Universität Rostock
CHALMERS UNIVERSITY OF TECHNOLOGY

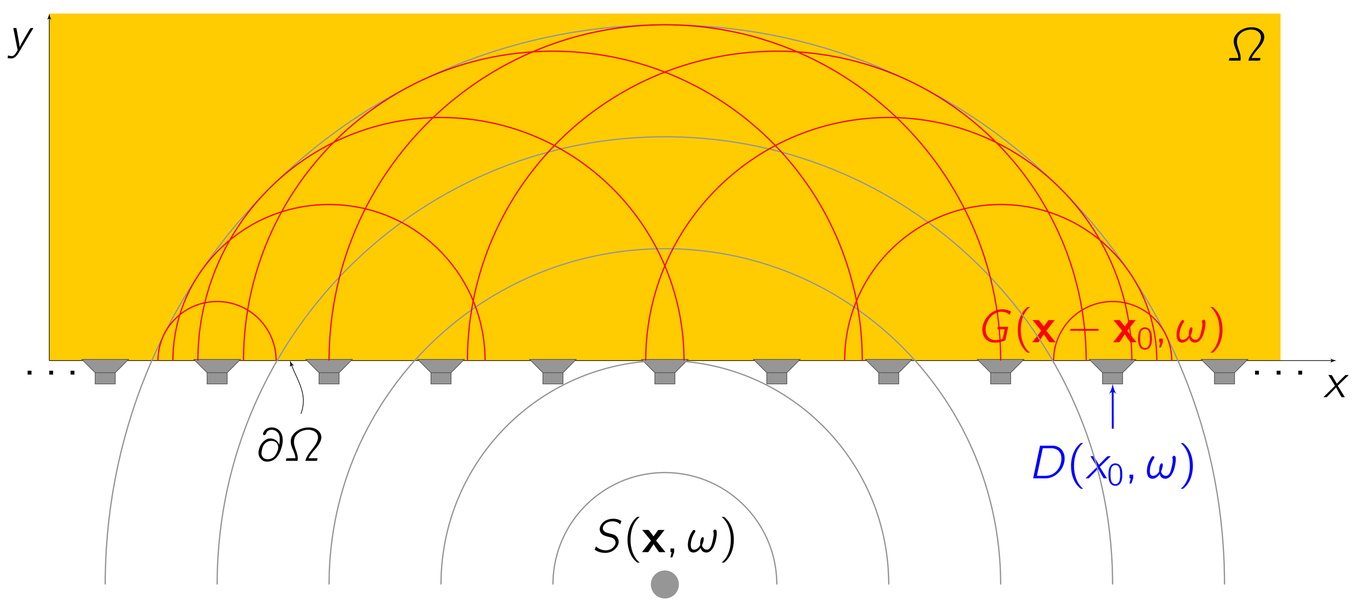
Fiete Winter[1], Jens Ahrens[2], Sascha Spors[1]
[1]Institute of Communications Engineering, University of Rostock, Germany
{fiete.winter; sascha.spors}@uni-rostock.de
[2]Division of Applied Acoustics, Chalmers University of Technology, Sweden
jens.ahrens@chalmers.se

A Geometric Model for Spatial Aliasing in Wave Field Synthesis

Abstract

Wave Field Synthesis aims at a physically accurate synthesis of a desired sound field inside a target region. Typically, the region is surrounded by a finite number of discrete loudspeakers. For practical loudspeaker setups, this spatial sampling causes spatial aliasing artefacts and does not allow for an accurate synthesis over the entire audible frequency range. In the past, different theoretical treatises of the spatial sampling process for simple loudspeaker geometries, e.g. lines and circles, led to anti-aliasing criteria independent of listener's position inside a target region. However, no inference about the spatial phenotype of the aliasing artefacts could be made by this models. This work presents a geometrical model based on high-frequency approximations of the underlying theory to describe the spatial occurrence and the propagation direction of the additional wave fronts caused by spatial aliasing. Combined with a ray-tracing algorithm, it can be used to predict position-dependent spatial aliasing artefacts for any convex loudspeaker geometry.

Wave Field Synthesis



- **goal**: accurate reproduction of desired sound source in target region Ω using a distribution of loudspeakers as so-called secondary sources along $\partial\Omega$ (loudspeaker symbols)
- determine driving signals for each secondary source such that

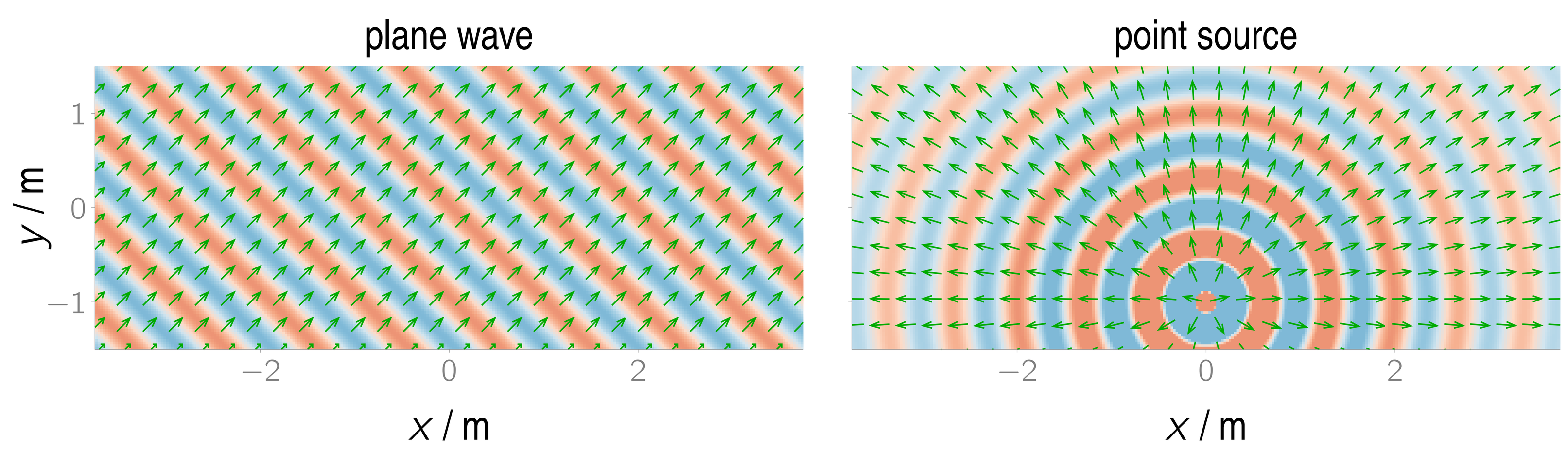
$$\underbrace{S(\mathbf{x},\omega)}_{\substack{\text{desired}\\\text{sound field}}} \overset{!}{=} \underbrace{P(\mathbf{x},\omega)}_{\substack{\text{reproduced}\\\text{sound field}}} = \int_{-\infty}^{\infty} \underbrace{D(x_0,\omega)}_{\substack{\text{driving}\\\text{signals}}} \underbrace{G(\mathbf{x}-\mathbf{x}_0,\omega)}_{\substack{\text{sound field of}\\\text{secondary source}}} \,\mathrm{d}x_0 \quad \forall \mathbf{x}\in\Omega\,. \tag{1}$$

- The continuous distribution is approximated by a equi-distantly (Δ_x) spaced loudspeaker array

$$P(\mathbf{x},\omega) \approx P^{\mathrm{S}}(\mathbf{x},\omega) = \sum_{n=-\infty}^{\infty} D(n\Delta_x,\omega) G(\mathbf{x}-\mathbf{x}_0,\omega)\Delta_x \text{ with } \mathbf{x}_0 = [n\Delta_x,0,0]^{\mathrm{T}} \tag{2}$$

Modelling of Spatial Aliasing

Local Wavenumber Vector

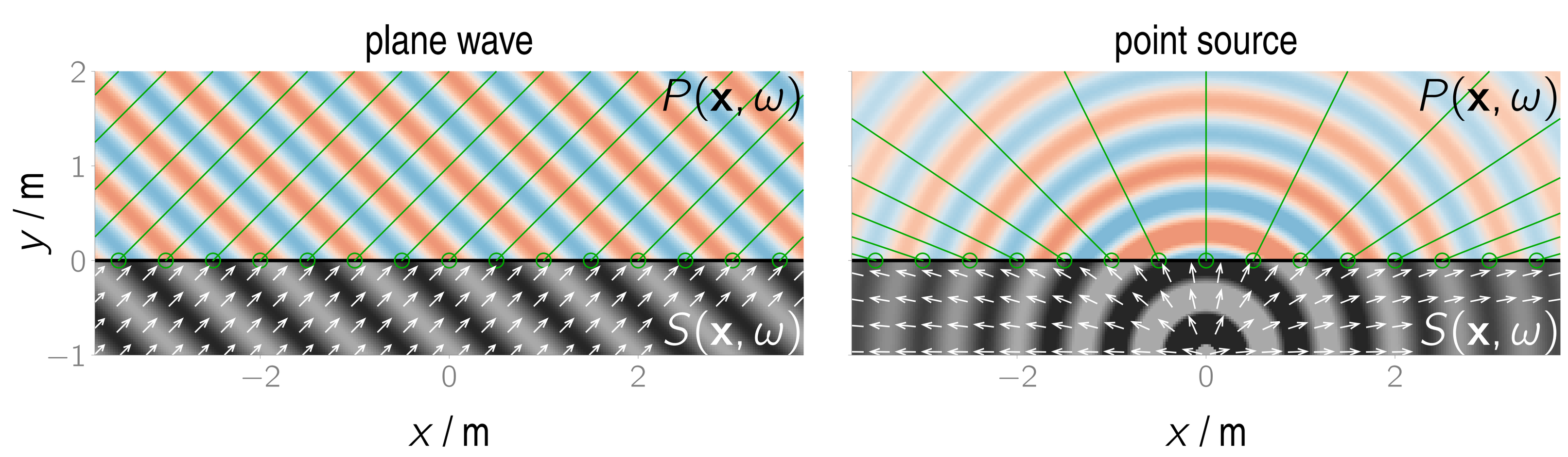


- Any sound field $P(\mathbf{x},\omega) = A_P(\mathbf{x},\omega)\,\mathrm{e}^{+\mathrm{j}\phi_P(\mathbf{x},\omega)}$ may be expressed in terms of its real-valued amplitude $A_P(\mathbf{x},\omega)$ and phase $\phi_P(\mathbf{x},\omega)$
- The 2D local wavenumber vector [?]

$$\mathbf{k}_P(\mathbf{x},\omega) := -\nabla\phi_P(\mathbf{x},\omega) \overset{\omega\to\infty}{\approx} \frac{\omega}{c}\underbrace{\hat{\mathbf{k}}_P(\mathbf{x},\omega)}_{\substack{\text{unit}\\\text{vector}}} = \frac{\omega}{c}\begin{bmatrix}\hat{k}_{P,x}(\mathbf{x},\omega)\\ \hat{k}_{P,y}(\mathbf{x},\omega)\\ 0\end{bmatrix} \tag{3}$$

describes the propagation direction of the sound field at a given coordinate (green arrows)

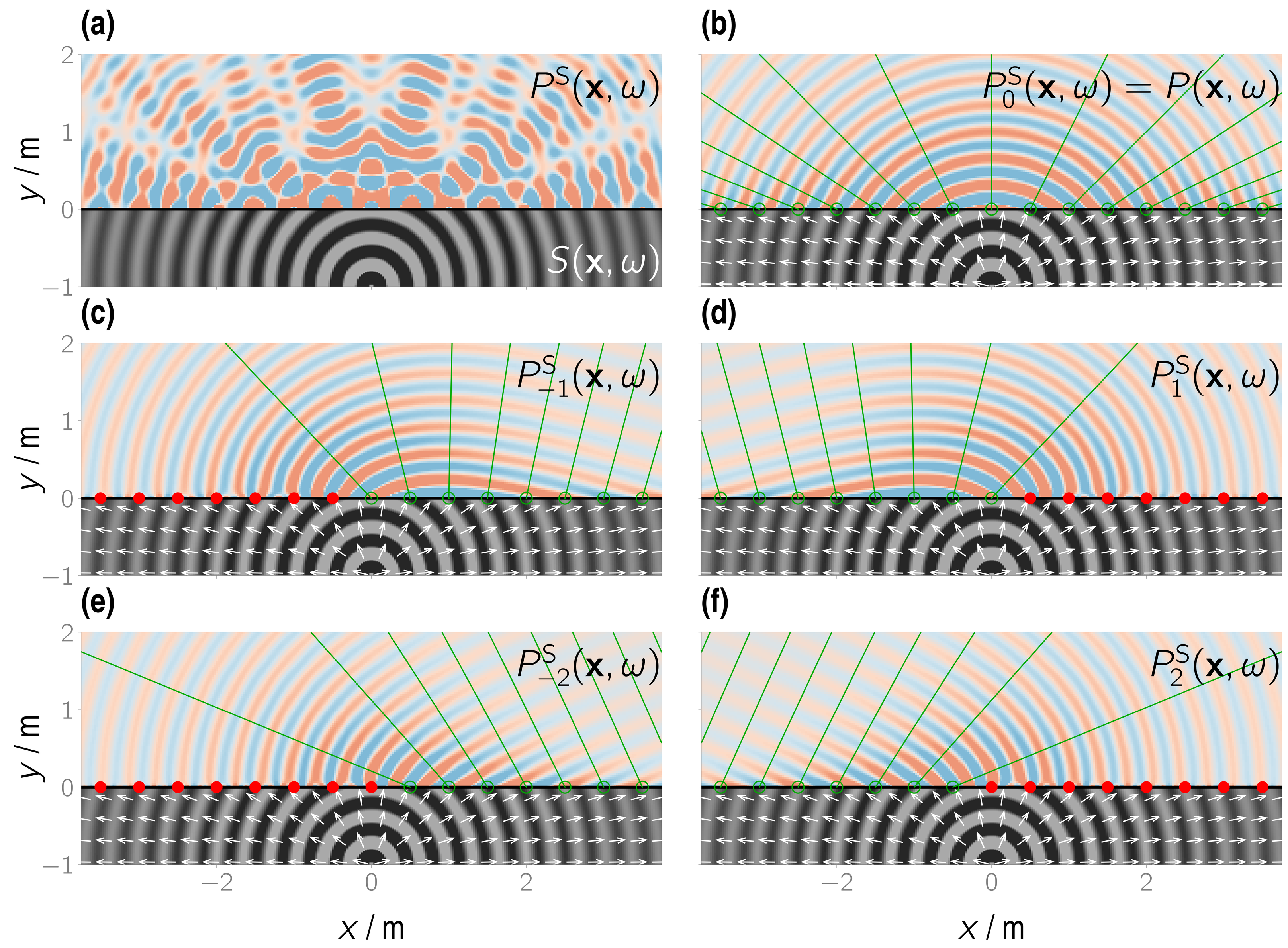
Ray-Approximation of Wave Field Synthesis



- For high frequencies ($\omega\to\infty$), the secondary source at $\mathbf{x}_0$ (green circles) completely determines the reproduced sound field $P(\mathbf{x},\omega)$ along the line/ray given by $\mathbf{x} = \mathbf{x}_0 + \gamma\,\hat{\mathbf{k}}_S(\mathbf{x}_0,\omega)$ (green lines).
- $\hat{\mathbf{k}}_S(\mathbf{x},\omega)$ is the normalised local wave vector of the desired sound field $S(\mathbf{x},\omega)$ (white arrows)
- The underlying calculus is referred to as the Stationary Phase Approximation and is used to approximate the synthesis integral in Eq. (1) by

$$P(\mathbf{x}_0+\gamma\,\hat{\mathbf{k}}_S(\mathbf{x}_0,\omega),\omega) \overset{\omega\to\infty}{\approx} D(x_0,\omega)G(\gamma\,\hat{\mathbf{k}}_S(\mathbf{x}_0,\omega),\omega) \tag{4}$$

Ray-Approximation of Aliasing Components



- Uniform sampling of the driving function is commonly modelled by multiplying $D(x_0,\omega)$ with a dirac comb $\text{Ш}\left(\frac{x_0}{\Delta_x}\right)$. The discrete driving function (2) can be expressed in the continuous domain via

$$D(n\Delta_x,\omega) \;\hat{=}\; D^{\mathrm{S}}(x_0,\omega) = D(x_0,\omega)\,\text{Ш}\left(\frac{x_0}{\Delta_x}\right) = D(x_0,\omega)\sum_{m=-\infty}^{\infty} \mathrm{e}^{-\mathrm{j}2\pi m\frac{x_0}{\Delta_x}}\,. \tag{5}$$

- The last equality allows to define the m-th aliasing component of the sampled driving function as

$$D_m^{\mathrm{S}}(x_0,\omega) := D(x_0,\omega)\mathrm{e}^{-\mathrm{j}2\pi m\frac{x_0}{\Delta_x}}\,. \tag{6}$$

- The sound field resulting from the m-th aliasing component (see Fig. (b)-(f)) can be written as

$$P_m^{\mathrm{S}}(\mathbf{x},\omega) = \int_{-\infty}^{\infty} D_m^{\mathrm{S}}(x_0,\omega)G(\mathbf{x}-\mathbf{x}_0,\omega)\,\mathrm{d}x_0\,. \tag{7}$$

- For high frequencies, the secondary source at $\mathbf{x}_0$ (green circles) completely determines $P_m^{\mathrm{S}}(\mathbf{x},\omega)$ along the line/ray given by $\mathbf{x} = \mathbf{x}_0 + \gamma\,\hat{\mathbf{k}}_m^{\mathrm{S}}(\mathbf{x}_0,\omega)$ (green lines) with

$$\hat{\mathbf{k}}_m^{\mathrm{S}}(\mathbf{x}_0,\omega) = \begin{bmatrix}\hat{k}_{S,x}(\mathbf{x}_0)+\frac{mc}{\Delta_x f}\\ \sqrt{1-\left(\hat{k}_{S,x}(\mathbf{x}_0)+\frac{mc}{\Delta_x f}\right)^2}\\ 0\end{bmatrix} \text{ constrained by } \left|\hat{k}_{S,x}(\mathbf{x}_0)+\frac{mc}{\Delta_x f}\right| \leq 1\,. \tag{8}$$

If latter condition is not fulfilled, the secondary source does not contribute to $P_m^{\mathrm{S}}(\mathbf{x},\omega)$ (red circles).

Prediction of the Spatial Aliasing Frequency

- Solving the inequality of Eq. (8) for f yields the frequency $f_m^{\mathrm{S}}(\mathbf{x}_0)$ above which the $\mathbf{x}_0$ contributes to $P_m^{\mathrm{S}}(\mathbf{x},\omega)$. No spatial aliasing is contributed, if f does not exceed this threshold for any m.

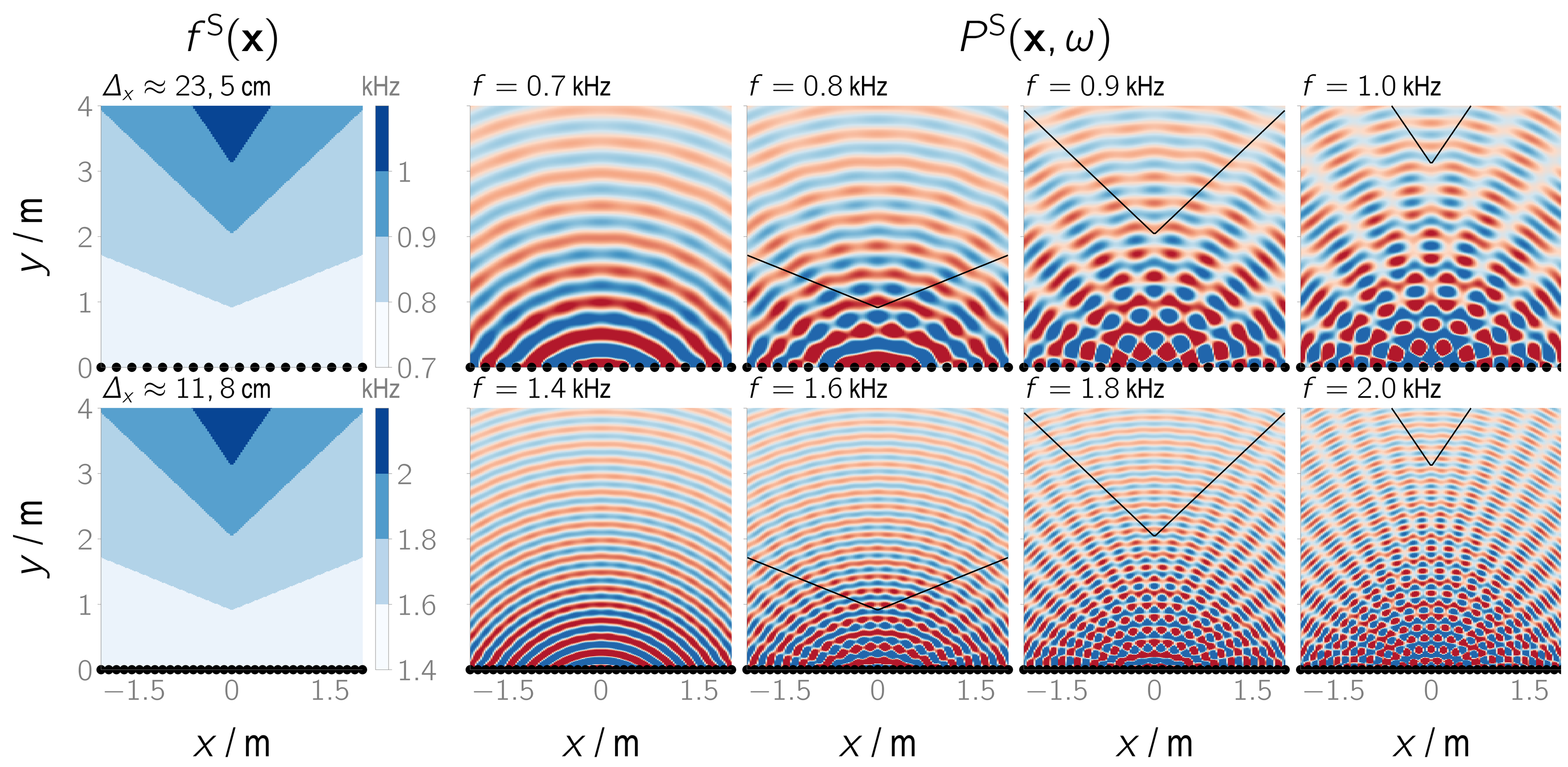
Anti-Aliasing Condition for individual Loudspeaker

$$f \leq f^{\mathrm{S}}(\mathbf{x}_0) = \min_m f_m^{\mathrm{S}}(\mathbf{x}_0) = \frac{c}{\Delta_x\left(1+\left|\hat{k}_{S,x}(\mathbf{x}_0)\right|\right)}$$

- Solving the line/ray equation given in (8) for f yields the frequency $f_m^{\mathrm{S}}(\mathbf{x},\mathbf{x}_0)$ at which a distinct loudspeaker $\mathbf{x}_0$ contributes the m-th aliasing component to $\mathbf{x}$. Taking the minimum over all loudspeakers and m defines the frequency up to which none of loudspeaker reproduces aliasing at $\mathbf{x}$.

Anti-Aliasing Condition for Listening Position

$$f \leq f^{\mathrm{S}}(\mathbf{x}) = \min_{\mathbf{x}_0} \frac{c}{\Delta_x\left|\hat{k}_{S,x}(\mathbf{x}_0)-\cos\alpha_{\mathbf{x}-\mathbf{x}_0}\right|} \quad \text{with } \cos\alpha_{\mathbf{x}-\mathbf{x}_0} = \frac{x-x_0}{\sqrt{(x-x_0)^2+y^2}}$$



References

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