Definition and Examples

Note 1

9080791fc8754b0bb88c381c10acbdfc

Let G be a group. If {{c2::H is a subgroup of G}} we shall write {{c1::

$$H \leq G.$$

}}

Note 2

66e7f23728af4c9d8839d172e59d716a

Let G be a group and $H \leq G$. We shall denote the operation for H by {{c1::the same symbol as the operation for G.}}

Note 3

e76ada2ee6da4b5fb71966e9f7ce3ded

Let G be a group. If {{c2::$H \leq G$ and $H \neq G$}} we shall write {{c1::$H < G$.}}

Note 4

1d28c11c52c84bd0b639505598bb1dce

If H is a subgroup of G then any equation in the subgroup H may also be viewed as {{c1::an equation in the group G.}}

Note 5

8f5b76596188446082314l645b5ea08b

Let G be a group and $H \leq G$. What is the identity of H?

> The identity of G.

Note 6

7c122a5400f64eba9a76438c1ff296ee

Let G be a group and $H \leq G$. The identity of H is the identity of G. What is the key idea in the proof?

> The identity is unique and it is the identity of G.

Note 7

83cba804764b43e2baf282ffee513694

Let G be a group. What is the minimal subgroup of G?

▌ The singleton $\{1\}$.

Note 8

d40df9f46a6b43ecb3fea9b5b37e5b1c

Let G be a group. What is the element that any subgroup of G must contain?

▌ The identity of G.

Note 9

bc95ad6358814213a6fff2eb0cfd544b

Let G be a group and $H \leq G$. What is the inverse of an element x in H?

▌ The inverse of x in G.

Note 10

be9f1756cf3449e8a6718069fd4aedf5

Let G be a group and $H \leq G$. Why is the notation x^{-1} unambiguous?

▌ The inverse in H is the same as the inverse in G.

Note 11

8aabd93df8a5437eb3e50c3e0d438381

Let G be a group. {{c2::The subgroup $\{1\}$ of G}} is called {{c1::the trivial subgroup.}}

Note 12

eb859714e1f34f4db3dc35755f562945

Let G be a group. {{c2::The trivial subgroup}} is denoted by {{c1::1.}}

Note 13

74ab3ceb9a36420fb53dc2b3a22eb5f2

Which subgroups does any group have?

▌ The trivial subgroup and the group itself.

Note 14

5683ff4198a74e9d988f501c925d85ad

If H is a subgroup of G and K is a subgroup of H, then {{c1::K is a subgroup of G.}}

Note 15

50a07cbb14aa4bed8866efcbedb0be4d

Which object is considered in the Subgroup Criterion?

> Any subset of a group.

Note 16

840038893a3642a18c3e43c4e89aed17

What are the conditions of the Subgroup Criterion?

> The subset is nonempty and closed under $(x, y) \mapsto x \cdot y^{-1}$.

Note 17

71291d04ca2941fca2fc08759d8fd302

What is the special case considered in the Subgroup Criterion?

> The subset is finite.

Note 18

a1e69be09e78402d989b3805b3dfc54f

What are the conditions of the Subgroup Criterion for a finite subset?

> The subset is nonempty and closed under the operation.

Note 19

5bcd55a73e184bcd9bcc32f1ee47da2e

What is the key idea in the proof of the Subgroup Criterion for a finite subset?

> Any element's inverse is it's n-th power.

Note 20

0e1ccaae016c4900ac96b733fb9e1764

Why is the set of 2-cycles in S_n not a subgroup of S_n?

> It does not contain the identity.

Note 21

587390d0450f4681a66bcbc8c0d5889c

Why is the set of reflection in D_{2n} not a subgroup of D_{2n}?

▌ It does not contain the identity.

Note 22

fc87d2283cb546708502ce325e326258

Why is the set of reflection in D_{2n} together with 1 not a subgroup of D_{2n}?

▌ Two distinct reflections induce a rotation.

Note 23

24b90e714649459ba38e6b40f07f6b2a

Is $\{1, r^2, s, sr^2\}$ a subgroup of D_8?

▌ Yes.

Note 24

eac99978715a4ec894b296f8e1ee52f3

Is $\{1, r, s, sr\}$ a subgroup of D_8?

▌ No.

Note 25

64ea968bdce94647b6fb2c351a60f2a2

Is $\{1, r^2, sr, sr^3\}$ a subgroup of D_8?

▌ Yes.

Note 26

678b87f890ac4d8da5be6a78cb619358

Is $\{1, r, r^2\}$ a subgroup of D_8?

▌ No.

Note 27

e036f3cc7667461b98e50e94ff3a8c80

Is $\{1, r, r^2, r^3\}$ a subgroup of D_8?

▌ Yes.

Note 28

209944ca7a524af3be44b398de974c2d

Give an example of a group and its infinite subset that is closed under the operations, but is not a subgroup of the original group.

> Positive integers under addition.

Note 29

547363a46106478187c20c5cbb868461

For what groups is the notion of the torsion subgroup introduced?

> For abelian groups.

Note 30

d29b9ffdb46c4c909fbfb2a438abb0a0

What is the torsion subgroup of an abelian group?

> The set of all the elements of a finite order.

Note 31

b2a854579339471d8ae41776f1661f29

Let G be an abelian group. What is the name of the set

$$\{g \in G : |g| < \infty\}\,?$$

> The torsion subgroup of G.

Note 32

2a685e6476b94b9eac539a17441574ef

Why is the notion of the torsion subgroup introduced only for abelian groups?

> For non-abelian groups the set is not guaranteed to form a subgroup.

Note 33

22a771a961c3498f88a030fabf778797

Give an example of a non-abelian group, who's "torsion subgroup" is not actually a subgroup.

> $GL_3(\mathbb{R})$

Note 34

f4edb9436c094103b0b9b82019185296

Give an example of two elements a, b in $GL_3(\mathbb{R})$ such that

$$|a|, |b| < \infty \quad \text{and} \quad |ab| = \infty.$$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & 1 \end{bmatrix}.$$

Note 35

f2c41a74f8a04bb892b471915e533055

What is the torsion subgroup of $\mathbb{Z} \times (\mathbb{Z}/n\mathbb{Z})$?

> The set of elements who's first component is 0.

Note 36

4df6b5997d30483fb469565c89630322

When is the union of two subgroups also a subgroup?

> If and only if one of the subgroups is a subset of the other.

Note 37

60129b39ceab4468915a6d2237915c1a

Let H and K be subgroups of G and $H \subseteq K$. What do we know about $H \cup K$?

> It is a subgroup of G.

Note 38

791301f78ecf4800a13e3a0299c57028

Let H and K be subgroups of G. If $H \cup K$ is a subgroup of G, then $H \subseteq K$ or $K \subseteq H$. What is the key idea in the proof?

> By contradiction.

Note 39

cc8decdf60194667b3b27ff0941c9fc0

What is the special linear group?

> The set of square matrices who's determinant is 1.

Note 40

1e420dd97e1942b3b7bc70d71fc0953e

{{c2::The special linear group of $n \times n$ matrices over a field F}} is denoted {{c1::$SL_n(F)$.}}

Note 41

941aeeb281da4b009ffdc95864eddb3b

When is the intersection of two subgroups also a subgroup?

> Always.

Note 42

887cf7600d994fcd9662e35fc9719c62

When is the intersection of an infinite number of subgroups also a subgroup?

> Always.

Note 43

3bdd7a0f0e044c6b9c3c1811d4478f10

Let $H_1 \leq H_2 \leq \cdots$ be an ascending chain of subgroups of G. Then {{c1::$\bigcup_{i=1}^{\infty} H_i$ is a subgroup of G.}}