Prerequisites

Note 1

621caffff9ce421bb4309fc0c1cf144c

A function is said to be {{c2::multilinear}} if and only if it is {{c1::linear separately in each variable.}}

Note 2

1a514ffb24744a278834d0048496a850

A function is said to be {{c2::bilinear}} if and only if {{c1::it is a multilinear function of two argument.}}

Note 3

6712178af383453faa8c5bad8aeabc89

{{c2::An endomorphism}} of a vector space is {{c1::a linear map from this space to itself.}}

Note 4

d6b4c6b47276475dbc8548d1c5240801

{{c2::The characteristic}} of a ring R is {{c1::the smallest positive number n such that

$$\underbrace{1+\cdots+1}_{n}=0,$$

or 0, if no such n exists.}}

Note 5

7ea5080a9df1419e88226f7df77af8db

{{c1::The characteristic}} of a ring R is denoted {{c2::char R.}}

Note 6

e270ce3bf39a4ad4b08726ec08e3353a

Let V be a vector space over a field K. {{c1::A linear map

$$V \to K$$

}} is called {{c2::a linear form on the vector space V.}}

Note 7

49d3a5be7380467e9f537fc3ce7b1197

Let V be a vector space over a field K. {{c1::The set of all the linear forms $V \to K$}} is called {{c2::the dual space of V.}}

Note 8

584e4213ae234a62a4eba3b726f581c0

{{c1::The dual space}} of a vector space V is denoted {{c2::V^*.}}

Note 9

c3c550f6f7284b649efd64a23b3fba07

Let V be a vector space over a field K. {{c1::A bilinear map

$$V \times V \to K$$

}} is called {{c2::a bilinear form on the vector space V.}}

Note 10

b578c7a8572d40ffbe4a3f132fda2107

A bilinear form $f : V \times V \to K$ is said to be {{c2::nondegenerate}} if {{c1::each of its corresponding linear maps $V \to V^*$ is nondegenerate.
}}

Note 11

7c1d2db5fbfb4cb2b6566a0787bf4475

Let $f : V \times V \to K$ be a bilinear form with a matrix A. Then, f is {{c2::nondegenerate}} {{c3::if and only if}} {{c1::$\det A \neq 0$.}}

Note 12

a4b1e02fac454bc682ea8f3d5c2dd6d6

Let V be a vector space over a field K and $e_1, \ldots, e_n$ be a basis in V. {{c1::The matrix

$$A = \Big(f(e_i, e_j) \Big) \sim n \times n$$

}} is called {{c2::the matrix of the bilinear form f on the basis $e_1, \ldots, e_n$.}}

Note 13

51ff92196aa24f35b1b42ac20f96306a

A linear transformation f : {{c3::$V \to V$}} is said to be {{c2::nilpotent
}} if {{c1::$f^k = 0$ for some integer k.}}

Note 14

42ce6cb27737499b9007da0c75d82e22

Let {{c3::N be a subspace of a vector space X.}} Two vectors $x_1, x_2 \in X$ are said to be {{c2::congruent modulo N,}} if {{c1::

$$x_1 - x_2 \in N.$$

}}

Note 15

405a3568dd064c36b532f8831937ffde

Let N be a subspace of a vector space X. The statement "{{c1::Two vectors $x_1, x_2 \in X$ are congruent modulo N}}" is denoted {{c2::

$$x_1 \equiv x_2 \pmod{N}.$$

}}

Note 16

ac00e08b0afb47349cf375268cda7ca0

Let {{c3::N be a subspace of a vector space X, $x \in X$.}} {{c1::The equivalence class of all vector congruent to x modulo N}} is denoted {{c2::$x + N$.}}

Note 17

92432098fb8e4ff5b95b22c8ae18884d

Let N be a subspace of a vector space X, $x \in X$.

$$\text{\{\{c2::}x + N\text{\}\}} \overset{\text{def}}{=} \text{\{\{c1::} \{x + n \mid n \in N\} .\text{\}\}}$$

Note 18

9bfda8056bdd49bfaf09c57ca40958ec

Let {{c3::N be a subspace of a vector space X.}} {{c2::The quotient space of X modulo N}} is {{c1::the set of all equivalence classes $x + N$ in X.}}

Note 19

408ab99bf03a4908829acecbdbaf5093

Let N be a subspace of a vector space X. {{c1::The quotient space of X modulo N}} is denoted {{c2::X/N.}}

1.1. The notion of Lie algebra

Note 1

686bbc96abfb46e883a4acb108450cc1

At the first place a Lie algebra is {{c1::a vector space L over a field $\mathbb{F}$}}.

Note 2

7a252531934f4c00829418ab1f3a1d01

What is the signature of the new operation in the definition of a Lie algebra?

> $$L \times L \to L.$$

Note 3

a1cc6426fa49471dad192df5295fb310

The operation $L \times L \to L$ from the definition of a Lie algebra is denoted {{c1::$(x, y) \mapsto [xy]$}}.

Note 4

8bb3c76247ab416a97f8f6e247a6c2a2

The operation $(x, y) \mapsto [xy]$ from the definition of a Lie algebra is called {{c1::the bracket or commutator of x and y}}.

Note 5

6c529b4b819a45c3b91755b1280be2a2

How many axioms are there in the definition of a Lie algebra?

> $(L1)$, $(L2)$, $(L3)$.

Note 6

f8d0434e7d3c404b8319bf527f96627c

What is the axiom $(L1)$ from the definition of a lie algebra?

> The bracket operation is bilinear.

Note 7

807fbd0c878541998eb3be30e870652c

What is the axiom $(L2)$ from the definition of a lie algebra?

▌ $[xx] = 0 \quad$ for all $x \in L$.

Note 8

d096a87546b14acfa601179c2ae323e8

What is the axiom $(L3)$ from the definition of a Lie algebra?

▌ $[x[yz]] + [y[zx]] + [z[xy]] = 0 \quad$ for all $x, y, z \in L$.

Note 9

db6289e2261549bcb58877ac4d6f36f7

{{c2::The axiom $(L3)$ from the definition of a Lie algebra}} is called {{c1::the Jacobi identity}}.

Note 10

fdd2c3d3027e4a34be5e9540f148a9cd

Let L, L' be two Lie algebras over F. {{c1::A vector space isomorphism $\phi : L \to L'$ satisfying

$$\phi([xy]) = [\phi(x)\phi(y)] \quad \forall x, y \in L$$

}} is called {{c2::an isomorphism of Lie algebras.}}

Note 11

cbd7fbe2d06c41e29e72d8075fc10e5f

We say that two Lie algebras L, L' over F are {{c2::isomorphic}} if {{c1::there exists a Lie algebra isomorphism $\phi : L \to L'$.}}

Note 12

fe5ab449614b4098a1e81d3d86903d64

Let L be a Lie algebra over F. {{c2::A subspace K of L satisfying

$$[xy] \in K \quad \forall x, y \in K.$$

}} is called {{c1::a subalgebra of L}}