1[st] September, 2016

Home Assignment – 5

1. Find the transfer function for the networks shown in **Fig. 1(a), Fig. 1(b)** and **Fig. 1(c).**

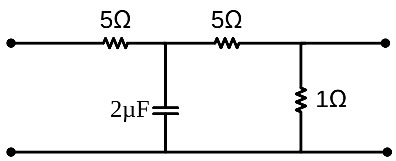


Fig. 1(a)

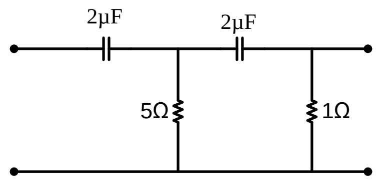


Fig. 1(b)

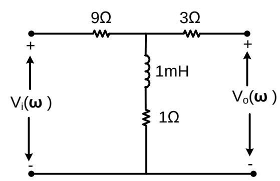


Fig. 1(c)

2. Determine the frequency response $\dfrac{V_o(j\omega)}{V_i(j\omega)}$ for the circuit shown in **Fig. 2.** Also plot the **Bode magnitude response** of the transfer function, showing all salient features. What kind of filter is it?

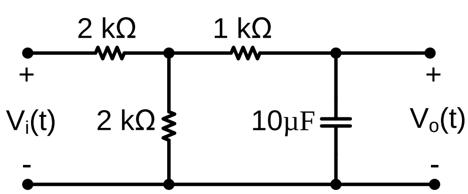


Fig. 2

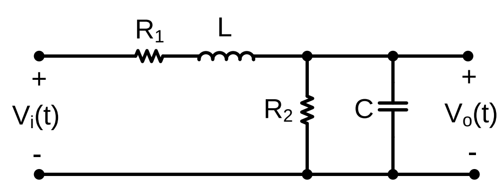


Fig. 3

3. For the circuit shown in **Fig. 3,** determine whether it is a low-pass, high-pass, band-pass, or band-reject (notch) filter from boundary value arguments. Determine the transfer function. (**No need to plot it**).

4. Draw the **Bode magnitude plot** for the transfer function given below, with 'ω' ranging from 0.1 rad/sec to 100 krad/sec.

$$H(j\omega) = 0.01\frac{(j\omega)^2}{(1+j\omega)(1+j0.1\omega)(1+j0.01\omega)}$$

5. A band–reject (notch) filter is shown in **Fig. 4**. Derive the expression of its transfer function H in the form $H(j\omega)=\frac{V_o(j\omega)}{V_i(j\omega} = K\left[\frac{(1+ja)}{(1+jb)}\right]$. Find out the expressions for the coefficients ***K***, ***a*** and ***b***. Determine the magnitudes of this transfer function at very low and very high frequencies from physical arguments. What is the resonance frequency of this circuit? What is the magnitude of the transfer function at this resonance frequency? Also calculate the level of rejection (in dB) at resonance frequency.

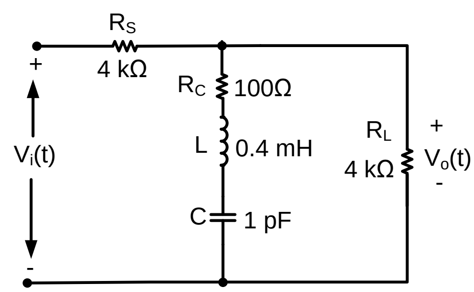


Fig. 4

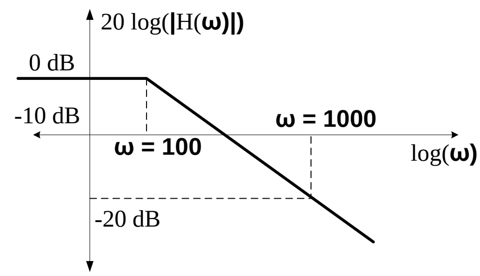
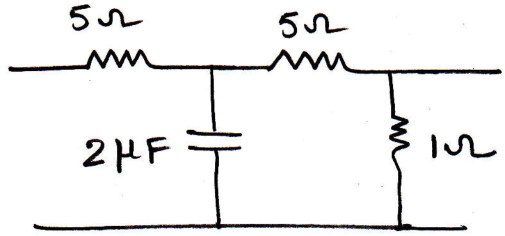


Fig. 5

6. Determine the voltage transfer function whose magnitude Bode plot is shown in **Fig. 5**. What type of filter is it? Design the complete circuit(s) using one 10 KΩ resistance and another passive element which can give such transfer function.

7. A coil of 15 Ω resistance and 0.75 H inductance is connected in series with a capacitor (C_1). The combination draws maximum current when a sinusoidal voltage source of 50 Hz is applied. Determine the value of C_1. A second capacitor (C_2) is now connected in parallel with the earlier combination. What should be the value for C_2 so that the combination will behave as purely resistive at 100 Hz? Calculate the current drawn by the combination if the applied voltage is 200 V.

(a)



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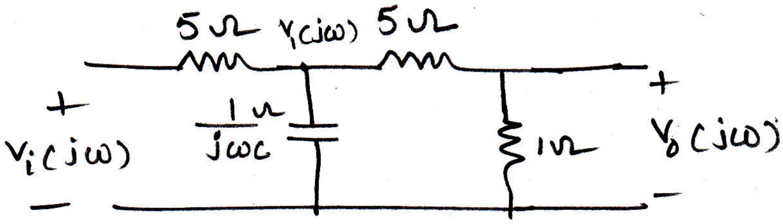


$\boxed{C = 2\,\mu F}$

$$V_1(j\omega) = \frac{(5+1) \parallel \left(\frac{1}{j\omega C}\right)}{5+\left[(5+1) \parallel \left(\frac{1}{j\omega C}\right)\right]} \times V_i(j\omega)$$

$$= \frac{1}{5+\left[\frac{6.\frac{1}{j\omega C}}{6+\frac{1}{j\omega C}}\right]} \times \left[\frac{6.\frac{1}{j\omega C}}{6+\frac{1}{j\omega C}}\right] \times V_i(j\omega)$$

$$= \frac{6(j\omega C)^{-1}}{30+\frac{5}{j\omega C}+\frac{6}{j\omega C}} \times V_i(j\omega) = \frac{6}{11+(j \times 30\omega C)} V_i(j\omega)$$

Now,

$$V_o(j\omega) = \frac{1}{5+1} \times V_1(j\omega) = \frac{1}{6} \times \frac{6}{11+(j\times 30\omega C)} V_i(j\omega)$$

$$\therefore \quad H(j\omega) = \frac{V_o(j\omega)}{V_i(j\omega)} = \frac{1}{11+j(30\omega C)} \text{ is the}$$

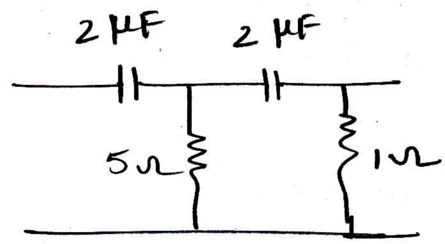
desired transfer function

$$H(j\omega) = \frac{1}{11+j(30\omega \times 2\times10^{-6})}$$

$$= \frac{1}{11+j(\omega 6\times10^{-5})}$$

(b)

P-2



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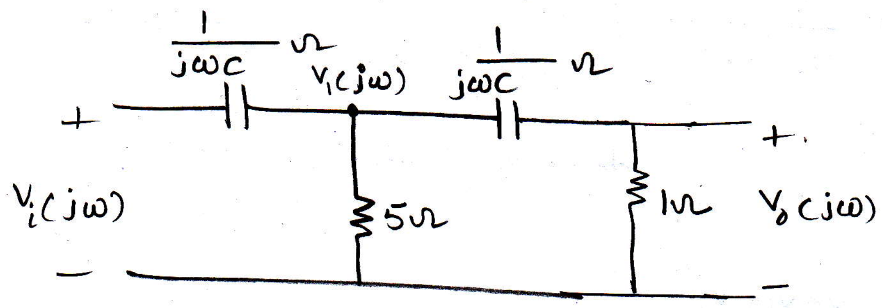


$\boxed{C = 2\,\mu F}$

$$V_1(j\omega) = \frac{5 \parallel \left(1 + \frac{1}{j\omega C}\right)}{\frac{1}{j\omega C} + \left[5 \parallel \left(1 + \frac{1}{j\omega C}\right)\right]} \times V_i(j\omega)$$

$$= \frac{1}{\left[\frac{1}{j\omega C} + \frac{5\left(1+\frac{1}{j\omega C}\right)}{5+1+\frac{1}{j\omega C}}\right]} \times \frac{5\left(1+\frac{1}{j\omega C}\right)}{5+1+\frac{1}{j\omega C}} \times V_i(j\omega)$$

$$= \frac{1}{\left[\frac{1}{j\omega C} + \frac{5+j5\omega C}{1+j6\omega C}\right]} \times \frac{5+j5\omega C}{1+j6\omega C} \times V_i(j\omega)$$

$$= \frac{5j\omega C\,(1+j\omega C)}{1+j\,6\omega C + j\,5\omega C - 5\omega^2 C^2}\, V_i(j\omega)$$

Now,

$$V_o(j\omega) = \frac{1}{\left(1+\frac{1}{j\omega C}\right)} \times V_1(j\omega)$$

$$= \frac{j\omega C}{1+j\omega C} \times \frac{5j\omega C\,(1+j\omega C)}{1+j\,11\omega C - 5\omega^2 C^2} \times V_i(j\omega)$$

$$\Rightarrow V_o(j\omega) = \frac{5(j\omega C)^2}{1 + j11\omega C - 5\omega^2 C^2} \times V_i(j\omega)$$

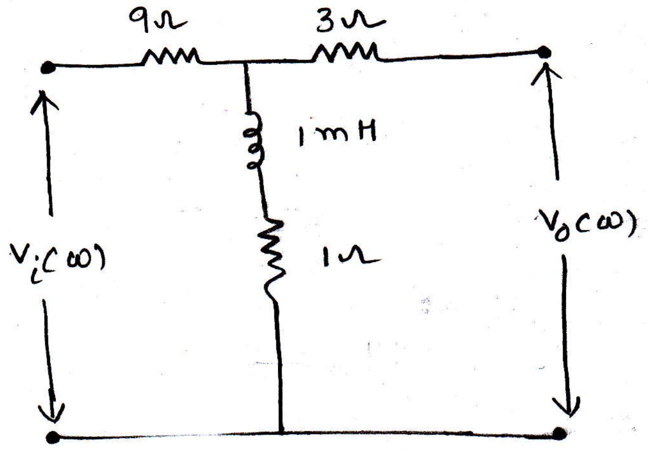
$$\therefore H(j\omega) = \frac{V_o(j\omega)}{V_i(j\omega)} = \frac{5(j\omega C)^2}{1 + j11\omega C - 5\omega^2 C^2}$$ is the

desired transfer function.

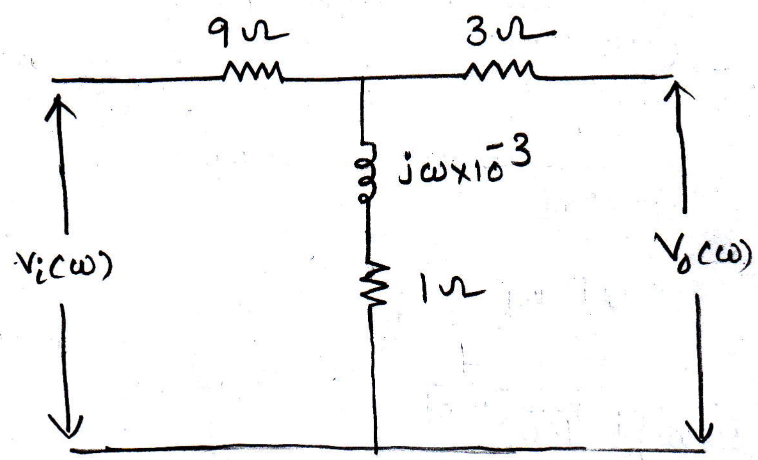
$$H(j\omega) = \frac{5.(j\omega \times 2\times10^{-6})^2}{1 + j.11\,\omega(2\times10^{-6}) - 5\omega^2(2\times10^{-6})^2}$$

$$= \frac{-2\times10^{-11}\,\omega^2}{1 + j(22\times10^{-6})\,\omega - (2\times10^{-11})\omega^2}$$

(c)



let us redraw the figure as shown below

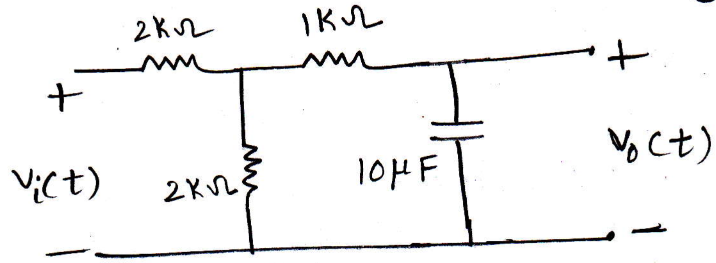


From the above figure,

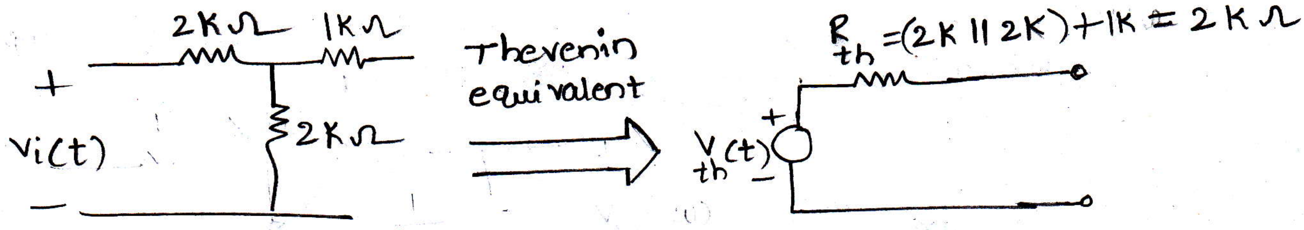
$$V_o(\omega) = \frac{1 + j\omega \times 10^{-3}}{9 + 1 + (j\omega \times 10^{-3})} \times V_i(\omega)$$

$$\Rightarrow H(\omega) = \frac{V_o(\omega)}{V_i(\omega)} = \frac{1 + j(\omega \times 10^{-3})}{10 + j(\omega \times 10^{-3})}$$

is the desired transfer function

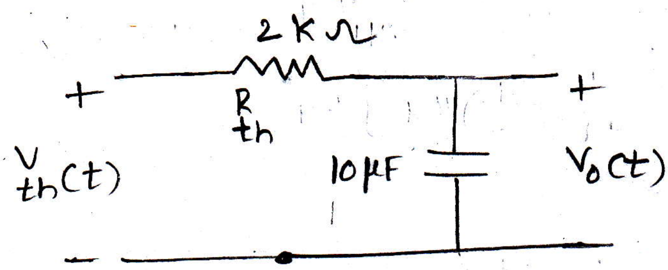


First let us consider a part of the circuit, as shown below

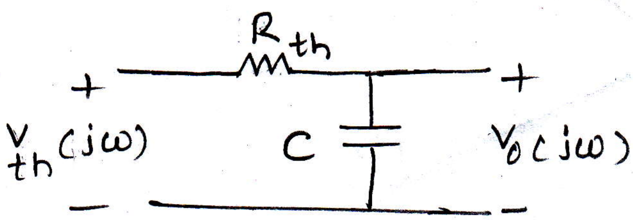


$$\text{where} \quad V_{th}(t) = \frac{2K}{2K+2K} \times V_i(t) = \frac{1}{2} V_i(t)$$

Then the actual circuit becomes,



To find the frequency response, let us redraw the above circuit as shown below.



where, $R_{th} = 2K\Omega$
$C = 10\,\mu F$

and $V_{th}(j\omega) = \frac{1}{2} V_i(j\omega)$

Now,

$$V_o(j\omega) = \frac{\frac{1}{j\omega C}}{\left(R_{th} + \frac{1}{j\omega C}\right)} \times V_{th}(j\omega) = \frac{1}{1+j\omega R_{th} C} \times \frac{V_i(j\omega)}{2}$$

$$\therefore \; H(j\omega) = \frac{V_o(j\omega)}{V_i(j\omega)} = \frac{0.5}{1+j(\omega/\omega_0)}$$

where,

$$\omega_0 = \frac{1}{R_{th}C} = \frac{1}{2\times10^{3}\times10\times10^{-6}} = \frac{1000}{20}$$

$$= 50 \text{ rad/sec.}$$

To plot the Bode plot,

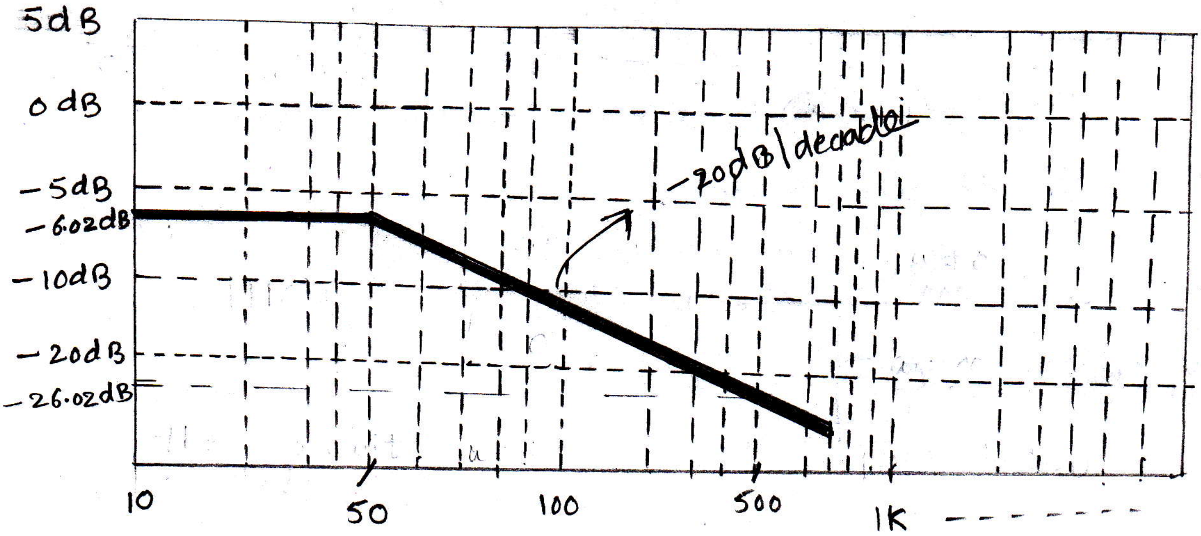
$$H(j\omega) = \frac{0.5}{1+j\left(\frac{\omega}{50}\right)} \Rightarrow |H(j\omega)| = \frac{0.5}{\sqrt{1+\left(\frac{\omega}{50}\right)^2}}$$

$$\Rightarrow H_{dB} = 20\log_{10}|H(j\omega)| = 20\log_{10}0.5 - 20\log_{10}\sqrt{1+\left(\frac{\omega}{50}\right)^2}$$

$$= -6.02 - 20\log_{10}\sqrt{1+\left(\frac{\omega}{50}\right)^2}$$

$$\Rightarrow H_{dB} = \begin{cases} -6.02 \text{ dB} & \text{for } \omega \ll 50 \text{ rad/sec} \\ -6.02 - 20\log_{10}\left(\frac{\omega}{50}\right) & \text{for } \omega \gg 50 \text{ rad/sec.} \end{cases}$$

The Bode magnitude plot (in dB) is,

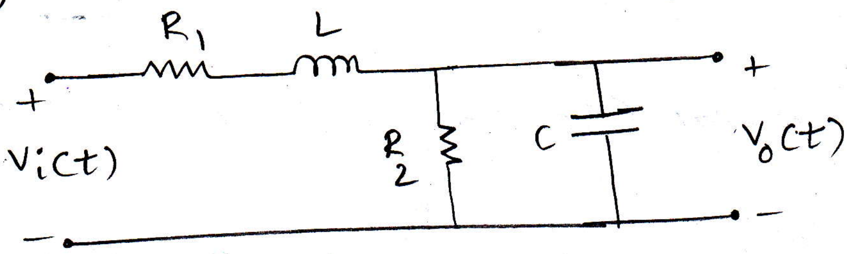


Since,

$$|H(j\omega)| = \begin{cases} 0.5, & \text{for extremely low frequencies} \\ 0, & \text{for extremely high frequencies,} \end{cases}$$

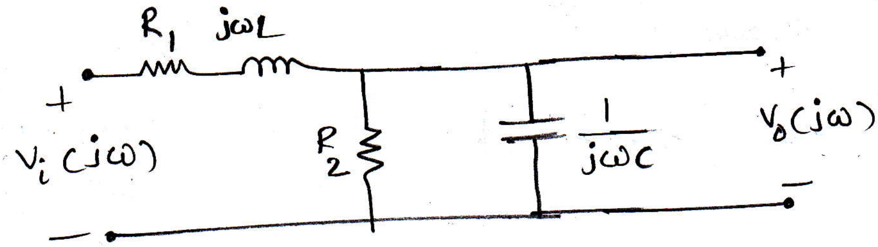
The given circuit works as a low pass filter.

SOLN. 3



Characterization of the filter can be done using the transfer function, $H(j\omega)$.

To find $H(j\omega)$,



Now,

$$V_o(j\omega) = \frac{\left(R_2 \parallel \frac{1}{j\omega C}\right)}{R_1 + j\omega L + \left(R_2 \parallel \frac{1}{j\omega C}\right)} \times V_i(j\omega)$$

$$\Rightarrow H(j\omega) = \frac{V_o(j\omega)}{V_i(j\omega)} = \frac{\left[R_2 \cdot \frac{1}{j\omega C} \Big/ \left(R_2 + \frac{1}{j\omega C}\right)\right]}{\left[R_1 + j\omega L + \frac{R_2 \cdot \frac{1}{j\omega C}}{R_2 + \frac{1}{j\omega C}}\right]}$$

$$\Rightarrow H(j\omega) = \frac{R_2}{(R_1 + j\omega L)(1 + j\omega C R_2) + R_2}$$

$$= \frac{R_2}{R_1 + j\omega C R_1 R_2 + j\omega L - \omega^2 L C R_2 + R_2}$$

$$\therefore \boxed{H(j\omega) = \frac{R_2}{(R_1 + R_2 - \omega^2 L C R_2) + j\omega(L + C R_1 R_2)}}$$

is the desired transfer function

Now,

when $\omega \rightarrow 0$ (ie., at very low frequencies)

$$H(j\omega)\Big|_{\omega \to 0} = \frac{R_2}{R_1 + R_2}, \text{ a constant}$$

when $\omega \rightarrow \infty$ (ie., at extremely high frequencies)

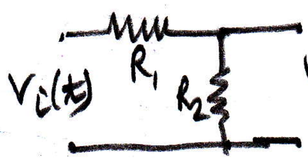
$$H(j\omega)\Big|_{\omega \to \infty} = 0$$

$\therefore$ Hence, The given filter is a low-pass filter.

Alternatively.

At low frequency, $\omega = 0$. $\therefore X_L = \omega L \to 0$ and $X_C \to \infty$.

$\therefore$ The circuit will be reduced to as follows:



$$\frac{V_o(t)}{V_i(t)} = \frac{R_2}{R_1 + R_2}$$

At high frequency, $\omega \to \infty$ $\therefore X_L \to \infty$ and $X_C \to 0$.

So, no $V_o(t)$ will be obtained.

$\therefore$ It will behave as a low pass filter.

SOLN. (4)

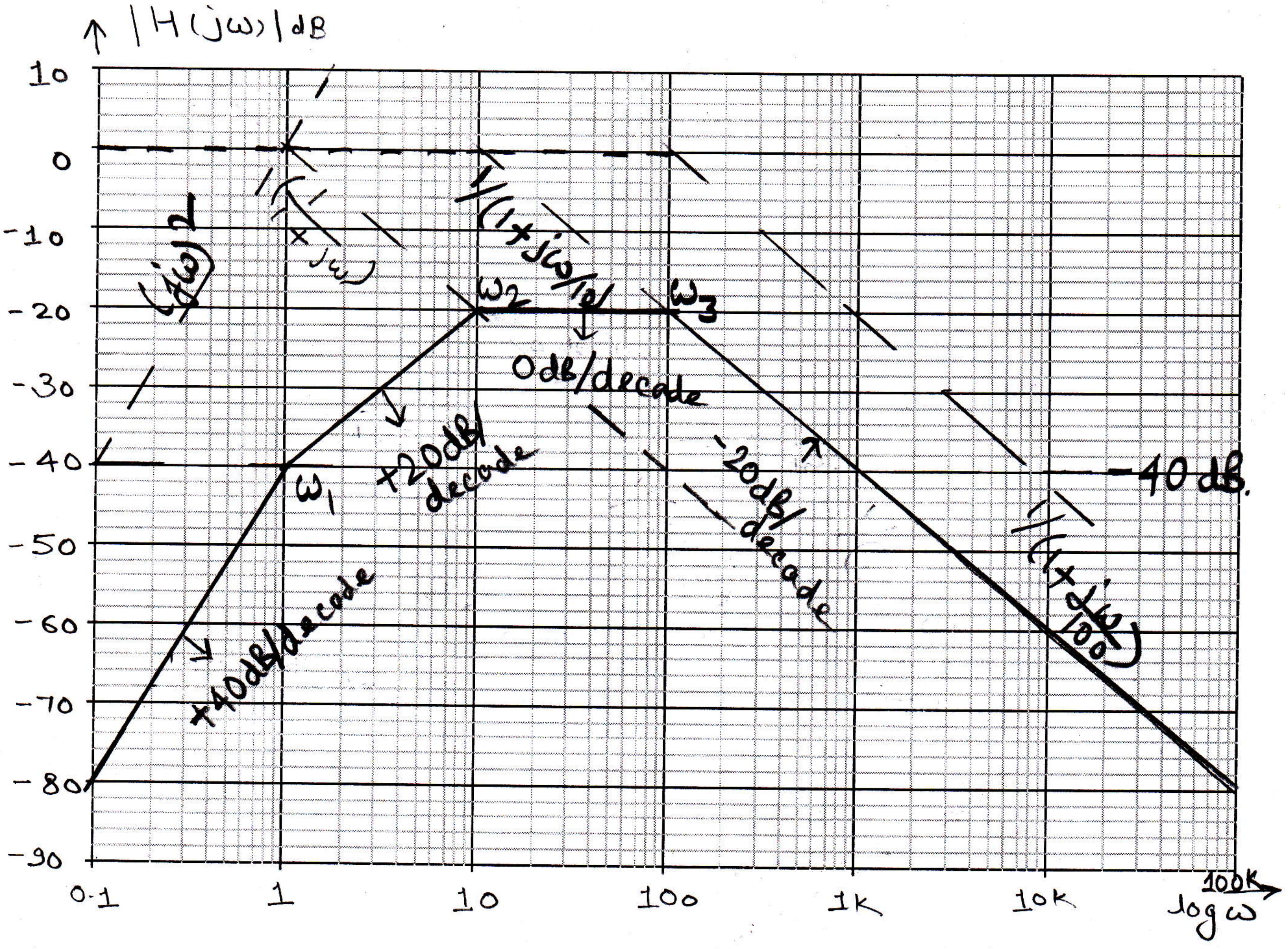
$$H(j\omega) = 0.01 \times \frac{(j\omega)^2}{(1+j\omega)(1+j0.1\omega)(1+j0.01\omega)}$$

$$\Rightarrow 20\log_{10}|H(j\omega)| = -40\text{ dB} + 40\log_{10}\omega - 20\log_{10}\sqrt{1+\omega^2}$$
$$- 20\log_{10}\sqrt{1+\left(\frac{\omega}{10}\right)^2} - 20\log_{10}\sqrt{1+\left(\frac{\omega}{100}\right)^2}.$$

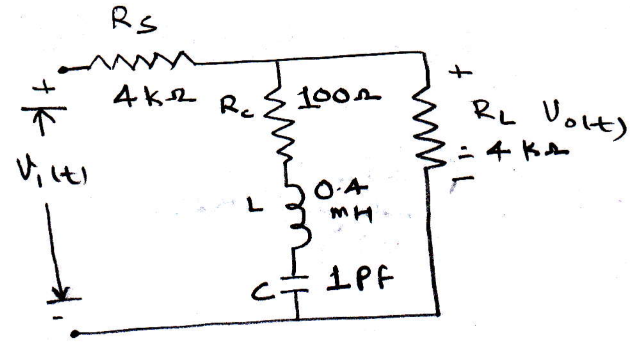
$\Rightarrow$ corner frequencies are

$\omega_1 = 1$ rad/sec $\quad \omega_2 = 10$ rad/sec. $\quad \omega_3 = 100$ rad/sec.

8/28/13 6cycle.gif (631×469)



$\because H(j\omega) = \dfrac{V_o(j\omega)}{V_i(j\omega)}$



let Z be the impedance of series R_c-L-c branch.

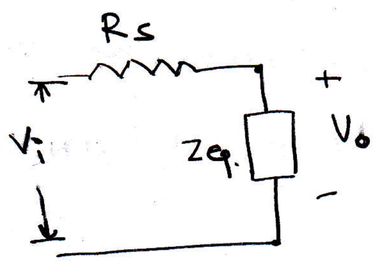
$$\Rightarrow Z = R_c + j\omega L + \frac{1}{j\omega C} = R_c + j\left(\omega L - \frac{1}{\omega C}\right)$$

$\Rightarrow$ let Z_{eq} be impedance of Z in parallel with R_L.

$$\Rightarrow Z_{eq} = \frac{Z \times R_L}{Z + R_L} = \frac{\left(R_c + j\left(\omega L - \frac{1}{\omega C}\right)\right) \times R_L}{R_c + j\left(\omega L - \frac{1}{\omega C}\right) + R_L}$$

$$\Rightarrow Z_{eq} = \frac{R_c R_L + j R_L\left(\omega L - \frac{1}{\omega C}\right)}{R_c + R_L + j\left(\omega L - \frac{1}{\omega C}\right)}$$

$$\because V_o(j\omega) = V_i(j\omega) \times \frac{Z_{eq}}{Z_{eq} + R_s}$$



$$\Rightarrow H(j\omega) = \frac{V_o(j\omega)}{V_i(j\omega)} = \frac{Z_{eq}}{Z_{eq} + R_s}$$

$$\Rightarrow H(j\omega) = \frac{R_c R_L + j R_L\left(\omega L - \frac{1}{\omega C}\right)}{R_c R_L + j R_L\left(\omega L - \frac{1}{\omega C}\right) + R_c R_s + R_L R_s + j R_s\left(\omega L - \frac{1}{\omega C}\right)}$$

$$= \frac{R_c R_L + j R_L\left(\omega L - \frac{1}{\omega C}\right)}{R_c R_L + R_c R_s + R_L R_s + j(R_s + R_L)\left(\omega L - \frac{1}{\omega C}\right)}.$$

$$= \left(\frac{R_c R_L}{R_c R_L + R_c R_s + R_L R_s} \right) \left(\frac{1 + \frac{j}{R_c}\left(\omega L - \frac{1}{\omega C}\right)}{1 + j \frac{R_s + R_L}{R_c R_L + R_c R_s + R_L R_s}\left(\omega L - \frac{1}{\omega C}\right)} \right).$$

$$= K \frac{(1+ja)}{(1+jb)} \text{ where.}$$

$$K = \frac{R_c R_L}{R_c R_L + R_L R_s + R_c R_s} \quad ; \quad a = \frac{1}{R_c}\left(\omega L - \frac{1}{\omega C}\right)$$

$$b = \frac{R_s + R_L}{R_c R_L + R_c R_s + R_L R_s}\left(\omega L - \frac{1}{\omega C}\right).$$

$\Rightarrow$ $\because$ at very low frequencies, $\omega \to 0$

$\therefore$ $X_L \to 0$; $X_c \to \infty$. So R_c–L–C branch will be open circuited. So

$$V_o(j\omega) = V_i(j\omega) \times \frac{R_L}{R_L + R_s}$$

$$\Rightarrow \underset{\omega \to 0}{H(j\omega)} = \frac{R_L}{R_L + R_s} = \frac{4K}{4K + 4K} = 0.5.$$

while at very high frequencies, $\omega \to \infty$.

$\therefore$ $X_L \to \infty$, $X_c \to 0$, so again R_c–L–C branch will be open circuited. so

$$\underset{\omega \to \infty}{H(j\omega)} = \frac{R_L}{R_L + R_s} = \frac{4K}{4K + 4K} = 0.5.$$

$$\because H(j\omega) = K\frac{(1+ja)}{(1+jb)}$$

$$= K\frac{(1+ja)}{(1+jb)}\frac{(1-jb)}{(1-jb)} = K\frac{(1+ja-jb+ab)}{1+b^2}$$

$$= K\left[\frac{1+ab+j(a-b)}{1+b^2}\right]$$

$$\therefore \text{Imag}[H(j\omega)] = K\times\frac{(a-b)}{(1+b^2)}$$

$\Rightarrow$ At resonance freq. $\text{Imag}(H(j\omega)) = 0$

$$\Rightarrow K\times\frac{(a-b)}{1+b^2} = 0 \quad \Rightarrow \quad a-b=0$$

$$\Rightarrow \frac{1}{R_c}\left(\omega L-\frac{1}{\omega C}\right) - \frac{R_s+R_L}{R_cR_L+R_cR_s+R_LR_s}\times\left(\omega L-\frac{1}{\omega C}\right)=0$$

$$\Rightarrow \left(\omega L-\frac{1}{\omega C}\right)\left(\frac{1}{R_c}-\frac{R_s+R_L}{R_cR_L+R_cR_s+R_LR_s}\right)=0$$

$$\Rightarrow \text{so} \quad \omega L-\frac{1}{\omega C}=0 \quad \Rightarrow \quad \omega=\frac{1}{\sqrt{LC}}$$

$$\Rightarrow \omega_0 = \frac{1}{\sqrt{0.4\times10^{-3}\times10^{-12}}} = 50\text{ M rad/sec}$$

$$\Rightarrow \boxed{f_0 = 7.96\text{ MHz}}$$

Transfer fn. at resonance frequency

$$H(\omega)\Big|_{\omega=\omega_0} = K \quad [\because a=b=0].$$

$\Rightarrow |H(j\omega)|_{\omega=\omega_0} = \dfrac{R_C R_L}{R_C R_S + R_L R_S + R_C R_L}$

$= \dfrac{100 \times 4K}{100 \times 4K + 4K \times 4K + 100 \times 4K} = 23.81 \times 10^{-3}$

$\Rightarrow$ Rejection of signal at resonant freq.

$= |H(j\omega)|_{\omega=\omega_0}$

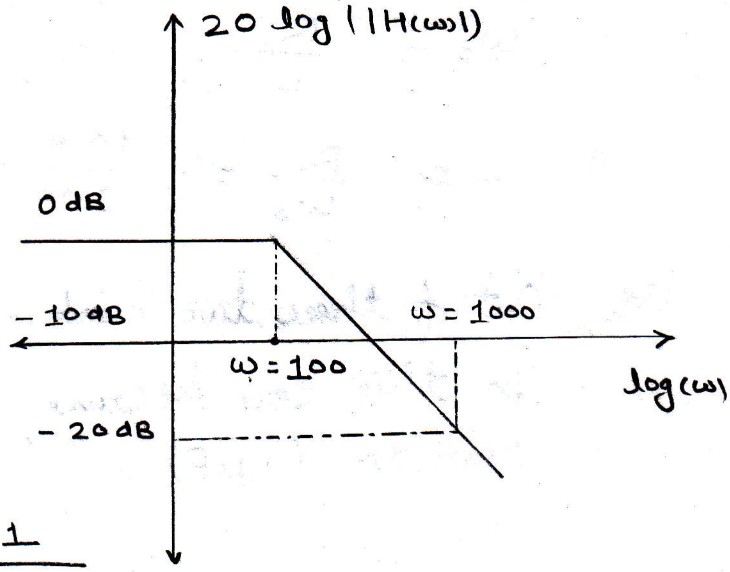
$= 23.81 \times 10^{-3}$

$= 20 \log_{10}(23.81 \times 10^{-3})$ dB

$= -32.46$ dB.

∵ corner frequency is at $\omega = 100$ rad/sec.

∵ slope of line for $\omega > 100 = -20$ dB/dec.

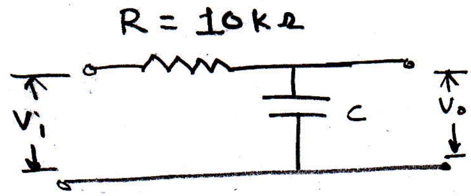


∴

$$H(j\omega) = \frac{1}{1 + j\frac{\omega}{100}} = \frac{1}{1 + j\frac{\omega}{\omega_0}}$$

This transfer function can be satisfied by two circuits (1) and (2).

For, the circuit (1),



(1)

$$H_1(j\omega) = \frac{V_o(j\omega)}{V_i(j\omega)}$$

$$= \frac{1/j\omega C}{R + \frac{1}{j\omega C}}$$

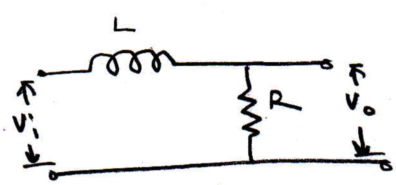
$$= \frac{1}{1 + j\omega RC} = \frac{1}{1 + j\frac{\omega}{\omega_0}}$$

$H_1(j\omega) = H(\omega)$ gives,

$$\omega_0 = \frac{1}{RC} \qquad \because \omega_0 = 100,\ R = 10k\Omega$$

$$\therefore C = \frac{1}{\omega_0 R} = 1\ \mu F.$$

Also,



(2)

$$\Rightarrow H_2(j\omega) = \frac{R}{R + j\omega L}$$

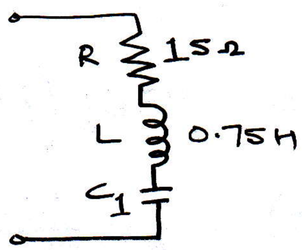
$$= \frac{1}{1 + j\frac{\omega L}{R}} = \frac{1}{1 + j\frac{\omega}{\omega_0}}$$

$H_2(\omega) = H(\omega)$ gives,

$$\Rightarrow \quad \omega_o = \frac{R}{L}$$

$$\Rightarrow \quad L = \frac{R}{\omega_o} = \frac{10\,k}{100} = 100\,H$$

Note: Out of these two combinations, R-C circuit is preferred in this case because of practical value of the capacitor ($1\,\mu F$).



∵ This is a series RLC circuit

∵ frequency of i/p signal = 50Hz.

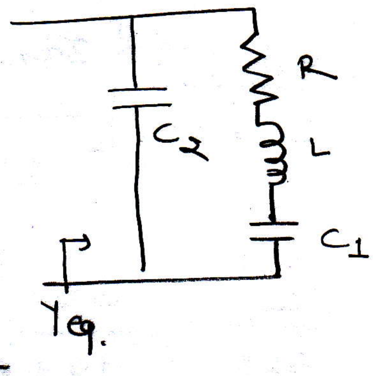
∵ Max. current is drawn at resonance freq. at which.

$$\omega L = \frac{1}{\omega C_1} \Rightarrow C_1 = \frac{1}{\omega^2 L}.$$

$$\Rightarrow C_1 = \frac{1}{(2\pi \times 50)^2 \times 0.75} = 13.52\ \mu F.$$

∵ now capacitor is connected in parallel. so

$$\Rightarrow Y_{eq} = Y_1 + Y_2.$$



$$\because Y_1 = \frac{1}{R + j\omega L + \frac{1}{j\omega C_1}}$$

$$= \frac{1}{R + j\left(\omega L - \frac{1}{\omega C_1}\right)} = \frac{R - j\left(\omega L - \frac{1}{\omega C_1}\right)}{R^2 + \left(\omega L - \frac{1}{\omega C_1}\right)^2}.$$

$$Y_2 = j\omega C_2.$$

$$\Rightarrow Y_{eq} = \frac{R - j\left(\omega L - \frac{1}{\omega C_1}\right)}{R^2 + \left(\omega L - \frac{1}{\omega C_1}\right)^2} + j\omega C_2.$$

$$= \frac{R + j\left[\omega C_2 \times \left\{R^2 + \left(\omega L - \frac{1}{\omega C_1}\right)^2\right\} - \left(\omega L - \frac{1}{\omega C_1}\right)\right]}{R^2 + \left(\omega L - \frac{1}{\omega C_1}\right)^2}$$

$\because$ at $f = 100$ Hz. imaginary part is zero so that the combination will be purely resistive.

$$\Rightarrow \omega C_2\left(R^2+\left(\omega L-\frac{1}{\omega C_1}\right)^2\right)-\left(\omega L-\frac{1}{\omega C_1}\right)=0$$

$$\Rightarrow \omega C_2-\frac{1}{R^2+\left(\omega L-\frac{1}{\omega C_1}\right)^2}\left[\omega L-\frac{1}{\omega C_1}\right]=0.$$

$$\Rightarrow \omega C_2 = \frac{\omega L-\frac{1}{\omega C_1}}{R^2+\left(\omega L-\frac{1}{\omega C_1}\right)^2}$$

$$\Rightarrow C_2 = \frac{1}{\omega}\left[\frac{\omega L-\frac{1}{\omega C_1}}{R^2+\left(\omega L-\frac{1}{\omega C_1}\right)^2}\right]$$

$$= \frac{1}{2\pi\times 100}\left[\frac{2\pi\times 100\times 0.75-\frac{1}{2\pi\times 100\times 13.5\times 10^{-6}}}{15^2+\left(2\pi\times 100\times 0.75-\frac{1}{2\pi\times 100\times 13.5\times 10^{-6}}\right)^2}\right]$$

$$= \frac{1}{628}\times\frac{353.047}{124.86\times 10^3}$$

$$= 4.5\ \mu F.$$

At $f = 100$ Hz,

$$Y_{eq} = \frac{R}{R^2+\left(\omega L-\frac{1}{\omega C_1}\right)^2} = \frac{15}{124.86\times 10^3} = 120.13\times 10^{-6}$$

$$\Rightarrow I = VY_{eq.} = 200\times 120.13\times 10^{-6}$$

$$= 24.026\ mA.$$