

Prerequisites

Note 1

621caffff9ce421bb4309fc0c1cf144c

A function is said to be $\{\{c2:\text{multilinear}\}\}$ if and only if it is $\{\{c1:\text{linear}\}$ separately in each variable. $\}$

Note 2

1a514ffb24744a278834d0048496a850

A function is said to be $\{\{c2:\text{bilinear}\}\}$ if and only if $\{\{c1:\text{it is a multilinear function of two argument.}\}$

Note 3

6712178af383453faa8c5bad8aeabc89

$\{\{c2:\text{An endomorphism}\}\}$ of a vector space is $\{\{c1:\text{a linear map from this space to itself.}\}$

Note 4

d6b4c6b47276475dbc8548d1c5240801

$\{\{c2:\text{The characteristic}\}\}$ of a ring R is $\{\{c1:\text{the smallest positive number } n \text{ such that}$

$$\underbrace{1 + \cdots + 1}_n = 0,$$

or 0, if no such n exists. $\}$

Note 5

7ea5080a9df1419e88226f7df77af8db

$\{\{c1:\text{The characteristic}\}\}$ of a ring R is denoted $\{\{c2:\text{char } R.\}\}$

1.1. The notion of Lie algebra

Note 1

686bbc96abfb46e883a4acb108450cc1

At the first place a Lie algebra is a vector space L over a field F .

Note 2

7a252531934f4c00829418ab1f3a1d01

What is the signature of the new operation in the definition of a Lie algebra?

$$L \times L \rightarrow L.$$

Note 3

a1cc6426fa49471dad192df5295fb310

The operation $L \times L \rightarrow L$ from the definition of a Lie algebra is denoted $(x, y) \mapsto [xy]$.

Note 4

8bb3c76247ab416a97f8f6e247a6c2a2

The operation $(x, y) \mapsto [xy]$ from the definition of a Lie algebra is called the bracket or commutator of x and y .

Note 5

6c529b4b819a45c3b91755b1280be2a2

How many axioms are there in the definition of a Lie algebra?

(L1), (L2), (L3).

Note 6

f8d0434e7d3c404b8319bf527f96627c

What is the axiom (L1) from the definition of a lie algebra?

The bracket operation is bilinear.

Note 7

807fbd0c878541998eb3be30e870652c

What is the axiom (L2) from the definition of a lie algebra?

$$\llbracket [xx] = 0 \quad \text{for all } x \in L. \rrbracket$$

Note 8

d096a87546b14acfa601179c2ac323e8

What is the axiom (L3) from the definition of a Lie algebra?

$$\llbracket [x[yz]] + [y[zx]] + [z[xy]] = 0 \quad \text{for all } x, y, z \in L. \rrbracket$$

Note 9

db6289e2261549bcb58877ac4d6f36f7

The axiom (L3) from the definition of a Lie algebra is called the Jacobi identity.

Note 10

fdd2c3d3027e4a34be5e9540f148a9cd

Let L, L' be two Lie algebras over F . A vector space isomorphism $\phi : L \rightarrow L'$ satisfying

$$\phi([xy]) = [\phi(x)\phi(y)] \quad \forall x, y \in L$$

is called an isomorphism of Lie algebras.

Note 11

cbd7fbc2d06c41e29e72d8075fc10e5f

We say that two Lie algebras L, L' over F are isomorphic if there exists a Lie algebra isomorphism $\phi : L \rightarrow L'$.

Note 12

fe5ab449614b4098a1e81d3d86903d64

Let L be a Lie algebra over F . A subspace K of L satisfying

$$[xy] \in K \quad \forall x, y \in K.$$

is called a subalgebra of L .