Problem 1

Sensitivity Analysis

Consider the following solution to a daily production mix optimization problem where:

- The decision variables are the number of units to produce of three different types of products
- The objective function coefficients (Objective coefficient in table) are the unitary profits (difference between production costs and sell price per unit) in euros
- The three constraint represent the total availability of three sections in minutes

Decision variables

Decision variable	Solution	Reduced cost	Objective Coefficient	Lower bound	Upper bound
product 1 units	300	0	30	24.44	inf
product 2 units	33.33	0	20	-0	90
product 3 units	0	-8.33	40	-inf	48.33

Constraints

Constraint	Right Hand Side	Shadow Price	Slack	Min RHS	Max RHS
Section 1 availability	400	6.67	0	300	525
Section 2 availability	600	11.67	0	0	800
Section 3 availability	600	0	166.67	433.33	inf

Answer the following questions. Motivate your response based on the provided results:

- Is it profitable to produce units of the three types of products? If any of the products is not profitable, what changes are needed to make it profitable? **(0.5 points)**
- In the actual economic context, the production costs are rising. Discuss how the rise of the production costs can affect your profit and identify the maximum unitary production cost increase that your daily production mix can support without changes in the base solution **(1 point)**
- In order to increase your production capacity, you need to evaluate 2 different improvement proposals from an external consultant. The first one considers a 12% increase of the availability of section 1, and the second a 7.5% increase of the availability of section 2. Which one is more profitable for the company? **(1 point)**

Problem 2

Precision parts

You work for a company that manufactures high quality polymer parts for OrgoCorp, a galactic bioengineering company. The quality requirements are so high that you need to use very sophisticated 3D printing machines. You need to develop a mathematical programming model to organize monthly production considering:

- **Fixed operational costs (euros):** Each 3D printing machine m has a different fixed operational costs Fm that needs to be considered when a printing machine is used to print any units.
- **Unitary production costs (euros):** Each 3D printing machine m produces parts of product type p at a different unitary costs C_{mp}
- **Capacity (minutes)**: Each 3D printing machine m has a different capacity S_m (minutes)
- **Speed (minutes/part)**: Each 3D printing machine m prints a unit of product type p at a different unitary speed
- **Demand (units)**: OrgoCorp has confirmed a demand for d_p units of every product type p

Build a model that takes into account these requirements, identifying indices, decision variables, objective function and constraints (**1.5 points**).

The CEO of OrgoCorp, The High Evolutionary, would like to set up a new contract allowing more flexibility in the delivery of units, such that:

- **Demand per period (units)**: Now you need to consider that the demand for every product type p is not constant and depends on the planning period t, D_{pt}
- **Delayed demand (units)**: You may delay the production for one period. For every unit of product type p that is delayed, OrgoCorp will charge an extra delayed demand cost of B_{pt}

Modify the model to take into account this new contract (**1 point**)

Problem 3

Mathematical Nirvana

Nathaniel Richards is a young, brilliant scientist that has developed the following Non-Linear Programming (NLP) problem to find the optimal balance between studying and meditating to maximize the overall satisfaction and achieve a state of enlightenment (Mathematical Nirvana).

Decision Variables: Let:

x_1 Time spent studying (in hours)

x_2 = Time spent meditating (in hours)

$x = [x_1, x_2]$ The set of decision variables

$x_1, x_2 \geq 0$

Objective Function: Maximize the overall satisfaction obtained from studying and meditating:

$\max z = f(x) = 2*x_1+0.5*\ln(1+x_1) + 0.7*x_2 + 0.3*\sqrt(x_2)$

Constraints: Subject to: Maximum amount of time available:

$x_1 + x_2 \leq 10$

Minimum amount of time studying to ensure academic performance:

$x_1 \geq 2$

Unfortunately Nathaniel misteriously dissappeared before he could completely analyse the problem, so you need to complete his work according to the following instructions:

1. Obtain the Kuhn-Tucker conditions **(1 point)**
2. Obtain the Hessian and determine if this solution ($x_1 = 3.9, x_2 = 6.1) is a global or local maximum **(0.75 points)**
3. Use the Kuhn-Tucker conditions to calculate the Lagrangian multipliers for this solution, can you explain what they mean? Discuss if this can be an optimal solution to the problem **(0.75 points)**

Problem 4

Strategies for political campaigns

Suppose there are two political parties, The Avengers Party (a coalition of superheroes who have come together to fight for the greater good. They believe in using their powers to protect the people and promote equality and justice) and the BrotherHood party (This party represents a group of mutants who feel oppressed by society and seek to overthrow the existing power structures). They are competing in an election. Each party has three possible strategies: they can focus their campaign on young citizens (below 30), middle-aged citizens (30-50), or senior citizens (over 50). The parties use national statistics to estimate the number of votes that can shift in each demographic group (undecided voters). Here are the estimated values for each demographic group:

- Young voters: They represent 60% of the undecided voters.
- Middle-aged voters: They represent 25% of the undecided voters.
- Senior citizens: They represent 15% of the undecided voters.

From past elections, the Avengers party has collected the following estimation about undecided voters on each scenario:

- **Young voters:** If both parties focus on young voters, the Avengers party will get 40% of the young voters and the Brotherhood party 60% of the young voters. If the Avengers party focuses on the young voters and the Brotherhood party does not, the Avengers party will get 70% of the young voters. If the Brotherhood party focuses on the young voters and the Avengers party does not, the Avengers party will get 10% of the young voters. If neither party focuses on the young voters, the Avengers party will get 40% of the young voters.
- **Middle-aged voters:** If both parties focus on middle-aged voters, the Avengers party will get 60% of the middle-aged voters and the Brotherhood party 40% of the middle-aged voters. If the Avengers party focuses on the middle-aged voters and the Brotherhood party does not, the Avengers party will get 90% of the middle-aged voters. If the Brotherhood party focuses on the middle-aged voters and the Avengers party does not, the Avengers party will get 30% of the middle-aged voters. If neither party focuses on the middle-aged voters, the Avengers party will get 60% of the middle-aged voters.
- **Senior citizens:** If both parties focus on senior citizens, the Avengers party will get 70% of the senior citizens and the Brotherhood party 30% of the senior citizens. If the Avengers party focuses on the senior citizens and the Brotherhood party does not, the Avengers party will get 90% of the senior citizens. If the Brotherhood party focuses on the senior citizens and the Avengers party does not, the Avengers party will get 20% of the senior citizens. If neither party focuses on the senior citizens, the Avengers party will get 70% of the senior citizens.

Let us use Game Theory to model the strategies of both political parties, following these steps:

1. Identify the alternatives for both players and draw a tabular representation of the game were Player A is the Avengers Party **(0.25 points)**
2. Complete the tabular representation of the game, estimating the total percentage of undecided voters that would vote for each party **(1.5 points)**
3. Calculate the strategy of each party using the MinMax criteria **(0.75 points)**.