# Basic Axioms and Examples

## Note 1

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Given a set $G$, {{c2::a binary operation $\star$ on $G$}} is {{c1::a function

$$\star : G \times G \to G.$$

}}

## Note 2

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Given a binary operation $\star$ on a set $G$, for any $a, b \in G$ we shall write {{c2::$a \star b$}} for {{c1::$\star(a, b)$.}}

## Note 3

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A binary operation $\star$ on a set $G$ is {{c2::associative}} if {{c1::for all $a, b, c \in G$ we have

$$a \star (b \star c) = (a \star b) \star c.$$

}}

## Note 4

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If $\star$ is a binary operation on a set $G$ we say elements $a$ and $b$ of $G$ {{c1::commute}} if {{c2::

$$a \star b = b \star a.$$

}}

## Note 5

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A binary operation $\star$ on a set $G$ is {{c2::commutative}} if {{c1::for all $a, b \in G$ we have

$$a \star b = b \star a.$$

}}

## Note 6

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Suppose that $\star$ is a binary operation on a set $G$ and $G \subseteq G$. If {{c2::the restriction of $\star$ to $H$ is a binary relation on $H$,}} then $H$ is said to be {{c1::closed under $\star$.}}

## Note 7

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{{c3::A group}} is {{c2::an ordered pair $(G, \star)$}} where {{c1::$G$ is a set and $\star$ is a binary operation on $G$}} satisfying {{c4::the group axioms.}}

## Note 8

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How many axiom are there in the definition of a group $(G, \star)$?

> Three.

## Note 9

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What is the first axiom from the definition of a group $(G, \star)$?

> $\star$ is associative.

## Note 10

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Given a binary operation $\star$ on a set $G$, {{c3::an element $e \in G$}} is called {{c2::an identity of $G$}} if {{c1::for all $a \in G$ we have

$$a \star e = e \star a = a.$$

}}

## Note 11

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What is the second axiom from the definition of a group $(G, \star)$?

> There exists an identity of $G$.

## Note 12

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Given a binary operation $\star$ on a set $G$ and $a \in G$, {{c3::an element $\tilde{a} \in G$}} is called {{c2::an inverse of $a$}} if {{c1::

$$a \star \tilde{a} = \tilde{a} \star a = e.$$

}}

## Note 13

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Given a binary operation $\star$ on a set $G$ and $a \in G$, {{c2::an inverse of $a$}} is usually denoted {{c1::$a^{-1}$.}}

## Note 14

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What is the third axiom from the definition of a group $(G, \star)$?

> For all $a \in G$ there exists $a^{-1}$.

## Note 15

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A group $(G, \star)$ is called {{c2::abelian}} if {{c1::$\star$ is commutative.}}

## Note 16

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We shall say $G$ is {{c2::a group under $\star$}} if {{c1::$(G, \star)$ is a group.}}

## Note 17

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We shall say a set $G$ is {{c2::a group}} if {{c1::$G$ is a group under an operation that is clear from the context.}}

## Note 18

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If $(A, \star)$ and $(B, \diamond)$ are {{c4::groups,}} then the group {{c2::$A \times B$,}} whose operation is {{c1::defined componentwise:

$$(a, b)(c, d) = (a \star c, b \diamond d),$$

}} is called {{c3::the direct product of the two groups.}}

## Note 19

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If $(G, \star)$ is a group, then {{c2::the identity of $G$}} is {{c1::unique.}}

## Note 20

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If $(G, \star)$ is a group, then the identity of $G$ is unique. What is the key idea in the proof?

Consider the product of two arbitrary identities.

## Note 21

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If $(G, \star)$ is a group, then for every $a \in G$, {{c2::$a^{-1}$}} is {{c1::uniquely determined.}}

## Note 22

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If $(G, \star)$ is a group, then for every $a \in G$, $a^{-1}$ is uniquely determined. What is the key idea in the proof?

Multiply an inverse on the right by $a \star a^{-1}$.

## Note 23

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If $(G, \star)$ is a group and $a \in G$, then

$$(a^{-1})^{-1} = \text{\{\{c1::}a.\text{\}\}}$$

## Note 24

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If $(G, \star)$ is a group and $a, b \in G$, then

$$\text{\{\{c2::}(a \star b)^{-1}\text{\}\}} = \text{\{\{c1::}(b^{-1}) \star (a^{-1}).\text{\}\}}$$

## Note 25

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If $(G, \star)$ is a group and $a_1, \ldots, a_n \in G$, then the value of

$$a_1 \star \cdots \star a_n$$

is {{c2::independent}} of {{c1::how the expression is bracketed.}}

«{{c3::The generalized associative law}}»

## Note 26

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What is the key idea in the proof of the generalized associative law for a group $(G, \star)$?

By induction.