18th August, 2016

Home Assignment – 3

Transient Response

1. For the network shown in **Fig. 1**, find the voltage across capacitor (initially uncharged) as a function of time (t > 0).

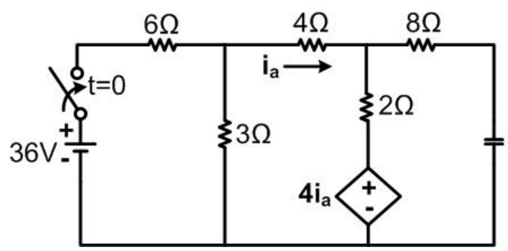


Fig. 1

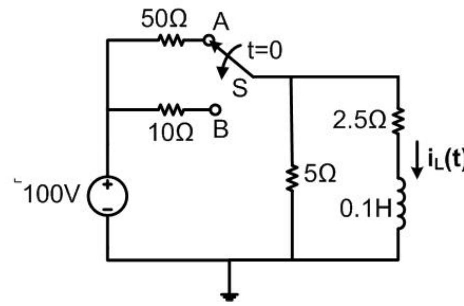


Fig. 2

2. In **Fig. 2**, the switch '**S**' was at position '**A**' for a long time, and changes its position from '**A**' to '**B**' at t = 0. Sketch and label $\mathbf{i_L(t)}$, and determine the time when it becomes equal to 5 A.

3. In **Fig. 3**, the switch '$\mathbf{S_1}$' was open and '$\mathbf{S_2}$' was closed for a long time. At t = 0, '$\mathbf{S_1}$' is closed and '$\mathbf{S_2}$' is opened simultaneously. Determine $\mathbf{V_c(t)}$ for t = 0, τ, 2 τ, 5 τ and 10 τ, where '**τ**' is the time constant of the circuit for t ≥ 0. Also determine the time constant '**τ**'.

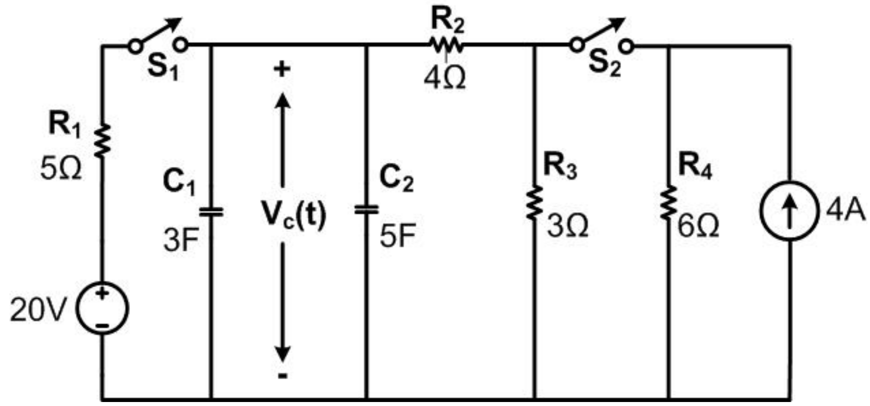


Fig. 3

Sinusoidal Response

4. For the circuit shown in **Fig. 4,** determine the voltage '$\mathbf{v_c}$'. Assume ω = 10,000 rad/s.

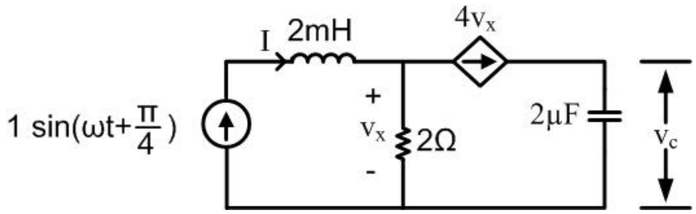


Fig. 4

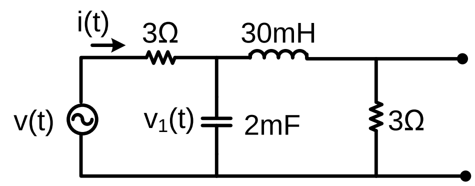


Fig. 5

5. The circuit shown in **Fig. 5** is operating in the sinusoidal steady state. The voltage across the capacitor is given by $v_1(t) = 5\angle 23.13°$ V at ω = 200 rad/s. Determine $i(t)$ and $v(t)$.

6. The circuit shown in **Fig. 6**, is a Wheatstone bridge that is used to find the reactance of an unknown element (L or C). The circuit is adjusted by changing R_1 and R_2 until v_{ab} becomes zero.
   a) Assuming that the circuit is balanced (i.e., v_{ab} = 0), determine X_4 in terms of the circuit elements.
   b) If C_3 = 5 µF, L_3 = 0.1 H, R_1 = 100 Ω, R_2 = 1 Ω, $v_S(t) = 24\sin(2000t)$ V and v_{ab} = 0, what is the reactance of the unknown element? Is it a capacitor or an inductor? What is its value?
   c) Is any particular measurement frequency prohibited? Why? Justify.

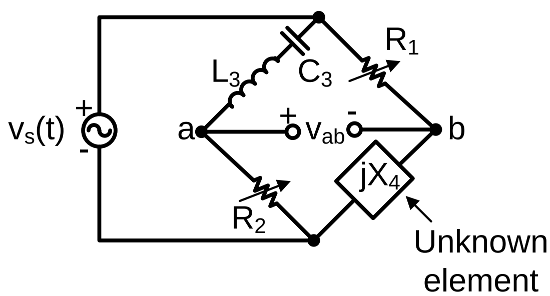


Fig. 6

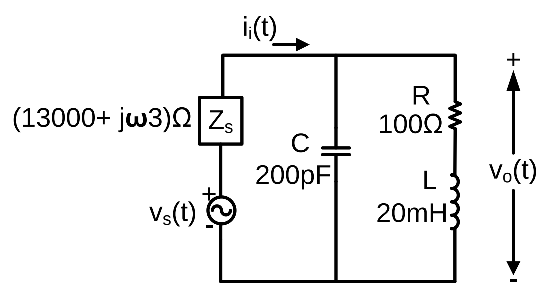
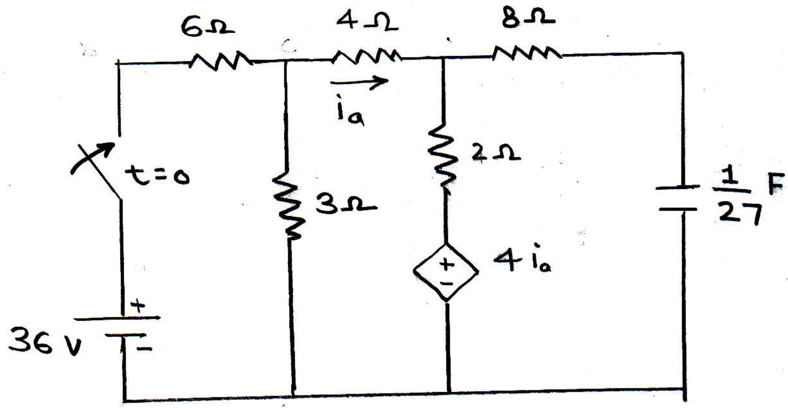


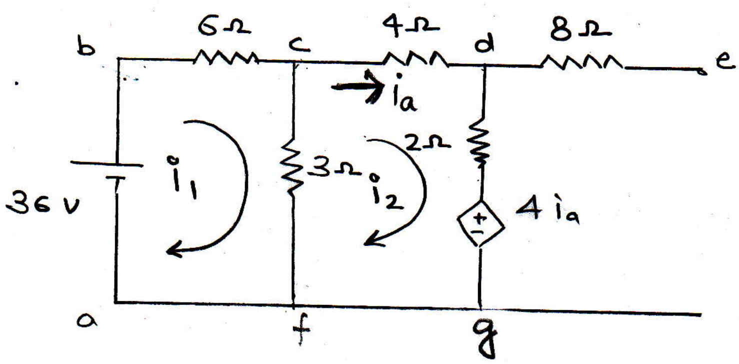
Fig. 7

7. Determine the angular frequency '**ω**' so that the current $\boldsymbol{i_i(t)}$ and the voltage $\boldsymbol{v_o(t)}$ in the circuit shown in **Fig. 7** are in same phase.



To find voltage across capacitor, equivalent Thevenin network is constructed after switch is closed.

1. Calculation of V_{Th} :-



here $i_a = i_2$ —(i)

Applying KVL in abcfa

$-36 + 6i_1 + 3(i_1 - i_2) = 0 \quad \Rightarrow -36 + 9i_1 - 3i_2 = 0$ —(ii)

Applying KVL in fcdgf.

$3(i_2 - i_1) + 4i_2 + 2i_2 + 4i_2 = 0$

$\Rightarrow 13i_2 - 3i_1 = 0 \quad \Rightarrow i_1 = \frac{13}{3} i_2$ —(iii)

Substituting value of i_1, from (iii) into (ii)

$\Rightarrow -36 + 9 \times \frac{13}{3} i_2 - 3i_2 = 0$

$\Rightarrow -36 + 39i_2 - 3i_2 = 0 \quad \Rightarrow 36i_2 = 36$

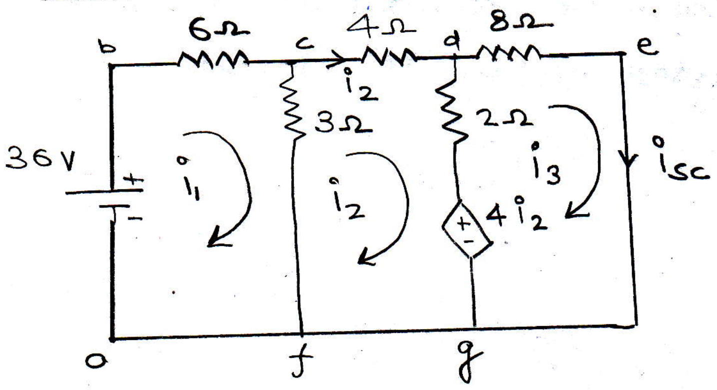
$\Rightarrow i_2 = 1$ A. —(iv)

so

$$V_{oc} = V_{eg} = V_{dg} = 2i_2 + 4i_2$$
$$= 6i_2$$
$$= 6\ V.$$

$\Rightarrow V_{Th} = 6\ V.$ —(v)

Calculation of R_{Th}:-



here $i_{sc} = i_3.$

applying KVL in loop abcfa

$\Rightarrow -36 + 6i_1 + 3(i_1 - i_2) = 0$

$\Rightarrow 9i_1 - 3i_2 = 36 \quad \Rightarrow i_1 = \dfrac{36 + 3i_2}{9}$ —(vi)

applying KVL in loop fcdgf

$\Rightarrow 3(i_2 - i_1) + 4i_2 + 2(i_2 - i_3) + 4i_2 = 0$

$\Rightarrow -3i_1 + 13i_2 - 2i_3 = 0$ —(vii)

Applying KVL in loop gdeg

$\Rightarrow -4i_2 + 2(i_3 - i_2) + 8i_3 = 0$

$\Rightarrow -6i_2 + 10i_3 = 0 \quad \Rightarrow i_3 = 0.6\ i_2$ —(viii)

$\Rightarrow$ on substituting the values of i_1 & i_3 from (vi) & (viii) in eq. (vii) we obtain,

$$\Rightarrow -3\times\left(\frac{36+3i_2}{9}\right)+13i_2-2\times0.6\,i_2=0$$

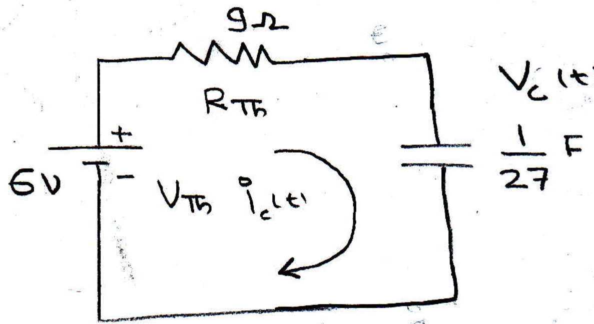
$$\Rightarrow -12-i_2+13i_2-1.2\,i_2=0$$

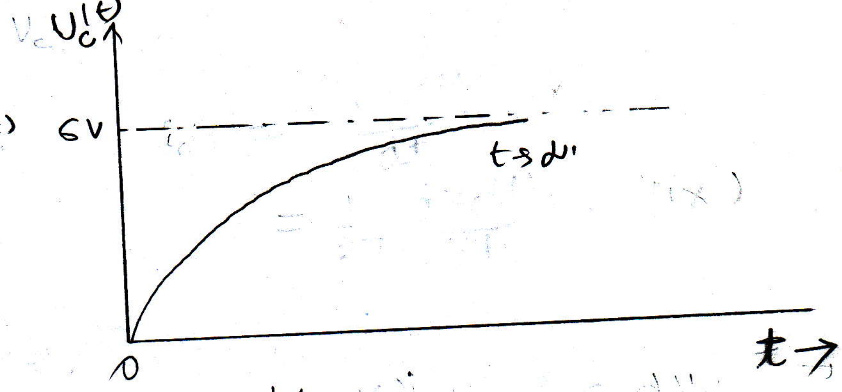
$$\Rightarrow 10.8\,i_2=12 \Rightarrow i_2=\frac{12}{10.8}\text{ A.}$$

So $i_{sc}=i_3=0.6\times\frac{12}{10.8}=\frac{2}{3}$ A.

$$\Rightarrow R_{Th}=\frac{V_{Th}}{i_{sc}}=\frac{6\times3}{2}=9\,\Omega.$$

So Thevenin equivalent circuit is:-





$$\therefore V_c(t)=V(\infty)+[V(0+)-V(\infty)]e^{-t/\tau}$$

$\because$ at $t=0^+$ capacitor is discharged, so $V(0+)=0$

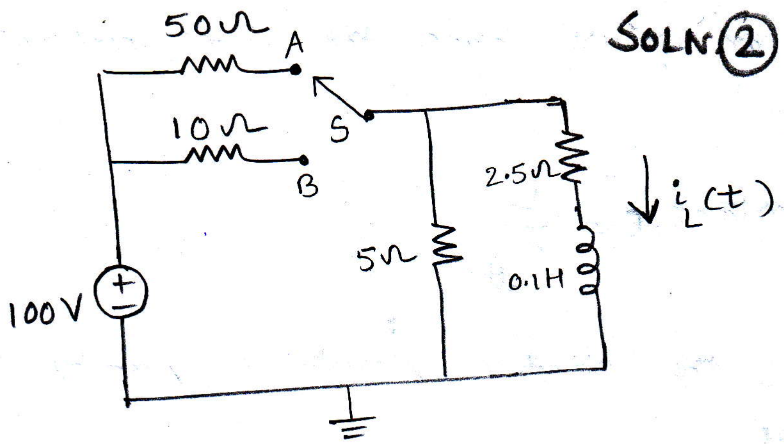
as, $t\to\infty$ capacitor is charged to 6 V. so $V(\infty)=6$ V.

$\Rightarrow$ Time Constant $\tau=RC=\frac{9}{27}=\frac{1}{3}$ sec

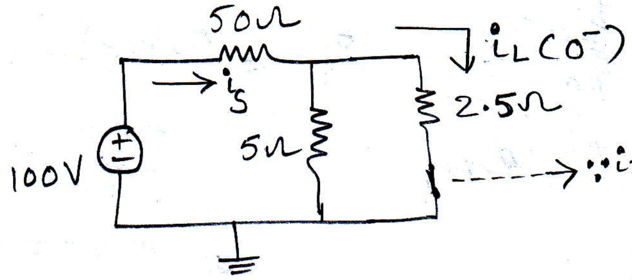
$$\Rightarrow V_c(t)=6+[0-6]e^{-3t}$$

$$=6(1-e^{-3t})\text{ V.}$$

SOLN (2) P-4



* given switch "S" was at position "A" for a long time. Then the circuit becomes,



(∵ inductor gets short circuited when steady state has been attained. $i_L(0^-)$ is the initial current through the inductor)

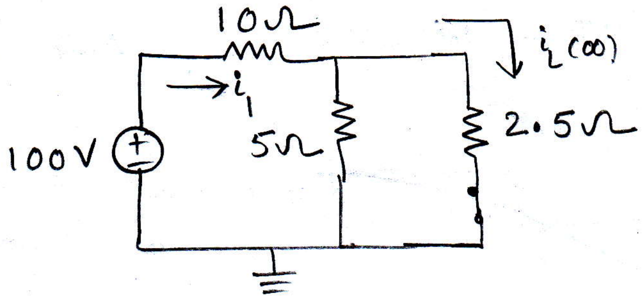
Then,

$$i_S = \frac{100}{50+[5 \parallel 2.5]} = \frac{100}{50+\left[\frac{5\times 2.5}{5+2.5}\right]} = \frac{100}{50+1.666}$$

$$= 1.935 \text{ A}$$

$$\therefore i_L(0^-) = i_S \times \frac{5}{5+2.5} = 1.935 \times \frac{5}{7.5} = 1.2903 \text{ A}$$

* At $t=0$ switch changes from "A" to "B". Then for $t\to\infty$, the circuit reduces to,



(∵ $i_L(\infty)$ being final current through inductor)

Then,

$$i_1 = \frac{100}{10+[5 \parallel 2.5]} = \frac{100}{10+\left[\frac{5\times 2.5}{5+2.5}\right]} = \frac{100}{10+1.666}$$

$$= 8.5715 \text{ A}$$

Now,

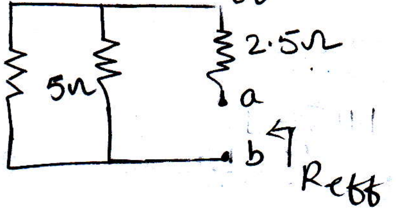
$$i_L(\infty) = i_1 \times \frac{5}{5+2.5} = 8.5715 \times \frac{5}{7.5} = 5.7143 \text{ A}$$

To find $i_L(t)$ we must have the "time-constant" of the circuit

$$\text{time-constant}, \tau = \frac{L}{R_{eff}}$$

where R_{eff} is the effective resistance seen by the inductor, $L = 0.1H$.

To compute τ we have to determine thevinin resistance across the inductor L. For that purpose 100 V source is short circuited and R_{eff} is determined across a-b.



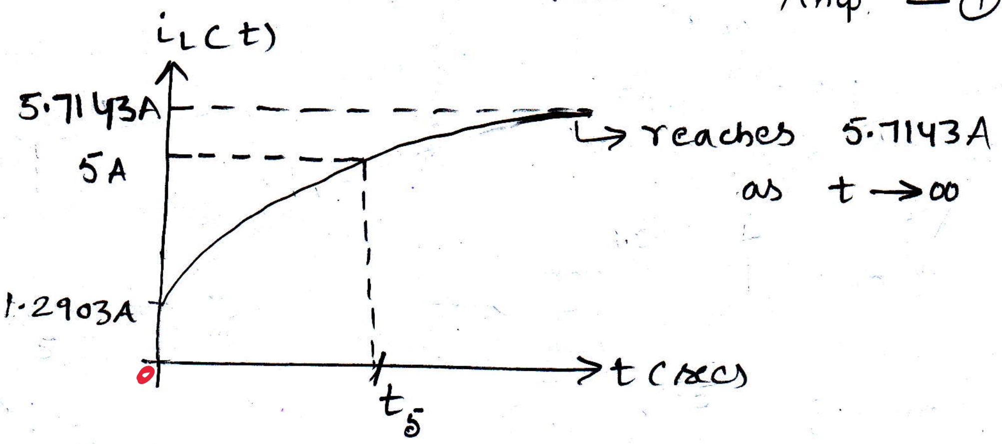
$$R_{eff} = [10 \| 5] + 2.5$$
$$= \frac{5 \times 10}{5+10} + 2.5$$
$$= 5.833\,\Omega$$

Then, $\tau = \dfrac{0.1\,H}{5.833\,\Omega} = 17.143\text{ ms}$

The current through the inductor is,

$$i_L(t) = i_{final} + [i_{initial} - i_{final}]\, e^{-t/\tau}$$
$$= i_L(\infty) + [i_L(0^-) - i_L(\infty)]\, e^{-t/\tau}$$
$$= 5.7143 + [1.2903 - 5.7143]\, e^{-\frac{t}{0.017143}}$$

$$\Rightarrow i_L(t) = 5.7143 - 4.424\, e^{-58.33t} \text{ Amp} \quad \text{— (1)}$$



let t_5 be the time at which $i_L(t) = 5A$

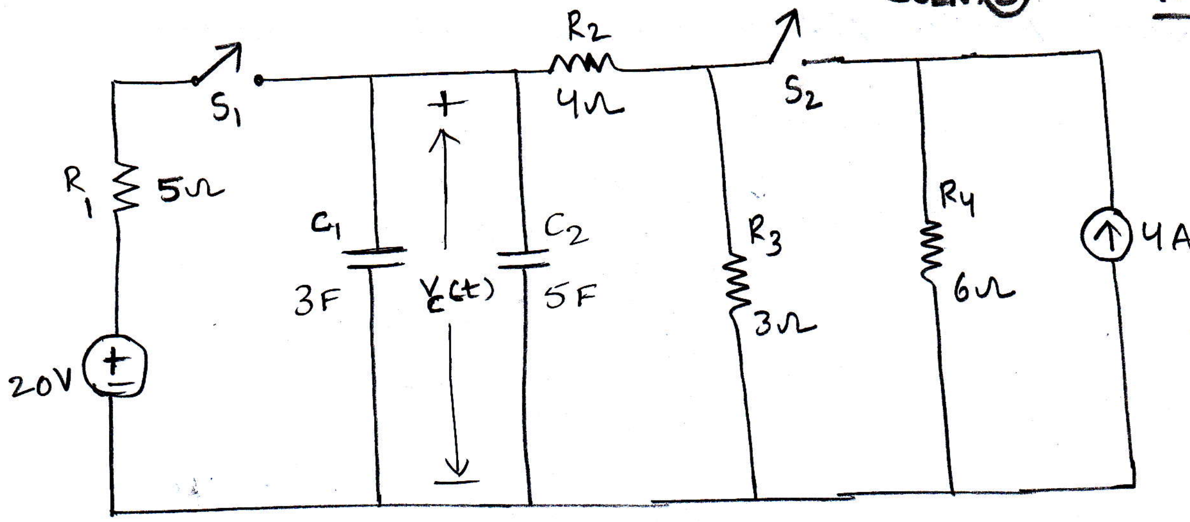
Then from ① we can write that,

$$5 = 5.7143 - 4.424\, e^{-58.33\, t_5}$$

$$\Rightarrow e^{-58.33 t_5} = \frac{5.7143 - 5}{4.424} = 0.1614$$

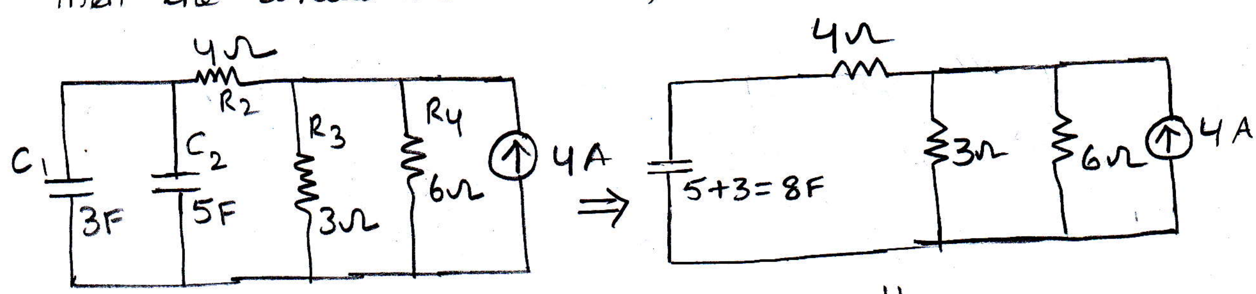
$$\Rightarrow t_5 = -\frac{1}{58.33} \ln(0.1614) = 31.26 \text{ ms}$$

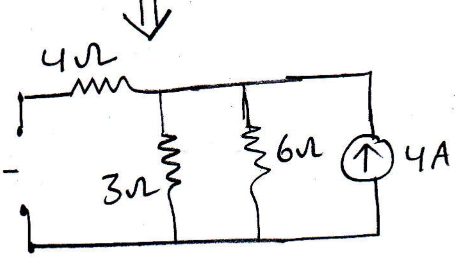
$\therefore$ At $t = 31.26$ ms, current through the inductor is 5A.



* given "S1" was open and "S2" was closed for a long time.

Then the circuit reduces to,





(∵ under initial conditions the capacitor gets open circuited when steady state is reached)

Hence,

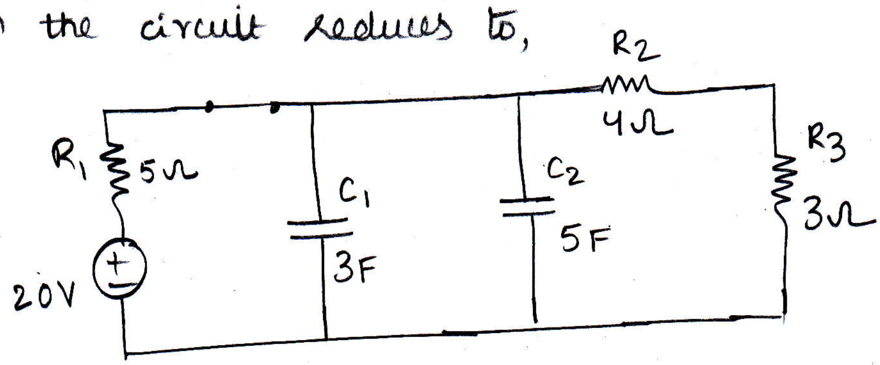
initial voltage across the capacitors is,

$$v_c(0^-) = 4A \times [3 \parallel 6]\Omega$$

$$= 4 \times \left[\frac{3 \times 6}{3+6}\right] = 4 \times \frac{18}{9} = 8\,V.$$

* At $t = 0$, "S1" is closed and "S2" is open

Then the circuit reduces to,



when steady state is reached, final voltage across the capacitor becomes,

$$v_c(\infty) = \frac{R_2 + R_3}{R_1 + R_2 + R_3} \times 20V$$

$$= \left(\frac{4+3}{5+4+3}\right) \times 20V$$

$$= \frac{7}{12} \times 20 = 11.67\ V.$$

Now,

$$V_c(t) = V_{final} + \left[V_{initial} - V_{final} \right] e^{-t/\tau}$$

$$= v_c(\infty) + \left[v_c(0^-) - v_c(\infty) \right] e^{-t/\tau}$$

$$= 11.67 + \left[8 - 11.67 \right] e^{-t/\tau}$$

$$= 11.67 - 3.67\, e^{-t/\tau}$$ Volts ; "τ being time constant for "$t \geq 0$"

at $t=0$,

$$V_c(0) = 11.67 - 3.67 = 8\ V$$ [Alternatively, $V_c(0^+) = V_c(0^-) = 8V$].

at $t=\tau$,

$$V_c(\tau) = 11.67 - 3.67 e^{-1} = 11.67 - [3.67 \times 0.3678]$$
$$= 10.319\ V.$$

at $t = 2\tau$,

$$V_c(2\tau) = 11.67 - 3.67 e^{-2} = 11.67 - [3.67 \times 0.1353]$$
$$= 11.173\ V$$

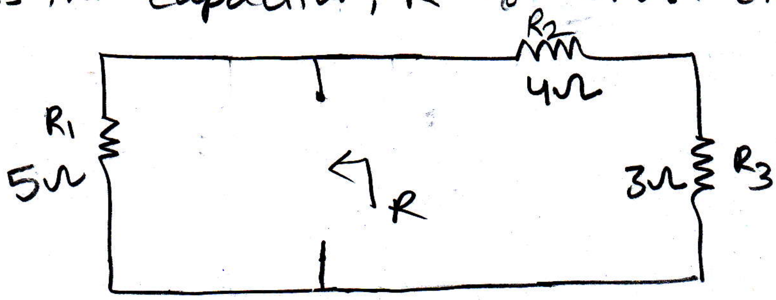
at $t = 5\tau$,

$$V_c(5\tau) = 11.67 - 3.67 e^{-5} = 11.67 - [3.67 \times 0.00674]$$
$$= 11.645\ V$$

at $t = 10\tau$,

$$V_c(10\tau) = 11.67 - 3.67 e^{-10} = 11.669\ V$$

To find the circuit time constant 'τ' for $t \geq 0$, we have to find the Thevinin's equivalent resistance across the capacitor, R by short circuiting 20V voltage source



($\because$ for $t \geq 0$)

$$R = (R_1 \parallel [R_2 + R_3])$$

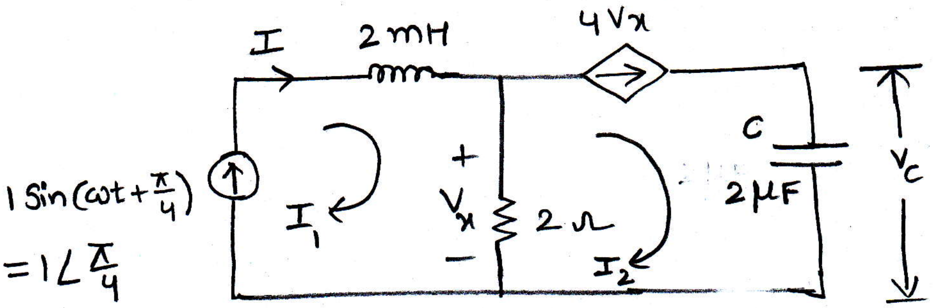
$$= (5 \parallel [4+3])$$

$$= \left[\frac{5 \times 7}{5+7}\right] = \frac{35}{12}$$

$$\tau = RC = \frac{35}{12} \times 8 = \frac{280}{12} = 23.33\ s$$

$\therefore$ for $t \geq 0$, circuit time constant $= 23.33$ sec.

SOLN. 4 P-10



$\omega = 10{,}000$ rad/s

For the mesh currents shown in the figure, we can write,

$$I_1 = I = 1\angle\frac{\pi}{4} \text{ Amp} \;\&\; I_2 = 4 V_x \text{ Amp}$$

where,

$$V_x = 2(I_1 - I_2) = 2\left(1\angle\frac{\pi}{4} - 4V_x\right)$$

$$\Rightarrow V_x = 2\angle\frac{\pi}{4} - 8V_x$$

$$\Rightarrow V_x + 8V_x = 2\angle\frac{\pi}{4}$$

$$\Rightarrow V_x = \frac{2}{9}\angle\frac{\pi}{4} \text{ Volts}$$

Then,

$$I_2 = 4V_x = 4\times\frac{2}{9}\angle\frac{\pi}{4} = \frac{8}{9}\angle\frac{\pi}{4} \text{ Amp}$$

Now from the figure,

$$V_c = \frac{1}{j\omega C}\times I_2 = \frac{1}{j\times 10^4\times 2\times 10^{-6}}\times\frac{8}{9}\angle\frac{\pi}{4}$$

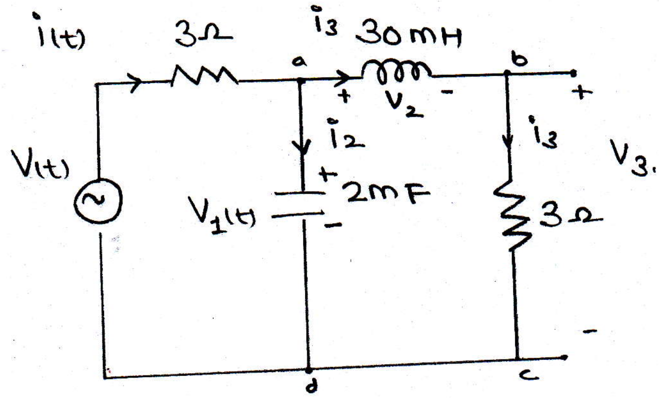
$$= \frac{8}{9}\times\frac{100}{2}\angle-\frac{\pi}{4}$$

$$= \frac{400}{9}\angle-\frac{\pi}{4} \text{ volts}$$

$$= 44.44\sin(\omega t - \pi/4)$$

$V_1(t) = 5\angle 23.13^\circ$

$\omega = 200$ rad/s.



∴ in loop abcd

$\Rightarrow V_1 = V_2 + V_3$

$V_2 = j\omega L i_3 = j6 i_3$; $V_3 = 3i_3$.

$\Rightarrow V_1 = i_3(3+j6) = 6.7\angle 63.4^\circ\, i_3$.

$\Rightarrow i_3 = \dfrac{5\angle 23.13^\circ}{6.7\angle 63.4^\circ} = 0.746\angle -40.27^\circ$ A.

$\Rightarrow i_2 = \dfrac{V_1}{\frac{1}{j\omega c}} = j0.4 \times 5\angle 23.13^\circ$

$= 2\angle 113.13^\circ$

$\Rightarrow \therefore i = i_2 + i_3 = 2\angle 113.13^\circ + 0.746\angle -40.27^\circ$

$i = -0.7856 + j1.8392 + 0.568 - j0.482$

$= -0.2176 + j1.3572$

$i = 1.3745\angle 99.1^\circ$ A.

$\Rightarrow V = 3i + V_1$

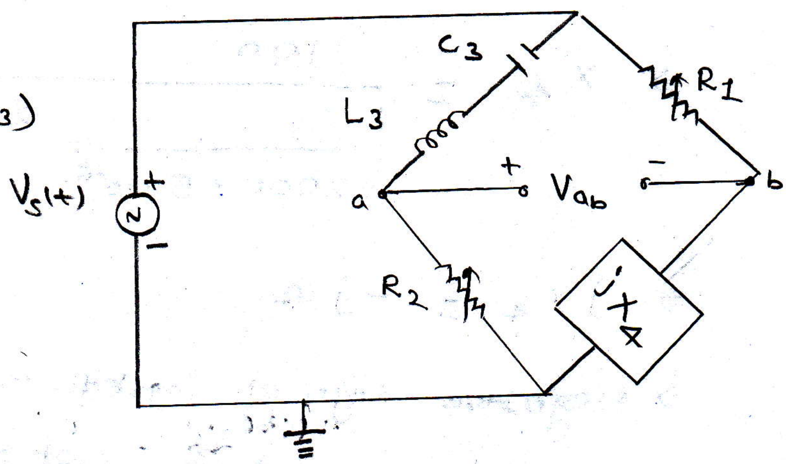
$= -0.6528 + j4.0716 + 4.598 + j1.964$

$= 3.9452 + j6.0356$

$V = 7.21\angle 56.8^\circ$. V.

a) ∵ circuit is balanced.

so $V_{ab} = V_a - V_b = 0$ —(i)



$\Rightarrow V_a = \dfrac{R_2 \times V_s(t)}{R_2 + j(X_{L_3} - X_{C_3})}$

where $X_{L_3} = \omega L_3$

$X_{C_3} = \dfrac{1}{\omega C_3}$

$\Rightarrow V_b = \dfrac{jX_4 \times V_s(t)}{R_1 + jX_4}$

⇒ substituting values of V_a & V_b in eq. (i)

$$\Rightarrow \frac{R_2 \times V_s(t)}{R_2 + j(X_{L_3} - X_{C_3})} - \frac{jX_4 \times V_s(t)}{R_1 + jX_4} = 0$$

$$\Rightarrow \frac{R_2}{R_2 + j(X_{L_3} - X_{C_3})} = \frac{jX_4}{R_1 + jX_4}$$

$$\Rightarrow R_1R_2 + \cancel{jX_4R_2} = \cancel{jX_4R_2} - X_4(X_{L_3} - X_{C_3})$$

$$\Rightarrow R_1R_2 = -X_4(X_{L_3} - X_{C_3})$$

$$\Rightarrow X_4 = \frac{R_1R_2}{(X_{C_3} - X_{L_3})} \quad \text{— ii)}$$

$$\Rightarrow X_4 = \frac{R_1R_2}{\left(\frac{1}{\omega C_3} - \omega L_3\right)}$$

b) $C_3 = 5\mu F$, $L_3 = 0.1H$, $R_1 = 100\Omega$

$R_2 = 1\Omega$

$V_s(t) = 24 \sin(2000t)$ V ∵ $\omega = 2000$ rad/s

$$\Rightarrow X_4 = \frac{100}{\left(\frac{1}{2000 \times 5 \times 10^{-6}} - 2000 \times 0.1\right)} = -1$$

$\Rightarrow jX_4 = -j\,\Omega$.

⇒ Negative sign of impedance mean that unknown circuit element is capacitor (c)

$$\Rightarrow X_4 = \frac{1}{\omega c} \quad \Rightarrow c = \frac{1}{X_4 \omega} = 500\,\mu F.$$

c) ∵ from eq i)

at particular ω', $\omega' L_3 = \frac{1}{\omega' C_3}$ where X_4 will be undefined.

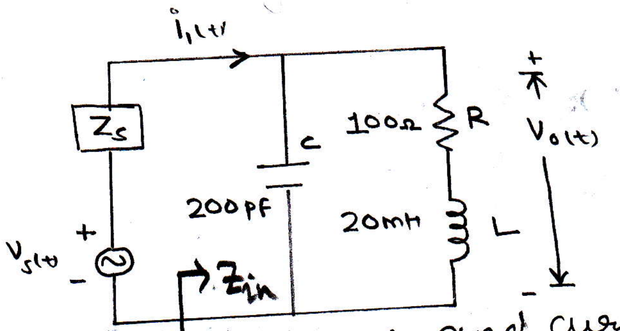
This ω is given by $\omega' = \frac{1}{\sqrt{L_3 C_3}}$.

$$\Rightarrow \omega' = \frac{1}{\sqrt{0.1 \times 5 \times 10^{-6}}} = 1.414 \times 10^{3} \text{ rad/s}$$

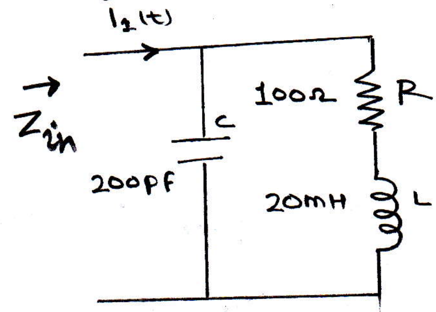
⇒ corresponding freq. is $f' = \frac{\omega'}{2\pi}$

⇒ $f' = 225.08$ Hz.

SOLN. 7



∵ Voltage $v_o(t)$ and current $i(t)$ are in phase
∴ Voltage $v_o(t)$ and current $i(t)$ are in phase (Note: Z_s has no role)
Input impedance Z_{in} is purely resistive.



$$\Rightarrow Z_{in} = (R + jX_L) \parallel (-jX_c)$$

$$= \frac{(R + jX_L)(-jX_c)}{(R + jX_L) - jX_c}$$

$$Z_{in} = \frac{-jRX_c + X_LX_c}{R + j(X_L - X_c)} \quad \text{—i)}$$

⇒ Multiplying both numerator & denominator of equation i) by $R - j(X_L - X_c)$.

$$\Rightarrow Z_{in} = \frac{(X_LX_c - jRX_c)}{[R + j(X_L - X_c)]} \times \frac{[R - j(X_L - X_c)]}{[R - j(X_L - X_c)]}$$

$$\Rightarrow Z_{in} = \frac{\{RX_LX_c - RX_c(X_L - X_c)\} - j\{R^2X_c + X_LX_c(X_L - X_c)\}}{R^2 + (X_L - X_c)^2}$$

∴ Z_{in} has to be purely resistive, so imaginary part of Z should be zero.

$$\Rightarrow \frac{R^2 X_c + X_L X_c (X_L - X_c)}{R^2 + (X_L - X_c)^2} = 0$$

$$\Rightarrow R^2 X_c = X_L X_c (X_c - X_L)$$

$$\Rightarrow R^2 = X_L (X_c - X_L) \quad [\because X_c \neq 0].$$

$$\because X_L = \omega L \text{ and } X_c = \frac{1}{\omega c}.$$

$$\Rightarrow R^2 = \omega L \left(\frac{1}{\omega c} - \omega L \right) = \frac{L}{c} - \omega^2 L^2 \Rightarrow \omega^2 L^2 = \frac{L}{c} - R^2.$$

$$\Rightarrow \omega^2 = \frac{1}{LC} - \frac{R^2}{L^2} \quad \text{— ii)}$$

on substituting $R = 100\Omega$, $L = 20\,mH$, $C = 200\,pF$ in eq. ii)

$$\Rightarrow \omega = \sqrt{\frac{1}{20\,mH \times 200\,pF} - \frac{(100\Omega)^2}{(20\,mH)^2}}$$

$$= \sqrt{\frac{1}{20 \times 10^{-3} \times 200 \times 10^{-12}} - \frac{(100)^2}{(20 \times 10^{-3})^2}}$$

$$= \sqrt{(2.5 \times 10^{11})} = 25\,0\,000000$$

$$= 499974.9994 \text{ rad/sec.}$$