MSO 203 B - PDE

sites.google.com/view/kaushal.

Discussion:- Thursday → 3:00 - 4:30 pm in FB 525

Exam:-

(i) End-sem - 45 marks (9-11) - 20th Nov.

(ii) Quiz - 25 marks (6:30-7:30) - 24th Oct !!

(iii) Atleast 2 surprise quiz.

Syllabus:-

ODE:
- Sturm-Liouville Problem
- Fourier Series.

PDE:-
- Defn (1st, 2nd, lin, non-lin)
- 1st Order (Method of Char)

- 2nd Order PDE - Classification.
- Canonical Form
- Laplace Eqn, Heat Eqn, Wave Eqn

(i) $\Delta u = 0 \sim \nabla^2 u = 0$ ← Laplace

$u_{xx} + u_{yy} = 0$

(ii) $u_t - \Delta u = 0$ - Heat

(iii) $u_{tt} - \Delta u = 0$ - Wave Eqn

Books:-

1. Lokenath - Myint - Linear PDE for Scientist & Engineer.

2. Kreyzig - Advanced Engineering Math

(i) Fourier Series

(ii) S-L. → Coddington (ODE)

Notations & Definitions :-

$I = [a,b]$ - is an interval in $\mathbb{R}$.

$C(I) = \{$ Set of all continuous fns on $I \}$.

$I = [0,1]$

(i) $C(I) = \{ \sin, \cos, x, x^2, e^x \dots \}$

- $f, g \in C(I)$

$(f+g) \in C(I)$

$c \in \mathbb{R}$, $cf \in C(I)$

$f(x) = x \in C([-1,1])$.

$f_0(x) = \frac{1}{x} \notin C([-1,1])$

(ii) $C^1(I)$ = Set of continuous diff fns on I $\approx$ f' exists and is continuous on I.

$C^1 \subset C$.

Ex :- Is $C(I) \subset C^1(I)$?? $f(x) = |x|$ on $[-1,1]$

$C^\infty(I)$ = Set of all infinitely diff fns.

$f(x) = e^x$, $P(x)$, $\sin, \cos x$ ||

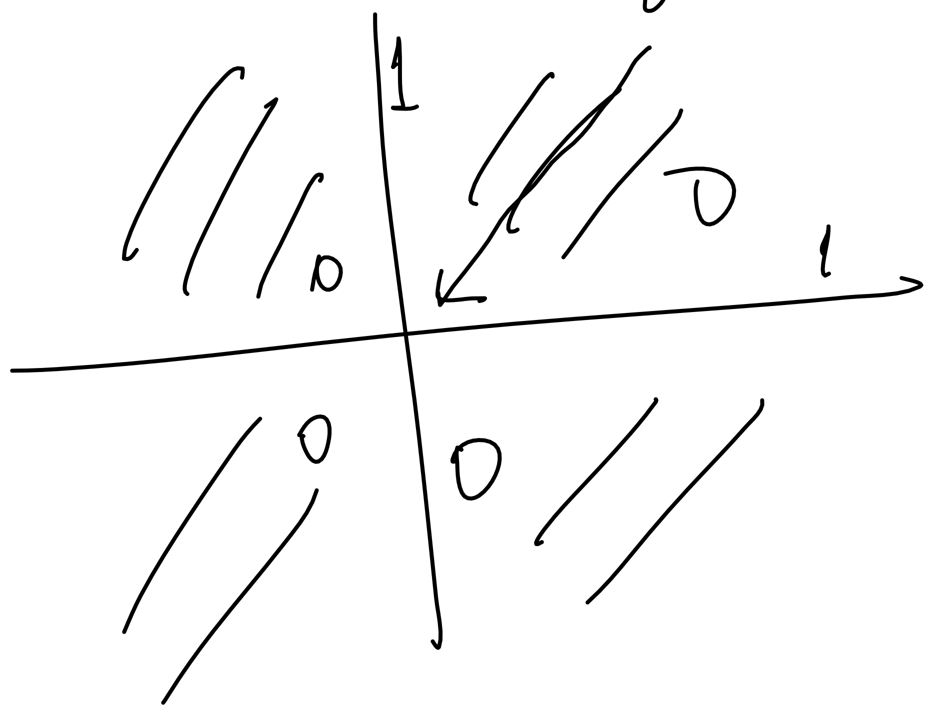
Smooth fns

Partial Derivative :- $f : \mathbb{R}^2 \to \mathbb{R}$

$\frac{\partial f}{\partial x}(x_0, y_0)$ = partial derivative of f w.r.t x = $f_x(x_0, y_0) = \lim_{t \to 0} \frac{f(x_0+t, y_0) - f(x_0, y_0)}{t}$ exist

$|||^{ly}$, $\frac{\partial f}{\partial y}(x_0, y_0)$ = partial derivative of f w.r.t y $= \lim_{t \to 0} \frac{f(x_0, y_0+t) - f(x_0, y_0)}{t}$ exists

$$f(x,y) = \begin{cases} 1 & , x = 0 \text{ or } y = 0 \\ 0 & \text{o/w} . \end{cases}$$



$f_x(0,0) = 0 = f_y(0,0)$

$x = y$

$f(0,0) = 1.$

$\lim_{x \to 0} f(x, x) = 0$

∇f = Gradient of $f = (f_x, f_y)$

$\nabla f(x_0, y_0) = (f_x(x_0, y_0), f_y(x_0, y_0))$

Definition:- We say f is diff at (x_0, y_0) if both f_x and f_y exists and is continuous at (x_0, y_0).

Moreover we define $Df(x_0, y_0)$ = Total Derivative of f at $(x_0, y_0) := \nabla f(x_0, y_0)$

Chain Rule:- Let $x = x(t)$ & $y = y(t)$ be diff at 't' and let $z = f(x, y)$ is diff at $(x(t), y(t))$.

Then $z = f(x(t), y(t))$ is diff at 't' and moreover

$$\frac{dz}{dt} = \frac{dx}{dt}\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} \cdot \frac{dy}{dt}$$

$f: \mathbb{R}^2 \to \mathbb{R}$.

$z: \mathbb{R} \to \mathbb{R}$

ODE:- $y'' + q(x)y' + r(x)y = 0$ in $I \ni x_0$

$y(x_0) = y_0$

$y'(x_0) = y_1$

(Initial Value Problem)

Th:- $\exists$ a unique soln for the I.V.P in a nbd of 'x_0'.

B.V.P :-

$$y'' + q(x)y' + r(x)y = 0 \quad \text{in } I = [x_0, x_1]$$

$$y(x_0) = y_0$$

$$y(x_1) = y_1$$

Ex :- $y'' + y = 0$

$y(x) = A\sin x + B\cos x$

$y(0) = 0$; $y(\pi/2) = 0 \Rightarrow y \equiv 0$ is a unique soln

$y(0) = 0$; $y(\pi) = 0 \Rightarrow y(x) = B\sin x$ → Infinitely many soln

$y(0) = 0$ & $y(\pi) = 1 \Rightarrow \nexists$ any soln.