20th October, 2016

Home Assignment – 11

1. Carry out the following conversions:
   (a) $(141)_8 = (?)_2$
   (b) $(168)_{16} = (?)_{10}$
   (c) $(10110.0101)_2 = (?)_8$
   (d) $(4050.6)_8 = (?)_{10}$
   (e) $(FAFA.B)_{16} = (?)_2$
   (f) $(31.8203125)_{10} = (?)_2$

2. Perform the following arithmetic operations using 2's complement method :
   (a) 50-21 (b) -50+21 (c) -50-21
   Use 8-bit representation of number.

3. Use Boolean identities to prove that
   $$(w.x.z + \overline{w}.x + \overline{x}.z + \overline{y}).(y + w.x + x.z) = x.(w + y).(\overline{w} + \overline{y}) + z.(x + y)$$

4. Simplify the 3-input Boolean function $\sum m(1,2,6,7)$ in terms of x, y and z using identities of Boolean algebra (Don't use K-maps).

5. Minimize the following functions using K-map:
   (a) $\overline{w}.\overline{x}.\overline{y}.z + \overline{w}.\overline{x}.y.\overline{z} + \overline{w}.x.\overline{y}.z + \overline{w}.\overline{x}.y.z + w.x.y.z + w.\overline{x}.y.z$
   (b) $w.x.\overline{y}.\overline{z} + \overline{w}.\overline{x}.\overline{y}.\overline{z} + w.\overline{x}.\overline{y}.z + \overline{w}.x.y.z + w.x.y.\overline{z} + \overline{w}.x.y.\overline{z}$
   (c) $f(x, y, z) = \sum m(0,1,2,3,4,5,6)$
   (d) $f(x, y, z) = \prod M(0,1,2,4)$
   (e) $f(w, x, y, z) = \sum m(0,2,4,9,12,15) + \sum d(1,5,7,10)$
   (f) $f(x, y, z) = x.\overline{y}.z + \overline{x}.y.z + x.y.\overline{z} + \overline{x}.y.\overline{z}$
   (g) $f(w, x, y, z) = \sum m(1,3,4,5,8,9,10,13)$
   (h) $f(w, x, y, z) = w.x + \overline{w}.x.y + \overline{w}.\overline{x}.\overline{y}.z$

6. (a) Minimize the function $f(w, x, y, z) = \sum m(1,3,11,15) + \sum d(5,7,12)$
   (b) Find the minimum in terms of product-of-sums of the expression mentioned in (a). Is your answer functionally equivalent to the minimum function obtained in (a)?

7. Four switches operate a lamp as follows: the lamp lights up if switches 1, 3 and 4 are closed and switch 2 is open, or if 2, 4 are closed and 3 is open, or if all the switches are kept closed. Express this as a Boolean function in a standard Sum-of-product form and solve it using K-map (Use bit '1' when switch is closed and bit '0' when switch is open).

(a) $(141)_8 = (\underbrace{001}_{1}\ \underbrace{100}_{4}\ \underbrace{001}_{1})_2$

$\therefore (141)_8 = (001100001)_2 = (1100001)_2$.

Alternatively,

First convert the given octal number to its decimal equivalent.

$$(141)_8 = (1\times 8^2 + 4\times 8^1 + 1\times 8^0)_{10}$$

$$= (64+32+1)_{10} = (97)_{10}.$$

Now convert this decimal to binary using the method of successive division by '2'.

97	remainder	
48	1	→ right most bit
24	0	
12	0	
6	0	
3	0	
1	1	
0	1	→ left most bit.

$\therefore (141)_8 = (97)_{10} = (1100001)_2$

(b) $(168)_{16} = (1\times16^{2} + 6\times16^{1} + 8\times16^{0})_{10}$

$= (1\times256 + 6\times16 + 8\times1)_{10}$

$= (256 + 96 + 8)_{10}$

$= (360)_{10}$

$\therefore (168)_{16} = (360)_{10}$

(c) $(10110.0101)_2$

$$= (\underline{010}\ \underline{110}.\underline{010}\ \underline{100})_2$$

$$= (2\ \ 6.\ 2\ \ 4)_8$$

$\therefore (10110.0101)_2 = (26.24)_8$

(d) $(4050.6)_8$

$= (4\times8^{3} + 0\times8^{2} + 5\times8^{1} + 0\times8^{0} + 6\times8^{-1})_{10}$

$= (2048 + 0 + 40 + 0 + 0.75)_{10}$

$= (2088.75)_{10}$

$\therefore (4050.6)_8 = (2088.75)_{10}$

(e) $(FAFA.B)_{16}$

$$= (\underbrace{1111}_{F}\ \underbrace{1010}_{A}\ \underbrace{1111}_{F}\ \underbrace{1010}_{A}.\ \underbrace{1011}_{B})_2$$

$\therefore (FAFA.B)_{16} = (1111101011111010.1011)_2$

(f) $(31.8203125)_{10}$

First we convert $(31)_{10}$ to binary using the method of successive division by 2.

31	remainder
15	1 ← right most bit
7	1
3	1
1	1
0	1 ← left most bit

$(31)_{10} = (11111)_2$

Now convert $(0.8203125)_{10}$ to binary as follows.

0.	8203125
	× 2
1.	6406250
	× 2
1.	2812500
	× 2
0.	5625
	× 2
1.	1250
	× 2
0.	2500
	× 2
0.	5000
	× 2
1.	0

$$\Rightarrow (0.8203125)_{10} = (1101001)_2$$

Now,

$$(31.8203125)_{10} = (1111.1101001)_2$$

(a) $50 - 21$

$50 = (00110010)_2$; $21 = (00010101)_2$

2's complement of $21 = (11101011)_2$

NOW,

$$\begin{array}{l} 50 \\ +(-21) \end{array} \rightarrow \begin{array}{r} 00110010 \\ (+)\ 11101011 \\ \hline 1\,00011101 \end{array}$$

$\hookrightarrow$ discard the carry

Since left most bit is zero, the number we get is positive.

$(00011101)_2 = (29)_{10}$

$\therefore$ Answer is $+29$

(b) $-50 + 21$

$21 = (00010101)_2$; $50 = (00110010)_2$

2's complement of $50 = (11001110)_2$

NOW,

$$\begin{array}{l} -50 \\ +21 \end{array} \rightarrow \begin{array}{r} 11001110 \\ (+)\ 00010101 \\ \hline 11100011 \end{array}$$

Since left most bit is 1, the number we get is negative.

2's complement of $(1110011)_2 = (0001101)_2$
$= (29)_{10}$

$\therefore$ Answer is -29.

(c) $-50-21$

$50 = (0011\,0010)_2$; $21 = (00010101)_2$

2's complement of 50 $= (1100\,1110)_2$

2's complement of 21 $= (11101011)_2$

Now,

$$\begin{array}{lcr} -50 & & 1100\,1110 \\ +(-21) & \longrightarrow & (+)\;11101011 \\ \hline & & 1\,1011\,1001 \end{array}$$

↳ discard the carry

Since left most bit is 1, the number we get is negative.

2's complement of $(10111001)_2 = (01000111)_2$

$= (71)_{10}$

∴ Answer is $= -71$

SOLN. 3

$$(w.x.z + \overline{w}.x + \overline{x}.z + \overline{y}).(y + w.x + x.z)$$

$$= (x.(w.z + \overline{w}) + \overline{x}.z + \overline{y}).(y + w.x + x.z)$$

$$= (x.(\overline{w} + z) + \overline{x}.z + \overline{y}).(y + w.x + x.z) \quad [\because A + \overline{A}B = A + B]$$

$$= (x.\overline{w} + x.z + \overline{x}.z + \overline{y}).(y + w.x + x.z)$$

$$= (x.\overline{w} + z + \overline{y}).(y + w.x + x.z) \quad [\because x.z + \overline{x}.z = z]$$

$$= x.\overline{w}.y + \underbrace{x.\overline{w}.w.x}_{=0} + x.\overline{w}.x.z + z.y + z.w.x + z.x.z + \underbrace{\overline{y}.y}_{=0} + \overline{y}.w.x + \overline{y}.x.z$$

$$= x.y.\overline{w} + x\overline{w}z + z.y + z.w.x + x.z + \overline{y}.w.x + x.z.\overline{y}$$

$$= x.y.\overline{w} + y.z + x.z(\overline{w} + w + 1 + \overline{y}) + \overline{y}.w.x$$

$$= x.y.\overline{w} + y.z + x.z + x.\overline{y}.w$$

$$= x.y.\overline{w} + x.\overline{y}.w + z.(x+y)$$

$$= x.(y.\overline{w} + \overline{y}.w) + z.(x+y)$$

$$= x.(y.\overline{w} + w.\overline{w} + \overline{y}.w + y.\overline{y}) + z.(x+y)$$

$$= x.(\overline{w}.(y+w) + \overline{y}.(w+y)) + z.(x+y)$$

$$= x.(y+w).(\overline{y}+\overline{w}) + z.(x+y)$$

$$\therefore (w.x.z + \overline{w}.x + \overline{x}.z + \overline{y}).(y + w.x + x.z) = x.(w+y).(\overline{w}+\overline{y}) + z.(x+y)$$

$\Sigma m(1,2,6,7) = f(x,y,z)$

The truth table is,

x	y	z	Decimal number
0	0	0	0
0	0	1	1
0	1	0	2
0	1	1	3
1	0	0	4
1	0	1	5
1	1	0	6
1	1	1	7

Now,

$$\Sigma m(1,2,6,7) = \bar{x}.\bar{y}.z + \bar{x}.y.\bar{z} + x.y.\bar{z} + x.y.z$$

$$= \bar{x}.\bar{y}.z + \bar{x}.y.\bar{z} + x.y.(\bar{z}+z)$$

$$= \bar{x}.\bar{y}.z + \bar{x}.y.\bar{z} + x.y \qquad [\because \bar{z}+z=1]$$

$$= \bar{x}.\bar{y}.z + y.(\bar{x}.\bar{z}+x)$$

$$= \bar{x}.\bar{y}.z + y.(\bar{z}+x) \qquad [\because \bar{A}B+A = B+A]$$

$$= \bar{x}.\bar{y}.z + y.\bar{z} + x.y$$

$$\therefore \Sigma m(1,2,6,7) = \bar{x}.\bar{y}.z + y.\bar{z} + x.y$$

a) $f = \overline{w}.\overline{x}.\overline{y}.z + \overline{w}.\overline{x}.y.\overline{z} + \overline{w}.x.\overline{y}.z + \overline{w}.\overline{x}.y\,z + w x y z + w\overline{x}yz$

$\Rightarrow$ 0001 0010 0101 0011 1111 1011

$\Rightarrow$ 1 2 5 3 15 11

$= \sum m(1, 2, 3, 5, 11, 15)$

=

wx \ yz	00	01	11	10
00	0	1	1	1
01	0	1	0	0
11	0	0	1	0
10	0	0	1	0

$= \overline{w}\overline{y}z + \overline{w}\,\overline{x}y + wyz$

b) $f = wx\overline{y}\,\overline{z} + \overline{w}\,\overline{x}\,\overline{y}\,\overline{z} + w\overline{x}\,\overline{y}z + \overline{w}xyz + wxy\overline{z} + \overline{w}xy\overline{z}$

$\Rightarrow$ 1100 0000 1001 0111 1110 0110

$\Rightarrow$ 12 0 9 7 14 6

$= \sum m(0, 6, 7, 9, 12, 14)$

wx \ yz	00	01	11	10
00	1	0	0	0
01	0	0	1	1
11	1	0	0	1
10	0	1	0	0

$= \overline{w}xy + wx\overline{z} + \overline{w}\,\overline{x}\,\overline{y}\,\overline{z}$
$+ w\overline{x}\,\overline{y}z$

c) $f(x,y,z) = \sum m(0,1,2,3,4,5,6)$

$= \bar{x} + \bar{y} + \bar{z}$

$x \backslash yz$	00	01	11	10
0	1	1	1	1
1	1	1	0	1

d) $f(x,y,z) = \prod M(0,1,2,4)$

$= (x+y)\cdot(y+z)\cdot(x+z)$

$x \backslash yz$	00	01	11	10
0	0	0	1	0
1	0	1	1	1

e) $f(w,x,y,z) = \sum m(0,2,4,9,12,15) + \sum d(1,5,7,10)$

$wx \backslash yz$	00	01	11	10
00	1	X	0	1
01	1	X	X	0
11	1	0	1	0
10	0	1	0	X

$= x\bar{y}\bar{z} + \bar{w}\bar{x}\bar{z} + \bar{x}\bar{y}z + xyz$

f) $f(x,y,z) = x\bar{y}z + \bar{x}yz + xy\bar{z} + \bar{x}y\bar{z}$

$= 101 \quad 011 \quad 110 \quad 010$

$= 5 \quad 3 \quad 6 \quad 2$

$= \sum m(2,3,5,6)$

x \ yz	00	01	11	10
0	0 [0]	0 [1]	1 [3]	1 [2]
1	0 [4]	1 [5]	0 [7]	1 [6]

$= \bar{x}y + y\bar{z} + x\bar{y}z$

g) $f(w,x,y,z) = \sum m(1,3,4,5,8,9,10,13)$

wx \ yz	00	01	11	10
00	0 [0]	1 [1]	1 [3]	0 [2]
01	1 [4]	1 [5]	0 [7]	0 [6]
11	0 [12]	1 [13]	0 [15]	0 [14]
10	1 [8]	1 [9]	0 [11]	1 [10]

$= \bar{y}z + \bar{w}x\bar{y} + w\bar{x}\bar{z} + \bar{w}\bar{x}z$

b) $f(w,x,y,z) = wx + \bar{w}xy + \bar{w}\bar{x}\bar{y}z$

$= wx(y+\bar{y})(z+\bar{z}) + \bar{w}xy(z+\bar{z}) + \bar{w}\bar{x}\bar{y}z$

$= wxy(z+\bar{z}) + wx\bar{y}(z+\bar{z}) + \bar{w}xyz + \bar{w}xy\bar{z} + \bar{w}\bar{x}\bar{y}z$

$= wxyz + wxy\bar{z} + wx\bar{y}z + wx\bar{y}\bar{z} + \bar{w}xyz$

$+ \bar{w}xy\bar{z} + \bar{w}\bar{x}\bar{y}z$.

$\Rightarrow$ 1111 1110 1101 1100 0111

0110 0001

$\Rightarrow$ 15 14 13 12 7 6 1

$\Rightarrow \sum m(1, 6, 7, 12, 13, 14, 15)$

wx \ yz	00	01	11	10
00	0^{0}	1^{1}	0^{3}	0^{2}
01	0^{4}	0^{5}	1^{7}	1^{6}
11	1^{12}	1^{13}	1^{15}	1^{14}
10	0^{8}	0^{9}	0^{11}	0^{10}

$= wx + xy + \bar{w}\bar{x}\bar{y}z$.

a) $f(w,x,y,z) = \sum m(1,3,11,15) + \sum d(5,7,12)$

wx \ yz	00	01	11	10
00	0	1	1	0
01	0	x	x	0
11	x	0	1	0
10	0	0	1	0

$= \bar{w}z + yz$

$f_1 = \bar{w}z + yz$

b) To find minimum product of sum, we combine the zeroes instead of 1 in K-map.

wx \ yz	00	01	11	10
00	0	1	1	0
01	0	x	x	0
11	x	0	1	0
10	0	0	1	0

$= (z)(\bar{w}+y)$; $f_2 = z\cdot(\bar{w}+y)$

Now from (a) & (b)

$f_2 = z\cdot(\bar{w}+y)$

$= \bar{w}z + yz$

$= f_1$

Both the functions are equivalent.

SOLN. 7 P-14

Let four switches are represented by w, x, y, z

For closed switch variable will have value 1 and for open switch variable will have value 0.

1, 3, 4 closed ; 2 open $\longrightarrow$ $w\bar{x}yz$

2, 4 closed ; 3 open $\longrightarrow$ $x\bar{y}z$

All closed $\longrightarrow$ $wxyz$

$wx \backslash yz$	00	01	11	10
00	0	0	0	0
01	0	1	0	0
11	0	1	1	0
10	0	0	1	0

$$f = x\bar{y}z + wyz$$