Chimera States in the Leaky Integrate-and-Fire Model with Non-Local Connectivity

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Outline

1. The leaky integrate-and-fire model
   - The single neuron model
   - Nonlocally coupled network of LIF neurons

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   - Minimum number of neurons and $\Delta\omega$ vs N
   - Varying σ
   - Dependence on initial conditions

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4. The modified LIF model
   - Chimeras states in the non-leaky integrate-and-fire model
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5. Conclusions

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- $\lambda = 1$ for the original LIF model

The single LIF neuron model

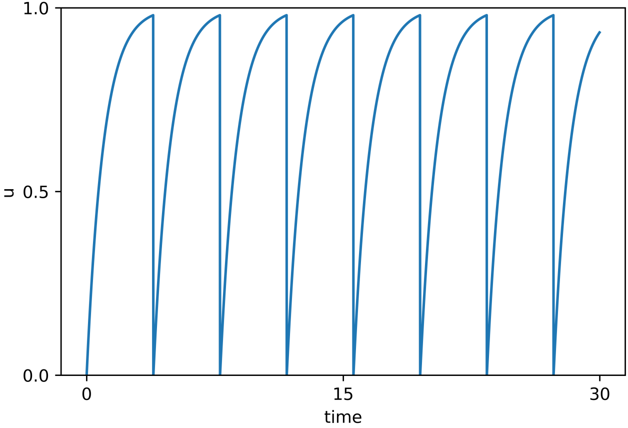

Figure: Dynamic evolution of the membrane potential in time according to Eq. (1) and (2).

Nonlocally coupled network of LIF neurons

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- index i taken modulo N

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- forward Euler method, step size of $dt = 0.01$

Minimum number of neurons

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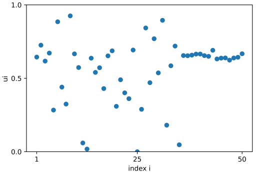
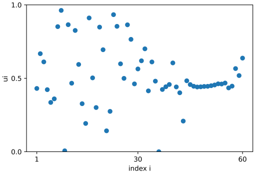
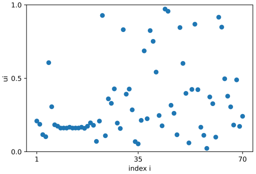


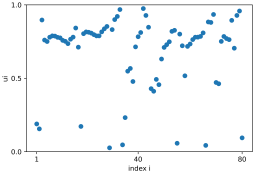
(a) $N = 50$



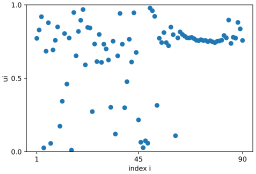
(b) $N = 60$



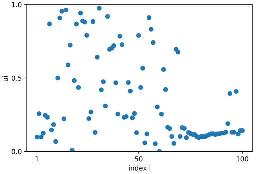
(c) $N = 70$



(d) $N = 80$



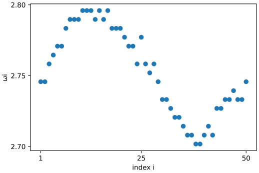
(e) $N = 90$



(f) $N = 100$

Figure: Snapshots of the membrane potential u_i at $t = 1000$ time units, for $\sigma = 0.7$, $r = 0.40$ and for various values of N.

Minimum number of neurons



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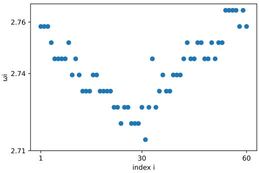
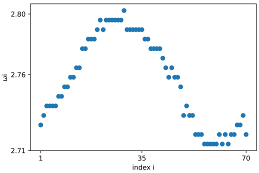
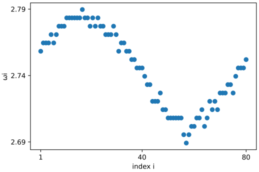


(b) $N = 60$



(c) $N = 70$



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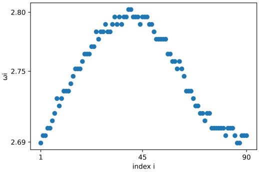

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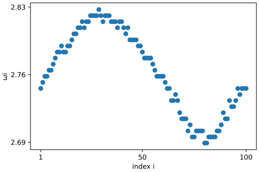

(f) $N = 100$

Figure: Mean phase-velocity profiles ω_i at $t = 1000$ time units, for $\sigma = 0.7$, $r = 0.40$ and for various values of N, as in Fig. (2).

Minimum number of neurons

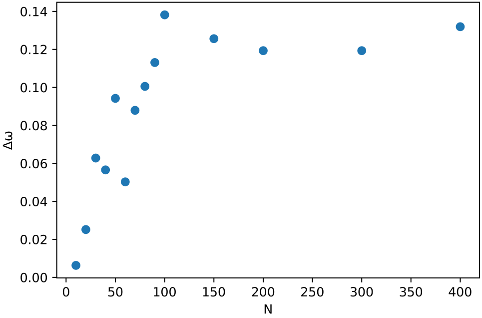

Figure: $\Delta\omega = \omega_{\text{max}} - \omega_{\text{min}}$ with varying system size N.

Varying σ

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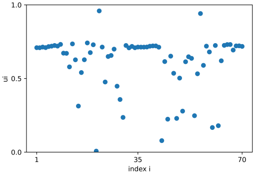
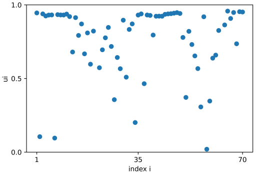
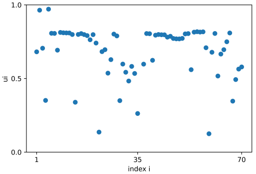


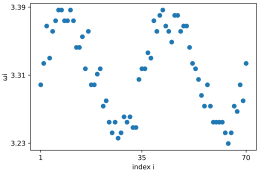
(a) $\sigma = 1.5$



(b) $\sigma = 1.6$



(c) $\sigma = 1.7$



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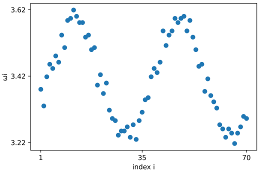

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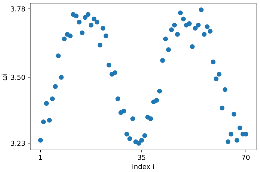

(f) $\sigma = 1.7$

Figure: Snapshots of the membrane potential u_i and mean phase-velocity profiles ω_i for the double chimeras observed, for $N = 70$, $r = 0.40$ at $t = 1000$. The first row are the u_i and the second row are the ω_i.

Varying σ

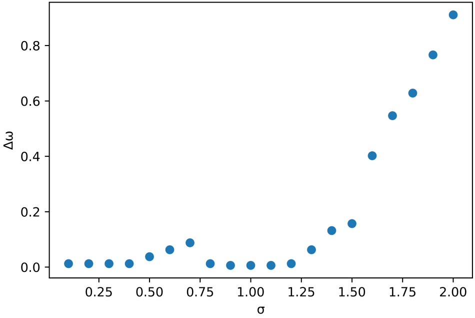

Figure: $\Delta\omega = \omega_{\text{max}} - \omega_{\text{min}}$ with varying σ.

Dependence on initial conditions

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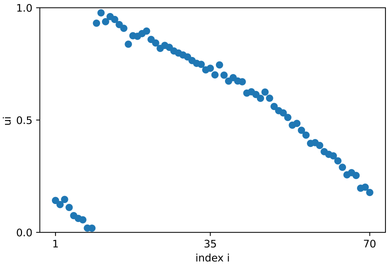
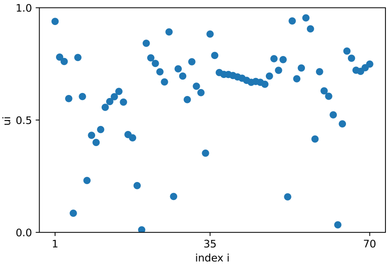


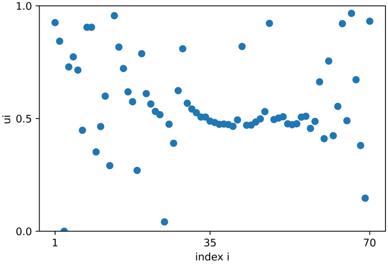
(a) seed $= 64856$



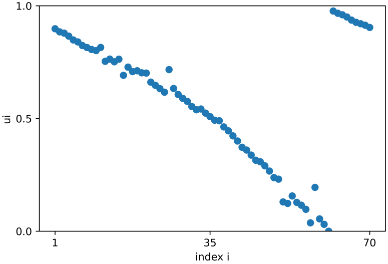
(b) seed $= 38958$

Figure: Snapshots of the membrane potential u_i at $t = 1000$, for $N = 70$, $\sigma = 0.7$ and $r = 0.35$, for two different initial conditions.

Dependence on initial conditions



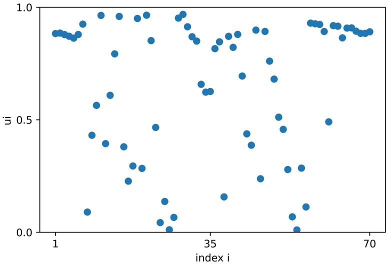
(a) $t = 1000$, seed $= 70893$



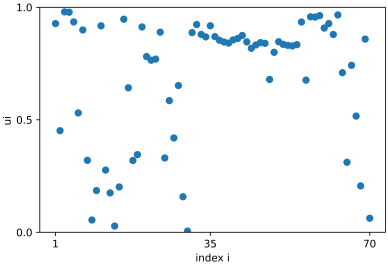
(b) $t = 4000$, seed $= 70893$

Figure: Snapshots of the membrane potential u_i, for $N = 70$, $\sigma = 0.7$ and $r = 0.35$, at $t = 1000$ and $t = 4000$.

Dependence on initial conditions



(a) $t = 1000$, seed $= 27027$



(b) $t = 4000$, seed $= 27027$

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Chimera states in the $\lambda = 0$ model

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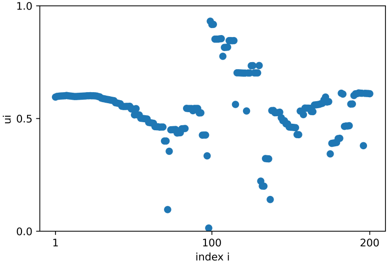
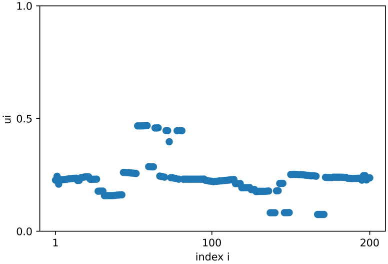


(a) $t = 1000$



(b) $t = 4000$

Figure: Snapshots of the membrane potential u_i at $t = 1000$ and $t = 4000$, for $N = 200$, $\sigma = 1.1$ and $r = 0.35$.

Chimera states in the $\lambda = 0$ model

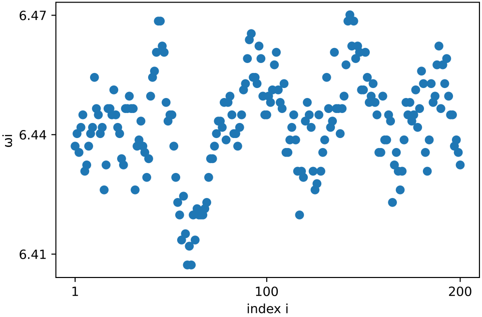


Figure: Mean phase-velocity profile ω_i at $t = 4000$, for $N = 200$, $\sigma = 1.1$ and $r = 0.35$.

Varying λ

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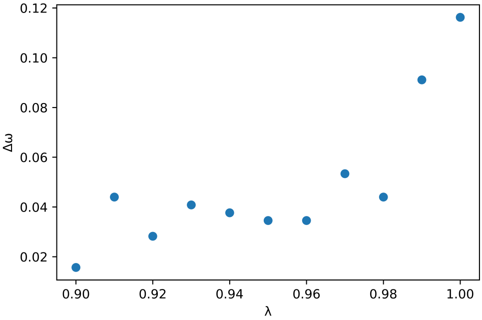

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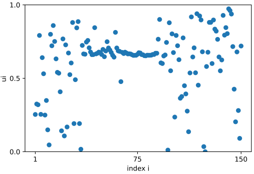
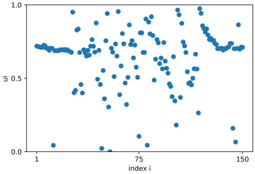
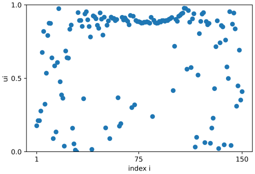


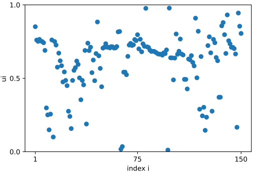
(a) $\lambda = 1.00$



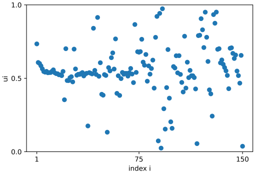
(b) $\lambda = 0.99$



(c) $\lambda = 0.98$



(d) $\lambda = 0.97$



(e) $\lambda = 0.96$

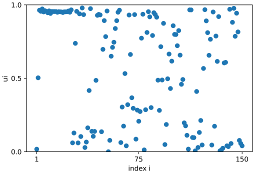


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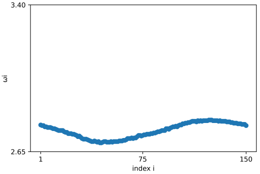
Figure: Snapshots of the membrane potential u_i at $t = 2000$ time units, for $N = 150$, $r = 0.4$ and $\sigma = 0.7$ and for various values of λ.

Varying λ



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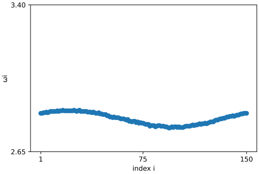

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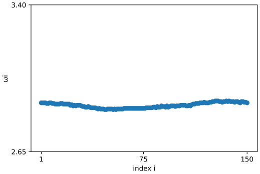

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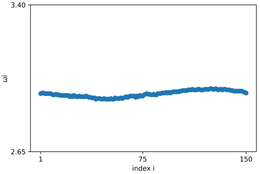

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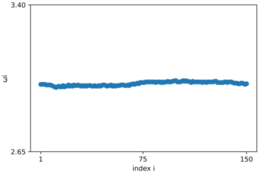

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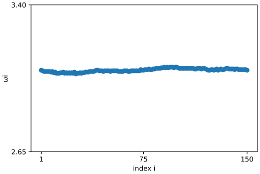

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Conclusions

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- high dependence on initial conditions
- future work: refractory period

References

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Thank you!

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