Universität Rostock — Traditio et Innovatio

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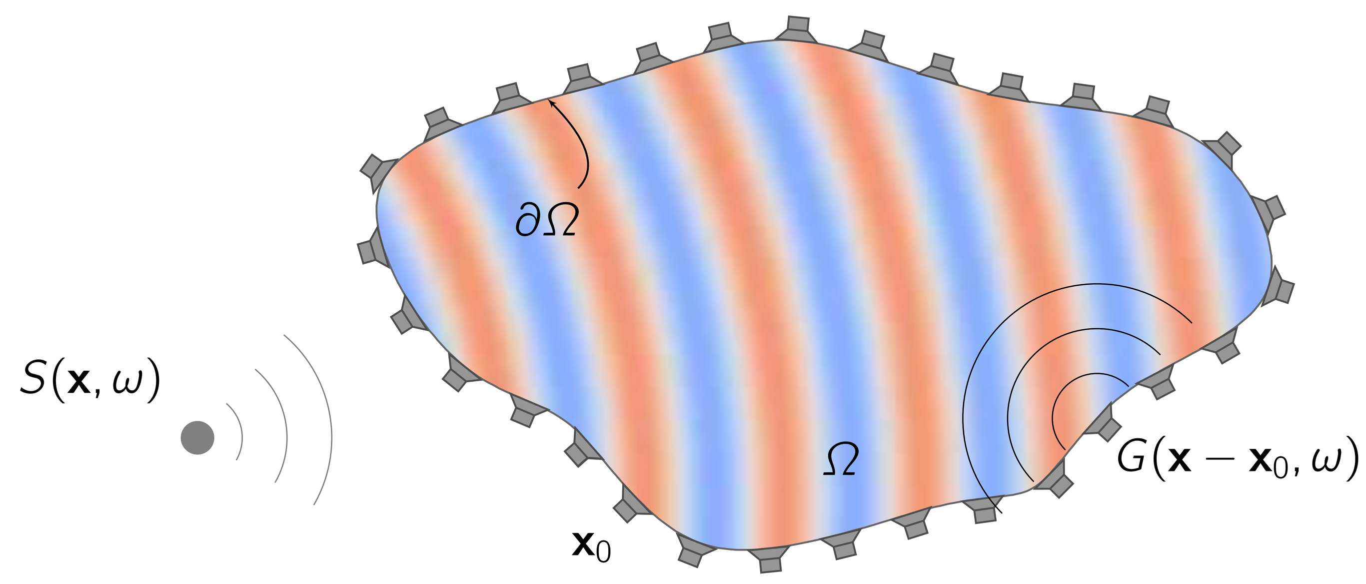
Array Design for Increased Spatial Aliasing Frequency in Wave Field Synthesis Based on a Geometric Model

Abstract

Wave Field Synthesis aims at a physically accurate synthesis of a desired sound field inside a target region. Typically, the region is surrounded by a finite number of discrete loudspeakers. For practical loudspeaker setups, this spatial sampling causes spatial aliasing artefacts and does not allow for an accurate synthesis over the entire audible frequency range. Recently, the authors proposed a geometric model to predict the so-called aliasing frequency up to which the spatial aliasing is negligible for a specific listening position or area. Besides its dependency on the desired sound field, this frequency is influenced by the spacing between individual loudspeakers. This work discusses the effects of non-uniform spacing on the aliasing frequency. We further propose optimal discretisation patterns for a given array geometry and desired sound field. The derived patterns are compared to a uniform sampling scheme via numerical simulations of the synthesised sound fields. The results show an increase of the aliasing frequency for the optimised patterns.

Wave Field Synthesis



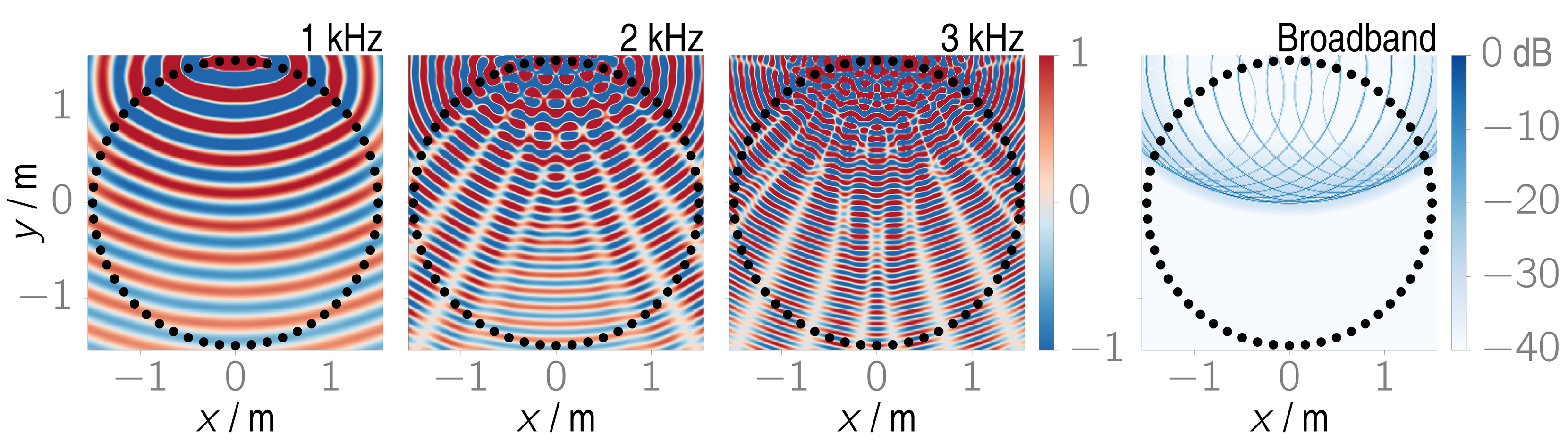
- **Goal**: Accurate synthesis of desired sound field $S(\mathbf{x}, \omega)$ inside the target region Ω using a loudspeaker distribution along the boundary $\partial\Omega$ (loudspeaker symbols)
- **Solution**: Determine driving signals $D(\mathbf{x}_0, \omega)$ for each loudspeaker such that

$$\underbrace{S(\mathbf{x}, \omega)}_{\substack{\text{desired}\\\text{sound field}}} \overset{!}{=} \underbrace{P(\mathbf{x}, \omega)}_{\substack{\text{synthesised}\\\text{sound field}}} = \int_{\partial\Omega} \underbrace{D(\mathbf{x}_0, \omega)}_{\substack{\text{driving}\\\text{signal}}} \underbrace{G(\mathbf{x} - \mathbf{x}_0, \omega)}_{\substack{\text{sound field emitted}\\\text{by loudspeaker at } \mathbf{x}_0}} \, \mathrm{d}A(\mathbf{x}_0) \qquad \forall \mathbf{x} \in \Omega\,.$$

- The continuous distribution is approximated by L loudspeakers located at discrete positions

$$P(\mathbf{x}, \omega) \approx P^{\mathrm{S}}(\mathbf{x}, \omega) = \sum_{n=0}^{L-1} D(\mathbf{x}_0^{(n)}, \omega) G(\mathbf{x} - \mathbf{x}_0^{(n)}, \omega) \underbrace{\Delta_{\mathbf{x}_0}(\mathbf{x}_0^{(n)})}_{\substack{\text{sampling}\\\text{distance}}} \qquad \mathbf{x}_0^{(n)} \in \partial\Omega$$

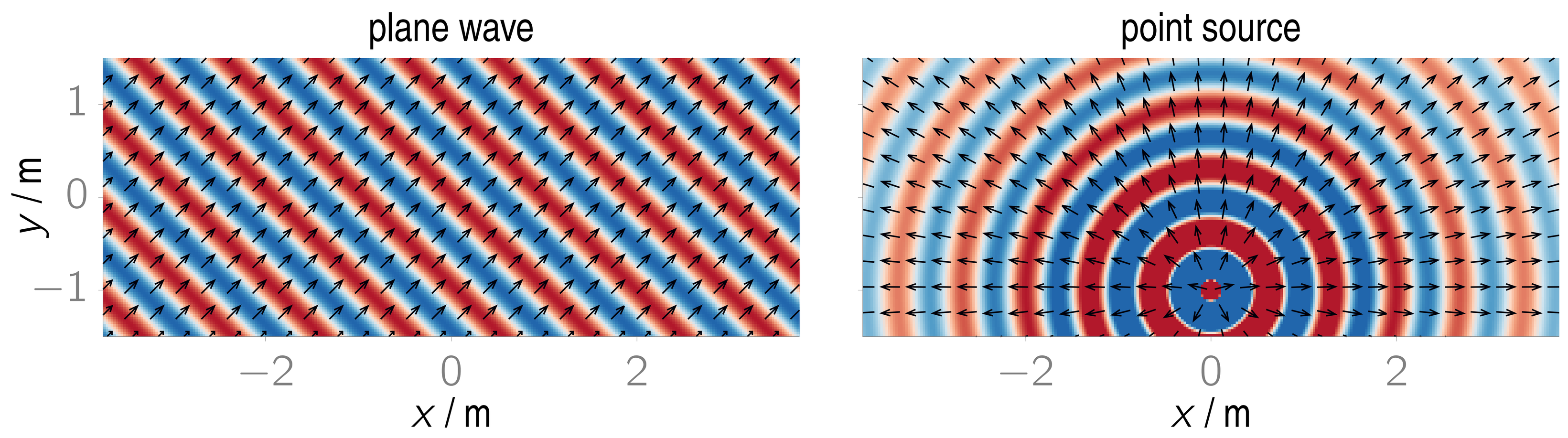
- The discretisation leads to spatial aliasing artefacts in the synthesised sound field



- The temporal frequency up to which the spatial aliasing artefacts are negligible is called the **spatial aliasing frequency** f^{S}

Geometric Model for Spatial Aliasing

- In [1], a geometric model was proposed to estimate the spatial aliasing frequency. It is based on the concept of the local wavenumber vector [2].

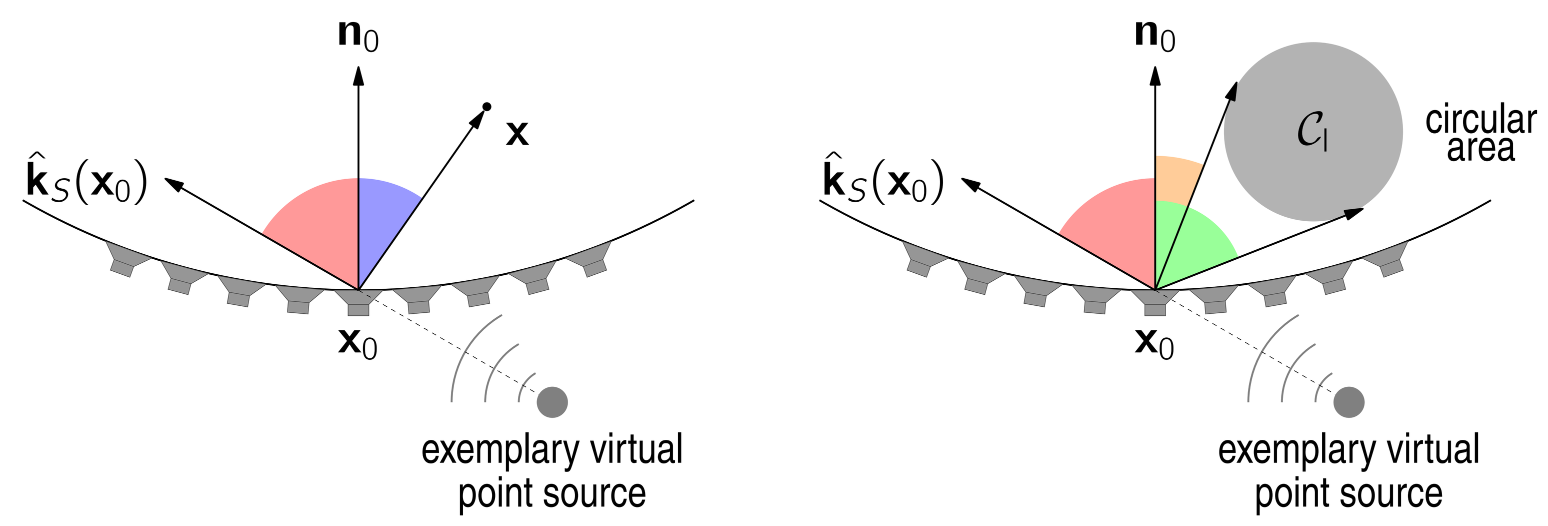


- The normalised local wavenumber vector $\hat{\mathbf{k}}_S(\mathbf{x})$ (black arrows) describes the propagation direction of the sound field $S(\mathbf{x}, \omega)$ at a given coordinate.

References

[1] F. Winter, F. Schultz, G. Firtha, and S. Spors, “A Geometric Model for Prediction of Spatial Aliasing in 2.5D Sound Field Synthesis,” *IEEE/ACM Transactions on Audio, Speech, and Language Processing*, 2019.

[2] G. Firtha, P. Fiala, F. Schultz, and S. Spors, “Improved Referencing Schemes for 2.5D Wave Field Synthesis Driving Functions,” *IEEE/ACM Transactions on Audio, Speech, and Language Processing*, vol. 25, no. 5, pp. 1117–1127, 2017.



- The aliasing frequencies f^{S} derived in [1] exhibit the same mathematical structure

$$f^{\mathrm{S}} = \min_{\mathbf{x}_0} \frac{c}{\Delta_{\mathbf{x}_0}(\mathbf{x}_0) \cdot \gamma(\mathbf{x}_0)}\,.$$

- $\gamma(\mathbf{x}_0)$ denotes a function depending on the scenario for which the aliasing frequency is calculated:

$$\gamma(\mathbf{x}_0) = \begin{cases} |\sin(\blacktriangle) - \sin(\blacktriangle)| & \text{for a single position } \mathbf{x} \in \Omega \\ 1 + |\sin(\blacktriangle)| & \text{for the entire target region } \Omega \\ \max\left(|\sin(\blacktriangle) - \sin(\blacktriangle)|; |\sin(\blacktriangle) - \sin(\blacktriangle)|\right) & \text{for a circular area } \mathcal{C}_\mathrm{l} \\ 1 + \max\left(|\sin(\blacktriangle)|; |\sin(\blacktriangle)|\right) & \text{for } \mathcal{C}_\mathrm{l} \text{ and arbitrary sound fields} \end{cases}$$

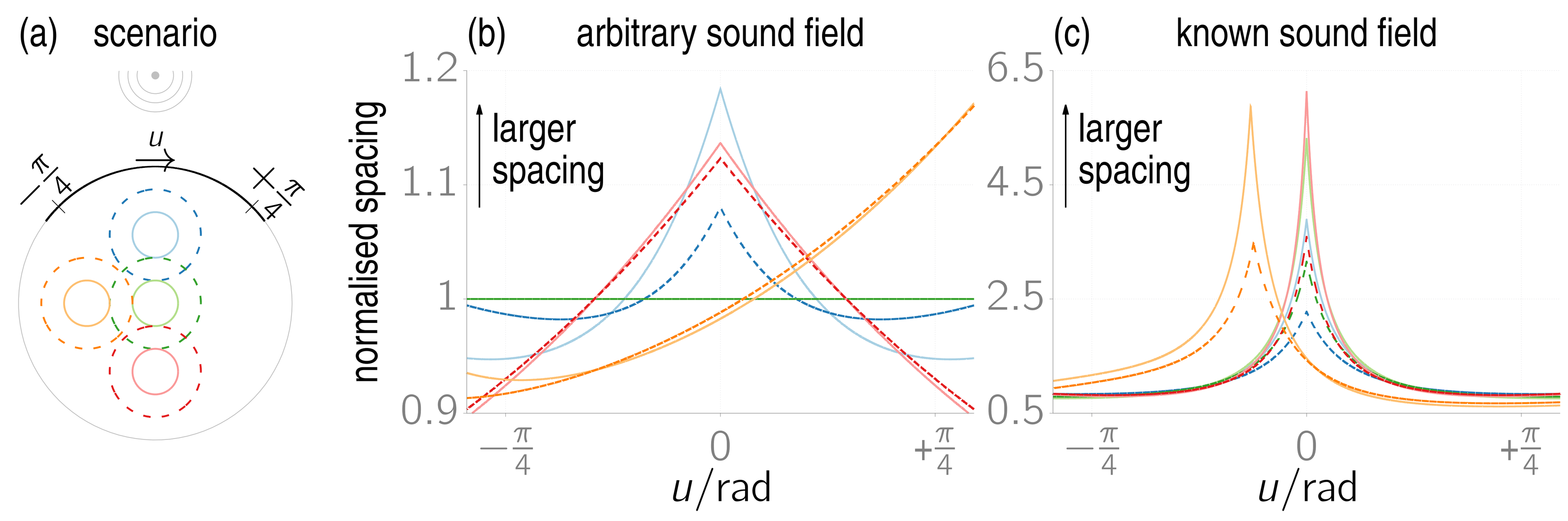
- Optimal spacing to maximise the spatial aliasing frequency reads

$$\Delta_{\mathbf{x}_0}(\mathbf{x}_0) \propto \frac{1}{\gamma(\mathbf{x}_0)}$$

- Proportionality factor results from the number of loudspeakers L and the length of the boundary $\partial\Omega$

Simulations

Optimal Spacing

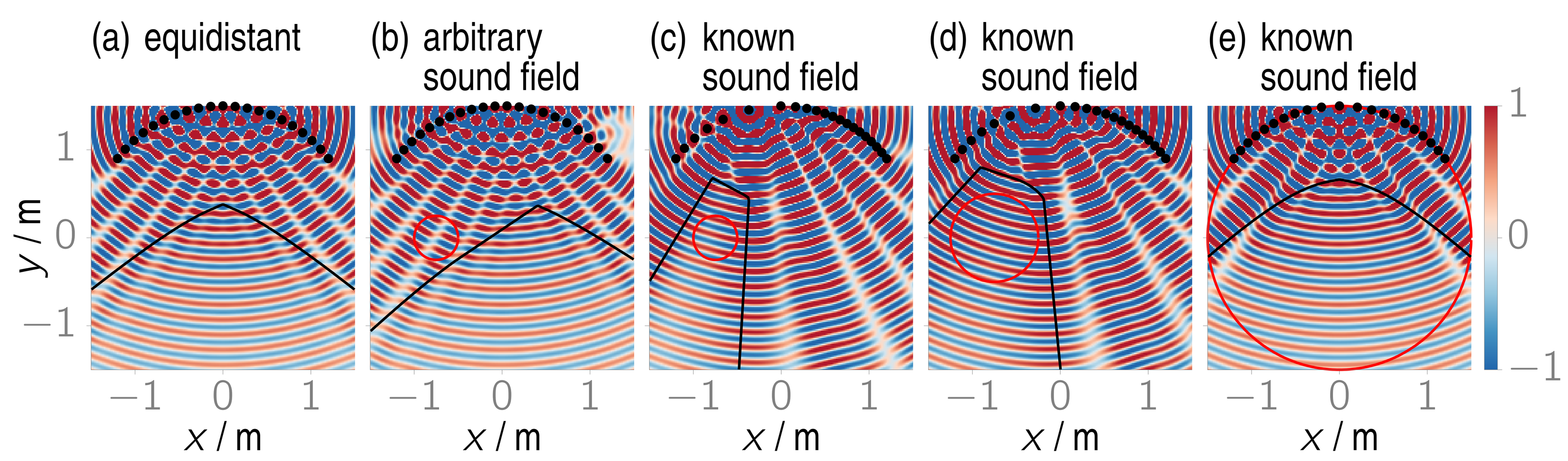


(a) The loudspeakers are positioned along a circular arc (black line) with 1.5 m radius and synthesise a virtual point source at $[0, 2.5, 0]^{\mathrm{T}}$ m (grey dot). Spacing is optimised for the coloured circular areas.

(b) Symmetric sampling schemes for the areas ○, ○, and ○. The larger the radius of the area, the more uniform the sampling. Antisymmetric pattern for ○ with only minor effect of the radius.

(c) The additional consideration of the virtual sound field leads to a larger spread of the spacing.

Synthesised Sound Field and Spatial Aliasing



(a) Reproduced sound field at $f = 2$ kHz using $L = 21$ loudspeakers (black dots) equiangularly distributed on the circular arc. Black line marks aliasing-free area estimated by the geometric model.

(b) Optimisation for arbitrary sound fields does not necessarily lead to an improvement of the aliasing properties inside the area (red circle) for a particular sound field.

(c) Aliasing frequency for the area (red circle) is 1.62 times as high as for equiangular sampling

(d) For the larger radius, the aliasing frequency is 1.48 times as high as for equiangular sampling

(e) For the entire target region Ω, the aliasing frequency 1.16 times as high as for equiangular sampling

Reproducible Research

The driving signals for Wave Field Synthesis and the sound fields are calculated with the Sound Field Synthesis Toolbox, Version 2.4.3. It is freely available under the MIT license. The code to reproduce the figures in the paper is published under the GNU General Public License v3 (GPL-3.0).

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