(const ma);
ismissing(const ma);
isnan(const ma);
```

`ma`	in:	arithmetic type

Return value

`isnan` always returns an integer: it returns 1 if *any* element in `ma` is `.NaN` (not a number), 0 otherwise. `.NaN` can be used to indicate a missing value.

`isdotnan` returns a matrix of the same dimensions if the input is a matrix; the returned matrix consists of 0's and 1's: 1 if the element is NaN, 0 otherwise. If the arguments is a double, `isdotnan` returns 1 if the double is NaN.

`ismissing` and `isdotmissing` are similar to `isnan` and `isdotnan` respectively. However, in addition to NaN, they also treat $+/-$ infinity and undefined (`.Null`) as a missing value.

See also

`deletec`, `deleter`, `selectc`, `selectr`

Example

```
#include <oxstd.oxh>
main()
{
    decl m1 = <1,2,3;4,5,6;7,8,9 >;
    decl m2 = <1,.,3;4,5,.;7,8,9 >;

    print( "m1 has ", isnan(m1) ? "" : "*** no *** ",
        "missing values\n");
    print( "m2 has ", isnan(m2) ? "" : "*** no *** ",
        "missing values\n");

    print(isdotnan(m2));
    print("m2", m2, "rows with NaN deleted",
        deleter(m2), deleteifr(m2, isdotnan(m2)));
}
```

produces

```
m1 has *** no *** missing values
m2 has missing values

      0.00000       1.0000      0.00000
      0.00000      0.00000       1.0000
      0.00000      0.00000      0.00000
m2
       1.0000         .NaN       3.0000
       4.0000       5.0000         .NaN
       7.0000       8.0000       9.0000
rows with NaN deleted
       7.0000       8.0000       9.0000

       7.0000       8.0000       9.0000
```

lag, lag0

```
lag(const ma);
lag(const ma, const ilag);
lag(const ma, const ilag, double dmisval);
lag0(const ma);
lag0(const ma, const ilag);
lag0(const ma, const ilag, double dmisval);
```

`ma`	in:	$T \times n$ matrix
`ilag`	in:	int, lag length, or matrix with lag lengths (default is 1)
`dmisval`	in:	(optional argument) double, value to set missing observations to (default is 0)

Return value

Returns a $T \times n$ matrix with the lags of the specified matrix, whereby missing values are replaced by zero. E.g. the result matrix `r` using two lags is:

```
r[0][0] = .NaN      r[0][1] = .NaN      ...
r[1][0] = .NaN      r[1][1] = .NaN      ...
r[2][0] = m[0][0]   r[2][1] = m[0][1]   ...
r[3][0] = m[1][0]   r[3][1] = m[1][1]   ...
...
```

The result has the same dimensions as `ma`.
The `lag0` function is the same, but using zero for the missing value (by default).
If `ilag` is a matrix the return value corresponds to `lag0(.,ilag[0]) ~ lag0(.,ilag[1]) ~...`

Description

Lags the specified matrix, missing values are replaced by `.NaN` (unless a missing value is specified as the third argument or `lag0` is used). Using the lag operator (also called backshift operator) L: this computes:

$$L^k a_t = a_{t-k} \quad \text{for} \quad t - k \geq 0,$$

and missing values for $t - k < 0$.
Note that a negative value for `ilag` will create leads.

See also

`diff`, `diff0`

Example

```
#include <oxstd.oxh>
#include <oxfloat.oxh>            // reguired for M_NAN
main()
{
    print( lag0(<1:5>', 2) ~ lag(<1:5>', 2) );
}
```

produces

```
      0.00000          .NaN
      0.00000          .NaN
       1.0000        1.0000
       2.0000        2.0000
       3.0000        3.0000
```

limits

`limits(const ma);`

`ma`	in:	$m \times n$ matrix

Return value

Returns a $4 \times n$ matrix:

1st row:	minimum of each column of `ma`
2nd row:	maximum of each column of `ma`
3rd row:	row index of minimum (lowest index if more than one exists)
4th row:	row index of maximum (lowest index if more than one exists)

See also

`max`, `maxc`, `maxcindex`, `min`, `mincindex`

Example

```
#include <oxstd.oxh>
main()
{
    decl m = rann(7,2);
    print( range(0, rows(m)-1)' ~ m,
        "%r", {"column min","column max",
        "row index of min","row index of max"}, limits(m) );
}
```

produces

```
      0.00000      0.22489       1.7400
       1.0000     -0.20426     -0.91760
       2.0000     -0.67417     -0.34353
       3.0000      0.22335     -0.14139
       4.0000     -0.18338      0.68035
       5.0000     0.090558     -0.83328
       6.0000      0.81350       1.1174

column min          -0.67417     -0.91760
column max           0.81350       1.7400
row index of min      2.0000       1.0000
row index of max      6.0000      0.00000
```

loadmat

```
loadmat(const sname);
loadmat(const sname, const iFormat);
loadmat(const sname, const aasNames);
```

`sname`	in:	string containing an existing file name
`iFormat`	in:	(optional argument, `.mat` matrix file only) 1: file has no matrix dimensions; then the matrix is returned as a column vector, and `reshape` can be used to create a differently shaped matrix.
	in:	(optional argument, `.xlsx,.xls` Excel files only) 1: strings are loaded as values and dates translated to Ox dates, as in OxMetrics or the database class. 0 (the default): strings are treated as empty cells, unless a dot or starting with #N/A), and dates are read using the Excel numbering instead of Ox. For a date after 1-Mar-1900: oxdate = exceldate + 2415019.
`aasNames`	in:	(optional argument, not for `.mat` matrix files) address of variable
	out:	array of strings, names of data columns.

Return value

Returns the matrix which was read, or `0` if the operation failed.

Description

The type of file read depends on the extension of the file name:

`.mat`	matrix file (text file), described below,
`.dat`	data file (text file) with load information,
`.in7`	PcGive 7 data file (with corresponding `.bn7` file),
`.xlsx`	Excel 2007 (or newer) workbook file (Office Open xml),
`.xls`	Excel worksheet or workbook file (binary file),
`.csv`	comma-separated spread sheet file (text file),
`.dta`	Stata data file (version 4–6 or 11),
`.dht`	Gauss data file (v89, with corresponding `.dat` file),
`.fmt`	Gauss matrix file (v89 and v96),
any other	as `.mat` file.

This function does not retrieve information on data frequency and sample periods. To retrieve such information, use the `Database` class.

A matrix file holds a matrix, preceded by two integers which specify the number of rows and columns of the matrix. It will normally have the `.mat` extension. White space and a comma between numbers are skipped. If a symbol is found which is not a number, then the rest of the line will be skipped (so, e.g. everything following ; or // is treated as comments). The exception to this is an isolated dot, the letters `m` and `M` or the words `.NaN` and `#N/A`: these are interpreted as a missing with value NaN (Not a Number); `.Inf` is interpreted as infinity.

If the `iFormat` argument equals 1, the file is assumed not to contain matrix dimension (if it does, they will be the first two elements in the matrix).

An example of a matrix file is:

```
2 3         //<-- dimensions, a 2 by 3 matrix
//comment   //<-- a line of comment
1 0 0       //<-- first row of the matrix
0 1 .5      //<-- second row of the matrix
```

The other file formats are described in more detail in the `Database` class (under the Load functions), and in the *OxMetrics* book. Note that all file formats work identically on whatever platform Ox runs on. So an `.xlsx` file could be written with Ox on OS X, then transferred (in binary mode) to a Windows machine, and read into Ox for Windows. Ox takes care of differences in byte ordering when writing and reading binary files (always using little-endian format). This also means that a v89 `.fmt` written by Ox on the Sun can be read by Ox under Windows. Gauss under Unix writes v89 `.fmt` files in a different format. The only exception are v96 .fmt files, which write the data in the format that is native to the platform on which Ox is running. The file stores information on the byte ordering, and such a file can be read on any platform.

Warning: Excel may write csv files with only single precision (9 significant digits, rather than the 17 that are needed for loss-less saving).

Error and warning messages

loadmat(): file not found
loadmat(): no matrix elements
loadmat(): not enough matrix elements

See also

`Database` class, `savemat`, `reshape`

Example

```
#include <oxstd.oxh>
main()
{
    decl m = unit(2), as;

    savemat("t.mat", m);
    print(m, loadmat("t.mat"));

    savemat("t.in7", m, {"AA", "BB"});
    loadmat("t.in7", &as);
    println("names", as);
}
```

produces

```
      1.0000      0.00000
     0.00000       1.0000

      1.0000      0.00000
     0.00000       1.0000
names
[0] = AA
[1] = BB
```

and a file called t.mat:

```
2 2
   1.0000000000000000e+000   0.0000000000000000e+000
   0.0000000000000000e+000   1.0000000000000000e+000
```

loadsheet

```
loadsheet(const sname);
loadsheet(const sname, const iSheet);
loadsheet(const sname, const iSheet, const bConvertDates);
```

`sname`	in:	string containing an existing file name
`iSheet`	in:	(optional argument), int: sheet number (default is zero)
`bConvertDates`	in:	(optional argument), int: 1: convert dates to Ox; 0: keep dates (default is one)

Return value

Returns a two-dimensional array with the elements of the spreadsheet. The elements are `.Null` (blank in the sheet), double or string.
Returns an empty array if the file cannot be read.

Description

The type of file read depends on the extension of the file name:

`.xlsx`	Excel 2007 (or newer) workbook file (Office Open xml),
`.csv`	comma-separated spread sheet file (text file),
any other	as `.xlsx` file.

This function does not retrieve information on data frequency and sample periods. To retrieve such information, use the `Database` class.
Warning: Excel may write csv files with only single precision (9 significant digits, rather than the 17 that are needed for loss-less saving).

See also

`Database` class, `loadmat`

Example

```
#include <oxstd.oxh>

main()
{
    decl convertdates = 1;
    decl as1 = loadsheet("nodata.xlsx", 0, convertdates);
    decl as2 = loadsheet("nodata.xlsx", 1, convertdates);

    println("xlsx sheet 1:\n", "%v", as1);
    if (convertdates)
    {
        println("date/time elements in sheet 1:");
        println("[1][3]=", "%C", as1[1][3]);
        println("[2][0]=", "%C", as1[2][0]);
        println("[3][3]=", "%C", as1[3][3]);
    }
    println("ismissing on cell 0,2: ", ismissing(as1[0][2]));

    println("xlsx sheet 2:\n", "%v", as2);

    decl as3 = loadsheet("nodata.csv", 0, convertdates);
    println("csv sheet:\n", "%v", as3);
}
```

produces (after inserting some additional line breaks):

```
xlsx sheet 1:
{   {"some text,","and text",.Null,.Null},
    {.Null,"12bb",.Null,0.5},
    {2455628.25,.Null,.NaN,.Null},
    {.Null,15.16,.Null,2455628}
}
date/time elements in sheet 1:
[1][3]=12:00:00
[2][0]=2011-03-07T06:00:00
[3][3]=2011-03-07
ismissing on cell 0,2: 1
xlsx sheet 2:
{   {"some more",.Null},
    {.Null,99.900000000000006},
    {"in second sheet",.Null}
}
csv sheet:
{   {"some text,","and text",.Null,.Null},
    {.Null,"12bb",.Null,0.5},
    {2455628.25,.Null,.NaN,.Null},
    {.Null,15.16,.Null,2455628}
}
```

log, log10

`log(const ma);`
`log10(const ma);`

`ma`	in:	arithmetic type

Return value

The `log` function returns the natural logarithm of each element of `ma`, of double or matrix type.

The `log10` function returns the logarithm (base 10) of each element of `ma`, of double or matrix type.

See also

`exp`

Example

```
#include <oxstd.oxh>
main()
{
    print( log(<1,10>) );
    print( log10(<1,10>) );

    // the following shows how to prevent log(0)
    // in the computation of y*log(y) using the
    // dot-conditional operator:
    decl y = range(0,4);
    print(y .* log(y .> 0 .? y .: 1));
}
```

produces

```
      0.00000    2.3026
      0.00000    1.0000
      0.00000   0.00000    1.3863    3.2958    5.5452
```

logdet

`logdet(const ma, const asign);`

`ma`	in:	$m \times m$ real matrix A
`asign`	in:	address of variable
	out:	int, the sign of the determinant of A; 0: singular; $-1, -2$: negative determinant; $+1, +2$: positive determinant; $-2, +2$: result is unreliable

Return value

Returns a double: the *logarithm* of the absolute value of the determinant of A (-.Inf if the matrix is singular).

Description

Computes the determinant (the log of the absolute value and the sign) of a matrix using the LU decomposition of the matrix (see `declu`). The exponent of log-absolute-determinant can only be computed for values $\leq$ `DBL_MAX_E_EXP` and $\geq$ `DBL_MIN_E_EXP` (see Ch. 9).

See also

`determinant`, `invert`

loggamma

`loggamma(const ma);`

`ma`	in:	arithmetic type

Return value

Returns the logarithm of the complete gamma function at the value of each element of `ma`, of double or matrix type.

Returns .Inf for argument zero, and .NaN for any argument less than zero.

Description

Computes the logarithm of the gamma function at the argument:

$$\log\Gamma(a) = \log\int_0^\infty x^{a-1}e^{-x}\mathrm{d}x \quad \text{for } a > 0.$$

If $a = i$ is integer then $\Gamma(i+1) = i!$.

Often the ratio of two gamma functions needs te be computed. This can be done as $\Gamma(a)/\Gamma(b) = \exp(\log\Gamma(a) - \log\Gamma(b))$, thus reducing the risk of overflow for large arguments.

The function is accurate to about 14 to 15 significant digits (a table is used to look up integer values up to 50). The approximation uses the recurrence relation to obtain an argument greater than 8.5; then an asymptotic formula with eight terms is applied (see Abramowitz and Stegun, 1984, §6.1.40).

See also

`gammafact`, `gammafunc`, `polygamma`

Example

```
#include <oxstd.oxh>
main()
{
    print( loggamma(<0.5,1,10>) );
}
```

produces

```
      0.57236      0.00000       12.802
```

lower

`lower(const ma);`

`ma`	in:	$m \times n$ matrix

Return value

Returns the lower diagonal (including the diagonal), i.e. returns a copy of the input matrix with strict upper-diagonal elements set to zero.

See also

`setdiagonal`, `setupper`, `setlower`, `upper`

Example

```
#include <oxstd.oxh>
main()
{
    print( lower(ones(3,3)) );
    print( upper(ones(3,3)) );
}
```

produces

```
      1.0000      0.00000      0.00000
      1.0000       1.0000      0.00000
      1.0000       1.0000       1.0000

      1.0000       1.0000       1.0000
     0.00000       1.0000       1.0000
     0.00000      0.00000       1.0000
```

matrix

`matrix(const ma);`

`ma`	in:	arithmetic type

Return value

Casts the argument to a matrix:

input	returns
integer	a 1×1 matrix
double	a 1×1 matrix
matrix	unchanged
string	a 1×1 matrix
other types	error

See also

`int`, `double`, §13.8.2.3

max

`max(const a, ...);`

`a`	in:	arithmetic type
...	in:	arithmetic type

Return value

Returns the maximum value in all the arguments. The return type is int if all arguments are of type int; otherwise the return type is double.

Description

Finds the maximum value in the arguments, ignoring missing values. Use the dot-relational operator to find the element-by-element maximum or mimimum, see Ch. 5.

See also

`limits`, `maxc`, `min`

Example

```
#include <oxstd.oxh>
main()
{
    print( min(<1.5,12.5>, 1, 6), " ", max(<1.5,12.5>, 1, 6) );
}
```

produces: `1 12.5`

maxc, maxcindex, maxr

```
maxc(const ma);
maxcindex(const ma);
maxr(const ma);
```

`ma`	in:	$m \times n$ matrix A

Return value

The `maxc` function returns a $1 \times n$ matrix holding the maximum of each column of `ma`.

The `maxcindex` function returns a $1 \times n$ matrix holding the row index of the maximum of each column of `ma`.

The `maxr` function returns a $m \times 1$ matrix holding the maximum of each row of `ma`.

Description

Finds the maximum value in each column (row for `minr`), ignoring missing values. If no maximum is found (a column has all missing values), then the maximum is `.NaN`, and the index -1.

See also

`limits`, `max`, `minc`, `mincindex`

Example

```
#include <oxstd.oxh>
main()
{
    decl x = <11,12;10,15>;
    print("x = ", x);
    println("maxc and maxcindex", maxc(x) ~ maxcindex(x));
    println("minc and mincindex", minc(x) ~ mincindex(x));
}
```

produces

```
x =
       11.000       12.000
       10.000       15.000
maxc and maxcindex
       11.000       15.000      0.00000       1.0000
minc and mincindex
       10.000       12.000       1.0000      0.00000
```

meanc, meanr

```
meanc(const ma);
meanr(const ma);
```

`ma`	in:	$T \times n$ matrix A

Return value

The `meanc` function returns a $1 \times n$ matrix holding the means of the columns of `ma`.
The `meanr` function returns a $T \times 1$ matrix holding the means of the rows of `ma`.

See also

`sumc`, `sumr`, `varc`, `variance` (for an example), `varr`

min

```
min(const a, ...);
```

`a`	in:	arithmetic type
`...`	in:	arithmetic type

Return value

Returns the minimum value in all the arguments, ignoring missing values. The return type is int if all arguments are of type int; otherwise the return type is double.

Description

Finds the minimum value in the arguments. Use the dot-relational operator to find the element-by-element maximum, see Ch. 5.

See also

`limits`, `max` (for an example), `minc`

minc, mincindex, minr

```
minc(const ma);
mincindex(const ma);
minr(const ma);
```

`ma`	in:	$m \times n$ matrix A

Return value

The `minc` function returns a $1 \times n$ matrix holding the minimum of each column of `ma`.
The `mincindex` function returns a $1 \times n$ matrix holding the row index of the minimum of each column of `ma`.
The `minr` function returns a $m \times 1$ matrix holding the minimum of each row of `ma`.

Description

Finds the minimum value in each column (row for *minr*), ignoring missing values. If no minimum is found (a column has all missing values), then the minimum is `.NaN`, and the index -1.

See also

`limits`, `maxc`, `maxcindex`, `min`

moments

```
moments(const ma);
moments(const ma, const k);
moments(const ma, const k, const fratio);
```

`ma`	in:	$T \times n$ matrix A
`k`	in:	(optional argument) no of moments k (default is $k = 4$)
`fratio`	in:	(optional argument) 0: no ratios (default is moment ratios)

Return value

Returns an $(k+1) \times n$ matrix holding in each column for the corresponding column of `ma`:

row	*holds*	*description*
0	T^*	effective sample size
1	m_1	sample mean
2	$m_2^{1/2}$	sample standard deviation
3	$\sqrt{b_1} = m_3/(m_2^{3/2})$	sample skewness
4	$b_2 = m_4/(m_2^2)$	sample kurtosis
...		
k	$m_k/(m_2^{k/2})$	sample kth central moment ratio (i.e. in deviation from mean)

If `fratio` equals 0, the moments are not divided:

row	*holds*	*description*
0	T^*	effective sample size
1	m_1	sample mean
2	m_2	sample variance
...		
k	m_k	sample kth central moment (i.e. in deviation from mean)

Description

Computes the central moment ratios or central moments. Skips missing values.

See also

`meanc`, `meanr`, `standardize`, `varc`, `varr`

Example

The normal distribution $N[\mu, \sigma^2]$ has central moments:

$$\mu_r = \mathsf{E}\left[X - \mathsf{E}X\right]^r = \begin{cases} 0 & \text{if } r \text{ is odd,} \\ \frac{r!}{(r/2)!}\frac{\sigma^r}{2^{r/2}} & \text{if } r \text{ is even.} \end{cases}$$

So the standard normal distribution has skewness

$$\sqrt{\beta_1} = \mu_3/\mu_2^{3/2} = 0,$$

and kurtosis

$$\beta_2 = \mu_4/\mu_2^2 = 3.$$

The exponential distribution $\exp(\lambda)$ has moments about zero:

$$\mu_r' = \mathsf{E}X^r = \frac{r!}{\lambda^r}.$$

Therefore, when $\lambda = 2$, the mean is $1/2$, the variance $1/2 - 1/4 = 1/4$, etc.

```
#include <oxstd.oxh>
#include <oxprob.oxh>
main()
{
    decl m1 = rann(10000,1) ~ ranexp(10000,1, 2);

    print("moment ratios",
          "%r", {"T","mean","std.dev.","skewness","kurtosis"},
          "%c", {"normal", "exp(2)"}, moments(m1));
    print("first 6 central moments",
          "%r", {"mean", "variance", "m3", "m4", "m5", "m6"},
          moments(m1, 6, 0)[1:][]);
}
```

produces

```
moment ratios
                   normal      exp(2)
T                  10000.      10000.
mean            -0.011605     0.49592
std.dev.           1.0033     0.50088
skewness         0.010556      1.9876
kurtosis           3.0314      8.4267
first 6 central moments
mean            -0.011605     0.49592
variance           1.0066     0.25088
m3               0.010660     0.24976
m4                 3.0713     0.53039
m5                0.13868      1.1581
m6                 15.774      2.9434
```

nans

```
nans(const r, const c);
nans(const ma);
```

`r`	in:	int
`c`	in:	int
`ma`	in:	matrix

Return value

`nans(r,c)` returns an `r` by `c` matrix filled with `.NaN`.

`nans(ma)` returns a matrix of the same dimension as `ma`, filled with `.NaN`.

See also

`constant`, `ones`, `zeros`

Example

```
#include <oxstd.oxh>
main()
{
    print( nans(2, 2) );
}
```

produces

```
         .NaN          .NaN
         .NaN          .NaN
```

norm

```
norm(const ma);
norm(const ma, const itype);
```

`ma`	in:	arithmetic type
`itype`	in:	int, type of norm (default is 0 for infinity norm)

Return value

Returns the norm of a matrix.

Description

Computes the norm of a matrix A. The type of norm depends on the `itype` argument. When A is a matrix:

`itype`	norm
0	$\|\|A\|\|_\infty = \max_{0\le i<m} \sum_{j=0}^{n-1} \|a_{ij}\|$,
1	$\|\|A\|\|_1 = \max_{0\le j<n-1} \sum_{i=0}^{m-1} \|a_{ji}\|$,
2	$\|\|A\|\|_2 =$ largest singular value,
'F'	$\|\|A\|\|_F = \left(\sum_i \sum_j \|a_{ij}\|^2)\right)^{1/2}$,
–1	$\|\|A\|\|_{-\infty} = \min_{0\le i<m} \sum_{j=0}^{n-1} \|a_{ij}\|$.

The last one is the Frobenius norm. `norm(x)` corresponds to `norm(x,0)`.
When A is a vector:

`itype`	norm
0	$\|\|a\|\|_\infty = \max_i \|a_i\|$,
1	$\|\|a\|\|_1 = \sum_i \|a_i\|$,
2	$\|\|a\|\|_2 = \left(\sum_i (a_i)^2\right)^{1/2}$,
p	$\|\|a\|\|_p = (\sum_i \|a_i\|^p)^{1/p}$,
–1	$\|\|a\|\|_{-\infty} = \min_i \|a_i\|$.

Again note that `norm(x)` corresponds to `norm(x,0)`.

See also

`decsvd`, `rank`

Example

```
#include <oxstd.oxh>
main()
{
    decl x = <1,2;3,4;5,6>;

    print( norm(x), " " );
    print( norm(x, 1), " " );
    print( norm(x, 2), " " );
    print( norm(x, 'F') );
}
```

produces: `11 12 9.52552 9.53939`

nullspace

`nullspace(const ma);`

`ma`	in:	$m \times n$ matrix A

Return value

Returns the null space of `ma`, or a conformant empty matrix if `ma` is square and full rank, or 0 if the SVD fails.

Description

Uses the SVD to compute the null space $A_\perp$ of an $m \times n$ matrix A, as explained in Appendix A1. If $\operatorname{rank}(A) = r$ and $m \geq n$, the rank of the null space is $p = m - r$, and $A_\perp$ is an $m \times p$ matrix such that $A_\perp' A_\perp = I$ and $A' A_\perp = 0$. The rank of A is the number of non-zero singular values, which is determined as explained under `inverteps`.

Error and warning messages

nullspace(): decomposition failed

See also

`decsvd`, `inverteps`

Example

```
#include <oxstd.oxh>
main()
{
    decl ma = zeros(4,2);
    ma[0][0] = ma[0][1] = 1;

    print(ma, nullspace(ma));
}
```

produces

```
       1.0000       1.0000
      0.00000      0.00000
      0.00000      0.00000
      0.00000      0.00000

      0.00000      0.00000      0.00000
      0.00000      0.00000      -1.0000
      0.00000       1.0000     -0.00000
       1.0000      0.00000     -0.00000
```

ols2c, ols2r, olsc, olsr

```
olsc(const my, const mx, const amb);
olsc(const my, const mx, const amb, const amxtxinv);
olsc(const my, const mx, const amb, const amxtxinv, const amxtx);
ols2c(const my, const mx, const amb);
ols2c(const my, const mx, const amb, const amxtxinv);
ols2c(const my, const mx, const amb, const amxtxinv, const amxtx);
```

`my`	in:	$T \times n$ matrix Y
`mx`	in:	$T \times k$ matrix X
`amb`	in:	address of variable
	out:	$k \times n$ matrix of OLS coefficients, B
`amxtxinv`	in:	(optional argument) address of variable
	out:	$k \times k$ matrix $(X'X)^{-1}$,
`amxtx`	in:	(optional argument) address of variable
	out:	$k \times k$ matrix $(X'X)$,

```
olsr(const my, const mx, const amb);
olsr(const my, const mx, const amb, const amxtxinv);
olsr(const my, const mx, const amb, const amxtxinv, const amxtx);
ols2r(const my, const mx, const amb);
ols2r(const my, const mx, const amb, const amxtxinv);
ols2r(const my, const mx, const amb, const amxtxinv, const amxtx);
```

`my`	in:	$n \times T$ matrix Y'
`mx`	in:	$k \times T$ matrix X', $T \geq k$
`amb`	in:	address of variable
	out:	$n \times k$ OLS coefficient matrix, B'
`amxtxinv`	in:	(optional argument) address of variable
	out:	$k \times k$ matrix $(X'X)^{-1}$,
`amxtx`	in:	(optional argument) address of variable
	out:	$k \times k$ matrix $(X'X)$,

Return value

0:	out of memory,
1:	success,
2:	ratio of diagonal elements of $X'X$ is large, rescaling is advised, (ratio of smallest to largest $\leq \epsilon_{inv}$)
-1:	$(X'X)$ is (numerically) singular, (decision made in `decqr` and `decldl` respectively).
-2:	combines 2 and -1.

The inversion epsilon, ϵ_{inv}, is set by the `inverteps` function.

Description

`olsc` and `olsr` do ordinary least squares using the Householder QR decomposition with pivoting (see, e.g., Golub and Van Loan, 1989, Ch. 5).

`ols2c` and `ols2r` form $(X'X)$ and solve the normal equations using the Choleski decomposition (see `decldl`).

The QR based method for computing OLS is more accurate, but about half as fast (unless $T \approx k$), and more memory intensive than the normal equations approach (the QR method uses a copy of the data to work on).

If $(X'X)$ is singular, the QR based method computes B and $(X'X)^{-1}$ with zeros at the positions corresponding to the singular variables; $X'X$ remains based on the full X. So $(X'X)^{-1}$ is not the normal generalized inverse when X does not have full column rank. The normal equation approach does not produce a meaningful result in case of singularity.

See also

`decldl`, `decqr`, `inverteps`

Example

```
#include <oxstd.oxh>
main()
{
    decl mx, my, cy = 2, ct = 50, ck = 3, mb, mxtx, mxtxi;
    mx = ranu(ct,ck);
    my = rann(ct,cy) / 10 + mx * ones(ck,1);

    olsc(my, mx, &mb);
    print(mb);
    olsr(my', mx', &mb, &mxtxi, &mxtx);
    print(mb, mxtx ~ mxtxi);

    print((1/mx)*my, mx'mx ~ invert(mx'mx));
}
```

produces:

```
  1.0992   0.98022
  1.1068   0.95734
 0.78966    1.0401

  1.0992    1.1068   0.78966
 0.98022   0.95734    1.0401

  16.842    13.139    12.740   0.23380  -0.11726  -0.10967
  13.139    15.095    11.872  -0.11726   0.24566 -0.098336
  12.740    11.872    14.467  -0.10967 -0.098336   0.24639

  1.0992   0.98022
  1.1068   0.95734
 0.78966    1.0401

  16.842    13.139    12.740   0.23380  -0.11726  -0.10967
  13.139    15.095    11.872  -0.11726   0.24566 -0.098336
  12.740    11.872    14.467  -0.10967 -0.098336   0.24639
```

ones

```
ones(const r, const c);
ones(const ma);
```

`r`	in:	int
`c`	in:	int
`ma`	in:	matrix

Return value

`ones(r,c)` returns an `r` by `c` matrix filled with ones.

`ones(ma)` returns a matrix of the same dimension as `ma`, filled with ones.

See also

`constant`, `nans`, `unit`, `zeros`

Example

```
#include <oxstd.oxh>
main()
{
    print( ones(2, 2) );
}
```

produces

```
   1.0000       1.0000
   1.0000       1.0000
```

outer

```
outer(const mx, const ms);
outer(const mx, const ms, const mode);
```

`mx`	in:	$m \times n$ matrix X
`ms`	in:	$n \times n$ symmetric matrix S or empty matrix
`mode`	in:	int, operation mode: 'd' or 'o' (optional argument)

Return value

`outer(mx,ms)` returns XSX' which is $m \times m$.

`outer(mx,<>)` returns XX' which is $m \times m$.

`outer(mx,ms,'d')` returns `diagonal(`XSX'`)` which is $1 \times m$. For large matrices this is much faster than using the `diagonal` function.

`outer(mx,<>,'o')` returns $\sum_{i=1}^{m} x_i x_i'$ which is $n \times n$, writing $X' = (x_1, \ldots, x_m)$.

See also

`diagonal`

Example

```
#include <oxstd.oxh>
main()
{
    decl x = rann(2,3), y = ranu(3,3), s = y'y;
    print( outer(x, s, 'd') | diagonal(outer(x, s))
        | diagonal(x * s * x') );
}
```

produces

```
       3.7646       4.2561
       3.7646       4.2561
       3.7646       4.2561
```

oxfilename

```
oxfilename(const itype);
```

`itype` in: int, determines output format

Return value

Returns a string with the name of the Ox source file from which it is called:

		example 1	example 2
`itype`	returns	`oxl D:\waste\func`	`oxl func`
0	full file name	`D:\waste\func.ox`	`func.ox`
1	path of file name	`D:\waste\`	
2	base name	`func`	`func`
3	file extension	`.ox`	`.ox`

In the first two cases the return value depends on how the program was started (the path may not have been specified).

oxprintlevel

`oxprintlevel(const ilevel);`

`ilevel`	in:	int, print level, see below

No return value.

Description

Controls printing:

oxprintlevel(1);	default: prints as normal,
oxprintlevel(0);	switches printing off (`print` and `println` have no output),
oxprintlevel(2);	disallows further calls to `oxprintlevel`,
oxprintlevel(-1);	switches printing off, including warnings.

This function can be useful in simulations (e.g.), where the code being simulated has no other mechanism for switching printing on and off (Modelbase derived code normally uses `SetPrint`).

See also

`oxwarning`

Example

```
#include <oxstd.oxh>
test()
{
    oxprintlevel(0);        // output off
    // do some simulations which otherwise have output
    for (decl i = 0; i < 1000; ++i)
        println("i=", i);
    oxprintlevel(1);        // output on
    // do some simulations which has output and warning
    oxprintlevel(-1);       // output and warnings off
    for (decl i = 0; i < 1000; ++i)
        println("i=", i, " invert(0):", invert(0));
    oxprintlevel(1);        // output on
    // do some simulations which have warnings
    decl oldwarnings = oxwarning(0);    // all warnings off
    for (decl i = 0; i < 1000; ++i)
        invert(0);
    oxwarning(oldwarnings);             // reset warning levels
}
main()
{
    // comment the next line in to overrule oxprintlevel calls
//  oxprintlevel(2);
    test();
}
```

Prints nothing unless the `oxprintlevel(2)` statement is commented in.

oxrunerror

```
oxrunerror(const smsg);
oxrunerror(const smsg, const i01);
```

`smsg`	in:	string, error message text
`i01`	in:	int, 0 (the default) or 1

No return value.

Description

Prints the specified run-time error message and location, exits the program. If `i01`=0, the standard call trace is printed; if `i01`=1, the top level function call is skipped (in case the error is in an error handler function). Use 2 to omit the trace.

oxversion

```
oxversion();
```

Return value

Returns an integer with the version of Ox multiplied by 100, e.g. 620 for Ox 6.2.

oxwarning

```
oxwarning(const smsg);
oxwarning(const flset);
```

`smsg`	in:	string, user-determined warning message
`flset`	in:	int, new warnings settings

Return value

Returns the previous warnings settings.

Description

When given a string as argument, the function will print a user-determined warning message. Otherwise, `oxwarning` controls the reporting of run-time warning messages. The following types of messages are controlled by this function:

flag	context
`WFL_DECFAILED`	decomposition failed,
`WFL_ITMAX`	maximum no. of iterations reached,
`WFL_CONCAT`	concatenation dimensions don't match,
`WFL_VECIDXMAT`	indexed a matrix as a vector,
`WFL_DETERMINANT`	determinant-related warning,
`WFL_USER`	user-determined warning message.

The first occurs when an inversion or decomposition fails, the second could happen in an eigenvalue based function. The concatenation message is printed when the dimensions don't match, and the results has been padded with zeros. The message related to `WFL_VECIDXMAT` is given when a matrix which is not a row or column vector is indexed with just one index. However, the message is not given when using an empty single index `[]`, which has the same effect as the `vecr` function.

You can add the flags together to specify warning settings. Use `oxwarning(0)` to switch all messages off, and `oxwarning(-1)` to switch them all on.

See also

`oxprintlevel` (for an example)

periodogram

```
periodogram(const ma);
periodogram(const ma, const itrunc, const cpoints,
    const imode);
```

`ma`	in:	arithmetic type, $T \times n$ matrix
`itrunc`	in:	int, truncation parameter m, if $\leq 0, \geq T$ then $T-1$ is used
`cpoints`	in:	int, no of points N at which to evaluate periodogram
`imode`	in:	0: (truncated) periodogram (multiplied by T), 1: smoothed periodogram (multiplied by T) using Parzen window, 2: estimated spectral density using Parzen window (as option 1, but divided by c_0).

Return value

- `periodogram(ma);`
  Returns T times the periodogram, evaluated at the Fourier frequencies $0, 2\pi/T, 4\pi/T, \ldots, (\text{int}(T/2)2\pi)/T$. The dimensions of the returned matrix are $\text{int}(T/2)+1 \times n$.
- `periodogram(ma, itrunc, N, 0);`
  Returns a $N \times n$ matrix with (T times) the periodogram of the columns of `ma` using autocovariances up to lag `itrunc`, computed at frequencies $0, \pi/(N-1), 2\pi/(N-1), \ldots, \pi$.
- `periodogram(ma, itrunc, N, 1);`
  Returns a $N \times n$ matrix with (T times) the smoothed periodogram of the columns of `ma` using autocovariances up to lag `itrunc`, computed at frequencies $0, \pi/(N-1), 2\pi/(N-1), \ldots, \pi$.
- `periodogram(ma, itrunc, N, 2);`
  Returns a $N \times n$ matrix with the spectral density of the columns of `ma` using autocorrelations up to lag `itrunc`, computed at frequencies $0, \pi/(N-1), 2\pi/(N-1), \ldots, \pi$.

Description

Computes the periodogram or spectral density of the columns of a $T \times n$ matrix $A = (a_0, a_1, \ldots, a_{n-1})$.

Define the autocovariance function of a T-vector $x = (x_0 \cdots x_{T-1})'$ up to lag k as $\mathsf{c} = (\hat{\mathsf{c}}_0 \cdots \hat{\mathsf{c}}_k)'$:

$$\hat{\mathsf{c}}_j = \tfrac{1}{T}\sum_{t=j}^{T-1}(x_t - \bar{x})(x_{t-j} - \bar{x}), \tag{8.3}$$

with the mean defined in the standard way as:

$$\bar{x} = \tfrac{1}{T}\sum_{t=0}^{T-1} x_t$$

Note that $\hat{\mathsf{r}}_j = \hat{\mathsf{c}}_j / \hat{\mathsf{c}}_0$, see equation (8.1) on page 74.

The sample periodogram is then defined as:

$$\hat{\mathsf{p}}(\omega) = \frac{1}{2\pi} \sum_{j=-T+1}^{T-1} \hat{\mathsf{c}}_{|j|} \cos(j\omega) = \frac{\hat{\mathsf{c}}_0}{2\pi} \sum_{j=-T+1}^{T-1} \hat{\mathsf{r}}_{|j|} \cos(j\omega), \quad 0 \leq \omega \leq \pi, \tag{8.4}$$

and the sample spectral density as:

$$\hat{\mathsf{s}}(\omega) = \frac{1}{2\pi} \sum_{j=-m}^{m} K(j)\, \hat{\mathsf{r}}_{|j|} \cos(j\omega), \quad 0 \leq \omega \leq \pi.$$

The $K(\cdot)$ function is called the *lag window*, m is called the *lag truncation parameter*.
The value of the `imode` parameter affects the computations as follows:
0: Computes $T\mathsf{p}(\omega)$.
1: Computes the smoothed periodogram $T\hat{\mathsf{c}}_0\hat{\mathsf{s}}(\omega)$. The smoothing is achieved using the Parzen window:

$$\begin{aligned} K(j) = \;& 1 - 6\left(\frac{j}{m}\right)^2 + 6\left|\frac{j}{m}\right|^3, & \left|\frac{j}{m}\right| \leq 0.5, \\ = \;& 2\left(1 - \left|\frac{j}{m}\right|\right)^3, & 0.5 \leq \left|\frac{j}{m}\right| \leq 1.0, \\ = \;& 0, & \left|\frac{j}{m}\right| > 1. \end{aligned}$$

2: Computes the estimated spectral density $\hat{\mathsf{s}}(\omega)$ using the Parzen window.
We have that $K(-j) = K(j)$, so that the sign of j does not matter. The c_js are based on fewer observations as j increases. The window function attaches decreasing weights to the autococorrelations, with zero weight for $j > m$. The larger m, the less smooth the spectrum becomes, but the lower the bias. For more information see Priestley (1981, Ch.6), Granger and Newbold (1986, §2.6) and Brockwell and Davis (1991, §10.1).
In each case, when $N =$ `cpoints` > 0, the periodogram is evaluated at N frequencies between 0 and π:

$$0, \frac{\pi}{N-1}, \frac{2\pi}{N-1}, \ldots, \frac{(N-1)\pi}{N-1} = \pi,$$

so that the horizontal axis could be computed as:

```
    M_PI * range(0, cpoints-1) / (cpoints-1)
```

When `cpoints` is 0 on input, or when the version with one argument is used, $N = \text{int}(T/2)$, and the periodogram is evaluated at:

$$0, \frac{2\pi}{T}, \frac{4\pi}{T}, \ldots, \frac{2N\pi}{T},$$

so that the horizontal axis could be computed as:

```
    M_2PI * range(0, int(ct/2)) / ct
```

See also

`fft1d`, `DrawSpectrum` (for another example).

Example

```
#include <oxstd.oxh>
#include <oxfloat.oxh>        // required for M_2PI

main()
{
    decl ct = 2^3 + 7, x, y, yzt, p1, p2;

    y = cumulate(rann(ct,1), 0.9);
    p1 = periodogram(y) / ct;
    x = M_2PI * range(0, int(ct/2))' / ct;

    yzt = (y - meanc(y))';// FFT expects data in row
    p2 = sqr(cabs(fft1d(yzt))') / (ct * M_2PI );
    print("%c", {"periodogram", "frequencies", "FFT"},
        p1 ~ x ~ p2);
}
```

produces (the zeros at the end of the periodogram and frequencies are added in the concatenation with fft):

```
  periodogram  frequencies          FFT
      0.00000      0.00000  1.1253e-033
      0.49542      0.41888      0.49542
     0.060270      0.83776     0.060270
     0.024741       1.2566     0.024741
      0.16432       1.6755      0.16432
     0.036133       2.0944     0.036133
     0.019385       2.5133     0.019385
     0.023846       2.9322     0.023846
      0.00000      0.00000     0.023846
      0.00000      0.00000     0.019385
      0.00000      0.00000     0.036133
      0.00000      0.00000      0.16432
      0.00000      0.00000     0.024741
      0.00000      0.00000     0.060270
      0.00000      0.00000      0.49542
```

polydiv

`polydiv(const ma, const mb, const cp);`

`ma`	in:	$1 \times m$ matrix $A = (a_0 \dots a_{m-1})$ specifying the A polynomial (see below)
`mb`	in:	$1 \times n$ matrix $B = (b_0 \dots b_{n-1})$ specifying the B polynomial (see below)
`cp`	in:	int, required length, p, of polynomial resulting from division

Return value

Returns a $1 \times p$ matrix with the coefficients of polynomial resulting from dividing the A polynomial by the B polynomial. The integer 0 is returned when b_0 is 0, or $p = 0$.

Description

Defining the two polynomials

$$\begin{aligned} A(x) &= a_0 + a_1 x + a_2 x^2 + \dots a_{m-1} x^{m-1}, \\ B(x) &= b_0 + b_1 x + b_2 x^2 + \dots b_{n-1} x^{n-1}, \end{aligned}$$

`polydiv` returns (p is specified in the function call):

$$D(x) = A(x)/B(x) = d_0 + d_1 x + d_2 x^2 + \dots d_{p-1} x^{p-1}.$$

See also

`polyeval`, `polymake`, `polymul` (for an example), `polyroots`

polyeval

`polyeval(const ma, const mx);`

`ma`	in:	$1 \times m$ matrix $A = (a_0 \dots a_{m-1})$ specifying the A polynomial (see below)
`mx`	in:	arithmetic type

Return value

Returns the polynomial evaluated at `mx`.

Description

Defining the polynomial

$$A(x) = a_0 + a_1 x + a_2 x^2 + \dots a_{m-1} x^{m-1},$$

`polyeval` returns $A(x)$.

See also

`polydiv`, `polymake`, `polymul`, `polyroots`

Example

```
#include <oxstd.oxh>
main()
{
    decl a = <1,-0.8,-0.1>;

    println("a(x)=a[0]+a[1]*x+a[2]*x^2; a(3)=", polyeval(a, 3));
}
```

produces

```
a(x)=a[0]+a[1]*x+a[2]*x^2; a(3)=-2.3
```

polygamma

```
polygamma(const ma, const mn);
```

`ma`	in:	arithmetic type, argument
`mn`	in:	arithmetic type, order of derivative: 0 = first derivative, 1 = second derivative, etc.

Return value

Returns the derivative of the logarithm of the complete gamma function at the value of each element of `ma`, of double or matrix type. The second argument specifies the order of the derivative.

Returns zero for derivative order less than 0 and `-.Inf` when the argument is zero or a negative integer.

The return type is derived as follows:

returns	`ma`	order arguments
$m \times n$ matrix	$m \times n$ matrix	scalar (int)
$m \times n$ matrix	scalar	$m \times n$ matrix
$m \times n$ matrix	$m \times n$ matrix	$m \times n$ matrix
double	scalar	scalar (int)

Description

Computes the derivatives of the loggamma function at the argument a:

$$\psi^{(n)}(a) = \frac{\mathrm{d}^{n+1}}{\mathrm{d}a^{n+1}} \log \Gamma(a) \quad \text{for } a > 0.$$

Most commonly used are:

$n = 0$	digamma (psi) function
$n = 1$	trigamma function
$n = 2$	tetragamma function
$n = \ldots$	etc.

The function is accurate to about 15 significant digits (except for arguments very close to a negative integer). The approximation uses the recurrence relation

$$\psi^{(n)}(a+1) = \psi^{(n)}(a) + (-1)^n n! z^{-n-1}.$$

to obtain an a value greater than 8.5; then an asymptotic formula with eight terms is applied (see Abramowitz and Stegun, 1984, §6.4.11).

See also

`loggamma`

Example

```
#include <oxstd.oxh>
#include <oxfloat.oxh>        // required for M_EULER

main()
{
    print(polygamma(<0.5,1>, 0), -M_EULER - 2*log(2) ~ -M_EULER);
    print("%12.7g", polygamma(0.5, <0,1,2,3>));
}
```

produces

```
      -1.9635     -0.57722
      -1.9635     -0.57722
     -1.96351     4.934802     -16.8288     97.40909
```

polymake

```
polymake(const roots);
```

`roots`	in:	$2 \times m$ matrix with (inverse) roots of the polynomial, first row is real part, second row imaginary part (or $1 \times m$ matrix if all roots are real).

Return value

Returns the coefficients of the polynomial ($a_0 = 1$) as a $2 \times (m + 1)$ matrix if the roots had a complex part, else $1 \times (m + 1)$.

Description

Computes the polynomial coefficients from the inverse roots. The constant term (a_0) is set to one, so returned is the a_i from:

$$1 + a_1 x + a_2 x^2 + \dots a_m x^m.$$

See also

`polyroots` (for an example)

polymul

`polymul(const ma, const mb);`

`ma`	in:	$1 \times m$ matrix $A = (a_0 \dots a_{m-1})$ with the A polynomial
`mb`	in:	$1 \times n$ matrix $B = (b_0 \dots b_{n-1})$ specifying the B polynomial

Return value

Returns a $1{\times}m{+}n{-}1$ matrix with the coefficients of the product of the polynomials.

Description

Defining the two polynomials

$$\begin{aligned} A(x) &= a_0 + a_1 x + a_2 x^2 + \dots a_{m-1} x^{m-1}, \\ B(x) &= b_0 + b_1 x + b_2 x^2 + \dots b_{n-1} x^{n-1}, \end{aligned}$$

the `polymul` function returns:

$$C(x) = A(x)B(x) = c_0 + c_1 x + c_2 x^2 + \dots c_{p-1} x^{p-1}, \quad p = m + n - 2.$$

The coefficients c_i correspond to the convolution of the coeficients a_i and b_i:

$$c_i = \sum_{j=\max(0,i-n+1)}^{\min(i,m-1)} a_j b_{i-j}, \quad i = 0, \dots, p-2.$$

The `polymul` function computes the sum directly. For large polynomials, faster computation can be based on the fast Fourier transform, as the example shows.

See also

`fft1d`, `polydiv`, `polyeval`, `polymake`, `polyroots`

Example

```
#include <oxstd.oxh>
main()
{
    decl a, b, c, ff;
    format("%10.4f");

    a = <1,-0.9>;  b = <1,-0.8,-0.1>;

    print(polymul(a, b));
    c = polymul(b, a);
    print(polydiv(c, a, 5));

       // multiply the two FFTs, padded with zeros
    ff = cmul( fft(a~zeros(b)), fft(b~zeros(a)) );
    ff = fft(ff, 2);     // apply inverse real FFT
    print( ff[][:columns(a)+columns(b)-2] );

         // divide the two FFTs, padded with zeros
    ff = cdiv( fft(c~zeros(a)), fft(a~zeros(c)) );
    ff = fft(ff, 2);     // apply inverse real FFT
    print( ff[][:4] );
}
```

produces

```
    1.0000   -1.7000    0.6200    0.0900
    1.0000   -0.8000   -0.1000   -0.0000   -0.0000
    1.0000   -1.7000    0.6200    0.0900
    1.0000   -0.8000   -0.1000    0.0000    0.0000
```

polyroots

```
polyroots(const ma, const amroots);
```

`ma`	in:	$1 \times (m+1)$ matrix $A = (a_0 \dots a_m)$ specifying the polynomial of order m (see below)
`amroots`	in:	address of variable
	out:	$2 \times m$ matrix with roots of the polynomial, first row is real part, second row imaginary part (all zeros if the roots are real). The roots are *not* sorted.

Return value

Returns the result of the eigenvalue decomposition:

0 no error;

1 maximum no of iterations (50) reached.

Description

Computes the inverse roots of a polynomial

$$a_0 + a_1 x + a_2 x^2 + \dots a_m x^m.$$

The inverse roots are found as the eigenvalues of the companion matrix (which is already in upper Hessenberg form), e.g. when $m = 4$ and $a_0 = 1$:

$$\begin{array}{cccc} -a_1 & -a_2 & -a_3 & -a_4 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}$$

Note that it is assumed that $a_0 \neq 0$. Also note that the inverse roots of $1 + a_1 x + a_2 x^2 + \dots a_m x^m$, correspond to the roots of $x^m + a_1 x^{m-1} + a_2 x^{m-2} + \dots a_1$.

Error and warning messages

polyroots(): maximum no. of iterations reached

See also

`cabs` (for another example), `eigen`, `polydiv`, `polyeval`, `polymake`, `polymul`

Example

```
#include <oxstd.oxh>
main()
{
    decl v1 = <-1, 1.2274, -0.017197, -0.28369, -0.01028>, roots;

    polyroots(v1, &roots);
    print(v1, "roots", roots, "inverse roots", cdiv(ones(roots),roots),
        "polynomial", polymake(roots) );
}
```

```
    -1.0000      1.2274   -0.017197    -0.28369   -0.010280
roots
    0.82865     0.82865    -0.39337   -0.036535
    0.16923    -0.16923     0.00000     0.00000
inverse roots
     1.1585      1.1585     -2.5422     -27.371
   -0.23659     0.23659     0.00000     0.00000
polynomial
     1.0000     -1.2274    0.017197     0.28369    0.010280
    0.00000     0.00000     0.00000     0.00000     0.00000
```

pow

`pow(const ma, const p);`

`ma`	in:	arithmetic type
`p`	in:	arithmetic type, power

Return value

Returns `ma .^ p`. This is identical to using the dot-power operator, with the exception that if both `ma` and `p` are an integer, the return type is a double.

See also

`sqr` (for an example), `^ .^` (§13.8.3)

print, println

```
print(const a, ...);
println(const a, ...);
```

`a`	in:	any type
`...`	in:	any type

Return value

Returns the number of arguments supplied to the function.

Description

Each argument is printed to stdout using default formatting. A formatting string can be input in the input stream: a formatting string starts with a % symbol, and is followed by one or more characters. If a formatting string is encountered, it is not printed, but applied to the next argument.

There is an additional option to add column and row labels for a matrix, specify a different format for each matrix column, or only print the lower diagonal:

`%r`	the next argument contains row labels (array of strings)
`%c`	the next argument contains column labels (array of strings)
`%cf`	the next argument contains column formats (array of strings)
`%lwr`	only print the lower diagonal of the matrix

The default format strings are:

no value	`"null"`
int	`"%d"`
double	`"%g"`
matrix	`"\n"`, then each element `"%#13.5g"`, 6 elements on a line (5 if row is labelled), no labels.
string	`"%s"`
array	`"&0x%p"`
function	`"&%d"`
class	`"&0x%p"`
library function	`"&0x%p"`

The `format` function may be used to set a different default format; it also lists the format options.

The format specification is similar to that for the printf function of the C language:

%[*flag*][*width*][.*precision*]*type*

The optional *flag* arguments are listed in Table 8.2. The optional *width* argument specifies the width of the output field. The optional *precision* argument specifies the number of significant digits (for type `gG`) or the number of digits after the decimal point (type `eEf`); the default is 6 if *precision* is absent. The possible values for *type* are listed in Table 8.3.

Table 8.2 Formatting flags for doubles and integers

flag	
`-`	left adjust in output field,
`+`	always print a sign,
space	prefix space if first character is not a sign
`0`	pad with leading zeros,
`#`	alternate output form:
	type is `o`: first digit will be 0,
	type is `xX`: prefix with 0x or 0X (unless value is 0),
	type is `eEfgG`: always print decimal point,
	type is `gG`: keep trailing zeros.
	type is `mM`: omit dimensions.

Table 8.3 explains the format strings; some notes:

- The `format` function allows setting a default format.
- Be careful with the `%f` format, for example, when printing 1e-300, the output field will need 302 characters.
- By default, integers are printed without leading spaces, to use a space as separator: `" %d"` alternatively specify a wider field: `"%6d"`.
- Matrices always use one space between elements.
- The `"%m"` and `"%M"` formats must be followed by a matrix. First the number of rows and columns is written, which is followed by the matrix, row by row; this corresponds to the format used by `savemat`. The dimensions are omitted by `"%#m"` and `"%#M"`.
  This format is most useful when the matrix has to be read from a file at a later stage.
- The `"%C"` format prints date and/or time. If there is no fraction the calendar date is printed as yyyy-mm-dd; if there is only a fraction the time is printed as hh:mm or hh:mm:ss or hh:mm:ss.hh. If both are present yyyy-mm-ddThh:mm[:ss[.hh]] is printed (so using the ISO standard for date/time formatting). Also see `dayofcalendar`.
- The `"%v"` format prints a variable in the format of an Ox constant, and can be used for any variable. It can be especially useful to read and write variables that consist of derived types, such as an array or a class object, but also for a matrix. An example is given below and in `ox/samples/inout/percent_v.ox`.

The `println` function is as `print` but ensures the next output will be on a new line.

Table 8.3 Formatting types for printing

double *type*:	(also used for matrices)
g,G	%e or %E if the exponent is < -4
	or $>=$ *precision*; else use %f,
e,e	scientific notation: with exponent,
f	print without exponent,
C	print as a calendar date
specials for matrices:	
r	followed by row labels (array of strings),
c	followed by column labels (array of strings),
cf	followed by column formats (array of strings), e.g.
	`print("%c",{"a","b"},"%cf",{"%8.4g","%6.2g"},m);`
integer *type*:	
d,i	signed decimal notation,
o	unsigned octal notation,
x,X	unsigned hexadecimal notation,
u	unsigned decimal notation,
c	print as a single character (i.e. one byte),
string *type*:	
s	string format,
matrix *type*:	
m	print matrix row by row using %25.26e,
M	print matrix row by row using default format,
any type:	
v	any variable in Ox constant format.

To print text starting with a percentage symbol that is also a format specifier, use either %s as the format, or a double percentage, as in `println("%s", "%GDP", " or:  ", "%%GDP")`.

See also

eprint, format, fprint, fscan, fwrite, sprint

Example

```
#include <oxstd.oxh>
main()
{
    print( "%r", {"row 1", "row 2"},
           "%c", {"col 1", "col 2"}, "%6.1g", unit(2) );

    decl xp = 9*rann(2,1)~ranu(2,1);
    print( "%c", {"x ", "p "},
           "%cf",{"%8.4g", " [%4.2f]"}, xp);

    decl x = rann(10,2);
    print("\nLower diagonal:", "%lwr", x'x);
}
```

produces

```
      col 1 col 2
```

```
row 1     1     0
row 2     0     1

      x      p
   2.024 [0.42]
   15.66 [0.16]

Lower diagonal:
       10.585
       3.1110       7.1178
```

In the second example we show the output from the "%v" format.

```
#include <oxstd.oxh>

class VClass
{
    decl m_mMatrix;
    decl m_aArray;
    VClass();
}
VClass::VClass()
{
    m_mMatrix = range(1,3);
    m_aArray = {"a", "b", "c"};
}
main()
{
    decl vc = new VClass();
    print("\nobject using %v:\n", "%v", vc);
}
```

produces

```
object using %v:
::VClass
{
.m_mMatrix = <1,2,3>;
.m_aArray = {"a","b","c"};
}
```

probchi, probf, probn, probt

```
probchi(const ma, const df);
probchi(const ma, const df, const nc);
probf(const ma, const df1, const df2);
probf(const ma, const df1, const df2, const nc);
probn(const ma);
probt(const ma, const df);
probt(const ma, const df, const nc);
```

`ma`	in:	arithmetic type
`df`	in:	arithmetic type, degrees of freedom
`df1`	in:	arithmetic type, degrees of freedom in the numerator
`df2`	in:	arithmetic type, degrees of freedom in the denominator
`nc`	in:	arithmetic type, non-centrality parameter

Return value

Returns the requested probabilities at `ma` (between zero and one):

`probchi`	probabilities from $\chi^2(df)$ distribution,
`probchi`	probabilities from non-central $\chi^2(df)$ distribution,
`probf`	probabilities from $\mathsf{F}(df1, df2)$ distribution,
`probf`	probabilities from non-central $\mathsf{F}(df1, df2)$ distribution,
`probn`	one-sided probabilities from the standard normal $\mathsf{N}(0, 1)$,
`probt`	one-sided probabilities from student-`t`(df) distribution,
`probt`	one-sided probabilities from non-central student-`t`(df) distribution.

The normal probabilities are accurate to 14-15 significant digits for probabilities $> 10^{-20}$. The other probabilities are accurate to at least 10 digits.
The return type is derived as follows:

returns	`ma`	degrees of freedom arguments
$m \times n$ matrix	$m \times n$ matrix	scalar
$m \times n$ matrix	scalar	$m \times n$ matrix
$m \times n$ matrix	$m \times n$ matrix	$m \times n$ matrix
double	scalar	scalar

Description

Sources: `probchi` uses `gammafunc` and Applied Statistics algorithm AS 275 (Mardia and Zemroch, 1975, and a modified version of Ding, 1992) for the non-central distribution; `probf` uses `betafunc`, `probn` and `tailn` use Ooura (1998) and AS 66 (Hill, 1973), `probt` uses AS 3 (Cooper, 1968) for two arguments and integer degrees of freedom, and a modification of AS 243 (Lenth, 1989 otherwise. The non-central F is based on a modified version of AS 266 (Lenth, 1987).

See also

`dens...`, `quan...`, `tail...`, Probability package (§11.3) for `probbvn`, `probmvn`

Example

```
#include <oxstd.oxh>
main()
{
    decl m = <0,4.61,5.99>;

    print("%r", {"chi:   "}, probchi(m, 2));
    print("%r", {"normal:"}, probn(<-1.96, 0, 1.96>) );
    print("%r", {"t:     "}, probt(<-1.96, 0, 1.96>, 4) );
                          /* additional argument types: */
    print("%r", {"chi:   "}, probchi(5.99, <2,3,4>),
          "%r", {"chi:   "}, probchi(<6,7,8>, <2,3,4>) );
    print("%r", {"nc chi:"}, probchi(m, 2, 5));
    print("%r", {"nc t:  "}, probt(<-1.96, 0, 1.96>, 4, 5));
}
```

produces

```
chi:              0.00000      0.90024      0.94996
normal:          0.024998      0.50000      0.97500
t:               0.060777      0.50000      0.93922
chi:              0.94996      0.88790      0.80010
chi:              0.95021      0.92810      0.90842
nc chi:           0.00000      0.37210      0.49621
nc t:         7.3581e-010  2.8665e-007    0.0052148
```

prodc, prodr

```
prodc(const ma);
prodr(const ma);
```

`ma` in: $T \times n$ matrix A

Return value

The `prodc` function returns a $1 \times n$ matrix which holds the product of all column elements of `ma`.

The `prodr` function returns a $T \times 1$ matrix which holds the product of all row elements of `ma`.

See also

`sumc`, `sumr`

Example

```
#include <oxstd.oxh>
main()
{
    print( prodc(<0:3;1:4;2:5>) );
    print( prodr(<0:3;1:4;2:5>) );
}
```

produces

```
      0.00000       6.0000       24.000       60.000

      0.00000
       24.000
       120.00
```

quanchi, quanf, quann, quant

```
quanchi(const ma, const df);
quanf(const ma, const df1, const df2);
quann(const ma);
quant(const ma, const df);
```

`ma`	in:	arithmetic type, probabilities: must be between 0 and 1
`df`	in:	arithmetic type, degrees of freedom
`df1`	in:	arithmetic type, degrees of freedom in the numerator
`df2`	in:	arithmetic type, degrees of freedom in the denominator

Return value

Returns the requested quantiles (inverse pdf; percentage points) at `ma`:

`quanchi`	quantiles from $\chi^2(df)$ distribution
`quanf`	quantiles from $\mathsf{F}(df1, df2)$ distribution
`quann`	standard normal quantiles
`quant`	quantiles from student-t(df) with integer degrees of freedom

The quantiles are accurate to about 10 digits. The return type is derived as follows:

returns	`ma`	degrees of freedom arguments
$m \times n$ matrix	$m \times n$ matrix	scalar
$m \times n$ matrix	scalar	$m \times n$ matrix
$m \times n$ matrix	$m \times n$ matrix	$m \times n$ matrix
double	scalar	scalar)

Description

Sources: `quanchi` uses a modified version of Applied Statistics algorithm AS 91 (Best and Roberts, 1975) and AS R85 (Shea, 1991), `quanf` uses AS 109 (Cran, Martin, and Thomas, 1977) and AS 64 (Majunder and Bhattacharjee, 1973) to obtain starting values for a Newton Raphson refinement (it does not use the iterative procedure from AS 109 because it is not accurate enough; AS R83 (Berry, Mielke Jr, and Cran, 1977) does not seem to solve this), `quann` uses AS 241 (Wichura, 1988), `quant` is based on Hill (1981), using Newton Raphson for refinement.

See also

`dens...`, `prob...`, `tail...`, `lib/Quantile.ox` (to compute quantiles of other distributions), Probability package (§11.3)

Example

```
#include <oxstd.oxh>
main()
{   decl t = range(1,10), tt = (t - 5) / 5;
    print("%14.10g", probf(t,10,10)' ~ quanf(probf(t,10,10),10,10)'
      ~ probt(tt,2)' ~ quant(probt(tt,2),2)' );
}
```

```
           0.5             1   0.253817018          -0.8
   0.855154194             2  0.3047166335          -0.6
  0.9510726929             3  0.3639172365          -0.4
    0.98041856             4  0.4299859958          -0.2
  0.9910499384             5           0.5             0
  0.9954702686             6  0.5700140042           0.2
  0.9975177199             7  0.6360827635           0.4
  0.9985507194             8  0.6952833665           0.6
    0.99910908             9   0.746182982           0.8
  0.9994284475            10  0.7886751346             1
```

quantilec, quantiler

```
quantilec(const ma);
quantiler(const ma);
quantilec(const ma, const mq);
quantiler(const ma, const mq);
```

ma	in:	$T \times n$ matrix A
mq	in:	(optional argument) $1 \times q$ matrix of quantiles

Return value

The `quantilec` function returns a $q \times n$ matrix holding the requested quantiles of the columns of `ma`. If no second argument is used the return value is a $1 \times n$ matrix holding the medians.

The `quantiler` function returns a $T \times q$ matrix holding the requested quantiles of the rows of `ma`. If no second argument is used the return value is a $T \times 1$ matrix holding the medians.

Description

The q-th quantile $\xi_q,\ 0 \leq q \leq 1$, of a random variable X is defined as the smallest ξ which satisfies $\mathsf{P}(X \leq \xi) = q$. So $\xi_{0.5}$, the median, divides the distribution in half.

For a sample of size T, $x = (x_0 \cdots x_{T-1})'$, the q-th quantile is found by interpolating the nearest two values. Write $(y_0 \cdots y_{T-1})$ for the ordered x-values, $y_0 \leq y_1 \leq \cdots \leq y_{T-1}$, the quantiles are computed as:

$$\xi_q = [k + 1 - q\,(T-1)]\,y_k + [q\,(T-1) - k]\,y_{k+1}, \tag{8.5}$$

where

$$k = \text{int}[q\,(T-1)].$$

when $q(T-1)$ is integer, the expression simplifies to $\xi_q = y_k$.

For example, for the quartiles ($\xi_{0.25}$, $\xi_{0.5}$ and $\xi_{0.75}$) when $T = 4$: $q(T-1) = 0.75, 1.5, 2.25$ and $k = 0, 1, 2$ respectively. In this case, the median is the average of the middle two observations: $\xi_{.5} = 0.5y_1 + 0.5y_2$, and the lower quartile: $\xi_{.25} = 0.25y_0 + 0.75y_1$.

The example below shows how to obtain quantiles without using interpolation.

See also

`meanc`, `meanr`, `varc`, `varr`

Example

```
#include <oxstd.oxh>
main()
{
    print( quantilec(<3;2;1;4>, <1/4,2/4,3/4>) );
    print( quantilec(<3;2;1;4>) );

    decl m = rann(2,10000);        /* generate m */

    print( quantiler(m, <0.8,0.9,0.95,0.975>) );
    print( quantilec(m', <0.8,0.9,0.95,0.975>) );

    m = sortr(m);                        /* sort m */
    print( m[][columns(m) * <0.8,0.9,0.95,0.975> ] );
}
```

produces:

```
       1.7500
       2.5000
       3.2500

       2.5000

      0.83516       1.2728       1.6457       1.9635
      0.84842       1.2740       1.6248       1.9570

      0.83516      0.84842
       1.2728       1.2740
       1.6457       1.6248
       1.9635       1.9570

      0.83536       1.2734       1.6459       1.9638
      0.84871       1.2744       1.6255       1.9585
```

range

```
range(const min, const max);
range(const min, const max, const step);
```

`min`	in:	int or double, first value m
`max`	in:	int or double, last value n
`step`	in:	int or double, (optional argument) increment

Return value

Returns a $1 \times (n - m + 1)$ matrix with the values with values $m, m + 1, \ldots, n$. If $n < m$, `range` returns a $1 \times (m - n + 1)$ matrix with the values with values m, $m - 1, \ldots, n$.

The version which uses the `step` argument uses that as the incrementor (rather than the default $+1$ or -1), the returned matrix is a row vector of the required length.

Description

When all arguments are integers, the incrementation arithmetic is done using integers, else using doubles. Integer arithmetic could be a bit more precise when using longer ranges. The following example illustrates the difference:

```
range(-1.1, 1.1, 0.11);
range(-110, 110, 11) / 100;
```

The first line has the loop using floating point arithmetic, and will not have exactly zero, but something like -1.9e-16 as its 11th element. In the second line, the loop is incremented in integer arithmetic before conversion to floating point numbers. Here the 11th number will be exactly zero. Because if these rounding errors, it is best to use the integer version, and scale afterwards.

See also

`constant`

Example

```
#include <oxstd.oxh>
main()
{
    print( range(1,4), range(4,1), range(1,6,2));
    print( range(1.2,4), range(1,6,2.1));
}
```

produces

```
       1.0000       2.0000       3.0000       4.0000
       4.0000       3.0000       2.0000       1.0000
       1.0000       3.0000       5.0000
       1.2000       2.2000       3.2000
       1.0000       3.1000       5.2000
```

ranloopseed

`ranloopseed(const iloop, const istage);`

`iloop`	in:	int, loop counter (if `istage` = 0)
`istage`	in:	int, -1: initialization, 0: iterating, 1: exiting

No return value.

Description

Colours the current seed with the loop iterator, so that parallel loops have the same random number stream independently of how they are partitioned among threads or processes. This is automatically done inside Ox for parallel for and foreach loops.

Note that nested calls to ranloopseed are ignored, as are calls inside a parallel loop.

See also

§4.7.

rank

`rank(const ma);`
`rank(const ma, const eps);`

`ma`	in:	arithmetic type
`eps`	in:	arithmetic type, optional tolerance

Return value

Returns the rank of a matrix, of type int. The rank of a scalar is 1, except for the rank of zero, which is zero.

Description

Computes the rank of a matrix A. The rank is the number of singular values $>$ $10\epsilon_{inv}||A||_\infty$, with ϵ_{inv} is set by the `inverteps` function (the default is the machine precision for doubles times 1000 $\approx 2 \times 10^{-13}$) and

$$||A||_\infty = \max_{0 \leq i < m} \sum_{j=0}^{n-1} |a_{ij}|.$$

Note that, by default, the rank is relative to the norm, so that, for example, the rank of `<1e-200>` is 1.

When the two argument version is used, the rank is computed as the number of singular values $>$ `eps`.

See also

`decsvd`, `inverteps`, `norm`

Example

```
#include <oxstd.oxh>
main()
{
    print( rank(<1,0;1,0>), " " );
    print( rank(<1e-200>), " " );
    print( rank(0), " " );
    print( rank(<1e-200>, inverteps(0)) );
}
```

produces: `1 1 0 0`

rann

```
rann(const r, const c);
```

`r`	in:	int, number of rows
`c`	in:	int, number of columns

Return value

Returns a `r` $\times$ `c` matrix of random numbers from the standard normal distribution. The matrix is filled by row. Note that, if both `r` and `c` are 1, the return value is a scalar of type double.

Description

The `rann` function generates pseudo-random draws from the standard normal distribution. This uses uniform random numbers as described under `ranu`.

Using `ranseed("MWC_52")` (the default uniform generator) or `ranseed("MWC_32")` will generate standard normal samples using the ziggurat method (Doornik, 2005), while the others use the polar-Marsaglia method. In the polar method, the draws are generated in pairs. As a consequence, the seed may be one state further advanced than expected.

See also

`ranseed`, `ranu`, Probability package (§11.3),

Example

```
#include <oxstd.oxh>
main()
{
    print( sumc( rann(1000,1) ) / 1000 );

    ranseed(-1);
    print(rann(1,5));
    ranseed(-1);
    print(rann(1,3) ~ rann(1,2));
}
```

produces

```
    -0.035817

   0.22489      1.7400    -0.20426    -0.91760    -0.67417
   0.22489      1.7400    -0.20426    -0.91760    -0.67417
```

ranseed

```
ranseed(const iseed);
```

`iseed`	in:	int (1 seed), or array of ints (multiple seeds), or
	in:	string, name of random number generator to use.

Return value

Returns the current seed(s) of the random number generator. If the generator only uses one seed, the return type is int. Otherwise it is an array holding all the seeds (all array elements are integers).

A call to `ranseed(0)` only returns the current seed, without changing it; `ranseed(-1)` resets to the initial seed and returns the initial seed.

A call with a string argument to set the RNG returns the name of the new RNG. Use `ranseed("")` to get the name of the current RNG without changing it.

Description

Sets and gets the seed(s); `ranseed` can also change the random number generator (see under `ranu` for more information). Some examples are:

`ranseed(0)`	just returns the seed(s)
`ranseed(-1)`	resets the initial seed(s)
`ranseed(111)`	sets seed to `111`
`ranseed(111, 1111)`	sets two seeds (e.g. for two seed rng, ”GE”)
`ranseed("MWC_52")`	MWC822_52 generator (the default generator)
`ranseed("PM")`	Park & Miller generator (the Ox 3 default)
`ranseed("GM")`	George Marsaglia’s generator
`ranseed("LE")`	Pierre L’Ecuyer’s generator

The seed is not set according to the date and time. Ox always uses a fixed seed, so that statistical results can be replicated on the next run. Thus, in many cases it is not necessary to set the seed explicitly. To set the seed according to the current time use `ranseed(today())`.

Note that each generator has its own set of seeds. When using L’Ecuyer, the four seeds must be $(>1, >7, >15, >127)$, otherwise the call is ignored.

Note that the `ranseed("MWC_32")` and `ranseed("MWC_52")` generators have 256 seeds and a state and carry, so `ranseed(0)` returns a vector with 258 elements. It is possible to set the seed with one element, for example `ranseed(111)`. In that case 111 is used as a starting point for a procedure that generates 256 seeds, and the default state and carry are used. For other RNGs which use more than one seed, if only one seed is set then all seeds are set to this value.

See also

`ran...`, `ranu`

Example

```
#include <oxstd.oxh>
main()
{
    decl seed = ranseed(0);
    print("RNG=", ranseed(""), " initial seeds: ",
        seed[0], " ... ", seed[sizeof(seed) - 1]);
    print( meanc(rann(10000,2)) | meanc(rann(10000,2)) );
    seed = ranseed(0);
    print("current seed: ",
        seed[0], " ... ", seed[sizeof(seed) - 1]);
    ranseed(-1);
    print( meanc(rann(10000,2)) );

    ranseed("GM");
    print("RNG=", ranseed(""), " initial seed: ",  ranseed(0) );
    print( meanc(rann(10000,2)) | meanc(rann(10000,2)) );
    ranseed(-1);
    print( meanc(rann(10000,2)) );
}
```

produces

```
RNG=MWC_52 initial seeds: 1013904223 ... 362436
    0.0011722   -0.0070313
   -0.0024659   -0.0065795
current seed: 866497328 ... 759508397
    0.0011722   -0.0070313
RNG=GM initial seed:
[0] = 362436069
[1] = 521288629

   -0.0046842     0.015912
    0.0037562     0.017064

   -0.0046842     0.015912
```

ranu

```
ranu(const r, const c);
```

`r`	in:	int
`c`	in:	int

Return value

Returns a `r` × `c` matrix of uniform random numbers. The matrix is filled by row. When both `r` and `c` are 1, the return value is a scalar of type double.

Description

Generates random numbers uniformly distributed in the range 0 to 1. Each call to `ranu` will produce a different set of numbers, unless the seed is reset (this is achieved through the `ranseed` function). There is a choice between five random number generators (made using `ranseed`). The following two tables list the origin and properties of the (pseudo) random number generators (see Doornik, 2006 for a more detailed discussion):

code	name	reference
`"PM"`	LCG31	modified version of Park and Miller (1988) (this was the Ox 1–3 default)
`"GM"`	MWC60	Marsaglia (1997)
`"LE"`	LFSR113	L'Ecuyer (1999)
`"MWC_32"`	MWC8222	Marsaglia (2003)
`"MWC_52"`	MWC8222_52	Marsaglia (2003) and Doornik (2007)
`"default"`		Set the default generator, same as `"MWC_52"`.

code	period	seeds	speed
`"PM"`	$2^{31} - 1 \approx 2 \times 10^9$	1	0.8
`"GM"`	$\approx 0.6 \times 2^{60} \approx 7 \times 10^{17}$	2	0.9
`"LE"`	$\approx 2^{113} \approx 4 \times 10^{34}$	4	1.1
`"MWC_32"`	$\approx 2^{8222} \approx 10^{2475}$	256	0.8
`"MWC_52"`	$\approx 2^{8221} \approx 10^{2475}$	256	1

The default `"MWC_52"` generates a random number that makes full use of the available floating point precision (this carries over to all other random number functions). The others only use 32 bits (instead of 52).

The relative speed ratio is only a rough indicator (and will be platform specific). All random number generators for the non-uniform distributions use the active uniform generator as input. A C-code listing of the generators is given in the Ox Appendices.

See also

`ran...`, `ranseed`

Example

```
#include <oxstd.oxh>
main()
{
    print( ranu(2,3) );
}
```

produces

```
      0.56444      0.76994      0.41641
      0.15881     0.098209      0.37477
```

reflect

`reflect(const ma);`

`ma`	in:	square $m \times m$ matrix

Return value

Returns the reflected version of `ma`.

Description

Reflects a matrix around its secondary diagonal (from element $m-1, 0$ to element $0, m-1$). A matrix which is unchanged under reflection is called *persymmetric*.

See also

transpose operator `'`

Example

```
#include <oxstd.oxh>
main()
{
    print( reflect(<2,1;1,4>) );
}
```

produces

```
   4.0000        1.0000
   1.0000        2.0000
```

replace

```
replace(const where, const what, const with);
replace(const where, const what, const with, const smode);
```

`where`	in:	object to replace in
`what`	in:	what to search for
`with`	in:	the replacement
`smode`	in	(optional argument), string controlling replace method

Return value

Returns `where`, with the requested replacements made (if any).

Description

`where`	`what`	`with`	action
array	string	string	replace string elements equal `where` with `with`
array	string	string	apply the replace action on each string element
string	string	string	replace one or more occurrences of substring `what`
arithmetic	int,double	int,double	replace every element that occurs in `what`
arithmetic	matrix	int,double	replace every element that occurs in `what`
arithmetic	matrix	matrix	replace every element that occurs in `what` with the corresponding element in `with`
array	arithmetic	arithmetic	apply the replace action on each arithmetic element

Arithmetic denotes a matrix, int, or double.

For string replacement, `smode` is an optional string consisting of the letters:

`"i"`	ignore case,
`"*"`	replace all,
`"a"`	replace all,
`"."`	dot-replacement: apply the string replacement to every string in the array (instead of replacing entire strings only),
`"1"` – `"9"`	perform from one to nine replacements.

The default is `"*"`.

The `smode` argument is ignored for arithmetic replacement.

See also

`find`, `vecindex`

Example

```
#include <oxstd.oxh>

main()
{
    decl sarr = {"Aa", "BbAaAa", "Aa", "Cc"};
    println(replace("aAaAbBaAa", "bB",  "xx"));
    println(replace("aAaAbBaAa", "bB",  ""  ));
    println(replace("aAaAbBaAa", "aAa", "1zzz"));
    println(replace("aAaAbBaAa", "AAA", "1zzz", "i"));
    println(replace(sarr,        "AA",  "1zzz", "i1"));
    println(replace(sarr,        "AA",  "1zzz", "i.1"));
    println(replace(unit(3),     <1,0>, <2,5>));
    println(unit(3) .== 0 .? 5 .: 2);
    println(replace(unit(3),     0,     2   ));
    println(replace({0,1,2,0},   0,     2   ));
}
```

produces:

```
    aAaAxxaAa
    aAaAaAa
    1zzzAbB1zzz
    1zzzAbB1zzz

    [0] = 1zzz
    [1] = BbAaAa
    [2] = Aa
    [3] = Cc

    [0] = 1zzz
    [1] = Bb1zzzAa
    [2] = 1zzz
    [3] = Cc

           2.0000       5.0000       5.0000
           5.0000       2.0000       5.0000
           5.0000       5.0000       2.0000

           2.0000       5.0000       5.0000
           5.0000       2.0000       5.0000
           5.0000       5.0000       2.0000

           1.0000       2.0000       2.0000
           2.0000       1.0000       2.0000
           2.0000       2.0000       1.0000

    [0] = 2
    [1] = 1
    [2] = 2
    [3] = 2
```

reshape

`reshape(const ma, const r, const c);`

`ma`	in:	arithmetic type
`r`	in:	int
`c`	in:	int

Return value

Returns an `r` × `c` matrix, filled by row from vecr(`ma`). If there are less than `rc` elements in `ma`, the input matrix is repeated.

Description

Reshapes a matrix. It runs through the rows of `ma` from top to bottom. When all the elements of `ma` are used, the function starts again at the begining of `ma`.

See also

`shape`, `vecr`

Example

```
#include <oxstd.oxh>
main()
{
    print( reshape(<1:3>, 4, 3)' );
}
```

```
      1.0000       1.0000       1.0000       1.0000
      2.0000       2.0000       2.0000       2.0000
      3.0000       3.0000       3.0000       3.0000
```

reversec, reverser

`reversec(const ma);`
`reverser(const ma);`

`ma`	in:	$m \times n$ matrix A

Return value

The `reversec` function returns an $m \times n$ matrix which is `ma`, except that the elements within each column are in reverse order.

The `reverser` function returns an $m \times n$ matrix which is `ma`, except that the elements within each row are in reverse order.

See also

`sortc`, `sortr`

Example

```
#include <oxstd.oxh>
main()
{
    decl m = <0:3;4:7;8:11;12:15>;
    print( reversec(m), reverser(m) );
}
```

```
      12.000       13.000       14.000       15.000
      8.0000       9.0000       10.000       11.000
      4.0000       5.0000       6.0000       7.0000
     0.00000       1.0000       2.0000       3.0000

      3.0000       2.0000       1.0000      0.00000
      7.0000       6.0000       5.0000       4.0000
      11.000       10.000       9.0000       8.0000
      15.000       14.000       13.000       12.000
```

round

```
round(const ma);
```

`ma`	in:	arithmetic type

Return value

Returns the rounded elements of `ma`, of double or matrix type. Rounds to the nearest integer.

See also

`ceil` (for an example), `floor`, `trunc`

rows

```
rows(const ma);
```

`ma`	in:	any type

Return value

Returns an integer value which is the number of rows in the argument:

type	returns
$m \times n$ matrix	m
string	number of characters in the string
array	number of elements in the array
file	number of rows in the file
	(only if opened with f format, see `fopen`)
other	0

Description

Computes the number of rows in the argument.

See also

`columns` (for an example), `sizec` (for an example), `sizeof`, `sizer`, `sizerc`

savemat

```
savemat(const sname, const ma);
savemat(const sname, const ma, const iFormat);
savemat(const sname, const ma, const asVarNames);
```

`sname`	in:	string containing a destination file name (with extension)
`ma`	in:	matrix
`iFormat`	in:	(optional argument) 1: omit the matrix dimensions (.mat file only) 1: save in universal v96 format (.fmt file only)
`asVarNames`	in:	(optional argument) array of strings with names for data columns

Return value

Returns 0 if the operation failed, 1 otherwise.

Description

The type of file saved depends on the extension of the file name:

`.mat`	matrix file (text file),
`.dat`	data file (text file) with load information,
`.in7`	PcGive 7 data file (with corresponding `.bn7` file),
`.xlsx`	Excel 2007 workbook file (Office Open xml),
`.xls`	Excel version 2.1 spreadsheet file,
`.csv`	comma-separated spread sheet file (text file),
`.fmt`	Gauss matrix file: extended v89 (default) or universal v96
`.dht`	Gauss data file: extended v89, with corresponding `.dat` file),
`.dta`	Stata 11 data file (version 114),
any other	as `.mat` file.

The .mat and .dat formats save the data in human readable (ascii) format, the rest in binary format. For general matrices, use .mat for flexibility and easy of use, and .fmt format for speed (it can be an order of magnitude faster than .mat for large files). The other formats are more appropriate for database style data, where the number of rows (observations) is larger than the number of columns (variables). In that case .in7 is the fastest. Old-format spreadsheet files (.xls) cannot save matrices larger than 65 536 $\times$ 256, although Ox allows up to 65 536 columns. For more information on spreadsheet files see `Database::LoadXlsx()`; for an example of a .mat file see `loaddata()`.

Where required, the sample start is set to 1 (1), the frequency to 1, and the variable names to `Var1`, `Var2`, ... The `Database` class allows proper treatment of sample periods and variable names.

When writing a matrix file (see `loadmat` for an example), the values are written to full precision (16 significant digits). A NaN (Not a Number) is written as a dot.

All written files (including `.fmt`) are identical on each platform, so that a file can be written under Windows, transferred to a Sun in binary mode, and then read again using `loadmat`. So, the files are written in Windows byte ordering (little endian; also see `fwrite`). Gauss under Unix writes `.fmt` files in a different format. The only exception are v96 .fmt files, which write the data in the format that is native to the platform on which Ox is running. The file stores information on the byte ordering, and such a file can again be read on any platform.

The `loadmat` function has a further discussion of the formats.

Error and warning messages

savemat(): cannot open file

Can only save ... variables

See also

`Database` class, `loadmat` (for an example)

scan

```
scan(const a, ...);
```

a	in:	any type
...	in:	any type

Return value

Returns the number of arguments successfully scanned and assigned.

Description

This function works as `fscan`, but reading from the console, not a file. Any text in the scanning string which does not have an input format is *echoed to the console* (this is different from the standard C `scanf` function).

See also

`fscan`, `fwrite`, `sscan`

Example

The following example reads one input line at a time (leading spaces in each line are skipped, because of the starting space in `" %z"`, and reads from that string using `scan`. The * in `"%*d"` suppresses assignment, so the integer is skipped in the file.

```
#include <oxstd.oxh>
main()
{
    decl c, i, d, m;

    c = scan("Enter an integer: %d", &i,
             "Enter a double:   %f", &d);
    print("items read=", c, " int=", i, " dbl=", d, "\n");

    c = scan("Enter a 2 x 2 matrix: %#m", 2, 2, &m);
    print("items read=", c, " mat=", m);

    c = scan("Enter a matrix with dimensions: %m", &m);
    print("items read=", c, " mat=", m);
}
```

This program produces (keyboard input is written in italics):

```
Enter an integer: 24
Enter a double:   25
items read=2 int=24 dbl=25
Enter a 2 x 2 matrix: 1 0 0 1
items read=1 mat=
       1.0000      0.00000
      0.00000       1.0000
Enter a matrix with dimensions: 2 2 1 0 0 1
items read=1 mat=
       1.0000      0.00000
      0.00000       1.0000
```

selectc, selectr, selectifc, selectifr, selectrc

```
selectc(const ma);
selectc(const ma, const mval);
selectr(const ma);
selectr(const ma, const mval);
selectifc(const ma, const mifc);
selectifr(const ma, const mifr);
selectrc(const ma, const mr, const mc);
```

`ma`	in:	$m \times n$ matrix to select from
`mval`	in:	$p \times q$ matrix with values to use for selection
`mifc`	in:	$p \times n$ boolean matrix specifying columns to select
`mifr`	in:	$m \times q$ boolean matrix specifying rows to select
`mc`	in:	$p \times n$ matrix with indices of columns to select
`mr`	in:	$m \times q$ matrix with indices of rows to select

Return value

The `selectc` function with one argument returns an $m \times s$ matrix, selecting columns from `ma` which have a missing value (`.NaN`: not a number).
The `selectr` function with one argument returns an $s \times n$ matrix, selecting rows from `ma` which have a missing value (`.NaN`: not a number).

The remaining forms do not have special treatment of missing values.

The `selectc` function with two arguments returns an $m \times s$ matrix, selecting the columns from `ma` which have at least one element equal to an element in `mval`.
The `selectr` function with two arguments returns an $s \times n$ matrix, selecting the rows from `ma` which have at least one element equal to an element in `mval`.
The selectif functions can be used to select rows or columns based on a logical expression: all rows (columns) which have a zero in the corresponding row (column) are dropped.
The `selectifc` function returns an $m \times s$ matrix, selecting columns from `ma` which have at least one non-zero element in the corresponding column of `mifc`.
The `selectifr` function returns an $s \times n$ matrix, selecting only those rows from `ma` which have at least one non-zero element in the corresponding row of `mifr`.
The `selectrc` function returns a $1 \times \max(pn, mq)$ matrix, which holds the selected elements. If an index is outside the matrix bounds of `ma` the corresponding element in the return value is NaN.

All functions return an empty matrix (<>) if the selection is empty.

See also

`deletec` (for an example involving NaNs), `deleter`, `deleteifc`, `deleteifr`, `isdotnan`, `vecindex`

Example

```
#include <oxstd.oxh>
main()
{
    decl m = <0:3;4:7;8:11;12:15>, sel = <1,9,10,14>;
    print(m, "select", selectc(m, sel), selectr(m, sel));
    print("selectif", selectifr(m, m .< 0 || m .> 14));
    print("selectrc", selectrc(m, <2,3,4>, <2,3,4>));
}
```

produces:

```
      0.00000       1.0000       2.0000       3.0000
       4.0000       5.0000       6.0000       7.0000
       8.0000       9.0000       10.000       11.000
       12.000       13.000       14.000       15.000
select
       1.0000       2.0000
       5.0000       6.0000
       9.0000       10.000
       13.000       14.000

      0.00000       1.0000       2.0000       3.0000
       8.0000       9.0000       10.000       11.000
       12.000       13.000       14.000       15.000
selectif
       12.000       13.000       14.000       15.000
selectrc
       10.000       15.000         .NaN
```

setbounds

```
setbounds(const ma, const dlo, const dhi);
```

`ma`	in:	$m \times n$ matrix
`dlo`	in:	scalar, lower bound (may be `-.Inf`)
`dhi`	in:	scalar, upper bound (may be `+.Inf`)

Return value

Returns the specified matrix, replacing values smaller than `dlo` by `dlo` and values greater that `dhi` by `dhi`. Missing values remain missing.

See also

Ch. 5 (censored random variates)

Example

```
#include <oxstd.oxh>
main()
{
    decl x = <1,2,3;4,5,6>;
    print( setbounds(x, 3, 4) );
    print( setbounds(x, -.Inf, 4) );
    print( setbounds(x, 2, .Inf) );
}
```

produces

```
       3.0000       3.0000       3.0000
       4.0000       4.0000       4.0000

       1.0000       2.0000       3.0000
       4.0000       4.0000       4.0000

       2.0000       2.0000       3.0000
       4.0000       5.0000       6.0000
```

setdiagonal, setlower, setupper

```
setdiagonal(const ma, const mdiag);
setlower(const ma, const ml);
setupper(const ma, const mu);
setlower(const ma, const ml, const mdiag);
setupper(const ma, const mu, const mdiag);
```

`ma`	in:	$m \times n$ matrix
`mdiag`	in:	$1 \times \min(m, n)$ or $\min(m, n) \times 1$ or $m \times n$ matrix or scalar
`ml`	in:	scalar, or $m \times n$ matrix, or vector, with new strict lower diagonal
`mu`	in:	scalar or $m \times n$ matrix with new strict upper diagonal

Return value

`setdiagonal` returns a matrix with the diagonal replaced by `mdiag`, which is either a vector with the new diagonal elements, or a matrix from which the diagonal is copied. If `mdiag` is scalar, all diagonal elements are set to that value.

`setlower` returns `ma` with the strict lower diagonal replaced by that of `ml`. If `ml` is a scalar, each element is set to that value. If `ml` is a row or column vector, the strict lower diagonal is set column-wise to the elements from the vector (until the vector runs out).

`setupper` returns `ma` with the strict upper diagonal replaced by that of `ml`. If `ml` is a scalar, each element is set to that value.

The following are equivalent:

```
setlower(ma,ml,mdiag)  setdiagonal(setlower(ma,ml),mdiag)
setupper(ma,ml,mdiag)  setdiagonal(setupper(ma,ml),mdiag)
```

See also

`diag`, `diagonal`, `diagonalize`, `lower`, `upper`

Example

```
#include <oxstd.oxh>
main()
{
    decl ma = ones(2,2), mb = rann(3,3);
    print(setdiagonal(ma, zeros(2,1)), setdiagonal(ma, 0),
          setdiagonal(ma, zeros(2,2)) );

    ma = ones(3,3);
    print(setlower(ma, mb, mb), setupper(ma, 0), setupper(ma, 0, 2));
}
```

```
     0.00000       1.0000
      1.0000      0.00000

     0.00000       1.0000
      1.0000      0.00000

     0.00000       1.0000
      1.0000      0.00000

     0.22489       1.0000       1.0000
    -0.91760     -0.67417       1.0000
     0.22335     -0.14139     -0.18338
```

```
      1.0000      0.00000      0.00000
      1.0000       1.0000      0.00000
      1.0000       1.0000       1.0000

      2.0000      0.00000      0.00000
      1.0000       2.0000      0.00000
      1.0000       1.0000       2.0000
```

shape

`shape(const ma, const r, const c);`

`ma`	in:	arithmetic type
`r`	in:	int
`c`	in:	int

Return value

Returns an `r` $\times$ `c` matrix, filled by column from vec(`ma`). If there are fewer than `rc` elements in `ma`, the value 0 is used for padding.

Description

Shapes a matrix. It runs through the columns of `ma` from left to right, and can be used e.g. to undo a `vec` operation. So `shape` puts the first `r` elements of `ma` in the first column of the return matrix, etc. To do the opposite, namely put the first `c` elements in the first row of the return matrx, use `shape(ma, c, r)'`.

Shape is closely related to `vec`:

```
    v = shape(x, rows(x)*columns(x), 1)
```

is the same as `v = vec(x)`.

```
    shape(v, rows(x), columns(x))
```

undoes the vectorization.

See also

`reshape`, `vec`

Example

```
#include <oxstd.oxh>
main()
{
    print( shape(<0:5>, 2, 4) );
    print( shape(<0:5>, 4, 2)' );
}
```

produces

```
      0.00000       2.0000       4.0000      0.00000
       1.0000       3.0000       5.0000      0.00000

      0.00000       1.0000       2.0000       3.0000
       4.0000       5.0000      0.00000      0.00000
```

sin, sinh

`sin(const ma);`
`sinh(const ma);`

`ma`	in:	arithmetic type

Return value

`sin` returns the sine of `ma`, of double or matrix type.
`sinh` returns the sine hyperbolicus of `ma`, of double or matrix type.

See also

`acos` (for examples), `asin`, `atan`, `cos`, `cosh`, `sinh`, `tan`, `tanh`

sizec, sizeof, sizer, sizerc

```
sizec(const ma);
sizeof(const ma);
sizer(const ma);
sizerc(const ma);
```

Return value

Returns an integer value which is the number of elements in the argument:

type	rows sizeof	columns	sizer	sizec	sizerc
int, double	0	0	1	1	1
$m \times n$ matrix	m	n	m	n	$m \times n$
string, length c	c	c	1	c	c
array, length c	c	c	1	c	c
file ($r \times c$)	r	c	c	c	$r \times c$
other	0	0	0	0	0

A file type variable only has dimensions if it was opened using the `'f'` format.

See also

`columns`, `rows`

Example

```
#include <oxstd.oxh>
main()
{
    decl i, d, m, s, a, res;
    i = 0; d = 0.0;
    m = unit(3,2);
    s = "aap", a = {"a", "b"};
    res = columns(i)~rows(i)~sizec(i)~sizer(i)~sizerc(i)
        | columns(d)~rows(d)~sizec(d)~sizer(d)~sizerc(d)
        | columns(m)~rows(m)~sizec(m)~sizer(m)~sizerc(m)
        | columns(s)~rows(s)~sizec(s)~sizer(s)~sizerc(s)
        | columns(a)~rows(a)~sizec(a)~sizer(a)~sizerc(a);

    print("%r",
          {"int","double","matrix[3][2]","string[3]","array[2]"},
          "%c",
          {"columns","rows","sizec","sizer","sizerc"},
          "%8.1g", res);
}
```

produces:

```
             columns    rows   sizec   sizer  sizerc
int                0       0       1       1       1
double             0       0       1       1       1
matrix[3][2]       2       3       2       3       6
string[3]          3       3       3       1       3
array[2]           2       2       2       1       2
```

solveldl

`solveldl(const ml, const md, const mb);`

`ml`	in:	$m \times m$ lower diagonal matrix L, $LDL' = A$
`md`	in:	$1 \times m$ matrix with reciprocals of D
`mb`	in:	$m \times n$ matrix B, the right-hand side

Return value

Returns the $m \times n$ matrix X from solving $AX = B$.

Description

Solves $AX = B$ for X following a square root free Choleski decomposition of A using `decldl` (A is symmetric and positive definite).

See also

`decldl` (for an example), `invertsym`

solveldlband

`solveldlband(const ml, const md, const mb);`

`ml`	in:	$p \times m$ vector specifying the L^b matrix
`md`	in:	$1 \times m$ matrix with reciprocals of D
`mb`	in:	$m \times n$ matrix B, the right-hand side

Return value

Returns the $m \times n$ matrix X from solving $AX = B$.
If `md` is the empty matrix, the return value is $m \times n$ matrix $X = L^{-1}B$.

Description

Solves $AX = B$ for X when A is a symmetric positive definite band matrix. A^b, the band form of A, must have been decomposed using `decldlband` first. See under `decldlband` for the storage format of A^b and examples to move between A^b and A.

See also

`decldlband` (for an example), `solvetoeplitz`

solvelu

`solvelu(const ml, const mu, const mp, const mb);`

`ml`	in:	$m \times m$ lower diagonal matrix L (use 0 to indicate absence of L)
`mu`	in:	$m \times m$ upper diagonal matrix U (use 0 to indicate absence of U)
`mp`	in:	$2 \times m$ matrix with interchange permutations in the second row (use 0 to indicate absence of permutations)
`mb`	in:	$m \times n$ matrix B, the right-hand side

Return value

Returns the $m \times n$ matrix X from solving $AX = B$, where A is supplied in decomposed form.

Description

Solves $AX = B$ for X following a LU decomposition of A using `declu`: $PA = LU$, where L is lower diagonal and U upper diagonal. First $LW = PB$ is solved for W by forward substitution, then $W = UX$ is solved for x by backward substitution. When a diagonal element of L or U is zero, the corresponding element of X will be set to zero.

This function may be used to only do the forward or backward substitution part:

`solvelu(L,0,0,B)` solves $LX = B$,
`solvelu(0,U,0,B)` solves $UX = B$.

So can be used to invert a triangular matrix.

See also

`declu` (for an example), `invert`

solvetoeplitz

`solvetoeplitz(const mr, const cm, const mb);`
`solvetoeplitz(const mr, const cm, const mb, alogdet);`

`mr`	in:	double, or $r \times 1$ or $1 \times r$ matrix, specifying the symmetric positive definite (band) Toeplitz matrix
`cm`	in:	dimension of complete Toeplitz matrix: $m \times m, m \geq r$
`mb`	in:	$m \times n$ matrix B, the right-hand side
`alogdet`	in:	(optional argument) address of variable
	out:	double, the *logarithm* of (the absolute value of) the determinant of A

Return value

Returns the $m \times n$ matrix X from solving $AX = B$, or 0 if the Toeplitz matrix is singular.

Description

Solves $AX = B$ for X when A is symmetric Toeplitz. A Toeplitz matrix has the same values along each diagonal (see under `toeplitz`). The algorithm is based on the Levinson algorithm in Golub and Van Loan (1989, algorithm 4.7.2, page 187). The algorithm also accepts a non-positive (non-singular) Toeplitz matrix, but note that it computes $\log[\mathrm{abs}(|A|)]$ for the optional third argument. The exponent of that can only be computed for values $\leq$ `DBL_MAX_E_EXP` and $\geq$ `DBL_MIN_E_EXP` (see Ch. 9).

See also

`pacf`, `toeplitz`

Example

```
#include <oxstd.oxh>
main()
{
    decl ct = 10, mb, mt, mx;

    mb = <2;3;4;5;6>;
    mx = solvetoeplitz(<3,.5,.2,.1>, 5, mb);
    print(mx');
    mx = invertsym( toeplitz(<3,.5,.2,.1>,5) ) * mb;
    print(mx');
}
```

produces

```
    0.46189      0.63974      0.88536      1.1737      1.7240
    0.46189      0.63974      0.88536      1.1737      1.7240
```

sortbyc, sortbyr

```
sortbyc(const ma, const icol);
sortbyr(const ma, const irow);
```

`ma`	in:	matrix
`icol`	in:	scalar: index of column to sort, or matrix: specifying the columns to sort by.
`irow`	in:	index of row to sort

Return value

The reordered (sorted in ascending order) matrix.

Description

The `sortbyc` function sorts the rows of a matrix according to the specified column; `sortbyr` sorts the columns of a matrix according to the specified row. Sorting is in ascending order using combsort (Lacey and Box, 1991).

If you want the sorting to be in descending order, you can use `reversec` after `sortbyc`, and `reverser` after `sortbyr`.

The `sortbyc` function can also sort on multiple columns. In that case specify a vector of columns on which to sort. The sorting is on the first specified column, within that on the second, etc. The elements in the `icol` argument when it is a matrix are processed by row, so corresponding to `vecr(icol)`.

See also

`reversec`, `reverser`, `sortc`, `sortr`

Example

```
#include <oxstd.oxh>
main()
{
    decl m = <1,0,3;0,4,4;4,3,0>;
    print( sortbyc(m,0), sortbyr(m,0) );

    m = <1,3;1,2;3,4;3,5;2,3;2,2>;

    print("%4.1g", m ~ sortbyc(m, 0) ~ sortbyc(m, 0~1));
}
```

produces

```
     0.00000       4.0000       4.0000
      1.0000      0.00000       3.0000
      4.0000       3.0000      0.00000

     0.00000       1.0000       3.0000
      4.0000      0.00000       4.0000
      3.0000       4.0000      0.00000

   1   3   1   3   1   2
   1   2   1   2   1   3
   3   4   2   2   2   2
   3   5   2   3   2   3
   2   3   3   5   3   4
   2   2   3   4   3   5
```

sortc, sortcindex, sortr

```
sortc(const ma);
sortr(const ma);
sortcindex(const mb);
```

`ma`	in:	matrix, array or string
`mb`	in:	row vector, column vector, array or string

Return value

If `ma` is a matrix, the return value is `ma` with each column (`sortc`) or row (`sortr`) sorted in ascending order. If `ma` is scalar the return type and value are that of `ma`.

If `ma` is an array of strings, the strings are sorted in increasing order (all non-string entries are pushed to the end, and will be in reverse order). If `ma` is a string, the string is returned unchanged.

The sorting method used is combsort.

The `sortcindex` returns a column vector with the sorted index which results from applying `sortc(mb)` (so `v[sortcindex(v)]` equals `sortc(v)`). A matrix argument to `sortcindex` must be a column vector or a row vector (the transpose is used in the latter case, so `sortcindex(v)` and `sortcindex(v')` are the same).

Applying `sortcindex` twice, as in `sortcindex(sortcindex(v))`, returns the ranking.

See also

`sortbyc`, `sortbyr`

Example

```
#include <oxstd.oxh>
main()
{
    decl m = <1,0,3;0,4,4;4,3,0>;
    print( sortc(m), sortr(m) );
    print( sortcindex(m[0][]) );
    print( sortc( {"x", "", 2, "aa", 1} ) );
}
```

produces

```
      0.00000      0.00000      0.00000
       1.0000       3.0000       3.0000
       4.0000       4.0000       4.0000

      0.00000       1.0000       3.0000
      0.00000       4.0000       4.0000
      0.00000       3.0000       4.0000

       1.0000
      0.00000
       2.0000

[0] =
[1] = aa
[2] = x
[3] = 1
[4] = 2
```

spline

```
spline(const my, const mx, const alpha);
spline(const my, const mx, const alpha, agcv);
```

`my`	in:	$T \times n$ matrix with variables (observations in columns) to smooth
`mx`	in:	0 for evenly spaced Y, else $T \times m$ matrix with X (where $m = 1$: same X used for all Ys, or $m = n$: corresponding X is used with Y)
`alpha`	in:	double, bandwidth α (also see below), 0: automatic bandwidth selection using GCV, < 0: absolute value is bandwidth, > 0: specifies equivalent number of parameters.
`agcv`	in:	(optional) address, returns GCV (generalized cross validation score) and k_e (equivalent number of parameters)

Return value

Returns a $T \times n$ matrix with the smooth from applying the natural cubic spline.
The optional `agcv` argument is a $2 \times n$ matrix, with the generalized cross validation (GCV) score in the first row, and the equivalent number of parameters in the second.

Description

The spline smoothes the cross plot of Y against time (`mx` argument is 0), or against an x variable. Consider a plot of y_t, against x_t, and sort the data according to x: $a < x_{[1]} < \ldots < x_{[T]} < b$. In a spline model, the sum of squared deviations from a function g is minimized, subject to a roughness penalty:

$$\min \sum_{t=1}^{T} \left[y_t - g\left(x_{[t]}\right)\right]^2 + \alpha \int_a^b \left[g''\left(x\right)\right]^2 \mathrm{d}x.$$

Ox uses a *natural cubic spline*, which is cubic because the function g is chosen as a third degree polynomial, and natural because the smooth is a straight line between a and $x_{[1]}$ and between $x_{[1]}$ and b. Two good references on splines and nonparametric regression are Green and Silverman (1994) and Hastie and Tibshirani (1994).
The α parameter is the bandwidth: the smaller α, the lower the roughness penalty, and hence the closer the smooth will track the actual data.
There are three ways of specifying the bandwidth α :

0 use automatic bandwidth selection based on GCV;
The GCV criterion is computed as:

$$GCV(\alpha) = T\left(\frac{RSS}{T - 1.25k_e + 0.5}\right).$$

A bracketing search algorithm is used to minimize GCV.

< 0 the absolute value is used for the bandwidth;
No iteration is required.

> 0 specifies the equivalent number of parameters k_e to be used.
A bracketing search algorithm is used to locate the specified k_e (k_e is approximately comparable to the number of regressors used in a linear regression)

The spline is evaluated at the data points, where missing y_t values (both in and outside sample) are estimated by the fit from the smooth. Observations where both y_t and x_t are missing are omitted in the calculations. The missing values used are .NaN.

The spline procedure handles ties in the x variable. The algorithm used to compute the spline is of order T, and consists of the Reinsch algorithm combined with the Hutchinson-de Hooch algorithm for computing the GCV score (see Green and Silverman, 1994, Chs. 2 & 3).

For evenly spaced data (e.g. cross plot against time), a natural cubic spline is very close to the Hodrick–Prescott filter which is popular in macro-economics. By default, the Hodrick–Prescott filter uses a bandwidth of 1600, in which case the smoothers from both methods are virtually identical. Also see the OxMetrics book.

See also

`lib/Spline3w.ox`, `lib/HPfilter.ox`,

Example

The following example first smoothes the four variables in the variable `my` using time as the X variable, and automatic bandwidth selection. The second observation of the first variable is set to a missing value.

The second spline smoothes the cross plot of the last three variables against the first, choosing the bandwidth as 12 equivalent parameters.

```
#include <oxstd.oxh>
#include <oxfloat.oxh>

main()
{   decl my, ms, gcv;

    my = loadmat("data/data.in7");
    my[1][0] = M_NAN;
    ms = spline(my, 0, 0);
    print( "%c", {"CONS", "smooth"}, my[:4][0] ~ ms[:4][0]);

    ms = spline(my[][1:], my[][0], 12, &gcv);
    print( "%r", {"GCV", "k_e"}, gcv);
}
```

produces

```
         CONS       smooth
       890.45       890.01
         .NaN       888.19
       886.33       886.58
       884.88       885.38
       885.25       884.66

GCV                 13.932       1.4645       24.309
k_e                 12.000       11.999       11.999
```

sprint

```
sprint(const a, ...);
```

`a`	in:	any type
`...`	in:	any type

Return value

Returns a string containing the written text, or 0 if the sprint buffer was too small (see `sprintbuffer`).

Description

Each argument is printed to a string. See `print` for a description of formatting. There is a maximum text length: this is documented under `sprintbuffer`.

Error and warning messages

sprint(): no string buffer
sprint(): string buffer length exceeded

See also

`eprint`, `print`, `sprintbuffer`

Example

```
#include <oxstd.oxh>
main()
{
    decl s = sprint("a", "_", "%0X", 10);
    print( s );
}
```

produces: `a_A`

sprintbuffer

```
sprintbuffer(const len);
```

`len`	in:	int

Return value

Returns 0 of type int.

Description

Sets the size of the internal sprint buffer. The default is 16×1024 characters, and this function is only needed if a larger buffer is needed for `sprint`.

See also

`sprint`

sqr, sqrt

`sqr(const ma);`
`sqrt(const ma);`

`ma`	in:	arithmetic type

Return value

`sqrt` returns the square root of the elements of `ma`, of double or matrix type.

`sqr` returns the square of the elements of `ma`. If the input to `sqr` is a double or matrix, the return type is a double or matrix. If the input is an integer, the return type is integer unless the result would overflow in integer computation. In that case the return type is double in order to represent the result.

Example

```
#include <oxstd.oxh>
main()
{
    print( sqrt(<2,3>), <2,3> .^ 0.5 );
    print( sqr(<2,3>), <2,3> .^ 2 );

    println( sqr(2^14), isint(sqr(2^14)) ? " int" : " double");
    println( sqr(2^15), isint(sqr(2^15)) ? " int" : " double");
    println( pow(2,15), isint(pow(2,15)) ? " int" : " double");
}
```

produces

```
       1.4142       1.7321
       1.4142       1.7321
       4.0000       9.0000
       4.0000       9.0000
268435456 int
1.07374e+009 double
32768 double
```

See also

`pow`, `^` `.^` (§13.8.3)

sscan

```
sscan(const string, const a, ...);
sscan(const astring, const a, ...);
```

`string`	in:	string to scan from
`astring`	in:	address of string to scan from, on return the scanned text has been removed from the string
`a`	in:	any type
...	in:	any type

Return value

Returns the number of arguments successfully scanned and assigned. If `s` is a string, then `sscan(s,...` will leave the string unchanged, whereas `sscan(&s,...` will remove the read characters from the string. Returns -1 when at the end of the string.

Description

This function works as `fscan`, but reading from a string, not a file. See `fscan` for a description of formatting; the `"%#m"` and `"%#M"` formats may not be used in `sscan`.

See also

`fscan`, `fwrite`, `scan`

Example

The following example (samples/inout/inout5.ox) reads one input line at a time (leading spaces in each line are skipped, because of the starting space in `" %z"`), and reads from that string using `sscan`. The `*` in `"%*d"` suppresses assignment, so the integer is skipped in the file.

```
#include <oxstd.oxh>
main()
{
    decl file, s, c;
    decl svar, address;

    file = fopen("data/data.in7");
    if (!isfile(file))
    {
        print("failed to open file\n");
        exit(1);
    }
    do
    {   c = fscan(file, " %z", &s);
        if (c > 0 && s[0] == '>')
        {
            sscan(&s, ">%s", &svar, "%*d", "%*d", "%*d",
                "%*d", "%*d", "%d", &address, " ");
            println("variable : ", svar, " address:", address);
            println("remainder: ", s);
        }
    } while (c > 0);

    fclose(file);
}
```

If the .in7 file can be found, this program produces:

```
variable : CONS address:32
```

```
remainder: data 10-04-1992 13:20:38.33
variable : INC address:1336
remainder: data 10-04-1992 13:20:38.33
variable : INFLAT address:2640
remainder: data 10-04-1992 13:20:38.33
variable : OUTPUT address:3944
remainder: data 10-04-1992 13:20:38.33
```

standardize

`standardize(const ma);`

`ma`	in:	$T \times n$ matrix A

Return value

Returns a $T \times n$ matrix holding the standardized columns of `ma`. If any variance is $\leq 10^{-20}$, then the corresponding column is set to 0.

Description

Standardization implies subtracting the mean, and then dividing by the standard deviation. A standardized vector has mean zero and variance one.

See also

`correlation` (for an example), `meanc`, `meanr`, `varc`, `varr`, `variance`

string

`string(const ma);`

`ma`	in:	arithmetic type

Return value

Casts the argument to a string, see §13.8.2.3.

See also

`double`, `sprint` (for printing to a string)

strfind, strfindr, strifind, strifindr

```
strfind(const where, const what);
strfindr(const where, const what);
strifind(const where, const what);
strifindr(const where, const what);
```

Return value

where	what	return type (-1 if not found)
array of strings	array of c string	$1 \times c$ matrix with indices of occurrence
array of strings	string	int: index of occurrence of string `what`
string	string	int: index of occurrence of substring `what`
string	$r \times c$ matrix with character values	$1 \times rc$ matrix with indices of occurrence (-1 if not found)
string	character	int: index of occurrence of character `what`

Example

```
#include <oxstd.oxh>
main()
{
    decl as1 = {"aa", "bb", "cc", "cc"};
    decl as2 = {"cc", "dd", "aa"};

    print("index = ", strfind(as1, "cc"), "\n",
          "index = ", strfindr(as1, "cc"), "\n",
          "index = ", strfind(as1, "ee"), "\n",
          "index = ", strfind(as1,  as2));
    println("first ox is at position ", strfind("ooxox", "ox"),
        " in \"ooxox\"");
    println("last  ox is at position ", strifindr("oOXoX", "ox"),
        " in \"oOXoX\" (no case)");
    println("x is at position ", strfind("ox", 'x'), " in \"ox\"");
    println("x is at position ", strfind("OX", 'x'), " in \"OX\"");
    println("x is at position ", strifind("OX", 'x'),
        " in \"OX\" (no case)");
    println("index of x,o in \"OX\" (no case):",
        strifind("OX", 'x'~'o'));
}
```

produces (remember that the first entry has index 0):

```
index = 2
index = 3
index = -1
index =
       2.0000      -1.0000      0.00000
first ox is at position 1 in "ooxox"
last  ox is at position 3 in "oOXoX" (no case)
x is at position 1 in "ox"
x is at position -1 in "OX"
x is at position 1 in "OX" (no case)
index of x,o in "OX" (no case):
       1.0000      0.00000
```

strlwr, strtrim, strupr

```
strlwr(const s);
strtrim(const s);
strupr(const s);
```

`s`	in:	the strings to convert

Return value

Returns a copy of the string, which is converted to lower case (`strlwr`) or uppercase (`strupr`). `strtrim` returns the string with leading and trailing white space removed.

Example

```
#include <oxstd.oxh>
main()
{
    decl s = "A StrinG\n";
    print( strlwr(s), strupr(s), s);
    s = "  aa  bb \t\n";
    print( "{", strtrim(s), "}");
}
```

produces

```
a string
A STRING
A StrinG
{aa bb}
```

submat

```
submat(const ma, const r1, const r2, const c1, const c2);
```

`ma`	in:	matrix
`r1,r2`	in:	int
`c1,c2`	in:	int

Return value

Returns the submatrix of `ma` from row indices `r1` to `r2` and column indices `c1` to `c2`. This is equivalent to `ma[r1:r2][c1:c2]`, apart from that indices below the lower bound are set to the lower bound, and indices above the upper bound set to the upper bound.

See also

`selectrc`, `[]` (§13.8.2.4)

sumc, sumr

`sumc(const ma);`
`sumr(const ma);`

`ma`	in:	$T \times n$ matrix A

Return value

`sumc` returns a $1 \times n$ matrix with the sum of the column elements of `ma`.

The `sumr` function returns a $T \times 1$ matrix with the sum of the row elements of `ma`.

See also

`meanc`, `meanr`, `prodc`, `prodr`, `sumsqrc`, `sumsqrr`, `varc`, `varr`

Example

```
#include <oxstd.oxh>
main()
{
    print( sumc(<0:3;1:4;2:5>) | sumsqrc(<0:3;1:4;2:5>));
    print( sumr(<0:3;1:4;2:5>) ~ sumsqrr(<0:3;1:4;2:5>));
}
```

produces

```
       3.0000       6.0000       9.0000       12.000
       5.0000       14.000       29.000       50.000

       6.0000       14.000
       10.000       30.000
       14.000       54.000
```

sumsqrc, sumsqrr

`sumsqrc(const ma);`
`sumsqrr(const ma);`

`ma`	in:	$T \times n$ matrix A

Return value

The `sumsqrc` function returns a $1 \times n$ matrix with the sum of the squares of the column elements of `ma`.

The `sumsqrr` function returns a $T \times 1$ matrix which holds the sum of the squares of the row elements of `ma`.

See also

`sumc` (for an example), `sumr`, `varc`, `varr`

systemcall

`systemcall(const s);`

`s`	in:	string with system command

Return value

Returns the exit code from the system call.

Description

Performs a operating system call, waiting for the call to finish.

For example, `systemcall("dir")` lists the directory under Windows, while `systemcall("ls")` does this under Linux. The output of the command is echoed to the standard output console.

See also

`chdir`, `getcwd`, `getenv`

tailchi, tailf, tailn, tailt

`tailchi(const ma, const df);`
`tailf(const ma, const df1, const df2);`
`tailn(const ma);`
`tailt(const ma, const df);`

`ma`	in:	arithmetic type
`df`	in:	arithmetic type, degrees of freedom
`df1`	in:	arithmetic type, degrees of freedom in the numerator
`df2`	in:	arithmetic type, degrees of freedom in the denominator

Return value

Returns the requested tail probabilities at `ma` (between zero and one):

`tailchi`	tail probabilities from $\chi^2(df)$ distribution
`tailf`	tail probabilities from $F(df1, df2)$ distribution
`tailn`	one-sided standard normal tail probability
`tailt`	one-sided tail probabilities from student-$t(df)$ distribution

The tail probabilities are accurate to about 10 digits. The return type is as follows:

returns	ma	degrees of freedom arguments
$m \times n$ matrix	$m \times n$ matrix	scalar (int for `tailt`)
$m \times n$ matrix	scalar	$m \times n$ matrix
$m \times n$ matrix	$m \times n$ matrix	$m \times n$ matrix
double	scalar	scalar (int for `tailt`)

See also

`dens...`, `prob...`, `quan...`

Example

```
#include <oxstd.oxh>
main()
{
    print("%r", {"chi(2):"}, tailchi(<0,4.61,5.99>, 2));
    print("%r", {"normal:"}, tailn(<-1.96, 0, 1.96>) );
    print("%r", {"t(4):  "}, tailt(<-1.96, 0, 1.96>, 4) );
    print("%r", {"t(50): "}, tailt(<-1.96, 0, 1.96>, 50) );
}
```

produces

```
chi(2):          1.0000     0.099759     0.050037
normal:         0.97500      0.50000     0.024998
t(4):           0.93922      0.50000     0.060777
t(50):          0.97221      0.50000     0.027790
```

tan, tanh

`tan(const ma);`
`tanh(const ma);`

`ma`	in:	arithmetic type

Return value

`tan` returns the tangent of `ma`, of double or matrix type.

`tanh` returns the tangent hyperbolicus of `ma`, of double or matrix type.

See also

`acos` (for examples), `asin`, `atan`, `cos`, `cosh`, `sin`, `sinh`, `tanh`

thinc, thinr

```
thinc(const ma, const c);
thinr(const ma, const r);
```

`ma`	in:	$m \times n$ matrix A
`c`	in:	int, desired number of columns to extract
`r`	in:	int, desired number of rows to extract

Return value

The `thinc` function returns an $m\times$ `c` matrix consisting of a selection of columns of the original matrix.

The `thinr` function returns an `r` $\times n$ matrix consisting of a selection of rows of the original matrix.

Description

The `thinc` function selects columns as follows:

$$0,\ g,\ 2g,\ 3g,\ldots,\ (\mathtt{c}-1)g,$$

$$\text{where}\quad g = 1 + \text{int}\left(\frac{n-\mathtt{c}}{\mathtt{c}-1}\right)\ \text{if } \mathtt{c} > 1.$$

The `thinr` function selects rows similarly.

The example below also indicates how to draw a random sample.

See also

`aggregatec`, `aggregater`

Example

Note in the example that, strictly speaking, it is not necessary to truncate the random indices in `idx`, as this is done automatically when using a matrix to index another matrix.

```
#include <oxstd.oxh>
main()
{
    decl m = rann(1000, 2), idx;
    print( thinr(m, 3) ~ m[<0,499,998>][] );
    print( thinc(m', 3)' ~ m[<0,499,998>][] );
                /* get three random indices in idx */
    idx = trunc(ranu(1,3) * rows(m));
    print(idx, m[idx][] ~ m[sortr(idx)][] );
}
```

produces

```
      0.22489       1.7400      0.22489       1.7400
     -0.21417      -1.0037     -0.21417      -1.0037
     0.084549      0.83591     0.084549      0.83591

      0.22489       1.7400      0.22489       1.7400
     -0.21417      -1.0037     -0.21417      -1.0037
     0.084549      0.83591     0.084549      0.83591

       408.00       852.00       877.00

       1.9639     0.073371       1.9639     0.073371
      0.25375      -1.2006      0.25375      -1.2006
      -1.1932     -0.52929      -1.1932     -0.52929
```

time

`time();`

Return value

A string holding the current time.

See also

`date` (for an example)

timeofday

```
timeofday();
timeofday(const index);
timeofday(const hours, const minutes);
timeofday(const hours, const minutes, const seconds);
timeofday(const hours, const minutes, const seconds, const h100s);
```

`index`	in:	in: arithmetic type, calendar index of a certain date with fractional time
`hours`	in:	arithmetic type, hours on 24-hour clock
`minutes`	in:	arithmetic type, minutes
`seconds`	in:	arithmetic type, seconds
`h100s`	in:	arithmetic type, hundreds

Return value

The `timeofday` function without any arguments returns the fraction of the calendar index representing the current time.

The `timeofday` function with two or more arguments returns the fraction of the calendar index of the specified time (see below). If all arguments are an integer, the return value will be an integer.

The `timeofday` function with one argument takes a calendar index (or vector of indices), as argument, returning a $n \times 4$ matrix with the quadruplet hours, minutes, seconds, hundreds in each row (n is the number of elements in the input).

Description

The calendar index is the Julian day number, with an optional fractional part to specifies the fraction of the day: 2453402.75 corresponds to 2005-01-01T18:00. If the day number is zero, it is interpreted as a time only, so 0.75 is just 18:00 (6 PM). The `"%C"` print format is available to print or scan a calendar index.

See also

`dayofcalendar`, `print`, `timing`

Example

```
#include <oxstd.oxh>
main()
{
    decl timeidx = range(0,4)' / 6 + range(0,4)' / 360;

    println("%cf", {"%5.0f","%5.0f","%5.0f","%5.0f","; %20C"},
        timeofday(timeidx) ~ timeidx + dayofcalendar(2005,1,1) );

    println("time today ", "%C", timeofday());
}
```

produces

```
    0    0    0    0;          2005-01-01
    4    4    0    0;  2005-01-01T04:04:00
    8    8    0    0;  2005-01-01T08:08:00
   12   12    0    0;  2005-01-01T12:12:00
   16   16    0    0;  2005-01-01T16:16:00
time today 22:11
```

timer, timespan

```
timer();
timespan(const time);
timespan(const time, const time0);
```

`time`	in:	double, value from previous call to `timer`
`time0`	in:	double, (optional argument) start time

Return value

The `timer` function returns a double representing the current elapsed processor time in one 100th of a second. (Under Windows this is the elapsed time since the process started; under Linux/Unix, it is the CPU time used so far, ignoring time taken by other processes.)

The `timespan(time)` function with returns a string holding the processor time lapsed since the `time` argument.

The `timespan(time, time0)` function with returns a string holding the time lapsed between `time` and `time0`. Both arguments must be measured in one 100th of a second.

See also

`today`

Example

```
#include <oxstd.oxh>
main()
{
    decl i, time, m = rann(100,10), m2;

    time = timer();

    for (i = 0; i < 1000; ++i)
        m2 = m'm;

    print("time lapsed: ", timespan(time), "\n");
    print("or in seconds: ", (timer() - time) / 100, "\n");
    print("time lapsed: ", timespan(time, timer()), "\n");
}
```

prints the time it took to do the for loop.

timestr, timing, today

```
timestr(const time);
timing(const mdates);
timing(const mtimes, const mode);
today();
```

`time`	in:	double, date expressed as number of seconds since 1 January 1970 at 00:00:00 (e.g. a value from `timing`)
`mdates`	in:	$T \times k$ matrix with date and time, in order: year, month, day, hour, minute, second (see below).
`mtimes`	in:	$m \times n$ matrix with dates expressed in seconds.
`mode`	in:	int, 0 (or absent): convert date/time to seconds; 1: convert seconds to date/time; 2: convert seconds to calendar index as used in `dayofcalendar` and `timeofday`.

Return value

The `timing` function with `mode` 0 (or no mode specified) converts a $T \times k$ matrix of year, month, ..., seconds (see below) to a $T \times 1$ vector with the date/time expressed as the number of seconds since 1 January 1970 at 00:00:00.

The `timing` function with `mode` 1 converts an $m \times n$ matrix of seconds, returning an $mn \times 6$ matrix with respectively year, month, day, hour, min, sec in the columns.

The `timing` function with `mode` 2 converts an $m \times n$ matrix (or a single double) of seconds to calendar indices, returning an $m \times n$ matrix (or a double).

The `timestr` function returns the date/time expressed as a text string: `"year-month-day hour:min:sec"`. The time is omitted if it is 00:00:00.

The `today` function returns a double with the current date/time expressed in seconds.

Description

These functions work with time in seconds: the number of seconds since 1 January 1970 at 00:00:00. This is more restrictive and less convenient than the calendar index (with fraction for time) that is used in `dayofcalendar` and `timeofday`.

The input matrix for `timing` with `mode` 0 (or no mode specified) has a specified data and time in each row, with the columns organized as:

column	item	values
0	year	full year (e.g. 1970)
1	month	month in year, 1 ... 12 (e.g. 2 for February)
2	day	day in month, 1 ... 31
3	hour	hour in day, 0 ... 23
4	min	minutes, 0 ... 59
5	sec	seconds, 0 ... 59

The actual input matrix may have fewer columns, in which case the remainder is assumed to be zero (one for month and day).

See also

dayofcalendar, `timeofday`, `timer`

Example

```
#include <oxstd.oxh>
main()
{
    decl time1, time2;

    time1 = timing(<1990, 12, 1; 1991, 1, 1>);
    time2 = timing(<1990, 12, 1, 12, 0, 1>);

    println("time1[0]: ", timestr(time1[0]));
    println("time1[1]: ", timestr(time1[1]));
    println("time2:    ", timestr(time2));
    println("today:    ", timestr(today()));
    println("today:    ", "%6.0f", timing(today(), 1));
    println("today:    ", "%C", timing(today(), 2));
}
```

which produces as output:

```
time1[0]: 1990-12-01
time1[1]: 1991-01-01
time2:    1990-12-01 12:00:01
today:    2012-06-26 14:27:44
today:
  2012     6    26    14    27    44
today:    2012-06-26T14:27:44
```

toeplitz

```
toeplitz(const ma);
toeplitz(const ma, const cm);
```

`ma`	in:	double, or $r \times 1$ or $1 \times r$ matrix
`cm`	in:	(optional argument) m: dimension of matrix to be created, $m \geq r$; if the argument is missing, $m = r$ is used.

Return value

Returns a symmetric Toeplitz matrix.

Description

Creates a symmetric Toeplitz matrix using the supplied argument. A Toeplitz matrix has the same values along each diagonal. Here we allow for a banded Toeplitz matrix, e.g. when $r = 3$ and $m = 5$:

$$\begin{pmatrix} a_0 & a_1 & a_2 & 0 & 0 \\ a_1 & a_0 & a_1 & a_2 & 0 \\ a_2 & a_1 & a_0 & a_1 & a_2 \\ 0 & a_2 & a_1 & a_0 & a_1 \\ 0 & 0 & a_2 & a_1 & a_0 \end{pmatrix}$$

When the bandwith equals the dimension (i.e. there are no zeros: $m = r$), we write $\mathcal{T}(a_0, a_1, \ldots, a_{m-1})$ for the Toeplitz matrix.

See also

`diag`, `pacf`, `solvetoeplitz` (for an example)

trace

```
trace(const ma);
```

`ma`	in:	arithmetic type

Return value

Returns the trace of `ma` (the sum of its diagonal elements). Return type is double.

See also

`determinant`

Example

```
#include <oxstd.oxh>
main()
{
    print( trace(<2,1;1,4>) );
}
```

produces: 6

trunc, truncf

```
trunc(const ma);
truncf(const ma);
```

`ma`	in:	arithmetic type

Return value

`trunc` returns the truncated elements of `ma`, of double or matrix type.
`truncf` is fuzzy truncation.

Description

Truncation is rounding towards zero, however, the result remains a double value. Note that conversion to an integer also results in truncation, but that in that case the result is undefined if the real number is too big to be represented as an integer.

`truncf` multiplies positive numbers by one plus the current fuzziness (one minus fuzziness for negative numbers) before truncation.

See also

`ceil`, `floor`, `fuzziness`, `round`,

Example

```
#include <oxstd.oxh>
main()
{
    print( trunc(<-2.0-1e-15, -2.0+1e-15, 2.0-1e-15, 2.0+1e-15>));
    print(truncf(<-2.0-1e-15, -2.0+1e-15, 2.0-1e-15, 2.0+1e-15>));
}
```

produces

```
      -2.0000      -1.0000       1.0000       2.0000
      -1.0000      -1.0000       2.0000       2.0000
```

union, unique

```
union(const ma);
unique(const ma);
```

`ma`	in:	matrix
`mb`	in:	matrix

Return value

`unique` returns the sorted unique elements of `ma` as a row vector.

`union` returns a row vector with the sorted unique elements of `ma` and `mb` combined. Returns an empty matrix if the result is empty. Missing values are skipped.

See also

`exclusion` (for an example), `intersection`

unit

```
unit(const rc);
unit(const r, const c);
```

`rc`	in:	int
`r`	in:	int
`c`	in:	int

Return value

Returns an `rc` by `rc` identity matrix (one argument), or a `r` by `c` matrix with ones on the diagonal (rest zero).

See also

`constant`, `unit`, `zeros`

Example

```
#include <oxstd.oxh>
main()
{
    print( unit(2) );
}
```

produces

```
      1.0000      0.00000
     0.00000       1.0000
```

unvech

```
unvech(const va);
```

`va`	in:	arithmetic type, (column or row) vector to make into symmetric matrix

Return value

Returns a symmetric matrix, given the vectorized lower diagonal of a symmetric matrix.

Description

Undoes the `vech` operation.

See also

`vech` (for an example)

upper

```
upper(const ma);
```

`ma`	in:	$m \times n$ matrix

Return value

Returns the upper diagonal (including the diagonal), i.e. returns a copy of the input matrix with strict lower-diagonal elements set to zero.

See also

`lower` (for an example), `setdiagonal`, `setlower`, `setupper`

va_arglist

```
va_arglist();
```

Return value

Returns an array holding the arguments starting with the first variable in the variable argument list.

Description

See §13.5.5.5.

Example

```
#include <oxstd.oxh>
test(const a, ...)
{
    decl i, args = va_arglist();

    println("number of extra arguments: ", sizeof(args));
    for (i = 0; i < sizeof(args); i++)
        println("vararg [", i, "] = ", args[i]);
}
main()
{
    test("tinker", "tailor", "soldier");
}
```

which prints

```
number of extra arguments: 2
vararg [0] = tailor
vararg [1] = soldier
```

varc, varr

`varc(const ma);`
`varr(const ma);`

`ma`	in:	$T \times n$ matrix A

Return value

The `varc` function returns a $1 \times n$ matrix with the variances of the columns of `ma`.

The `varr` function returns a $T \times 1$ matrix holding the variances of the rows of `ma`.

Description

The variance of $x_t, t = 1, \ldots T$ is computed as:

$$\frac{1}{T}\sum_{t=1}^{T}(x_t - \bar{x})^2, \quad \text{where } \bar{x} = \frac{1}{T}\sum_{t=1}^{T} x_t.$$

See also

`meanc`, `meanr`, `sumc`, `sumr`, `variance`

Example

```
#include <oxstd.oxh>
main()
{
    decl m1 = rann(100,2), m2;
    print( variance(m1), varc(m1) | varr(m1') ' );
}
      1.0356    -0.037133
   -0.037133      0.86569

      1.0356      0.86569
      1.0356      0.86569
```

variance

`variance(const ma);`

`ma`	in:	$T \times n$ matrix A

Return value

Returns an $n \times n$ matrix holding variance-covariance matrix of `ma`.

Description

The variance-covariance matrix of a $T \times n$ matrix $A = (a_0, \ldots, a_{n-1})$ is:

$$T^{-1}\breve{A}'\breve{A}, \text{ where } \breve{A} = (a_0 - \bar{a}_0, \ldots a_{n-1} - \bar{a}_{n-1}), \text{ and } \bar{a}_i = \frac{1}{T}\sum_{t=0}^{T-1} a_{it}.$$

See also

`acf`, `correlation`, `meanc`, `meanr`, `standardize`, `varc`, `varr`

Example

```
#include <oxstd.oxh>
main()
{
    decl m1 = rann(100,2), m2 = m1 - meanc(m1);
    print( variance(m1), m2'm2/rows(m2) );
}
      1.0356    -0.037133
   -0.037133      0.86569

      1.0356    -0.037133
   -0.037133      0.86569
```

vec

```
vec(const ma);
```

ma	in:	arithmetic type

Return value

If ma is an $m \times n$ matrix, the return value is an $mn \times 1$ matrix consisting of the stacked columns of ma. If ma is scalar, the return value is an 1×1 matrix with the value ma.

Description

Vectorizes a matrix by stacking columns. The shape function can be used to undo the vectorization.

See also

shape, vech, vecr

Example

```
#include <oxstd.oxh>
main()
{
    print( vec(<0,1;2,3>) );
}
```

produces

```
  0.00000
   2.0000
   1.0000
   3.0000
```

vech

```
vech(const ma);
```

ma	in:	arithmetic type

Return value

If ma is an $m \times n$ matrix, the return value is an $(m(m+1)/2 - j(j+1)/2) \times 1$ matrix, where $j = \max(m-n, 0)$, consisting of the stacked columns of the lower diagonal of ma. If ma is scalar, the return value is a 1×1 matrix with the value ma.

Description

Vectorizes the lower diagonal of a matrix by stacking columns. use unvech to undo this vectorization.

See also

unvech, vec, vecr

Example

```
#include <oxstd.oxh>
main()
{
    decl m = <0,1;2,3>;
    print( vech(m), unvech(vech(m)) );
}
```

produces

```
      0.00000
       2.0000
       3.0000

      0.00000       2.0000
       2.0000       3.0000
```

vecindex

```
vecindex(const ma);
vecindex(const ma, const mfind);
```

`ma`	in:	matrix
`mfind`	in:	matrix (optional argument)

Return value

`vecindex` with one argument returns a $p \times 1$ matrix holding the row index of the non-zero elements of vec(`ma`), where p is the number of non-zero elements in `ma`. If there is no non-zero element, the function returns the empty matrix (`<>`). A `.NaN` in `ma` is treated as a non-zero.

`vecindex` with two arguments returns a $p \times 1$ matrix holding the sorted row indices of the elements of vec(`ma`) which appear in `mfind`. If none are found, the function returns the empty matrix (`<>`). The second argument can also be used to find the `.NaN`s in `ma`.

Description

The one argument version is often used with a boolean expression:

```
    vecindex(x .< 0 .|| x .== 10)
```

A scalar second argument locates all indices that have that value:

```
    vecindex(x, 5)
```

If the second argument is a matrix, `vecindex` returns the sorted indices of elements of the first argument that occur in the second. Use `find` instead to determine the location of the elements of one vector in another.

See also

`find`, `shape`, `vec`

Example

```
#include <oxstd.oxh>
main()
{
    decl x = <0,1,2;0,2,0>;
    print(vec(x), vecindex(x)', vecindex(x, 0)' );
}
```

produces

```
      0.00000
      0.00000
       1.0000
       2.0000
       2.0000
      0.00000

       2.0000       3.0000       4.0000

      0.00000       1.0000       5.0000
```

vecr

`vecr(const ma);`

`ma`	in:	arithmetic type

Return value

If `ma` is an $m \times n$ matrix, the return value is an $mn \times 1$ matrix consisting of the stacked transposed rows of `ma`. If `ma` is scalar, the return value is a 1×1 matrix consisting of the value `ma`.

Description

Vectorizes a matrix by stacking rows into a column vector. This is compatible with using one empty index on a matrix (see the example).

See also

`reshape`, `vec`, `vech`, `vecrindex`

Example

```
#include <oxstd.oxh>
main()
{
    decl x = <0,1;2,3>;
    print( vecr(x) ~ x[] );
}
```

produces

```
      0.00000      0.00000
       1.0000       1.0000
       2.0000       2.0000
       3.0000       3.0000
```

vecrindex

```
vecrindex(const ma);
vecrindex(const ma, const mfind);
vecrindex(const ma, const mfind, const bunique);
```

`ma`	in:	matrix
`mfind`	in:	matrix (optional argument)
`bunique`	in:	int (optional argument when `mfind` is present)

Return value

`vecrindex` with one argument returns a $p \times 1$ matrix holding the row index of the non-zero elements of vecr(`ma`), where p is the number of non-zero elements in `ma`. If there is no non-zero element, the function returns the empty matrix (`<>`). A `.NaN` in `ma` is treated as a non-zero.

`vecrindex` with two arguments returns a $p \times 1$ matrix holding the row indices of the elements of vecr(`ma`) which appear in `mfind`. If none are found, the function returns the empty matrix. The second argument can also be used to find the `.NaN`s. If the optional third argument is `TRUE`, then the return value has at most the dimension of `vecr(mfind)`; for each element in `vecr(mfind)` the first occurrence in `ma` is reported (if any). This can be faster when `ma` is large, and you know that each occurrence occurs only once.

See also

`find`, `vecindex`, `vecr`

Description

`vecrindex` is compatible with using one index on a matrix. When `ma` is a vector, `vecrindex` and `vecindex` will give identical results.

Example

```
#include <oxstd.oxh>
main()
{
    decl x = <0,1,2;0,2,0>;
    print(vecr(x),  "1 argument:",  vecrindex(x)',
                    "2 arguments:", vecrindex(x, 0)',
                    "3 arguments:", vecrindex(x, <0,2>, 1)');
    print("non-zeros:", vecr(x)[vecrindex(x)]',
          "zeros:", vecr(x)[vecrindex(x, 0)]' );
}
```

```
      0.00000
       1.0000
       2.0000
      0.00000
       2.0000
      0.00000
1 argument:
       1.0000       2.0000       4.0000
2 arguments:
      0.00000       3.0000       5.0000
3 arguments:
      0.00000       2.0000
non-zeros:
       1.0000       2.0000       2.0000
zeros:
      0.00000      0.00000      0.00000
```

zeros

```
zeros(const r, const c);
zeros(const ma);
```

`r`	in:	int
`c`	in:	int
`ma`	in:	matrix

Return value

`zeros(r,c)` returns an `r` by `c` matrix filled with zeros.

`zeros(ma)` returns a matrix of the same dimension as `ma`, filled with zeros.

See also

`nans`, `ones`, `unit`, `new`

Example

```
#include <oxstd.oxh>
main()
{
    print( zeros(2, 2) );
}
```

produces

```
   0.00000      0.00000
   0.00000      0.00000
```

Chapter 9

Predefined Constants

oxstd.oxh defines (requires `#include <oxstd.oxh>`):

`FALSE`	0
`TRUE`	1

oxfloat.oxh defines (requires `#include <oxfloat.oxh>`):

`M_PI`	π
`M_2PI`	2π
`M_PI_2`	$\pi/2$
`M_1_PI`	$1/\pi$
`M_SQRT2PI`	$\sqrt(2\pi)$
`M_E`	$e = \exp(1)$
`M_EULER`	Euler's constant, γ
`M_NAN`	.NaN (Not a Number), also see `isnan` and `isdotnan`
`M_INF`	.Inf (Infinity)
`M_INF_POS`	+.Inf (Infinity)
`M_INF_NEG`	-.Inf (minus Infinity)
`DBL_DIG`	number of decimal digits of precision
`DBL_EPSILON`	machine precision ϵ_m, smallest number such that $1.0 + \epsilon_m \mathrel{!}= 1.0$
`DBL_MANT_DIG`	number of bits in mantissa
`DBL_MAX`	maximum double value
`DBL_MIN`	minimum positive double value
`DBL_MIN_EXP`	minimum 2 exponent
`DBL_MAX_EXP`	maximum 2 exponent
`DBL_MIN_E_EXP`	minimum e exponent
`DBL_MAX_E_EXP`	maximum e exponent
`DBL_MIN_10_EXP`	minimum 10 exponent
`DBL_MAX_10_EXP`	maximum 10 exponent
`INT_MAX`	maximum integer value
`INT_MIN`	minimum integer value

The following constants are predefined by the Ox compiler:

`OX_64_BIT`	when running 64-bit Ox
`OX_AIX`	when running on IBM/AIX
`OX_BIG_ENDIAN`	only on a big-endian machine (Unix workstations)
`OX_DecUNIX`	when running on Dec/UNIX
`OX_HPUX`	when running on HP-UX
`OX_Irix`	when running on SGI/Irix
`OX_Linux`	when running on Linux/PC
`OX_PARALLEL`	indicates that parallel and serial keywords are supported (Ox 7 onwards)
`OX_OS_X`	when running on Sun
`OX_Sun`	when running on Mac/OS X
`OX_Windows`	when running under Windows

9.1 Missing values (NaN)

The hardware-defined missing value is called *Not a Number*, or `.NaN` for short. Any computation involving a `.NaN` results in a `.NaN`. The format used when printing output is `.NaN`.

In a matrix constant, either `.NaN`, `M_NAN` or a dot may be used to represent a missing value (`M_NAN` requires `oxfloat.oxh`). If the dot is the first or last element, an extra space is required to avoid confusion with dot-greater/less than.

In a double constant, either `.NaN` or `M_NAN` may be used to represent a missing value (`M_NAN` requires `oxfloat.oxh`).

A number of procedures are available to deal with missing values, most importantly:

- `deletec()`: deletes all columns which have a `.NaN`,
- `deleter()`: deletes all rows which have a `.NaN`,
- `isdotnan()`: returns matrix of 0's and 1's: 1 if the element is a `.NaN`, 0 otherwise,
- `isnan()`: returns 1 if *any* element is a `.NaN`, 0 otherwise.
- `selectc()`: selects all columns which have a `.NaN`,
- `selectr()`: selects all rows which have a `.NaN`,

9.2 Infinity

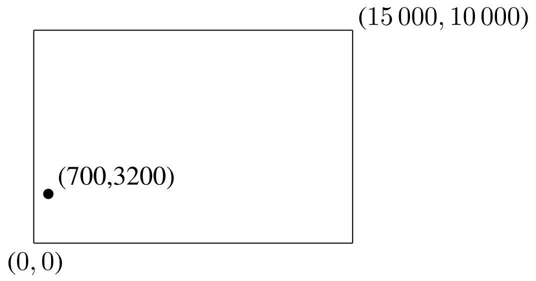
Infinity also exists as a special value supported by the hardware. Infinity can be positive or negative (printed as `+.Inf` and `-.Inf`), and can be used in comparisons as any normal number. You can use `.Inf`, `+.Inf` and `-.Inf` in your code. Alternatively, the predefined constants `M_INF`, `M_INF_POS`, and `M_INF_NEG` are defined in `oxfloat.oxh`. The `isdotinf()` function tests for infinity.

Chapter 10

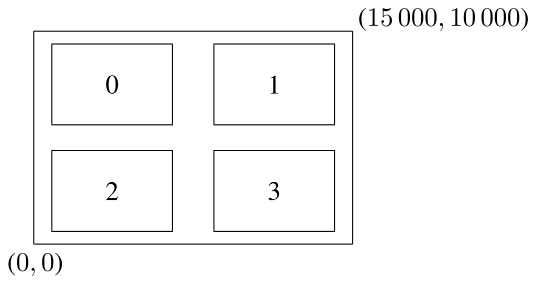
Graphics function reference

10.1 Introduction

Graphs in Ox are drawn on a graphics worksheet, consisting of 15 000 by 10 000 pixels, with (0,0) in the bottom left corner:



Positions can be specified in pixel coordinates, as for example $(p_x, p_y) = (70, 3200)$. More often it is convenient to use real world coordinates. This is done by specifying an area on the graphics worksheet, and attaching real world coordinates to it. These areas are allowed to overlap, but need not:



Suppose we have set up all areas as being from $(x, y) = (0.0, 0.0)$ to $(x, y) = (1.0, 1.0)$ (again within each area the origin is the lower left corner). Then we can draw a line through area 2 in two ways:

1. in real coordinates within an area
   step 1: select area 2;
   step 2: move to (0.0, 0.0);
   step 3: draw a line to (1.0,1.0).
2. using pixel coordinates on the worksheet
   step 1: move to pixel coordinates (600,600);
   step 2: draw a line to pixel coordinates (3600, 3600),

where we assume that (600,600) to (3600,3600) are the pixel coordinates chosen for area 2. Drawing in real world coordinates has the advantage that it corresponds more closely to our data.

In general we use high level drawing functions. These select an area, and a type of graph, and give the data to plot. Note that the supplied matrix must have the data in *rows* (unlike, for example,. the `Database`, where it is in columns). Several functions documented below expect an $m \times T$ matrix for T observations on m variables. The header file to be included for graphics is `oxdraw.oxh`.

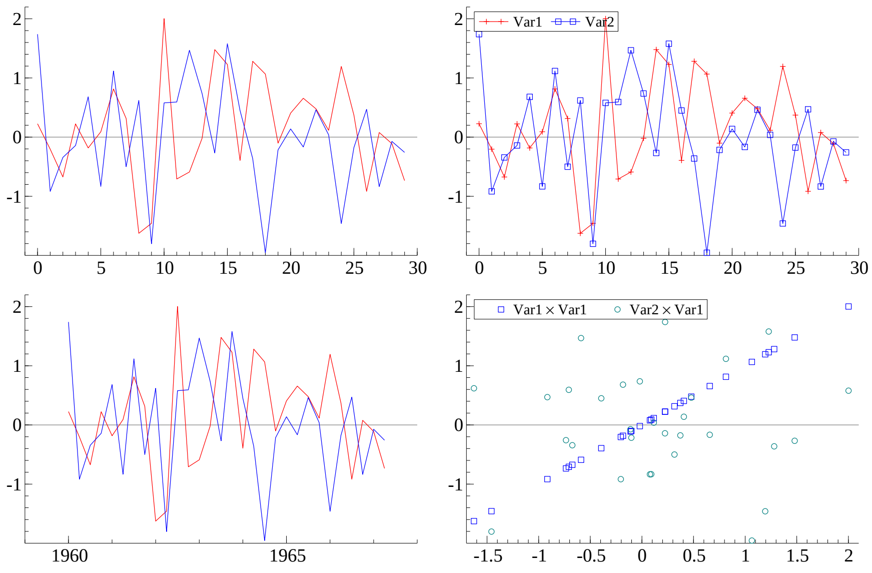


Figure 10.1 PDF file from `draw1.ox`

Example
. . . . *samples/graphics/draw1.ox*
```
#include <oxstd.oxh>
#include <oxdraw.oxh>

main()
{
    decl m = rann(30,2);

    Draw(0, m', 0, 1);
    DrawMatrix(1, m', {"Var1", "Var2"}, 0, 1, 2);
    DrawT(2, m', 1960, 1, 4);
    DrawXMatrix(3, m', {"Var1", "Var2"}, m', "Var1", 1, 3);

    SetDrawWindow("draw1");
    ShowDrawWindow();
    SaveDrawWindow("draw1.ps");
}
```
. . . .

The file `draw1.pdf` produces Fig. 10.1. The `SetDrawWindow` function is only relevant when you use *OxRun* to run the program. Then it may be used to specify the name of the graphics window in *OxMetrics*.

Example
. . . . *samples/graphics/draw2.ox*
```
#include <oxstd.oxh>
#include <oxdraw.oxh>

main()
{
    decl m = rann(100,2);

    DrawAcf(0, m[][0]', "var", 9);
    DrawDensity(1, m[][0]', "var", TRUE, TRUE, TRUE);
    DrawQQ(2, m[][0]', "var", QQ_N, 0, 0);
    DrawQQ(3, m[][0]', "var", QQ_U, 0, 0);

    ShowDrawWindow();
    SaveDrawWindow("draw2.pdf");
}
```
. . . .

The file `draw2.pdf` produces Fig. 10.2.

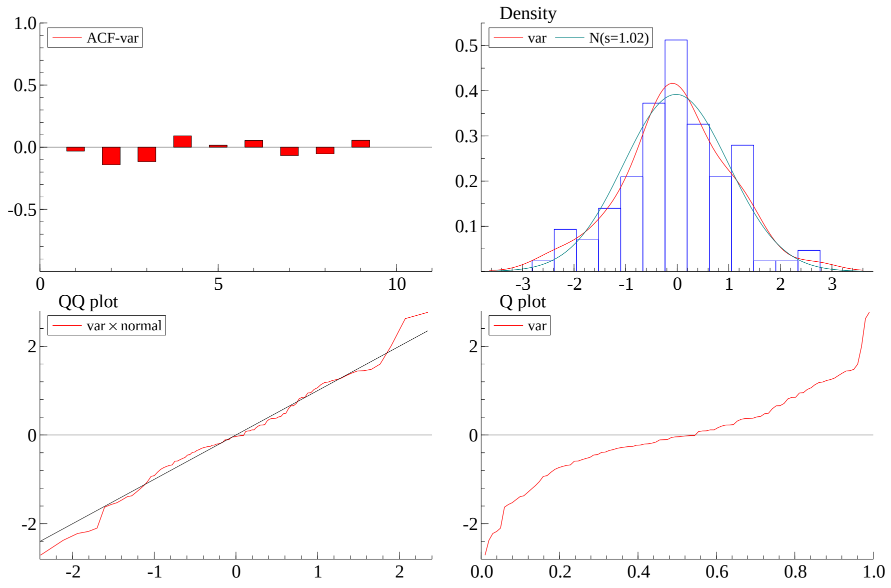


Figure 10.2 `draw2.pdf`

10.2 Symbol and line types

Table 10.1 Symbol types in graphics

`PL_FILLBOX`	filled box	`PL_FILLCIRCLE`	filled circle
`PL_BOX`	open box	`PL_TRIANGLE`	triangle
`PL_PLUS`	plus	`PL_FILLTRIANGLE`	filled triangle
`PL_DASH`	dash	`PL_DIAMOND`	diamond
`PL_CIRCLE`	circle	`PL_FILLDIAMOND`	filled diamond
`PL_LINE`	line	`PL_CROSS`	cross

Table 10.2 Line types in graphics

`TP_SOLID`	solid line
`TP_DOTTED`	dotted line
`TP_DASHED`	dashed line
`TP_LDASHED`	long-dashed line
`TP_USER`	user-defined line

Table 10.3 Plotting styles in graphics

`ST_LINE`	line (points connected)
`ST_SYMBOLS`	symbols
`ST_LINESYMBOLS`	line and symbols
`ST_INDEX`	index line
`ST_INDEXSYMBOLS`	index line with symbols
`ST_BARS`	bars
`ST_SHADING`	shading

Table 10.4 Default line attributes in graphics

index	color	line type	width	symbol	size
0	white	solid	10	plus	90
1	black	solid	6	plus	90
2	red	solid	10	plus	90
3	blue	solid	10	box	90
4	blue/green	solid	10	circle	90
5	purple	dotted	10	plus	90
6	green	dotted	10	plus	90
7	brown/yellow	long dash	10	plus	90
8	dark purple	long dash	10	plus	90
9	pastel yellow	dotted	10	plus	90
10	pastel green	dotted	10	plus	90
11	pastel blue	solid	10	plus	90
12		solid	10	plus	90
13	light grey	solid	10	plus	90
14	grey	solid	10	plus	90
15	light grey	solid	10	plus	90

Table 10.5 Default palette and error fan attributes in graphics

index	color	index	color	index	color
-1	wireframe	2	red	5	purple
0	white	3	blue	6	green
1	black	4	blue/green	7	brown/yellow

10.3 Function reference

CloseDrawWindow

```
CloseDrawWindow();
```

No return value.

Description

Starts a new draw window for subsequent graphs. Note that the *OxMetrics* graphics window will remain active there. A call to `ShowDrawDrawing` also clears the graphics buffer, so does not need to be followed by a call to `CloseDrawWindow`.

Draw

```
Draw(const iArea, const mYt);
Draw(const iArea, const mYt, const dXfirst, const dXstep);
```

`iArea`	in:	int, area index
`mYt`	in:	matrix, $m \times T$ matrix with m rows of data
`dXfirst`	in:	(optional) double, X-value of first observation, x, default is 1
`dXstep`	in:	(optional) double, gap between X-values, d_x, default is 1

No return value.

Description

This function draws m variables against an X variable, where the X variable consists of evenly spaced observations $x, x + d_x, x + 2d_x, x + 3d_x, \ldots$. Each variable is drawn by linking up the points. The first line index is 2.

DrawAcf

```
DrawAcf(const iArea, const vY, const sY, const cLag, ...);
DrawAcf(const iArea, const vY, const sY, const cLag, const fAcf,
    const fPacf, const fErrorBand, int iIndex, const fBar);
```

`iArea`	in:	int, area index
`mY`	in:	$k \times T$ matrix, each *row* is a new plot
`sY`	in:	string, variable name, or array of strings ($k > 1$)
`cLag`	in:	int, highest lag to be used in the ACF
`fAcf`	in:	int, `TRUE`: draw ACF (optional argument, drawn by default)
`fPacf`	in:	int, `TRUE`: draw PACF (optional argument, not drawn by default)
`fErrorBand`	in:	int, `TRUE`: draw error bands (optional argument, not drawn by default)
`iIndex`	in:	int, line index, see Table 10.4, (optional, default is 2).
`fBar`	in:	int, `TRUE`: draw bar plot, else draw index plot (optional argument, using bars by default)

No return value.

Description

Draws the autocorrelation function and/or partial autocorrelation function. The autocorrelation at lag zero is always one, and not included in the graph. The y-axis is $[0, 1]$ if all autocorrelations are positive, $[-1, 1]$ otherwise. The acf is computed similarly to the `acf()` library function.

See also

`acf`, `DrawCorrelogram` for an example.

DrawAdjust

```
DrawAdjust(const iType, ...);
```

`iType`	in:	int, type of adjustment
`d1, ..., d4`	in:	optional extra arguments, int or double (defaults to -1 if missing)

No return value.

Description

This function adjust the *most recently created* graphics object. For example, immediately after a call to `Draw()`, you can use `DrawAdjust` to change the line type. The `iType` argument specifies the type of adjustment:

`ADJ_AREA_3D`	coordinates of the 3D view point of the specified area,
`ADJ_AREA_P`	pixel coordinates of the specified area,
`ADJ_AREA_X`	X world coordinates of the specified area,
`ADJ_AREA_Y`	Y world coordinates of the specified area,
`ADJ_AREA_Z`	Z world coordinates of the specified area,
`ADJ_AREAMATRIX`	area layout (area matrix), boxing and margin,
`ADJ_AXISCENTRE`	centre the axis labels between the large tick marks,
`ADJ_AXISGRID`	set grid lines for the current axis,
`ADJ_AXISHIDE`	hide the axis,
`ADJ_AXISLABEL`	set the label rotation, font size and tick mark size
`ADJ_AXISLINE`	control the axis line,
`ADJ_AXISSCALE`	set the axis scaling type
`ADJ_COLOR`	change line type and colour,
`ADJ_COLORMODEL`	change display or saved PostScript/PDF colour model,
`ADJ_INDEX`	make into index line,
`ADJ_MINMAX`	adjust minimum and maximum y value (also affects area),
`ADJ_PAPERCOLOR`	adjust the colour of the paper (RGB),
`ADJ_PAPERSCALE`	adjust the Y scale of the paper (default is 100%),
`ADJ_SCALE`	adjust scale and shift factor for the vector line,
`ADJ_SYMBOLUSE`	change symbol/line drawing mode,
`ADJ_SYMBOL`	change symbol type and size.

The expected number of arguments depends on the type of adjustment (use -1 to keep the default value):

constant	d1	d2	d3	d4	d5
`ADJ_AREA_3D`	area	azimuth	elevation	distance	twist
`ADJ_AREA_P`	area	$x_{\min}$	$y_{\min}$	width	height
`ADJ_AREA_X`	area	$x_{\min}$	$x_{\max}$	grow	
`ADJ_AREA_Y`	area	$y_{\min}$	$y_{\max}$	grow	
`ADJ_AREA_Z`	area	$z_{\min}$	$z_{\max}$	grow	
`ADJ_AREASCOLOR`	*red*:0–255	*green*:0–255	*blue*:0–255		
`ADJ_AREAMATRIX`	Y areas	X areas	box	margin	
`ADJ_AXISCENTRE`	0,1				
`ADJ_AXISGRID`	0,1	colour	type		
`ADJ_AXISHIDE`	0,1				
`ADJ_AXISLABEL`	rotation	font size	tick size		
`ADJ_AXISLINE`	line at $y = 0$	above	no line	no small	
`ADJ_AXISSCALE`	type	scale	shift		
`ADJ_COLOR`	colour	type			
`ADJ_COLORMODEL`	display:0,1	print:0-3			
`ADJ_INDEX`	0,1,2	base			
`ADJ_LEGEND`	area	no columns	font size	resize	box all
`ADJ_MINMAX`	minimum	maximum			
`ADJ_PAPERCOLOR`	*red*:0–255	*green*:0–255	*blue*:0–255		
`ADJ_PAPERSCALE`	percentage				
`ADJ_SCALE`	scale	shift			
`ADJ_SYMBOLUSE`	style				
`ADJ_SYMBOL`	type	size			

Some notes and examples:

`ADJ_AREA_3D` expects the area number as the first argument. The azimuth is the rotation along the Z axis (or, more precisely orthogonal to the line of view). Elevation is the angle with the X–Y plane, and twist the rotation along the line of view. Azimuth, elevation and twist are specified in degrees, distance is in area units. The default values of azimuth, elevation, distance and twist for Fig. 10.9 correspond approximately to: $-125, 25, 1800, 0$;

`ADJ_AREA_P` expects the area number as the first argument.

`ADJ_AREA_X`, `ADJ_AREA_Y`, `ADJ_AREA_Z` all expect the area number as the first argument. Real-world area adjustment does currently not work properly for 3D graphs. All have an optional argument grow; set this to one if the area should only grow if it already has dimensions fixed.

`ADJ_AREAMATRIX` Adjust the rows and columns of the area matrix from the default For example, when there are two areas, the default layout is 2×1. To put the graphs next to each other:

```
DrawAdjust(ADJ_AREAMATRIX, 1, 2);
```

Set the third argument to one to box all areas. The margin size can be changed with the fourth argument (the default is 640)

`ADJ_AXIS...` Unless explicitly created, axes are only made once the graph is displayed. Therefore, adjustments to an axis need to be preceeded by an explicit creation, as for example in:

```
DrawT(1, x, 1960, 1, 4);
DrawAxisAuto(1, 1);        // create a default X axis
DrawAdjust(ADJ_AXISCENTRE, 1);// and centre the dates
```

`ADJ_AXISGRID` Use -1 for default colour and line type:

```
DrawAxisAuto(0, 1);
DrawAdjust(ADJ_AXISGRID, 1, -1, -1);
```

`ADJ_AXISLABEL` Rotation changes the label rotation relative to the axis, the value is 0, 1, or -1 to leave the default. The default font size is 300 and tick size 6; use -1 to leave the default.

`ADJ_AXISLINE` All arguments are 0, 1, or -1 to leave the default. The second ('above') puts the labels on the opposite side of the axis; ' no line' omits the base line (leaving the tick marks); 'no small' removes the small tick marks.

`ADJ_AXISSCALE` The type for is one of:

`AXIS_LINEAR`	– standard axis,
`AXIS_LOG`	– log-scale (data is in natural logarithms),
`AXIS_LOG10`	– log10-scale (data is in base-10 logarithms),
`AXIS_SCALED`	– scaled: set scale and shift as 2nd and 3rd value,
`AXIS_DATE`	– dated: interpret as Julian date/time values, see Fig. 10.7 for an example.

The following example illustrates `AXIS_LOG` . Logarithms are taken, and then undone along the axis to show in the original units:

```
DrawT(0, log(x), 1960, 1, 4);
DrawAxisAuto(0, 0);                   // default Y axis
DrawAdjust(ADJ_AXISSCALE, AXIS_LOG); // use log scale
```

`ADJ_COLOR` Colour is $0 \ldots 15$: 0 = background (white), 1 = foreground (black), $2 \ldots 15$ are remaining colours. Type is $0 \ldots 15$. By default, the colour and type are equal, with settings given in Table 10.4. Use -1 to leave the default.

`ADJ_COLORMODEL` The display mode can be 0 (colour) or 1 (b&w).
The print mode can take the values $0 - 3$: 0 = black & white, 1 = black, white & gray, 2 = gray levels, 3 = colour. This defines the colour model that is used for saving graphs in PDF and PostScript.

`ADJ_INDEX` Use `d1=1` to change to an index line (in that case `d2` defaults to 0), and `d1=2` to change to a bar. The base argument is the point to which the index lines or bars are drawn. When omitted, it is assumed to be zero. An index line is a single vertical line, centred on the observation values — multiple index lines will overwrite each other. The bar type is centred on the observation value, and will make space if multiple bars are drawn. If the bars become too thin, they will become a single line drawn in the colour, instead of a black outline filled with the colour.

`ADJ_MINMAX` Sets the minimum and maximum of the previous vector or histogram object. This implies that the area will encompass these values, and therefore differs from `ADJ_AREA_Y`, which enforces a Y range.

`ADJ_PAPERCOLOR` Sets the color of the paper (not the areas).

`ADJ_PAPERSCALE` This adjust the Y scale as a percentage of the original. To set half size (50%; an example is given under `DrawXYZ`):

```
DrawAdjust(ADJ_PAPERSCALE, 50);
```

`ADJ_SYMBOL` Symbol types are listed in Table 10.1.

`ADJ_SYMBOLUSE` Style can be as per Table 10.3: 0 = draw line, 1 = draw symbols, 2 = draw both. An example drawing both:

```
DrawT(0, x, 1960, 1, 4);
```

```
        DrawAdjust(ADJ_SYMBOL, PL_CIRCLE, 150);
        DrawAdjust(ADJ_SYMBOLUSE, ST_LINESYMBOLS);
```

Example

A selection of adjustments is used in the following listing, producing Figure 10.3. Another example is given under `DrawCorrelogram()`, and `DrawXYZ()`.

. . . . *samples/graphics/draw4.ox*

```
#include <oxstd.oxh>
#include <oxdraw.oxh>
main()
{
    decl m = rann(30,2);
    Draw(0, m', 0, 1);                      // draw 2 variables
    DrawAdjust(ADJ_MINMAX, -5, 5);          // adjust y min/max
    DrawTitle(0, "A title");                   // set the title
    DrawText(0, "Maths: $\\theta_i$", 1, 3); // text at (1,3)
                           // add text at pixel coordinates
    DrawPText(0, "at pixel (1000,3000)", 1000, 3000, 0, 400);

    DrawBoxPlot(1, m[][0]', "Var1");         // draw a box plot
    DrawLine(1, 0, -1, 1, 1, 4);      //and a line in the plot
         // draw a circle, is in area 2 but belongs to area 1
    DrawPSymbol(1, 3000, 3000, 4000, 4000, PL_CIRCLE,3);
    DrawAxisAuto(1, FALSE);                // draw default y axis
                                           // add a second y axis
    DrawAxis(1, FALSE, 2, -1, 1, -1, 0.5, 0.1, 0);
    DrawAdjust(ADJ_AXISLINE, 0, 1, 1, 0);        // adjust axis
                                             // draw a cross plot
    DrawXMatrix(2, m[][0]', "Var1", m[][1]', "Var2");
    DrawAdjust(ADJ_INDEX, 1, -1);       // change to index line
    DrawLegend(2, 0, 0, 1);                     // hide the legend

    DrawTMatrix(3, m[][0]', "Var1", 1960, 1, 4);        // draw
    DrawZ(m[][1]');                   // add 2nd var as error bar
    DrawLegend(3, 100, 50, 0);                  // draw the legend
    DrawAdjust(ADJ_AREA_X, 3, 1959, 1968);      //fix x(world)
                        // also fix pixel location of area 3
    DrawAdjust(ADJ_AREA_P, 3, 9000, 1500, 5000, 3000);

    DrawTMatrix(4, m[][1]', "Var2", 1960, 1, 4, 0, 3);
                          // draw area 4 on top of area 3
    DrawAdjust(ADJ_AREA_P, 4, 9000, 1500, 5000, 3000);
    DrawAdjust(ADJ_AREA_X, 4, 1959, 1968);      //same x world
    DrawAxisAuto(4, TRUE, FALSE);             // remove x axis
    DrawAxisAuto(4, FALSE, TRUE, ANCHOR_MAX);      //move axis
    DrawAdjust(ADJ_AXISLINE, TRUE, TRUE);      // labels right
    DrawLegend(4, 550, 50, 0);                   // move legend

    DrawPText(4, "Var1 on left scale",  9100, 700);
    DrawPText(4, "Var2 on right scale", 9100, 400);

    DrawAdjust(ADJ_AREAMATRIX, 2, 2);      // 5 areas, use 2x2

    // leave display setting, but save in black,white&gray
    DrawAdjust(ADJ_COLORMODEL, -1, 1);
    ShowDrawWindow();                    // show this concoction
    SaveDrawWindow("draw4.pdf");
}
```
....

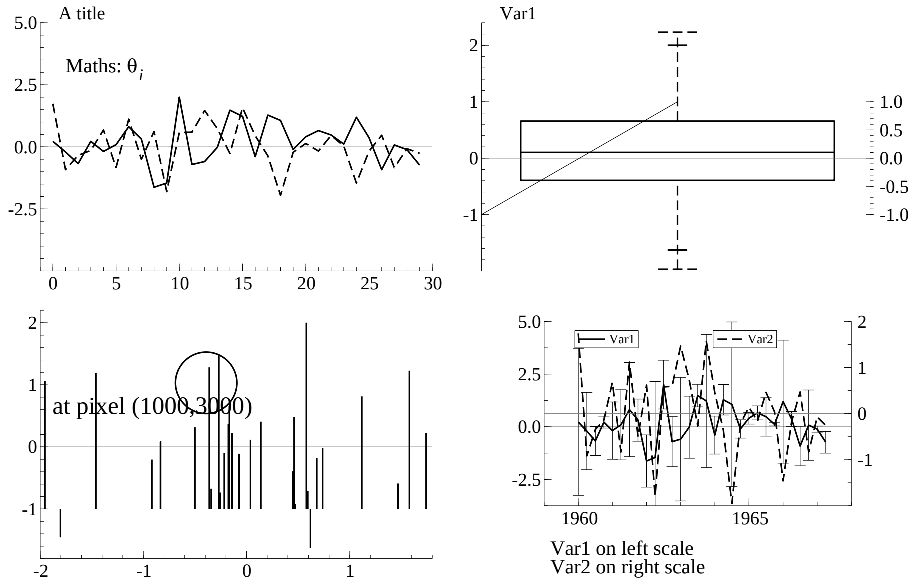


Figure 10.3 Illustration of `DrawAdjust`

DrawAxis, DrawAxisAuto

```
DrawAxis(const iArea, const iIsXaxis, const dAnchor, const dAxmin,
    const dAxmax, const dFirstLarge, const dLargeStep,
    const dSmallStep, const iFreq);
DrawAxis(const iArea, const iIsXaxis, const dAnchor, const dAxmin,
    const dAxmax, const dFirstLarge, const dLargeStep,
    const dSmallStep, const iFreq, const dAnchor2);
DrawAxisAuto(const iArea, const iIsXaxis, ...);
DrawAxisAuto(const iArea, const iIsXaxis, const fShow,
    const iAnchor, const dAnchor, const dAnchor2);
```

`iArea`	in:	area index
`iIsXAxis`	in:	1: X axis, 0: Y axis, 2: Z axis
`dAnchor`	in:	if `iAnchor=ANCHOR_USER`: anchor of the axis (Y location of X, X location of Y and Z axis)
`dAnchor2`	in:	if `iAnchor=ANCHOR_USER`: anchor of the 3D axis (Z location of X and Y axis, Y location of Z)
`dAxmin`	in:	axis minimum
`dAxmax`	in:	axis maximum
`dFirstLarge`	in:	location of first large tick
`dLargeStep`	in:	step size between large ticks
`dSmallStep`	in:	step size between small ticks
`iFreq`	in:	frequency (for time series X-axis, set to 0 otherwise)
`fShow`	in:	`TRUE`: show the axis
`iAnchor`	in:	axis anchor location, `ANCHOR_MIN`: at minimum, `ANCHOR_MAX`: at maximum, `ANCHOR_USER`: at `dAnchor`

No return value.

Description

`DrawAxis` draws an axis, fully specified.

`DrawAxisAuto` draws an axis with automatic design.

See also

`DrawAdjust` (for examples)

Example

.... *samples/drawaxis_log10.ox*

```
#include <oxstd.oxh>
#include <oxdraw.oxh>
main()
{
    decl x = 1000 * ranu(1000, 1);
    decl y = x + 10 * rann(1000, 1);

    // plot 1: normal X-axis (taking log10 of data in log10)
    DrawX(0, y', log10(x)');
    // Put an unusual Y-axis in the graph
    DrawAxis(0, 0, 1, 100, 1000, 100, 100, 0, 0);
    // plot 1: log10 axis (taking log10 of data in log10)
    DrawX(1, y', log10(x)');
    // add axis so that it can be manipulated
    DrawAxisAuto(1, 1);
    // now change it to log10
    DrawAdjust(ADJ_AXISSCALE, AXIS_LOG10);

    // change layout to 1x2, boxed
    DrawAdjust(ADJ_AREAMATRIX, 1, 2, 1);
    // and shrink
    DrawAdjust(ADJ_PAPERSCALE, 50);

    // and show
    ShowDrawWindow();
}
```
....

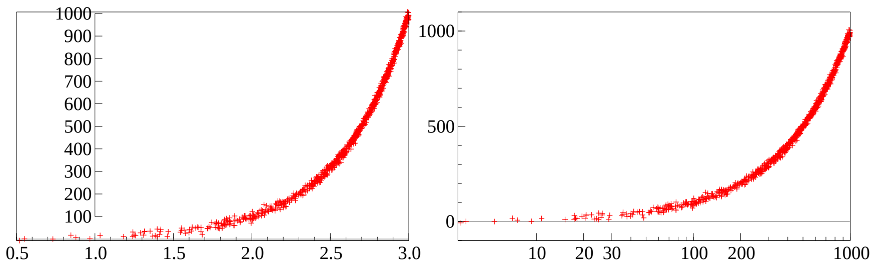


Figure 10.4 PDF file from drawaxis_log10.ox

DrawBoxPlot

```
DrawBoxPlot(const iArea, const mY, const sY);
DrawBoxPlot(const iArea, const vY, const sY, const iIndex);
```

`iArea`	in:	int, area index
`mY`	in:	$k \times T$ matrix, each *row* is a new plot (needs $T > 5$)
`sY`	in:	string, variable name, or array of strings ($k > 1$)
`iIndex`	in:	int, line index, see Table 10.4, (optional, default is 2).

No return value.

Description

Draws a box plot of the data in the specified area.

A box plot shows the distribution of a variable in terms of its quartiles, labelled Q_1, Q_2, Q_3 (the 25%, 50% and 75% quartiles). Define the interquartile range as $IQR = 1.5(Q_3 - Q_1)$. The box plot consists of the following elements:

- a box, with horizontal lines at Q_1, Q_2 (the median) and Q_3;
- a vertical line from $Q_1 - IQR$ to $Q_3 + IQR$ (omitted inside the box);
- individual observations: all observations outside the $(Q_1 - IQR, Q_3 + IQR)$ range, plus the two observations on either end which just fall inside this range.

See also

`DrawAdjust` (for an example)

DrawCorrelogram

```
DrawCorrelogram(const iArea, const mY, const sY, const cLag);
DrawCorrelogram(const iArea, const mY, const sY, const cLag,
    const iIndex);
```

`iArea`	in:	int, area index
`mY`	in:	$k \times T$ matrix, each *row* is a new plot
`sY`	in:	string, variable name, or array of strings ($k > 1$)
`cLag`	in:	int, highest lag to be used in correlogram
`iIndex`	in:	int, line index, see Table 10.4, (optional, default is 2).

No return value.

Description

Draws a correlogram which plots the autocorrelation function. The autocorrelation at lag zero is always one, and not included in the graph. The y-axis is $[0, 1]$ if all autocorrelations are positive, $[-1, 1]$ otherwise. The acf is computed differently from that in the `acf()` library function. The difference is that `DrawCorrelogram` uses the running mean:

$$\hat{\mathsf{r}}_j^* = \frac{\sum_{t=j+1}^{T}(x_t - \bar{x}_0)(x_{t-j} - \bar{x}_j)}{\sqrt{\sum_{t=j+1}^{T}(x_t - \bar{x}_0)^2 \sum_{t=j+1}^{T}(x_{t-j} - \bar{x}_j)^2}}.$$

Here $\bar{x}_0 = \frac{1}{T-j}\sum_{t=j+1}^{T} x_t$ is the sample mean of x_t, $t = j+1, \ldots, T$, and $\bar{x}_j = \frac{1}{T-j}\sum_{t=j+1}^{T} x_{t-j}$ is the sample mean of x_{t-j}, so that $\hat{\mathsf{r}}_j^*$ corresponds to a proper sample correlation coefficient. The difference with the definition of the sample autocorrelations in (8.1) tends to be small, and vanishes asymptotically.

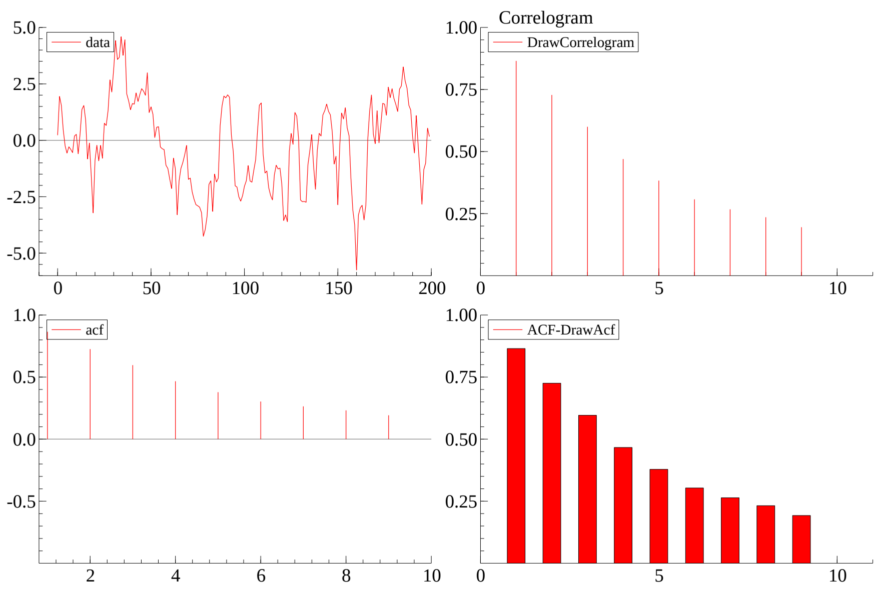


Figure 10.5 Autocorrelation functions

See also

`acf`, `DrawAcf` to draw the standard ACF.

Example

The following example compares the two correlograms, with the bottom graph holding the standard ACF, computed using the `acf()` function.

. . . . *samples/graphics/draw5.ox*

```
#include <oxstd.oxh>
#include <oxdraw.oxh>
main()
{
    decl lag = 9;
    decl m = cumulate(rann(200,1), 0.9);

    DrawMatrix(0, m', "data", 0, 1);
    DrawCorrelogram(1, m', "DrawCorrelogram", lag);

    decl macf = acf(m, lag);         // compute standard ACF
    DrawMatrix(2, macf[1:][]', "acf", 1, 1);// draw the ACF
    DrawAdjust(ADJ_INDEX, 1);        // change to index line
    DrawAdjust(ADJ_MINMAX, -1, 1); // set y range to [-1 1]

    DrawAcf(3, m', "DrawAcf", lag);

    ShowDrawWindow();
}
```

. . . .

DrawDensity

```
DrawDensity(const iArea, const vY, const sY, ...);
DrawDensity(const iArea, const vY, const sY, const fDens,
    const fHist, const fNormal, BOOL fCdf, BOOL fStand,
    const cBar, const iIndex);
```

`iArea`	in:	int, area index
`mY`	in:	$k \times T$ matrix, each *row* is a new plot
`sY`	in:	string, variable name, or array of strings ($k > 1$)
`fDens`	in:	int, TRUE: draw estimated density (optional, default)
`fHist`	in:	int, TRUE: draw histogram (optional, not drawn by default)
`fNormal`	in:	int, TRUE: add the normal density with same mean and variance for reference (optional, not drawn by default)
`fCdf`	in:	int, TRUE: plot CDF in separate area (optional, not drawn by default); this is drawn as a QQ plot against the normal with same mean and variance (unless `fStand=TRUE`)
`fStand`	in:	int, TRUE: use standardized data (optional, default uses original data)
`cBar`	in:	int, number of bars (0: use default; optional argument)
`iIndex`	in:	int, line index for density, see Table 10.4, (optional, default is 2).

No return value.

Description

Draws the histogram and/or density of the data in the specified area. When `fNormal` is `TRUE`, a normal density with the same mean and variance as the data will be drawn. The density estimate is based on a kernel density estimation, with Gaussian kernel, and optimal bandwidth (if the data are indeed from a normal density) of $1.06\hat{\sigma}T^{-0.2}$. The density is estimated at 128 points using the fast Fourier transform due to B.W. Silverman (see Silverman, 1986) and Applied Statistics algorithm AS 176). Also see the *OxMetrics* book.

See also

`samples/lib/DensEst.ox` (which gives examples of the use of density estimation code, in comparison with DrawDensity),
`DrawHistogram`.

DrawHistogram

```
DrawHistogram(const iArea, const vBar, ...);
DrawHistogram(const iArea, const vBar, const dMin, const dStep,
    const iIndex, const iColorIn);
```

`iArea`	in:	int, area index
`vBar`	in:	$k \times T$ matrix with bar heights, each *row* is a new plot
`dMin`	in:	double, first X-coordinate of histogram (optional argument, default is 1)
`dStep`	in:	double, bar step size (optional argument, default is 1)
`iIndex`	in:	int, line index for outline, see Table 10.4, (optional, default is 2).
`iColorIn`	in:	int, colour index for inside of bars, see Table 10.4, (optional argument, default is 0: white).

No return value.

Description

Draws a histogram when the data is already in histogram format, i.e. `vBar` contains the bar heights.

See also

`DrawDensity`

DrawLegend

```
DrawLegend(const iArea, const iOffsX, const iOffsY,
    const fHidden);
```

`iArea`	in:	area index
`iOffsetX`	in:	X pixel offset from top left of area
`iOffsetY`	in:	Y pixel offset from top left
`fHidden`	in:	`TRUE`: hide the legend

No return value.

Description

`DrawLegend` determines the location of the legend. By default, a legend is drawn in the top left-hand corner, with a scale that adjusts automatically to the area size. `DrawLegend` can also be used to hide the legend. The content of the legend is determined by the variable names that are used when drawing vectors.

See also

`DrawAdjust` (for an example)

DrawLine

```
DrawLine(const iArea, const dX1, const dY1, const dX2, const dY2,
    const iIndex);
DrawLine(const iArea, const dX1, const dY1, const dZ1,
    const dX2, const dY2, const dZ2, const iIndex);
```

`iArea`	in:	area index
`dX1,dY1`	in:	real-world coordinates of starting point
`dX2,dY2`	in:	real-world coordinates of end point
`dZ1,dZ2`	in:	real-world Z coordinates for symbol in 3D graph
`iIndex`	in:	int, line index for first row, see Table 10.4.

No return value.

Description

`DrawLine` draws a line between the specified coordinates.

See also

`DrawAdjust` (for an example)

DrawMatrix

```
DrawMatrix(const iArea, const mYt, const asY, const dXfirst,
    const dXstep, ...);
DrawMatrix(const iArea, const mYt, const asY, const dXfirst,
    const dXstep, const iSymbol, const iIndex);
```

`iArea`	in:	int, area index
`mYt`	in:	$m \times T$ matrix with m rows of data
`asY`	in:	array of strings (holds variable names), or 0 (no names), or a string (when only one variable to graph)
`dXfirst`	in:	double, X-value of first observation, x
`dXstep`	in:	double, gap between X-values, d_x
`iSymbol`	in:	int, 0: draw line, 1: draw symbols, 2: draw both (optional argument, default is 0), see Table 10.3. Or vector with value for each row of data.
`iIndex`	in:	int, line index for first row, see Table 10.4, (optional, default is 2). Each subsequent row will have the next index. Or vector with value for each row of data.

No return value.

Description

This is a more flexible version of the `Draw()` function. `DrawMatrix` draws the m variables in the rows of `mYt`. The X variable consists of evenly spaced observations $x, x + d_x, x + 2d_x, x + 3d_x, \ldots$.

The following table gives the default settings for each line index. Note that index 0 is the background colour, and 1 the foreground colour.

DrawPLine, DrawPSymbol, DrawPText

```
DrawPLine(const iArea, const iX1, const iY1, const iX2,
    const iY2, const iIndex);
DrawPSymbol(const iArea, const iX1, const iY1, const iX2,
    const iY2, const iSymType, const iIndex);
DrawPText(const iArea, const sText, const iPx1,
    const iPy1, ...);
DrawPText(const iArea, const sText,const iPx1,const iPy1,
    const iFontNo, const iFontSize, const iTitle, const iRotation);
```

iX1,iY1	in:	pixel coordinates
iX2,iY2	in:	pixel coordinates

No return value.

Description

These are pixel coordinate equivalents of DrawLine, DrawSymbol and DrawText respectively. See under those functions for a description of the remaining arguments.

DrawQQ

```
DrawQQ(const iArea, const mY, const sY, const iDens,
    const df1, const df2);
DrawQQ(const iArea, const mY, const sY, const iDens,
    const df1, const df2, const iIndex);
```

iArea	in:	int, area index
mY	in:	$k \times T$ matrix, each *row* is a new plot
sY	in:	string, variable name, or array of strings ($k > 1$)
iDens	in:	int, one of: QQ_CHI, QQ_F, QQ_N, QQ_T, QQ_U
df1	in:	double, first parameter for distribution
df2	in:	double, second parameter for distribution
iIndex	in:	int, line index for first row, see Table 10.4, (optional, default is 2).

No return value.

Description

Draws a QQ plot. Each row of mY would normally hold critical values which are hypothesized to come from a certain distribution. This function then draws a cross plot of these observed values (sorted), against the theoretical quantiles. The 45^o line is drawn for reference (the closer the cross plot to this line, the better the match). The following distributions are supported:

QQ_CHI	$\chi^2(df1)$,
QQ_F	$\mathsf{F}(df1, df2)$,
QQ_N	$\mathsf{N}(0, 1)$,
QQ_N_SE	$\mathsf{N}(0, 1)$ with pointwise asymptotic 95% standard error bands, as derived in Engler and Nielsen (2009),
QQ_T	$\mathsf{t}(df1)$,
QQ_U	Uniform$(0, 1)$, resulting in a quantile plot.

DrawSpectrum

```
DrawSpectrum(const iArea, const mY, const sY, const iOrder);
DrawSpectrum(const iArea, const mY, const sY, const iOrder,
    const iIndex);
```

`iArea`	in:	int, area index
`mY`	in:	$k \times T$ matrix, each *row* is a new plot
`sY`	in:	string, variable name, or array of strings ($k > 1$)
`iOrder`	in:	int, lag truncation parameter m
`iIndex`	in:	int, line index for first row, see Table 10.4, (optional, default is 2).

No return value.

Description

Draws the estimated spectral density, which is a smoothed function of the autocorrelations $\mathtt{r}_j$. The graph corresponds to the results computed with the `periodogram` library function using `imode = 2`, and `cpoints = 128`. Note that the horizontal axis in the graph is scaled by π, thus transforming the scale from $[0, \pi]$ to $[0, 1]$.

See also

`periodogram`

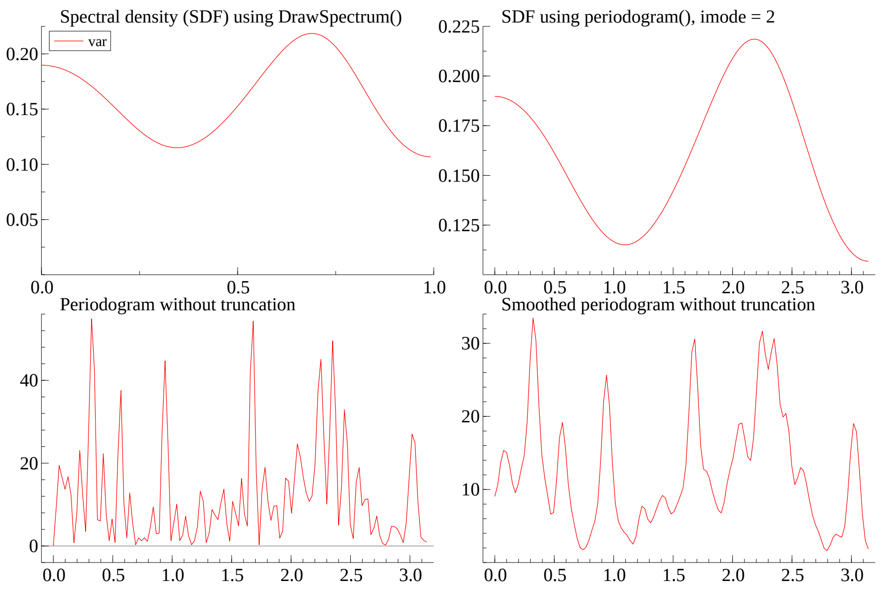


Figure 10.6 Periodograms and spectral density estimates

Example

. . . .*samples/draw6.ox*

```
#include <oxstd.oxh>
#include <oxfloat.oxh>
```

```
#include <oxdraw.oxh>
main()
{
    decl m = rann(100,1), cp = 128;

    DrawSpectrum(0, m', "var", 10);
    DrawTitle(0, "Spectral density (SDF) using DrawSpectrum()");

    Draw(1, periodogram(m,10, cp,2)', 0, M_PI / (cp-1));
    DrawTitle(1, "SDF using periodogram(), imode = 2");

    Draw(2, periodogram(m,100,cp,0)', 0, M_PI / (cp-1));
    DrawTitle(2, "Periodogram without truncation");

    Draw(3, periodogram(m,100,cp,1)', 0, M_PI / (cp-1));
    DrawTitle(3, "Smoothed periodogram without truncation");

    ShowDrawWindow();
}
```

produces a graph like Fig. 10.6.

DrawSymbol

```
DrawSymbol(const iArea, const dX1, const dY1, const dX2,
    const dY2, const iSymType, const iIndex);
DrawSymbol(const iArea, const dX1, const dY1, const dZ1, const dX2,
    const dY2, const dZ2, const iSymType, const iIndex);
```

`iArea`	in:	area index
`dX1,dY1`	in:	real-world coordinates, lower-left corner of bounding box
`dX2,dY2`	in:	real-world coordinates, upper-right corner of bounding box
`dZ1,dZ2`	in:	real-world Z coordinates for symbol in 3D graph
`iSymType`	in:	symbol type, see Table 10.1
`iIndex`	in:	int, line index for first row, see Table 10.4, (optional, default is 2).

No return value.

Description

`DrawSymbol` draws a symbol in the specified bounding box.

See also

`DrawAdjust` (for an example)

DrawT

```
DrawT(const iArea, const mYt, const mnYear, const mnPeriod,
    const iFreq);
DrawT(const iArea, const mYt, const vDates, 0, 0);
```

`iArea`	in:	int, area index
`mYt`	in:	$m \times T$ matrix with m y variables
`mnYear`	in:	int, year of first observation
`mnPeriod`	in:	int, period of first observation
`iFreq`	in:	int, frequency of observations
`vDates`	in:	$1 \times T$ matrix with Julian dates (and/or times, see `dayofcalendar` and `timeofday`)

No return value.

Description

Draws m variables in the specified area against time. Each variable is drawn by linking up the points. The first line index is 2.

DrawText, DrawTitle

```
DrawText(const iArea, const sText, const dX1, const dY1, ...);
DrawText(const iArea, const sText, const dX1, const dY1,
    const iFontNo, const iFontSize, const iTitle, const iRotation,
    const dZ1);
DrawTitle(const iArea, const sText);
```

`iArea`	in:	area index
`sText`	in:	text to draw, this may include LaTeX-style formatting
`dX1,dY1`	in:	real-world coordinates of text anchor
`iFontNo`	in:	font number (0 for first font; use -1 for the default font)
`iFontSize`	in:	font size (e.g. 330; use -1 for the default size)
`iTitle`	in:	`TEXT_TEXT` or `0`: normal text, else is graph title (coordinates are ignored): `TEXT_TITLE` – graph title; `TEXT_XLABEL` – label along X-axis; `TEXT_YLABEL` – label along Y-axis; `TEXT_ZLABEL` – label along Z-axis
`iRotation`	in:	rotation (in degrees, default is 0), only effective if the `iTitle` argument is zero
`dZ1`	in:	real-world Z coordinate of text anchor (for text in 3D graphs; default is 0)

No return value.

Description

`DrawText` draws text at the specified location. There is optional control of font and font size.

For a summary of the LaTeX-style features, see the OxMetrics book. Note that the forward slash for LaTeX commands must be doubled, for example:

```
    DrawText(0, "$\\leftarrow\\arrowext$", 1962, 1, -1,
        -1, 0, 45);
```

`DrawTitle` draws text at the title location. This corresponds to

```
    DrawText(iArea, sText, 0, 0, -1, -1, TEXT_TITLE).
```

Text can also be rotated, by specifying the angle in degrees. This will not work well for multiple line text blocks.

See also

`DrawAdjust` (for an example)

DrawTMatrix

```
DrawTMatrix(const iArea, const mYt, const asY, ...);
DrawTMatrix(const iArea, const mYt, const asY, const mnYear,
    const mnPeriod, const iFreq, const iSymbol, const iIndex);
DrawTMatrix(const iArea, const mYt, const asY, const vDates, ...);
DrawTMatrix(const iArea, const mYt, const asY, const vDates,
    0, 0, const iSymbol, const iIndex);
```

`iArea`	in:	int, area index
`mYt`	in:	$m \times T$ matrix with m y variables
`asY`	in:	array of strings (holds variable names), or 0 (no names), or a string (when only one variable to graph)
`mnYear`	in:	int, year of first observation (optional argument, default is 1)
`mnPeriod`	in:	int, period of first observation (optional argument, default is 1)
`iFreq`	in:	int, frequency of observations (optional argument, default is 1)
`iSymbol`	in:	int, 0: draw line, 1: draw symbols, 2: draw both (optional argument, default is 0), see Table 10.3 Or vector with value for each row of data.
`iIndex`	in:	int, line index for first row, see Table 10.4, (optional, default is 2) Each subsequent row will have the next index. Or vector with value for each row of data.
`vDates`	in:	$1 \times T$ matrix with Julian dates (and/or times, see `dayofcalendar` and `timeofday`)

No return value.

Description

This is a more flexible version of the `DrawT()` function. Draws m variables in the specified area against time. See under `DrawMatrix` for the default settings for each line index.

See `Modelbase::DbDrawTMatrix` for a version that uses the database sample information for the horizontal axis.

Example

The code of `draw10.ox` draws a data against the Julian time values which are representing dates (Fig. 10.7b), and against time (Fig. 10.7c).

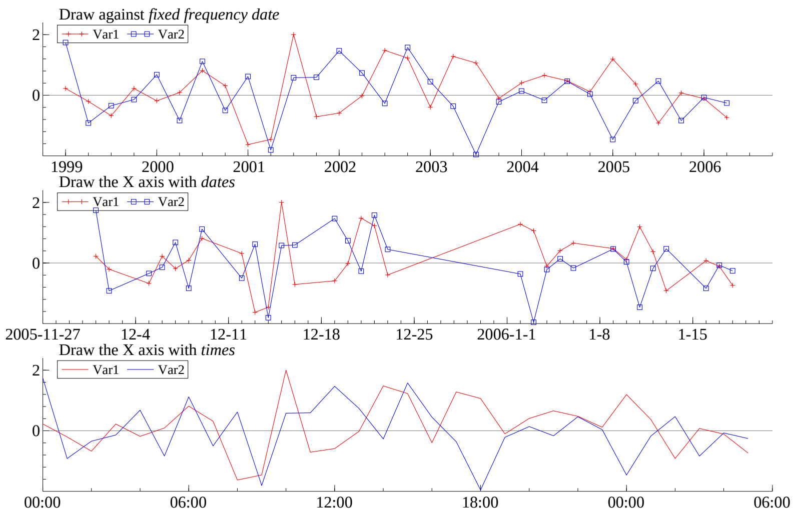


Figure 10.7 `DrawTMatrix` example with dates and times

. . . . *samples/draw10.ox*
```
#include <oxstd.oxh>
#include <oxdraw.oxh>

main()
{
    decl c = 30, m = rann(30,2);

    decl dates = dayofcalendar(2005, 12, 1) + range(0, 300);
    // drop weekends
    decl drop = dayofweek(dates) .== 1 .|| dayofweek(dates) .== 7;
    // drop period between christmas and newyear
    drop = drop .|| (dates .>= dayofcalendar(2005, 12, 25) .&&
        dates .<= dayofcalendar(2006,  1, 1));

    // drop those dates, and get c dates
    dates = deleteifc(dates, drop)[ : c - 1];
    println("%C", dates');

    DrawTitle(0, "Draw against {\it fixed frequency date}");
    DrawTMatrix(0, m', {"Var1", "Var2"}, 1999, 1, 4);

    DrawTitle(1, "Draw the X axis with {\it dates}");
    DrawTMatrix(1, m', {"Var1", "Var2"}, dates, 0, 0, 2);

    DrawTitle(2, "Draw the X axis with {\it times}");
    DrawTMatrix(2, m', {"Var1", "Var2"}, range(0, c - 1) / 24);
    ShowDrawWindow();
}
```
. . . .

DrawX

```
DrawX(const iArea, const mYt, const vX);
```

`iArea`	in:	int, area index
`mYt`	in:	$m \times T$ matrix with m y variables
`vX`	in:	$1 \times T$ matrix with x variable

No return value.

Description

Draws m y variables in the specified area against an x variable. Each point is marked, but the points are not linked, resulting in a cross plot. The first line index is 2.

DrawXMatrix

```
DrawXMatrix(const iArea, const mYt, const asY, const vX, const sX,
    ...);
DrawXMatrix(const iArea, const mYt, const asY, const vX, const sX,
    const iSymbol, const iIndex);
```

`iArea`	in:	int, area index
`vX`	in:	$1 \times T$ matrix with x variable
`iSymbol`	in:	int, 0: draw line, 1: draw symbols, 2: draw both (optional argument, default is 0). Or vector with value for each row of data.
`iIndex`	in:	int, line index for first row, see Table 10.4, (optional, default is 2), see Table 10.3. Each subsequent row will have the next index. Or vector with value for each row of data.

No return value.

Description

This is a more flexible version of the `DrawX()` function. Draws m variables in the specified area against an x variable See under `DrawMatrix` for the default settings for each line index and a description of the remaining arguments.

Example

The code of `draw11.ox` draws a data against the Julian time values which are representing dates (Fig. 10.8b).

.... *samples/draw11.ox*

```
#include <oxstd.oxh>
#include <oxdraw.oxh>

main()
{
    decl c = 30, m = rann(30,2);

    decl dates = dayofcalendar(2005, 12, 1) + range(0, 300);
    // drop weekends
    decl drop = dayofweek(dates) .== 1 .|| dayofweek(dates) .== 7;
    // drop period between christmas and newyear
```

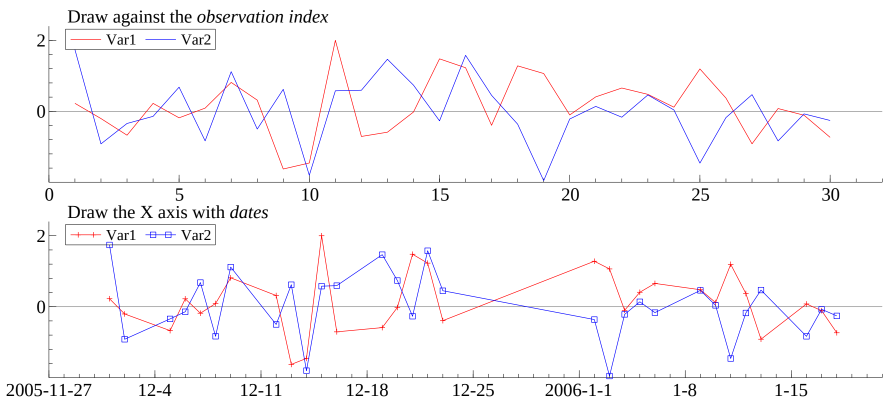


Figure 10.8 `DrawXMatrix` example with dates and times

```
    drop = drop .|| (dates .>= dayofcalendar(2005, 12, 25) .&&
        dates .<= dayofcalendar(2006,  1, 1));

    // drop those dates, and get c dates
    dates = deleteifc(dates, drop)[ : c - 1];

    println("%C", dates');

    DrawTitle(0, "Draw against the {\it observation index}");
    DrawTMatrix(0, m', {"Var1", "Var2"}, 1, 1, 1);

    DrawTitle(1, "Draw the X axis with {\it dates}");
    DrawAxisAuto(1, 1);
    DrawAdjust(ADJ_AXISSCALE, AXIS_DATE);
    DrawXMatrix(1, m', {"Var1", "Var2"}, dates, "", 2);

    DrawAdjust(ADJ_PAPERSCALE, 70);
    DrawAdjust(ADJ_AREAMATRIX, 2, 1);

    ShowDrawWindow();
}
```
....

DrawXYZ

```
DrawXYZ(const iArea, const vX, const vY, const mZ, ...);
DrawXYZ(const iArea, const vX, const vY, const mZ,
    const iMode, const sX, const sY, const sZ,
    const iPalette, const iIndex);
```

`iArea`	in:	int, area index
`vX`	in:	$1 \times k$ matrix with X variable
`vY`	in:	$1 \times n$ matrix with Y variable
`mZ`	in:	$k \times n$ matrix with Z variable, heights above XY plane
or	in:	$1 \times n = k$ matrix with Z coordinates for points (X, Y, Z), creates rough approximating surface (scatter format)
`iMode`	in:	int, type of plot (optional argument): -1: triangulation (only for scatter format) 0: surface plot only (default) 1: unsupported: surface with contours on ground level 2: 2-dimensional contour plot
`sX`	in:	string, name of X variable (optional argument)
`sY`	in:	string, name of Y variable (optional argument)
`sZ`	in:	string, name of Z variable (optional argument)
`iPalette`	in:	int, palette index, see Table 10.5, (optional, default is 2: red).
`iIndex`	in:	int, line index for mesh, see Table 10.4, (optional, default is 1: black).

No return value.

Description

This function draws a 3-dimensional surface.

Example

The first example shows a simple 3-dimensional plot of a bivariate independent normal density (without the normalizing constant). In first plot of the second example, the tabular format is different for x and y: x is 1×61, y is 1×14, z is 61×14. The second plot of Fig. 10.10 is drawn from a random scatter: the X, Y, Z vectors have the same dimension. It keeps the azimuth, elevation and distance at the approximate default values, but adds a twist of about $25°$.

See `samples/draw8contour.ox` for an example involving contour plots.

.... *samples/draw7.ox*

```
#include <oxstd.oxh>
#include <oxdraw.oxh>

main()
{
    decl x, y, z;
    x = y = range(-30,30,3) / 10;
    z = exp(-sqr(x') / 2) .* exp(-sqr(y) / 2);
    DrawXYZ(0, x, y, z);
    DrawAdjust(ADJ_PAPERSCALE, 60);
    ShowDrawWindow();
}
```

....

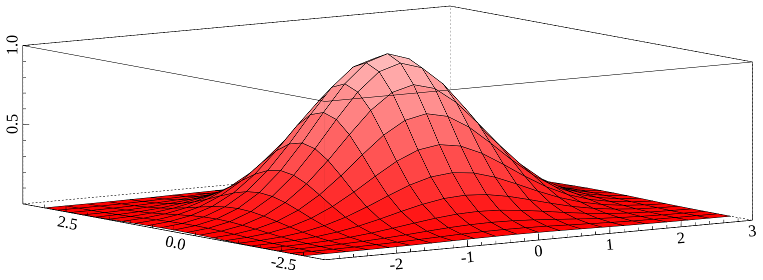


Figure 10.9 Three-dimensional plot

samples/draw8.ox

```
#include <oxstd.oxh>
#include <oxdraw.oxh>

main()
{
    decl x = range(-30,30,1) / 10;                    // k row vector
    decl y = quann(range(1,14) / 15) * 2;             // n row vector
    decl z = exp(-sqr(x') / 2) .* exp(-sqr(y) / 2); // (kxn) table
    DrawXYZ(0, x * 10, y, z, 0, "X", "Y", "Z");

    x = (ranu(500, 1) - 0.5) * 6;
    y = (ranu(500, 1) - 0.5) * 6;
    z = exp(-sqr(x) / 2) .* exp(-sqr(y) / 2);  // vector!
    DrawXYZ(1, x, y, z);
    DrawAdjust(ADJ_AREA_3D, 1, -125, 25, 1000, 25);
    DrawAdjust(ADJ_PAPERSCALE, 60);
    ShowDrawWindow();
}
```

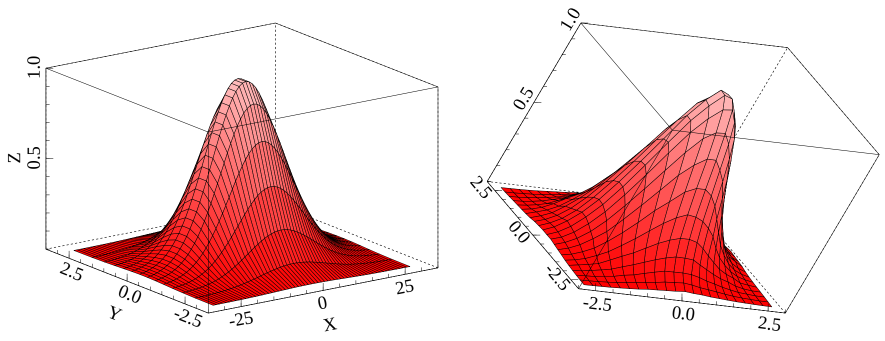


Figure 10.10 Three-dimensional plot

DrawZ

```
DrawZ(const vZ, ...);
DrawZ(const vZ, const sZ, const iMode, const dFac, const iIndex);
```

`vZ`	in:	$1 \times T$ matrix
`sZ`	in:	Z variable name (optional argument)
`iMode`	in:	type of Z variable (optional argument)
`dFac`	in:	bar/band factor (optional argument, 2.0 is default)
`iIndex`	in:	int, line index for first row, see Table 10.4, (optional, default is 2).

No return value.

Description

`DrawZ` adds a Z component to the most recent graphics object. `DrawZ` should be used immediately after a call to one of the draw functions `Draw`, `DrawMatrix`, `DrawX`, etc.). The `iMode` argument can have one of the following values:

`ZMODE_SYMBOL`	use values as symbol size,
`ZMODE_VALUE`	draw value as text,
`ZMODE_BAR`	draw error bars (the default),
`ZMODE_BAND`	draw error bands,
`ZMODE_FAN`	draw error fans,
`ZMODE_HILO`	draw high-low plot,
`ZMODE_3D`	draw 3-D points.

To draw a sequence of 3D points, use for example:

```
DrawX(0, y, x);
DrawZ(z, "Z", ZMODE_3D);
```

See also

`DrawAdjust` (for an example)

SaveDrawWindow

```
SaveDrawWindow(const sFilename);
```

`sFilename`	in:	valid file name

No return value.

Description

Saves the current graph to the specified file. The file format for saving is derived from the extension. The following formats are supported:

extension	format
`.eps`	Encapsulated PostScript;
`.gwg`	*OxMetrics* graphics file;
`.pdf`	PDF: Portable document format;
`.png`	Portable Network Graphics, which is a bitmap format;
`.ps`	PostScript.

See the *OxMetrics* book for a description of these formats. When saving in a format other than `.gwg`, the `.gwg` file is automatically saved as well (using the same file name with `.gwg` appended).

SetDraw

```
SetDraw(const iOption, ...);
SetDraw(const iOption, const i1, const i2, const i3, const i4,
    const i5);
```

`iOption`	in:	int, option to set
`i1,...,i5`	in:	int, optional extra arguments

No return value.

Description

This function changes the default settings used in graphics. When run via OxMetrics, this will affect the persistent OxMetrics settings. Check first under `DrawAdjust` if the required customization is available there. The following constants may be used for `iOption`:

option	changes	*option*	changes
`SET_AXISFORMAT`	axis label format	`SET_LEGENDHIDE`	legend hiding
`SET_AXISLINE`	axis options	`SET_LEGENDRESIZE`	resize legends
`SET_AXIS`	axis fonts/ticks	`SET_LEGEND`	legend style
`SET_BOX`	box and grid	`SET_LINEBWG`	b&w line settings
`SET_BWG`	b&w setting	`SET_LINE`	line settings
`SET_COLORMODEL`	PostScript/PDF model	`SET_MARGIN`	paper margins
`SET_COLOR`	colour settings	`SET_PALETTE_MAX`	palette max colour
`SET_DEFAULT`	reset all defaults	`SET_PALETTE_MIN`	palette min colour
`SET_FONT`	font	`SET_PAPERCOLOR`	paper colour
`SET_GRID`	grid style	`SET_PRINTPAGE`	PostScript paper
`SET_HISTOGRAM`	bar colours	`SET_SYMBOL`	symbol settings
`SET_LEGENDFONTSIZE`	legend font size	`SET_XYSTYLE`	labels along axes

The following table lists the integer arguments for each option, with the range of possible values. If no range is given, the argument is a size in pixel coordinates (see §10.1).

option	i1	i2	i3	i4	i5
`SET_AXISFORMAT`	*width*:8	*precision*:6	*same prec*:0,1	*lead zero*:0,1	
`SET_AXISLINE`	*no X-line*	*no Y-line*	*center dates*	*no small Y*	
`SET_AXIS`	*fontsize*	*step*	*tick*		
`SET_BOX`	*box*:0–1	*X-grid*:0–1	*Y-grid*:0–1		
`SET_BWG`	*lineno*:0–15	*red*:0–255	*green*:0–255	*blue*:0–255	
`SET_COLORMODEL`	*model*:0–3				
`SET_COLOR`	*lineno*:0–15	*red*:0–255	*green*:0–255	*blue*:0–255	
`SET_FONT`	*fontno*:0–3	*fontsize*			
`SET_GRID`	*color*:0–15	*type*:0–15			
`SET_HISTOGRAM`	*inside*:0–15	*outside*:0–15			
`SET_LEGENDFONTSIZE`	*fontsize*				
`SET_LEGENDHIDE`	*hide*:0–1				
`SET_LEGENDRESIZE`	*resize*:0,1				
`SET_LEGEND`	*boxed*:0–1	*columns*			

option	i1	i2	i3	i4	i5
SET_LINEBWG	*lineno*:0–15	*linetype*:0–4	*width*	*on*	*off*
SET_LINE	*lineno*:0–15	*linetype*:0–4	*width*	*on*	*off*
SET_MARGIN	*left*	*top*			
SET_PALETTE_MAX	*lineno*:0–7	*red*:0–255	*green*:0–255	*blue*:0–255	
SET_PALETTE_MIN	*lineno*:0–7	*red*:0–255	*green*:0–255	*blue*:0–255	
SET_PAPERCOLOR	*red*:0–255	*green*:0–255	*blue*:0–255		
SET_PRINTPAGE	*papertype*:0–2	*orientation*:0–1	*X-size*	*Y-size*	
SET_SYMBOL	*lineno*:0–15	*symtype*:0–4	*size*		
SET_XYSTYLE	*2D-style*:0,1	*3D-style*:0,1			

For *symtype* see Table 10.1 and for *linetype* see Table 10.2. All SET_AXISLINE arguments are 0 or 1. Paper, orientation and model arguments for SET_PRINTPAGE and SET_COLORMODEL are:

papertype	*orientation*	*model*	
PAGE_A4	PAGE_PORTRAIT	0	black & white
PAGE_LETTER	PAGE_LANDSCAPE	1	black, white, gray
PAGE_USER		2	gray
		3	color

Papertype and orientation are used when saving as a PostScript (.ps) file. The colour model is used for all PostScript and PDF files.

Example

....*samples/draw3.ox*

```
#include <oxstd.oxh>
#include <oxdraw.oxh>
main()
{
    decl m = rann(30,2);

    Draw(0, m', 0, 1);
    DrawMatrix(1, m', {"Var1", "Var2"}, 0, 1, 2);
    DrawT(2, m', 1960, 1, 4);
    DrawXMatrix(3, m', {"Var1", "Var2"}, m', "Var1", 1, 3);

    ShowDrawWindow();
    SetDraw(SET_PRINTPAGE, PAGE_LETTER, PAGE_PORTRAIT);
    SaveDrawWindow("draw3.ps");
}
```
....

SetDrawWindow

```
SetDrawWindow(const sTitle);
```

`sTitle`	in:	string, name of window

No return value.

Description

This function is only relevant when interacting with *OxMetrics* otherwise it does nothing. It sets the name of the *OxMetrics* window in which the graphs of the Ox program appear to `sTitle`.

SetTextWindow

```
SetTextWindow(const sTitle);
```

`sTitle`	in:	string, name of window

No return value.

Description

This function is only relevant when interacting with *OxMetrics* otherwise it does nothing. It sets the name of the *OxMetrics* window in which the output (from the `print()` function) of the Ox program appears to `sTitle`.

ShowDrawWindow

```
ShowDrawWindow();
```

No return value.

Description

Shows the drawing. Note that in some implementations the graphs cannot be displayed. Then a message is printed (`SaveDrawWindow()` will still work in that case!).

A call to `ShowDrawWindow` also clears the drawing buffer, so does not need to be followed by a call to `CloseDrawWindow`. Therefore, two subsequent calls to `ShowDrawWindow` first show, then clear the graph from the active window.

Chapter 11

Packages

Packages are extensions and additions to the Ox language. Whereas the core of Ox contains the general purpose functions, packages often solve a specific problem or are ports of existing code to Ox. Sometimes part of the code is available through a Dynamic Link Library (DLL) with accompanying header file. Many packages are third party contributions to Ox, and documented and maintained by their respective authors. The Ox web site maintains an up to date list of available packages. This chapter only describes the packages which are part of the standard release of Ox.

11.1 Arma package

The Arma package implements functions which are commonly used in autoregressive-moving average models. The Arma package requires the header file `arma.h`. Note that the Arma package uses the convention of writing the AR and MA coefficients on the right-hand side with a positive sign.

arma0

`arma0(const ma, const vp, const cp, const cq);`

`ma`	in:	$T \times n$ matrix A
`vp`	in:	$1 \times s$ matrix with autoregressive coefficients $\phi_1, \phi_2, \ldots, \phi_p$ followed by the moving average coefficients $\theta_1, \theta_2, \ldots, \theta_q$, $s \geq p + q$
`cp`	in:	int, no of autoregressive coefficients (could be 0)
`cq`	in:	int, no of moving average coefficients (could be 0)

Return value

Returns the residual from applying the ARMA(p, q) filter to each column of A. The result has the same dimensions as `ma`. The first p rows of the return value will be zero.

Description

For a column $a = (a_0, \ldots, a_{T-1})'$ of A, this function computes (see e.g. Harvey, 1993, §3.3):

$$\begin{aligned} \epsilon_t &= 0 & t &= 0, \ldots, p-1, \\ \epsilon_t &= a_t - \phi_1 a_{t-1} \ldots - \phi_p a_{t-p} - \theta_1 \epsilon_{t-1} \ldots - \theta_q \epsilon_{t-q} & t &= p, \ldots, T-1, \end{aligned}$$

using $\epsilon_t = 0$ for $t < 0$. For example when $p = 1$ and $q = 2$:

$$\begin{aligned} \epsilon_0 &= 0 \\ \epsilon_1 &= a_1 - \phi_1 a_0 - \theta_1 \epsilon_0 \\ \epsilon_2 &= a_2 - \phi_1 a_1 - \theta_1 \epsilon_1 - \theta_2 \epsilon_0 \\ \epsilon_t &= a_t - \phi_1 a_{t-1} - \theta_1 \epsilon_{t-1} - \theta_2 \epsilon_{t-2} \quad t = p, \ldots, T-1. \end{aligned}$$

Comparison with the `cumulate` function shows that in the univariate case `cumulate(y,a0,a1)` corresponds to `arma0(y,-(a0~a1),0,2)`.

See also

`armagen`, `armaforc`, `armavar`, `diff0`, `diffpow`, `pacf`

Example

```
#include <oxstd.oxh>
#include <arma.oxh>
main()
{
    decl mx = <1:5>';
    print(arma0(mx,<0.5, 0.5>, 1, 1) ~ arma0(mx,<0.5>, 0, 1));
}
```

produces

```
     0.00000       1.0000
      1.5000       1.5000
      1.2500       2.2500
      1.8750       2.8750
      2.0625       3.5625
```

armaforc

```
armaforc(const mx, const vp, const cp, const cq, ...);
armaforc(const mx, const vp, const cp, const cq,
    const ma, const me);
```

`mx`	in:	$H \times n$ matrix X, fixed part of forecasts
`vp`	in:	$1 \times s$ matrix with autoregressive coefficients $\phi_1, \phi_2, \ldots, \phi_p$ followed by the moving average coefficients $\theta_1, \theta_2, \ldots, \theta_q$, $s \geq p+q$
`cp`	in:	int, no of autoregressive coefficients (could be 0)
`cq`	in:	int, no of moving average coefficients (could be 0)
`ma`	in:	(optional argument) $T \times n$ matrix A, pre-forecast data values (default is zero)
`me`	in:	(optional argument) $T \times n$ matrix E, pre-forecast residual values (default is zero)

Return value

Returns the forecasts from an ARMA(p, q) model, as an $H \times n$ matrix. The same model is applied to each column of `mx`.

Description

For a column $x = (x_0, \ldots, x_{H-1})'$ of X, as the first argument, and assuming the `ma` and `me` arguments are omitted, this function computes:

$$\begin{aligned}
&\hat{a}_0 = x_0 \\
&\hat{a}_1 = x_1 + \phi_1 \hat{a}_0 \\
&\hat{a}_2 = x_2 + \phi_1 \hat{a}_1 + \phi_2 \hat{a}_0 \\
&\ldots \\
&\hat{a}_h = x_h + \phi_1 \hat{a}_{h-1} + \ldots + \phi_p \hat{a}_{h-p} \quad h = p, \ldots, H-1,
\end{aligned}$$

The `ma` argument can be used to specify actual values $a = (a_0, \ldots, a_{T-1})'$, which are used in the beginning stages of the forecasting, e.g. when $p = 2$:

$$\begin{aligned}
&\hat{a}_0 = x_0 + \phi_1 a_{T-1} + \phi_2 a_{T-2} \\
&\hat{a}_1 = x_1 + \phi_1 \hat{a}_0 + \phi_2 a_{T-1} \\
&\hat{a}_2 = x_2 + \phi_1 \hat{a}_1 + \phi_2 \hat{a}_0 \\
&\hat{a}_h = x_h + \phi_1 \hat{a}_{h-1} + \phi_2 \hat{a}_{h-2} \quad h = 2, \ldots, H-1,
\end{aligned}$$

Note that the actual values are taken from the end of `ma`: the first forecast will use the last two values, the second forecast the last value.
When a moving average component is present, it is necessary to specify the actual values for the error term. The `me` argument is used for this. As for the actual values, the errors are taken from the end of `me`, and are only used when lagged errors fall in

the pre-forecast period. For an ARMA(2,2) model (see e.g. Harvey, 1993, §2.6):

$$\begin{aligned}
\hat{a}_0 &= x_0 + \phi_1 a_{T-1} + \phi_2 a_{T-2} + \theta_1 \epsilon_{T-1} + \theta_2 \epsilon_{T-2} \\
\hat{a}_1 &= x_1 + \phi_1 \hat{a}_0 + \phi_2 a_{T-1} + \theta_2 \epsilon_{T-1} \\
\hat{a}_2 &= x_1 + \phi_1 \hat{a}_1 + \phi_2 \hat{a}_0 \\
\hat{a}_h &= x_h + \phi_1 \hat{a}_{h-1} + \phi_2 \hat{a}_{h-2} && h = 2, \ldots, H-1,
\end{aligned}$$

See also

arma0, armavar, cumulate, modelforc

Example

We use an example from Harvey (1993, p.35):

$$y_t = 0.6 y_{t-1} + 0.2 y_{t-2} + \epsilon_t + 0.3\epsilon_{t-1} - 0.4\epsilon_{t-2}.$$

Using $y_T = 4$, $y_{T-1} = 5$, $\epsilon_T = 1$ and $\epsilon_{T-1} = 0.5$ four forecasts are computed. The two entries of 100 are ignored, because values are taken from the end:

```
#include <oxstd.oxh>
#include <arma.oxh>
main()
{
    print( armaforc(zeros(4,1), <0.6,0.2,0.3,-0.4>, 2, 2,
           <100;100;5.0;4.0>, <0.5;1>) );
}
```

produces

```
       3.5000
       2.5000
       2.2000
       1.8200
```

armagen

armagen(const mx, const me, const vp, const cp,const cq);

mx	in:	$T \times n$ matrix of known component X
me	in:	$T \times n$ matrix of errors E
vp	in:	$1 \times s$ matrix with autoregressive coefficients $\phi_1, \phi_2, \ldots, \phi_p$ followed by the moving average coefficients $\theta_1, \theta_2, \ldots, \theta_q$, $s \geq p + q$
cp	in:	int, no of autoregressive coefficients (could be 0)
cq	in:	int, no of moving average coefficients (could be 0)

Return value

Generates a an ARMA(p, q) series from an error term (me) and a mean term (mx). The result has the same dimensions as mx. The first p rows of the return value will be identical to those of mx; the recursion will be applied from the pth term onward (missing lagged errors are set to zero).

Description

For a column $(x_0, \ldots, x_{T-1})'$ of X, and a column $(\epsilon_0, \ldots, \epsilon_{T-1})'$ of E, this function computes:

$$\begin{aligned}
a_t &= x_t && t = 0, \ldots, p-1, \\
a_t &= x_t + \phi_1 a_{t-1} \ldots \phi_p a_{t-p} + \epsilon_t + \theta_1 \epsilon_{t-1} \ldots \theta_q \epsilon_{t-q} && t = p, \ldots, T-1,
\end{aligned}$$

using $\epsilon_t = 0$ for $t < 0$. For example when $p = 1$ and $q = 2$:

$$
\begin{aligned}
a_0 &= x_0 \\
a_1 &= x_1 + \phi_1 a_0 + \epsilon_1 + \theta_1 \epsilon_0 \\
a_2 &= x_2 + \phi_1 a_1 + \epsilon_2 + \theta_1 \epsilon_1 + \theta_2 \epsilon_0 \\
a_t &= x_t + \phi_1 a_{t-1} + \epsilon_t + \theta_1 \epsilon_{t-1} + \theta_2 \epsilon_{t-2} \quad t = p, \ldots, T-1.
\end{aligned}
$$

This function could be used to generate an ARMA(p, q) series from random numbers. In that case it is common to discard intitial observations to remove the effect of starting up the recursion.

See also

`arma0`, `armaforc`, `armavar`, `cumsum`, `cumulate`

Example

```
#include <oxstd.oxh>
#include <arma.oxh>
main()
{
    decl mx = ones(5,1), meps = rann(5,1) / 10;
    print( armagen(mx, meps, <0.5, 0.5>, 1, 1)
         ~ armagen(mx, meps, <0.5>, 0, 1) );
}
```

produces

```
       1.0000       1.0225
       1.6852       1.1852
       1.9092       1.0666
       1.8526      0.89803
       1.8130      0.88670
```

armavar

```
armavar(const vp, const cp, const cq, const dvar,
    const ct);
```

`vp`	in:	$1 \times s$ matrix with autoregressive coefficients $\phi_1, \phi_2, \ldots, \phi_p$ followed by the moving average coefficients $\theta_1, \theta_2, \ldots, \theta_q$, $s \geq p + q$
`cp`	in:	int, no of autoregressive coefficients (could be 0)
`cq`	in:	int, no of moving average coefficients (could be 0)
`dvar`	in:	double, variance of disturbance, σ^2_ϵ.
`ct`	in:	int, number of autocovariance terms required

Return value

Returns a $1\times$ `ct` matrix with the autocovariances of the ARMA(p, q) process. Or `0` if the computations failed (e.g. when all autoregressive coefficients are zero).

Description

Computes the theoretical autocovariances $\mathsf{c}(i)$, $i = 0, \ldots, T-1$ (see equation (8.3) on page 188 for a definition) of the ARMA(p, q) process specified as

$$
a_t = \phi_1 a_{t-1} + \ldots + \phi_p a_{t-p} + \epsilon_t + \theta_1 \epsilon_{t-1} + \ldots + \theta_q \epsilon_{t-q}, \quad \mathsf{E}\epsilon_t = 0, \ \mathsf{E}\epsilon_t^2 = \sigma^2_\epsilon.
$$

using $\epsilon_t = 0$ for $t < 0$. Stationary is assumed, but not verified. The computations are based on the algorithm given in McLeod (1975).

See also

`arma0`, `pacf`

Example

In the example below, we set σ_ϵ^2 such that we obtain the autocorrelation function:

```
#include <oxstd.oxh>
#include <arma.oxh>
main()
{
    print( armavar(<0.5>, 1, 0, (1 - 0.5^2), 5)'
         ~ armavar(<-0.5>, 1, 0, (1 - (-0.5)^2), 5)'
         ~ armavar(<0.5>, 0, 1, 1 / (1 + 0.5^2), 5)' );
}
```

produces

```
       1.0000        1.0000        1.0000
      0.50000      -0.50000       0.40000
      0.25000       0.25000       0.00000
      0.12500      -0.12500       0.00000
     0.062500      0.062500       0.00000
```

diffpow

```
diffpow(const ma, const d);
diffpow(const ma, const d, const dmisval);
```

`ma`	in:	$T \times n$ matrix A
`d`	in:	double, length of difference d, $\|d\| \leq 10000$
`dmisval`	in:	(optional argument) double, value to set missing observations to (default is 0)

Return value

Returns a $T \times n$ matrix with $(1-L)^d A$. The result has the same dimensions as `ma`.

Description

Differences the specified matrix, missing values are replaced by zero (unless a missing value is specified as the third argument). For a column $a = (a_0, \ldots, a_{T-1})'$ of A, this function computes $(1-L)^d a$, defined as:

$$a_t = \sum_{j=0}^{t} \frac{(-d)_j}{j!} a_{t-j}, \quad t = 0, \ldots, T-1,$$

where the $(\cdot)_j$ symbol is defined as:

$$\begin{array}{ll} (z)_0 = 1, & \\ (z)_j = z(z+1)\ldots(z+j-1) & \text{for } j > 0 \\ (z)_j = 1/\left((z-1)(z-2)\ldots(z-j)\right) & \text{for } j < 0 \end{array}$$

and using $a_k = 0$ for $k < 0$.

See also

`arma0`, `diff0`

Example

In this example, `fracdiff` replicates the functionality of the library function `diffpow`.

```
#include <oxstd.oxh>
#include <arma.oxh>

fracdiff(const mY, const d)
{
    decl i, mu = mY, fac = -d;

    for (i = 1; i < rows(mY); ++i, fac *= (-d+i-1)/i)
        mu += fac * lag0(mY,i);

return mu;
}
main()
{
    decl mx = <1:5>';
    print( diffpow(mx,2) ~ diff0(diff0(mx,1),1) ~
        diffpow(mx,-2) ~ diff0(diff0(mx,-1),-1) );
    print( diffpow(mx,0.2) ~ fracdiff(mx,0.2) ~
        diffpow(mx,-0.2) ~ fracdiff(mx,-0.2) );
}
```

produces

```
      1.0000      0.00000       1.0000      0.00000
     0.00000       1.0000       4.0000      0.00000
     0.00000      0.00000       10.000      0.00000
     0.00000      0.00000       20.000      -1.0000
     0.00000      0.00000       35.000      0.00000

      1.0000       1.0000       1.0000       1.0000
      1.8000       1.8000       2.2000       2.2000
      2.5200       2.5200       3.5200       3.5200
      3.1920       3.1920       4.9280       4.9280
      3.8304       3.8304       6.4064       6.4064
```

modelforc

```
modelforc(const mU, const mData, const miDep,
    const miSel, const miLag, const mPi, const iTmin);
```

`mU`	in:	0, or $(T_2 - T_1 + 1) \times n$ matrix $\mathbf{U}$, optional error term
`mData`	in:	$T(= T_2 + 1) \times d$ matrix $\mathbf{D}$, database
`miDep`	in:	$1 \times n$ matrix with indices in $\mathbf{D}$ of dependent variables
`miSel`	in:	$1 \times k$ matrix with indices in $\mathbf{D}$ of explanatory variables
`miLag`	in:	$1 \times k$ matrix with lag lengths of explanatory variables
`iTmin`	in:	T_1, observation to start forecasting from (this may be zero)

Return value

Returns the dynamic forecasts from a linear dynamic model as a $(T - T_1 = T_2 - T_1 + 1) \times n$ matrix.

Description

This function forecasts from a dynamic model, which may be an estimated (reduced form) model or a DGP:

$$\mathbf{y}_t = \mathbf{\Pi w}_t + \mathbf{u}_t, \quad t = T_1, \ldots, T_2$$

where $\mathbf{w}$ contains $\mathbf{z}$, r lags of $\mathbf{z}$ and m lags of $\mathbf{y}$:

$$\mathbf{w}_t' = \left(\mathbf{y}_{t-1}', \ldots, \mathbf{y}_{t-m}', \mathbf{z}_t', \ldots, \mathbf{z}_{t-r}'\right).$$

Take $\mathbf{y}_t$ as an $n \times 1$ vector, $\mathbf{z}_t$ as $q \times 1$, and $\mathbf{w}_t$ as $k \times 1$.
Given data on $\mathbf{z}_t$ for $t = 0, \ldots, T_2$, and on $\mathbf{y}_t$ for $t = 0, \ldots, T_1 - 1$, `modelforc` will produce forecasts for $t = T_1 \ldots T_2$. No actual $\mathbf{y}_t$ data is used for $t \geq T_1$, only previously forecasted values. If lagged data is missing (z_t, y_t for $t < 0$), it is assumed to be zero. If the error term is not given (`mU` argument 0 implies $\mathbf{u}_t = 0$), the output corresponds to model forecasts. Otherwise it could e.g. be the fitted values from a DGP. Note that in that case the first observation in the `mU` matrix is $\mathbf{u}_{T_1}$.

See also

`armaforc`, `cumulate`, PcFimlDgp class.

pacf

```
pacf(const macf);
pacf(const macf, const alogdet);
pacf(const macf, const alogdet, const my);
pacf(const macf, const meps);
```

`macf`	in	arithmetic type, $T \times 1$ matrix of autocovariances or autocorrelations
`alogdet`	in:	(optional argument) address of variable
	out:	double, the *logarithm* of the the determinant of the filter
`my`	in:	(optional argument) $T \times n$ data matrix Y to apply filter to
`meps`	in:	(optional argument) $T \times n$ data matrix Y to apply inverse filter to

Return value

- `pacf(macf);`
- `pacf(macf, alogdet);`
  Returns a $T \times 1$ matrix with the partial autocorrelation function of the first column of `macf`.
- `pacf(macf, alogdet, my);`
  Returns a $T \times (n+1)$ matrix with the residuals from the filter based on the specified ACF applied to the columns of `my`. The last column contains the standard devations of the filter.
- `pacf(macf, meps);`
  Returns a $T \times n$ matrix with the fitted values from applying the inverse filter based on the specified ACF applied to the columns of `my`.

Returns 0 if the computations fail (the stochastic process has a root on the unit circle).

Description

Given autocovariance (or autocorrelation) functions in the first column of `macf`, this function computes the partial autocorrelations using Durbin's method as described in Golub and Van Loan (1989, §4.7.2). This corresponds to recursively solving the Yule-Walker equations. For example, with autocorrelations, $\rho_0, \rho_1, \rho_2, \ldots$, the first reported partial correlation is 1. The second is the solution p_1 from $(\rho_0\rho_1)' = \mathcal{T}(\rho_0\rho_1)(p_0p_1)'$, the third is p_2 from $(\rho_0\rho_1\rho_2)' = \mathcal{T}(\rho_0\rho_1\rho_2)(p_0p_1p_2)'$. This may be verified by repeatedly using the function `solvetoeplitz`. See under `toeplitz` for the $\mathcal{T}(\cdot)$ notation.

For the theoretical PACF of an ARMA(p, q) process, use the results from `armavar` as input. For the sample PACF, use the results from `acf`.

When a data matrix is specified, the filter (corresponding to the specified ACF) is applied to the data, returning the residuals $\mathbf{E}$. This corresponds to applying the inverse Choleski factor to the data matrix:

$$\mathcal{T}(\rho_0\rho_1\ldots) = LDL' = PP', \quad \mathbf{E} = D^{-1/2}L^{-1}Y = P^{-1}Y.$$

As in `decldl`, L is lower diagonal, with ones on the diagonal. D contains the squared diagonal values, which here correspond to the residual variances. The last column of the return value holds the diagonal of $D^{1/2}$. The log-determinant of $\mathcal{T}(\cdot)$ corresponds to twice the sum of the log of the last column of the return value.

When `logdet` is absent, and a data matrix (e.g. white noise) is specified, the inverse filter (corresponding to the specified ACF) is applied to the data, returning the generated data $\mathbf{Y}$. This corresponds to applying the Choleski factor to the data matrix:

$$\mathcal{T}(\rho_0\rho_1\ldots) = LDL' = PP', \quad \mathbf{Y} = PE.$$

This allows for generating data according to the specified ACF when the input is standard normal random data. In general, this is slower than applying P directly. However, for large T, storage of P may become prohibitive.

See also

`acf`, `arma0`, `armavar`, `solvetoeplitz`

Example

```
#include <oxstd.oxh>
#include <arma.oxh>
main()
{
    decl ct = 5;
    decl acf1 = armavar(<0.5>, 1, 0, (1 - 0.5^2),  ct)';
    decl acf2 = armavar(<-0.5>, 1, 0, (1 - (-0.5)^2), ct)';
    decl acf3 = armavar(<0.5>, 0, 1, 1 / (1 + 0.5^2), ct)';
    decl y = rann(ct,1), logdet, e;

    print(pacf(acf1) ~ pacf(acf2) ~ pacf(acf3));

    e = pacf(acf3, &logdet, y);
    print(e ~ invert(choleski(toeplitz(acf3))) * y);
    print("logdet = ", logdet, " ",
        2 * double(sumc(log(e[][1]))) );

    e = pacf(acf1, &logdet, y);
    print(arma0(y, <0.5>, 1, 0) ~ e[][0] .* e[][1]);
    e = pacf(acf3, &logdet, y);
    //differ, but will be the same beyond approx. 10 obs:
    print(arma0(y, <0.5>, 0, 1) ~ e[][0] .* e[][1]);
}
```

produces

```
        1.0000        1.0000        1.0000
       0.50000      -0.50000       0.40000
       0.00000       0.00000      -0.19048
       0.00000       0.00000      0.094118
       0.00000       0.00000     -0.046921

       0.22489        1.0000       0.22489
        1.8004       0.91652        1.8004
       -1.1003       0.89974       -1.1003
      -0.47828       0.89574      -0.47828
      -0.51476       0.89476      -0.51476
logdet = -0.82828 -0.82828
       0.00000       0.22489
        1.6276        1.6276
       -1.0743       -1.0743
      -0.81547      -0.81547
      -0.21537      -0.21537

       0.22489       0.22489
        1.6276        1.6501
       -1.0181      -0.99002
      -0.40857      -0.42842
      -0.46988      -0.46059
```

11.2 Maximization package

The maximization package implements maximization of functions of (several) parameters, as well as numerical differentiation. The maximization package requires the header file `maximize.oxh`, and linking in of `maximize.oxo`. This is achieved by adding `#import <maximize>` at the top of your code.

11.2.1 Maximization control

Several aspects of maximization can be changed from the default settings, including convergence tolerances, the number of iterations and the amount of output.

Three methods are available

1. by changing the global settings (`GetMaxControl`, `GetMaxControlEps`, `MaxControl`, `MaxControlEps`);
2. through additional arguments to `MaxBFGS` etc.;
3. using a `CMaxControl` object as argument.

CMaxControl

`CMaxControl(const iOptions=0);`
Constructor. Only possible option is `CMaxControl::PARALLEL_SCORE`.

`GetControl();`
returns `{ mxIter, iPrint, bCompact }`.

`GetEps();`
returns `{ dEps1, dEps2 }`.

`GetIterationCount();`
returns the iteration count.

`GetResult();`
returns the convergence code.

`SetControl(const mxIter, const iPrint=-1, const bCompact=-1);`
See `MaxControl`.

`SetEps(const dEps1, const dEps2=-1);`
See `GetMaxControl`.

`SetIterationCount(const cIter);`
Sets the iteration count.

`SetOptions(const iOptions);`
Only possible option is `CMaxControl::PARALLEL_SCORE`.

`SetResult(const iResult);`
Sets the convergence code.

Description

The `CMaxControl` class manages the configurable maximization options in a more convenient way. The added flexibility is that parallel numerical scores can be used, and the number of iterations retrieved upon convergence.

GetMaxControl, GetMaxControlEps

```
GetMaxControl();
GetMaxControlEps();
```

Return value

Return an array with three values and two values respectively.

`GetMaxControl` returns `{ mxIter, iPrint, bCompact }`.

`GetMaxControlEps` returns `{ dEps1, dEps2 }`.

See also

`MaxControl`, `MaxControlEps`

MaxControl, MaxControlEps

```
MaxControl(const mxIter, const iPrint);
MaxControl(const mxIter, const iPrint, const bCompact);
MaxControlEps(const dEps1, const dEps2);
```

`mxIter`	in:	int, maximum number of iterations; default is 1000, use -1 to leave the current value unchanged
`iPrint`	in:	int, print results every iPrint'th iteration; default is 0, use -1 to leave the current value unchanged
`bCompact`	in:	int, if `TRUE` uses compact format for iteration results (optional argument)
`dEps1`	in:	double, ϵ_1, default is 10^{-4}, use ≤ 0 to leave the current value unchanged
`dEps2`	in:	double, ϵ_2, default is 5×10^{-3}, use ≤ 0 to leave the current value unchanged

Return value

No return value.

Description

The `MaxControl` and `MaxControlEps` functions provide control over some iteration parameters. Use a value of –1 for `mxIter`, `iPrint`, `dEps1` or `dEps2` to leave the current value unchanged.

See also

`GetMaxControl`, `GetMaxControlEps`, `MaxBFGS` (for an example), `MaxSimplex`

MaxConvergenceMsg

```
MaxConvergenceMsg(const iCode);
```

`iCode`	in:	int, code returned by `MaxBFGS`, `MaxNewton`, etc.

Return value

Returns the text corresponding to the convergence code listed under the return values of `MaxBFGS`.

See also

`MaxBFGS` (for an example), `MaxNewton`, `MaxSimplex`, `MaxSQP`, `MaxSQPF`

11.2.2 Maximization functions

MaxBFGS

```
MaxBFGS(const func, const avP, const adFunc, const amInvHess,
    const fNumDer);
MaxBFGS(const func, const avP, const adFunc, const amInvHess,
    const fNumDer, const objMaxCtrl);
```

`func`	in:	a function computing the function value, optionally with derivatives
`avP`	in:	address of $p \times 1$ matrix with starting values
	out:	$p \times 1$ matrix with final coefficients
`adFunc`	in:	address
	out:	double, final function value
`amInvHess`	in:	address of $p \times p$ matrix, initial (inverse negative) quasi-Hessian $\mathbf{K}$; a possible starting value is the identity matrix
		or: `0`, in which case the identity matrix is used
	out:	if not 0 on input: final $\mathbf{K}$ (not reliable as estimate of actual Hessian)
`fNumDer`	in:	0: `func` provides analytical first derivatives
		1: use numerical first derivatives
`objMaxCtrl`	in:	`CMaxControl` object (optional argument)
	out	updated to reflect status and iteration count.

The supplied `func` argument should have the following format:

```
func(const vP, const adFunc, const avScore, const amHessian);
```

`vP`	in:	$p \times 1$ matrix with coefficients
`adFunc`	in:	address
	out:	double, function value at `vP`
`avScore`	in:	0, or an address
	out:	if !0 on input: $p \times 1$ matrix with first derivatives at `vP`
`amHessian`	in:	always 0 for MaxBFGS, as it does not need the Hessian
returns		1: successful, 0: function evaluation failed

Return value

Returns the status of the iterative process:

`MAX_CONV` *Strong convergence*

Both convergence tests (11.2) and (11.3) were passed, using tolerance $\epsilon = \epsilon_1$.

`MAX_WEAK_CONV` *Weak convergence (no improvement in line search)*

The step length s_i has become too small. The convergence test (11.2) was passed, using tolerance $\epsilon = \epsilon_2$.

`MAX_MAXIT` *No convergence (maximum no of iterations reached)*

`MAX_LINE_FAIL` *No convergence (no improvement in line search)*

The step length s_i has become too small. The convergence test (11.2) was not passed, using tolerance $\epsilon = \epsilon_2$.

`MAX_FUNC_FAIL` *No convergence (function evaluation failed)*

The chosen default values for the tolerances are:

$$\epsilon_1 = 10^{-4},\ \epsilon_2 = 5 \times 10^{-3}.$$

Description

MaxBFGS maximizes a function, using the quasi-Newton method developed by Broyden, Fletcher, Goldfarb, Shanno (BFGS). The function either uses supplied analytical first derivatives, or numerical first derivatives (in which case only the function values need to be available: this uses the function Num1Derivative).

Using numerical derivatives saves programming (and thinking) time, but analytical dervatives tend to be computable with higher accuracy and over a wider parameter range. The iteration process is unaffected by this choice, other than caused by the slight numerical differences between the two methods (and the lower robustness of numerical derivatives).

A Newton scheme is used to maximize the unconstrained function $f(\boldsymbol{\theta})$:

$$\boldsymbol{\theta}(k+1) = \boldsymbol{\theta}(k) + s(k)\mathbf{Q}(k)^{-1}\mathbf{q}(k), \tag{11.1}$$

with

$\boldsymbol{\theta}(k)$	parameter values at iteration k;
$s(k)$	step length, normally 1;
$\mathbf{Q}(k)$	symmetric positive definite matrix (at iteration k);
$\mathbf{q}(k)$	first derivative of the function (the score vector);
$\boldsymbol{\delta}(k)$	$= \boldsymbol{\theta}(k) - \boldsymbol{\theta}(k-1)$, the change in the parameters;
$\boldsymbol{\gamma}(k)$	$= \mathbf{q}(k) - \mathbf{q}(k-1)$, the change in the score.

The BFGS method updates $\mathbf{K} = \mathbf{Q}^{-1}$ directly, avoiding the need for second derivatives. A linear line search is used when necessary.

Owing to numerical problems it is possible (especially close to the maximum) that the calculated $\boldsymbol{\delta}_i$ does not yield a higher likelihood. Then an $s_i \in [0, 1]$ yielding a higher function value is determined by a line search. Theoretically, since the direction is upward, such an s_i should exist; however, numerically it might be impossible to find one. When using BFGS with numerical derivatives, it often pays to scale the data so that the initial gradients are of the same order of magnitude.

The *convergence* decision is based on two tests. The first uses likelihood elasticities ($\partial\ell/\partial \log \boldsymbol{\theta}$, switching notation from $f(\boldsymbol{\theta})$ to $\ell(\boldsymbol{\theta})$):

$$\begin{aligned} |q_{i,j}\theta_{i,j}| &\le \epsilon \quad \text{for all } j \text{ when } \theta_{i,j} \neq 0, \\ |q_{i,j}| &\le \epsilon \quad \text{for all } j \text{ with } \theta_{i,j} = 0. \end{aligned} \tag{11.2}$$

The second is based on the one-step-ahead relative change in the parameter values:

$$\begin{aligned} |\delta_{i+1,j}| &\le 10\epsilon\ |\theta_{i,j}| \quad \text{for all } j \text{ with } \theta_{i,j} \neq 0, \\ |\delta_{i+1,j}| &\le 10\epsilon \quad \text{for all } j \text{ when } \theta_{i,j} = 0. \end{aligned} \tag{11.3}$$

The final inverse negative quasi-Hessian $\mathbf{K}$ can not reliably used to estimate standard errors. When, for example, iteration starts in the maximum with an identity matrix as initial quasi-Hessian, the final-Hessian will also be the identity matrix. Instead, it is possible to take the inverse of minus the numerical second derivatives.

Note that the code resides in src/maximize.ox. To use this function, either include the code, or link the corresponding maximize.oxo file using #import <maximize>.

See also

MaxControl, MaxConvergenceMsg, MaxNewton, Num1Derivative, Num2Derivative

Example

The following example minimizes the so-called Rosenbrock function (see Fletcher, 1987):

$$f(\alpha, \beta) = 100 * \left(\beta - \alpha^2\right)^2 + (1 - \alpha)^2 .$$

No data are involved. It is easily seen that the minimum is at $(1, 1)$ with function value 0. The contours are rather banana-shaped. The program maximizes the function twice, starting from (0,0), once with analytical derivatives, once without:

....*samples/maximize/maxbfgs.ox*

```
#include <oxstd.oxh>
#import <maximize>

fRosenbrock(const vP, const adFunc, const avScore, const amHessian)
{
    adFunc[0] = -100 * (vP[1] - vP[0] ^ 2) ^ 2
        - (1 - vP[0]) ^ 2;                // function value

    if (avScore)                      // if !0: compute score
    {  // this bit is not needed for numerical derivatives
        (avScore[0])[0] = 400 * (vP[1] - vP[0]^2)
             * vP[0] + 2 * (1 - vP[0]);
        (avScore[0])[1] = -200 * (vP[1] - vP[0]^2);
    }
return 1;                              // 1 indicates success
}

main()
{
    decl vp, dfunc, ir;
    MaxControl(1000, 50);

    vp = zeros(2, 1);                      // starting values
    ir = MaxBFGS(fRosenbrock, &vp, &dfunc, 0, FALSE);

    print("\n", MaxConvergenceMsg(ir),
        " using analytical derivatives",
        "\nFunction value = ", dfunc, "; parameters:",vp);

    vp = zeros(2, 1);                      // starting values
    ir = MaxBFGS(fRosenbrock, &vp, &dfunc, 0, TRUE);

    print("\n", MaxConvergenceMsg(ir),
        " using numerical derivatives",
        "\nFunction value = ", dfunc, "; parameters:",vp);
}
```

....

This produces:

```
Starting values
parameters
      0.00000      0.00000
gradients
       2.0000      0.00000
Initial function =                  -1

Position after 20 BFGS iterations
Status: Strong convergence
parameters
       1.0000      0.99999
gradients
 -6.7948e-005  3.8365e-005
function value = -2.29573829351e-011

Strong convergence using analytical derivatives
Function value = -2.29574e-011; parameters:
       1.0000
      0.99999

Starting values
parameters
      0.00000      0.00000
gradients
       2.0000      0.00000
Initial function =                  -1

Position after 20 BFGS iterations
Status: Strong convergence
parameters
       1.0000      0.99999
gradients
 -6.7948e-005  3.8365e-005
function value = -2.30014575614e-011

Strong convergence using numerical derivatives
Function value = -2.30015e-011; parameters:
       1.0000
      0.99999
```

MaxNewton

```
MaxNewton(const func, const avP, const adFunc, const amInvHess,
    const fNumDer);
MaxNewton(const func, const avP, const adFunc, const amInvHess,
    const fNumDer, const objMaxCtrl);
```

`func`	in:	a function computing the function value, optionally with derivatives
`avP`	in:	address of $p \times 1$ matrix with starting values
	out:	$p \times 1$ matrix with final coefficients
`adFunc`	in:	address
	out:	double, final function value
`amHessian`	in:	address, or `0`
	out:	if not 0 on input: final Hessian $\mathbf{H}$
`fNumDer`	in:	0: `func` provides analytical second derivatives
		1: use numerical second derivatives
`objMaxCtrl`	in:	`CMaxControl` object (optional argument)
	out	updated to reflect status and iteration count.

The supplied `func` argument should have the following format:

```
func(const vP, const adFunc, const avScore, const amHessian);
```

`vP`	in:	$p \times 1$ matrix with coefficients
`adFunc`	in:	address
	out:	double, function value at `vP`
`avScore`	in:	0, or an address
	out:	if !0 on input: $p \times 1$ matrix with first derivatives at `vP`
`amHessian`	in:	0, or an address
	out:	if !0 on input: $p \times p$ matrix with second derivatives (Hessian matrix) at `vP`
returns		1: successful, 0: function evaluation failed

Return value

Returns the status of the iterative process, see `MaxBFGS`.

Description

`MaxNewton` maximizes a function, using the Newton method. The function expects analytical first derivatives (scores), and either uses supplied analytical second derivatives (Hessian), or computes the Hessian numerically. The numerical second derivatives are computed using forward differences on the scores.

Using numerical derivatives saves programming (and thinking) time, but analytical dervatives tend to be computable with higher accuracy and over a wider parameter range. The iteration process is unaffected by this choice, other than caused by the small numerical differences between the two methods (and the lower robustness of numerical derivatives).

`MaxNewton` uses a scheme like (11.1) to maximize $f(\boldsymbol{\theta})$:

$$\boldsymbol{\theta}(k+1) = \boldsymbol{\theta}(k) - s(k)\mathbf{H}(k)^{-1}\mathbf{q}(k),$$

where $\mathbf{H}$ is the user supplied Hessian matrix. This requires that $\mathbf{H}$ is negative definite at each step. If this is not the case, a steepest descent step with line search is taken. Otherwise the line search is as discussed in MaxFBGS. The *convergence* decision is also the same as for MaxBFGS.

Since the Hessian matrix is user supplied, this function can be used to implement various methods, for example:

$\mathbf{H}$	description
$\partial^2 f(\boldsymbol{\theta})/\partial\boldsymbol{\theta}\partial\boldsymbol{\theta}'$	Newton's method
$\mathsf{E}[\mathbf{H}]$	method of scoring
$\mathbf{I}$	steepest descent
OPG	outer product of gradients: BHHH method, see Berndt, Hall, Hall, and Hausman, 1974

Note that the code resides in src/maximize.ox. To use this function, either include the code, or link the corresponding maximize.oxo file using #import <maximize>.

See also

MaxBFGS, MaxControl, MaxConvergenceMsg, Num1Derivative, Num2Derivative

Example

The following program extends the MaxBFGS example by adding second derivatives to the Rosenbrock function. Note that we always should check whether the score and Hessian arguments are of type array. For example, during the line search neither are required, and both will be zero. When numerical second derivatives are used the amHessian arguments to fRosenbrock will always be zero.

.... *samples/maximize/maxnewt.ox*

```
#include <oxstd.oxh>
#import <maximize>

fRosenbrock(const vP, const adFunc, const avScore, const amHessian)
{
    decl ab2 = vP[1] - vP[0] ^ 2, a1 = 1 - vP[0];

    adFunc[0] = -100 * ab2 ^ 2 - a1 ^ 2;
    if (avScore)                      // if !0: compute score
    {
        (avScore[0])[0] = 400 * ab2 * vP[0] + 2 * a1;
        (avScore[0])[1] = -200 * ab2;
    }
    if (amHessian)                  // if !0: compute Hessian
    {  // this bit is not needed for numerical derivatives
        (amHessian[0])[0][0] =
            400 * vP[1] - 1200 * vP[0]^2 - 2;
        (amHessian[0])[1][1] = -200;
        (amHessian[0])[1][0] =
            (amHessian[0])[0][1] = 400 * vP[0];
    }
return 1;                            // 1 indicates success
}

main()
{
    decl vp, dfunc, ir, mhess;

//    MaxControl(100, 1, 1);

    vp = zeros(2, 1);                  // starting values
    ir = MaxNewton(fRosenbrock, &vp, &dfunc,&mhess, TRUE);

    print("\n", MaxConvergenceMsg(ir),
        " using numerical 2nd derivatives",
        "\nFunction value = ", dfunc, "; parameters:", vp,
        "final Hessian:", mhess);

    vp = zeros(2, 1);                  // starting values
    ir = MaxNewton(fRosenbrock, &vp, &dfunc,&mhess,FALSE);

    print("\n", MaxConvergenceMsg(ir),
        " using analytical 2nd derivatives",
        "\nFunction value = ", dfunc, "; parameters:", vp,
        "final Hessian:", mhess);
}
```

produces

```
Strong convergence using numerical 2nd derivatives
Function value = -2.45742e-009; parameters:
       1.0000
       1.0000
final Hessian:
     -0.49900     -0.99799
     -0.99799      -2.0010

Strong convergence using analytical 2nd derivatives
Function value = -1.22009e-012; parameters:
       1.0000
       1.0000
final Hessian:
     -0.50000     -0.99999
     -0.99999      -2.0050
```

MaxSimplex

```
MaxSimplex(const func, const avP, const adFunc, vDelta);
MaxSimplex(const func, const avP, const adFunc, vDelta,
    const objMaxCtrl);
```

`func`	in:	a function computing the function value
`avP`	in:	address of $p \times 1$ matrix with starting values
	out:	$p \times 1$ matrix with coefficients at convergence
`adFunc`	in:	address
	out:	double, function value at convergence
`vDelta`	in:	0, or a $p \times 1$ matrix with the initial simplex (if 0 is specified, the score is used for the initial simplex)
`objMaxCtrl`	in:	`CMaxControl` object (optional argument)
	out	updated to reflect status and iteration count.

The supplied `func` argument should have the same format as in `MaxBFGS`.

Return value

Returns the status of the iterative process, as documented under `MaxBFGS`.

Description

Maximizes a function using the simplex method, see for example Applied Statistics algorithm AS 47 (O'Neil, 1971). The simplex method can be rather slow. For reasonably well behaved functions, a preferred derivative free method is MaxBFGS using numerical derivatives.

Note that the code resides in `src/maximize.ox`. To use this function, either include the code, or link the corresponding `maximize.oxo` file using `#import <maximize>`.

See also

`MaxBFGS`

Example

.... *samples/maximize/maxboth.ox*

```
#include <oxstd.oxh>
#include <oxfloat.h>
#import <maximize>

fRosenbrock(const vP, const adFunc, const avScore, const amHessian)
{
    adFunc[0] =
        -100 * (vP[1][0] - vP[0][0] ^ 2) ^ 2 - (1 - vP[0][0]) ^ 2;

return 1;
}
fPowell(const vP, const adFunc, const avScore, const amHessian)
{
    adFunc[0] =
    -((vP[0][0] + 10*vP[1][0]) ^ 2 + 5 * (vP[2][0] - vP[3][0]) ^ 2
     +(vP[1][0] - 2*vP[2][0]) ^ 4 + 10 * (vP[0][0] + vP[3][0]) ^ 4);

return 1;
}
fQuad(const vP, const adFunc, const avScore, const amHessian)
```

```
{
    adFunc[0] = -double(sumc(vP .^ 4));

return 1;
}

main()
{
    decl vp, vf, mh;

    format(66);            // shorter lines than normal
    MaxControl(-1,1000);

    vp = <-1.2;1>;  mh = unit(2);
    MaxBFGS(fRosenbrock, &vp, &vf, &mh, TRUE);
    vp = <-1.2;1>;  mh = unit(2);
    MaxSimplex(fRosenbrock, &vp, &vf, 0 /*<1;1>*/);

    vp = <3;-1;0;1>;  mh = unit(4);
    MaxBFGS(fPowell, &vp, &vf, &mh, TRUE);
    vp = <3;-1;0;1>;  mh = unit(4);
    MaxSimplex(fPowell, &vp, &vf, 0 /*<1;1;1;1>*/);

    vp = ones(10,1); mh = unit(10);
    MaxBFGS(fQuad, &vp, &vf, &mh, TRUE);
    vp = ones(10,1); mh = unit(10);
    MaxSimplex(fQuad, &vp, &vf, 0 /*vp*/);
}
```

produces after some editing of the output:

```
Starting values
parameters
      -1.2000       1.0000
gradients
       215.60       88.000
Initial function =               -24.2

Position after 33 BFGS iterations
Status: Strong convergence
parameters
       1.0000       1.0000
gradients
 -6.6755e-008  4.8263e-008
function value =  -4.0124066543e-016

Starting values
parameters
      -1.2000       1.0000
Initial function =               -24.2

Position after 132 Simplex iterations
Status: Strong convergence
parameters
       1.0000       1.0000
gradients
```

```
  3.1028e-005 -1.5521e-005
function value = -6.02226722279e-013

Starting values
parameters
       3.0000      -1.0000      0.00000       1.0000
gradients
      -2546.0       144.00       2.0000      -2570.0
Initial function =               -2615

Position after 50 BFGS iterations
Status: Strong convergence
parameters
 -4.2609e-005  4.2609e-006  -0.00017248  -0.00017248
gradients
  6.0617e-010  1.9090e-009 -2.5012e-010  9.8933e-010
function value = -3.62744789919e-014

Starting values
parameters
       3.0000      -1.0000      0.00000       1.0000
Initial function =               -2615

Position after 239 Simplex iterations
Status: Strong convergence
parameters
  -0.00081637  8.1527e-005   0.00029861   0.00029848
gradients
  2.2137e-006  2.2082e-005 -1.3407e-006  1.3451e-006
function value = -2.09880254028e-012

Starting values
parameters
       1.0000       1.0000       1.0000       1.0000       1.0000
       1.0000       1.0000       1.0000       1.0000       1.0000
gradients
      -4.0000      -4.0000      -4.0000      -4.0000      -4.0000
      -4.0000      -4.0000      -4.0000      -4.0000      -4.0000
Initial function =                 -10

Position after 1 BFGS iterations
Status: Strong convergence
function value =                   0  steplen = 0.25
Initial function =                 -10

Position after 454 Simplex iterations
Status: Strong convergence
parameters
   0.00012390  -0.00040964   0.00099913  7.2798e-005  -0.00027496
   0.00085512  -0.00076729  -0.00081975   0.00052821  -0.00060839
gradients
 -7.6214e-012  2.7501e-010 -3.9896e-009 -1.5505e-012  8.3175e-011
 -2.5012e-009  1.8070e-009  2.2036e-009 -5.8956e-010  9.0080e-010
function value =  -2.5783761224e-012
```

MaxSQP, MaxSQPF

```
#import <maxsqp>
MaxSQP(const func, const avP, const adFunc, const amHessian,
    const fNumDer, const cfunc_gt0, const cfunc_eq0, vLo, vHi, ...);
MaxSQP(const func, const avP, const adFunc, const amHessian,
    const fNumDer, const cfunc_gt0, const cfunc_eq0, vLo, vHi,
    const cfunc_gt0_jac, const cfunc_eq0_jac, const objMaxCtrl);
MaxSQPF(const func, const avP, const adFunc, const amHessian,
    const fNumDer, const cfunc_gt0, const cfunc_eq0, vLo, vHi, ...);
MaxSQPF(const func, const avP, const adFunc, const amHessian,
    const fNumDer, const cfunc_gt0, const cfunc_eq0, vLo, vHi,
    const cfunc_gt0_jac, const cfunc_eq0_jac, const objMaxCtrl);
```

`func`	in:	a function computing the function value, optionally with derivatives
`avP`	in:	address of $p \times 1$ matrix with starting values
	out:	$p \times 1$ matrix with final coefficients
`adFunc`	in:	address
	out:	double, final function value
`amHessian`	in:	address, or `0`
	out:	if not 0 on input: final Hessian (BFGS-style) approximation $\mathbf{B}$
`fNumDer`	in:	0: `func` provides analytical first derivatives 1: use numerical first derivatives
`vLo`	in:	$p \times 1$ matrix with lower bounds, or `<>`
`vHi`	in:	$p \times 1$ matrix with upper bounds, or `<>`
`objMaxCtrl`	in:	`CMaxControl` object (optional argument)
	out	updated to reflect status and iteration count.

The supplied `func` argument should have the same format as in `MaxBFGS`.

The `cfunc_gt0` argument can be zero, or a function evaluating the nonlinear constraints (which will be constrained to be positive) with the following format:

```
cfunc_gt0(const avF, const vP);
```

`avF`	in:	address
	out:	$m \times 1$ matrix with inequality constraints at `vP`
`vP`	in:	$p \times 1$ matrix with coefficients
returns		1: successful, 0: constraint evaluation failed

The `cfunc_eq0` argument can be zero, or a function evaluating the nonlinear constraints (which will be constrained to zero) with the following format:

```
cfunc_eq0(const avF, const vP);
```

`avF`	in:	address
	out:	$m_e \times 1$ matrix with equality constraints at `vP`
`vP`	in:	$p \times 1$ matrix with coefficients
returns		1: successful, 0: constraint evaluation failed

The `cfunc_gt0_jac` and `cfunc_eq0_jac` are optional functions that return the analytical Jacobian matrix of the constraints. They have the same format, returning in `avF` an $m \times 1$ and an $m_e \times p$ matrix respectively.

Return value

Returns the status of the iterative process, see `MaxBFGS`.

Description

`MaxSQP` implements a sequential quadratic programming technique to maximize a non-linear function subject to non-linear constraints, similar to Algorithm 18.7 in Nocedal and Wright (1999).

`MaxSQPF` enforces all iterates to be feasible, using the Algorithm by Lawrence and Tits (2001). The current version does not support equality constraints. If a starting point is infeasible, `MaxSQPF` will try to minimize the squared constraint violations to find a feasible point.

Note that the code resides in `src/maxsqp.ox`. To use these functions add the line `#import <maxsqp>` at the top of the file.

See also

`MaxBFGS`, `MaxControl`, `MaxConvergenceMsg`, `Num1Derivative`, `Num2Derivative`

Example

See `ox\samples\maximize`.

Num1Derivative, Num2Derivative

```
Num1Derivative(const func, vP, const avScore);
Num1Derivative_parallel(const func, vP, const avScore);
Num2Derivative(const func, vP, const amHessian);
Num2Derivative_parallel(const func, vP, const amHessian);
```

`func`	in:	a function computing the function value, optionally with derivatives
`vP`	in:	$p \times 1$ matrix with parameter values
`mHessian`	in:	$p \times p$ matrix, initial Hessian
`avScore`	in:	an address
	out:	$p \times 1$ matrix with 1st derivatives at `vP`
`amHessian`	in:	an address
	out:	$p \times p$ matrix with 2nd derivatives at `vP`

The supplied `func` argument should have the format as documented under `MaxBFGS`.

Return value

Returns 1 if successful, 0 otherwise.

Description

These functions take numerical first and second differences of a function based on a central finite difference approximation. The numerical derivatives are calculated using:

$$\frac{f(\theta + \epsilon\imath) - f(\theta - \epsilon\imath)}{\mu} \simeq \frac{\partial f(\theta)}{\partial(\imath'\theta)}$$

where $\imath$ is a unit vector (for example, $(1\ 0 \ldots 0)'$ for the first element of θ), ϵ is a suitably chosen step length. Thus, ϵ represents a compromise between round-off error (cancellation of leading digits when subtracting nearly equal numbers) and truncation error (ignoring terms of higher order than ϵ in the approximation). Although the Ox code chooses ϵ carefully, there may be situations where the numerical derivative performs poorly.

If in `Num1Derivative` one-side fails, the procedure will use a one-sided difference.

The numerical values of second derivatives can be computed in a corresponding way using:

$$\frac{f(\theta + \epsilon_1\imath + \epsilon_2\jmath) + f(\theta - \epsilon_1\imath - \epsilon_2\jmath) - f(\theta - \epsilon_1\imath + \epsilon_2\jmath) - f(\theta + \epsilon_1\imath - \epsilon_2\jmath)}{4\epsilon_1\epsilon_2}$$

where $\imath$ or $\jmath$ is zero except for unity in the i^{th} or j^{th} position.

`Num1Derivative_parallel` and `Num2Derivative_parallel` use a parallel for loop over the parameters.

Note that the code resides in `src/maximize.ox`. Add `#import <maximize>` to use this function.

See also

`MaxBFGS`

Example

The following example is based on the Rosenbrock function (see `MaxBFGS`):

samples/maximize/numder.ox

```
#include <oxstd.oxh>
#import <maximize>

fRosenbrock(const vP, const adFunc, const avScore, const amHessian)
{
    adFunc[0] = -100 * (vP[1][0] - vP[0][0] ^ 2) ^ 2
        - (1 - vP[0][0]) ^ 2;            // function value

    if (avScore)                   // if !0: compute score
    {  // this bit is not needed for numerical derivatives
        (avScore[0])[0][0]= 400 * (vP[1][0] - vP[0][0]^2)
             * vP[0][0] + 2 * (1 - vP[0][0]);
        (avScore[0])[1][0]=-200 * (vP[1][0] - vP[0][0]^2);
    }
return 1;
}

main()
{
    decl vp, dfunc, vscore, mhess;

    vscore = vp = zeros(2, 1);          // starting values

    fRosenbrock(vp, &dfunc, &vscore, 0);
    print("analytical first derivative at <0;0>", vscore);

    if (Num1Derivative(fRosenbrock, vp, &vscore))
        print("numerical 1st derivative at <0;0>", vscore);

    if (Num2Derivative(fRosenbrock, vp, &mhess))
        print("numerical 2nd derivative at <0;0>", mhess);
}
```

produces

```
analytical first derivative at <0;0>
       2.0000
      0.00000
numerical 1st derivative at <0;0>
       2.0000
      0.00000
numerical 2nd derivative at <0;0>
      -2.0000      0.00000
      0.00000      -200.00
```

NumJacobian

```
NumJacobian(const func, vU, const amJacobian);
```

`func`	in:	function mapping from restricted to unrestricted parameters
`vU`	in:	of $u \times 1$ matrix with parameters
`amJacobian`	in:	address
	out:	$r \times u$ Jacobian matrix corresponding to mapping

The supplied `func` argument should have the following format:

```
func(const avR, const vU);
```

`avR`	in:	address
	out:	$r \times 1$ matrix with restricted coefficients
`vU`	in:	$u \times 1$ matrix with unrestricted coefficients
returns		1: successful, 0: function evaluation failed

Return value

Returns 1 if successful, 0 otherwise.

Description

Computes the Jacobian matrix of the restrictions imposed of the form $\theta = f(\phi)$: $J = \partial f(\phi)/\partial\theta'$; $f(\cdot)$ is an r-vector, ϕ is an u-vector.

Use `#import <maximize>` (the code is in `src/maximize.ox`).

See also

`Num1Derivative`

Example

.... *samples/maximize/jacobian.ox*

```
#include <oxstd.oxh>
#import <maximize>

fMap(const avR, const vU)
{
    avR[0] = vU[ : rows(vU) - 2][] .^ 2; // drop last row, square
return 1;
}
main()
{
    decl vp, mjacob;

    if (NumJacobian(fMap, ones(4, 1), &mjacob))
        print("numerical Jacobian at <1;1;1;1>", mjacob);
    if (NumJacobian(fMap, zeros(4, 1), &mjacob))
        print("numerical Jacobian at <0;0;0;0>", mjacob);
}
```

....

```
numerical Jacobian at <1;1;1;1>
       2.0000      0.00000      0.00000      0.00000
      0.00000       2.0000      0.00000      0.00000
      0.00000      0.00000       2.0000      0.00000
numerical Jacobian at <0;0;0;0>
      0.00000      0.00000      0.00000      0.00000
      0.00000      0.00000      0.00000      0.00000
      0.00000      0.00000      0.00000      0.00000
```

SolveNLE

```
#import <solvenle>
SolveNLE(const func, const avX);
SolveNLE(const func, const avX, iMode, funcJac, const objMaxCtrl);
SolveNLE(const func, const avX, iMode, funcJac, dEps1, dEps2,
    mxIter, iPrint, mxItInner);
```

`func`	in:	Ox function evaluating the nonlinear equations (see below)
`avX`	in:	address of $n \times 1$ matrix with starting values
	out:	$n \times 1$ matrix with final coefficients
`iMode`	in:	int, mode of operation: -1 (default): mode 1 if $n < 80$, else mode 3 0: Newton's method using analytical Jacobian 1: Newton's method using numerical Jacobian 2: using Broyden's approximation to Jacobian 3: large scale problem (tensor-gmres method, avoiding $n \times n$ Jacobian matrix)
`funcJac`	in:	a function computing the function value, optionally with derivatives
`dEps1`	in:	double, ϵ_1, default is 10^{-4}, use ≤ 0 to leave the current value unchanged (can also be set with `MaxControlEps`)
`dEps2`	in:	double, ϵ_2, default is 5×10^{-3}, use ≤ 0 to leave the current value unchanged (can also be set with `MaxControlEps`)
`mxIter`	in:	int, maximum number of iterations; default is 1000, use -1 to leave the current value unchanged (can also be set with `MaxControl`)
`iPrint`	in:	int, print results every iPrint'th iteration; default is 0, use -1 to leave the current value unchanged (can also be set with `MaxControl`)
`mxItInner`	in:	int, number of inner iterations for large scale problems, use -1 for the default $\max(50, 10 * \log_{10}(n))$
`objMaxCtrl`	in:	`CMaxControl` object (optional argument)
	out	updated to reflect status and iteration count.

- The supplied `func` argument should have the following format:

  ```
  func(const avF, const vX)
  ```

`avF`	in:	address
	out:	$n \times 1$ matrix with with nonlinear system $f(x)$ evaluated at x
`vX`	in:	$n \times 1$ matrix with coefficients x
returns		1: successful, 0: function evaluation failed

  Return value

  Returns 1 if successful, 0 otherwise.

- When the analytical Jacobian is used, the `funcJac` argument should have the following format:

  ```
  funcJac(const amJac, const vX)
  ```

`amJac`	in:	address
	out:	$n \times n$ Jacobian matrix evaluated at x
`vX`	in:	$n \times 1$ matrix with coefficients x
returns		1: successful, 0: function evaluation failed

Return value

Returns 1 if successful, 0 otherwise.

Return value

Returns the status of the iterative process:

`MAX_CONV` *Strong convergence*

norm$(f(x)) < 0.001\epsilon_1$.

`MAX_WEAK_CONV` *Weak convergence (no improvement in line search)*

The step length has become too small, but norm$(f(x)) < \epsilon_2$.

`MAX_MAXIT` *No convergence (maximum no of iterations reached)*

`MAX_LINE_FAIL` *No convergence (no improvement in line search)*

The step length has become too small and weak convergence was not achieved.

`MAX_FUNC_FAIL` *No convergence (function evaluation failed)*

`MAX_NOCONV` *No convergence*

Probably not yet attempted to solve the system.

The chosen default values for the tolerances are:

$$\epsilon_1 = 10^{-4},\ \epsilon_2 = 5 \times 10^{-3}.$$

Description

Solves a system $f(x)$ of n nonlinear equations in n unknowns. The principle method implemented is the tensor–Newton method, using either a numerical or analytical Jacobian matrix. The tensor–Newton method is similar (but not identical) to that discussed in Schnabel and Frank (1985). There is an option to use the Broyden approximation to the Jacobian instead, but that often works less well in practice. Finally, a large scale option avoids the $n \times n$ Jacobian matrix and uses the gmres method to approximately solve the linear system, and nonlinear gmres to solve the tensor system (so is different from Feng and Pulliam, 1997).

For a general overview see, e.g., Dennis Jr. and Schnabel (1983) and Nocedal and Wright (1999).

Note that the code resides in `src/solvenle.ox`. Add `#import <solvenle>` to use this function.

See also

`CMaxControl`, `MaxControl`, `MaxControlEps`

Example

.... *samples/maximize/solvenle1.ox (part of)*

```
#include <oxstd.oxh>
#import <maximize>
#import <solvenle>

test813(const avF, const vX)
{
    avF[0] = vX[0] + vX[1] - 3 | sqr(vX[0]) + sqr(vX[1]) - 9;
    return 1;
}
```

```
test813_jac(const amJac, const vX)
{
    amJac[0] = (1 ~ 1) | (2 * vX[0] ~ 2 * vX[1]);
    return 1;
}

main()
{
    decl x;
    MaxControl(-1, 1, 1);

    x = <1;5>;
    println("==== Using numerical Jacobian:");
    SolveNLE(test813, &x);
    println("x=", x);

    x = <1;5>;
    println("\n==== Using analytical Jacobian:");
    SolveNLE(test813, &x, 0, test813_jac);
    println("x=", x);
}
```
....

produces

```
==== Using numerical Jacobian:
it0     f'f/2=     149.0000 ||f||=     17.000
it1     f'f/2=     10.26612 ||f||=     4.5313 slope=     -298.00
it2     f'f/2=    0.1598528 ||f||=    0.56543 slope=     -20.550
it3     f'f/2= 0.0001087224 ||f||=   0.014746 slope=    -0.32032
it4     f'f/2=3.744902e-022 ||f||=2.2677e-011 slope= -0.00021762
SolveNLE(1): Strong convergence
x=
  1.1541e-011
       3.0000

==== Using analytical Jacobian:
it0     f'f/2=     149.0000 ||f||=     17.000
it1     f'f/2=     10.26611 ||f||=     4.5312 slope=     -298.00
it2     f'f/2=    0.1598527 ||f||=    0.56543 slope=     -20.550
it3     f'f/2= 0.0001087224 ||f||=   0.014746 slope=    -0.32032
it4     f'f/2=3.243347e-023 ||f||=8.0540e-012 slope= -0.00021762
SolveNLE(0): Strong convergence
x=
 -1.3427e-012
       3.0000
```

SolveQP

```
#import <solveqp>
SolveQP(const mG, const vG, const mA, const vB, const mC,
    const vD, const vLo, const vHi);
SolveQPE(const mG, const vG, const mC, const vD);
SolveQPS(const sFile, const iVerbose)
SolveQPS(const sFile, const iVerbose, const fnSolveQP)
```

`mG`	in:	$n \times n$ matrix G with quadratic weights, or $n \times 1$ vector with diagonal of G
`vG`	in:	$n \times 1$ vector g with linear weights
`mA`	in:	$m \times n$ matrix A with linear inequality constraints $Ax \geq b$ (may be empty)
`vB`	in:	$m \times 1$ vector b with right-hand side for linear inequality constraints (empty if A is empty)
`mC`	in:	$m_e \times n$ matrix C with linear equality constraints (may be empty)
`vD`	in:	$m_e \times 1$ vector d with right-hand side for linear equality constraints (empty if C is empty)
`vLo`	in:	$n \times 1$ vector with lower bounds (may be empty)
`vHi`	in:	$n \times 1$ vector with upper bounds (may be empty)
`sFile`	in:	string with `.qps` file name
`iVerbose`	in:	int, 0 for no output, 1 for one line summary output, 2 to print all matrices and results
`fnSolveQP`	in:	(optional argument) QP solver with call syntax as `SolveQP`. If absent `SolveQP` is used.

Return value

`SolveQP` returns an array with three elements:

`[0]` integer return value:

- 0 success
- 1 initial point not feasible (should only be possible when `SolveQPIF` is called directly)
- 2 maximum number of iterations reached

`[1]` $n \times 1$ vector with solution x

`[2]` $m^* \times 1$ vector with Lagrange multipliers λ, $m^* = m_e + m + 2n$
in order: equality constraints, inequality constraints, lower bounds, upper bounds.

`SolveQPE` returns an array with three elements:

`[0]` $n \times 1$ vector with solution x

`[1]` $m_e \times 1$ vector with Lagrange multipliers λ

`[2]` $p \times 1$ vector with index of redundant constraint ($p = 0$ if all constraints were used)

`SolveQPS` returns an array with four elements: the first three as *SolveQP*, the fourth is the value of the objective function $f(x)$.

Description

SolveQP solves the quadratic program

$$\begin{aligned} \min f(x) = x'Gx/2 + x'g, &\ \text{subject to:} \\ Ax \geq b, \\ Cx = d, \\ x_{lo} \leq x \leq x_{hi}. \end{aligned}$$

using an active set method based on a QR decomposition of $G^{-1}A'$. This is updated using `decqrupdate` to achieve reasonable speed. If G is not positive definite, a small number is added to its diagonal. Sparseness is not taken into account, so `SolveQP` is not appropriate for large problems (say more than 1000 variables or constraints). `SolveQP` implements a pre-processing step, where bounds which are part of A are moved to the explicit bound variables, and the remaining restrictions are checked for the tightest bounds. See, for example, Nocedal and Wright (1999) or Fletcher (1987) for an overview.

Note that the code resides in `src/solveqp.ox`. Add `#import <solveqp>` to use this function.

See also

`MaxFSQP`

Example

....*samples/maximize/solveqp1.ox*

```
#include <oxstd.oxh>
#import  <solveqp>

main()
{
    decl mg, vg, ma, vb, x, iret;
    mg = <4,2,2;2,4,0;2,0,2>;
    vg = <-8;-6;-4>;
    ma = <-1,-1,-2>;
    vb = <-3>;

    [iret,x] = SolveQP(mg, vg, ma, vb, <>, <>, <>, <>);
    println("HS35    result from SolveQP: ", iret, " (0=OK)",
        " x'= ", x', "f=", (x'mg*x) / 2 + vg'x + 9);

    [iret,x] = SolveQP(mg, vg, ma, vb, <>, <>,
        <-.Inf;0.5;-.Inf>, <.Inf;0.5;.Inf>);
    println("HS35MOD result from SolveQP: ", iret, " (0=OK)",
        " x'= ", x', "f=", (x'mg*x) / 2 + vg'x + 9);

    [iret,x] = SolveQP(mg, vg, ma, vb, <0,1,0>, <0.5>, <>, <>);
    println("HS35MOD result from SolveQP: ", iret, " (0=OK)",
        " x'= ", x', "f=", (x'mg*x) / 2 + vg'x + 9);

    mg = <0.02;2>;
    vg = <0;0>;
    ma = <10,-1;1,0;-1,0;0,1;0,-1>;
    vb = <10;2;-50;-50;-50>;
    [iret,x] = SolveQP(mg, vg, ma, vb, <>, <>, <>, <>);
    println("HS21    result from SolveQP: ", iret, " (0=OK)",
        " x'= ", x', "f=", (x'(mg.*x)) / 2 + vg'x - 10);
}
```

produces

```
HS35    result from SolveQP: 0 (0=OK) x'=
       1.3333      0.77778      0.44444
f=
      0.11111
HS35MOD result from SolveQP: 0 (0=OK) x'=
       1.5000      0.50000      0.50000
f=
      0.25000
HS35MOD result from SolveQP: 0 (0=OK) x'=
       1.5000      0.50000      0.50000
f=
      0.25000
HS21    result from SolveQP: 0 (0=OK) x'=
       2.0000      0.00000
f=
      -9.9600
```

11.3 Probability package

The probability package contains various probability distributions (the standard library only defines the standard sampling distributions). It also contains random number generators for many distributions. This package requires `#include <oxprob.h>`.

dens...

```
densbeta(const ma, const a, const b);
densbinomial(const ma, const n, const p);
denscauchy(const ma);
densexp(const ma, const lambda);
densextremevalue(const ma, const alpha, const beta);
densgamma(const ma, const dr, const da);
densgeometric(const ma, const p);
densgh(const ma, const nu, const delta, const gamma, const beta);
densgig(const ma, const nu, const delta, const gamma);
denshypergeometric(const ma, const n, const k, const m);
densinvgaussian(const ma, const mu, const lambda);
denskernel(const ma, const itype);
denslogarithmic(const ma, const alpha);
denslogistic(const ma, const alpha, const beta);
denslogn(const ma);
densmises(const ma, const mu, const kappa);
densnegbin(const ma, const k, const p);
denspareto(const ma, const k, const a);
denspoisson(const ma, const mu);
densweibull(const ma, const a, const b);
```

`ma`	in:	arithmetic type
`a,b`	in:	arithmetic type, arguments for Beta distribution
`alpha,beta`	in:	arithmetic type, location and scale parameter
`lambda`	in:	arithmetic type, parameter of exponential distribution
`mu`	in:	arithmetic type, von Mises: mean direction (use `M_PI` for symmetric between 0 and π); Poisson: mean
`kappa`	in:	arithmetic type, dispersion

Return value

The return type is derived as follows:

returns	`ma`	degrees of freedom arguments
$m \times n$ matrix	$m \times n$ matrix	scalar
$m \times n$ matrix	scalar	$m \times n$ matrix
$m \times n$ matrix	$m \times n$ matrix	$m \times n$ matrix
double	scalar	scalar

Returns the requested density at ma (the returned densities are positive):

function	*density (for discrete distributions: Pr$\{X = x\}$)*	
`densbeta`		
Beta (a, b),	$\frac{1}{B(a,b)} x^{a-1}(1-x)^{b-1}$	$0 < x < 1; a > 0, b > 0$
`densbinomial`		
Binomial(n,p)	$\binom{n}{x} p^x q^{n-x}$	$x = 0, 1, \ldots, n;\ 0 \le p \le 1$
`denscauchy`		
Cauchy,	$\left(\pi\left(1+x^2\right)\right)^{-1}$	
`densexp`		
Exponential,	$\lambda e^{-\lambda x}$	$x > 0; \lambda > 0$
`densextremevalue`		
Extreme Value,	$\frac{e^{-(x-\alpha)/\beta}}{\beta} F(x)$ where	$\beta > 0$
(Type I or Gumbel)	$F(x) = \exp\left[-e^{-(x-\alpha)/\beta}\right]$	
`densgamma`		
Gamma	$\frac{a^r}{\Gamma(r)} x^{r-1} e^{-ax}$	$x > 0; r > 0, a > 0$
`densgeometric`		
Geometric	pq^x	$x = 0, 1, \ldots;\ \mu > 0$
`densgh`		
Generalized hyperbolic, see (11.5)		
`densgig`		
Generalized inverse Gaussian, see (11.4)		
`denshypergeometric`		
Hypergeometric	$\binom{K}{x}\binom{M-K}{n-x} / \binom{M}{n}$	$x = 0, 1, \ldots, n$
Pr[x white balls $\mid$ sample n without replacement from K white balls and M in total]		
`densinvgaussian`		
Inverse Gaussian,	$\left(\frac{\lambda}{2\pi x^3}\right)^{1/2} \exp\left[-\frac{\lambda(x-\mu)^2}{2\mu^2 x}\right]$	$x > 0;\ \lambda > 0, \mu > 0$
`denskernel`		
kernel,	see below	
`denslogarithmic`		
Logarithmic	$\frac{-\alpha^x}{x \log(1-\alpha)}$	$x = 1, 2 \ldots;\ 0 < \alpha < 1$
`denslogistic`		
Logistic,	$\frac{F(x)(1-F(x))}{\beta}$	$F(x) = \left[1 + e^{-(x-\alpha)/\beta}\right]^{-1}, \beta > 0,$
`denslogn`		
Lognormal,	$\frac{1}{x(2\pi)^{1/2}} \exp\left[-(\log x)^2/2\right]$	$x > 0$
`densmises`		
von Mises,	see (11.7) below	
`densnegbin`		
Negative Binomial	$\binom{k+x-1}{x} p^k q^x$	$x = 0, 1, \ldots;\ 0 < p \le 1,\ k > 0$
`denspareto`		
Pareto(k, a)	$ak^a x^{-(a+1)}$	$x \ge k > 0;\ a > 0$
`denspoisson`		
Poisson	$\frac{e^{-\mu}\mu^x}{x!}$	$x = 0, 1, \ldots;\ \mu > 0$
`densweibull`		
Weibull	$abx^{b-1} \exp\left(-ax^b\right)$	$x > 0;\ a > 0, b > 0$

`denskernel` arguments:

`itype`	kernel name	form	
'e'	Epanechnikov	$0.75(1-x^2)$	$\lvert x\rvert < 1$
'b'	Biweight (Quartic)	$(15/16)(1-x^2)^2$	$\lvert x\rvert < 1$
't'	Triangular	$1-\lvert x\rvert$	$\lvert x\rvert < 1$
'g'	Gaussian (Normal)	$(2\pi)^{-1/2}\exp\left[-x^2/2\right]$	
'r'	Rectangular (Uniform)	0.5	$\lvert x\rvert < 1$

Description

The information regarding the generalized inverse Gaussian and generalized hyperbolic distributions is based on Barndorff-Nielsen and Shephard (2001). The generalized inverse Gaussian distribution is a rather general model for positive random variables.

If $X \sim GIG(\nu,\delta,\gamma)$ then it has a generalized inverse Gaussian density:

$$\frac{(\gamma/\delta)^\nu}{2K_\nu(\delta\gamma)}x^{\nu-1}\exp\left\{-\frac{1}{2}(\delta^2x^{-1}+\gamma^2x)\right\},\quad \gamma,\delta\geq 0,\quad \nu\in\mathbb{R},\quad x>0, \tag{11.4}$$

where K_ν is a modified Bessel function of the third kind.

The generalized hyperbolic distribution with $\mu=0$, $GH(\nu,\delta,\gamma,\beta)$ has support on the real line. The density is :

$$\frac{(\gamma/\delta)^\nu}{\sqrt{2\pi}\alpha^{\nu-\frac{1}{2}}K_\nu(\delta\gamma)}\left\{\delta^2+x^2\right\}^{\frac{1}{2}\left(\nu-\frac{1}{2}\right)}K_{\nu-\frac{1}{2}}\left(\alpha\left[\delta^2+x^2\right]^{1/2}\right)e^{\beta x}, \tag{11.5}$$

where $\alpha=\sqrt{\beta^2+\gamma^2}$. For $\mu\neq 0$ replace x by $x-\mu$.

Some special cases of the GIG distribution are:

Gamma:	$\Gamma(\nu,\gamma^2/2)$	$=$	$GIG(\nu>0,0,\gamma)$,
Reciprocal Gamma:	$R\Gamma(\nu,\delta^2/2)$	$=$	$GIG(-\nu,\delta,0)$,
Inverse Gaussian:	$IG(\delta,\gamma)$	$=$	$GIG(-\frac{1}{2},\delta,\gamma)$,
Reciprocal inverse Gaussian:	$RIG(\delta,\gamma)$	$=$	$GIG(\frac{1}{2},\delta,\gamma)$,
Positive hyperbolic:	$PH(\delta,\gamma)$	$=$	$GIG(1,\delta,\gamma)$.
Reciprocal positive Hyperbolic:	$PH(\delta,\gamma)$	$=$	$GIG(-1,\delta,\gamma)$.

Some special cases of the GH distribution are:

Normal	$N(0,\sigma^2)$	$=$	$\lim_{\gamma\to\infty}GH(\nu,\gamma,0,\sigma^2\gamma)$,
Normal inverse Gaussian	$NIG(\alpha,\beta,\delta)$	$=$	$GH\left(-\frac{1}{2},\alpha,\beta,\delta\right)$,
Reciprocal NIG	$NRIG(\alpha,\beta,\delta)$	$=$	$GH\left(\frac{1}{2},\alpha,\beta,\delta\right)$,
Hyperbolic	$H(\alpha,\beta,\delta)$	$=$	$GH(1,\alpha,\beta,\delta)$,
Skewed Student's t	$T(\nu,\delta,\beta)$	$=$	$GH(-\nu,\beta,\beta,\delta)$,
Student's t		$=$	$\lim_{\alpha\to\infty}GH(-\nu,\beta,\beta,\delta)$,
Laplace	$La(\alpha,\beta)$	$=$	$GH(1,\alpha,\beta,0)$
Normal Gamma	$N\Gamma(\nu,\delta,\beta)$	$=$	$GH(\nu,\beta,\beta,\delta)$,
Reciprocal hyperbolic	$RH(\alpha,\beta,\delta)$	$=$	$GH(-1,\alpha,\beta,\delta)$.

See also

`prob...`, `quan...`, `tail...`

prob...

```
probbeta(const ma, const a, const b);
probbinomial(const ma, const n, const p);
probbvn(const da, const db, const drho);
probcauchy(const ma);
probexp(const ma, const lambda);
probextremevalue(const ma, const alpha, const beta);
probgamma(const ma, const dr, const da);
probgeometric(const ma, const p);
probhypergeometric(const ma, const n, const k, const m);
probinvgaussian(const ma, const mu, const lambda);
problogarithmic(const ma, const alpha);
problogistic(const ma, const alpha, const beta);
problogn(const ma);
probmises(const ma, const mu, const kappa);
probmvn(const mx, const msigma);
probnegbin(const ma, const k, const p);
probpareto(const ma, const k, const a);
probpoisson(const ma, const mu);
probweibull(const ma, const a, const b);
```

`ma`	in:	arithmetic type
`a,b`	in:	arithmetic type, arguments for Beta distribution
`dr`	in:	arithmetic type
`da`	in:	arithmetic type
`mu`	in:	arithmetic type, von Mises: mean direction (use `M_PI` for symmetric between 0 and π); Poisson: mean
`alpha,beta`	in:	arithmetic type, location and scale parameter
`lambda`	in:	arithmetic type, parameter of exponential distribution
`kappa`	in:	arithmetic type, dispersion
`nc`	in:	arithmetic type, non-centrality parameter
`da,db`	in:	arithmetic type, upper limits of integration
`drho`	in:	arithmetic type, correlation coefficient
`mx`	in:	$m \times n$ matrix for n-variate normal
`msigma`	in:	$n \times n$ variance matrix $\mathbf{\Sigma}$

Return value

The return type for `probbvn` is a double if all arguments are scalar, or an $m \times n$ matrix if one or more arguments are an $m \times n$ matrix.

The return type for `probbeta`, `probgamma`, `probmises`, `probpoisson` is derived as follows:

returns	ma	degrees of freedom arguments
$m \times n$ matrix	$m \times n$ matrix	scalar
$m \times n$ matrix	scalar	$m \times n$ matrix
$m \times n$ matrix	$m \times n$ matrix	$m \times n$ matrix
double	scalar	scalar

The return type for `probmvn` is a double if $m = 1$, or an $m \times 1$ vector if $m > 1$, where m is the number of rows of the first argument. *Note that* `probmvn` *currently only computes up to a trivariate normal distribution.*

Returns the requested cumulative distribution functions at `ma` ($P[X \leq x]$; the returned probabilities are between zero and one):

`probbvn`	bivariate normal distribution,
`probbinomial`	Bin(n, p) distribution,
`probbeta`	Beta(a, b) distribution,
`probcauchy`	Cauchy distribution,
`probexp`	exp(λ) distribution with mean $1/\lambda$,
`probextremevalue`	Extreme Value (type I or Gumbel) distribution,
`probgamma`	Γ distribution,
`probgeometric`	Geometric distribution,
`probhypergeometric`	Hypergeometric distribution,
`probinvgaussian`	Inverse Gaussian distribution,
`problogarithmic`	Logarithmic distribution,
`problogistic`	Logistic distribution,
`problogn`	Lognormal distribution,
`probmises`	$\mathsf{VM}(\mu, \kappa)$ distribution,
`probmvn`	normal distribution $\mathsf{N}_n(0, \Sigma), n \leq 3$,
`probnegbin`	Negative Binomial distribution,
`probpareto`	Pareto distribution,
`probpoisson`	Poisson μ distribution,
`probweibull`	Weibull distribution.

The functional forms are listed under the density functions.
The probabilities are accurate to about 10 digits, except for `probbvn` and `probmvn` which are accurate to 10^{-15}.

Description

The bivariate normal distribution with mean zero and correlation ρ is defined as:

$$\left(2\pi\sqrt{1-\rho^2}\right)^{-1}\int_{-\infty}^{a}\int_{-\infty}^{b}\exp\left(-\frac{1}{2}\frac{x^2-2\rho xy+y^2}{1-\rho^2}\right)\mathrm{d}x\mathrm{d}y.$$

The Beta distribution is defined as $I_x(a, b)$ under `betafunc`.
The Gamma distribution, $\Gamma(z; r, a)$, is defined as:

$$\Gamma(z; r, a) = \int_0^z \frac{a^r}{\Gamma(r)} x^{r-1} e^{-ax}\mathrm{d}x, \quad z > 0, r > 0, a > 0. \tag{11.6}$$

so that $\Gamma(z; r, 1)$ corresponds to the incomplete gamma function. Note that $\chi(df)$ can be computed as $\Gamma(\cdot; 0.5df, 0.5)$.
The von Mises distribution $\mathsf{VM}(\mu, \kappa)$ is defined as:

$$F(z) = \int_0^z \frac{[2\pi I_0(\kappa)]^{-1}}{e^{\kappa\cos(x-\mu)}}\mathrm{d}x, \quad 0 \leq z < 2\pi, \kappa \geq 0, \tag{11.7}$$

where $I_0(\kappa)$ is the modified Bessel function. Note that the density is defined from 0 to 2π, which means that the mean direction is π and not zero. Usually, $\mathsf{VM}(0, \kappa)$

is written for the symmetric von Mises distribution. In the current notation, that corresponds to $\mathsf{VM}(\pi, \kappa)$. For applications of the von Mises distribution, see e.g. Fisher (1993).

The multivariate normal distribution with mean zero and $n \times n$ variance matrix $\mathbf{\Sigma}$ is defined as:

$$[(2\pi)^n|\mathbf{\Sigma}|]^{-1/2} \int_{-\infty}^{a} \int_{-\infty}^{b} \int_{-\infty}^{c} \exp\left(-\frac{1}{2}\mathbf{x}'\mathbf{\Sigma}^{-1}\mathbf{x}\right) \mathrm{d}\mathbf{x},$$

where $\mathbf{x}' = (x_1, x_2, \ldots, x_n)$.

Sources: `probmises` uses AS 86 (Mardia and Zemroch, 1975). The bivariate and trivariate normal distributions are derived from Genz (2000).

See also

`bessel`, `betafunc`, `gammafunc`, `dens...`, `quan...`, `tail...`

Example

```
#include <oxstd.oxh>
#include <oxprob.h>
main()
{
    decl m = <0,4.61,5.99>;

    print("%r", {"chi:  "}, probchi(m, 2));
    print("%r", {"gamma:"}, probgamma(m, 1, 0.5));

    println("Bivariate normal probabilities (rho=0 and 1):");
    println("BVN=", probbvn(<0,0>, <1.645,1.645>, <0,1>));

    println("Multivariate normal probabilities (unit variance):");
    println("TVN=", probmvn(<0,0,0;1.645,1.645,1.645>, unit(3))');
    println("BVN=", probmvn(<0,0;1.645,1.645>, unit(2))');
    println("N=", probmvn(<0;1.645>, unit(1))');
}
```

produces

```
chi:                   0.00000      0.90024      0.94996
gamma:                 0.00000      0.90024      0.94996
Bivariate normal probabilities (rho=0 and 1):
BVN=
      0.47501      0.50000
Multivariate normal probabilities (unit variance):
TVN=
      0.12500      0.85742
BVN=
      0.25000      0.90253
N=
      0.50000      0.95002
```

quan...

```
quanbeta(const ma, const a, const b);
quanbinomial(const ma, const n, const p);
quancauchy(const ma);
quanexp(const ma, const lambda);
quanextremevalue(const ma, const alpha, const beta);
quangamma(const ma, const dr, const da);
quangeometric(const ma, const p);
quanhypergeometric(const ma, const n, const k, const m);
quaninvgaussian(const ma, const mu, const lambda);
quanlogarithmic(const ma, const alpha);
quanlogistic(const ma, const alpha, const beta);
quanlogn(const mx);
quanmises(const mp, const mu, const kappa);
quannegbin(const ma, const k, const p);
quanpareto(const ma, const k, const a);
quanpoisson(const ma, const mu);
quanweibull(const ma, const a, const b);
```

`ma`	in:	arithmetic type, probabilities: all values must be between 0 and 1
`a,b`	in:	arithmetic type, arguments for Beta distribution
`dr`	in:	arithmetic type
`da`	in:	arithmetic type
`alpha,beta`	in:	arithmetic type, location and scale parameter
`lambda`	in:	arithmetic type, parameter of exponential distribution
`mu`	in:	arithmetic type, mean direction (use `M_PI` for symmetric between 0 and π)
`kappa`	in:	arithmetic type, dispersion

Return value

The return type is derived as follows:

returns	`ma`	degrees of freedom arguments
$m \times n$ matrix	$m \times n$ matrix	scalar
$m \times n$ matrix	scalar	$m \times n$ matrix
$m \times n$ matrix	$m \times n$ matrix	$m \times n$ matrix
double	scalar	scalar

Returns the requested quantiles (inverse probability function; percentage points) at `ma`:

`quanbeta`	quantiles from Beta (a, b) distribution
`quanbinomial`	quantiles from Bin(n, p) distribution,
`quancauchy`	quantiles from the Cauchy distribution,
`quanexp`	quantiles from the exp(λ) distribution with mean $1/\lambda$,
`quanextremevalue`	quantiles from the Extreme Value (type I or Gumbel),

`quangamma`	quantiles from $\Gamma(r, a)$ distribution
`quangeometric`	quantiles from the Geometric distribution,
`quanhypergeometric`	quantiles from the Hypergeometric distribution,
`quaninvgaussian`	quantiles from the Inverse Gaussian distribution,
`quanlogarithmic`	quantiles from the Logarithmic distribution,
`quanlogistic`	quantiles from the Logistic distribution,
`quanlogn`	quantiles from the Lognormal distribution,
`quanmises`	quantiles from $\mathsf{VM}(\mu, \kappa)$ distribution
`quannegbin`	quantiles from the Negative Binomial distribution,
`quanpareto`	quantiles from the Pareto distribution,
`quanpoisson`	quantiles from the Poisson μ distribution,
`quanweibull`	quantiles from the Weibull distribution.

The functional forms are listed under the density functions.

The quantiles are accurate to about 10 digits.

See also

`dens...`, `prob...`, `tail...` `lib/Quantile.ox` (to compute quantiles of other distributions)

ran...

```
ranbeta(const r, const c, const a, const b);
ranbinomial(const r, const c, const n, const p);
ranbrownianmotion(const r, const times);
ranchi(const r, const c, const df);
rancauchy(const r, const c);
randirichlet(const r, const valpha);
ranexp(const r, const c, const lambda);
ranextremevalue(const r, const c, const alpha, const beta);
ranf(const r, const c, const df1, const df2);
rangamma(const r, const c, const dr, const da);
rangeometric(const r, const c, const p);
rangh(const r, const c, const nu, const delta, const gamma,
    const beta);
rangig(const r, const c, const nu, const delta, const gamma);
ranhypergeometric(const r, const c, const n, const k, const m);
ranindex(const c);
ranindex(const c, const n);
raninvgaussian(const r, const c, const mu, const lambda);
ranlogarithmic(const r, const c, const alpha);
ranlogistic(const r, const c);
ranlogn(const r, const c);
ranmises(const ma, const kappa);
ranmultinomial(const n, const vp);
rannegbin(const r, const c, const k, const p);
ranpareto(const r, const c, const k, const a);
ranpoisson(const r, const c, const mu);
ranpoissonprocess(const r, const times, const mu);
ranshuffle(const c, const x);
ranstable(const r, const c, const alpha, const beta);
ransubsample(const c, const n);
rant(const r, const c, const df);
ranuorder(const c);
ranweibull(const r, const c, const a, const b);
ranwishart(const n, const p);
```

`r`	in:	int, number of rows
`c`	in:	int, number of columns
`a,b`	in:	double or $r \times c$ matrix, arguments for Beta distribution
`n`	in:	int, number of trials
`p`	in:	double, probability of success (rangeometric also allows $r \times c$ matrix)
`vp`	in:	vector with c probabilities of success (must sum to one)
`lambda`	in:	double or $r \times c$ matrix
`df`	in:	double or $r \times c$ matrix, degrees of freedom
`df1`	in:	double or $r \times c$ matrix, degrees of freedom in the numerator
`df2`	in:	double or $r \times c$ matrix, degrees of freedom in the denominator

`dr`	in:	double or $r \times c$ matrix
`da`	in:	double or $r \times c$ matrix
`mu`	in:	double or $r \times c$ matrix, mean
`kappa`	in:	double or $r \times c$ matrix, dispersion (mean direction is π)
`alpha`	in:	double or $r \times c$ matrix
`beta`	in:	double or $r \times c$ matrix
`nu`	in:	double, parameter for GH and GIG distributions
`valpha`	in:	vector with $c + 1$ shape parameters for Dirichlet distribution
`times`	in:	vector with c time points (must be non-decreasing)
`x`	in:	column or row vector to sample from

Return value

The following return a `r` $\times$ `c` matrix of random numbers which is filled by row. Note that, if both `r` and `c` are 1, the return value is a scalar of type double! The functional forms are listed under the density functions.

function	Generates random numbers from
`ranbeta`	Beta(a, b) distribution,
`ranbinomial`	Binomial(n, p) distribution,
`ranbrownianmotion`	r realizations of the Brownian motion,
rancauchy	equals `rant(r, c, 1)`,
`ranchi`	$\chi^2(df)$ distribution,
`randirichlet`	Dirichlet$(\alpha_1, \ldots, \alpha_{c+1})$ distribution (each row is a realization of the c-variate random variable),
`ranexp`	$\exp(\lambda)$ distribution with mean $1/\lambda$,
`ranextremevalue`	Extreme Value (type I or Gumbel) distribution,
`ranf`	$F(df1, df2)$ distribution,
`rangamma`	Gamma(r, a) distribution, see (11.6), p. 330,
`rangeometric`	Geometric distribution,
`rangh`	$GH(\nu, \delta, \gamma, \beta)$ distribution (see `densgh`)
`rangig`	$GIG(\nu, \delta, \gamma)$ distribution (see `densgig`)
`ranhypergeometric`	Hypergeometric distribution,
`raninvgaussian`	Inverse Gaussian(μ, λ) distribution,
`ranlogarithmic`	logarithmic distribution,
`ranlogistic`	logistic distribution,
`ranlogn`	log normal distribution,
`ranmises`	VM(π, κ) distribution, see (11.7), p. 330,
`rannegbin`	Negative binomial(k, p) distribution,
`ranpareto`	Pareto(k, a) distribution,
`ranpoisson`	Poisson(μ) distribution,
`ranpoissonprocess`	r realizations of the Poisson process,
`ranstable`	Stable distribution, $S(\alpha, \beta),\ 0 < \alpha \leq 2,\ -1 \leq \beta \leq 1$ with location 0, and scale 1, $S(2, 0) \sim N(0, 2)$,
`rant`	Student $t(df)$, df need not be integer.
`ranweibull`	Weibull distribution.
`ranwishart`	Wishart$(n, \mathbf{I}_p)$ distribution, returns a $p \times p$ matrix. Let $V = \sum_{i=1}^{n} x_i x_i'$ where $x_i \sim N_p(0, \mathbf{I}_p)$, then $V \sim$ Wishart$(n, \mathbf{I}_p)$.

The following return a $1\times$ c matrix of random numbers:

function	Generates random numbers from
`ranindex(c)`	draws c numbers from $0,\ldots,c-1$ without replacement,
`ranindex(c,n)`	draws c numbers from $0,\ldots,n-1$ without replacement (this is the same as `ranshuffle(c, range(0,n-1))`),
`ranmultinomial`	Multinomial$(n,p_1,p_2,\ldots,p_c)$ distribution, `vp` must hold the m probabilities which sum to one,
`ranshuffle`	draws c elements from `x` without replacement,
`ransubsample`	draws c numbers from the integers $0,\ldots,n-1$ without replacement (the return value is sorted, so `ransubsample(n,n)` returns $0, ..., n-1$),
`ranuorder`	generates c uniform order statistics.

Description

All these functions use uniform random numbers generated as described under `ranu`.

The `rangamma` function uses algorithms 3.19 and 3.20 from Ripley (1987), `rangamma` is used for `ranchi`: `rangamma`$(n_1/2,\ 1/2)$, and `ranf`: `ranchi`(n_1) n_2 / $(n_1$ `ranchi`$(n_2))$, `ranbinomial` is based on a simple execution of the Bernoulli trials, `rannegbin` sums k independent geometric random numbers, `ranmultinomial` generates n order statistics and counts the bin contents, `ranpoisson` uses algorithms 3.3 and 3.15 from Ripley (1987). Drawings from the Beta and Dirichlet distributions are generated as a ratio of Gamma's.

The `ranmises` function generates random numbers between 0 and 2π from the von Mises distribution with mean direction π. For a different mean use:

`fmod(ranmises(r, c, kappa) + mu, M_2PI)`

(`M_2PI` requires `oxfloat.h`). The algorithm is given in Best and Fisher (1979).

The inverse Gaussian distribution is generated according to Michael, Schucany, and Haas (1976). The logistic distribution uses algorithm LBM from Kemp (1981). The stable distribution with location zero and scale 1 has characteristic function:

$$\phi(t) = \exp\left[|t|^\alpha \left\{1 + i\beta\frac{t}{|t|}w(|t|,\alpha)\right\}\right]$$

where

$$w(|t|,\alpha) = \begin{cases} \tan(\frac{1}{2}\pi\alpha), & \alpha \neq 1, \\ \frac{2}{\pi}\log|t|, & \alpha = 1. \end{cases}$$

The skewness parameter is β $(-1 \leq \beta \leq 1)$, and the characteristic component α $(0 < \alpha \leq 2)$. Stable random number generation is implemented according to Chambers, Mallows, and Sturk (1976) (but without the corrections for α close to but not equal to one).

The uniform random order statistics are generated using the method of exponential spacing (see, e.g., Ripley, 1987, p.97). This may be combined with a quantile function to generate random order statistics of other distributions, e.g. for 100 standard normal order statistics use `quann(ranuorder(100))`.

The Wishart$(n, \mathbf{I}_p)$ random numbers are generated as in Applied Statistics algorithm AS 53 (Smith and Hocking, 1972). To generate from a Wishart$(n, \mathbf{\Sigma}_p)$ use $\mathbf{PWP}'$ where $\mathbf{PP}' = \mathbf{\Sigma}_p$ and $\mathbf{W}$ is generated as Wishart$(n, \mathbf{I}_p)$.
Several generators use rejection methods (notably `rann`, `rangamma`, and hence `rannt` and `ranchi`). Such generators may suffer from a lattice structure in the uniform rng (i.e. correlation between successive values). This may be noticeable in the higher moments (skewnewss and kurtosis) of the generated data. If this is a problem, use quantiles of the uniform rng, such as `quanchi(ranu(), ...)`. (Also see Ripley, 1987, p.55–59.)
`ransubsample` draws without replacement. To draw c numbers with replacement from $0, \ldots, n-1$, simply use `int(ranu(1,c) * n)`. Note that the return value from `ransubsample` is ordered (so `ransubsample(n,n)` just returns $0, \ldots, n-1$). Use `ranshuffle(c, range(0,n-1)))` if a random ordering is required.
A simple generic method to draw random numbers from the GIG distribution (11.4) has been derived by Dagnapur (1988, pp. 133-5), and adjusted by Lehner (1989). This technique is used in `rangig`. If $\sigma^2 \sim GIG(\nu, \delta, \gamma)$ and is independent of $\varepsilon \sim N(0,1)$, then $\beta\sigma^2 + \sigma\varepsilon$ has the generalized hyperbolic distribution (11.5). This is used in `rangh`.
For `ranbrownianmotion`, the increment has an $\mathsf{N}\left[0, \Delta\tau\right]$ distribution. Defining τ as the T-vector of time steps:

$$\begin{aligned} y_0 &= \varepsilon_0 * \tau_0^{1/2}, \\ y_t &= y_{t-1} + \varepsilon_t * (\tau_t - \tau_{t-1})^{1/2}, \quad t = 1, \ldots, T-1, \end{aligned}$$

where ε_t is $\mathsf{IN}[0,1]$. In the case of `ranpoissonprocess`, the increment has a Poisson$(\mu\Delta\tau)$ distribution:

$$\begin{aligned} y_0 &= z_0 & z_0 &\sim \text{Poisson}(\mu\tau_0), \\ y_t &= y_{t-1} + z_t, & z_t &\sim \text{Poisson}\left(\mu\left[\tau_t - \tau_{t-1}\right]\right), \quad t = 1, \ldots, T-1. \end{aligned}$$

The function argument `times` represents the vector τ. If the `r` argument is set to one, one column of length `vec(times)` is generated. If `r` is greater than one, r independent columns are generated, and the return value is a matrix with r columns.

See also

`rann`, `ranseed`, `ranu`

Example

```
#include <oxstd.oxh>
#include <oxprob.h>
main()
{
    print( double(sumc( ranchi(1000,1,5) )) / 1000, " " );
    print( double(sumc( ranexp(1000,1,5) )) / 1000, " " );
    print( double(sumc( rann(1000,1) )) / 1000 );

    ranseed(-1);
    print(rann(1,5));
    ranseed(-1);
    print(rann(1,3) ~ rann(1,2));
    ranseed(-1);
    println("%4.0f", ransubsample(5, 9));
    println("%4.0f", ranshuffle(5, range(0,9)));
}
```

produces

```
4.97999 0.206975 0.0173497
      0.22489       1.7400     -0.20426     -0.91760     -0.67417

      0.22489       1.7400     -0.20426     -0.91760     -0.67417

   2   3   4   5   7

   4   7   6   5   8
```

11.4 QuadPack

QuadPack (documented in Piessens, de Donker-Kapenga, Überhuber, and Kahaner, 1983) is a Fortran library for univariate numerical integration ('quadrature') using adaptive rules. The main driver functions are exported to Ox from `quadpack.dll`, using the header file `quadpack.h`. At the end of this section is a sample program using several of these functions. Full documentation is in Piessens, de Donker-Kapenga, Überhuber, and Kahaner, 1983.

QNG, QAG, QAGS, QAGP, QAGI

```
QNG (const func, const a, const b, const aresult,
    const aabserr);
QAG (const func, const a, const b, const key,
    const aresult, const aabserr);
QAGS(const func, const a, const b, const aresult,
    const aabserr);
QAGP(const func, const a, const b, const vpoints,
    const aresult, const aabserr);
QAGI(const func, const bound, const inf, const aresult,
    const aabserr);
```

func	in:	function to integrate; func must be a function of one argument (a double), returning a double
a	in:	double, lower limit of integration
b	in:	double, upper limit of integration
key	in:	int, key for choice of local integration rule, which determines the number of points in the Gauss-Kronrod pair: ≤ 1 (7–15 points), 2 (10–21 points), 3 (15–31 points), 4 (20–41 points), 5 (25–51 points), ≥ 6 (30–61 points).
vpoints	in:	row vector with singularities of integrand
bound	in:	double, lower bound (`inf` == 1) or upper bound (`inf` == –1)
inf	in:	int, $1 : \int_b^\infty$, $-1 : \int_{-\infty}^b$, $2 : \int_{-\infty}^\infty$
aresult	in:	address of variable
	out:	double, approximation to the integral
aabserr	in:	address of variable
	out:	double, estimate of the modulus of the absolute error

Return value

Result of the QuadPack routine:

0	normal and reliable termination of routine;
1	maximum number of steps has been executed;
2	roundoff error prevents reaching the desired tolerance;
3	extremely bad integrand behaviour prevents reaching tolerance;
4	algorithm does not converge;
5	integral is probably convergent or slowly divergent;
6	invalid input;
10	not enough memory;

An error message greater than 0 is reported unless switched off with `QPWARN`.

Description

`QNG`: simple non-adaptive automatic integrator for a smooth integrand.

`QAG`: simple globally adaptive Gauss-Kronrod-based integrator, with choice of formulae.

`QAGS`: globally adaptive integrator with extrapolation, which can handle integrand singularities of several types.

`QAGP`: as `QAGS`, but allows the user to specify singularities, discontinuities and other difficulties of the integrand.

`QAGI`: as `QAGS`, but handles integration over infinite integrals.

QAWO, QAWF, QAWS, QAWC

```
#include <quadpack.h>
QAWO(const func, const a, const b, const omega, const fcos,
    const maxp1, const aresult, const aabserr);
QAWF(const func, const a, const omega, const fcos, const limlst,
    const maxp1, const aresult, const aabserr);
QAWS(const func, const a, const b, const alpha, const beta,
    const type, const aresult, const aabserr);
QAWC(const func, const a, const b, const c, const aresult,
    const aabserr);
```

func	in:	function to integrate; func must be a function of one argument (a double), returning a double
a	in:	double, lower limit of integration
b	in:	double, upper limit of integration
omega	in:	double, factor in cosine or sine function
fcos	in:	int, 1: function to integrate is $\cos(\omega x)f(x)$, else it is $\sin(\omega x)f(x)$
maxp1	in:	in: int, upper bound on the number of Chebyshev moments which can be stored.
limlst	in:	int, upper bound on the number of cycles (must be 3).
alpha,beta	in:	double, powers in $w(x)$, both > -1.
itype	in:	int, 1: $v(x) = 1$; 2: $v(x) = \log(x-a)$; 3: $v(x) = \log(b-x)$; 4: $v(x) = \log(x-a) * \log(b-x)$.
c	in:	double, term for Cauchy principal value (!= a and != b).
aresult	in:	address of variable
	out:	double, approximation to the integral
aabserr	in:	address of variable
	out:	double, estimate of the modulus of the absolute error

Return value

Result of the QuadPack routine:

0	normal and reliable termination of routine;
1	maximum number of steps has been executed;
2	roundoff error prevents reaching the desired tolerance;
3	extremely bad integrand behaviour prevents reaching tolerance;
4	algorithm does not converge;
5	integral is probably convergent or slowly divergent;
6	invalid input;
10	not enough memory;

An error message greater than 0 is reported unless switched off with `QPWARN`.

Description

`QAWO`: integrates $\cos(\omega x)f(x)$ or $\sin(\omega x)f(x)$ over a finite interval (a, b).

`QAWF`: Fourier cosine or Fourier sine transform of $f(x)$, from a to infinity. (`QAWF` returns error 6 if epsabs is zero, use `QPEPS` to change the value of epsabs.)

`QAWS`: integrates $w(x) * f(x)$ over a finite interval (a, b), where $w(x) = [(x - a)^\alpha][(b - x)^\beta]v(x)$, and $v(x)$ depends on the `itype` argument.

`QAWC`: Cauchy principal value of $f(x)/(x - c)$ over a finite interval (a, b) and for user-determined c.

QPEPS, QPWARN

```
QPEPS(const epsabs, const epsrel);
QPWARN(const ion);
```

`epsabs`	in:	double, absolute accuracy requested (the default value is $\epsilon_a = 0$)
`epsrel`	in:	double, relative accuracy requested (the default value is $\epsilon_r = 10^{-10}$)
`ion`	in:	1: print warning and error messages (the default), or 0: don't print

No return value.

Description

`QPEPS` Sets the accuracy which the integration routines should try to achieve. Let $\hat{I}$ be the approximation from the QuadPack routines to the integral:

$$I = \int_a^b f(x)\mathrm{d}x,$$

then the result will hopefully satisfy:

$$\left|I - \hat{I}\right| \leq \texttt{abserr} \leq \max\{\epsilon_a, \epsilon_r\,|I|\}.$$

`QPWARN` controls whether warning/error messages are printed or not.

Example

```
#include <oxstd.oxh>
#include <quadpack.h>

output(const sFunc, const result, const abserr)
{
    print(sFunc, result, " abserr=", abserr, "\n");
}
```

```
mydensn(const x)
{
    return densn(x);
}
main()
{
    decl result, abserr, pn = probn(1) - probn(0);

    QNG(densn, 0.0, 1.0, &result, &abserr);
    output("QNG: ", result, abserr);

    QAG(densn, 0.0, 1.0, 5, &result, &abserr);
    output("QAG: ", result, abserr);

    QAG(densn, 0.0, 1.0, 15, &result, &abserr);
    output("QAG: ", result, abserr);

    QAGS(densn, 0.0, 1.0, &result, &abserr);
    output("QAGS:", result, abserr);

    QAGP(densn, 0.0, 1.0, <0.1,0.9>, &result, &abserr);
    output("QAGP:", result, abserr);

    QAGI(mydensn, 0, 1, &result, &abserr);
    output("QAGI:", result, abserr);

    print("using probn(): ", probn(1) - probn(0),
          " and ", probn(0), "\n");
}
```

produces

```
QNG: 0.341345 abserr=3.78969e-015
Quadpack warning 1
QAG: 0.330835 abserr=0.0101865
QAG: 0.341345 abserr=3.78969e-015
QAGS:0.341345 abserr=3.78969e-015
QAGP:0.341345 abserr=3.78969e-015
QAGI:0.5 abserr=1.24255e-011
using probn(): 0.341345 and 0.5
```

Chapter 12

Class reference

This chapter documents the preprogrammed classes which are provided with the Ox system. All classes in this chapter are located as follows in the Ox installation:

`ox/include`	header file (.oxh)
`ox/include`	compiled code (.oxo file)
`ox/src`	source code (.ox file)

To use these classes, it is necessary to include the header file, and import the `.ox` or `.oxo` file. This is most easily achieved using the `#import <...>` statement (see 13.9.3). For example:

```
#import <database>
```

which automatically inserts `database.oxh` at that point, and links `database.oxo` when the program is executed (or `database.ox` if the `.oxo` file does not exist).

12.1 Database and Sample class

12.1.1 Introduction

The Sample class stores a time interval, and the frequency, e.g. 1980 (1) – 1990 (1), with frequency 4 (i.e. quarterly observations). Although we talk about year and period to denote a point in time, the year denotes the major time period, and the period the minor, so that, for example, 20 (3) could be day 3 in week 20, when the frequency is 7 (daily data). The member functions of Sample return information about the sample. Use frequency 1 for cross-section data.

The Sample class forms the basis for the Database class and has no constructor function of its own. Because it will be mostly used as part of the Database class, the documentation of the two is presented together.

The Database class stores a matrix of data, together with the sample period (the class derives from the Sample class), and the names of the variables. Functions to create a database from disk files (ASCII, *OxMetrics*, *PcGive*, and Excel spreadsheet formats) are provided. The Database class supports the use of daily and weekly data.

In addition, the Database class has built-in support to select variables (for modelling) from the database. Variables are selected by name, optionally with a lag length, and allocated to a group (e.g. to distinguish between dependent and independent variables). A sample period for the selection can be set. This selection can then be extracted from the database. The selected sample is always adjusted so as not to include missing values.

Some examples follow. Remember to import the database code when using this class. This is achieved using the #import <database> statement, which also automatically inserts database.oxh.

Example

....*samples/database/dbclass.ox*

```
#include <oxstd.oxh>
#import <database>        // required to use Database class

main()
{   decl dbase, y, dy, names;

    dbase = new Database();           // create new object
    dbase.Load("data/data.in7");              // load data

    dbase.Info();                   // print database info
                                        // select variables
    dbase.Select(0, { "CONS", 0, 0, "INC", 0, 0 } );
    dbase.Select(1, { "CONS", 1, 1, "INC", 1, 1 } );
    dbase.SetSelSample(1953, 1, 1992, 3);     // and sample

    y = dbase.GetGroup(0);              // extract group 0
    dy  = y - dbase.GetGroup(1);

    names = {"CONS", "INC", "DCONS", "DINC"};
    print("\nsample variance over ",
          dbase.GetSelSample(),
          "%r", names, "%c", names, variance(y ~ dy) );
    println("\nnumber of observations:  ", dbase.GetSize());
    println("period of observation 9: ", dbase.ObsYear(9),
        " (", dbase.ObsPeriod(9), ")");
    println("database index 1985(4):  ", dbase.GetIndex(1985,4));

    delete dbase;            // done with object, delete it
}
```

The program produces:

```
---- Database information ----
Sample:    1953 (1) - 1992 (3) (159 observations)
Frequency: 4
Variables: 4

Variable #obs #miss   type       min      mean       max  std.dev
CONS      159     0 double     853.5    875.94    896.83   13.497
INC       159     0 double    870.22    891.69    911.38   10.725
INFLAT    159     0 double   -0.6298    1.7997    6.4976   1.2862
OUTPUT    159     0 double    1165.9    1191.1    1213.3   10.974

sample variance over 1953 (2) - 1992 (3)
               CONS          INC        DCONS         DINC
CONS         181.97       135.71       2.9314       3.7989
INC          135.71       114.01       1.9820       5.4127
DCONS        2.9314       1.9820       4.8536       5.5060
DINC         3.7989       5.4127       5.5060       11.183

number of observations:  159
period of observation 9: 1955 (2)
database index 1985(4):  131
```

The following code uses the Dow Jones data to give an example involving weekly data. This is different from the previous database, because some years have 52 and others 53 weeks. Therefore, the method of using a fixed frequency, as implemented in the `Sample` class from which the `Database` class derives, does not work. Instead, a database can now be dated:

- the first column must be of type `DB_DATE`,
- the first column holds date indices as created by `dayofcalendar`,
- the optional fractional part of this indicates time,
- the first and last observation must be valid, i.e. cannot be missing.

These criteria are satisfied in `dowjones.xls`, and the Excel dates are translated in Ox dates when reading the file[1]

Note that the underlying fixed frequency information is set to a frequency of one (equivalent to undated data), so that `GetSize` and other `Sample` functions still work.

As the following example illustrates, there are several functions to facilitate the handling of dated data.

[1]Excel inherits a mistake that was made by the Lotus developers, assuming wrongly that 1900 was a leap year. Instead, the rule for centuries is that they are only a leap year when divisible by 400. Ox takes this into account when loading and saving Excel files.

Example

.... *samples/database/dbdates.ox*

```
#include <oxstd.oxh>
#import <database>        // required to use Database class

main()
{   decl dbase, y, dy, names;

    dbase = new Database();            // create new object
    dbase.Load("data/dowjones.xls");           // load data

    dbase.Info();                    // print database info

    if (!dbase.IsDated())
    {
        println("Expecting a dated database");
        return;
    }
    decl dum = dbase.GetVar("d408");

    println("\nSome dates: ", "%r", {"start","dummy","end"},
        "%C", dbase.GetDateByIndex(
            0 ~ vecindex(dum) ~ dbase.GetSize() - 1));

    dbase.Select(0, { "DLDOWJONES", 0, 1 } );// select vars
    dbase.SetSelDates(1987, 1, 1, 1987, 12, 31);// & sample
    println("\nSelecting dates: 1987-01-01 - 1987-12-31");
    println("Selected sample: ", dbase.GetSelSample(),
        " (database is weekly)");

    delete dbase;            // done with object, delete it
}
```
....

The program produces:

```
---- Database information ----
Sample:    1980-01-02 - 1994-09-28 (770 observations)
Frequency: 1
Variables: 5

Variable  #obs #miss   type      min       mean     max  std.dev
Date       770     0   date   1980-01-02     1994-09-28
DOWJONES   770     0 double   762.12       2055  3975.5   961.68
LDOWJONES  770     0 double   6.6361     7.5076  8.2879  0.50376
DLDOWJONES 769     1 double -0.17377  0.0020133 0.07242 0.021404
d408       770     0 double        0  0.0012987       1 0.036014

Some dates:
start    1980-01-02
dummy    1987-10-21
end      1994-09-28

Selecting dates: 1987-01-01 - 1987-12-31
Selected sample: 1987-01-07 - 1987-12-30 (database is weekly)
```

A choice variable has type DB_CHOICE. In that case, there is an array of text labels associated with values: value zero corresponds to index zero in the array, etc. The final example illustrates the use of choice labels, as well as the Recode and Tabulate functions.

Example

.... *samples/database/dbchoice.ox*

```
#include <oxstd.oxh>
#import <database>                  // required to use Database class

main()
{
    decl db = new Database();       // create new object

    db.Create(16);                  // 16 observations
    db.Append(rann(db.GetSize(), 1), "y");
    db.Shrink(4);                   // remove last four obs
    db.Append(range(-2, db.GetSize() - 4)' | .NaN, "x");

    println("#variables: ", db.GetVarCount(),
        " #observations: ", db.GetSize());
                                    // copy x to xnew and recode
    db.SetVar(db.GetVar("x"), "xnew");
    db.Recode("xnew", <-.Inf,1>,1, 2,4, <3,5>,2, <7,.Inf>,3,
        6,4, .NaN,5);
    db.SetVarChoices("xnew", {"", "One","Two","Three","Four","Five"});
    println("%c", {"x", "xnew"}, db.GetVar("x") ~ db.GetVar("xnew"));

    db.Info();                      // print database info

    db.Tabulate("x");               // tabulations
    db.Tabulate("xnew");
    db.Tabulate("x", "xnew");
    db.Tabulate("x", "xnew", "%");
    delete db;                      // tidy up: delete object
}
```

....

Part of the program output is:

```
#variables: 2 #observations: 12

           x        xnew
     -2.0000      1.0000
     -1.0000      1.0000
     0.00000      1.0000
      1.0000      1.0000
      2.0000      4.0000
      3.0000      2.0000
      4.0000      2.0000
      5.0000      2.0000
      6.0000      4.0000
      7.0000      3.0000
      8.0000      3.0000
        .NaN      5.0000
```

```
---- Database information ----
Sample:    1 - 12 (12 observations)
Frequency: 1
Variables: 3

Variable   #obs  #miss   type      min       mean   max   std.dev
y            12      0  double  -0.9176  -0.028201  1.74   0.69486
x            11      1  double       -2          3     8    3.1623
xnew         12      0  choice        1     2.4167     5    1.3202

Tabulation of xnew
        Counts       %
One          4    33.3
Two          3    25.0
Three        2    16.7
Four         2    16.7
Five         1     8.3
Total       12   100.0

Tabulation of x against xnew
x \         xnew
          One      Two   Three     Four     Five   Total
-2          1        0       0        0        0       1
-1          1        0       0        0        0       1
0           1        0       0        0        0       1
1           1        0       0        0        0       1
2           0        0       0        1        0       1
3           0        1       0        0        0       1
4           0        1       0        0        0       1
5           0        1       0        0        0       1
6           0        0       0        1        0       1
7           0        0       1        0        0       1
8           0        0       1        0        0       1
Total       4        3       2        2        0      11


Tabulation of x against xnew
x \         xnew
           One      Two   Three     Four     Five  Total%
-2         9.1      0.0     0.0      0.0      0.0     9.1
-1         9.1      0.0     0.0      0.0      0.0     9.1
0          9.1      0.0     0.0      0.0      0.0     9.1
1          9.1      0.0     0.0      0.0      0.0     9.1
2          0.0      0.0     0.0      9.1      0.0     9.1
3          0.0      9.1     0.0      0.0      0.0     9.1
4          0.0      9.1     0.0      0.0      0.0     9.1
5          0.0      9.1     0.0      0.0      0.0     9.1
6          0.0      0.0     0.0      9.1      0.0     9.1
7          0.0      0.0     9.1      0.0      0.0     9.1
8          0.0      0.0     9.1      0.0      0.0     9.1
Total%    36.4     27.3    18.2     18.2      0.0   100.0
```

12.1.2 Database and Sample overview

Creation/Information

`Database`	Constructor function.
`Create`	create a database (not needed when using `Load`...)
`Empty`	empties the database and sample
`GetDbName`	get the database name
`GetSize`	number of observations in the database
`GetVarCount`	returns the number of variables
`Info`	prints summary of database contents
`IsEmpty`	returns TRUE if the database is empty
`SetDbName`	set the database name
`Tabulate`	cross-tabulation of discrete variables

Data input/output

`Load`	load data set
`LoadCsv`	load CSV spreadsheet file
`LoadDht`	load Gauss data file
`LoadDta`	load Stata data file
`LoadFmtVar`	load ASCII formated by variable
`LoadIn7`	load PcGive 7 data set
`LoadObs`	load ASCII file by observation
`LoadVar`	load ASCII file by variable
`LoadXls`	load Excel old-format spreadsheet file
`LoadXlsx`	load Excel Open XML spreadsheet file
`Save`	save the database
`SaveCsv`	save as CSV spreadsheet file
`SaveFmtVar`	save as ASCII formated by variable
`SaveIn7`	save as PcGive 7 data set
`SaveObs`	save as ASCII file by observation
`SaveVar`	save as ASCII file by variable
`SaveXls`	save as Excel spreadsheet file
`SaveXlsx`	save as Excel Open XML spreadsheet file

Adding/removing variables/observations

`Append`	append variable(s) to the database
`Deterministic`	create Constant, Trend and Seasonals
`Grow`	grows the database sample size
`Recode`	recode observations of a variable
`Remove`	remove variable(s) from the database
`RemoveObsIf`	remove observations from the database
`Rename`	rename a variable
`Renew`	renew the observations of a variable (append if new variable)
`RenewBlock`	renews a block of variables
`SetVar`	sets variable(s) by name
`Shrink`	shrinks the database

`SortBy`	sort the database by a variable

Extraction

`GetAll`	returns the whole database data matrix
`GetAllNames`	returns all the variable names
`GetSample`	returns text with database sample
`GetSampleByIndex`	virtual funcrion returning sample text
`GetVar`	gets variable(s) by name
`GetVarByIndex`	gets variable(s) by database index
`GetVarIndex`	gets the database index of a named variable
`GetVarNameByIndex`	gets variable name(s) by database index

Database sample information: `Sample` class

`GetFrequency`	data frequency
`GetIndex`	index of time point
`GetPeriod1`	period of first observation
`GetPeriod2`	period of last observation
`GetSize`	number of observations in sample (the database)
`GetYear1`	year of first observation
`GetYear2`	year of last observation
`ObsPeriod`	finds period of observation index
`ObsYear`	finds year of observation index

Dated data, choice data, and variable types

`GetDateByIndex`	get the date of a database index
`GetDates`	returns date variable or <> if undated
`GetIndexByDate`	get the index for a date (must be dated)
`GetIndexByDates`	get the indices for a date range (must be dated)
`GetVarType`	gets the variable type
`GetVarTypeByIndex`	gets the variable type
`IsDated`	returns TRUE if the database is dated
`SetDates`	sets the date variable
`GetObsLabel`	gets the string representation of an observation
`GetVarChoices`	gets the choice labels (array of strings)
`GetVarChoicesByIndex`	gets the choice labels (array of strings)
`SetVarChoices`	sets the choice labels (array of strings)
`SetVarChoicesByIndex`	sets the choice labels (array of strings)
`SetVarType`	sets the variable type
`SetVarTypeByIndex`	sets the variable type

Variable selection

`DeSelect`	remove the current variable and sample selection
`DeSelectByIndex`	remove a variable (by database index) from selection
`DeSelectByName`	remove a named variable from selection
`FindSelection`	find a variable name with lag in the selection
`GetGroup`	get a group selection matrix

`GetGroupLag`	get group with specific lag range
`GetGroupLagNames`	get the names of group with specific lag range
`GetGroupNames`	get the names of variables in a group
`GetMaxGroupLag`	gets maximum lag length in group
`GetMaxSelLag`	get maximum lag length
`GetSelInfo`	get array with selection info
`Select`	select variables into a group
`SelectByIndex`	select variables by their database index
`SetSelInfo`	set array with selection info

Variable and sample selection

`ForceSelSample`	forces a selection sample
`ForceSelSampleByIndex`	forces a selection sample
`GetSelEnd`	index of last selection observation
`GetSelSample`	get text with selected sample
`GetSelSampleMode`	returns the current selection sample mode
`GetSelStart`	index of first selection observation
`SetSelDates`	select a sample by year, month, day
`SetSelSample`	select a sample (fixed frequency)
`SetSelSampleByDates`	select a sample by date value
`SetSelSampleByIndex`	select a sample by datebase indices
`SetSelSampleMode`	set the selection sample mode

Database data members (all protected)

`m_mData`	data matrix ($T \times k$)
`m_sDbName`	database name (string)
`m_asNames`	variable names (array with k strings)
`m_iSampleSelMode`	sample selection mode (argument to `findsample` in `SetSelSample`)
`m_vVarType`	variable types ($1 \times k$)
`m_aasChoices`	array$[k]$ to store array of strings if choice type
	Remainder is for sample selection:
`m_mLagsel`	lag length of each entry in `m_mVarsel` ($1 \times s$ matrix)
`m_mSelgroup`	group number of each entry in `m_mVarsel` ($1 \times s$ matrix)
`m_iT1sel`	row index in `m_mData` of first selected observation (int)
`m_iT2sel`	row index in `m_mData` of last selected observation (int)
`m_mVarsel`	variable selection ($1 \times s$ matrix with selection)
	the selection consists of indices in `m_mData` and `m_asNames`

Sample data members

`m_iFreq`	data frequency (int)
`m_iYear1`	year of first observation (int)
`m_iPeriod1`	period of first observation (int)
`m_iYear2`	year of last observation (int)
`m_iPeriod2`	period of last observation (int)

12.1.3 Database and Sample function members

Database::Append

```
Database::Append(const mNew, const asNew);
Database::Append(const mNew, const asNew, const iT1);
```

`mNew`	in:	$T \times k$ matrix with the new variables
`asNew`	in:	array with k variable names of the new variables, may be a single string if $k = 1$
`iT1`	in:	starting observation index in database (0 if missing)

No return value.

Description

Appends the k new variables to the database, storing the observations and variable names. It is an error if the variable already exists (use `Renew` or `SetVar` instead), or if the new variable has more observations than the database (use `Grow` first or `SetVar` instead).

The first observation has database index `iT1` (omit the third argument, or use 0 if the variables start at the same sample point as the database), the last is the end of the database sample, or the end of `mNew`, whichever comes first.

The following error and warning messages can occur:

One data column expected
Need same number of names as variables
Argument asNew has wrong type
Variable(s) already exist(s), use Renew()
Append() warning: cannot grow sample, use Grow()

Example

The following example shows how you could load a matrix file into a database, assuming that that matrix file contains a $T \times 2$ matrix:

```
    decl dbase, mx;

    dbase = new Database();
    mx = loadmat("./mydata.mat");
    dbase.Create(1,1,1,rows(mx),1);
    dbase.Append(mx, {"Y1", "Y2"}, 0);
```

Here the database is created with frequency 1 (annual data), and first observation year 1, period 1. We give the two variables the names "Y1" and "Y2", and match the first observation of `mx` to the first in the database (which has index 0).

Database::Create

```
Create(const iFreq, const iYear1, const iPeriod1, const iYear2,
    const iPeriod2);
Database::Create(const cT);
```

`iFreq`	in:	int, frequency
`iYear1`	in:	int, start year
`iPeriod1`	in:	int, start period
`iYear2`	in:	int, end year
`iPeriod2`	in:	int, end period
`cT`	in:	int, number of observations

No return value.

Description

Creates a database. Use this when the database is not to be loaded from disk. The `Append` member function allows adding data to the database. Use `Empty` first if the database is not empty.
`Create(cT)` is equivalent to `Create(1, 1, 1, cT, 1)`.

Database::Database

```
Database::Database();
```

No return value.

Description

Constructor. Calls `Empty` and sets the sample selection mode to `SAM_ALLVALID`.

Database::DeSelect

```
Database::DeSelect();
```

No return value.

Description

Clears the current variable and sample selection completely.

Database::DeSelectByIndex, Database::DeSelectByName

```
Database::DeSelectByIndex(const iSel);
Database::DeSelectByName(const sVar, const iGroup, const iLag);
```

`iSel`	in:	int or matrix: selection indices of variables to delete
`sVar`	in:	string: database name of variable to delete
`iGroup`	in:	int: group identifier of variable to delete
`iLag`	in:	int: lag length of variable to delete

No return value.

Description

Delete specific variable(s) from the current selection. The selection sample is not changed.

Database::Deterministic

```
Database::Deterministic(const iCseason);
```

`iCseason`	in:	0: create n normal seasonals 1: create n centred seasonals 2: create 1 normal seasonal 3: create 1 centred seasonal $<$ 0: do not create any seasonals

No return value.

Description

Appends constant, trend and seasonals to the database. These variables are named `Constant`, `Trend` and `Season` `Season_1`, ..., `Season_`x, where x is the frequency. `Season` has a 1 in quarter 1 (for quarterly data), and zeros elsewhere, `Season_1` has a 1 in quarter 2, etc.

If `iCseason` is 1, the seasonals are centred (with quarterly observations, for quarter 1: 0.75, -0.25, -0.25, -0.25, $\ldots$), in which case the names are `CSeason`, `CSeason_1`, ..., `CSeason_`x.

When a single variable is created, the name is `Seasonal` and `CSeasonal` respectively.

Database::Empty

```
Database::Empty();
```

No return value.

Description

Empties the database.

Database::FindSelection

```
Database::FindSelection(const sVar, const iLag);
```

`sVar`	in:	string, variable name
`iLag`	in:	int, lag length

Return value

Returns the selection index of the specified variable with the specified lag, or –1 if it is not selected.

Database::ForceSelSample, Database::ForceSelSampleByIndex

```
Database::ForceSelSample(const iYear1, const iPeriod1,
    const iYear2, const iPeriod2);
Database::ForceSelSampleByIndex(const iT1, const iT2);
```

`iYear1`	in:	int, start year of selection, use –1 for earliest year and period
`iPeriod1`	in:	int, start period of selection
`iYear2`	in:	int, end year of selection, use –1 for latest year and period
`iPeriod2`	in:	int, end period of selection
`iT1`	in:	int, starting observation index in database
`iT2`	in:	int, final observation index in database

Return value

Returns the number of observations in the sample.

Description

Sets a selection a sample for the variables previously selected with the `Select` function. This function does not check for missing values. Use `SetSelSample()` to set a sample with checking for missing values.

Database::GetAll, Database::GetAllNames

```
Database::GetAll();
Database::GetAllNames();
```

Return value

`GetAll` returns the whole database matrix. `GetAllNames` returns an array of strings with all the variable names.

Database::GetDateByIndex

```
Database::GetDateByIndex(const iT1)
```

`iT1`	in:	int, observation index in database

Return value

Returns the date at the specified index (the same as `GetDates()`

$$iT1$$

). This can be printed with the `"%C"` format, or translated using `dayofcalendar`. The database must be dated.

Database::GetDates

```
Database::GetDates();
```

Return value

Returns a column vector with the date variable or <> if the database is undated.

Database::GetDbName

```
Database::GetDbName();
```

Return value

Returns the current database name.

Sample::GetFrequency

```
Sample::GetFrequency();
```

Return value

The data frequency.

Database::GetGroup, Database::GetGroupLag

```
Database::GetGroup(const iGroup);
Database::GetGroupLag(const iGroup, const iLag1, const iLag2);
```

`iGroup`	in:	int, group number
`iLag1`	in:	int, first lag
`iLag2`	in:	int, last lag

Return value

`GetGroup` returns a $T \times n$ matrix with all selected variables of group `iGroup`. `GetGroupLag` returns only those with the specified lag length. If no database sample has been selected yet, the return value is a 0.

Description

`GetGroup` extracts all selected variables of group `iGroup`.
`GetGroupLag` extracts all selected variables of group `iGroup` which have a lag in `iLag1` ... `iLag2`. The selection sample period must have been set.

Database::GetGroupLagNames, Database::GetGroupNames

```
Database::GetGroupLagNames(const iGroup, const iLag1, const iLag2,
  aasNames);
Database::GetGroupNames(const iGroup, const aasNames);
```

`iGroup`	in:	int, group number
`iLag1`	in:	int, first lag
`iLag2`	in:	int, last lag
`aasNames`	in:	array
	out:	will hold an array of strings with the names of the variables with specified group and lag

No return value.

Description

`GetGroupLagNames` gets the names of all selected variables of group `iGroup` which have a lag in `iLag1` ... `iLag2`. The selection sample period must have been set. `GetGroupNames` gets all the variables of the specified group.
The following code section gets all names of `X_VAR` variables and prints them.

```
decl as, i;
db.GetGroupNames(X_VAR, &as);
for (i = 0; i < columns(as); ++i)
    println(as[i]);
```

Sample::GetIndex

```
Sample::GetIndex(const iYear, const iPeriod);
```

`iYear`	in:	int, year
`iPeriod`	in:	int, period

Return value

The index of the specified time point.

Database::GetIndexByDate, Database::GetIndexByDates

`Database::GetIndexByDate(const dDate1)`
`Database::GetIndexByDates(const dDate1, const dDate2)`

`dDate1`	in:	double, date value
`dDate2`	in:	double, date value

Return value

`GetIndexByDate` returns the index closest to the specified date.

`GetIndexByDates` returns the start and end indices of the specified period as an array of two integers. This can be used, e.g., as `[t1,t2]= GetIndexByDates( dayofcalendar(1990, 1, 1), dayofcalendar(1990, 12, 31))`.

Database::GetMaxGroupLag, Database::GetMaxSelLag

`Database::GetMaxSelLag();`
`Database::GetMaxGroupLag(iGroup);`

`iGroup`	in:	int, group number

Return value

`GetMaxSelLag` returns the highest lag in all selected variables.

`GetMaxGroupLag` returns the highest lag in selected variables of the specified group.

Description

Gets lag information on the selection.

Database::GetObsLabel

`Database::GetObsLabel(sVar, const iT);`

`sVar`	in:	string, variable name
`iT`	in:	int, observation index

Return value

Return a string with the text of the observation, taking into account whether it is a choice, date or normal value.

Sample::GetPeriod1, Sample::GetPeriod2

`Sample::GetPeriod1();`
`Sample::GetPeriod2();`

Return value

`GetPeriod1` returns the period of the first observation.

`GetPeriod2` returns the period of the last observation.

Database::GetSample

`Database::GetSample();`
`virtual Database::GetSampleByIndex(const iT1, const iT2)`

`iT1`	in:	int, first observation index in database
`iT2`	in:	int, last observation index in database

Return value

`GetSample` returns a string with the full database sample, e.g. `"1980(1) - 1990(2)"`. `GetSampleByIndex` is called to create the text.

`GetSampleByIndex` writes the sample text for the sample with database indices `iT1`, `iT2`.

If `iT1`< 0 the output is `"no sample"`; if `iT2`< 0 the end-period is omitted, so only a sample date is returned.

Database::GetSelEnd, Database::GetSelStart

```
Database::GetSelStart();
Database::GetSelEnd();
```

Return value

`GetSelStart` returns the database index of the first observation of the selected sample.

`GetSelEnd` returns the database index of the last observation of the selected sample.

Database::GetSelInfo

```
Database::GetSelInfo();
```

Return value

Returns a 1×5 array with the selection information as follows (c is the number of selected variables):

0	$1 \times c$ matrix with database indices of selected variables
1	$1 \times c$ matrix with group index of selected variables
2	$1 \times c$ matrix with lag lengths of selected variables
3	integer, first selection observation
4	integer, last selection observation

Database::GetSelSample

```
Database::GetSelSample();
```

Return value

`GetSelSample` returns a string with the text of the selected database sample, e.g. `"1980(1) - 1984(2)"`. `GetSampleByIndex` is called to create the text.

Database::GetSelSampleMode

```
Database::GetSelSampleMode();
```

Return value

`GetSelSampleMode` returns the current sample selection mode (also see `findsample`), one of: `SAM_ALLVALID` , `SAM_ENDSVALID`, `SAM_ANYVALID` .

Sample::GetSize

```
Sample::GetSize();
```

Return value

The number of observations in the sample.

Database::GetVar, Database::GetVarByIndex

```
Database::GetVar(const sName);
Database::GetVarByIndex(const iVar);
```

`sName`	in:	string or array of strings with variable names
`iVar`	in:	int or matrix of database indices of variables

Return value

Returns a matrix with the specified variable(s), or <> if the variable(s) cannot be found.

Database::GetVarChoices, Database::GetVarChoicesByIndex

```
Database::GetVarChoices(const sVar);
Database::GetVarChoicesByIndex(const iVar);
```

`iVar`	in:	int or matrix of database indices of variables
`sVar`	in:	string or array of strings with variable names

Return value

Return an array of strings with the choice labels; if multiple variables are specified the return value is an array of arrays of strings.

Database::GetVarCount

```
Database::GetVarCount();
```

Return value

Returns the number of variables in the database.

Database::GetVarIndex

```
Database::GetVarIndex(const asName);
```

`asName`	in:	string, or array of strings: variable names

Return value

Returns the database indices of the specified variable(s), or the empty matrix if none are found.

Database::GetVarNameByIndex, Database::GetVarType, GetVarTypeByIndex

```
Database::GetVarNameByIndex(const iVar);
Database::GetVarType(const sVar);
Database::GetVarTypeByIndex(const iVar);
```

`iVar`	in:	int or matrix of database indices of variables
`sVar`	in:	string or array of strings with variable names

Return value

`GetVarNameByIndex` returns an array with the names of the specified variable(s). If `iVar` is a scalar, a single string is returned.

`GetVarTypeByIndex` and `GetVarType` return the variable type, one of: `DB_DOUBLE`, `DB_DATE` , `DB_CHOICE`.

Sample::GetYear1, Sample::GetYear2

```
Sample::GetYear1();
Sample::GetYear2();
```

Return value

`GetYear1` returns the year of the first observation.
`GetYear2` returns the year of the last observation.

Database::Grow

```
Database::Grow(const cTadd);
```

`cTadd`	in:	int, number of observations to grow database sample by (> 0: `cTadd` observations are added at the end; <0: `-cTadd` observations are added at the beginning)

No return value.

Database::Shrink

```
Database::Shrink(const cTdel);
```

`cTdel`	in:	int, number of observations to shrink database sample by (> 0: `cTdel` observations are removed at the end; <0: `-cTdel` observations are removed at the beginning)

No return value.

Database::Info

```
Database::Info();
```

No return value.

Description

Prints information on the contents of the database.

Database::IsDated

```
Database::IsDated();
```

Return value

`TRUE` if the database is dated, `FALSE` otherwise.

Database::IsEmpty

```
Database::IsEmpty();
```

Return value

`TRUE` if the database is empty, `FALSE` otherwise.

Database::Load

```
Database::Load(const sFilename);
```

`sFilename`	in:	string, filename

Return value

`FALSE` if the loading failed, `TRUE` otherwise.

Description

`Load` creates the database and loads the specified data file from disk. The file type is derived from the extension. Supported are: `.csv`, `.dat` (see `LoadFmtVar`), `.dht`, `.dta`, `.in7`, `.xlsx`, `.xls`, `.xlsx`.

See also

`loadmat`

Database::LoadDht, Database::LoadDta

```
Database::LoadDht(const sFilename, const iYear1, const iPeriod1,
    const iFreq);
Database::LoadDta(const sFilename, const iYear1, const iPeriod1,
    const iFreq);
```

`sFilename`	in:	string, filename
`iYear1`	in:	int, start year
`iPeriod1`	in:	int, start period
`iFreq`	in:	int, frequency

Return value

`FALSE` if the loading failed, `TRUE` otherwise.

Description

`LoadDht` creates the database and loads the specified *Gauss* (small and extended v86) data file from disk. Such files come in pairs: the .dht is a binary file which specifies the number of columns, the corresponding .dat file (with the same base name) is a binary file with the data.

`LoadDta` creates the database and loads the specified *Stata* (version 4–6) data file from disk.

Database::LoadFmtVar, Database::LoadIn7

```
Database::LoadFmtVar(const sFilename);
Database::LoadIn7(const sFilename);
```

`sFilename`	in:	string, filename

Return value

`FALSE` if the loading failed, `TRUE` otherwise.

Description

`LoadIn7` creates the database and loads the specified *OxMetrics* file (which is the same as a PcGive 7 data file) from disk.

`LoadFmtVar` creates the database and loads the ASCII file with formatting information from disk. In *OxMetrics* this is called 'Data with load info'. Such a file is human-readable, with the data ordered by variable, and each variable preceded by a line of the type:

> *name year1 period1 year2 period2 frequency*.

For example:

```
>CONS 1953 1 1955 4 4
    890    886    886    884
    885    884    884    884
    887    889    890    894
```

See also

`loadmat`

Database::LoadObs, Database::LoadVar

```
Database::LoadObs(const sFilename, const cVar,const cObs,
    const iYear1, const iPeriod1, const iFreq, const fOffendMis);
Database::LoadVar(const sFilename, const cVar,const cObs,
    const iYear1, const iPeriod1, const iFreq, const fOffendMis);
```

`sFilename`	in:	string, filename
`cVar`	in:	int, number of variables
`cObs`	in:	int, number of observations
`iYear1`	in:	int, start year
`iPeriod1`	in:	int, start period
`iFreq`	in:	int, frequency
`fOffendMis`	in:	int, TRUE:offending text treated as missing value; FALSE: offending text skipped

Return value

`FALSE` if the loading failed, `TRUE` otherwise.

Description

Creates the database and loads the specified human-readable data file from disk. The data is ordered by observation (`LoadObs`), or by variable. Since there is no information on the sample or the variable names in these files, the sample must be provided as function arguments. The variable names are set to Var1, Var2, etc., use `Rename` to rename the variables.

As the name suggests, a human-readable (or ASCII) data file is a file that can be read using a file viewer or editor. (A binary file cannot be read in this way.) The default extension is .DAT.

Each variable must have the same number of observations. So variables that have too short a sample have to be padded by missing values (`M_NAN`). Text following ; or // up to the end of the line is considered to be comment, and skipped. Data files can be ordered by observation (first observation on all variables, second observation on all variables, etc.) or by variable (all observations of first variable, all observations of second variable, etc.). Examples are:

```
// by variable
// cons
883 884 885
889 891 900
// inflat
2.7 3.5 3.9
2.6 2.8 3.4
```

```
//by observation
891 2.8 //1953 (1)
883 2.7 //1953 (2)
884 3.5 // etc.
891 2.8
885 3.9
889 2.6
891 2.8
```

The `fOffendMis` argument gives additional flexibility in reading human- readable files, by giving the option to treat offending words as missing values, or to skip them. The former can be used to read files with a . or a word for missing values, the latter for comma-separated files. Treating offending words or symbols as missing values (`fOffendMis` is TRUE) can be visualized as:

10 M 30	read as →	10 . 30
20 . 40		20 . 40

When read by observation (`LoadObs`), the second variable will be removed (consisting of missing values only), and the database variables will be labelled Var1 and Var3.
And for a comma separated example using the skip option (`fOffendMis` is FALSE):

10,5,30,	read as →	10 5 30
20,6,40,		20 6 40

Database::LoadCsv, Database::LoadXls, Database::LoadXlsx

```
Database::LoadCsv(const sFilename);
Database::LoadXls(const sFilename);
Database::LoadXlsx(const sFilename);
```

`sFilename`	in:	string, filename

Return value

`FALSE` if the loading failed, `TRUE` otherwise.

Description

Creates the database and loads the specified spreadsheet file from disk. An `.xls` file is an old-format Excel worksheet, while the newer Excel Open XML format is in an `.xlsx` file.

The `Database` class can read and write the following spreadsheet files:

- comma-separated: `.csv`;
- old-format Excel: `.xls`;
- Excel Open XML: `.xlsx`;

provided the following convention is adopted:

- Ordered by observation (that is, variables are in columns).
- Columns with variables are labelled.
- There is an *unlabelled column* with the dates (as a string), in the form year–period (the – can actually be any single character), for example, 1980–1 (or: 1980Q1 1980P1 1980:1 etc.). This doesn't have to be the first column.
- The data form a contiguous sample (non-numeric fields are converted to missing values, so you can leave gaps for missing observations).

`Database` class can read the following types of old-style Excel file:

- Excel 2.1, 3.0, 4.0 worksheets;
- Excel 5.0, 95, 97, XP, 2003, 2007 workbooks.

When saving an Excel file, it is written as an Excel 2.1 worksheet. Workbooks are compound files, and only the first sheet in the file is read. If OxMetrics cannot read a workbook file, it is recommended to retry with a worksheet file.

For example, the format for writing is (this is also the optimal format for reading):

	A	**B**	**C**	**D**
1		CONS	INFL	DUM
2	1980-1	883	2.7	3
3	1980-2	884	3.5	5
4	1980-3	885	3.9	1
5	1980-4	889	2.6	9
6	1981-1	900	3.4	2

If these conventions are not adopted the file can still be read, but you will have to check the final result.

See also

`loadmat`

Sample::ObsPeriod

`Sample::ObsPeriod(iObs);`

`iObs`	in:	int, observation index

Return value

The period of the observation index.

Sample::ObsYear

`Sample::ObsYear(iObs);`

`iObs`	in:	int, observation index

Return value

The year of the observation index.

Database::Recode

`Database::Recode(const sVar, ...);`

`sVar`	in:	string, name of variable to recode
`...`	in:	comma-separated pairs of recoding values: old-value, new value

No return value.

Description

recoding arguments	example
scalar old-value, scalar new value	`2, 1`
1×2 matrix m, scalar new value old value is closed interval $[m[0], m[1]]$	`<-.Inf,2>, 1`

Note that the intervals are closed, and that recoding processes the arguments from left to right.

For example

```
db.Recode("x", <-.Inf,2>,1, <3,5>,2, 6,3);
```

Is equivalent to

```
    decl x = db.GetVar("x");
    x = x .<= 2 .? 1 .: x .>= 3 .|| x .<= 5 .? 2 .: x.== 6 .? 3 .: x;
    db.SetVar(x, "x");
```

A further example is given in `samples/database/dbchoice.ox`, see §12.1.1.

Database::Remove, Database::RemoveObsIf

```
Database::Remove(const sName);
Database::RemoveObsIf(const vRemove)
```

`sName`	in:	string or array of strings, variable name(s)
`vRemove`	in:	matrix $T \times 1$ or $1 \times T$ matrix, non-zero at position of observations to remove, 0 for observations to keep

No return value.

Description

Removes the named variable or specified observations from the database.

Database::Rename

```
Database::Rename(const sNewName, const sOldName);
```

`sNewName`	in:	string or array of strings, new name(s)
`sOldName`	in:	string or array of strings, old name(s) of database variable(s)

No return value.

Description

Renames a database variable. To rename more than one variable at once, both most be arrays of the same size, and all old names must exist in the database.

Database::Renew

```
Database::Renew(const mNew, const asName);
Database::Renew(const mNew, const asName, const iT1);
```

`mNew`	in:	$T \times k$ matrix
`asName`	in:	array with k variable names, may be a single string if $k = 1$
`iT1`	in:	first observation (0 if argument is missing)

No return value.

Description

Renews the observations on the named variable. The first new observation has database index `iT1`, the last is the end of the database sample, or the end of `mNew`, whichever comes first.

If a non-existent variable is renewed, the variable is created first using `Append`. The database sample can be changed by `Grow` or `SetVar`, not by `Renew` or `Append`. If that fails, the following error message will appear:

Renew(): could not append variable(s)

Database::RenewBlock

```
RenewBlock(const mNew, const iVarIndex);
```

`mNew`	in:	$T \times k$ matrix
`iVarIndex`	in:	int, database index of first variable to renew

No return value.

Description

Renews the observations on the k variables starting from the first, without any checking for existence.

Sample::Resample

```
Sample::Resample(const iFreq, const iYear1, const iPeriod1);
```

`iFreq`	in:	int, frequency
`iYear1`	in:	int, start year
`iPeriod1`	in:	int, start period

No return value.

Description

Changes the frequency and start year(period). The sample size is unchanged, so the end year(period) is derived from that.

Database::Save, Database::SaveFmtVar, Database::SaveIn7

```
Database::Save(const sFilename);
Database::SaveIn7(const sFilename);
Database::SaveFmtVar(const sFilename);
```

`sFilename`	in:	string, filename

No return value.

Description

`Save` derives the file type from the file extension (using `.in7/.bn7` if no extension is given). Supported are: `.csv`, `.dat` (see `SaveFmtVar`), `.dht`, `.dta`, `.in7`, `.xls`, `.xlsx`.

`SaveIn7` saves the database as a *OxMetrics* file.

`SaveFmtVar` saves the database as a formatted ASCII file. Also see under `LoadFmtVar`.

See also

`savemat`

Database::SaveObs, Database::SaveVar

```
Database::SaveObs(const sFilename);
Database::SaveVar(const sFilename);
```

`sFilename`	in:	string, filename

No return value.

Description

Saves the database as a human-readable data file, ordered by observation, or by variable. Also see under `LoadObs`, `LoadVar`.

Database::SaveCsv, Database::SaveXls, Database::SaveXlsx

```
Database::SaveCsv(const sFilename);
Database::SaveXls(const sFilename);
Database::SaveXlsx(const sFilename);
```

`sFilename`	in:	string, filename

No return value.

Description

Saves the database as a comma-separated, old-style Excel, or Excel Open XML spreadsheet file.

See also

`Database::LoadCsv`, `Database::LoadXls`, `Database::LoadXlsx`, `savemat`

Database::Select, Database::SelectByIndex

```
Database::Select(const iGroup, const aSel);
Database::SelectByIndex(const iGroup, const iVar, const iLag0,
    const iLag1);
```

`iGroup`	in:	int, group number
`aSel`	in:	$3k$ array, specifying name, start lag, end lag
`iVar`	in:	int: database index of variable to select
		matrix: database index of k variables to select
`iLag0`	in:	int: initial lag length of variables to select
		matrix: k initial lag lengths of variables to select
`iLag1`	in:	int: final lag length of variables to select
		matrix: k final lag lengths of variables to select

No return value.

Description

Selects variables by name and with specified lags, and assigns the `iGroup` number to the selection. The `aSel` argument of `Select` is an array consisting of sequences of three values: name, start lag, end lag. For example:

```
Select(0, {"CONS", 0, 0});    // select CONS as group 0
                                           // from lag 0 to 0
Select(0, {"INC", 0, 0}); // also select INC as group 0
Select(1, {"CONS", 1, 1, "INC", 1, 1});
            // the first lag of CONS and INC as group 1
```

After a sample period is set, the selection can be extracted from the database.

If `CONS` and `INC` are variables 0 and 1 in the database, the same selection could be written as:

```
          // select CONS,INC as group 0 from lag 0 to 0
SelectByIndex(0, <0,1>, 0, 0);
            // the first lag of CONS and INC as group 1
SelectByIndex(1, <0,1>, 1, 1);
```

Database::SetDbName

```
Database::SetDbName(const sName);
```

No return value.

Description

Sets the current database name.

Database::SetDates

```
Database::SetDates(const vDates);
```

No return value.

Description

If the database is not yet dated, `vDates` is set as the date column (the first column). Otherwise `vDates` replaces the current date column.

Database::SetSelInfo

```
Database::SetSelInfo(const asInfo);
```

`asInfo`	in:	1×5 array with selection info

Description

Sets the selection based on the specified input array. No checking is done on the input values.

The selection information should be organized as follows (c is the number of selected variables):

0	$1 \times c$ matrix with database indices of selected variables
1	$1 \times c$ matrix with group index of selected variables
2	$1 \times c$ matrix with lag lengths of selected variables
3	integer, first selection observation
4	integer, last selection observation

The last two arguments may be omitted, in which case a call to `SetSelSample` may be required.

Database::SetSelDates

```
Database::SetSelDates(const iYear1, const iMonth1, const iDay1,
    const iYear2, const iMonth2, const iDay2)
```

`iYear1`	in:	int, start year of selection
`iMonth1`	in:	int, start month of selection
`iDay1`	in:	int, start day of selection
`iYear2`	in:	int, end year of selection
`iMonth2`	in:	int, end month of selection
`iDay2`	in:	int, end day of selection

Return value

Returns the number of observations in the sample.

Description

This is the equivalent of SetSelSample that can be used when the database is dated. Selects a sample for the variables previously selected with the `Select` function. The actually selected sample will be the largest starting from the specified starting date (but not exceeding the specified end date) without any missing values when using the default selection mode. Use `SetSelSampleMode` to change the selection mode. Use `DeSelect` to deselect the current sample and variables.

```
Database::SetSelSample(const iYear1, const iPeriod1,
    const iYear2, const iPeriod2);
```

`iYear1`	in:	int, start year of selection, use –1 for earliest year and period
`iPeriod1`	in:	int, start period of selection
`iYear2`	in:	int, end year of selection, use –1 for latest year and period
`iPeriod2`	in:	int, end period of selection

Return value

Returns the number of observations in the sample.

Description

Selects a sample for the variables previously selected with the `Select` function. The actually selected sample will be the largest starting from the specified starting date (but not exceeding the specified end date) without any missing values when using the default selection mode. Use `SetSelSampleMode` to change the selection mode. Use `DeSelect` to deselect the current sample and variables.

Database::SetSelSampleByDates, Database::SetSelSampleByIndex

```
Database::SetSelSampleByDates(const dDate1, const dDate2);
Database::SetSelSampleByIndex(const iT1, const iT2);
```

`dDate1`	in:	double, date value
`dDate2`	in:	double, date value
`iT1`	in:	int, first observation index in database
`iT2`	in:	int, last observation index in database

Return value

Returns the number of observations in the sample.

Database::SetSelSampleMode

```
Database::SetSelSampleMode(const iMode);
```

`iMode`	in:	int, the new sample selection mode, see `findsample`

No return value.

Database::SetVar

```
Database::SetVar(const mNew, const asName);
```

`mNew`	in:	$T \times k$ matrix
`asName`	in:	array with k variable names, may be a single string if $k = 1$

No return value.

Description

If any of the named variables exist in the database, the content is changed, otherwise the new variables are appended. If T is larger than the sample size of the database, the database is extended (unlike `Renew`); if it is shorter, the new (or changed) variable will have missing values for the remainder. If the database has not been created yet, it is created with frequency of unity (annual/undated).

Database::SetVarChoices, Database::SetVarChoicesByIndex

```
Database::SetVarChoices(const sVar, const asChoices);
Database::SetVarChoicesByIndex(const iVar, const asChoices);
```

`iVar`	in:	int, database index of variable
`sVar`	in:	string, name of variable
`asChoices`	in:	array of strings, choice labels

No return value.

Description

Makes the variable in a choice variable (`DB_CHOICE`), and assigns the array of labels. The label for value i is `asChoices`$[i]$. Choice labels are only preserved in in7, xlsx and dta files.

If `asChoices` is an empty array, the choice labels are removed and the variable is changed into a normal variable (`DB_DOUBLE`).

Database::SetVarType, Database::SetVarTypeByIndex

```
Database::SetVarType(const sVar, const iType);
Database::SetVarTypeByIndex(const iVar, const iType);
```

Database::SetVarTypeByIndex(const iVar, const iType); `sVar`	in:	string or array of strings with variable names
`iVar`	in:	int or matrix of database indices of variables
`iType`	in:	int, type `DB_DOUBLE` (the default), `DB_DATE`, `DB_CHOICE`

No return value.

Description

Sets the variable type.

Database::SortBy

```
Database::SortBy(const sVar)
```

`sVar`	in:	string, name of variable

No return value.

Description

Sort the database by a variable.

Database::Tabulate

```
Database::Tabulate(const sX);
Database::Tabulate(const sX, const sY);
Database::Tabulate(const sX, const sY, const sOption);
```

`sX`	in:	string, name of first variable
`sY`	in:	string, name of second variable
`sOption`	in:	strings, ”%” to print percentages instead of counts

Return value

Returns an array of two elements with the vector of values and corresponding counts (tabulation), or an array of three elements with the vector of x-values (row), y-values (column) and corresponding matrix of counts (cross-tabulation).

Description

Tabulates one variable or cross-tabulates two variables. An example is given in `samples/database/dbchoice.ox`, see §12.1.1.

12.2 Modelbase : Database class

12.2.1 Introduction

The Modelbase class derives from the Database class to implement model estimation features. Modelbase is not intended to be used directly, but as a base for a more specialized class. A range of virtual member functions allows for customization of the class. Modelbase facilitates interactive use with OxMetrics through the **OxPack** program. Dialogs and a test menu are easily created by overriding just a few virtual functions. More information is on using Modelbase with OxPack is in the separate Ox Appendices.

In most cases, model estimation involves the following steps (key virtual functions are given in parentheses):

- Call constructor (Modelbase), specify package name and version (GetPackage, GetVersion).
- Initialize data: extract estimation data from underlying database (InitData).
- Initialize parameters (InitPar): specify the number of parameters; set fixed parameters (if any); determine starting values (if necessary).
- Estimate model (Estimate or DoEstimation).
- Produce model output and evaluation (GetParNames, Covar, Output, etc.).

Modelbase has a few essential properties to track this procedure:

Model status	GetModelStatus, SetModelStatus
Maximization method	GetMethod, SetMethod
Estimation result	GetResult, SetResult
Parameters	GetPar, SetPar, GetParCount, SetParCount
Fixed/Free parameters	GetFreeParCount, GetFreePar, SetFreePar, FixPar, FreePar
Covariance	Covar

The following example shows a minimal Modelbase implementation.

Example

.... *samples/database/mbclass.ox*

```
#include <oxstd.oxh>
#import  <modelbase>

class Ols : Modelbase
{
    decl m_mRes;
    decl m_dSigmaSqr;

    Ols();
    GetPackageName();
    GetPackageVersion();
    DoEstimation(vP);
};

Ols::Ols()
{
    Modelbase();
}
Ols::GetPackageName()
```

```
{
    return "Ols";
}
Ols::GetPackageVersion()
{
    return "1.0";
}
Ols::DoEstimation(vP)
{
    decl cp = columns(m_mX);
    SetParCount(cp);

    olsc(m_mY, m_mX, &vP, &m_mCovar);
    m_mRes = m_mY - m_mX * vP;
    m_dSigmaSqr = m_mRes'm_mRes / (rows(m_mY) - cp);
    m_mCovar *= m_dSigmaSqr;

    SetResult(MAX_CONV);

    return vP;
}

main()
{
    decl ols = new Ols();

    ols.Load("data/data.in7");
    ols.Deterministic(FALSE);

    ols.Select(Y_VAR, {"CONS", 0, 2});
    ols.Select(X_VAR, {"Constant", 0, 0, "INC" , 0, 2});

    ols.Estimate();
}
```
....

The program produces:

```
Ols package version 1.0, object created on  8-12-2005

---- Ols ----
The estimation sample is:  1953(3) -  1992(3)
The dependent variable is: CONS  (data/data.in7)
                  Coefficient  Std.Error  t-value  t-prob
CONS_1                1.31039    0.07564     17.3   0.000
CONS_2              -0.352108    0.07915    -4.45   0.000
Constant             -2.17250      11.19   -0.194   0.846
INC                  0.508481    0.03606     14.1   0.000
INC_1               -0.577251    0.05816    -9.92   0.000
INC_2                0.112122    0.05325     2.11   0.037

log-likelihood            .NaN
no. of observations        157  no. of parameters           6
AIC.T                     .NaN  AIC                      .NaN
mean(CONS)              875.78  var(CONS)             182.397
```

At first sight it may be somewhat surprising how much this program achives with so little coding. But, with an understanding of virtual functions, the documentation below, and the actual source code of `Modelbase` (in `ox\src`), it should be possible to implement a `Modelbase` derived package. Other examples of the use of `Modelbase` are the Arfima and DPD packages.

12.2.2 Modelbase overview

Functions which are used in a minimal implementation are marked as follows:
* virtual function to override,
** need to be called as part of the estimation procedure.

general

`Modelbase`	constructor
`ClearEstimation`	removes results fom previous estimation
`ClearModel`	sets model status to `MS_NONE`
`GetMethod`	get the estimation method
`GetMethodLabel`	get the label for the current estimation method
`GetModelLabel`	get the label for the model
`GetModelStatus`	get the model status (`MS_`....)
`GetPackageName`*	returns name of the package
`GetPackageVersion`*	returns version of the package
`GetResult`	get the estimation result
`Grow`	extend database and update the deterministic terms
`Init`	resets all variables to default
`IsUnivariate`	returns `TRUE` if only one `Y_VAR` allowed
`SetForecasts`	set the number of forecasts
`SetMethod`	set the estimation method
`SetModelStatus`**	set the model status (`MS_`....)
`SetPrint`	switch printing on or off
`SetResult`**	set the estimation result
`SetRecursive`	set the number of recursive steps
`ShowBanner`	static method to switch off creation banner

parameter related

`FixPar`	fixes parameters
`FreePar`	frees parameters
`GetFreePar`	get the vector of free parameters, $p \times 1$
`GetFreeParCount`	get number of free parameters p
`GetFreeParNames`	get the names of free parameters, array of length p
`GetPar`	get the vector of all parameters, $q \times 1$
`GetParCount`	get number of parameters q (including fixed)
`GetParNames`*	get the names of all parameters, array of length q
`GetParStatus`	returns full parameter info
`GetParTypes`	returns array of type letters for each model variable
`MapParToFree`	return the free parameters from the argument
`ResetFixedPar`	reset the values of the fixed parameters
`SetFreePar`	set the free parameters

`SetPar`**	set the full parameter vector
`SetParCount`**	set the number of parameters q (including fixed)
move up in model status	
`DoEstimation`	low level estimate
`Estimate`	high level estimate
`InitData`	get the data: Y, X
`InitPar`	initializes the parameter values
`SetStartPar`	set the starting values
covariance evaluation	
`Covar`*	sets `m_mCovar`
get model results	
`GetCovar`	returns $p \times p$ covariance matrix
`GetCovarRobust`	returns $<>$ or $p \times p$ robust covariance matrix
`GetLogLik`	return the log-likelihood, `m_dLogLik`
`GetResVar`	returns residual variance, $n \times n$
`GetResiduals`	returns residual matrix, $T \times n$
`GetStdErr`	returns the std.errors (0 for fixed) $q \times 1$
`GetStdErrRobust`	returns $<>$ or robust standard errors
`GetcDfLoss`	returns degrees of freedom lost (for tailt, AIC)
`GetcT`	returns actual no of variables to use in output, `m_cT`
other get functions	
`GetPrint`	returns current print setting
`GetX`	returns X matrix
`GetY`	returns Y matrix
`GetcX`	returns no of X variables
`GetcY`	returns no of Y variables
`GetcYlag`	returns no of lags of Y
post estimation	
`DbDrawTMatrix`	draws using the database sample information
`GetForecastData`	returns avilable data over a forecast period
`Output`	prints output
`OutputHeader`	prints output header, returns `TRUE` to print rest
`OutputPar`	prints parameter estimates
`OutputLogLik`	prints log-likelihood, AIC, etc.
`OutputMax`	prints maximization result and starting values
`PrintTestVal`	prints a test statistic
`TestRestrictions`	tests restrictions on the parameters
OxPack related, see the separate Ox Appendices for documentation	
`GetLongRunInfo`	returns 0 or info on long-run
`GetLongRunNames`	returns 0 or names of long-run parameters

`ReceiveData`	receive the data for estimation
`ReceiveDialog`	receive output from a dialog
`ReceiveModel`	receive the model specification
`SendDialog`	send a dialog
`SendFunctions`	send specification of special functions
`SendMenu`	send a menu list
`SendMethods`	send the estimation methods
`SendResults`	send an output variable
`SendSpecials`	send the names of special variables
`SendVarStatus`	send the types of variables

12.2.3 Modelbase function members

Modelbase::ClearEstimation, Modelbase::ClearModel

```
virtual ClearEstimation()
virtual ClearModel();
```

No return value.

Description

`ClearEstimation()` clears the model estimation settings. `ClearModel()` sets the model status to `MS_NONE`, and calls `ClearEstimation()`.

Modelbase::Covar

```
virtual Covar();
```

No return value.

Description

In some models, the evaluation of the variance-covariance matrix of the estimated parameters is costly, therefore, this matrix is only computed on demand: when the covariance matrix does not yet exist, `Covar()` is called to compute it.

By default, the `m_mCovar` member variable of `Modelbase` is -1 when estimation commences (through a call to `ClearEstimation()`). `Covar()` should set `m_mCovar` to the variance-covariance matrix (but `Estimate()` or `DoEstimation()` may also do this, as in the example above). `Covar()` can optionally set `m_mCovarRobust` as well. If the covariance matrix does not exist, `Covar()` is automatically called when using `GetCovar()`, `GetCovarRobust()`, `GetStdErr()`, or `GetStdErrRobust()`.

This procedure ensures that the covariance is only computed once when required, and not at all when not required (in some Monte Carlo experiments, for example).

Modelbase::DbDrawTMatrix

```
DbDrawTMatrix(const iArea, const mYt, const asY, const iT1);
DbDrawTMatrix(const iArea, const mYt, const asY, const iT1,
        const iSymbol, const iIndex);
```

`iArea`	in:	int, area index
`mYt`	in:	$m \times T$ matrix with m y variables
`asY`	in:	array of strings (holds variable names), or 0 (no names), or a string (when only one variable to graph)
`iT1`	in:	int, database index of first observation
`iSymbol`	in:	int, 0: draw line, 1: draw symbols, 2: draw both (optional argument, default is 0).
`iIndex`	in:	int, line index for first row, see Table 10.4, (optional argument, default is 2). Each subsequent row will have the next index.

No return value.

Description

This is equivalent to `DrawTMatrix`, but using sample information from the underlying database. The function will automatically draw a proper date axis if the database is dated.

Modelbase::DoEstimation, Modelbase::Estimate

```
virtual DoEstimation(vPar);
virtual Estimate();
```

`vPar`	in:	matrix, vector of starting values (free parameters), $p \times 1$

Return value

`DoEstimation()` returns:

- Direct estimation: $p \times 1$ matrix with the estimated parameters.
- Iterative estimation: array of length 3, with respectively:
  - $p \times 1$ matrix with the estimated parameters,
  - string, name of the iterative procedure (`"BFGS"` for example),
  - `TRUE` if numerical derivatives were used, `FALSE` otherwise.

`Estimate()` returns `TRUE` if estimation was successful, `FALSE` otherwise.

Description

There are two ways to implement estimation:

- Override `DoEstimation()`, which is called from `Modelbase::Estimate`. In this case, the derived `DoEstimation()` returns the estimated parameters, and sets `m_iResult`, see `SetResult()`. Prior to calling `DoEstimation()`, `Modelbase::Estimate()` will call `InitData()`, `InitPar()` and `ClearEstimation()`. Afterwards, it will update the model status, and, if estimation was successful, `Output()`, and, if iterative estimation was used: `OutputMax()`.
- Override `Estimate()`, in which case `DoEstimation()` is not automatically called. This provides complete control, but requires more code. For example, a slightly simplified version of `Modelbase::Estimate()` is given below, showing the essential properties which must be set:
  - estimated free parameters: `SetFreePar()`,
  - `m_iResult`, see `SetResult()`
  - `m_iModelStatus`, see `SetModelStatus()`.

```
Modelbase::Estimate()
{   decl  vpstart, vpfree, estout;

    if (!InitPar())           // calls InitData() if necessary
        return FALSE;

    vpstart = GetFreePar();  // map pars to estimation format
    estout = DoEstimation(vpstart); // do the estimation
    vpfree = isarray(estout) ? estout[0] : estout;

    SetFreePar(vpfree);// map estimated pars to normal format

    if (m_iResult >= MAX_CONV && m_iResult < MAX_MAXIT)
        m_iModelStatus = MS_ESTIMATED;
    else
        m_iModelStatus = MS_EST_FAILED;

    if (m_fPrint)
    {   Output();
        if (isarray(estout))
            OutputMax(estout[1],m_iResult,vpstart,estout[2]);
    }
    return m_iModelStatus == MS_ESTIMATED;
}
```

Modelbase::FixPar

`FixPar(const iP, const dFix);`

`iP`	in:	int, index of parameter to fix
`dFix`	in:	double, value to fix parameter at

No return value.

Description

`FixPar()` is used to fix a parameter at the specified value. Subsequently, this parameter is omitted from the vector returned by `GetFreePar()`.

Modelbase::FreePar

`FreePar(const iP)`

`iP`	in:	int, index of parameter to free, use –1 to free all

No return value.

Description

Frees a parameter which was previously fixed by `FixPar()`.

Modelbase::GetcDfLoss

`virtual GetcDfLoss();`

Return value

The loss in degrees of freedom in the estimated model. The default is the number of estimated parameters.

Description

Only override this function if the number to be used in the output is different from that number of free parameters in the estimation.

Modelbase::GetCovar, Modelbase::GetCovarRobust

```
GetCovar();
GetCovarRobust();
```

Return value

Returns the $p \times p$ variance-covariance matrix of the free parameters.

Description

See `Covar` for an explanation of the implementation.

Modelbase::GetcT

```
virtual GetcT();
```

Return value

Returns an integer with the actual number of observations to be used in the output.

Description

Only override this function if the number reported in the output is different from that used in the estimation (`m_cT`).

Modelbase::GetcX, Modelbase::GetcY, Modelbase::GetcYlag

```
GetcX();
GetcY();
GetcYlag();
```

Return value

Returns respectively: no of X variables, no of Y variables, lag length of Y.

Modelbase::GetForecastData

```
GetForecastData(const iGroup, const mnLag, const mxLag,
    const cTforc);
GetForecastData(const iGroup, const mnLag, const mxLag,
    const cTforc, const iT1forc);
```

`iGroup`	in:	int, group number
`mnLag`	in:	int, start lag
`mxLag`	in:	int, end lag
`cTforc`	in:	int, number of forecasts
`iT1forc`	in:	int, first forecasts observation (default is `m_iT2est+1`)

Return value

Returns a matrix with the available forecasts data (or an empty matrix if no there is no data).

Modelbase::GetFreePar, Modelbase::GetFreeParCount, Modelbase::GetFreeParNames

```
GetFreePar();
GetFreeParCount();
GetFreeParNames();
```

Return value

`GetFreePar` returns the $p \times 1$ vector with free parameters.

`GetFreeParCount` returns the free parameter count p.

`GetFreeParNames` returns an array of length p with the names of the free parameters.

Description

`GetFreePar` returns the current values of the free parameters. Parameters are fixed with `FixPar()`. The value of free parameters is set with `SetFreePar()`.

Modelbase::GetLogLik

```
GetLogLik();
```

Return value

Returns the log-likelihood, which is the value of the `m_dLogLik` member variable.

Modelbase::GetMethod, Modelbase::GetMethodLabel

```
GetMethod();
virtual GetMethodLabel();
```

Return value

`GetMethod` returns the integer representing the estimation method, which is the value of the `m_iMethod` member variable.

`GetMethodLabel` returns the text label for the current estimation method `m_iMethod`.

Modelbase::GetModelLabel, Modelbase::GetModelStatus

```
virtual GetModelLabel();
GetModelStatus();
```

Return value

`GetModelLabel` returns the text label for the current model.

`GetModelStatus` returns the model estimation status:

value	description
`MS_NONE`	no model preparatory action has been taken,
`MS_DATA`	estimation data has been extracted from the database,
`MS_PARAMS`	the starting values for estimation have been set,
`MS_ESTIMATED`	the model has been estimated,
`MS_EST_FAILED`	model estimation has failed.

This value is stored in the `m_iModelStatus` member variable.

Modelbase::GetPackageName

```
virtual GetPackageName();
```

Return value

Name of the modelling package.

Description

This virtual function should be overridden by the derived class.

Modelbase::GetPackageVersion

```
virtual GetPackageVersion();
```

Return value

Version number of the modelling package.

Description

This virtual function should be overridden by the derived class.

Modelbase::GetPar, Modelbase::GetParCount, Modelbase::GetParNames

```
GetPar();
GetParCount();
virtual GetParNames();
```

Return value

`GetPar` returns the $q \times 1$ vector with the current parameter values (both fixed and free).

`GetParCount` returns the total parameter count q (both fixed and free parameters).

`GetParNames` returns an array of length q with the names of the parameters.

Description

`GetParNames` should be overridden to use proper labels in the output.

Modelbase::GetParStatus

```
GetParStatus();
```

Return value

Returns array with:

0 total number of parameters q,

1 $q \times 1$ matrix with 1 in position of free parameters, and 0 for fixed,

2 $q \times 1$ matrix with fixed value in position of fixed parameters (free positions are unused),

Description

This function is infrequently used.

Modelbase::GetParTypes

```
virtual GetParTypes();
```

Return value

Override the default to return an array of strings indicating the type of each model variable, e.g. `"Y","X","X","U"`. The default returns 0, so that no types are indicated in the output.

Modelbase::GetPrint

`GetPrint();`

Return value

Returns the current print setting.

Modelbase::GetResiduals

`virtual GetResiduals();`

Return value

Returns the $T \times n$ matrix with residuals (n equals 1 for univariate models).

Description

Must be overridden by the derived class to return residuals.

Modelbase::GetResult

`GetResult();`

Return value

The estimation result (normally a value from `MaxBFGS`), which is the value of the `m_iResult` member variable.

Modelbase::GetResVar

`virtual GetResVar();`

Return value

Returns the $n \times n$ matrix with the residual variance (n equals 1 for univariate models).

Description

Must be overridden by the derived class.

Modelbase::GetStdErr, Modelbase::GetStdErrRobust

`GetStdErr();`
`GetStdErrRobust();`

Return value

Returns the $q \times 1$ vector with standard errors (0 at position of fixed parameters).

Description

See `Covar` for an explanation of the implementation.

Modelbase::GetX, Modelbase::GetY

```
GetX();
GetY();
```

Return value

Returns the X and Y matrix.

Modelbase::Grow

```
Modelbase::Grow(const cTadd);
```

`cTadd`	in:	int, number of observations to grow database sample by ($>$ 0: `cTadd` observations are added at the end; $<$0: `-cTadd` observations are added at the beginning)

No return value.

Description

Calls `Database::Grow` and updates the deterministic variables (”Constant”, ”Trend”, ”Seasonal”, ”CSeasonal”).

Modelbase::InitData

```
virtual InitData();
```

Return value

`TRUE` if successful.

Description

Extracts the data for estimation from the underlying database. Sets the model status to `MS_DATA` if successful.

Modelbase::InitPar

```
virtual InitPar();
```

Return value

`TRUE` if successful.

Description

Gets starting values for the estimation procedure. Sets the model status to `MS_PARAMS` if successful.

Modelbase::IsUnivariate

```
virtual IsUnivariate();
```

Return value

`TRUE` if only one dependent variable (`Y_VAR`) is allowed.

Description

This virtual function should be overridden by the derived class if multivariate models are implemented.

Modelbase::MapParToFree

```
MapParToFree(const vPar);
```

`vPar`	in:	int, q vector with parameter values (both fixed and free)

Return value
Returns a $p \times 1$ vector with free parameter values.
Description
Extracts and returns the free parameter values from a full parameter vector.

Modelbase::Modelbase

```
Modelbase();
```

No return value.
Description
Constructor function.

Modelbase::Output

```
virtual Output();
```

No return value.
Description
Prints the estimation output.

Modelbase::OutputHeader

```
OutputHeader(const sTitle);
```

`sTitle`	in:	string, title

No return value.
Description
Called by `Output` to print the header section.

Modelbase::OutputLogLik

```
OutputLogLik();
```

No return value.
Description
Called by `Output` to print the loglikelihood and other summary statistics.

Modelbase::OutputMax

```
OutputMax(const sMethod, const iResult, const vPstart,
    const bNumerical);
```

`sMethod`	in:	maximization method
`iResult`	in:	int, maximization result
`vPstart`	in:	vector with starting values
`bNumerical`	in:	int, `TRUE` if using numerical derivatives

No return value.

Description

Called by `Estimate` to print the starting values and method used for iterative estimation.

Modelbase::OutputPar

```
OutputPar();
```

No return value.

Description

Called by `Output` to print the parameter estimates.

Modelbase::PrintTestVal

```
static PrintTestVal(const dTest, const cR, const cTdf,
    const sLabel);
```

`dTest`	in:	test statistic
`cR`	in:	first degrees of freedom
`cTdf`	in:	second degrees of freedom
`sLabel`	in:	name of test

No return value.

Description

Prints a test statistic and its significance. If `cTdf` is zero, the test is assumed to have $\chi^2(\mathtt{cR})$ distribution, otherwise an $F(\mathtt{cR},\mathtt{cTdf})$ distribution.

Modelbase::ResetFixedPar

```
ResetFixedPar();
```

No return value.

Description

Resets the fixed parameters to their prespecified values.

Modelbase::SetForecasts

```
virtual SetForecasts(const cForc, const bIsLessForecasts);
```

`cForc`	in:	int, number of forecasts,
`bIsLessForecasts`	in:	int, `TRUE`: the forecasts are subtracted from the selection sample.

No return value.

Description

The `Modelbase` version sets `m_cTforc`.

Modelbase::SetFreePar

```
SetFreePar(const vParFree);
```

`vParFree`	in:	p vector with free parameter values

No return value.

Modelbase::SetMethod

```
SetMethod(const iMethod);
```

`iMethod`	in:	int, estimation method (no values are predefined in Modelbase)

No return value.

Description

Sets `m_iMethod`.

Modelbase::SetModelStatus

```
SetModelStatus(const iModelStatus);
```

`iModelStatus`	in:	int, model status to set, one of: `MS_NONE`, `MS_DATA`, `MS_PARAMS`, `MS_ESTIMATED`, `MS_EST_FAILED`

No return value.

Description

Sets `m_iModelStatus`.

Modelbase::SetPar, Modelbase::SetParCount

```
SetPar(const vPar);
SetParCount(const cPar);
SetParCount(const cPar, const bAdd);
```

`vPar`	in:	$q \times 1$ vector with new parameter values (both fixed and free)
`cPar`	in:	int, total number of parameters (fixed and free)
`bAdd`	in:	(optional) int, `TRUE`: add parameters to current count; else set the count.

No return value.

Description

`SetParCount()` be called for the other parameter functions to work.

Modelbase::SetPrint

```
SetPrint(fPrint);
```

`fPrint`	in:	int, `TRUE` to switch printing on, `FALSE` to switch off.

No return value.

Description

For Monte Carlo experiments, it can be useful to switch off printing.

Modelbase::SetRecursive

```
virtual SetRecursive(const bSet, const cInit);
```

`bSet`	in:	int, `TRUE` to switch recursive estimation on
`cInit`	in:	int, number of initialization steps

No return value.

Description

The Modelbase version sets m_bRecursive and m_cTinit.

Modelbase::SetResult

```
SetResult(const iResult);
```

No return value.

Description

Sets the estimation result (normally a value from MaxBFGS), which is the value of the m_iResult member variable.

Modelbase::SetStartPar

```
virtual SetStartPar(const vParFree);
```

vParFree	in:	p vector with the starting values for the free parameters

No return value.

Description

This is an alternative to InitPar, allowing for direct setting of the starting parameters prior to estimation.

Modelbase::ShowBanner

```
static ShowBanner(const bSet);
```

bSet	in:	int, FALSE: suppress creation banner

No return value.

Modelbase::TestRestrictions

```
virtual TestRestrictions(vSel);
virtual TestRestrictions(mR, const vR);
```

vSel	in:	p vector, with a 1 for the coefficients which tested to be zero, 0 otherwise
mR	in:	$s \times p$ matrix R
vR	in:	s vector r

No return value.

Description

The one-argument version tests whether one or more coefficients are zero. The second form tests restrictions of the type $R\theta = r$. Both are implemented as a Wald test with a $\chi^2(s)$ distribution.

This function requires that Covar() and GetParNames() are implemented, and SetPar() or SetParFree() are used to set the estimated parameters.

12.3 PcFiml : Modelbase : Database class

The PcFiml class provides part of the advanced computations available in the menu driven computer program *PcGive*, see Doornik and Hendry (2009). The class is derived from the Database class, and provides model formulation using variable names.

The class allows for estimating a Vector Autoregression (VAR), cointegration analysis ('Johansen procedure'), and multivariate regression model (such as an unrestricted reduced form, URF), as well as a simultaneous equations model (2SLS, 3SLS, FIML). Identities equations are currently not supported. Mis-specification tests include: vector autoregression, vector normality, vector heteroscedasticity, vector portmanteau, as well as a Chow test.

The documentation here is rather cursory, the actual source code (pcfiml.ox) gives more documentation. The required header file is pcfiml.h, which is imported here (togther with the actual code) using #import <pcfiml>.

Example

.... *samples/pcfiml/pcf1.ox*

```
#include <oxstd.oxh>
#import <pcfiml>

main()
{
    decl system;

    system = new PcFiml();

    system.LoadIn7("data/data.in7");
    system.Deterministic(FALSE);
                                    // formulate the system
    system.Select(Y_VAR, { "CONS", 0, 2, "INC", 0, 2 } );
    system.Select(X_VAR, { "INFLAT", 0, 0 } );
    system.Select(U_VAR, { "Constant", 0, 0 } );

    system.SetSelSample(1953, 1, 1992, 3);
    system.Estimate();         // estimate the system (VAR)
    system.Cointegration();       // cointegration analysis

    system.Chow(1980, 1);                     // some tests
    system.Portmanteau(12);
    system.NormalityTest();
    system.ArTest(1, 5);
    system.HeteroTest(FALSE, FALSE);
    system.HeteroTest(FALSE, TRUE);

    delete system;
}
```
....

The output of this program is (omitting the χ^2 form of some tests):

```
---- System estimation by OLS ----
The estimation sample is 1953(3) - 1992(3)
CONS         lag 0 status Y
CONS         lag 1 status Y
CONS         lag 2 status Y
INC          lag 0 status Y
INC          lag 1 status Y
INC          lag 2 status Y
INFLAT       lag 0
Constant     lag 0 status U

coefficients
                      CONS          INC
CONS_1             0.90553     0.083906
CONS_2            0.039957      0.17361
INC_1             0.060179      0.73816
INC_2            -0.033528    -0.089942
INFLAT            -0.95629    0.0023221
Constant            25.505       87.920

coefficient standard errors
                      CONS          INC
CONS_1             0.13261      0.21549
CONS_2             0.12260      0.19923
INC_1             0.086063      0.13986
INC_2             0.075989      0.12349
INFLAT             0.17341      0.28179
Constant            15.216       24.727

equation standard errors
         CONS          INC
       1.9275       3.1323
residual covariance
               CONS          INC
CONS         3.7152       4.9906
INC          4.9906       9.8111

log-likelihood=-185.911118 det-omega=10.6792 T=157

Cointegration analysis
  eigenvalues    trace   [pval]   max-eval   [pval]
      0.40306   101.97   0.0000     81.002   0.0000
      0.12502   20.967   0.0000     20.967   0.0000

beta
CONS               0.22102      0.17651
INC               -0.24747     -0.25253
INFLAT              1.0903     -0.22638

alpha
CONS        -0.74698      0.62647
INC          0.24209       1.1558

standardized beta
CONS                1.0000     -0.69898
INC                -1.1197       1.0000
INFLAT              4.9332      0.89643

standardized alpha
CONS        -0.16510     -0.15820
```

```
INC        0.053507     -0.29187

long run matrix
                CONS           INC       INFLAT
CONS      -0.054518     0.026651     -0.95629
INC         0.25752     -0.35178    0.0023221
Unrestricted constant

Chow test for break after 1980(1) in sample up to 1992(3):
Scalar Chow tests:  F(50,101)=
       1.2555       1.0587
Vector Chow test:   F(100,200)=1.18615 [0.1558]
Vector portmanteau:   Chi(38)=45.8927 [0.1776]
Vector normality:     Chi(4)=3.49129 [0.4792]
Vector AR 1-5 test:   F(20,280)=1.74601 [0.0265]
Vector hetero test:   F(30,405)=0.977499 [0.5030]
Vector hetero-X test: F(60,382)=1.07614 [0.3357]
```

The next example involves simultaneous equations estimation.

.... *samples/pcfiml/pcf3.ox*

```
#include <oxstd.oxh>
#import <pcfiml>
main()
{
    decl system = new PcFiml();

    system.LoadIn7("data/data.in7");
    system.Deterministic(FALSE);
                                    // formulate the system
    system.Select(Y_VAR, { "CONS", 0, 2, "INC", 0, 2 } );
    system.Select(X_VAR, { "INFLAT", 0, 0 } );
    system.Select(U_VAR, { "Constant", 0, 0 } );

    system.SetSelSample(1953, 1, 1992, 3);
    system.SetPrint(FALSE);      // don't print URF results
    system.Estimate();                      // estimate URF

    system.SetPrintUrf(FALSE);
    system.SetPrint(TRUE);        // but print model output
                                       // formulate a model
    system.SetEquation("CONS", {"CONS",1,2, "INC",0,0 });
    system.SetEquation("INC",  {"INC", 1,2 } );

    system.Fiml();            // estimate the model by FIML
    system.Portmanteau(12);                // do some tests
    system.EgeArTest(1, 1);
    system.EgeArTest(1, 5);
    system.NormalityTest();
    system.HeteroTest(FALSE, FALSE);
    system.HeteroTest(FALSE, TRUE);

    delete system;                  // done with the system
}
```

....

```
---- Model estimation by FIML ----
The estimation sample is 1953 (3) 1992 (3)

coefficients
                      CONS           INC
CONS               -1.0000       0.00000
INC             -0.0024770       -1.0000
CONS_1              1.2238       0.00000
CONS_2            -0.24947       0.00000
INC_1              0.00000       0.99701
INC_2              0.00000     -0.044041
INFLAT             0.00000       0.00000
Constant            24.527        41.792

coefficient standard errors
                      CONS           INC
CONS               0.00000       0.00000
INC               0.035193       0.00000
CONS_1            0.063435       0.00000
CONS_2            0.062315       0.00000
INC_1              0.00000      0.065539
INC_2              0.00000      0.063625
INFLAT             0.00000       0.00000
Constant            16.305        22.300

equation standard errors
         CONS           INC
       2.1822        3.3125
residual covariance
               CONS           INC
CONS         4.7620        5.5795
INC          5.5795        10.972

log-likelihood=-236.414419 det-omega=20.3209 T=157
FIML estimation: Strong convergence

Vector portmanteau:     Chi(43)=77.2487 [0.0010]
Vector EGE-AR 1-1 test: F(4,302)=1.53528 [0.1918]
Vector EGE-AR 1-5 test: F(20,286)=2.23427 [0.0022]
Vector normality:       Chi(4)=4.07116 [0.3965]
Vector hetero test:     F(30,405)=2.10415 [0.0008]
Vector hetero-X test:   F(60,382)=2.67075 [0.0000]
```

PcFiml function members

`ArTest(const iAr1, const iAr2);`
: System vector AR test for lags `iAr1`...`iAr2`.

`Chow(const iYear, const iPeriod);`
: Forecast Chow tests for break on or after `iYear` (`iPeriod`).

`Cointegration();`
: Estimate cointegrating space.

`CointegrationI2();`
: I(2) cointegration analysis.

`EgeArTest(const iAr1, const iAr2);`
: Model vector AR test for lags `iAr1`...`iAr2`.

`Estimate();`
: Estimate the system (NB: use `.SetSelSample()` first).

`Fiml();`
: Do FIML estimation.

`GetOmega();`
: Returns $n \times n$ matrix of URF/RRF residual variance $\mathbf{V}'\mathbf{V}/(T-k)$.

`GetPi();`
: Returns $n \times k$ matrix of URF/RRF coefficients.

`GetResiduals();`
: Returns $T \times n$ matrix $\mathbf{V}$ of URF/RRF residuals.

`GetResult();`
: Returns results from FIML estimation (return code from `MaxBFGS`).

`GetStatus(const aiConst, const aiTrend);`
: Returns status of Constant & Trend (0: no constant; 1: restricted constant; 2: unrestricted constant; 4: unrestricted trend; 3: restricted trend)

`GetVarNames(const aasY, const aasW);`
: Returns n n1 k (n1 is no of lagged Ys); puts list of varnames in arguments.

`GetVarPi();`
: Returns $n \times k$ matrix with variances of RRF/URF coefficients.

`GetVarRf();`
: System: returns full $nk \times nk$ variance-covariance matrix of URF coefficients; Model: returns full $nk \times nk$ variance-covariance matrix of RRF coefficients.

`GetVarTheta();`
: System: returns full $nk \times nk$ variance-covariance matrix of URF coefficients; Model: returns full $np \times np$ variance-covariance matrix of model coefficients.

`HeteroTest(const fStand, const fCross);`
: Vector heteroscedasticity test.

`NormalityTest();`
: Vector normality test.

`Output(const fSys, const fCoint);`
: Print System and/or Cointegration results.

`PcFiml();`
: Constructor.

`Portmanteau(const iLag);`
  Vector portmanteau test up to lag `iLag`.
`SetEquation(const sEquation, const aModel);`
  Delete or add variable from model.
`SetPrint(fPrint);`
  Toggles print switch.
`SetPrintUrf(fPrintUrf);`
  Toggles URF print switch.
`ThreeSLS();`
  Do 3SLS estimation.
`TwoSLS();`
  Do 2SLS estimation.

12.4 PcFimlDgp class

The `PcFimlDgp` class is a data generation process (DGP), designed for use in dynamic econometric Monte Carlo experiments. Unlike the `PcNaiveDgp` class, it derives from `Database` to formulate the DGP and store the generated data. This makes the DGP more general, but somewhat more complex. The class is used through the header file `pcfimldgp.h` .

The form of the DGP in mathematical formulation is a reduced form model:

$$\begin{aligned} \mathbf{y}_t &= \mathbf{\Pi}\mathbf{w}_t + \mathbf{u}_t, \;\; t = T_1, \ldots, T_2, \\ \mathbf{z}_t &= \mathbf{C}_0\mathbf{z}_{t-1} + \mathbf{v}_t, \;\; t = T_1, \ldots, T_2. \end{aligned}$$

where $\mathbf{w}$ contains $\mathbf{z}$, r lags of $\mathbf{z}$ and m lags of $\mathbf{y}$:

$$\mathbf{w}_t' = \left(\mathbf{y}_{t-1}', \ldots, \mathbf{y}_{t-m}', \mathbf{z}_t', \ldots, \mathbf{z}_{t-r}'\right).$$

Take $\mathbf{y}_t$ as an $n \times 1$ vector, $\mathbf{z}_t$ as $q \times 1$, and $\mathbf{w}_t$ as $k \times 1$.

The database is constructed as follows:

$0 \ldots s-1$	initial values for lagged observations, $s \geq \max(1, m, r)$
$T_1 = s \ldots s+d-1$	space to allow for discarded observations,
$T_1 + d \ldots T_2^*$	remainder of generated data.

T_2^*, the sample size of the database, is determined by the call to `Create()`. T_2, the endpoint for data generation, is determined by the call to `GenerateTo()`; $T_2 \leq T_2^*$.

Example

.... *samples/simula/pcfdgp.ox*

```
#include <oxstd.oxh>
#import <pcfimldgp>

main()
{
    decl dgp = new PcFimlDgp(2,1);

    dgp.Create(1, 1980, 1, 0, 1, 100);

    dgp.Select(Y_VAR, {"Ya", 0, 1});
    dgp.Select(Y_VAR, {"Yb", 0, 1});
    dgp.Select(Z_VAR, {"Za", 0, 0});
    dgp.Select(Z_VAR, {"Constant", 0, 0});

    dgp.SetYParameter((<0.9,0;0.1,0.8> ~ <0.2;0.2> ~ <1;0>)');
    dgp.SetZParameter(<0.5>);
    dgp.SetDistribution(U_DGP, MVNORMAL, zeros(2,1),
        ones(2,2)/10 + unit(2)/5);

    dgp.Prepare();
    dgp.Print();

    print("%c", {"Ya", "Yb", "Ua", "Ub"}, dgp.GenerateTo(6));

    delete dgp;
}
```

. . . .

produces (all non specified parameters are zero by default):

```
---- PcFiml (2.00) DGP ----
y is (2 x 1), z is (1 x 1) and fixed.

DGP: y[t] = e[t] + Pi w
Ya           Y_VAR: dependent variable
Yb           Y_VAR: dependent variable
Ya_1         Y_VAR: lagged dependent variable
Yb_1         Y_VAR: lagged dependent variable
Za           X_VAR: regressor
Constant     X_VAR: regressor
Database sample: 1979 - 2079

Coefficients, Pi'=
                        Ya           Yb
Ya_1               0.90000      0.10000
Yb_1               0.00000      0.80000
Za                 0.20000      0.20000
Constant            1.0000      0.00000
e ~ MVN(0,sigma)
sigma=
      0.30000      0.10000
      0.10000      0.30000

z[t] = v[t] + C0 z[t-1]
C0 =
      0.50000
v ~ N(0,1)


           Ya           Yb           Ua           Ub
       1.1673     0.012740      0.12233    -0.032237
       2.3206      0.81527     -0.10044      0.31785
       3.2826      0.61491     0.049601     -0.41377
       4.2886       1.4344      0.44557      0.72554
       4.8417       1.1850      0.17252     -0.20085
       4.3026       1.2911     -0.89102     0.022867
```

PcFimlDgp::Asymp

```
Asymp();
```

No return value.

Description

Prints an asymptotic analysis of the current DGP.

PcFimlDgp::Create

```
Create(const iFreq, const iYear1, const iPeriod1, const cTdiscard,
    const mxDgpLag, const mxT);
```

`iFreq`	in:	int, database frequency
`iYear1`	in:	int, start year of observation $T_1 + d$
`iPeriod1`	in:	int, start period of observation $T_1 + d$
`cTdiscard`	in:	int, number of discards, d
`mxDgpLag`	in:	int, maximum lag s to be used in DGP
`mxT`	in:	int, maximum sample size to be used, $= T_2^* - T_1 - d + 1$ (this excludes lags and discards)

No return value.

Description

Creates the database. After this, `Select` may be used to formulate the DGP, with group identifier `Y_VAR` or `Z_VAR`. The database name of the variables are `"Ya"`, `"Yb"`, ..., and `"Za"`, `"Zb"`, ... The `Constant`, `Trend` and normal `Seasonals` are automatically created.

PcFimlDgp::DiscardZ

```
DiscardZ();
```

No return value.

Description

Discards the current $\mathbf{z}_t$; the next call to `Generate()` will generate new observations on $\mathbf{z}_t$.

PcFimlDgp::GenerateTo

```
GenerateTo(const cT);
```

`cT`	in:	int, sample size T

Return value

`GenerateTo` returns generated $\mathbf{Y} : \mathbf{U}$, as a $T \times 2n$ matrix.

Description

Generates `cT` observation of the current DGP.

PcFimlDgp::GenerateU, GenerateV, GenerateY, GenerateZ

```
virtual GenerateU(const cT);
virtual GenerateV(const cT);
virtual GenerateZ(const cT, const mC0t, const mV);
virtual GenerateY(const cT, const mPit, const mU);
```

`cT`	in:	int, sample size T
`mPit`	in:	$k \times n$ matrix $\mathbf{\Pi}'$
`mC0t`	in:	$q \times q$ matrix $\mathbf{C}_0'$
`mV`	in:	$T \times q$ matrix $\mathbf{V}$
`mU`	in:	$T \times n$ matrix $\mathbf{U}$

Return value

`GenerateU` returns generated $\mathbf{U} = (\mathbf{u}_{T-1} \dots \mathbf{u}_{T_2})'$.
`GenerateV` returns generated $\mathbf{V} = (\mathbf{v}_{T-1} \dots \mathbf{u}_{T_2})'$.
`GenerateY` returns generated $\mathbf{Y} = (\mathbf{y}_{T-1} \dots \mathbf{y}_{T_2})'$.
`GenerateZ` returns generated $\mathbf{Z} = (\mathbf{z}_{T-1} \dots \mathbf{z}_{T_2})'$.

Description

These virtual functions are called by `GenerateTo` to generate the data using matrix expressions (the default).

PcFimlDgp::GenerateU_t, GenerateV_t, GenerateY_t, GenerateZ_t

```
virtual GenerateU_t(const iT);
virtual GenerateV_t(const iT);
virtual GenerateZ_t(const iT, const mC0t);
virtual GenerateY_t(const iT, const mPit);
```

`iT`	in:	int, observation t
`mPit`	in:	$k \times n$ matrix $\mathbf{\Pi}'$
`mC0t`	in:	$q \times q$ matrix $\mathbf{C}_0'$

Return value

`GenerateU_t` returns generated $\mathbf{u}_t'$.
`GenerateV_t` returns generated $\mathbf{v}_t'$.
`GenerateY_t` returns generated $\mathbf{y}_t'$.
`GenerateZ_t` returns generated $\mathbf{z}_t'$.

Description

These virtual functions are called by `GenerateTo` to generate the data when using a for loop. This is the case after a call to `UseObsLoop(TRUE)`.

PcFimlDgp::GetU, GetV, GetY, GetZ

```
GetU();
GetV();
GetY();
GetZ();
```

Return value

`GetU` returns current $\mathbf{U} = (\mathbf{u}_{T_1} \ldots \mathbf{u}_{T_2})'$, as a $T \times n$ matrix.
`GetV` returns current $\mathbf{V} = (\mathbf{v}_{T_1} \ldots \mathbf{u}_{T_2})'$, as a $T \times q$ matrix.
`GetY` returns current $\mathbf{Y} = (\mathbf{y}_{T_1} \ldots \mathbf{y}_{T_2})'$, as a $T \times n$ matrix.
`GetZ` returns current $\mathbf{Z} = (\mathbf{z}_{T_1} \ldots \mathbf{z}_{T_2})'$, as a $T \times q$ matrix.

PcFimlDgp::PcFimlDgp

```
PcFimlDgp(const cY, const cZ);
```

`cY`	in:	int, n, dimension of $\mathbf{y}_t$
`cZ`	in:	int, q, dimension of $\mathbf{z}_t$

No return value.

Description

Constructor.

PcFimlDgp::Prepare

```
virtual Prepare();
```

No return value.

Description

Virtual function which must be called prior to data generation.

PcFimlDgp::Print

```
Print();
```

No return value.

Description

Prints the setup of the current DGP.

PcFimlDgp::SetDistribution

```
SetDistribution(const iEqn, const iDist, mM, mS);
```

Description

See `RanPcNaive::SetDistribution()`.

PcFimlDgp::SetFixedZ

```
SetFixedZ(const fSetting);
```

Description

See `RanPcNaive::SetFixedZ()`.

PcFimlDgp::SetInit

```
SetInit(const iDgp, const mInit);
```

`iEqn`	in:	one of: `Y_DGP`, `Z_DGP`
`mInit`	in:	0, or
		`Y_DGP`: $s \times n$ matrix
		`Z_DGP`: $s \times q$ matrix

No return value.

Description

This function is used to specify initial values for the data generation. By default the initial values are 0.

The first row of `mInit` will be stored at observation $0 = T_1 - d - s$ in the database.

PcFimlDgp::SetU, SetV, SetY, SetZ

```
SetU(const m);
SetY(const m);
```

`m`	in:	$T \times n$ matrix

```
SetV(const m);
SetZ(const m);
```

`m`	in:	$T \times q$ matrix

No return value.

Description

`SetU` sets $\mathbf{U} = \mathbf{u}_{T_1} \dots \mathbf{u}_{T_1+T-1}$.
`SetV` sets $\mathbf{V} = \mathbf{v}_{T_1} \dots \mathbf{v}_{T_1+T-1}$.
`SetY` sets $\mathbf{Y} = \mathbf{y}_{T_1} \dots \mathbf{y}_{T_1+T-1}$.
`SetZ` sets $\mathbf{Z} = \mathbf{z}_{T_1} \dots \mathbf{z}_{T_1+T-1}$.

PcFimlDgp::SetYParameter

```
SetYParameter(const mPit);
```

`mPit`	in:	$k \times n$ matrix $\mathbf{\Pi}'$

No return value.

Description

Sets the parameters for the $\mathbf{y}_t$ equation.

PcFimlDgp::SetZParameter

```
SetZParameter(const mC0);
```

`mC0`	in:	$q \times q$ matrix $\mathbf{C}_0$

No return value.

Description

Sets the parameters for the $\mathbf{z}_t$ equation.

PcFimlDgp::UseObsLoop

`UseObsLoop(const bUseObsLoop);`

`bUseObsLoop`	in:	`TRUE`: generate data by looping over observations

No return value.

Description

By default, the data are generated using matrix expressions. Use this to generate the data in a `for`-loop. This is considerably slower, but gives more flexibility.

12.5 PcNaiveDgp : RanPcNaive class

The `PcNaiveDgp` class is a data generation process (DGP), designed for use in dynamic econometric Monte Carlo experiments. The class is used through the header file `ranpcnaive.oxh`.

The class derives from RanPcNaive, see §12.5. Unlike RanPcNaive, the generated data are stored inside the object, and retrieved using Get functions.

PcNaiveDgp::DiscardZ

```
PcNaiveDgp::DiscardZ();
```

No return value.

Description

Discards the current $\mathbf{z}_t$; the next call to `Generate()` will generate new observations on $\mathbf{z}_t$.

PcNaiveDgp::Generate, PcNaiveDgp::GenerateTo

```
PcNaiveDgp::Generate(const cT);
PcNaiveDgp::GenerateTo(const cT);
```

`cT`	in:	int, sample size T

Return value

`Generate` returns generated $\mathbf{Y} = (\mathbf{y}_0 \ldots \mathbf{y}_T)'$, as a $T \times n$ matrix.

`GenerateTo` returns generated $\mathbf{Y} : \mathbf{U}$, as a $T \times 2n$ matrix.

Description

Generates `cT` observation of the current DGP and stores it in the current object.

PcNaiveDgp::GenerateBreakTo

```
PcNaiveDgp::GenerateBreakTo(const cT,const iTbreak,const iTreset,
    const mA0, const mA1, const mA2, const mA3, const mA5);
```

`cT`	in:	int, sample size T
`iTbreak`	in:	int, T_1, first observation with break
`iTreset`	in:	int, T_2, first observation after the break
`mA0`	in:	$n \times n$ matrix $\mathbf{A}_0^*$ *must have zeros on the diagonal*
`mA1`	in:	$n \times n$ matrix $\mathbf{A}_1^*$
`mA2`	in:	$n \times q$ matrix $\mathbf{A}_2^*$
`mA3`	in:	$n \times 1$ matrix $\mathbf{a}_3^*$
`mA5`	in:	$n \times n$ matrix $\mathbf{A}_5^*$

Return value

Returns generated generated $\mathbf{Y} : \mathbf{U}$, as a $T \times 2n$ matrix.

Description

Generates `cT` observation of the current DGP and stores it in the current object. For observations $[0, T_1 - 1]$ and $[T_2, T - 1]$ the original DGP is used. For observations $[T_1, T_2 - 1]$ the DGP as specified in the arguments is used. Note that only the Y equation can have a break.

PcNaiveDgp::GetU, GetY, GetZ

```
PcNaiveDgp::GetU();
PcNaiveDgp::GetY();
PcNaiveDgp::GetZ();
```

Return value

`GetU` returns current $\mathbf{U} = (\mathbf{u}_0 \ldots \mathbf{u}_{T-1})'$, as a $T \times n$ matrix.
`GetY` returns current $\mathbf{Y} = (\mathbf{y}_0 \ldots \mathbf{y}_{T-1})'$, as a $T \times n$ matrix (as does `Generate`).
`GetZ` returns current $\mathbf{Z} = (\mathbf{z}_0 \ldots \mathbf{z}_{T-1})'$, as a $T \times q$ matrix.

PcNaiveDgp::PcNaiveDgp

```
PcNaiveDgp::PcNaiveDgp(const cY, const cZ);
```

`cY`	in:	int, n, dimension of $\mathbf{y}_t$
`cZ`	in:	int, q, dimension of $\mathbf{z}_t$

No return value.

Description

Constructor.

12.6 RanMC class

The `RanMC` class provides random number generation of specific distribution for use by the RanPcNaive, PcNaiveDgp and PcFimlDgp classes. All member functions are static, and can be used without constructing an object, for example as:

```
    x = RanMC::Choleski(x);
```

RanMC::Choleski

```
static Choleski(const mSig);
```

`mSig`	in:	square symmetric matrix

Return value

The Choleski decomposition of `mSig`. `mSig` may have zeros on the diagonal; the corresponding rows and columns are ignored in the decomposition, and will be zero in the return value.

RanMC::CheckDist

```
static CheckDist(const sFunc, iDist, mPar1, mPar2);
```

`iDist`	in:	int, see `RanPcNaive::SetDistribution()`
`mPar1`	in:	matrix, see `RanPcNaive::SetDistribution()`
`mPar2`	in:	matrix, see `RanPcNaive::SetDistribution()`

Return value

Returns an array of three values:

`iDist`	int, distribution
`mPar1`	matrix, adjusted input value
`mPar2`	matrix, adjusted input value

Description

The following adjustments are made:

- column vectors are made into row vectors;
- `MVNORMAL_CORR`: matrix with standard deviations/correlations is translated to covariance matrix; distribution is set to `MVNORMAL`;
- `MVNORMAL`: second argument returned as transposed Choleski factor;
- `NORMAL`: second argument returned as square root of input value.

RanMC::RanDist

```
RanDist(const iDist, const cT, const cY, const mDf1, const mDf2);
```

`iDist`	in:	int, distribution, may not be `MVNHETERO`
`cT`	in:	int, desired sample size T
`cY`	in:	int, number of variables n
`mDf1`	in:	matrix, first parameter
`mDf2`	in:	matrix, second parameter

Return value

Returns a $T \times n$ matrix of random numbers from the specified distribution. The distribution parameters must be as returned from `CheckDist()`.

Example

The following program generates the same bivariate normal random numbers twice (also see page 5):

```
#include <oxstd.oxh>
#import <ranmc>

main()
{
    decl x, par1, par2, idist, mu = <9,3>, sigma = <4,1;1,2>;
    [idist, par1, par2] =
        RanMC::CheckDist("text", MVNORMAL, mu, sigma);
    // use RanMC class, note: calling CheckDist first
    ranseed(-1);
    x = RanMC::RanDist(MVNORMAL, 5, 2, par1, par2);
    // or use as described in How to chapter
    ranseed(-1);
    x ~= rann(5, 2) * choleski(sigma)' + mu;
    print(x);
}
```

RanMC::RanDist1

```
static RanDist1(const iDist, const cY, const mDf1, const mDf2,
    const mUlag, const mYlag);
```

`iDist`	in:	int, distribution
`cY`	in:	int, number of variables n
`mDf1`	in:	matrix, first parameter
`mDf2`	in:	matrix, second parameter
`mUlag`	in:	matrix, last period error term, used for `MVNARCH`
`mYlag`	in:	matrix, last period generated numbers, used for `MVNHETERO`

Return value

Returns a $1 \times n$ matrix of random numbers from the specified distribution. The distribution parameters must be as returned from `CheckDist()`.

RanMC::WriteDist

```
static WriteDist(const sPar, const iDist, const mDf1, const mDf2);
```

`sPar`	in:	string, name of generated variable (e.g. `"Y"`)

No return value.

Description

Writes the used distribution. The distribution parameters must be as returned from `CheckDist()`.

12.7 RanPcNaive class

The `RanPcNaive` class is a data generation process (DGP), designed for use in dynamic econometric Monte Carlo experiments. The class is used through the header file `ranpcnaive.oxh`. The design is an n-variate version of the DGP used in Hendry, Neale, and Ericsson (1991). The form of the DGP in mathematical formulation is:

$$\begin{aligned}
\mathbf{y}_t &= \mathbf{A}_0\mathbf{y}_t + \mathbf{A}_1\mathbf{y}_{t-1} + \mathbf{A}_2\mathbf{z}_t + \mathbf{a}_3 + \mathbf{A}_5\mathbf{y}_{t-2} + \mathbf{u}_t, \\
\mathbf{u}_t &= \mathbf{B}_0\mathbf{u}_{t-1} + \mathbf{e}_t + \mathbf{B}_1\mathbf{e}_{t-1}, \\
\mathbf{z}_t &= \mathbf{C}_0\mathbf{z}_{t-1} + \mathbf{c}_1 + \mathbf{c}_2 t + \mathbf{v}_t.
\end{aligned} \tag{12.1}$$

The vectors $\mathbf{y}_t, \mathbf{u}_t, \mathbf{e}_t$ are $n \times 1$, so that the coefficient matrices $\mathbf{A}_0, \mathbf{A}_1, \mathbf{B}_0, \mathbf{B}_1$ are $n \times n$, and $\mathbf{a}_3$ is $n \times 1$. The $\mathbf{z}_t$ vector is $q \times 1$, making $\mathbf{a}_2$ $n \times q$, $\mathbf{C}_0$ $q \times q$, and $\mathbf{c}_1, \mathbf{c}_2$ $q \times 1$. The zs can be kept fixed between experiments, or regenerated for the experiment. A distribution for $\mathbf{e}_t$ and $\mathbf{v}_t$ can be specified.

The DGP can also be formulated in equilibrium correction form:

$$\Delta\mathbf{y}_t = \boldsymbol{\alpha}\boldsymbol{\beta}'\mathbf{y}_{t-1} + \mathbf{A}_2\mathbf{z}_t + \mathbf{a}_3 + \mathbf{A}_5^*\Delta\mathbf{y}_{t-1} + \mathbf{u}_t. \tag{12.2}$$

Example

.... *samples/simulation/pcndgp.ox*

```
#include <oxstd.oxh>
#import <ranpcnaive>

main()
{
    decl dgp = new RanPcNaive(2,1);

    dgp.SetYParameter(zeros(2,2), <0.9,0;0.1,0.8>,
        <0.2;0.2>, <1;0>);
    dgp.SetZParameter(<0.5>, <0>, <0>);
    dgp.SetDistribution(U_DGP, MVNORMAL, zeros(2,1),
        ones(2,2)/10 + unit(2)/5);

    dgp.Print();
    decl y, x, u;
    [y, x, u] = dgp.GenerateTo(6);
    print("%c", {"Ya", "Yb", "Ua", "Ub"}, y ~ u);

    delete dgp;
}
```

produces (all non specified parameters are zero by default):

```
---- PcNaive (2.00) DGP ----
y is (2 x 1), z is (1 x 1) and fixed.

y[t] = e[t] + A1 y[t-1] + A2 z[t] + a3
A1 =
      0.90000       0.00000
      0.10000       0.80000
A2 =
      0.20000
      0.20000
a3 =
       1.0000
      0.00000

e ~ MVN(0,sigma)
sigma=
      0.30000      0.10000
      0.10000      0.30000

z[t] = v[t] + C0 z[t-1]
C0 =
      0.50000

v ~ N(0,1)

           Ya           Yb           Ua           Ub
       1.1673     0.012740      0.12233    -0.032237
       2.3206      0.81527     -0.10044      0.31785
       3.2826      0.61491     0.049601     -0.41377
       4.2886       1.4344      0.44557      0.72554
       4.8417       1.1850      0.17252     -0.20085
       4.3026       1.2911     -0.89102     0.022867
```

RanPcNaive::Asymp

`RanPcNaive::Asymp();`

No return value.

Description

Prints an asymptotic analysis of the current DGP: companion matrix with eigenvalues, together with cointegrating space and level of integration of DGP: I(0), I(1) or I(2).

RanPcNaive::GenerateTo

`RanPcNaive::GenerateTo(const cT);`

`cT`	in:	int, sample size T

Return value

`GenerateTo` returns an array of three elements, holding the generated $\{\mathbf{Y}, \mathbf{Z}, \mathbf{U}\}$. If **Z** is fixed, the fixed value is used, unless none has been set.

Description

Generates `cT` observation of the current DGP.

RanPcNaive::GenerateBreakTo

```
RanPcNaive::GenerateBreakTo(const cT,const iTbreak,const iTreset,
    const mA0, const mA1, const mA2, const mA3, const mA5);
```

`cT`	in:	int, sample size T
`iTbreak`	in:	int, T_1, first observation with break
`iTreset`	in:	int, T_2, first observation after the break
`mA0`	in:	$n \times n$ matrix $\mathbf{A}_0^*$ *must have zeros on the diagonal*
`mA1`	in:	$n \times n$ matrix $\mathbf{A}_1^*$
`mA2`	in:	$n \times q$ matrix $\mathbf{A}_2^*$
`mA3`	in:	$n \times 1$ matrix $\mathbf{a}_3^*$
`mA5`	in:	$n \times n$ matrix $\mathbf{A}_5^*$

Return value

Returns an array of three elements, holding generated $\{\mathbf{Y}, \mathbf{Z}, \mathbf{U}\}$.

Description

Generates `cT` observation of the current DGP. For observations $[0, T_1 - 1]$ and $[T_2, T - 1]$ the original DGP is used. For observations $[T_1, T_2 - 1]$ the DGP as specified in the arguments is used. Note that only the Y equation can have a break.

RanPcNaive::HasFixedZ

`RanPcNaive::HasFixedZ();`

Return value

`TRUE` if **Z** is fixed.

RanPcNaive::GetFixedZValue

`RanPcNaive::GetFixedZValue();`

Return value

Returns the current fixed $\mathbf{Z}$ matrix.

RanPcNaive::RanPcNaive

```
RanPcNaive::RanPcNaive(const cY, const cZ);
```

cY	in:	int, n, dimension of $\mathbf{y}_t$
cZ	in:	int, q, dimension of $\mathbf{z}_t$

No return value.

Description

Constructor.

RanPcNaive::Print

```
RanPcNaive::Print();
```

No return value.

Description

Prints the setup of the current DGP.

RanPcNaive::SetDistribution

```
RanPcNaive::SetDistribution(const iEqn, const iDist, mM, mS);
```

iEqn	in:	one of: `U_DGP`, `Z_DGP`
iDist	in:	one of: `NO_DIST`, `NORMAL`, `MVNORMAL`, `MVNORMAL_CORR`, `LOGNORMAL`, `T_DIST`, `F_DIST`, `EXPONENTIAL`, `MVNARCH`, `MVNHETERO`
mM	in:	first parameter of distribution, $\boldsymbol{\alpha}$ `MVNARCH`, `MVNHETERO`: $n \times n$ for $\mathbf{y}_t, \mathbf{u}_t$; $q \times q$ for $\mathbf{z}_t$ others: $n \times 1$ for $\mathbf{y}_t, \mathbf{u}_t$; $q \times 1$ for $\mathbf{z}_t$
mS	in:	second parameter of distribution, $\boldsymbol{\beta}$ `MVNORMAL`, `MVNARCH`, `MVNHETERO`: $n \times n$ for $\mathbf{y}_t, \mathbf{u}_t$; $q \times q$ for $\mathbf{z}_t$ others: $n \times 1$ for $\mathbf{y}_t, \mathbf{u}_t$; $q \times 1$ for $\mathbf{z}_t$

No return value.

Description

Specifies the distribution for the $\mathbf{u}$, or $\mathbf{z}$ equation in (12.1). The first argument indicates the equation, the second the distribution. The last two arguments parametrize the distribution.

Write $\boldsymbol{\epsilon}_t$ for either $\mathbf{e}_t$ or $\mathbf{v}_t$, then:

argument	distribution
NO_DIST	0 (no distribution)
NORMAL	$\epsilon_{it} \sim \mathsf{N}(\alpha_i, \beta_i) = \mathsf{N}(0,1) \times \sqrt{\beta_i} + \alpha_i$
MVNORMAL	$\boldsymbol{\epsilon}_t \sim \mathsf{N}_n(\boldsymbol{\alpha}, \boldsymbol{\beta})$
MVNORMAL_CORR	$\boldsymbol{\epsilon}_t \sim \mathsf{N}_n(\boldsymbol{\alpha}, \boldsymbol{\beta})$ specified with standard deviations on diagonal, correlations on lower diagonal
LOGNORMAL	$\epsilon_{it} \sim \Lambda(\alpha_i, \beta_i) = \exp\{N(0,1)\} \times \sqrt{\alpha_i} + \beta_i$
T_DIST	$\epsilon_{it} \sim \mathsf{t}(\alpha_i)$
F_DIST	$\epsilon_{it} \sim \mathsf{F}(\alpha_i, \beta_i)$
EXPONENTIAL	$\epsilon_{it} \sim \mathsf{exp}(\alpha_i)$
MVNARCH	$\boldsymbol{\epsilon}_t \sim \mathsf{N}_n(\mathbf{0}, \boldsymbol{\alpha} + \boldsymbol{\beta}\boldsymbol{\epsilon}_{t-1}\boldsymbol{\epsilon}'_{t-1}\boldsymbol{\beta}')$
MVNHETERO	$\mathbf{e}_t \sim \mathsf{N}_n(\mathbf{0}, \boldsymbol{\alpha} + \boldsymbol{\beta}\mathbf{y}_{t-1}\mathbf{y}'_{t-i}\boldsymbol{\beta}')$

RanPcNaive::SetFixedZ

`RanPcNaive::SetFixedZ(const fSetFixed);`

`fSetFixed`	in:	0: $\mathbf{z}_t$ is fixed, 1: $\mathbf{z}_t$ not fixed

No return value.

Description

Specifies whether $\mathbf{z}_t$ is fixed or not, and clears the currently stored fixed value.

A new value for fixed $\mathbf{z}_t$ can be set by `SetFixedZValue` or generated by `SetNewFixedZValue`.

RanPcNaive::SetFixedZValue

`RanPcNaive::SetFixedZValue(const mZ);`

`mZ`	in:	$q \times T$ matrix $\mathbf{Z}$

No return value.

Description

Sets a new value for fixed $\mathbf{z}_t$ in the object.

RanPcNaive::SetInit

`RanPcNaive::SetInit(const iDgp, const mInit);`

`iEqn`	in:	one of: `Y_DGP`, `Z_DGP`
`mInit`	in:	0, or
		`Y_DGP`: $1 timesn$ or $2 timesn$ matrix
		`Z_DGP`: $1 timesq$ matrix

No return value.

Description

This function is used to specify initial values for the data generation. By default the initial values are 0.

The Z equation has only one lag, and `mInit` specifies $\mathbf{z}_{-1}$.

The Y equation can have up to two lags. If `mInit` has two rows, the first row specifies $\mathbf{y}_{-2}$, and the second $\mathbf{y}_{-1}$. If `mInit` has one row, that row is used for both $\mathbf{y}_{-2}$ and $\mathbf{y}_{-1}$.

RanPcNaive::SetNewFixedZValue

```
RanPcNaive::SetNewFixedZValue(const cT);
```

cT	in:	int, sample size T

No return value.

Description

Generates a new $\mathbf{Z}$ value and stores it in the object for subsequent use.

RanPcNaive::SetUParameter

```
RanPcNaive::SetUParameter(const mB0, const mB1);
```

mB0	in:	$n \times n$ matrix $\mathbf{B}_0$
mB1	in:	$n \times n$ matrix $\mathbf{B}_1$

No return value.

Description

Sets the parameters for the $\mathbf{e}_t$ equation.

RanPcNaive::SetYParameter

```
RanPcNaive::SetYParameter(const mA0, const mA1, const mA2,
    const mA3);
RanPcNaive::SetYParameter(const mA0, const mA1, const mA2,
    const mA3, const mA5);
```

mA0	in:	$n \times n$ matrix $\mathbf{A}_0$ *must have zeros on the diagonal*
mA1	in:	$n \times n$ matrix $\mathbf{A}_1$
mA2	in:	$n \times q$ matrix $\mathbf{A}_2$
mA3	in:	$n \times 1$ matrix $\mathbf{a}_3$
mA5	in:	(optional argument) $n \times n$ matrix $\mathbf{A}_5$

No return value.

Description

Sets the parameters for the $\mathbf{y}_t$ equation.

RanPcNaive::SetYParameterEcm

```
RanPcNaive::SetYParameterEcm(const mAlpha, const mBeta, const mA2,
    const mA3);
RanPcNaive::SetYParameterEcm(const mAlpha, const mBeta, const mA2,
    const mA3, const mA5);
```

mAlpha	in:	$n \times p$ matrix $\boldsymbol{\alpha}$
mBeta	in:	$n \times p$ matrix $\boldsymbol{\beta}$
mA2	in:	$n \times q$ matrix $\mathbf{A}_2$
mA3	in:	$n \times 1$ matrix $\mathbf{a}_3$
mA5	in:	(optional argument) $n \times n$ matrix $\mathbf{A}_5^*$

No return value.

Description

Sets the parameters for the $\mathbf{y}_t$ equation in equilibrium correction form. The rank of the cointegration space is p.

RanPcNaive::SetZCustom

```
RanPcNaive::SetZCustom(mCZ);
```

`mCZ`	in:	$T \times q$ matrix with custom Z

No return value.

Description

Installs a custom Z. This is added to $\mathbf{Z}$ after generation of $\mathbf{Z}$, but before Z is used in the Y equation.

RanPcNaive::SetZParameter

```
RanPcNaive::SetZParameter(const mC0, const mC1, const mC2);
```

`mC0`	in:	$q \times q$ matrix $\mathbf{C}_0$
`mC1`	in:	$q \times 1$ matrix $\mathbf{c}_1$
`mC2`	in:	$q \times 1$ matrix $\mathbf{c}_2$

No return value.

Description

Sets the parameters for the $\mathbf{z}_t$ equation.

RanPcNaive::StoreInDatabase

```
RanPcNaive::StoreInDatabase(const amYZU, const oDb, const iY0,
    const iZ0, const iU0, const cTDiscard)
```

`amYZU`	in:	array[3], holding $\{\mathbf{Y}, \mathbf{Z}, \mathbf{U}\}$ (e.g. as returned by `GenerateTo`)
`oDb`	in:	object of Database type
`iY0`	in:	int, -1 or index in Database object of first $\mathbf{Y}$ variable
`iZ0`	in:	int, -1 or index in Database object of first $\mathbf{Z}$ variable
`iU0`	in:	int, -1 or index in Database object of first $\mathbf{U}$ variable
`cTDiscard`	in:	int, 0 or number of initial observations to remove from $\mathbf{Y}, \mathbf{Z}, \mathbf{U}$

No return value.

Description

Stores generated data in a database object. If the index is -1, the corresponding variable is not changed in the Database. It is assumed that the $\mathbf{Y}$ variables are in a consecutive block, similar for $\mathbf{Z}$ and $\mathbf{U}$.

12.8 Simulator : SimulatorBase class

The `Simulator` class can be used to set up Monte Carlo experiments. Derive your own simulation experimentation class from this, overriding the virtual functions. `Simulator` will handle the replications and storage, and print the final results. The type of data it can handle are coefficients, test statistics and p-values of test statistics. The class is used through the header file `simulator.oxh`.

An extensive example, using the `PcFiml` class for estimation, is given in the file `samples/simulation/artest.ox`. An example more in line with the one here is `samples/simulation/simnor.ox`. This program compares the small sample size of two tests for normality. When run in *OxRun*, it will plot the distribution of the test statistics as the Monte Carlo experiment proceeds. A more elaborate example can be found in the *Introduction to Ox.*

The example discussed here generates data from a standard normal distribution, and estimates the mean and variance. It also tests whether the mean is different from zero. The properties of the estimated coefficients and test statistic are studied by repeating the experiment M times, and averaging the outcome of the M experiments. So the data generation process is:

$$y_t = \mu + \epsilon_t \text{ with } \epsilon_t \sim N(0, \sigma^2),$$

together with $\mu = 0$ and $\sigma^2 = 1$. We estimate the parameters from a sample of size T by:

$$\hat{\mu} = T^{-1} \sum_{t=0}^{T-1} y_t, \quad \hat{\sigma}^2 = (T)^{-1} \sum_{t=0}^{T-1} (y_t - \hat{\mu})^2,$$

and

$$\hat{s} = \left\{ (T-1)^{-1} \sum_{t=0}^{T-1} (y_t - \hat{\mu})^2 \right\}^{\frac{1}{2}} = \left\{ \frac{T}{T-1} \hat{\sigma}^2 \right\}^{\frac{1}{2}}.$$

The t-test which tests the hypothesis H_0: $\hat{\mu} = 0$ is:

$$T^{\frac{1}{2}} \frac{\hat{\mu}}{\hat{s}}.$$

The code for this Monte Carlo experiment is in `simtest.ox` (remember that the simula code needs to be imported in):

Example

.... *samples/simulation/simtest.ox*

```
#include <oxstd.oxh>
#import <simulator>

/*----------------- SimNormal : Simulator -----------------*/
class SimNormal : Simulator        // inherit from simulation
{
    SimNormal();                                // constructor
    Generate(const iRep, const cT, const mxT);
};
SimNormal::SimNormal()
{
    Simulator(<50>, 100, 10000, TRUE, -1,
        <0.2,0.1,0.05,0.01>,       // p-values to investigate
        <0,1>);                    // true coefs: mean=0, sd=1
    SetTestNames({"t-value"});
    SetCoefNames({"constant", "std.dev"});
    SetTwoSided(<1>);
}
SimNormal::Generate(const iRep, const cT, const mxT)
{
    decl my, sdevy, meany, test;

    my = rann(cT, 1);                           // generate data

    meany = meanc(my);                              // mean of y
    sdevy = sqrt(cT * varc(my) / (cT-1));      // std.dev of y
    test = meany / (sdevy / sqrt(cT));

    return
    {   1,                                  // indicates success
        meany | sdevy,                         // mean,sdev of y
        tailt(test, cT-1),                 // t(T-1) distributed
        test                                  // t-value on mean
    };
}
/*-------------- END SimNormal : Simulator ----------------*/

main()
{
    decl experiment = new SimNormal();          // create object
    experiment.Simulate();                     // do simulations
    delete experiment;                          // remove object
}
```

produces

```
T=50, M=10000, RNG=MWC_52: loop seed, common seed=-1

moments of test statistics
                      mean      std.dev     skewness  ex.kurtosis
t-value         -0.0051694       1.0128     0.012870     0.020042

critical values (two sided: left tail quantiles)
                       10%           5%         2.5%         0.5%
t-value            -1.2989      -1.6651      -1.9692      -2.5689

critical values (two sided: right tail quantiles)
                       10%           5%         2.5%         0.5%
t-value             1.3061       1.6563       2.0005       2.5952

rejection frequencies
                       20%          10%           5%           1%
t-value            0.20090      0.10080     0.048400    0.0079000
[ASE]            0.0040000    0.0030000    0.0021794   0.00099499

moments of estimates
                      mean         MCSD
```

```
constant       -0.00070079      0.14042
std.dev            0.99472      0.10023

biases of estimates
                 mean bias         MCSE         RMSE   true value
constant       -0.00070079    0.0014042      0.14042      0.00000
std.dev         -0.0052756    0.0010023      0.10037       1.0000
```

The sample size is $T = 50$, with $M = 10000$ experiments. Setting the seed enables us to use common random numbers (i.e. the same random numbers in different experiments). Note that Ox always starts with a fixed seed, so exactly the same results will be obtained when rerunning the program. The first table gives the empirical critical values for the test statistic, at the p-values we provided. These should correspond to the theoretical distribution, namely $\mathtt{t}(49)$. The value 1.306 is the 9000th number in the 10000 `t`-values after sorting the `t` values (computed using `quantiler`). The empirical rejection frequencies give the percentage of experiments which were rejected at the specified probability points, based on the p-values returned by `GetPvalues`. The final table gives the results for the coefficients. If $\hat{\mu}_m$ is the estimated mean for experiment m, and μ the true parameter then:

mean	$\bar{\hat{\mu}} = M^{-1}\sum_{m=0}^{M-1}\hat{\mu}_m,$
std.dev	$\hat{\sigma}_{\hat{\mu}} = \left\{M^{-1}\sum_{m=0}^{M-1}(\hat{\mu}_m - \bar{\hat{\mu}})^2\right\}^{\frac{1}{2}},$
mean bias	$\bar{\hat{\mu}} - \mu,$
se mean bias	$\hat{\sigma}_{\bar{\hat{\mu}}} = M^{-\frac{1}{2}}\hat{\sigma}_{\hat{\mu}},$
rmse	$\left\{M^{-1}\sum_{m=0}^{M-1}(\hat{\mu}_m - \mu)^2\right\}^{\frac{1}{2}} = \left\{(\text{std.dev})^2 + (\text{mean bias})^2\right\}^{\frac{1}{2}},$

where RMSE is the root of the mean squared error. The standard deviation of the simulated coefficient is also called MCSD (Monte Carlo Standard Deviation). When simulating coefficients, it is also possible to compute the mean of the estimated coefficient standard error, this is called the MCSE.

Note that the functions are documented as belonging to the Simulator class, but are actually mostly in SimulatorBase. The source code of both classes can be found in `ox/src`. A further example is given in Chapter 4.

Simulator succeeds the Simulation class which was used up to Ox 6 (this is still available through `simula.oxo` and `simula.oxh`, but now deprecated).

Simulator::Generate

```
virtual Simulator::Generate(const iRep, const cT, const mxT);
```

`iRep`	in:	int, index of current replication (0 is first)
`cT`	in:	int, sample size to be used for replication
`mxT`	in:	int, maximum sample size to be used for replication (this is only relevant when using common random numbers)

Return value

Upon failure, `Generate` should return an empty matrix or array, or an array which has integer 0 as the first element.

Upon success, `Generate` should return an array with four elements:

1. integer, value 1
2. coefficients or <>,
3. p-values or <>,
4. test statistics or <>.

If the call to the `Generate` function fails, additional experiments are run in an attempt to reach the required number of replications. The number of rejected replications is reported in the output.

Description

Virtual function which the derived class must override. It is called for every replication, and must perform the actual replication.

This function should be re-entrant unless `Simulate_serial` is called.

Simulator::Plot

```
virtual Simulator::Plot(const iRep, const iT)
```

`iRep`	in:	int, index of current replication (0 is first)
`iT`	in:	int, sample size of current replication

No return value.

Description

Virtual plot function. The default version does nothing.

Simulator::Prepare

```
virtual Simulator::Prepare(const cT, const mxT);
```

`cT`	in:	int, sample size to be used for replication
`mxT`	in:	int, maximum sample size to be used for replication (this is only relevant when using common random numbers)

No return value.

Description

Virtual function which the derived class can override if necessary. It is called just before the replications for sample size `cT` commence. It can be used to initialize common regressors (e.g.), and is not run in a parallel section.

Simulator::SaveIn7, Simulator::SaveRecIn7

```
Simulator::SaveIn7(const sFilename);
Simulator::SaveRecIn7(const sFilename);
```

`sFilename`	in:	destination file name

Return value

Returns `TRUE` if results were saved.

Description

Saves simulation results to the named file.

`SaveIn7` stores the test and coefficient values.

`SaveRecIn7` stores: coefficients, MCSE, Bias, RMSE, test critical values (right tail), rejection frequencies and moments.

Simulator::SetCoefNames, Simulator::SetTestNames

```
Simulator::SetCoefNames(const asNames);
Simulator::SetTestNames(const asNames);
```

`asNames`	in:	`SetCoefNames`: array with s_c names `SetTestNames`: array with s_t names

No return value.

Description

Installs the names of tests statistics and coefficients, to determine dimensions of the collected information, and to make the report more readable.

Simulator::SetPlotRep, SetRecursive, SetStore

```
Simulator::SetPlotRep(const iPlotRep);
Simulator::SetRecursive(const bRecursive);
Simulator::SetStore(const bStore);
```

`iPlotRep`	in:	call `Plot()` every `iPlotRep` replications (default is 0)
`bRecursive`	in:	int, `TRUE`: do recursive Monte Carlo (default is `FALSE`)
`bStore`	in:	int, store results of all replications for later access (default is `FALSE`)

No return value.

Simulator::SetTwoSided

```
Simulator::SetTwoSided(const mIsTwoSided);
```

`mIsTwoSided`	in:	$1 \times s_t$ matrix of 0–1 values, with a 1 for each test statistics which is two-sided.

No return value.

Description

Should be called before `Prepare` is called to indicate which tests are two-sided. Need not be called if all tests are one-sided.

The value of s_t is derived from the call to `SetTestNames`.

Simulator::Simulate

```
Simulator::Simulate()
Simulator::Simulate_parallel()
Simulator::Simulate_serial()
```

No return value.

Description

Runs the Monte Carlo experiment, and prints the results. `Simulate` runs the experiment in parallel unless plots are made (and if there are multiple cores and using Ox Professional).

`Simulate_parallel` runs in parallel, while `Simulate_serial` runs serially.

Simulator::Simulator

```
Simulator::Simulator(const mT, const mxT, const cRep,
    const fCommon, const dSeed, const mPvalue, const mTrueParam);
```

`mT`	in:	$1 \times r$ matrix of sample sizes
`mxT`	in:	int, maximum sample size
`cRep`	in:	int, number of replications
`fCommon`	in:	1: reset seed for each experiment; else 0
`dSeed`	in:	double, resets seed to `dSeed` if `fCommon == TRUE`
`mPvalue`	in:	$1 \times s_p$ matrix with p-values to test at, only used if `GetPvalues` returns p-values
`mTrueParam`	in:	$1 \times s_c$ matrix with true parameters, only used if `GetCoefficients` returns coefficients

No return value.

Description

Constructor function. The `mT`, `mPvalue`, and `mTrueParam` arguments are automatically changed to a row vector if they are a column vector on input.

Calls to `SetCoefNames` (if coefficients are generated) and/or `SetTestNames` (if p-values or test statistics are generated) are also required.

Chapter 13

Language reference

13.1 Introduction

The Ox syntax is formalized in a fashion similar to Kernighan and Ritchie (1988) and Stroustrup (1997). These two books describe the C and C++ languages on which the Ox language is modelled (although the object-oriented features in Ox are closer to those of Java than C++).

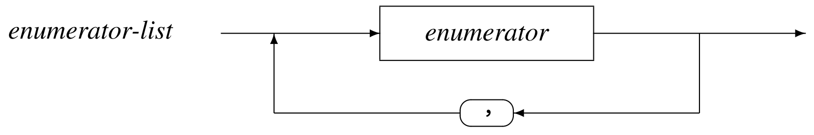
As an example, consider the syntax of `enum` declaration statements:

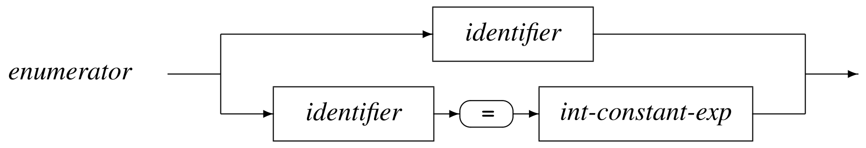
`enum` { *enumerator-list* } ;

enumerator-list:
  enumerator
  enumerator-list **,** *enumerator*

enumerator:
  identifier
  identifier = *int-constant-expression*

Symbols which have to be typed literally are given in `typewriter` font; these are called terminal symbols. *Italic* symbols are non-terminal, and require further definition. Ultimately, the whole syntax can be reduced to terminal statements. The subscript $_{opt}$ denotes an optional element. In this example, *identifier* and *int-constant-expression* remain as yet undefined. An *enumerator-list* is defined recursively: consisting of one or more enumerators, separated by columns. This can be visualized as follows:





13.2 Lexical conventions

13.2.1 Tokens

The first action of a compiler is to divide the source code into units it can understand, so-called tokens. There are four kinds of tokens: identifiers, keywords, constants (also called literals) and operators. White space (newlines, formfeeds, tabs, comments) is ignored, but can serve to separate tokens.

13.2.2 Comment

Anything between `/*` and `*/` is considered comment. This comment *can* be nested (unlike C and C++). Everything following `//` up to the end of the line is also comment, but is ignored inside `/*` ... `*/` type comment. So nested comment is possible:

```
one = cons + 1;        // comment
/* two = cons + 1;     // comment
*/
```

This is also legal:

```
two = cons + 1;    /* comment /* nested comment */ */
```

Note that code can also be removed using preprocessor statements, see §13.9.4.

13.3 Identifiers

Identifiers are made up of letters and digits. The first character must be a letter. Underscores (_) count as a letter. Valid names are `CONS`, `cons`, `cons_1`, `_a_1_b`, etc. Invalid are `#CONS`, `1_CONS`, `log(X)`, etc. Ox is case sensitive, so `CONS` and `cons` are different identifiers. It is better not to use identifiers with a leading underscore, as several compilers use these for internal names. The maximum length of an identifier is 60 characters; additional characters are ignored.

13.3.1 Keywords

The following keywords are reserved:

keyword: one of

```
array      default   foreach    operator   static
break      delete    goto       parallel   string
case       do        if         private    struct
char       double    inline     protected  switch
class      else      int        public     switch_single
const      enum      matrix     return     this
continue   extern    namespace  serial     virtual
decl       for       new        short      while
```

13.3.2 Constants

Arithmetic types, string type and array type (see §13.4.1) have corresponding constants.

constant:
  scalar-constant:
    int-constant
    double-constant
  vector-constant:
    matrix-constant
  string-constant
  array-constant

13.3.2.1 Integer constants

A sequence of digits is an integer constant. A hexadecimal constant is a sequence of digits and the letters `A` to `F` or `a` to `f`, prefixed by `0x` or `0X`. Examples are:

`1236`	
`0x1a`	(26 decimal)
`0xFF`	(255 decimal)
`0xffffffff`	(–1 decimal using 32 bit integers)

13.3.2.2 Character constants

Character constants are interpreted as an integer constant. A character constant is an integer constant consisting of a single character enclosed in single quotes (e.g. `'a'` and `'0'`) or an escape sequence enclosed in single quotes.

escape-sequence: one of

`\"`	double quote (`"`)	`\'`	single quote (`'`)
`\0`	null character	`\\`	backslash (\)
`\a`	alert (bel)	`\b`	backspace
`\f`	formfeed	`\n`	newline
`\r`	carriage return	`\t`	horizontal tab
`\v`	vertical tab	`\x`*hh*	hexadecimal number (*hh*)

So `'\n'` is the integer constant corresponding to the newline character. On most systems the newline character has decimal value 10, and in that case could also be written as `'\x0A'` or `'\x0a'`, but not `'\X0a'`.

13.3.2.3 Double constants

A double constant consists of an integer part, a decimal point, a fraction part, an `e`, `E`, `d` or `D` and an optionally signed integer exponent. Either the integer or the fraction part may be missing (not both); either the decimal point or the full exponent may be missing (not both). A hexadecimal double constant is written as `0x.`*hhhhhhhhhhhhhhhh*. The format used is an 8 byte IEEE real. The hexadecimal string is written with the most significant byte first (the sign and exponent are on the left). If any hexadecimal digits are missing, the string is left padded with 0's. Examples of correct double constants:

`0.`		`1.2`	
`.5`		`-.5e-10`	
`2.1E-112`		`1D-1`	(0.1)
`1E1`	(10.0)	`0x.7FF0000000000000`	(infinity)
`0x.3ff0000000000000`	(1)	`0x.3fb999999999999a`	(–0.1)

The last example shows that most numbers which can be expressed exactly in decimal notation, cannot be represented exactly on the computer.

Double constants in an external declaration (see §13.5.4) may use a dot to represent a missing values. This sets the variable to `.NaN` (Not a Number).

13.3.2.4 Matrix constants

A matrix constant lists within < and > the elements of the matrix, row by row. Each row is delimited by a semicolon, successive elements in a row are separated by a comma. For example:

```
< 00, 01, 02; 10, 11, 12 >
< 0.0, 0.1, 0.2 >
< 1100 >
```

which are respectively a 2×3 matrix, a 1×3 matrix and a 1×1 matrix:

$$\begin{pmatrix} 00 & 01 & 02 \\ 10 & 11 & 12 \end{pmatrix} \quad \begin{pmatrix} 0.0 & 0.1 & 0.2 \end{pmatrix} \quad \begin{pmatrix} 1100 \end{pmatrix}$$

Elements in a matrix constant can be specified as:

matrix element:

constant-expression
constant-expression : *constant-expression*
constant-expression : [*constant-expression*] *constant-expression*
[*constant-expression*] [*constant-expression*] = *constant-expression*
[*constant-expression*] * *constant-expression*

The constant expressions must evaluate to an integer or a double. The index of each row is one higher than the previous row. Within each row, the column index of an element is one higher than that created with the previous element in the same row.

We have seen examples of the first element type. The second specifies an integer range, e.g. `2:5` corresponds to `2,3,4,5`. The range may decrease, so that `5.3:2.8` corresponds to `5.3,4.3,3.3`. It is also possible to specify a step size as in `2:[2]8`, which gives `2,4,6,8`. The third form sets a specific element in the matrix (which overrides the location implicit in the position of the element in the matrix constant). Note that the top left element is `[0][0]`, the second element in the first row `[0][1]`, etc. Consider for example:

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix} \quad \text{indexed as} \quad \begin{matrix} \texttt{[0][0]} & \texttt{[0][1]} & \texttt{[0][2]} \\ \texttt{[1][0]} & \texttt{[1][1]} & \texttt{[1][2]} \\ \texttt{[2][0]} & \texttt{[2][1]} & \texttt{[2][2]} \end{matrix}$$

Finally, it is possible to specify a number of identical elements, e.g. `[3]*0` corresponds to `0,0,0`. Unspecified elements are set to zero.

As an example involving all types, consider:

```
< [4]*1,2; 10,11,14-2; 1:4; [3][4]=99,2;8:[-2-1]2 >
```

The 2 in the first row will be in column 4, as columns 3 was the last created previously. The 2 in the penultimate row gets column 5. The last specified row is equivalent to `8:[-3]2`. The result is:

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 2 & 0 \\ 10 & 11 & 12 & 0 & 0 & 0 \\ 1 & 2 & 3 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 99 & 2 \\ 8 & 5 & 2 & 0 & 0 & 0 \end{pmatrix}$$

Further examples are given in §13.5.4.

Missing values in a matrix constant could be represented with a dot, or `.NaN` which represents NaN (Not a Number), e.g.: `< .,2,3; 4,.,6 >` Similarly, `.Inf` represents infinity. An empty matrix can be written as: `<>`

13.3.2.5 String constants

A string constant is a text enclosed in double quotes. Adjacent string constants are concatenated. A null character is always appended to indicate the end of a string. The maximum length of a string constant is 1024 characters. Escape sequences can be used to represent special characters, as in §13.3.2.2. At least one and at most two hexadecimal digits must be given for the hexadecimal escape sequence. A single quote need not be escaped. Some examples of string constants:

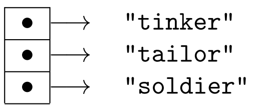
```
"a simple string"
"two strings" " joined together"
"with double quote \" and a newline character:\n"
"three ways to include a tab: \t, \x9 and \x09"
"use \\ to include a backslash,e.g. c:\\ox\\include"
```

13.3.2.6 Array constants

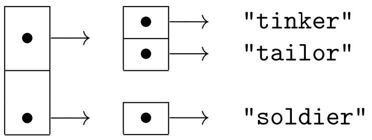
An array constant is a list of constants in braces, separated by a comma. This is a recursive definition, because the constant can itself be an array constant. The terminating level consists of non-array constants. Each level of array constants creates an array of references. An empty array is written as `{}`. For example:

```
{ "tinker", "tailor", "soldier" }
{{ "tinker", "tailor"}, {"soldier"} }
```

The first creates an array of three references to strings, the second an array of two references, the first references an array of two references to strings, the second to an array of one reference to the word soldier:



and:



Remember that `{ "tinker" "tailor" "soldier" }` is identical to an array consisting of one string: `{ "tinkertailorsoldier" }`.

13.4 Objects

13.4.1 Types

Variables in Ox are implicitly typed, and can change type during their lifetime. The life of a variable corresponds to the level of its declaration. Its scope is the section of the program in which it can be seen. Scope and life do not have to coincide.

There are three basic types and four derived types. The integer type int is a signed integer. The double precision floating point type is called double. A matrix is a two-dimensional array of doubles which can be manipulated as a whole. A string-type holds a string, while an array-type is an array of references.

arithmetic-type:	int, double, matrix
string-type:	string
scalar-type:	int, double
vector-type:	string, matrix
derived-type:	array, function, class, reference to class object

13.4.1.1 Type conversion

When a double is converted to an int, the fractional part is discarded; if the resulting value cannot be represented, the behaviour is undefined. When an int is converted to a double, the nearest representation will be used. For example, conversion to int of 1.3 and 1.7 will be 1 on both occasions. Explicit type conversion is discussed in §13.8.2.3.

A single element of a string (a character) is of type int. An int or double can be assigned to a string element, which first results in conversion to int, and then to a single byte character.

13.4.2 Lvalue

An lvalue is an object to which an assignment can be made.

13.4.3 Scope

Variables declared at the start of a statement block have scope and life restricted to the block. These variables are called automatic: they are created and initialized whenever the block is entered, and removed as soon as the block is exited. Variables declared outside any statement block have global scope and life; these are called static. Note that Ox assignment of arithmetic types and string type implies copying over the contents from the right-hand side to the left-hand side. Automatic variables of any type can be assigned to variables with broader scope.

13.5 External declarations

external-declaration:
  `enum` { *enumerator-list* } ;$_{opt}$
  specifier$_{opt}$ *type-qualifier*$_{opt}$ `decl` *ext-variable-decl-list* ;
  specifier$_{opt}$ *function-declaration* ;
  specifier$_{opt}$ *function-definition*
  `inline`$_{opt}$ *function-definition*
  `inline`$_{opt}$ *member-function-definition*
  class-specifier ;$_{opt}$

An Ox program consists of a sequence of external declarations. These either reserve storage for an object, or serve to inform of the existence of objects created elsewhere. Each program must define one function called `main`, where execution of the program will start. The return value from `main` (if any) is returned to the console window.

13.5.1 Enumerations

`enum` { *enumerator-list* } ;$_{opt}$
enumerator-list:
  enumerator
  enumerator-list , *enumerator*
enumerator:
  identifier
  identifier = *int-constant-expression*

An enumeration defines a list of integer constants. They provide a convenient way of centralizing parameters which have a constant value. Members of an enumeration cannot be assigned to, but can occur in a constant expression. By default, the first member will have value 0, and each successive member will have a value of one plus that of the previous member. The value of a member can be set by assigning it a constant integer value. The names of enumerators cannot coincide with names of other objects in the same scope (but a previously defined scalar constant may be redefined, as long as it is set to the same value). Enumerator names only exist in the file in which they occur. Enumerations should be placed in header files if they need to be shared between several source files.

Here are some examples with corresponding values:

```
enum { C_FIRST, C_SECOND, C_THIRD };                    // 0,1,2
enum { T_INT, T_DBL=2, T_STR, T_MAT=C_THIRD };        // 0,2,3,2
enum { FLAG0,FLAG1, FLAG2=FLAG1*2, FLAG3=FLAG2*2};     //0,1,2,4
enum { T_ERR = 1.0 } ;                                  // error
```

13.5.2 Storage class specifiers

specifier: one of
  `static`
  `extern`

External variable declarations (i.e. declared outside a function) create global variables: such variables exist while the program runs. The `static` specifier restricts the scope of the declared object to the remainder of the file. Although it will exist throughout the program's life, it cannot be seen from other files. In classes (§13.5.6), the `static` keyword is used with a different meaning.

The `extern` specifier informs the remainder of the file that the object can be accessed, although defined (created) in another file. The `extern` and `static` specifiers are mutually exclusive. External declarations are most conveniently placed in header files.

13.5.3 Type qualifiers

type-qualifier: one of
    `const`
    `serial`

A `const` object can only be initialized once, and not changed thereafter. The use of `serial` is explained in §13.7.7. The `const` and `serial` qualifiers are mutually exclusive.

13.5.4 External variable declarations

specifier$_{opt}$ *type-qualifier*$_{opt}$ `decl` *ext-variable-decl-list* ;

ext-variable-decl-list:
    ext-init-declarator
    ext-variable-decl-list , *ext-init-declarator*

ext-init-declarator:
    identifier
    identifier = *constant-expression*
    mat-identifier
    mat-identifier = *int-constant-expression*

mat-identifier:
    identifier [*int-constant-expression*] [*int-constant-expression*]

The `static` or `extern` specifier and the `const` qualifier preceding an external variable declaration list applies to all variables in the list. Each identifier creates space for an object with global lifetime, unless declared `extern` or `const`.

A `const` object must be initialized (unless declared `extern`) but its value may not be changed thereafter. Unless declared `extern`, a `const` object cannot be accessed from other files. If of scalar type (see §13.4.1), a `const` can appear in a constant-expression.

At the external level of declarations, as treated here, it is possible to specify a matrix size, and initialize that matrix to zero. If an external variable is created without explicit value and without dimensions, it will default to an int with value 0. Here are some examples:

```
decl a, b;                      // default to type int, value 0
```

```
enum { AAP, NOOT, MIES, WIM };
const decl ia = NOOT, ib = NOOT + WIM;              // type: int
const decl ma = < NOOT, AAP; 0, 1 >;             // type: matrix
const decl aa = {"tinker", "tailor"};             // type: array
decl id = ia * (WIM - 1) * MIES + ib;               // type: int
decl da = ia + 0.;                               // type: double
decl mb = <0:3; 4:7; 8:11>;                      // type: matrix
decl ab = { ma, ma};                              // type: array
extern decl elsewhere;                     // defined in other file

decl mc[3][3] = 1.5;              // 3 x 3 matrix with values 1.5
static serial decl s_md[2][1];           // 3 x 1 matrix of zeros

enum { ZUS = id };                        // error: id is not const
decl ih = id;                             // error: id is not const
decl ia;                                  // error: already defined
```

13.5.5 Functions

13.5.5.1 Function declarations

> *specifier*$_{opt}$ *function-declaration* ;
> `extern` `serial`$_{opt}$ *string-constant function-declaration* ;
> *function-declaration:*
>   *identifier* (*argument-type-list*$_{opt}$)
> *argument-type-list:*
>   *argument-list* , ...
>   ...
> *argument-list:*
>   *argument*
>   *argument-list* , *argument*
> *argument:*
>   `const`$_{opt}$ `decl`$_{opt}$ *identifier*
>   `const`$_{opt}$ `decl`$_{opt}$ *identifier* = *constant-expression*

A function declaration communicates the number of arguments and their types to a file, so that the function can be called correctly from that file. The actual creation of the function is done through a function definition (which at the same time declares the function). A function can be declared many times, but type and number of arguments must always be identical:

```
test0();                             // function takes no arguments
test1(const a1);                                // one const argument
test2(const a2, a3);               // two arguments, first is const
static test3(a1);               // cannot be used outside this file
extern test4(a1);             // function defined outside this file
print(a1, ...);                     // variable number of arguments
test1(a1);          // error: previous declaration was different
```

The second form, which uses `extern` *string-constant*, provides dynamic linking of extension functions (which could be written in C, FORTRAN, etc.; creation of dynamic link libraries is platform dependent). In the following example, `test5` corresponds to the external function MyCFunc(), located in the dynamic library mydll.[1] When the Ox program is linked, mydll will be automatically loaded, and the function imported.

```
extern "mydll,MyCFunc" test5(a1);
```

13.5.5.2 Function definitions

specifier$_{opt}$ *function-definition*
inline$_{opt}$ *function-definition*
function-definition:
  identifier (*argument-type-list*$_{opt}$) *compound-statement*

A function definition specifies the function header and body, and declares the function so that it can be used in the remainder of the file. A function can be declared many times, but defined only once.

The use of const is optional but useful: arguments declared const can be referenced, but cannot be changed inside the function. If the argument is a const reference, the reference cannot be changed, but what it references can. The decl keyword is optional in front of an argument. An empty argument list indicates that the function takes no arguments at all. The ... indicates a variable number of arguments; it must have the last position in the header, but cannot be the first.

```
test1(const a1);                            // declaration of test1
print(a1, ...);                     // variable number of arguments
test2(const a1, a2)                          // definition of test2
{
    test1(a2);                                 // call function test1
    print(a1, 1, 2, "\n");                   // at least one argument
    test1(a2, 1);             // error: wrong number of arguments
    a2 = 1;                                     // a2 may be changed
    a1 = 1;                                   // error: a1 is const
    /* ... */
}
```

All function arguments are passed by value. This means that a copy of the actual object is made (although the compiler will avoid this internally if the argument is not assigned to). For int, double, matrix and string types the whole object is copied. Any changes to the copy are lost as soon as the function returns. Derived types (see §13.4.1) are accessed through a reference, and that reference is passed by value. However, what is referenced may be changed, and that change will remain in effect after function return. So passing references allows a function to make a permanent change to a variable,

[1]The 64-bit version will try to load mydll_64 first, then try mydll; the appropriate extension is appended automatically. The following table lists the defaults that are searched first (thus allowing the folder structure to be shared between platforms):

mydll.dll	Windows 32-bit
mydll_64.dll	Windows 64-bit
mydll.so	Linux 32-bit
mydll_64.so	Linux 64-bit
mydll_osx.so	OS X 32-bit
mydll_sparc.so	Solaris on Sparc, 32-bit
mydll_sunx86.so	Solaris on x86, 32-bit

for examples see §13.8.2.2. It is good practice to label an argument `const` if a function doesn't change the variable.

Lambda functions are introduced in §13.8.1.2.

13.5.5.3 Returning a value

All functions may have a return value, but this return value need not be used by the caller. *If a function does not return a value, its actual return value is undefined.*

The `return` statement returns a value from the function, *and also exits the function*. So, when the program flow reaches a `return` statement, control returns to the caller, without executing the remainder of the function. The syntax of the return statement is:

`return` *return_value* ;

Or, to exit from a function which does not have a return value:

`return;`

The following example illustrates the use of `return`:

```
threes(const r, const c)                // definition of threes
{
    return constant(3, r, c);
}
otherfunc()
{
    println(threes(2, 2));
}
```

Multiple returns can be implemented through the multiple assignment statement, see §13.8.1.1:

```
func(const r, const c)                  // definition of threes
{
    return {zeros(r,c), ones(r,c)};    // array with 2 elements
}
otherfunc()
{
    decl x1, x2;
    [x1, x2] = func(3, 3);//get element [0] in x1 and [1] in x2
}
```

13.5.5.4 Default values for function arguments

Default values for function arguments can be supplied, subject to the following constraints

1. A default value cannot be replaced by another default.
2. The value must be within scope when the call is made, so that it can be substituted when compiling.
3. When a default value is supplied for an argument, all subsequent arguments must have a default value.

Default values for member calls and functions that are called as a string are injected at run-time. This is possible, because the default values become a property of the function.

The following example illustrates the use of a default argument:

```
#include <oxstd.oxh>
func(arg=<1,1>);              // forward declaration

main()
{
    func();                   // same as func(<1,1>);
}
func(arg)                     // definition
{
    println("func arg=", arg);
}
```

13.5.5.5 Variable length argument list

A special library function `va_arglist()` is used to access arguments in the variable argument list. It returns the arguments supplied for the ellipse as an array. An example illustrates:

```
test(const a, ...)
{
    decl i, args = va_arglist();

    for (i = 0; i < sizeof(args); i++)
        print (" vararg ", i, ": ", args[i]);
}
main()
{
    test("tinker", "tailor", "soldier");
}
```

which prints `vararg 0:  tailor vararg 1:  soldier`.

13.5.5.6 Inline function definitions

A function can be defined as `inline`. This instructs the compiler to expand the function body wherever it is called, and tends to be used for very small functions. The `inline` qualifier is currently ignored.

13.5.6 Classes

A class is a collection of data objects combined with functions operating on those objects. Access to data members from outside the class is through member functions: only member functions can access data directly (at least, that is the default, see §13.5.6.3 below). So by default, all data members are protected, and all function members public, using C++ parlance.

class-specifier ;$_{opt}$

class-specifier:

  `class` *identifier base-class*$_{opt}$ { *member-list* }

  `struct` *identifier base-class*$_{opt}$ { *member-list* }

base-class:

  : *identifier*

member-list:

  member

  member-list member

  `public:`

    member-list member

  `protected:`

    member-list member

member:

  `static`$_{opt}$ `decl` *member-variable-decl-list* ;

  `const`$_{opt}$ `decl` *member-variable-decl-list* ;

  `serial`$_{opt}$ `decl` *member-variable-decl-list* ;

  `static const decl` *ext-variable-decl-list* ;

  `static`$_{opt}$ *function-declaration* ;

  `virtual`$_{opt}$ *function-declaration* ;

  `enum` { *enumerator-list* } ;$_{opt}$

member-variable-decl-list:

  identifier

  member-variable-decl-list , *identifier*

Consider a simple line class, which supports drawing lines from the current cursor position to the next, and moving the cursor:

```
class Line                              // Line is the class name
{
    decl m_x, m_y;                          // two data members
    const decl m_origin;                    // const data member
    static decl sm_cLines;                 // static data member
    Line(const orig);                             // constructor
    moveto(const x, const y);                     // move cursor
    lineto(const x, const y);      // draw line and move cursor
    static getcLines();                       // static function
    static setcLines(c);                      // static function
public:
    static const decl M_CONST = 1;    // value must be set here
    enum { M_AA, M_BB = -1};
};                            // ; is optional in Ox (unlike C++)
```

All member names within a class must be unique. A class declaration introduces a type, and can be shared between source files through inclusion in header files. Ox accesses an object through a reference to the object which is created using the `new` operator. An object is removed from memory using the `delete` operator (if there is no matching `delete`, the object will exist until the program terminates). Both `new` and `delete` are unary operators.

A member function declaration can specify default arguments, subject to the restriction that, when a default value is supplied for an argument, all subsequent arguments must have a default value. Default arguments are added to the call at run time.

Data members that are `static const` must be initialized in the class declaration. Data members that are not `static` but are `const` can only be initialized in the constructor function, see §13.5.6.2. Otherwise data members can be initialized in the constructor function, or anywhere else they are accessible.

Enumerations of constants can be defined within the class through the `enum` keyword (§13.5.4). Constants defined through `enum` behave the same as `static const decl` member variables. In the example above, the `public` keyword means that `M_CONST`, `M_AA` and `M_BB` can be accessed from outside the class as `Line::M_CONST`, etc.

13.5.6.1 Member function definitions

`inline`$_{opt}$ *member-function-definition*

member-function-definition:
  identifier `::` *identifier* `(`*argument-type-list*$_{opt}$ `)` *compound-statement*

A member function provides access to data members of an object. It is defined as its class name, followed by `::` and the function name. The function name must have been declared in the class. Member functions cannot be declared outside a class; the class declaration contains the member function declaration. Only a member function can use data members of its own class directly.

Function member definitions cannot specify default arguments; they must be specified in the declarations instead (which is usually in a header file).

Here are the definitions of the member functions of class `Line`:

```
Line::Line(const orig)
{
    m_x = m_y = orig;                // set cursor at the origin
    m_origin = orig;                // only allowed in constructor
    sm_cLines++;                   // count number of Line objects
}
Line::moveto(const x, const y)
{
    m_x = x;  m_y = y;
    println("moved to ", x, " ", y);
    return this;
}
Line::lineto(const x, const y)
{
                      // draw the line from (x,y) to (ax,ay) ...
    m_x = x;  m_y = y;
    println("line to ", x, " ", y);
    return this;
}
```

The `new` operator creates an object of the specified class, calls the constructor function, and returns a reference to it. A member function is called through a member reference, which is a class object name followed by `->` or a dot. For example:

```
decl lineobj;
lineobj = new Line(0);                      // create object and
```

```
                                              // set cursor to (0,0)
    lineobj.lineto(10, 10);                 // draw line to (10, 10)
    lineobj->Line::lineto(10, 10);                       // same call
    lineobj::lineto(10, 10);                 // error, needs -> or .

    delete lineobj;          // delete object from memory when done
```

Since lineobj is of class Line, both calls to lineto are to the same function. The only difference is one of efficiency. Ox has implicit typing, so can only know the class of lineobj at run time. In the second case the class is specified, and the function address can be resolved at compile time.

13.5.6.2 Constructor and destructor functions

The member function with the same name as the class is called the constructor, and is automatically invoked when creating an object of the class. If the constructor function is absent, a default constructor function will be assumed which takes no arguments. A constructor may not be static. A constructor always returns a reference to the object for which it was called and may not specify a return value. Only the constructor function may set const data members. In the Line class, the origin is only set during construction, and not thereafter. However, each Line object has its own origin (unless origin is made static).

A destructor is called after a request to delete an object, and before the object is actually removed. It may be used to clear up any allocated objects inside the object to be deleted. A destructor function has the same name as the class, is prefixed by ~, and may neither take arguments, nor return a value. It does however receive the this reference.

```
    class Line
    {   /* ... */
        Line(const orig);                                // constructor
        ~Line();                                          // destructor
        /* ... */
    };
    test()
    {
        decl lineobj;

        lineobj = new Line(0);    //create object, call constructor
        delete lineobj;           // call destructor, delete object
    }
```

13.5.6.3 public and protected members, structs

All function members are public and data members are protected by default in a class. This means that function members can be called from anywhere by accessing an object, while data members can only be accessed from inside a class or derived class:

```
    class Line
    {   /* ... */
        decl m_x;
        Func();
    };
```

```
Line::Func()
{
    m_x = 0;              // can access data member from inside
}
test()
{
    decl lineobj = new Line(0);
    lineobj.Func();               // can access function member
    lineobj.m_x = 1;        // error: cannot access data member
}
```

A `struct` differs from a `class` only in that *all members are public*. So, if in the above example we would have used `struct Line`, then the last line (`lineobj.m_x = 1`) would have been allowed.

More fine-grained control is available using the `public` and `protected` specifiers: some variables can be made accessible, and others not. The following code illustrates:

```
class Line
{   /* ... */
public:
    decl m_x;
    decl m_y;
protected:
    decl m_z;
    Func();
};
test()
{
    decl lineobj = new Line(0);
    lineobj.Func();               // can access function member
    lineobj.m_y = 1;                       // OK: m_y is public
    lineobj.m_z = 1;                  // error: m_z is protected
}
```

Note, however, that in Ox, the addition of `public` and `protected` only applies to variables. Functions remain public.

13.5.6.4 The `this` reference and member scope

All non-static member functions receive a hidden argument called `this`, which points to the object for which the function is called. So the constructor function `Line` obtains in `this` a reference to the newly created object. The assignment to `m_x` and `m_y` refer to the members of the `this` object. When accessing a variable in a member function, it is determined first whether the function is a local variable or an argument. Next it is considered as a member of `this`. If all these fail, it is considered as a global variable. So local variables and arguments hide members, together these hide global variables. The following example shows how the scope resolution operator `::` may be used to resolve conflicts:

```
decl x, y;                                   // global variables
extern moveto(x, y);                        // external function

Line::moveto(const x, const y)
{
    ::x = x;             // assign arguments to global variables
```

```
        ::y = y;
        this.m_x = x;           // assign arguments to data members
        this.m_y = y;     // this. needed if these were als x and y

        ::moveto(x, y);                 // call non-member function
        moveto(x, y);                 // error: call to itself will
    }                                        // cause infinite loop
```

13.5.6.5 Static members

There is only one copy of a static member, shared by all objects of a class. A static member may not have the same name as the class it is in. A static member function can only make direct access to static data members.

```
    Line::getcLines()
    {   return sm_cLines;
    }
    Line::setcLines(c)
    {   sm_cLines = c;
        m_x = 0;                    // error: must be static member
        lineto(1, 1);               // error: must be static member
    }
```

A static member function can be called directly, and indirectly:

```
    Line::setcLines(0);                      // no Line objects yet
    lineobj = new Line(0);                         // create object
    lineobj2 = new Line(3);                // create another object
    i = Line::getcLines();                                // i = 2
    i = lineobj.getcLines();                              // i = 2
    i = lineobj2.getcLines();                             // i = 2
    Line::moveto(0, 0);             // error: function is not static
    Line.getcLines();                             // error, needs ::
```

Since there is only one instance of the static function, in all cases the same `getcLines` function is called (assuming both `lineobj` and `lineobj2` are an object of class `Line`).

13.5.6.6 Derived classes

A class may derive from a previously declared class. A derived class will inherit all members from its base class, and can access these inherited members as its own members. However, if the derived class has members with the same name as members of the base class, the former take precedence. In this way, a class can redefine functionality of its base class. If a function is redefined, the base class name followed by `::` may be used to refer to the base class function.

Deriving from the `Line` class:

```
    class Angle : Line                     // Line is the base class
    {
        Angle();                                     // constructor
        lineto(const x, const y);          // draw dash, move cursor
    };
    Angle::Angle()
    {
        Line(0);                                   // starts at zero
    }
```

```
Angle::lineto(const x, const y)
{
    Line::lineto(x, m_y);                          // horizontal line
    Line::lineto(x, y);                              // vertical line
    print("is angle to ", x, " ", y, "\n");
    moveto(x, y);
}
```

`Angle`'s constructor just calls the base class constructor, as the body may be read as `this.Line(0);`. Note that the base class constructor and destructor functions are *not* called automatically (unlike in C++). In the new `lineto` object, `Line::lineto` is used to make sure that we call the correct function (otherwise it would make a recursive call). For the `moveto` that is no problem, `moveto` calls the base function, as it was not redefined in the `Angle` class. Non-static member functions may be declared as virtual (that is, they can be redefined by a derived class), this is discussed in the next section.

New classes may be derived from a class which is itself derived, but Ox only supports single inheritance: a class can only derive from one other class at a time.

13.5.6.7 Virtual functions

Virtual functions allow a derived class to supply a new version of the virtual function in the derived class, replacing the version of the base class. When the base class calls the virtual function, it will actually use the function of the derived class. For a virtual function, the call can only be resolved at run time. Then, the object type is known, and the called function is the one first found in the object, when moving from the highest class towards the base class.

A virtual function cannot be static.

The effect of using virtual functions is most easily explained by the following example.

```
#include <oxstd.oxh>
class Base
{
    basefunc();
    virtual vfunc();
};
Base::basefunc()
{
    vfunc();                          // call the virtual function
}
Base::vfunc()
{   print("Base vfunc()\n");
}

class Derived : Base
{
    derfunc();
    vfunc();
};
Derived::derfunc()
{
    this.Base::basefunc();
    Base::basefunc();
```

```
        basefunc();                        // three equivalent calls
    }
    Derived::vfunc()
    {   print("Derived vfunc()\n");
    }

    main()
    {
        decl obj = new Derived();
        obj.basefunc();
        obj.derfunc();
    }
```

The output is:

```
    Derived vfunc()
    Derived vfunc()
    Derived vfunc()
    Derived vfunc()
```

Even though `Base` has its own `vfunc()`, the derived class provides a new version of this function. This is used whenever `Basefunc()` is called for an object of class `Derived`. Were we to remove the `virtual` keyword, the output would be four times `Base vfunc()`. If we replace `vfunc()` with `Base::vfunc()` inside `Base::basefunc`, the result would also be four times `vfunc()` from `Base`.

13.6 Namespace

```
namespace identifier
{
        external-declaration
}
```

A namespace surrounds a section of external declarations, separating it from functions and variables in other namespaces, or from those outside the namespace. If the namespace is called `ns`, then identifiers inside the namespace are first resolved within that namespace, and then in the unnamed space. From another namespace, access is by prefix `ns::`.

Namespaces in Ox cannot be nested, and unnamed namespaces are unsupported.

```
foo()
{
    println("foo");
}
bar()
{
    println("bar");
}
namespace test
{
bar()
{
    println("test::bar");
}
foo()
{
    println("in test::foo");
    bar();      // calls test::bar
    ::bar();    // calls bar
}
}   // end of namespace
main()
{
    println("calling ::foo");
    foo();
    println("calling test::foo");
    test::foo();
}
```

which prints:

```
calling ::foo
in foo
calling test::foo
in test::foo
in test::bar
in bar
```

13.7 Statements

statement-list:
  statement
  statement-list statement

statement:
  labelled-statement
  expression-statement
  compound-statement
  serial-compound-statement
  selection-statement
  switch-statement
  iteration-statement
  jump-statement
  declaration-statement

expression-statement:
  expression$_{opt}$;

compound-statement:
  { *statement-list*$_{opt}$ }

serial-compound-statement:
  `serial` { *closed-statement-list* }

iteration-statement:
  while-iteration-statement
  `parallel`$_{opt}$ *for-iteration-statement*

labelled-statement:
  :*label statement*

The executable part of a program consists of a sequence of statements. Expression statements are expressions or function calls. It can be a do-nothing expression, as in:

```
for (i = 0; i < 10; i++)
    ;
```

A compound statement groups statements together in a block, e.g.:

```
for (i = 0; i < 10; i++)
{
    a = test(b);
    b = b + 10;
}
```

A statement can be prefixed by a label as in:

```
:L001
    for (i = 0; i < 10; i++)
        ;
```

Labels are the targets of `goto` statements (see §13.7.4); labels are local to a function and have separate name spaces (which means that variables and labels may have the same name). Note that labels are defined in a non-standard way: the colon is prefixed, rather than suffixed as in C or C++.

13.7.1 Selection statements

selection-statement:
  `if` (*expression*) *statement*
  `if` (*expression*) *statement* `else` *statement*

The conditional expression in an `if` statement is evaluated, and if it evaluates to true (*for a matrix: no element evaluates to false*), the statement is executed. Zero (0), the empty matrix (<>) and a missing value (`.NaN`) all evaluate to false. The conditional expression may not be a declaration statement. Some examples for the `if` statement:

```
if (i == 0)
    i++;                               // do only if i equals 0

if (i >= 0)
    i = 1;                                 // do only if i >= 0
else
    i = 0;                               // set negative i to 0

if (i == 0)
    if (k > 0)
        j = 1;                   // do only if i != 0 and k > 0
    else                      // this else matches the inner if
        j = -1;                 // do only if i != 0 and k <= 0

if (i == 0)
{   if (k > 0)
        j = 1;                   // do only if i != 0 and k > 0
}
else                          // this else matches the outer if
    j = -1;                                // do only if i != 0
```

Each `else` part matches the closest previous `if`, but this can be changed by using braces. When coding nested `if`s, it is advisable to use braces to make the program more readable and avoid potential mistakes.

Further examples involving matrices are given in §13.8.9.

13.7.2 Switch statements

switch-statement:
  `switch` (*expression*) { *case-list* $default_{opt}$ }
  `switch_single` (*expression*) { *case-list* $default_{opt}$ }

case-list:
  case
  case-list case

case:
  `case` *expression* : *statement-list*

default:
  `default` : *statement-list*

A `switch` statement is a compact way of writing a sequence of `if` statements involving the same variable for comparison:

```
decl i = 1;
switch (i)
{
    case 0:
        println("zero");
        break;
    case 1:
        println("one");
        break;
    default:
        println("not zero, not one");
        break;
}
```

which prints: ”one”. There is a sequence of `case` blocks, and an optional `default` block, which must be the last. The `break` statement jumps out of the `switch` statement.

Here, the value of `i` is compared to each value in turn, until a comparison is true. Then all the statements for that case *and all subsequent cases* are executed (including the default) until a `break` is encountered. If no case is true, the default statements are executed. So, once inside a case, we automatically fall through to the next case. The advantage is that several cases can be grouped together:

```
switch (i)
{
    case 0:
        println("zero");
        break;
    case 1:
    case 2:
        println("one,two");
        break;
    default:
        println("default");
        break;
}
```

printing `one,two` when `i` is 1 or 2.

The drawback is that is easy to forget the `break` statements, and get unexpected results. The following code

```
switch (i)
{
    case 0:
        println("zero");
    case 1:
    case 2:
        println("one or two");
    default:
        println("default");
}
```

will print when `i` equals zero:

```
zero
one or two
default
```

To emphasize that distinction, and allow for more readable code, Ox also has the `switch_single` statement. Then, one and only one case (or default) is executed:

```
switch_single (i)
{
    case 0:
        println("zero");
    case 1:
        println("one");
    case 2:
        println("two");
    default:
        println("default");
}
```

13.7.3 Iteration statements

while-iteration-statement:
  `while (` *expression* `)` *statement*
  `do` *statement* `while (` *expression* `) ;`
for-iteration-statement:
  `for (` *expression*$_{opt}$ `;` *expression*$_{opt}$ `;` *expression*$_{opt}$ `)` *statement*
  `for (` *declaration-statement*$_{opt}$ `;` *expression*$_{opt}$ `;` *expression*$_{opt}$ `)` *statement*
  `foreach (` *identifier* `in` *identifier foreach-index-expression*$_{opt}$ `)` *statement*
  `foreach (decl` *identifier* `in` *identifier foreach-index-expression*$_{opt}$ `)` *statement*
foreach-index-expression:
  `[` *identifier* `]`
  `[` *identifier* `] [` *identifier* `]`
  `[` *identifier* `] [ ]`
  `[ ] [` *identifier* `]`

The `while` statement excutes the substatement as long as the test expression is nonzero (for a matrix: all elements are nonzero). The test is performed before the substatement is executed.

The `do` statement excutes the substatement, then repeats this as long as the test expression is nonzero (for a matrix: all elements are nonzero). The test is performed after the substatement is executed. So for the `do` statement the substatement is executed one or more times, whereas for the `while` statement this is zero or more times.

The `while` and `do` statements can be envisaged respectively as:

```
:startwhile                        :startdo
    if (expression)                    statement
    {                                  if (expression)
        statement                          goto startdo;
        goto startwhile;
    }
```

The `for` expression:

  `for (`*init_expr*`;` *test_expr* `;` *increment_expr*`)` *statement*

corresponds to:

```
{
    init_expr;
    while (test_expr )
    {
        statement
        increment_expr;
    }
}
```

Note that, when the *init_expr* is a declaration statement, the declaration is local to the `for` statement.

The `foreach` expression is used to loop over all elements in a matrix, array or string. The most simple form:

```
        foreach (element-identifier in collection-identifier) statement
```

implements a loop over all elements in the collection. For an array or string, this is equivalent to:

```
        for (decl i = 0; i < sizeof(collection-identifier); ++i)
        {
            element-identifier = collection-identifier[i];
            statement           // no access to i
        }
```

While for a matrix:

```
        for (decl i = 0; i < sizer(collection-identifier); ++i)
        {
            for (decl j = 0; j < sizec(collection-identifier); ++j)
            {
                element-identifier = collection-identifier[i][j];
                statement           // no access to i,j
            }
        }
```

The following restrictions apply to the `foreach` loop:

1. The *collection-identifier* must be an lvalue; it can be an object member, but may not contain an index, because that would be interpreted as the *foreach-index-expression*.
2. The *element-identifier* and the identifiers in the *foreach-index-expression* must be local variables
3. The dimension of the *collection-identifier* must be fixed during the loop, but its contents may change.
4. Assigning to the *element-identifier* does not change the *collection-identifier*.
5. When the loop terminates, the *element-identifier* is undefined.

The *foreach-index-expression* part determines how the loop is performed:

`foreach (el in a)` — loop over all elements (matrix, array, string), no access to iterators;

`foreach (el in a[i][j])` — loop over all elements of a matrix with access to iterators i and j;

`foreach (el in a[i][])` — loop over all rows i, with access to i;

`foreach (el in a[][j])` — loop over all columns j, with access to j;

`foreach (el in a[i])` — loop over all elements i (row/column vector, string or array).

Some examples:

```
decl x, m = rann(2,2), i, j;
// Example 1: print all elements
foreach (x in m)
    println(x);
foreach (x in m[i][j])
    println("element ", i, ",", j, ": ", x);

// Example 2: create a Toeplitz matrix
decl c = zeros(10, 10);
foreach (x in c[i][j])
    c[i][j] = fabs(i - j) + 1;

// Example 3: print all strings in an array:
decl a = {"aaa", m, "BBB"}, s;
foreach (s in a)
    if (isstring(s))
        println(s);
```

13.7.4 Jump statements

jump-statement:
`break ;`
`continue ;`
`goto` *label*;
`return` *expression*$_{opt}$;

The `return` statement exits the function; if it is followed by an expression, the value of the expression is returned to the caller, see §13.5.5.3.

A `continue` statement may only appear within an iteration statement and causes control to pass to the loop-continuation portion of the smallest enclosing iteration statement.

The use of `goto` should be kept to a minimum, but could be useful to jump out of a nested loop, jump to the end of a routine or when converting Fortran code. It is always possible to rewrite the code such that no `goto`s are required.

A `break` statement may only appear within an iteration statement and terminates the smallest enclosing iteration statement.

Two examples:

```
for (i = 0; i < 10; i++)
{
    if (test1(i))
        continue;
    test2();                 // only done if test1(i) returns 0
}
for (i = 0; i < 10; i++)
{
    if (test1(i) == 0)
        break;        // jump out of loop if test1(i) returns 0
    test2();
}
```

13.7.5 Declaration statements

declaration-statement:
  `decl` *declaration-list* ;
  `serial decl` *declaration-list* ;

declaration-list:
  init-declaration
  declaration-list , *init-declaration*

decl-statement-list:
  identifier
  identifier = *expression*

Declarations at the external level were discussed in §13.5. Here we treat declaration within a block. The use of `serial` is explained in §13.7.7.

Declaration statements create a ‘local’ variable for further manipulation as long as it stays within scope. The created object is removed as soon as the block in which it was created is exited. Variables can be intitialized in a declaration statement. Variables in Ox are implicitly typed, and their type can change during program execution. Non-externally declared variables must be initialized before they can be used in an expression. It is not possible to specify matrix dimension as can be done at the external level, so instead of `decl ma[3][3] = 1.5` write `decl ma = constant(1.5,3,3)`. Unlike C, declaration statements do not have to occur at the start of a block. Consider for example:

```
test1(arg0)
{
    decl k, a = arg0;
    decl ident = <1, 0; 0, 1>;
    decl identsq = ident * ident;

    print("test\n");

    decl i, j;
    for (i = 0; i < 10; i++)
    {
        test2(i);
        test3(j);                      // error: j has no value
    }
```

Variables declared in an inner block hide variables in the outer block:

```
decl i = 3;                             // external declaration

test2(a)
{
    print(i, "\n");                                       // 3

    {   decl i = 0;
        print(i, "\n");                                   // 0
        if (i == 0)                                 // is true
```

```
            {
                decl i = 1;
                print(i, "\n");                                   // 1
                print(::i, "\n");                                 // 3
            }
            decl a;                 // error: conflict with argument
        }
    }
```

13.7.6 Closed statement list

A statement list is closed if the only possible entry is at the top of the block, and the only exit at the bottom. So a closed block may not contain `return` or `break` to terminate a loop (thus leaving the block; but continue is allowed). Neither may there be a jump statement into or out of the block.

13.7.7 Parallel programming

This section gives a summary of the use of `parallel` and `serial`. Examples are given in Chapter 4.

13.7.7.1 Canonical `for` and `foreach` loops

A `for` loop is *canonical* if:

1. the iterator is a local variable,
2. the iterator is an integer,
3. the iterator is not changed in the loop body,
4. the iterator is incremented (or decremented) by an integer constant,
5. the upperbound can be computed before the loop starts,
   In particular, it is either the value of a variable, or `sizer`, `sizec`, `sizerc`, `sizeof`, `rows`, `columns` of a variable.
6. the upperbound is fixed while the loop executes,
7. the loop body is a closed statement list.
   Except for the last condition, all are automatically satisfied by a `foreach` loop.

Ox can determine whether a `for` or `foreach` loop is canonical, and use compiled code for the iteration aspect, which is more efficient. If you use the `-v` command line switch, a message will indicate if a loop was optimized this way.

13.7.7.2 Parallel `for` and `foreach` loops

A canonical `for` or `foreach` loop can be run in parallel (Ox Professional only) if there is no dependency between iterations, i.e. if the ordering of the iterations does not matter. This condition is not verified by Ox, but the user can label a loop as `parallel`.

When Ox starts running code in parallel, n threads are created. Each thread gets its own space for local variables. Initially these are the same as the main thread (integers and doubles are copied, the remainder are references to the value in the main thread). As the threads proceed in parallel, the local variables may be different in each thread. When the parallel section is finished, only the local variables in the main thread survive, the others are removed. This is useful because it separates local variables, but a problem for reduction operations such as accumulating a sum.

There is just one version of global variables. These are safe for reading, but writing (or writing and reading) in parallel is unsafe, resulting in a race condition. Or even a crash when memory allocation and deallocation overlaps.

Ox variables can be declared as `serial`, in which case only one thread at a time is able to modify the variable through the following compound assignment operations: `*=   /=   += -=   ~=   |=   .*=   ./=   ++   --`. Note that simple assignment (=) is unaffected by the `serial` declaraion.

```
decl i, j, crep = 10;

decl sum1 = 0;
parallel for (i = 0; i < crep; ++i)
{
    sum1 += 1;
}
println("sum1=", sum1);

serial decl sum2 = 0;
parallel for (i = 0; i < crep; ++i)
{
    sum2 += 1;
}
println("sum2=", sum2);
```

prints

```
sum1=3
sum2=10
```

The precise value of `sum1` depends on the number of threads, i.e. what part is executed in the main thread. However, it clearly has not the intended result.

The value of `sum2` is correct though: only one thread at a time was allowed to update, so, while one was doing this, the others had to wait. the price we pay for this is slower code.

Note that updating matrix elements is safe, provided the matrix is pre-allocated, and each iteration updates a different element.

Note that functions written in Ox code cannot be labelled as serial, but calls to dynamic-link libraries can.

Sections of code may need to be executed together in serial mode. This can be achieved by creating a `serial` block. For example, to keep the print statements together:

```
parallel for (i = 0; i < crep; ++i)
{
    // lengthy computation running in parallel
    // ....
    serial
    {
        print("i=");
        println(i);
    }
}
```

Parallel computations are not-nested: if a parallel loop contains another parallel loop, the latter is executed serially. Relatedly, any parallel loops inside a serial section will not be executed in parallel. Specifying the `-rp1` Ox command line switch also forces the program to run serially.

13.8 Expressions

Table 13.1 Ox operator precedence

Category	operators	associativity
primary	`() :: [] {}`	left to right
postfix	`-> . () [] ++ -- '`	left to right
power	`^ .^`	left to right
unary	`++ -- + - ! & new delete`	right to left
multiplicative	`** * .* / ./`	left to right
additive	`+ -`	left to right
horizontal concatenation	`~`	left to right
vertical concatenation	`|`	left to right
relational	`< > <= >= .< .> .<= .>=`	left to right
equality	`== != .== .!=`	left to right
logical dot-and	`.&&`	left to right
logical-and	`&&`	left to right
logical dot-or	`.||`	left to right
logical-or	`||`	left to right
conditional	`? : .? .:`	right to left
assignment	`= *= /= += -= ~= |= .*= ./=`	right to left
comma	`,`	left to right

Table 13.1 gives a summary if the operators available in Ox, together with their precedence (in order of decreasing precedence) and associativity. The precedence is in decreasing order. Operators on the same line have the same precedence, in which case the associativity gives the order of the operators. Note that the order of evaluation of expressions is not fully specified. In:

```
    i = a() + b();
```

it is unknown whether `a` or `b` is called first.

Subsections below give a more comprehensive discussion. Several operators require an *lvalue*, which is a region of memory to which an assignment can be made. Note that an object which was declared `const` is not an lvalue. Many operators require operands of arithmetic type, that is int, double or matrix.

The most common operators are *dot-operators* (operating element-by-element) and relational operators (element by element, but returning a single boolean value). The resulting value is given Tables 13.2 and 13.3 respectively. In addition, there are special matrix operations, such as matrix multiplication and division; the result from these operators is explained below. A scalar consists of: int, double or 1×1 matrix.

Table 13.2 Result from dot operators

left a	operator	right b	result	computes
int	*op*	int	int	a *op* b
int/double	*op*	double	double	a *op* b
double	*op*	int/double	double	a *op* b
scalar	*op*	matrix $m \times n$	matrix $m \times n$	a *op* b_{ij}
matrix $m \times n$	*op*	scalar	matrix $m \times n$	a_{ij} *op* b
matrix $m \times n$	*op*	matrix $m \times n$	matrix $m \times n$	a_{ij} *op* b_{ij}
matrix $m \times n$	*op*	matrix $m \times 1$	matrix $m \times n$	a_{ij} *op* b_{i0}
matrix $m \times n$	*op*	matrix $1 \times n$	matrix $m \times n$	a_{ij} *op* b_{0j}
matrix $m \times 1$	*op*	matrix $m \times n$	matrix $m \times n$	a_{i0} *op* b_{ij}
matrix $1 \times n$	*op*	matrix $m \times n$	matrix $m \times n$	a_{0j} *op* b_{ij}
matrix $m \times 1$	*op*	matrix $1 \times n$	matrix $m \times n$	a_{i0} *op* b_{0j}
matrix $1 \times n$	*op*	matrix $m \times 1$	matrix $m \times n$	a_{0j} *op* b_{i0}
string n	*op*	string n	matrix $1 \times n$	a_j *op* b_j
string n	*op*	int	matrix $1 \times n$	a_j *op* i
int	*op*	string n	matrix $1 \times n$	i *op* b_j

Table 13.3 Result from relational operators

left a	operator	right b	result	computes
int	*op*	int	int	a *op* b
int/double	*op*	double	int	a *op* b
double	*op*	int/double	int	a *op* b
scalar	*op*	matrix $m \times n$	int	a *op* b_{ij}
matrix $m \times n$	*op*	scalar	int	a_{ij} *op* b
matrix $m \times n$	*op*	matrix $m \times n$	int	a_{ij} *op* b_{ij}
matrix $m \times n$	*op*	matrix $m \times 1$	int	a_{ij} *op* b_{i0}
matrix $m \times n$	*op*	matrix $1 \times n$	int	a_{ij} *op* b_{0j}
matrix $m \times 1$	*op*	matrix $m \times n$	int	a_{i0} *op* b_{ij}
matrix $1 \times n$	*op*	matrix $m \times n$	int	a_{0j} *op* b_{ij}
string	*op*	string	int	a *op* b

Table 13.4 Result from operators involving an empty matrix as argument

operator	a *op* <>	<> *op* b	<> *op* <>
`==`	`FALSE`	`FALSE`	`TRUE`
`!=`	`TRUE`	`TRUE`	`FALSE`
`>=`	`FALSE`	`FALSE`	`TRUE`
`>`	`FALSE`	`FALSE`	`FALSE`
`<=`	`FALSE`	`FALSE`	`TRUE`
`<`	`FALSE`	`FALSE`	`FALSE`
other	`<>`	`<>`	`<>`

Table 13.5 Result from relational operators involving missing values

a op b	one of a, b missing	both a, b missing
`==`	0	1
`<=`	0	1
`>=`	0	1
`<`	0	0
`>`	0	0
`!=`	1	0

13.8.1 Primary expressions

primary-expression:
- `(` *expression* `)`
- `[` *assignment-expression-list* `]`
- `{` *expression-list* `}`
- *constant*
- *identifier*
- `::` *identifier*
- *class-name* `::` *identifier*
- `this`
- `[=]` `(` *argument-type-list*$_{opt}$ `)` *compound-statement*

An expression in parenthesis is a primary expression. Its main use is to change the order of evaluation, or clarify the expression.

An expression in curly braces creates an array of the comma-separated expressions.

All types of constants discussed in §13.3.2 form a primary expression.

The operator `::` followed by an identifier references a variable declared externally (see §13.5). Section 13.5.6.4 gives examples. A class name followed by `::` and a function member of that class references a static function member, or any function member if preceded by an object reference, see sections 13.5.6.5 and 13.5.6.1.

The `this` reference is only available inside non-static class member functions, and points to the object for which the function was called.

13.8.1.1 Multiple assignment

A comma-separated list of lvalues in square brackets can be used for multiple assignments. When the right-hand side is an array, each array value in turn is assigned to the next value of the left-hand side. The return value of a multiple assignment expression is zero (the examples below illustrate). When there is one lvalue in the square brackets, the right-hand side need not be an array. Fewer array elements on the right than lvalues on the left leads to a runtime error. The converse is no problem. A multiple assignment expression can be used to implement multiple returns from a function.
The following examples illustrate multiple assignments:

```
decl x1, x2, x3, x4, as;
as = {"a", <10,11>, "b"};
[x1, x2, x3] = as;
println("x1=", x1, " x2=", x2, "x3=", x3);

[x1] = 10;
[x2, x3] = {11,12,13};
    //[x2, x3, x4] = {11,12};         // error
println("x1=", x1, " x2=", x2, " x3=", x3);

x3 = 10 + ([x1, x2] = as[<0,2>]);
println("x1=", x1, " x2=", x2, " x3=", x3);

x1=<1,2,3,4>;
[x1[0], x1[3]] = {-1, -3};
println("x1=", x1);
```

Which prints:

```
x1=a x2=
       10.000       11.000
x3=b
x1=10 x2=11 x3=12
x1=a x2=b x3=10
x1=
      -1.0000      2.0000      3.0000     -3.0000
```

13.8.1.2 Lambda function

A lambda function can be useful to create a local function with a different signature, or to provide access to local variables when the signature is proscribed (as e.g. the function for maximization).[2]

A lambda function can have arguments and local variables. It is somewhat different from a normal function: it has no function name (it is also called an anonymous function) but can be stored in a variable. Moreover, it has access to all the local variables that are in the scope of its definition:

```
decl a, b;
decl fnlam = [=](arg) { println("a=", a, " arg=", arg); return b; };
```

There are some restrictions:

[2]This can also be achieved through a class, because a function member can access the members of the object to which it belongs, even when passed as an argument.

- Variable and default arguments are not allowed.
- `[=]` captures local variables and arguments of the lambda context by value: they cannot be modified. This implies that a lambda function can be safely used in a parallel setting.
- However, internally, the local variables are captured by reference. So a lambda function that accesses local variables can only be used when these are in existence. As a consequence, care is required returning a lambda function: when the function surrounding the lambda returns, the local variables disappear, and the lambda is invalid. In other words: the closure is incomplete, because it is by reference. This implementation is less flexible than, say, JavaScript but is efficient, because large matrices are not unneccesarily copied.

  A lambda function can also be created in place, from `samples/maximize/probit1a`:

```
ir = MaxBFGS(
    [=](const vP, const adFunc, const avScore, const amHessian)
    {
        return fProbit(vP, adFunc, mx, my);
    }, &vp, &dfunc, 0, TRUE);
```

  But as a variable may be easier to read, see `samples/maximize/probit1b`:

```
decl fprobit_max = [=](const vP, const adFunc, const avScore,
    const amHessian)
{
    return fProbit(vP, adFunc, mx, my);
};
ir = MaxBFGS(fprobit_max, &vp, &dfunc, 0, TRUE, maxctrl);
```

13.8.2 Postfix expressions

postfix-expression:
  primary-expression
  postfix-expression `->`
  postfix-expression `.`
  postfix-expression `(` *expression-list*$_{opt}$ `)`
  postfix-expression `[` *index-expression*$_{opt}$ `]`
  postfix-expression `++`
  postfix-expression `--`
  postfix-expression `'`

expression-list:
  assignment-expression
  expression-list `,` *assignment-expression*

13.8.2.1 Member reference

The `.` operator selects a member from an object reference (`->` may also be used). The left-hand expression must evaluate to a reference to an object, the right-hand expression must result in a member of that object. See section 13.5.6.

13.8.2.2 Function calls

A function call is a postfix expression consisting of the function name, followed in parenthesis by a possibly empty, comma-separated list of assignment expressions. All argument passing is by value, but when an array is passed, its contents may be changed by the function (unless they are `const`). The order of evaluation of the arguments is unspecified; all arguments are evaluated before the function is entered. Recursive function calls are allowed. A function must be declared before it can be called, and the number of arguments in the call must coincide with the number in the declaration, unless the declaration has `...` as the last argument, see §13.5.5.1.

Some examples:

```
func1(a0, a1, a2, a3)
{   print("func1(", a0, ",", a1, ",", a2, ",", a3, ")\n");
}
func2()
{   return 0;
}
func3(a0)
{   a0[0] = 1;
}
test1()
{   decl a, b;

    a = 1;
    func1(a, b = 10, func2(), a != 0);      // func1(1,10,0,1)
    a = func2();                                     // a = 0
    func3(&a);                                       // a = 1
    func3(a);                                        // error
}
```

In the latter example a will have been changed by `func3`. Function arguments are passed by giving the name of the function:

```
func4(a0, a1)
{   a1(a0);                              // make function call
}
func5(a0)
{   print("func5(", a0, ")\n");
}
test2()
{   decl a = func5;

    func4(1, func5);                      // prints "func5(1)"
    func4(1, a);                          // prints "func5(1)"
    func4(1, func5(a));               // error: requires function
    func4(1, func2);               // error: func2 takes incorrect
}                                  //           number of arguments
```

Note that the parentheses in `func5()` indicate that it is a function call, whereas lack of brackets just passes the function itself.

13.8.2.3 Explicit type conversion

Explicit type conversion has the same syntax as a function call, using types `int`, `double`, `matrix` and `string`:

	int	double	matrix	string
	`v=0;`	`v=0.6;`	`v=<0.6,1>;`	`v="tinker";`
`matrix(v)`	$< 0 >$	$< 0.6 >$	v	$< 116 >$
`double(v)`	0.0	v	0.6	see below
`int(v)`	v	0	0	116

The double to string conversion function and its reverse are for packing string values in a double and subsequently extracting it. This is usually better avoided, as it restricts the string length to eight characters (more flexibility is offered by using arrays of strings). Use the `sprint` library function to express double (or any other) value as a string.

For example, `double("tinker")` packs the string in a double value. Since a double is 8 bytes, the string is truncated at 8 characters (or padded by null characters). Conversely, `string(dbl)` extracts the string from a double value, automatically appending a null character.

Calling `string` on a function returns the function name; on an object the class name.

13.8.2.4 Indexing vector and array types

Vector types (that is, string or matrix) and array types are indexed by postfixing square brackets. A matrix can have one or two indices, a string only one. For an array type it depends on the level of indirection. *Note that indexing always starts at zero.*[3] So a 2×3 matrix has elements:

[0][0]	[0][1]	[0][2]
[1][0]	[1][1]	[1][2]

Three ways of indexing are distinguished:

indexing type	matrix, string	array	example
scalar	$\surd$	$\surd$	`m[0][0]`
matrix	$\surd$	$\surd$	`m[0][<0,1,2>]`
range	$\surd$	$\surd$	`m[][1:]`

In the first indexing case (allowed for all non-scalar types), the expression inside square brackets must have scalar type, whereby double is converted to integer.

Vector types may also be indexed by a matrix or have a range expression inside the brackets. In a matrix index to a string the first *column* of the matrix specifies the selected elements of the string.

It is possible to use only one index to a matrix. If a matrix `x` is a column or row vector, `x[i]` it will pick the ith element from the vector. If `x` is a matrix, it will treat the matrix as a vector (row by row, which corresponds to the `vecr`).

If a matrix is used as an index to a matrix, then each element (row by row, i.e. the `vecr` of the argument) is used as an index. As a consequence, indexing by a column vector or its transpose (a row vector) has the same effect. A matrix in the first index selects rows, a matrix in the second index selects columns. The resulting matrix is the intersection of those rows and columns.

[3]But see §13.9.5 for the option to change that.

A range index has the form *start-index* : *end-index*. Either the start-index or the end-index may be missing, which results in the lower-bound or upper-bound being used respectively. An empty index selects all elements. The resulting type from a range or empty index is always a vector type.

Indexing beyond the end will result in a fatal run-time error. An exception is indexing a string for reference: this can be done one position beyond the end, which returns 0. For example, `i=s[sizeof(s)]` sets `i` to 0.

Some examples:

```
decl mat = < 0:3; 10:13 >, d, m;
decl str = "tinkertailor", s;
decl arr = { "tinker", "tailor", "soldier" };
                                // mat = <0,1,2,3; 10,11,12,13>
d = mat[0][0];                                          // d = 0
d = mat[1][2];                                         // d = 12
m = mat[1][];                               // m = <10,11,12,13>
i = 1;
m = mat[1][i:];                                // m = <11,12,13>

d = m[1];                                            // d = <11>
d = m'[1];                                 // the same: d = <11>
d = mat[5];                                          // d = <11>

m = mat[][2];                                    // m = <2; 12>
m = mat[][];                                // same as: m = mat;
m = mat[0][<1:3>];       // matrix indexes columns: m = <1,2,3>
m = mat[<1,0,1>][<1,3>];         // m = < 11,13; 1,3; 11,13 >
mat[0][1:3] = 9;                       // range indexes columns:
                                // mat = <0,9,9,9; 10,11,12,13>
s = str[6:11];                                  // s = "tailor"
str[6:11] = 'a';                         // str = "tinkeraaaaaa"
s = arr[1];                                     // s = "tailor"
arr[1][0] = 'a';                            // arr[1] = "aailor"
```

13.8.2.5 Postfix incrementation

A postfix expression followed by ++ or -- leads to the value of the expression being evaluated and then incremented or decremented by 1. The operand must be an lvalue and must have arithmetic type. For a matrix the operator is applied to each element separately. The result of the expression is the value prior to the increment/decrement operation.

```
decl mat = < 0:3; 10:13 >, m, i, j;
decl str = "tinkertailor", s;
j = 0;
i = j++;                                      // i = 0, j = 1
m = mat++;                      // mat = <1,2,3,4; 11,12,13,14>
                                // m   = <0,1,2,3; 10,11,12,13>
str[0]++;                                // str = "uinkertailor"
str++;                                                  // error
```

13.8.2.6 Transpose

The postfix operator `'` takes the transpose of a matrix. It has no effect on other arithmetic types of operands. The following translations are made when parsing Ox code:

`'` *identifier*	into	`' *` *identifier*
`' (`	into	`' * (`
`' this`	into	`' * this`

A single quote is also used in a character constant; the context avoids ambiguity:

```
mat = m' * a';
mat = m'a';                                // interpreted as m' * a'
mat = m'';                                       // two '' cancel out
mat = m + 'a';                        // 'a' is a character constant
```

13.8.3 Power expressions

power-expression:
  postfix-expression
  power-expression `^` *unary-expression*
  power-expression `.^` *unary-expression*

The operands of the power operator must have arithmetic type, and the result is given in the table. If the first operand is not a matrix `.^` and `^` are the same. A scalar consists of: int, double or 1×1 matrix.

left a	operator	right b	result	computes
int	`^` `.^`	int or double	int	a^b
int/double	`^` `.^`	double	double	a^b
double	`^` `.^`	scalar	double	a^b
scalar	`^` `.^`	matrix $m \times n$	matrix $m \times n$	$a^{b_{ij}}$
matrix $m \times n$	`.^`	scalar	matrix $m \times n$	a_{ij}^b
matrix $m \times n$	`.^`	matrix $m \times n$	matrix $m \times n$	$a_{ij}^{b_{ij}}$
matrix $m \times m$	`^`	scalar	matrix $m \times m$	$a^{\text{int}(b)}$

When a and b are integers, then `a ^ b` is an integer if $b \geq 0$ and if the result can be represented as a 32 bit signed integer. If $b < 0$ and $a \neq 0$ or the integer result would lead to overflow, the return type is double, giving the outcome of the floating point power operation.

The first line in the example shows that power has higher precedence than unary minus:

```
i = - 2 ^ 2;                                            // i = -4
decl r, m1 = <1,2; 2,1>, m2 = <2,3; 3,2>;
r = m1 .^ 3;                                       // <1,8; 8,1>
r = m1 .^ 3.7;                           // <1,12.996; 12.996,1>
r =  3 .^ m1;                                      // <3,9; 9,3>
r =  3 ^  m1;                                      // <3,9; 9,3>
r = m1 .^ m2;                                      // <1,8; 8,1>
r = m1 ^  3;                                   // <13,14; 14,13>
r = m1 ^  3.7;                                 // <13,14; 14,13>
r = m1 ^  -3;                // equivalent to: r = (1 / m1) ^ 3;
```

```
r = m1 ^  m2;                                          // error
```

The following code prints 14 zero matrices of dimension 2×2:

```
decl i, ma, m1 = <1,2; 2,1>;

for (i = 0, ma = <1,0; 0,1>; i <= 13; i++, ma *= m1)
     print("i = ", i, ma - m1^i);
```

13.8.4 Unary expressions

unary-expression:
 power-expression
 `++` *unary-expression*
 `--` *unary-expression*
 `+` *unary-expression*
 `-` *unary-expression*
 `!` *unary-expression*
 `&` *unary-expression*
 `new` *class-name* `(` *expression-list* `)`
 `new matrix [` *expression-list* `]`
 `new matrix [` *expression-list* `] [` *expression-list* `]`
 `new string [` *expression-list* `]`
 `new array [` *expression-list* `]`
 `new array [` *expression-list* `] [` *expression-list* `]`
 `delete` *unary-expression*

13.8.4.1 Prefix incrementation

A prefix expression preceded by `++` or `--` leads to the lvalue being incremented or decremented by 1. This new value is the result of the operation. The operand must be an lvalue and must have arithmetic type. For a matrix the operator is applied to each element separately.

```
j = 0;
i = ++j;                                     // i = 1, j = 1
```

13.8.4.2 Unary minus and plus

The operand of the unary minus operator must have arithmetic type, and the result is the negative of the operand. For a matrix each element is set to its negative. Unary plus is ignored.

13.8.4.3 Logical negation

The operand of the logical negation operator must have arithmetic type, and the result is 1 if the operand is equal to 0 and 0 otherwise. For a matrix, logical negation is applied to each element. Negating a missing value returns 0, and negating an empty matrix returns an empty matrix.

```
j = 0;  k = 10;
i = !j;                                             // i = 1
i = !k;                                             // i = 0
```

13.8.4.4 Address operator

The operand of the address operator & must be an lvalue. In addition, it must be an object: it is possible to take the address of a class object, a function, or an array element, but not of a matrix element. The result is a reference to the operand as an array of one element, pointing to the region of space occupied by the lvalue. Referencing works through arrays; unlike C and C++ (but like the Java programming language), Ox does not have pointers.

```
test5(const arrstring)
{
    arrstring[0][0] = 'x';
}
test6(astring)
{
    astring[0] = 'a';
}
test4()
{
    decl a, str = "spy";

    a = &str;
    a[0][0]--;                                  // str="rpy"
    test5(&str);                                // str="xpy"
    test6(str);                             // str unchanged
}
```

13.8.4.5 New and delete

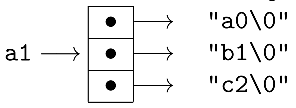
The `new` operator can be used to create an object of a class, or to create a matrix, string or array. The `delete` operator removes an object created by `new`. Note that matrices, strings and arrays are automatically removed when they go out of scope; this is not the case for objects. A class object, on the other hand, must be removed explicitly using the `delete` operator. If not, it will exist until the program terminates (which may be acceptable).

Only one or two array levels at a time can be created by `new`; however, `delete` removes all sublevels. A string created by `new` consists of null characters, a matrix will have all elements zero. Matrix, string and array objects with dimension zero are allowed (this can be useful to start concatenation in an iterative loop; remember that an empty matrix constant is `<>`, and an empty array `{}`). Matrices and arrays can be created with either one or two dimensions.

```
decl i, m1, a1;

m1 = new matrix[2][2];                     // m1 = <0,0; 0,0>
m1[0][0] = 1;

delete m1;
```

```
a1 = m1[0][0];                // error: contents of m1 deleted

a1 = new array[3];

for (i = 0; i < sizeof(a1); i++)
{
    a1[i] = new string[3];
    a1[i][0] = 'a' + i;
    a1[i][1] = '0' + i;
}
```

The `a1` variable has the following structure:



Examples involving objects of classes are given in §13.5.6.

13.8.5 Multiplicative expressions

multiplicative-expression:
 power-expression
 multiplicative-expression `**` *power-expression*
 multiplicative-expression `*` *power-expression*
 multiplicative-expression `.*` *power-expression*
 multiplicative-expression `/` *power-expression*
 multiplicative-expression `./` *power-expression*

The operators `**`, `*`, `.*`, `/`, and `./` group left-to-right and require operands of arithmetic type. A scalar consists of: int, double or 1×1 matrix. Strings are not allowed. These operators conform to Table 13.2 on page 449, except for:

left a	operator	right b	result	computes
matrix $m \times n$	`*`	matrix $n \times p$	matrix $m \times p$	$a_{i.}b_{.k}$
matrix $m \times n$	`**`	matrix $p \times q$	matrix $mp \times nq$	$a_{ij}b$
scalar	`*`	matrix $n \times p$	matrix $n \times p$	ab_{ij}
matrix $m \times n$	`*`	scalar	matrix $m \times n$	$a_{ij}b$
matrix $m \times n$	`/`	matrix $p \times n$	matrix $p \times m$	$a_{i.}b^{+}_{.k}$
scalar	`/`	matrix $m \times n$	matrix $n \times m$	ab^{+}_{ij}
matrix $m \times n$	`/`	scalar	matrix $m \times n$	a_{ij}/b
scalar	`/` `./`	scalar	double	a/b

This implies that `*` `**` are the same as `.*` when one or both arguments are scalar, and similarly for `/` and verb./ when the right-hand operand is not a matrix.

Kronecker product is denoted by `**`. If neither operand is a matrix, this is identical to normal multiplication.

The binary `*` operator denotes multiplication. If both operands are a matrix and neither is scalar, this is matrix multiplication and the number of columns of the first operand has to be identical to the number of rows of the second operand.

The `.*` operator defines element by element multiplication. It is only different from `*` if both operands are a matrix (these must have identical dimensions, however, if one or both of the arguments is a 1×1 matrix, `*` is equal to `.*`).

The product of two integers remains an integer. This means that overflow could occur (when it would not occur in operations where one of the argument is a double). For example `5000 * 50000` fits in an integer and yields 250 000 000, but `50000 * 50000` overflows, yielding $-1.794\,967\,296$. When using double arithmetic: `50000.0 * 50000` = 2500 000 000.0.

The binary / operator denotes division. If the second operand is a non-scalar matrix, this is identical to post-multiplication by the inverse (if the matrix is square the matrix is inverted using the `invert()` library function; if that fails, or the matrix is non-square, the generalized inverse is used, see §13.8.5.1). If the second operand is a scalar, each element of the first is divided by it. If the first operand is a scalar, it is multiplied by the inverse of the second argument.

The `./` operator defines element by element division. If either argument is not a matrix, this is identical to normal division. It is only different from / if both operands are a matrix (these must have identical dimensions).

Note that / does not support integer division (such as e.g. 3 / 2 resulting in 1). In Ox, the result of dividing two integers is a double (3 / 2 gives 1.5). Integer division can be performed using the `idiv` library function. The remainder operator (`%` in C and C++) is supported through the library function `imod`. Multiplication of two integers returns an integer.

Some examples of multiplication and division involving matrices:

```
decl m1 = <1,2; 2,1>, m2 = <2,3; 3,2>, r;

r = m1 * 2.;                                         // <2,4; 4,2>
r = 2. * m2;                                         // <4,6; 6,4>
r = m1 * m2;                                         // <8,7; 7,8>
r = m1 .* m2;                                        // <2,6; 6,2>
r = m1 .* <2,3>;                                     // <2,6; 4,3>
r = m1 ** m2;            // <2,3,4,6; 3,2,6,4; 4,6,2,3; 6,4,3,2>
r = 2 / 3;                                             // 0.666667
r = 2 / 3.;                                            // 0.666667
r = m1 / 2.;                                     // <0.5,1; 1,0.5>
r = m1 ./ <2;3>;                      // <0.5,1; 0.66667,0.33333>
r = 2./ m2;                                  // <-0.8,1.2; 1.2,-0.8>
r = 2 ./ m2;                            // <1,0.66667; 0.66667,1>
r = m2 / m2;                                         // <1,0; 0,1>

r = 1/<1;2>;                                          // <0.2,0.4>
r = 1/<1,2>;                                         // <0.2; 0.4>
r = 1/<0,0;0,0>;                                     // <0,0; 0,0>
```

Notice the difference between `2./ m2` and `2 ./ m2`. In the first case, the dot is interpreted as part of the real number `2.`, whereas in the second case it is part of the `./` dot-division operator. The white space is used here to change the syntax (as in the example in §13.8.2.6); it would be more clear to write the second case as `2.0 ./ m2`. The same difference applies for dot-multiplication, but note that `2.0*m2` and `2.0.*m2` give the same result.

13.8.5.1 Generalized inverse

The $n \times m$ generalized inverse A^+ of an $m \times n$ matrix A is determined using the singular value decomposition:

$$A = UWV',$$

with:

U is $m \times n$ and $U'U = I_n$,
W is $r \times n$ and diagonal, with non-negative diagonal elements w_i,
V is $n \times n$ and $V'V = I_n$.

The generalized inverse A^+ is computed as:

$$A^+ = VW^+U',$$

where the diagonal elements of W^+ are given by:

$$w_i^{-1} = \begin{array}{ll} 1/w_i & \text{if } w_i > 10\epsilon_{inv}||A||_\infty, \\ 0 & \text{otherwise.} \end{array}$$

The rank of A is the number of non-zero w_i. The inversion epsilon, ϵ_{inv}, is set by the `inverteps` function. By default $\epsilon_{inv} = 1000\epsilon_m$, where ϵ_m is the machine precision for doubles ($\approx 2 \times 10^{-16}$) and

$$||A||_\infty = \max_{0 \le i < m} \sum_{j=0}^{n-1} |a_{ij}|.$$

When $n > m$ the singular value decomposition is applied to A' to avoid a large V matrix:

$$A^+ = UW^+V',$$

where U and V derive from $A' = UWV'$.

Note that the generalized inverse of a square non-singular matrix corresponds to the normal inverse. The generalized inverse of a matrix consisting of zeros only is a matrix of zeros. This follows from the four Moore–Penrose conditions for A^+:

$$AA^+A = A,\ A^+AA^+ = A^+,\ \left(AA^+\right)' = AA^+,\ \left(A^+A\right)' = A^+A.$$

13.8.6 Additive expressions

additive-expression:
  multiplicative-expression
  additive-expression + *multiplicative-expression*
  additive-expression - *multiplicative-expression*

The additive operators + and - are dot-operators, conforming to Table 13.2 on page 449. The exception is that adding strings amounts to concatenation, and subtraction involving strings is not allowed. Both operators group left-to-right. They respectively return the sum and the difference of the operands, which must both have arithmetic type. Matrices must be conformant in both dimensions, and the operator is applied element by element. For example:

```
decl m1 = <1,2; 2,1>, m2 = <2,3; 3,2>;

r = 2 - m2;                                       // <0,-1; -1,0>
r = m1 - m2;                                     // <-1,-1; -1,-1>
```

13.8.7 Concatenation expressions

horizontal-concatenation-expression:
  additive-expression
  horizontal-concatenation-expression ˜ *additive-expression*

vertical-concat-expression:
  horizontal-concatenation-expression
  vertical-concat-expression | *horizontal-concatenation-expression*

left	operator	right	result
int/double	˜	int/double	matrix 1×2
int/double	˜	matrix $m \times n$	matrix $m \times (1+n)$
matrix $m \times n$	˜	int/double	matrix $m \times (n+1)$
matrix $m \times n$	˜	matrix $p \times q$	matrix $\max(m,p) \times (n+q)$
int/double	|	int/double	matrix 2×1
int/double	|	matrix $m \times n$	matrix $(1+m) \times n$
matrix $m \times n$	|	int/double	matrix $(m+1) \times n$
matrix $m \times n$	|	matrix $p \times q$	matrix $(m+p) \times \max(n,q)$
int	˜ |	string	string
string	˜ |	int	string
string	˜ |	string	string
array	˜ |	array	array
array	˜ |	any basic type	array

If both operands have arithmetic type, the concatenation operators are used to create a larger matrix out of the operands. If both operands are scalar the result is a row vector (for ˜) or a column vector (for |). If one operand is scalar, and the other a matrix, an extra column (˜) or row (|) is pre/appended. If both operands are a matrix, the matrices are joined. Note that the dimensions need not match: missing elements are set to zero (however, a warning is printed of non-matching matrices are concatenated). Horizontal concatenation has higher precedence than vertical concatenation.

Two strings or an integer and a string can be concatenated, resulting in a longer string. Both horizontal and vertical concatenation yield the same result.

The result is most easily demonstrated by examples:

```
print(1 ~ 2 ~ 3 | 4 ~ 5 ~ 6);                   // <1,2,3; 4,5,6>
print("tinker" ~ '&' ~ "tailor" );            // "tinker&tailor"
print(<1,0; 0,1> ~ 2);                          // <1,0,2; 0,1,2>
print(2 | <1,0; 0,1>);                         // <2,2; 1,0; 0,1>
print(<2> ~ <1,0; 0,1>);                        // <2,1,0; 0,0,1>
```

The first two lines could have been written as:

```
print(<1,2,3; 4,5,6>);
print("tinker" "&" "tailor" );
```

In the latter case, the matrix and string are created at compile time, whereas in the former case this is done at run time. Clearly, the compile time evaluation is more efficient. However, only the concatenation expressions can involve non-constant variables:

```
decl  i1 = 1, i2 = 2, s1 = "tinke";

print(i1 ~ i2);                                      // <1,2>
print(s1 ~ 'r');                                   // "tinker"
```

Array concatenation results in an array with combined size, with assignment of each member of both arrays to the new array.

```
decl  i, a1 = {"tinker", "tailor"}, a2 = {"soldier"};

a1 ~= a2;
print(a1);
```

prints:

```
[0] = tinker
[1] = tailor
[2] = soldier
```

Often, concatenation is required in a loop. In that case, it is convenient to start from a matrix of dimension zero, for example:

```
decl m, i;

for (i = 0, m = <>; i < 4; ++i)
    m ~= i;
print(m);                             // m = <0, 1, 2, 3>
```

13.8.8 Relational expressions

relational-expression:
  vertical-concat-expression
  relational-expression < *vertical-concat-expression*
  relational-expression > *vertical-concat-expression*
  relational-expression <= *vertical-concat-expression*
  relational-expression >= *vertical-concat-expression*
  relational-expression .< *vertical-concat-expression*
  relational-expression .> *vertical-concat-expression*
  relational-expression .<= *vertical-concat-expression*
  relational-expression .>= *vertical-concat-expression*

The relational operators are <, <=, >, >=, standing for 'less', 'less or equal', 'greater', 'greater or equal'. They all yield 0 if the specified relation is false, and 1 if it is true. The type of the result is always an integer, see Table 13.3. If both operands are a matrix, the return value is true if the relation holds for each element. If one of the operands is of scalar-type, and the other of matrix-type, each element in the matrix is compared to the scalar, and the result is true if each comparison is true.

The dot relational operators are .<, .<=, .>, .>=, standing for 'dot less', 'dot less or equal', 'dot greater', 'dot greater or equal'. They conform to Table 13.2 on page 449.

If both arguments are scalar, the result type inherits the higher type, so `1 >= 1.5` yields a double with value `0.0`. If both operands are a matrix the return value is a matrix with a 1 in each position where the relation is true and zero where it is false. If one of the operands is of scalar-type, and the other of matrix-type, each element in the matrix is compared to the scalar returning a matrix with 1 at each position where the relation holds.

String-type operands can be compared in a similar way. If both operands are a string, the results is int with value 1 or 0, depending on the case sensitive string comparison.

Examples are given in the next section.

13.8.9 Equality expressions

equality-expression:
  relational-expression
  equality-expression == *relational-expression*
  equality-expression != *relational-expression*
  equality-expression .== *relational-expression*
  equality-expression .!= *relational-expression*

The == (is equal to), != (is not equal to), .== (is dot equal to) and .!= (is not dot equal to) are analogous to the relational operators, but have lower precedence.

The non-dotted versions conform to Table 13.3. The dotted versions conform to Table 13.2 on page 449. String comparison is case sensitive (also see `isfeq`).

For example:

```
decl m1 = <1,2; 2,1>, m2 = <2,3; 3,2>, s1 = "tinke";

print(m1 == 1);                                          // 0
print(m1 != 1);                                          // 0
print(!(m1 == 1));                                       // 1
print(m1 > m2);                                          // 0
print(m1 < m2);                                          // 1
print(s1 <= "tinker");                                   // 1
print(s1 <= "tink"  );                                   // 0
print(s1 == "tinker");                                   // 0
print(s1 >= "tinker");                                   // 0
print(s1 == "Tinke");                                    // 0

print(m1 .== 1);                                 // <1,0; 0,1>
print(m1 .!= 1);                                 // <0,1; 1,0>
print(m1 .> m2);                                 // <0,0; 0,0>
print(m1 .< m2);                                 // <1,1; 1,1>

print("AACGTGGC" .== "ACCTTGGC");          // <1,0,1,0,1,1,1,1>
print("AACGTGGC" .== 'A');                 // <1,1,0,0,0,0,0,0>
```

The non-dotted versions only return true if the relation holds for each element. In the first two examples neither `m1 == 1` nor `m1 != 1` is true for each element, hence the return value 0. The third example shows how to test if a matrix is not equal to a value. The parenthesis are necessary, because `!` has higher precedence than ==, and `!m1 == 1` results in `<0,0; 0,0> == 1` which is false.

The last four examples use dot-relational expressions, resulting in a matrix of zeros and ones. In `if` statements, it is possible to use such matrices. Remember that a matrix is true if all elements are true (i.e. no element is zero). In the example below, both `if (m1 .== 1)` and `if (m1 .!= 1)` result in the `else` part being executed:

	evaluates to	leads to
`if (m1 .== 1)`	`if (<1,0;0,1>)`	`else` part
`if (m1 .!= 1)`	`if (<0,1;1,0>)`	`else` part
`if (m1 == 1)`	`if (0)`	`else` part
`if (m1 != 1)`	`if (0)`	`else` part

and both have at least one zero, so that both test statements are false.

The `any` library function evaluates to TRUE if any element is TRUE, e.g.

	evaluates to	leads to
`if (any(m1 .== 1))`	`if (any(<1,0;0,1>))`	`if` part
`if (any(m1 .!= 1))`	`if (any(<0,1;1,0>))`	`if` part
`if (m1 == 1)`	`if (0)`	`else` part
`if (m1 != 1)`	`if (0)`	`else` part

Consider a few more examples, using the matrix `m2 = <2 2;2 2>`:

	evaluates to	leads to
`if (m2 .== 2)`	`if (<1,1;1,1>)`	`if` part
`if (m2 .!= 2)`	`if (<0,0;0,0>)`	`else` part
`if (m1 .== <1,2; 2,1>)`	`if (<1,1;1,1>)`	`if` part
`if (m1 - 1)`	`if (<0,1;1,0>)`	`else` part
`if (m1 .>= 1)`	`if (<1,1;1,1>)`	`if` part
`if (m1 .> 1)`	`if (<0,1;1,0>)`	`else` part
`if (m2 == 2)`	`if (1)`	`if` part
`if (m2 != 2)`	`if (0)`	`else` part
`if (m1 >= 1)`	`if (1)`	`if` part
`if (m1 > 1)`	`if (0)`	`else` part

13.8.10 Logical dot-AND expressions

logical-dot-and-expression:
  equality-expression
  logical-dot-and-expression `.&&` *equality-expression*

The `.&&` operator groups left-to-right. It returns 1 if both of its operands compare unequal to 0, 0 otherwise. Both operands must have arithmetic type. Handling of matrix-type is as for dot-relational operators: if one or both operands is a matrix, the result is a matrix of zeros and ones. Unlike the non-dotted version, both operands will always be executed. For example, in the expression `func1() .&& func2()` the second function is called, regardless of the return value of `func1()`.

13.8.11 Logical-AND expressions

logical-and-expression:
  logical-dot-and-expression
  logical-and-expression `&&` *logical-dot-and-expression*

The `&&` operator groups left-to-right.

It returns the integer 1 if both of its operands compare unequal to 0, and the integer 0 otherwise. Both operands must have arithmetic type. First the left operand is evaluated, if it is false (for a matrix: there is at least one zero element), the result is false, and the right operand will not be evaluated. So in the expression `func1() && func2()` the second function will *not* be called if the first function returned false.

13.8.12 Logical dot-OR expressions

logical-dot-or-expression:
  logical-and-expression
  logical-dot-or-expression `.||` *logical-and-expression*

The `.||` operator groups left-to-right. It returns 1 if either of its operands compares unequal to 0, 0 otherwise. Both operands must have arithmetic type. Handling of matrix-type is as for dot-relational operators: if one or both operands is a matrix, the result is a matrix of zeros and ones. Unlike the non-dotted version, both operands will always be executed. For example, in the expression `func1() .|| func2()` the second function is called, regardless of the return value of `func1()`.

13.8.13 Logical-OR expressions

logical-or-expression:
  logical-dot-or-expression
  logical-or-expression `||` *logical-dot-or-expression*

The `||` operator groups left-to-right. It returns the integer 1 if either of its operands compares unequal to 0, integer value 0 otherwise. Both operands must have arithmetic type. First the left operand is evaluated, if it is true (for a matrix: no element is zero), the result is true, and the right operand will not be evaluated. So in the expression `func1() || func2()` the second function will *not* be called if the first function returned true.

13.8.14 Conditional expression

conditional-expression:
  logical-or-expression
  logical-or-expression `?` *expression* `:` *conditional-expression*
  logical-or-expression `.?` *expression* `.:` *conditional-expression*

Both the conditional and the dot-conditional expression are ternary expressions. For the conditional expression, the first expression (before the `?`) is evaluated. If it is unequal to 0, the result is the second expression, otherwise the third expression.

The dot-conditional expression only differs from the conditional expression if the first expression evaluates to a matrix, here called the test matrix. In that case the result is a matrix of the same size as the test matrix, and the test matrix can be seen as a filter: non zero elements get a value corresponding to the second expression, zero elements corresponding to the third expression. If the second or third expression is scalar, each matrix element will get the appropriate scalar value. If it is a matrix, the corresponding matrix element will be used, unless the matrix is too small, in which case the value 0. will be used. *Note that in the dot-conditional expression both parts are executed, whereas in the conditional expression only one of the two parts is executed.*

```
decl r, m2;

r = <1,0; 0,1> ? 4 : 5;  // 5, matrix is true if no element is 0
r = <1,0; 0,1> .? 4 .: 5;                          // <4,5; 5,4>
m2 = <1>;
r = r .== 4 .? m2 .: 0;                            // <1,0; 0,0>
```

13.8.15 Assignment expressions

assignment-expression:
  conditional-expression
  unary-expression assignment-operator assignment-expression

assignment-operator: one of
  `=   *=   /=   +=   -=   ~=   |=   .*=   ./=`

The assignment operators are the simple assignment = as well as the compound `*=  /=  +=  -=  ~=  |=  .*=  ./=` assignment operators. An lvalue is required as the left operand. The type of an assignment is that of its right operand. The compound assignment *l op*= *r* is equivalent to *l* = *l op (r)*.

If the left-hand side is a comma-separated list in square brackets, the statement is a multiple assignment expression, see §13.8.1.1.

The following code:

```
decl i, k;
for (i = 0, k = 1; i < 5; i += 2)
    k *= 2, print("i = ", i, " k = ", k, "\n");
```

writes:

```
i = 0 k = 2
i = 2 k = 4
i = 4 k = 8
```

Assignment of an array to part of an array (i.e. using selection on the left-hand side) uses the array contents of right-hand side. So, when both as1 and as2 are arrays (of strings e.g.)

```
as1[0:1] = as2
```

is executed as:

```
as1[0] = as2[0], as1[1] = as2[1];
```

thus preserving the array level in `as1`.

Assigning an object to another variable only passes a reference: both will refer to the same object. The `clone` library function makes a copy which should be removed using `delete`.

13.8.16 Comma expression

expression:
  assignment-expression
  expression , *assignment-expression*

A pair of expressions separated by a comma is evaluated left to right, and the value of the left expression is discarded. The result will have type and value corresponding to the right operand. The example in the previous section has two instances of the comma operator. The second could be omitted as follows:

```
for (i = 0, k = 1; i < 5; i += 2)
{   k *= 2;
    print("i = ", i, " k = ", k, "\n");
}
```

or as:

```
for (i = 0, k = 1; i < 5; i += 2)
    print("i = ", i, " k = ", k *= 2, "\n");
```

13.8.17 Constant expressions

An expression that evaluates to a constant is required in initializers and certain preprocessor expressions. A constant expression can have the operators `* / + -`, but only if the operands have scalar type. Some examples were given in sections 13.5.1 and 13.5.4.

13.9 Preprocessing

Preprocessing in Ox is primarily used for inclusion of files and conditional compilation of code. As such it is more restricted than the options available in C or C++. Escape sequences in strings literals are interpreted when used in preprocessor statements.

13.9.1 File inclusion

A line of the form

`#include "`*filename*`"`

will insert the contents of the specified file at that position. The file is searched for as follows:

1. in the directory containing the source file (if just a filename, or a filename with a relative path is specified), or in the specified directory (if the filename has an absolute path);
2. the directories specified on the compiler command line (if any);
3. the directories specified in the `OX7PATH` environment string (if any).
4. in the current directory.

A line of the form

`#include <`*filename*`>`

will skip the first step, and search as follows:

1. the directories specified on the compiler command line (if any);
2. the directories specified in the `OX7PATH` environment string (if any);

3. in the current directory.

The quoted form is primarily for inclusion of user created header or code files, whereas the second form will be mainly for header files that are an integral part of Ox. The default extension for Ox header files is `.oxh`.[4]

For example if `OX7PATH` is defined as (which corresponds to the Windows default if it is not set, and the executable is in `c:\ox\bin`):

```
set OX7PATH=c:\ox\include;c:\ox;
```

Then `include <maximize.oxh>` will look for:

```
c:/ox/include/maximize.oxh
c:/ox/maximize.oxh
maximize.oxh
```

In this case the first try finds the file. The method in angular brackets is used for Ox header files, whereas the other method is appropriate for your own header files. If the source file is specified to Ox as `d:\mycode\test.ox`, and run from `c:\user`, the search for `include "test.oxh"` is:[5]

```
d:/mycode/test.oxh
c:/user/test.oxh
c:/ox/include/test.oxh
c:/ox/test.oxh
test.oxh
```

13.9.2 Using file names in Ox

Note that escape sequences *are* interpreted in the include string, but not in the version which uses `<...>` (so in `#include "dir\nheader.oxh"`, the `\n` is replaced by a newline character). Both forward and backslashes are allowed (use `#include "dir/nheader.oxh"`, to avoid the newline character).

13.9.3 Import of modules

The `#import` preprocessor statement makes it easier to import compiled code modules. The statement can only happen at the external level, and has the form:

`#import` <*modulename*>

For example

```
#import <pcnaive>
```

has the following effect:

1. `#include <pcnaive.oxh>`
   The header file is inserted at that location.
2. link the `pcnaive.oxo` file when the program is run, or if this is not found:
3. compile and link the `pcnaive.ox` file when the program is run.

[4]Up to version 6 the `.h` extension was used. For compatibility with older code, when a `.h` is included, the search is first for the file with a `.oxh` extension, and, if that fails, for the `.h` file.

[5]The `-v2` commandline switch can be used to see what files were tried; this can be useful to find out why an include or import file is not found.

Similarly:

```
#import "pcnaive"
```

has the following effect:

1. `#include "pcnaive.oxh"`
   The header file is inserted at that location.
2. link `pcnaive.oxo` (or `pcnaive.ox` if the `.oxo` file is not found) when the program is run.

The import statement marks the file for linking, but that linking only happens when the file is executed. Even when a module is imported multiple times, it will only be linked in once. Similarly, the header file will not be included more than once in the same source code file.

If the import name ends in a backward/forward slash, no header file is included, but the path will be searched when trying to find a DLL or loading a data file into Ox. For example, when `OX7PATH` is set to `c:/ox/include;c:/ox;`, and the source file has:

```
#import <packages/arfima/arfima>
#import <packages/arfima/data/>
```

Then `include arfima.oxh` is searched for:

```
c:/ox/include/packages/arfima/arfima.oxh
c:/ox/packages/arfima/arfima.oxh
arfima.oxh
```

when a DLL or data file is needed in the code (say ukm1.in7 from source file `d:\mycode\test.ox`), it will be searched as:

```
ukm1.in7
d:/mycode/ukm1.in7
c:/ox/include/ukm1.in7
c:/ox/ukm1.in7
c:/ox/include/packages/arfima/ukm1.in7
c:/ox/packages/arfima/ukm1.in7
c:/ox/include/packages/arfima/data/ukm1.in7
c:/ox/packages/arfima/data/ukm1.in7
```

13.9.4 Conditional compilation

The first step in conditional compilation is to define (or undefine) identifiers:

`#define` *identifier*
`#undef` *identifier*

Identifiers so defined only exist during the scanning process of the input file, and can subsequently be used by `#ifdef` and `#ifndef` preprocessor statements:

`#ifdef` *identifier*
`#ifndef` *identifier*
`#else`
`#endif`

As an example, consider the following header file:

```
#ifndef OXSTD_INCLUDED
#define OXSTD_INCLUDED
```

```
        // header statements

    #endif
```

Now multiple inclusion of the header file into a source code file will only once include the actual header statements; on second inclusion, OXSTD_INCLUDED will be defined, and the code skipped.

Another example uses some predefined constants (see Ch. 9):

```
#include <oxstd.oxh>

main()
{
#ifdef OX_BIG_ENDIAN
    print("This is a big endian machine.\n");
#else
    print("This is a little endian machine.\n");
#endif

#ifdef OX_Windows
    print("This program is running under Windows.\n");
#endif
}
```

13.9.5 Pragmas

Pragmas influence the parsing process of the Ox compiler. Pragmas may only occur at the level of external declarations. Defined is:

`#pragma array_base(`*integer*`)`

As discussed at various points, indices in matrices, arrays and strings always start at 0. This is the C and C++ convention. Ox, however, allows circumventing this convention by using the array_base pragma. Library functions which return a set of indices, are aware of the array_base settings, and will return appropriate values. *It is recommended to adopt the zero-based convention, and not use the* array_base *pragma.* The following example shows the difference:

```
    #include <oxstd.oxh>

    base0(const m)
    {   decl i;
        i = m[0][0];            // first row, first element: i = 0
        i = m[][1:2];                                 // i = <1,2>
    }
    #pragma array_base(1)
    base1(const m)
    {   decl i;
        i = m[1][1];            // first row, first element: i = 0
        i = m[][1:2];                                 // i = <0,1>
        i = m[0][0];                                     // error
    }
    #pragma array_base(0)                        // reset to base 0

    main()
```

```
{   decl m = <0,1,2,3>;

    base0(m);
    base1(m);
}
```

The link pragma may be found in older code, but has been superseded by the `#import` preprocessor statement. The link pragma can be used to include the named file (which should be compiled source code, with default extension `.oxo`) at the point of the pragma. This provides an alternative to specifying the link files on the command line. The search machanism is the same as for `#include "`*filename*`"`. Link pragmas will normally occur in the same file as the `main` function; multiple linking of the same file will lead to errors:

```
#include <oxstd.oxh>
#pragma  link("test.oxo")

main()
{                                                   // main code
}
```

The _ox_stdlib pragma is for internal use only. It ensures that function optimizations and arraybase adjustments are only applied to internal library functions.

13.10 Difference with ANSI C and C++

This section lists some of the differences between Ox and C/C++ which might cause confusion:

- /* */ type comments can be nested in Ox.
- `sizeof` is a function in Ox, not an operator (and not a reserved word).
- Labels (targets of `goto` statements) have the colon prefixed, rather than suffixed.
- By default, all data members of a class are private, all function members public.
- The base class constructor and destructor functions are *not* called automatically.
- Integer division is not used, so `1 / 2` yields 0.5, instead of 0. Use `idiv(1, 2)` for integer division of 1 by 2.
- The preprocessor does not allow: `#define XXX value`, for integer constants, enums could be used, but more convenient is: `const decl XXX = value;`.

Part III

Appendix

Appendix A1

Some matrix algebra

This chapter summarizes the matrix algebra necessary to understand the matrix capabilities of Ox. For a more thorough overview consult Magnus and Neudecker (1988), Dhrymes (1984), Rao (1973, Chapter 1) or Anderson (1984, Appendix A), among many others.

To define the elementary operators on matrices we shall write $(a_{ij})_{m,n}$ for the $m \times n$ matrix $\mathbf{A}$ when this is convenient:

$$\mathbf{A} = (a_{ij})_{m,n} = \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \vdots \\ a_{m1} & \cdots & a_{mn} \end{pmatrix}.$$

So, for example the 3×2 matrix of ones is:

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}.$$

- *addition,* $\mathbf{A}$ is $m \times n$, $\mathbf{B}$ is $m \times n$:

$$\mathbf{A} + \mathbf{B} = (a_{ij} + b_{ij})_{m,n}.$$

- *multiplication,* $\mathbf{A}$ is $m \times n$, $\mathbf{B}$ is $n \times p$, c is a scalar:

$$\mathbf{AB} = \left(\sum_{k=1}^{n} a_{ik} b_{kj} \right)_{m,p}, \; c\mathbf{A} = (ca_{ij})_{m,n}.$$

- *dot-multiplication* (hadamard product), $\mathbf{A}$ is $m \times n$, $\mathbf{B}$ is $m \times n$:

$$\mathbf{A} \odot \mathbf{B} = (a_{ij} b_{ij})_{m,n}.$$

For example:

$$\mathbf{\Omega} \odot \mathbf{S} = \begin{pmatrix} \omega_{11} s_{11} & \omega_{12} s_{12} \\ \omega_{21} s_{21} & \omega_{22} s_{22} \end{pmatrix}.$$

- *kronecker product,* $\mathbf{A}$ is $m \times n$, $\mathbf{B}$ is $p \times q$:

$$\mathbf{A} \otimes \mathbf{B} = (a_{ij}\mathbf{B})_{mp,nq}\,.$$

For example, with $\boldsymbol{\Omega} = (\omega_{ij})_{2,2}\,,\ \mathbf{S} = (s_{ij})_{2,2}$:

$$\boldsymbol{\Omega} \otimes \mathbf{S} = \begin{pmatrix} \omega_{11}s_{11} & \omega_{11}s_{12} & \omega_{12}s_{11} & \omega_{12}s_{12} \\ \omega_{11}s_{21} & \omega_{11}s_{22} & \omega_{12}s_{21} & \omega_{12}s_{22} \\ \omega_{21}s_{11} & \omega_{21}s_{12} & \omega_{22}s_{11} & \omega_{22}s_{12} \\ \omega_{21}s_{21} & \omega_{21}s_{22} & \omega_{22}s_{21} & \omega_{22}s_{22} \end{pmatrix}.$$

- *transpose,* $\mathbf{A}$ is $m \times n$:

$$\mathbf{A}' = (a_{ji})_{n,m}\,.$$

- *determinant,* $\mathbf{A}$ is $n \times n$:

$$|\mathbf{A}| = \sum (-1)^{c(j_1,\ldots,j_n)} \prod_{i=1}^{n} a_{ij_i}$$

where the summation is over all permutations $(j_1, \ldots, j_n)$ of the set of integers $(1, \ldots, n)$, and $c(j_1, \ldots, j_n)$ is the number of transpositions required to change $(1, \ldots, n)$ into $(j_1, \ldots, j_n)$. In the 2×2 case the set $(1, 2)$ can be transposed once into $(2, 1)$, so $|\boldsymbol{\Omega}| = (-1)^0\omega_{11}\omega_{22} + (-1)^1\omega_{12}\omega_{21}$.
- *trace,* $\mathbf{A}$ is $n \times n$:

$$\mathrm{tr}\mathbf{A} = \sum_{i=1}^{n} a_{ii}.$$

- *rank,* $\mathbf{A}$ is $m \times n$: the rank of $\mathbf{A}$ is the number of linearly independent columns (or rows, row rank always equals column rank) in $\mathbf{A}$, $\mathrm{r}(\mathbf{A}) \leq \min(m, n)$. If $\mathbf{A}$ is $n \times n$ and of full rank then:

$$\mathrm{r}\,(\mathbf{A}) = n.$$

- *symmetric matrix*, $\mathbf{A}$ is $n \times n$: $\mathbf{A}$ is symmetric if:

$$\mathbf{A}' = \mathbf{A}.$$

- *matrix inverse,* $\mathbf{A}$ is $n \times n$ and of full rank (non-singular, which is equivalent to $|\mathbf{A}| \neq 0$) then $\mathbf{A}^{-1}$ is the unique $n \times n$ matrix such that:

$$\mathbf{A}\mathbf{A}^{-1} = \mathbf{I}.$$

This implies that $\mathbf{A}^{-1}\mathbf{A} = \mathbf{I}$; $\mathbf{I}$ is the $n \times n$ identity matrix:

$$\begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & 1 \end{pmatrix}.$$

- *orthogonal matrix*, $\mathbf{A}$ is $n \times n$: $\mathbf{A}$ is orthogonal if:

$$\mathbf{A}'\mathbf{A} = \mathbf{I}.$$

  Then also $\mathbf{A}\mathbf{A}' = \mathbf{I}$; further: $\mathrm{r}(\mathbf{A}) = n$, $\mathbf{A}' = \mathbf{A}^{-1}$.
- *orthogonal complement*, $\mathbf{A}$ is $m \times n$, $m > n$ and $\mathrm{r}(\mathbf{A}) = n$, define the orthogonal complement $\mathbf{A}_\perp$ as the $m \times (m-n)$ matrix such that: $\mathbf{A}'\mathbf{A}_\perp = \mathbf{0}$ with $\mathrm{r}(\mathbf{A}_\perp) = m - n$ and $\mathrm{r}(\mathbf{A} : \mathbf{A}_\perp) = m$. $\mathbf{A}_\perp$ spans the *null space* of $\mathbf{A}$; $\mathrm{r}(\mathbf{A}_\perp)$ is called the *nullity* of $\mathbf{A}$.
- *idempotent matrix*, $\mathbf{A}$ is $n \times n$: $\mathbf{A}$ is idempotent if:

$$\mathbf{A}\mathbf{A} = \mathbf{A}.$$

  An example is the projection matrix $\mathbf{M}_X = \mathbf{I} - \mathbf{X}\left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{X}'$.
- *vectorization,* $\mathbf{A}$ is $m \times n$:

$$\mathrm{vec}\mathbf{A} = \begin{pmatrix} a_{11} \\ \vdots \\ a_{m1} \\ \vdots \\ a_{1n} \\ \vdots \\ a_{mn} \end{pmatrix},$$

  which is an $mn \times 1$ vector consisting of the stacked columns of $\mathbf{A}$.
  If $\mathbf{A}$ is $n \times n$ and symmetric, we can use the vech operator to vectorize the unique elements, thus ignoring the elements above the diagonal:

$$\mathrm{vech}\mathbf{A} = \begin{pmatrix} a_{11} \\ \vdots \\ a_{n1} \\ a_{22} \\ \vdots \\ a_{n2} \\ \vdots \\ a_{nn} \end{pmatrix},$$

  which is a $\frac{1}{2}n(n+1) \times 1$ vector.
- *diagonalization,* $\mathbf{A}$ is $n \times n$:

$$\mathrm{dg}\mathbf{A} = \begin{pmatrix} a_{11} & 0 & \cdots & 0 \\ 0 & a_{22} & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & a_{nn} \end{pmatrix} = \mathrm{diag}\left(a_{11}, a_{22}, \ldots, a_{nn}\right).$$

- *positive definite,* $\mathbf{A}$ is $n \times n$ and symmetric: $\mathbf{A}$ is positive definite if $\mathbf{x}'\mathbf{A}\mathbf{x} > 0$ for all $n \times 1$ vectors $\mathbf{x} \neq \mathbf{0}$, positive semi-definite if $\mathbf{x}'\mathbf{A}\mathbf{x} \geq 0$ for all $\mathbf{x} \neq \mathbf{0}$, and negative definite if $\mathbf{x}'\mathbf{A}\mathbf{x} < 0$ for all $\mathbf{x} \neq \mathbf{0}$.
- *eigenvalues and eigenvectors*, $\mathbf{A}$ is $n \times n$: the eigenvalues of $\mathbf{A}$ are the roots of the characteristic equation:
$$|\mathbf{A} - \lambda\mathbf{I}| = 0.$$
If λ_i is an eigenvalue of $\mathbf{A}$, then $\mathbf{x}_i \neq \mathbf{0}$ is an eigenvector of $\mathbf{A}$ if it satisfies:
$$(\mathbf{A} - \lambda_i\mathbf{I})\,\mathbf{x}_i = \mathbf{0}.$$
- *Choleski decomposition*, $\mathbf{A}$ is $n \times n$ summetric and positive definite, then:
$$\mathbf{A} = \mathbf{P}\mathbf{P}',$$
where $\mathbf{P}$ is a unique lower triangular matrix with positive diagonal elements.
- *LU decomposition*, $\mathbf{A}$ is $n \times n$, then:
$$\mathbf{A} = \mathbf{L}\mathbf{U}',$$
where $\mathbf{L}$ is a lower triangular matrix with ones on the diagonal and $\mathbf{U}$ is upper diagonal.
- *singular value decomposition*, decomposes an $m \times n$ matrix $\mathbf{A}$, $m \geq n$, into:
$$\mathbf{A} = \mathbf{U}\mathbf{W}\mathbf{V}',$$
with:

  $\mathbf{U}$ is $m \times n$ and $\mathbf{U}'\mathbf{U} = \mathbf{I}_n$,
  $\mathbf{W}$ is $n \times n$ and diagonal, with non-negative diagonal elements,
  $\mathbf{V}$ is $n \times n$ and $\mathbf{V}'\mathbf{V} = \mathbf{I}_n$.

  The diagonal of $\mathbf{W}$ holds the singular values. The number of non-zero singular values is the rank of $\mathbf{A}$, also see §13.8.5.1.
  The SVD can be used to find the orthogonal complement of $\mathbf{A}$. Assume $\mathrm{r}(\mathbf{A}) = n$ and compute the singular value decomposition of the $(m \times m)$ matrix $\mathbf{B} = (\mathbf{A} : \mathbf{0})$. The last $m - n$ diagonal elements of $\mathbf{W}$ will be zero. Corresponding to that are the last $m - n$ columns of $\mathbf{U}$ which form $\mathbf{A}_\perp$:
$$\mathbf{B} = (\mathbf{A} : \mathbf{0}) = \mathbf{U}\mathbf{W}\mathbf{V}' = (\mathbf{U}_1 : \mathbf{U}_2)\begin{pmatrix} \mathbf{W}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{pmatrix}\begin{pmatrix} \mathbf{V}_1' \\ \mathbf{V}_2' \end{pmatrix}.$$
Here $\mathbf{U}$, $\mathbf{V}$ and $\mathbf{W}$ are $(m \times m)$ matrices; $\mathbf{U}_2'\mathbf{U}_1 = \mathbf{0}$ so that $\mathbf{U}_2'\mathbf{A} = \mathbf{U}_2'\mathbf{U}_1\mathbf{W}_1\mathbf{V}_1' = \mathbf{0}$ and $\mathrm{r}(\mathbf{A} : \mathbf{U}_2) = m$ as $\mathbf{U}_2'\mathbf{U}_2 = \mathbf{I}$.
- *differentiation*, define $f(\cdot) : \mathbb{R}^m \mapsto \mathbb{R}$ then:
$$\nabla f = \frac{\partial f(\mathbf{a})}{\partial \mathbf{a}} = \begin{pmatrix} \frac{\partial f(\mathbf{a})}{\partial a_1} \\ \vdots \\ \frac{\partial f(\mathbf{a})}{\partial a_m} \end{pmatrix}, \quad \nabla^2 f = \frac{\partial^2 f(\mathbf{a})}{\partial \mathbf{a}\partial \mathbf{a}'} = \left(\frac{\partial^2 f(\mathbf{a})}{\partial a_i \partial a_j}\right)_{m,m}.$$

If $f(\cdot)$ is a log-likelihood function we shall write $\mathbf{q}(\cdot)$ for the first derivative (or score), and $\mathbf{H}(\cdot)$ for the second derivative (or Hessian) matrix.
For $f(\cdot) : \mathbb{R}^{m\times n} \mapsto \mathbb{R}$ we define:

$$\frac{\partial f(\mathbf{A})}{\partial \mathbf{A}} = \left(\frac{\partial f(\mathbf{A})}{\partial a_{ij}} \right)_{m,n}.$$

- *Jacobian matrix*, for a vector function $\mathbf{f}(\cdot) : \mathbb{R}^m \mapsto \mathbb{R}^n$ we define the $n \times m$ Jacobian matrix $\mathbf{J}$:

$$\frac{\partial \mathbf{f}(\mathbf{a})}{\partial \mathbf{a}'} = \begin{pmatrix} \frac{\partial f_1(\mathbf{a})}{\partial a_1} & \cdots & \frac{\partial f_1(\mathbf{a})}{\partial a_m} \\ \vdots & & \vdots \\ \frac{\partial f_n(\mathbf{a})}{\partial a_1} & \cdots & \frac{\partial f_n(\mathbf{a})}{\partial a_m} \end{pmatrix} = \begin{pmatrix} (\nabla f_1)' \\ \vdots \\ (\nabla f_m)' \end{pmatrix} = (\nabla \mathbf{f})'.$$

The transpose of the Jacobian is called the gradient, and corresponds to the $\mathbf{q}(\cdot)$ above for $n = 1$ (so in that case the Jacobian is $1 \times m$ and the score $n \times 1$). The Jacobian is the absolute value of the determinant of $\mathbf{J}$ when $m = n$: $||\mathbf{J}||$.
Normally we wish to compute the Jacobian matrix for a transformation of a coefficient matrix: $\boldsymbol{\Psi} = \mathbf{F}(\boldsymbol{\Pi}')$ where $\mathbf{F}$ is a matrix function $\mathbf{F}(\cdot) : \mathbb{R}^{m\times n} \mapsto \mathbb{R}^{p\times q}$:

$$\mathbf{J} = \frac{\partial \text{vec}\boldsymbol{\Psi}}{\partial (\text{vec}\boldsymbol{\Pi}')'},$$

with $\boldsymbol{\Pi}$ $n \times m$ and $\boldsymbol{\Psi}$ $p \times q$ so that $\mathbf{J}$ is $pq \times mn$.

References

Abramowitz, M. and I. A. Stegun (1970). *Handbook of Mathematical Functions*. New York: Dover Publications Inc. [156]

Abramowitz, M. and I. A. Stegun (1984). *Pocketbook of Mathematical Functions*. Frankfurt/Main: Verlag Harri Deutsch. [153, 172, 192]

Anderson, T. W. (1984). *An Introduction to Multivariate Statistical Analysis,* (2nd ed.). New York: John Wiley & Sons. [475]

Barndorff-Nielsen, O. E. and N. Shephard (2001). *Lévy Based Dynamic Models for Financial Economics*. Oxford: Draft book, Nuffield College. [328]

Barnett, S. (1990). *Matrices – Methods and Applications*. Oxford: Clarendon Press. [57]

Berndt, E. K., B. H. Hall, R. E. Hall, and J. A. Hausman (1974). Estimation and inference in nonlinear structural models. *Annals of Economic and Social Measurement 3*, 653–665. [310]

Berry, K. J., P. W. Mielke Jr, and G. W. Cran (1977). Remark AS R83: A remark on algorithm AS 109: Inverse of the incomplete beta function ratio. *Applied Statistics 39*, 309–310. [203]

Best, D. J. and N. I. Fisher (1979). Efficient simulation of the von Mises distribution. *Applied Statistics 28*, 152–157. [336]

Best, D. J. and D. E. Roberts (1975). Algorithm AS 91: The percentage points of the χ^2 distribution. *Applied Statistics 24*, 385–389. [203]

Brockwell, P. J. and R. A. Davis (1991). *Time Series: Theory and Methods*. New York: Springer-Verlag. [189]

Chambers, J. M., C. L. Mallows, and B. W. Sturk (1976). A method for simulating stable random variables. *Journal of the American Statistical Association 71*, 340–344. [336]

Cooper, B. E. (1968). Algorithm AS 3: The integral of student's t-distribution. *Applied Statistics 17*, 189–190. [200]

Cran, G. W., K. J. Martin, and G. E. Thomas (1977). Remark AS R19 and algorithm AS 109: A remark on algorithms AS 63: The incomplete beta integral; AS 64: Inverse of the incomplete beta function ratio. *Applied Statistics 26*, 111–114. [203]

Dagnapur, J. (1988). *Principles of Random Variate Generation*. Oxford: Clarendon Press. [337]

Dennis Jr., J. E. and R. B. Schnabel (1983). *Numerical Methods for Nonlinear Equations and Unconstrained Optimization,*. Englewood Cliffs, NJ: Prentice Hall. [321]

Devroye, L. (1986). *Non-Uniform Random Variate Generation*. New York: Springer-Verlag. [52]

Dhrymes, P. J. (1984). *Mathematics for Econometrics,* (2nd ed.). New York: Springer-Verlag. [475]

Ding, C. G. (1992). Algorithm AS 275: Computing the non-central χ^2 distribution function. *Applied Statistics 41*, 478–483. [200]

Doornik, J. A. (2005). An improved ziggurat method to generate normal random samples. Mimeo, Nuffield College. [208]

Doornik, J. A. (2006). The role of simulation in econometrics. In T. Mills and K. Patterson (Eds.), *Palgrave Handbook of Econometrics*, pp. 787–811. Basingstoke: Palgrave MacMillan. [211]

Doornik, J. A. (2007). Conversion of high-period random numbers to floating point. *ACM Transactions on Modeling and Computer Simulation 17*. [211]

Doornik, J. A. and D. F. Hendry (2009). *Modelling Dynamic Systems using PcGive: Volume II* (4th ed.). London: Timberlake Consultants Press. [388]

Doornik, J. A. and M. Ooms (2006). *Introduction to Ox* (2nd ed.). London: Timberlake Consultants Press. [3, 6, 15, 16, 19]

Dubrulle, A. (1970). A short note on the implicit *ql* algorithm for symmetric tridiagonal matrices. *Numerische Mathematik 15*, 450. [126]

Engler, E. and B. Nielsen (2009). The empirical process of autoregressive residuals. *Econometrics Journal 12*, 367–381. [279]

Feng, R. B. and P. D. Pulliam (1997). Tensor-gmres method for large systems of nonlinear equations. *SIAM Journal of Optimization 7*, 757–779. [321]

Fisher, N. I. (1993). *Statistical Analysis of Circular Data*. New York: Cambridge University Press. [331]

Fletcher, R. (1987). *Practical Methods of Optimization,* (2nd ed.). New York: John Wiley & Sons. [307, 324]

Genz, A. (2000). Numerical computation of bivariate and trivariate normal probabilities. Mimeo, Washington State University, Pullman, WA. [331]

Golub, G. H. and C. F. Van Loan (1989). *Matrix Computations*. Baltimore: The Johns Hopkins University Press. [103, 105, 182, 229, 300]

Granger, C. W. J. and P. Newbold (1986). *Forecasting Economic Time Series,* (2nd ed.). New York: Academic Press. [189]

Green, P. J. and B. W. Silverman (1994). *Nonparametric Regression and Generalized Linear Models. A Roughness Penalty Approach*. London: Chapman and Hall. [232, 233]

Harvey, A. C. (1993). *Time Series Models,* (2nd ed.). Hemel Hempstead: Harvester Wheatsheaf. [294, 296]

Hastie, T. J. and R. J. Tibshirani (1994). *Generalized Additive Models*. London: Chapman and Hall. [232]

Hendry, D. F., A. J. Neale, and N. R. Ericsson (1991). *PC-NAIVE, An Interactive Program for Monte Carlo Experimentation in Econometrics. Version 6.0*. Oxford: Institute of Economics and Statistics, University of Oxford. [405]

Hill, G. W. (1981). Remark on Algorithm 396: Student's t-quantiles [s14]. *ACM Transactions on Mathematical Software 7*, 250–251. [203]

Hill, I. D. (1973). Algorithm AS 66: The normal integral. *Applied Statistics 22*, 424–427. [200]

Kemp, A. W. (1981). Efficient generation of logarithmically distributed pseudo-random variables. *Applied Statistics 30*, 249–253. [336]

Kernighan, B. W. and D. M. Ritchie (1988). *The C Programming Language* (2nd ed.). Englewood Cliffs, NJ: Prentice Hall. [28, 418]

Kiefer, N. M. (1989). The ET interview: Arthur S. Goldberger. *Econometric Theory 5*, 133–160. [56]

Lacey, S. and R. Box (1991). A fast, easy sort. *Byte April*. [230]

LAPACK (1999). *LAPACK Users' Guide* (3rd ed.). Philadelphia: SIAM. By Anderson, E. and Bai, Z. and Bischof, C. and Blackford, S. and Demmel, J. and Dongarra, J. and Du Croz, J. and Greenbaum, A. and Hammarling, S. and McKenney, A. and Sorensen, D. [112, 126]

Lawrence, C. T. and A. L. Tits (2001). A computationally efficient feasible sequential quadratic programming algorithm. *SIAM Journal of Optimization 11*, 1092–1118. [316]

L'Ecuyer, P. (1999). Tables of maximally-equidistributed combined LFSR generators. *Mathematics of Computation 68*, 261–269. [211]

Lehner, K. (1989). Erzeugung von zufallszahlen für zwei exotische verteilungen. Diplomarbeit, Techn. Universität Graz. [337]

Lenth, R. V. (1987). Algorithm AS 226: Computing noncentral Beta probabilities. *Applied Statistics 36*, 241–244. [200]

Lenth, R. V. (1989). Algorithm AS 243: Cumulative distribution function of the non-central t distribution. *Applied Statistics 38*, 185–189. [200]

Longley, G. M. (1967). An appraisal of least-squares for the electronic computer from the point of view of the user. *Journal of the American Statistical Association 62*, 819–841. [54]

Magnus, J. R. and H. Neudecker (1988). *Matrix Differential Calculus with Applications in Statistics and Econometrics*. New York: John Wiley & Sons. [475]

Majunder, K. L. and G. P. Bhattacharjee (1973). Algorithm AS 64. Inverse of the incomplete beta function ratio. *Applied Statistics 22*, 411–414. [203]

Mardia, K. V. and P. J. Zemroch (1975). Algorithm AS 86: The von Mises distribution function. *Applied Statistics 24*, 268–272. [200, 331]

Marsaglia, G. (1997). A random number generator for C. Posting, Usenet newsgroup sci.stat.math. 29-Sep-1997. [211]

Marsaglia, G. (2003). Re: good C random number generator. Posting, Usenet newsgroup sci.lang.c. 13-May-2003. [211]

Martin, R. S., C. Reinsch, and J. H. Wilkinson (1968). Householder's tridiagonalization of a symmetric matrix. *Numerische Mathematik 11*, 181–195. [126]

Martin, R. S. and J. H. Wilkinson (1968a). The implicit *ql* algorithm. *Numerische Mathematik 12*, 377–383. [126]

Martin, R. S. and J. H. Wilkinson (1968b). Similarity reduction of a general matrix to Hessenberg form. *Numerische Mathematik 12*, 349–368. [126]

McLeod, I. (1975). Derivation of the theoretical autocovariance function of autoregressive-moving average time series. *Applied Statistics 24*, 255–256. Correction in *Applied Statistics*, **26**, 194. [297]

Michael, J. R., W. R. Schucany, and R. W. Haas (1976). Generating random variates using transformations with multiple roots. *American Statistician 30*, 88–90. [336]

Nocedal, J. and S. J. Wright (1999). *Numerical Optimization*. New York: Springer-Verlag. [316, 321, 324]

Olde Daalhuis, A. B. (2010). Hypergeometric function. See Olver, Lozier, Boisvert, and Clark (2010), Chapter 15. [156]

Olver, F., D. W. Lozier, R. F. Boisvert, and C. W. Clark (2010). *NIST Handbook of Mathematical Functions*. New York: Cambridge University Press. [84, 483]

O'Neil, R. (1971). Algorithm AS 47: Function minimization using a simplex procedure. *Applied Statistics 20*, 338–345. Improved version in Griffiths, P. and Hill, I. D. (eds) (1985), *Applied Statistics Algorithms*. Chichester: Horwood. [312]

Ooura, T. (1998). Special functions – gamma/error functions. mimeo, Kyoto University. www.kurims.kyoto-u.ac.jp/ ooura. [200]

Park, S. K. and K. W. Miller (1988). Random number generators: Good ones are hard to find. *Communications of the ACM 31*, 1192–1201. [211]

Parlett, B. N. and C. Reinsch (1969). Balancing a matrix for calculation of eigenvalues and eigenvectors. *Numerische Mathematik 13*, 293–304. [126]

Peters and J. H. Wilkinson (1970). Eigenvectors of real and complex matrices by *lr* and *qr* triangulazations. *Numerische Mathematik 16*, 181–204. [126]

Petzold, C. (1992). *Programming Windows 3.1*. Redmond: Microsoft Press. [33]

Piessens, R., E. de Donker-Kapenga, C. W. Überhuber, and D. K. Kahaner (1983). *QUADPACK, A Subroutine Package for Automatic Integration*. Heidelberg: Springer-Verlag. [339]

Press, W. H., B. P. Flannery, S. A. Teukolsky, and W. T. Vetterling (1988). *Numerical Recipes in C*. New York: Cambridge University Press. [81]

Priestley, M. B. (1981). *Spectral Analysis and Time Series*. London: Academic Press. [189]

Rao, C. R. (1973). *Linear Statistical Inference and its Applications,* (2nd ed.). New York: John Wiley & Sons. [475]

Ripley, B. D. (1987). *Stochastic Simulation*. New York: John Wiley & Sons. [336, 337]

Schnabel, R. B. and P. D. Frank (1985). Tensor methods for nonlinear equations. *SIAM Journal of Numerical Analysis 21*, 815–843. [321]

Shea, B. L. (1988). Algorithm AS 239: Chi-squared and incomplete gamma integral. *Applied Statistics 37*, 466–473. [154]

Shea, B. L. (1991). Algorithm AS R85: A remark on algorithm AS 91: The percentage points of the χ^2 distribution. *Applied Statistics 40*, 233–235. [203]

Silverman, B. W. (1986). *Density Estimation for Statistics and Data Analysis*. London: Chapman and Hall. [276]

Smith, W. B. and R. R. Hocking (1972). Algorithm AS 53: Wishart variate generator. *Applied Statistics 21*, 341–345. [337]

Stroustrup, B. (1997). *The C++ Programming Language* (3rd ed.). Reading, MA: Addison Wesley. [418]

Wichura, M. J. (1988). Algorithm AS 241: The percentage points of the normal distribution. *Applied Statistics 37*, 477–484. [203]

Wilkinson, J. H. (1965). *The Algebraic Eigenvalue Problem*. Oxford: Oxford University Press. [103]

Wirth, N. (1987). *Compilerbouw*. Schoonhoven, The Netherlands: Academic Service. Dutch version. Original German version published in 1986 by B.G. Teubner Verlag, Stuttgart. [xxxi]

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