Basics

Note 1

21e64c2f0430467f8a36481045e172b3

{{c2::$\mathbb{Z}^+$, $\mathbb{Q}^+$ and $\mathbb{R}^+$}} denote {{c1::the positive (nonzero) elements in $\mathbb{Z}$, $\mathbb{Q}$ and $\mathbb{R}$, respectively.}}

Note 2

ad32571de5d04cafb2a7d6a27aee4b14

Given a function $f : A \to B$, {{c1::the set B}} is called the codomain of f.

Note 3

0cd492ad876a4dfbbb22c9210039fcc1

Given a function $f : A \to B$ and {{c3::a $b \in B$,}} {{c2::the preimage of $\{b\}$ under f}} is called {{c1::the fiber of f over b.}}

Note 4

b2a02c66209140a591b43dede69ffbf1

If $f : A \to B$ and $g : B \to C$, then the {{c1::composite map}}

$$g \circ f : A \to C$$

is defined by

$$(g \circ f)(a) = g(f(a)).$$

Note 5

1b2bf2fe79dc4063a151a960f45698d9

A function $f : A \to B$ {{c3::has a left inverse}} if there is a function $g :$ {{c2::$B \to A$}}, such that {{c1::

$$g \circ f = id_A.$$

}}

Note 6

d9a63bd7866e44ab83172cf9189e9b9a

A function $f : A \to B$ {{c3::has a right inverse}} if there is a function $g :$ {{c2::$B \to A$}}, such that {{c1::

$$f \circ g = id_B.$$

}}

Note 7

8098015b57774529a721663e39eabb18

A map f is {{c1::injective}} if and only if f has a {{c2::left}} inverse.

Note 8

205100b0fd6447a9bcc94e4d7711a606

A map f is {{c1::surjective}} if and only if f has a {{c2::right}} inverse.

Note 9

8e4ddf27550f4aa9a1daf0b67cd2f7e4

A {{c2::permutation}} of a set A is {{c1::a bijection from A to itself.}}

Note 10

1feef80fbcdd48618084ce93c88df83b

If $A \subseteq B$ and $f : B \to C$, {{c2::the restriction of f to A}} is denoted {{c1::$f|_A$.}}

Note 11

01a5a3b0e5f24f6782e689090b17c437

If $A \subseteq B$ and $g : A \to C$ and there is a function $f : B \to C$ such that {{c2::$f|_A = g$,}} we shall say f is {{c1::an extension of g to B.}}

Note 12

6ca7e478954c4c898718ce116219822f

{{c2::A binary relation on a set A}} is {{c1::a subset R of $A \times A$.}}

Note 13

50bb82cd97cb40bf8621065845545d18

Let R be a binary relation on a set A. We write {{c2::$a \sim b$}} if {{c1::$(a, b) \in R$.}}

Note 14

65287096376a47f399a0048c0d8092d0

A binary relation R on A is said to be {{c2::reflexive}} if {{c1::

$$a \sim a, \text{ for all } a \in A.$$

}}

Note 15

71b961a1f8f347dcbf7b9c7c8dee303c

A binary relation R on A is said to be {{c2::symmetric}} if {{c1::

$$a \sim b \text{ implies } b \sim a \text{ for all } a, b \in A.$$

}}

Note 16

40964931d9594b2997437cc9e3e150cc

A binary relation R on A is said to be {{c2::transitive}} if {{c1::

$$a \sim b \text{ and } b \sim c \text{ implies } a \sim c \text{ for all } a, bc \in A.$$

}}

Note 17

54a959a8e36045c1aea2d838ce8998b8

A binary relation is {{c2::an equivalence relation}} if {{c1::it is reflexive, symmetric and transitive.}}

Note 18

7c28d643ddd74509b88cfe2f75e6d743

If $\sim$ defines an {{c3::equivalence}} relation on A, then {{c2::the equivalence class}} of $a \in A$ is defined to be {{c1::

$$\{x \in A \mid x \sim a\}.$$

}}

Note 19

323fae73cd4b47ddb8c19cb515ffd4cf

If C is an equivalence class, {{c2::any element of C}} is called {{c1::a representative of the class C.}}

Note 20

3a597e1d5c48420490d792b972a38fe6

{{c2::A partition of a set A}} is {{c3::any collection $\{A_i \mid i \in I\}$ of nonempty subsets of A}} such that {{c1::A is the disjoint union of all A_i.}}

Note 21

c2216701429649b7a262afdd5c85a72d

If $\sim$ defines an equivalence relation on A then {{c2::the set of equivalence classes of $\sim$}} form {{c1::a partition of A.}}

Properties of the Integers

Note 1

f535d29c343f494fa35bccefce9d6988

Let $a, b \in \mathbb{Z}$. We write {{c2::$a \mid b$}} if {{c1::a divides b.}}

Note 2

96293ae3b76348d8ba9f0b02c8b49a94

Let $a, b \in \mathbb{Z}$ with $a \neq 0$. We write {{c2::$a \nmid b$}} if {{c1::a does not divide b.}}

Note 3

533403fe830341a39cee216314b861e8

Let $a, b \in$ {{c3::$\mathbb{Z} - \{0\}$ }}. {{c2::The greatest common divisor of a and b}} is denoted by {{c1::(a, b).}}

Note 4

20b204b896884b6b9d07ca3023b7cf4a

Let $a, b \in$ {{c3::$\mathbb{Z} - \{0\}$ }}. If {{c2::$(a, b) = 1$,}} we say that a and b are {{c1::relatively prime.}}

Note 5

69adfe8820204997a5aa44c50b353a40

If $a, b \in \mathbb{Z} - \{0\}$, then there exists unique $q, r \in \mathbb{Z}$ such that

$$a = qb + r \text{ and } 0 \leqslant r < |b|,$$

where q is {{c1::the quotient}} and r {{c1::the remainder.}}

«{{c2::Division Algorithm}}»

Note 6

6267be99c4884a09b1282d041ac05e18

If $a, b \in \mathbb{Z} - \{0\}$, then there exist $x, y \in$ {{c3::$\mathbb{Z}$}} such that

$$\text{\{\{c2::}(a, b)\text{\}\}} = \text{\{\{c1::}xa + yb.\text{\}\}}$$

Note 7

e30ea564f2ce479391e71512867aea51

If p is prime and $p \mid ab$, for some $a, b \in \mathbb{Z}$, then {{c1::

$$\text{either } p \mid a \text{ or } p \mid b.$$

}}

Note 8

3cc931ead0ec4cddae21114d84f1de0c

{{c3::The Euler φ-function}} is defined as follows: for $n \in$ {{c2::$\mathbb{Z}^+$}} let $\varphi(n)$ be {{c1::the number of positive integers $a \leq n$ with a relatively prime to n.}}

Note 9

03f37e11eb9d40d29ca92031ac27d9ed

Let φ stand for the Euler φ-function. If p is {{c3::prime}} and $a \geq 1$, then

$$\{\{c2::\varphi(p^a)\}\} = \{\{c1::p^a - p^{a-1}.\}\}$$

Note 10

7dc766a783c04a309951678711bd8317

Let φ stand for the Euler φ-function. Then

$$\{\{c1::\varphi(ab) = \varphi(a)\varphi(b)\}\} \quad \text{if } \{\{c2::(a, b) = 1.\}\}$$