Definition and Examples

Note 1

9080791fc8754b0bb88c381c10acbdfc

Let G be a group. If {{c2::H is a subgroup of G}} we shall write {{c1::

$$H \leq G.$$

}}

Note 2

66e7f23728af4c9d8839d172e59d716a

Let G be a group and $H \leq G$. We shall denote the operation for H by {{c1::the same symbol as the operation for G.}}

Note 3

e76ada2ee6da4b5fb71966e9f7ce3ded

Let G be a group. If {{c2::$H \leq G$ and $H \neq G$}} we shall write {{c1::$H < G$.}}

Note 4

1d28c11c52c84bd0b639505598bb1dce

If H is a subgroup of G then any equation in the subgroup H may also be viewed as {{c1::an equation in the group G.}}

Note 5

8f5b76596188446082314l645b5ea08b

Let G be a group and $H \leq G$. What is the identity of H?

> The identity of G.

Note 6

7c122a5400f64eba9a76438c1ff296ee

Let G be a group and $H \leq G$. The identity of H is the identity of G. What is the key idea in the proof?

> The identity is unique and it is the identity of G.

Note 7

83cba804764b43e2baf282ffee513694

Let G be a group. What is the minimal subgroup of G?

▌ The singleton $\{1\}$.

Note 8

d40df9f46a6b43ecb3fea9b5b37e5b1c

Let G be a group. What is the element that any subgroup of G must contain?

▌ The identity of G.

Note 9

bc95ad6358814213a6fff2eb0cfd544b

Let G be a group and $H \leq G$. What is the inverse of an element x in H?

▌ The inverse of x in G.

Note 10

be9f1756cf3449e8a6718069fd4aedf5

Let G be a group and $H \leq G$. Why is the notation x^{-1} unambiguous?

▌ The inverse in H is the same as the inverse in G.

Note 11

8aabd93df8a5437eb3e50c3e0d438381

Let G be a group. {{c2::The subgroup $\{1\}$ of G}} is called {{c1::the trivial subgroup.}}

Note 12

eb859714e1f34f4db3dc35755f562945

Let G be a group. {{c2::The trivial subgroup}} is denoted by {{c1::1.}}

Note 13

74ab3ceb9a36420fb53dc2b3a22eb5f2

Which subgroups does any group have?

▌ The trivial subgroup and the group itself.

Note 14

5683ff4198a74e9d988f501c925d85ad

If H is a subgroup of G and K is a subgroup of H, then {{c1::K is a subgroup of G.}}

Note 15

50a07cbb14aa4bed8866efcbedb0be4d

Which object is considered in the Subgroup Criterion?

> Any subset of a group.

Note 16

840038893a3642a18c3e43c4e89aed17

What are the conditions of the Subgroup Criterion?

> The subset is nonempty and closed under $(x, y) \mapsto x \cdot y^{-1}$.

Note 17

71291d04ca2941fca2fc08759d8fd302

What is the special case considered in the Subgroup Criterion?

> The subset is finite.

Note 18

a1e69be09e78402d989b3805b3dfc54f

What are the conditions of the Subgroup Criterion for a finite subset?

> The subset is nonempty and closed under the operation.

Note 19

5bcd55a73e184bcd9bcc32f1ee47da2e

What is the key idea in the proof of the Subgroup Criterion for a finite subset?

> Any element's inverse is it's n-th power.

Note 20

0e1ccaae016c4900ac96b733fb9e1764

Why is the set of 2-cycles in S_n not a subgroup of S_n?

> It does not contain the identity.

Note 21

587390d0450f4681a66bcbc8c0d5889c

Why is the set of reflection in D_{2n} not a subgroup of D_{2n}?

- It does not contain the identity.

Note 22

fc87d2283cb546708502ce325e326258

Why is the set of reflection in D_{2n} together with 1 not a subgroup of D_{2n}?

- Two distinct reflections induce a rotation.

Note 23

24b90e714649459ba38e6b40f07f6b2a

Is $\{1, r^2, s, sr^2\}$ a subgroup of D_8?

- Yes.

Note 24

eac99978715a4ec894b296f8e1ee52f3

Is $\{1, r, s, sr\}$ a subgroup of D_8?

- No.

Note 25

64ea968bdce94647b6fb2c351a60f2a2

Is $\{1, r^2, sr, sr^3\}$ a subgroup of D_8?

- Yes.

Note 26

678b87f890ac4d8da5be6a78cb619358

Is $\{1, r, r^2\}$ a subgroup of D_8?

- No.

Note 27

e036f3cc7667461b98e50e94ff3a8c80

Is $\{1, r, r^2, r^3\}$ a subgroup of D_8?

- Yes.

Note 28

209944ca7a524af3be44b398de974c2d

Give an example of a group and its infinite subset that is closed under the operations, but is not a subgroup of the original group.

> Positive integers under addition.

Note 29

547363a46106478187c20c5cbb868461

For what groups is the notion of the torsion subgroup introduced?

> For abelian groups.

Note 30

d29b9ffdb46c4c909fbfb2a438abb0a0

What is the torsion subgroup of an abelian group?

> The set of all the elements of a finite order.

Note 31

b2a854579339471d8ae41776f1661f29

Let G be an abelian group. What is the name of the set

$$\{g \in G : |g| < \infty\}\,?$$

> The torsion subgroup of G.

Note 32

2a685e6476b94b9eac539a17441574ef

Why is the notion of the torsion subgroup introduced only for abelian groups?

> For non-abelian groups the set is not guaranteed to form a subgroup.

Note 33

22a771a961c3498f88a030fabf778797

Give an example of a non-abelian group, who's "torsion subgroup" is not actually a subgroup.

> $GL_3(\mathbb{R})$

Note 34

f4edb9436c094103b0b9b82019185296

Give an example of two elements a, b in $GL_3(\mathbb{R})$ such that

$$|a|\,, |b| < \infty \quad \text{and} \quad |ab| = \infty.$$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & 1 \end{bmatrix}.$$

Note 35

f2c41a74f8a04bb892b471915e533055

What is the torsion subgroup of $\mathbb{Z} \times (\mathbb{Z}/n\mathbb{Z})$?

> The set of elements who's first component is 0.

Note 36

4df6b5997d30483fb469565c89630322

When is the union of two subgroups also a subgroup?

> If and only if one of the subgroups is a subset of the other.

Note 37

b523dcaec101461e902f34191451e112

When is the union of an infinite number of subgroups also a subgroup?

> It depends.

Note 38

60129b39ceab4468915a6d2237915c1a

Let H and K be subgroups of G and $H \subseteq K$. What do we know about $H \cup K$?

> It is a subgroup of G.

Note 39

791301f78ecf4800a13e3a0299c57028

Let H and K be subgroups of G. If $H \cup K$ is a subgroup of G, then $H \subseteq K$ or $K \subseteq H$. What is the key idea in the proof?

> By contradiction.

Note 40

cc8decdf60194667b3b27ff0941c9fc0

What is the special linear group?

▌ The set of square matrices who's determinant is 1.

Note 41

1e420dd97e1942b3b7bc70d71fc0953e

{{c2::The special linear group of $n \times n$ matrices over a field F}} is denoted {{c1::$SL_n(F)$.}}

Note 42

941aeeb281da4b009ffdc95864eddb3b

When is the intersection of two subgroups also a subgroup?

▌ Always.

Note 43

887cf7600d994fcd9662e35fc9719c62

When is the intersection of an infinite number of subgroups also a subgroup?

▌ Always.

Note 44

3bdd7a0f0e044c6b9c3c1811d4478f10

Let $H_1 \leq H_2 \leq \cdots$ be an ascending chain of subgroups of G. Then {{c1::$\bigcup_{i=1}^{\infty} H_i$ is a subgroup of G.}}

Centralizers and Normalizers, Stabilizers and Kernels

Note 1

93de251693e74655a5752529379e7081

For what do we define centralizers in groups?

> For nonempty subsets of the group.

Note 2

b46233e067ea4c24b38af57081ef1db3

Let G be a group and A be a nonempty subset of G. {{c1::The set

$$\{g \in G \mid gag^{-1} = a \text{ for all } a \in A\}$$

}} is called {{c2::the centralizer of A in G.}}

Note 3

3c2adb104b55494a8a248b4e6cf72980

Let G be a group and A be a nonempty subset of G. {{c2::The centralizer of A in G}} is denoted {{c1::

$$C_G(A)\,.$$

}}

Note 4

aeea9d02d1a8429ab94927313c1e2194

How can centralizers be redefined in terms of commutativity?

> As the set of all the elements that commute with every element of the subset.

Note 5

588fd51b4281485c87a74faa9ddbf8f5

Let G be a group and A be a nonempty subset of G. The centralizer of A in G forms {{c1::a subgroup of G.}}

Note 6

18e60ffefbe647a6aa9b7a9feeb58ef1

Let G be a group and A be a nonempty subset of G. When is the centralizer of A in G a subgroup of G?

▌ Always.

Note 7

23eee6bafc20447987eaab729108324e

Let G be a group and A be a nonempty subset of G. In the special case when $A = \{a\}$ we shall write {{c1::simply $C_G(a)$}} instead of {{c2::$C_G(\{a\})$.}}

Note 8

92e1f52031224232bf8ac69f4014862c

Let G be a group and $a \in G$. Then

$$\text{\{\{c1::}\langle a \rangle\text{\}\}} \subseteq C_G(a)\,.$$

Note 9

656f69d38b7e4f41882f7feda5410dde

$$C_{Q_8}(i) = \text{\{\{c1::}\ \{1, -1, i, -i\}\ .\text{\}\}}$$

Note 10

1fd69e94ef324e30a0054ea4860105e4

$$C_{Q_8}(1) = \text{\{\{c1::}Q_8\ .\text{\}\}}$$

Note 11

1ec04d8a609e442690da0ee9332a9647

For what do we define centers in groups?

▌ For the group itself.

Note 12

936431cf3df24996965ce022800fa1bc

Let G be a group. {{c2::The set of elements of G commuting with all elements of G}} is called {{c1::the center of G.}}

Note 13

4fdf7ab9640d453a9eb90b77b45f35b2

Let G be a group. {{c2::The center of G}} is denoted {{c1::$Z(G)$.}}

Note 14

b7d3c0377db64825b9428611799a938c

Let G be a group. The center of G forms {{c1::a subgroup of G.}}

Note 15

3c3e0bd81b194850bbeed0d6688646ea

Let G be a group. When is the center of G a subgroup of G?

> Always.

Note 16

10b56b05fab34d6a91d494a9c515f2a4

Let G be a group. {{c2::The center of G}} is the centralizer of {{c1::G in G.}}

Note 17

4c598c0713ef4d649fe3629dfcd8a0c7

For what do we define normalizers in groups?

> For nonempty subsets.

Note 18

ea05e39de520479892867fd132778337

Let G be a group and A be a nonempty subset of G. {{c2::The set

$$\{g \in G \mid gAg^{-1} = A\}$$

}} is called {{c1::the normalizer of A in G.}}

Note 19

c4b4c4424c6549c6b9afccb2945d17ee

Let G be a group and A be a nonempty subset of G. {{c2::The normalizer of A in G}} is denoted {{c1::

$$N_G(A)\,.$$

}}

Note 20

f831383807f34538b90b130d417dfb95

Let G be a group and A be a nonempty subset of G. The normalizer of A in G forms {{c1::a subgroup of G.}}

Let G be a group and A be a nonempty subset of G. When is the normalizer of A in G a subgroup of G?

> Always.