

# Properties of Infinite Series

## Note 1

51836e3c068e4688891ad60f449bd6

Let  $\sum_{k=1}^{\infty} a_k = A$  and  $c \in \mathbf{R}$ . Under which condition does

$$\sum_{k=1}^{\infty} ca_k$$

converge?

■ Always.

## Note 2

548101004aba462b8e81b2c4f7cbd1b9

If  $\sum_{k=1}^{\infty} a_k = A$  and  $c \in \mathbf{R}$ , then  $\sum_{k=1}^{\infty} ca_k = \{c1:cA\}$ .

## Note 3

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Let  $\sum_{k=1}^{\infty} a_k = A$  and  $\sum_{k=1}^{\infty} b_k = B$ . Under which condition does

$$\sum_{k=1}^{\infty} a_k + b_k$$

converge?

■ Always.

## Note 4

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If  $\sum_{k=1}^{\infty} a_k = A$  and  $\sum_{k=1}^{\infty} b_k = B$ , then

$$\sum_{k=1}^{\infty} a_k + b_k = \{c1:A+B\}$$

## Note 5

6795efea2a204bfb90bf19f3ac01f60a

The series  $\sum_{k=1}^{\infty} a_k$   $\{c5:converges\}$   $\{c4:if\ and\ only\ if,\}$  given  $\{c3: \epsilon > 0,\}$  there exists  $\{c2:an\ N \in \mathbf{N}\}$  such that whenever  $\{c2:n > m \geq N\}$  it follows that  $\{c1:$

$$|a_{m+1} + \cdots + a_n| < \epsilon.$$

$\}$

## Note 6

f83e35fa266b4b71ae674a5ae53196aa

The series  $\sum_{k=1}^{\infty} a_k$  converges if and only if, given  $\epsilon > 0$ , there exists an  $N \in \mathbb{N}$  such that whenever  $n > m \geq N$  it follows that

$$|a_{m+1} + \cdots + a_n| < \epsilon.$$

«[c1:Cauchy Criterion]»

## Note 7

255fd1a8d1ca40ddbe4706f396dcaad5

What is the key idea in the proof of the Cauchy Criterion for Series?

■ Cauchy Criterion for the sequence of partial sums.

## Note 8

2cccd666d0d4025a48baaa6ac297e88

If the series  $\sum_{k=1}^{\infty} a_k$  converges, then  $(a_k) \rightarrow 0$ .

## Note 9

e553a27c1b0240b4a08a2d2e1291a1c5

If the series  $\sum_{k=1}^{\infty} a_k$  converges, then  $(a_k) \rightarrow 0$ . What is the key idea in the proof?

■ Apply the Cauchy Criterion with  $n = m + 1$ .

## Note 10

0314d6d2761e4bd1b24b1b858e9c5086

Assume  $(a_k)$  and  $(b_k)$  are sequences satisfying  $0 \leq a_k \leq b_k$  for all  $k \in \mathbb{N}$ . If  $\sum_{k=1}^{\infty} b_k$  converges, then  $\sum_{k=1}^{\infty} a_k$  converges.

## Note 11

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Assume  $(a_k)$  and  $(b_k)$  are sequences satisfying  $0 \leq a_k \leq b_k$  for all  $k \in \mathbb{N}$ . If  $\sum_{k=1}^{\infty} a_k$  diverges, then  $\sum_{k=1}^{\infty} b_k$  diverges.

## Note 12

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Assume  $(a_k)$  and  $(b_k)$  are sequences satisfying  $0 \leq a_k \leq b_k$  for all  $k \in \mathbb{N}$ . If  $\sum_{k=1}^{\infty} b_k$  converges, then  $\sum_{k=1}^{\infty} a_k$  converges.

«[c1: Comparison Test]»

## Note 13

7f40a1b03ff44e75af1465ca5e329e3e

What is the key idea in the proof of the Comparison Test for Series?

■ Use the Cauchy Criterion explicitly.

## Note 14

f49c77a313a747e9b024dd5189511f35

$$\sum_{k=1}^{\infty} \frac{1}{k} = \{[c1::\infty,)\}$$

## Note 15

184fe5e5e62b4c3f8a49c4ea6d26c240

$$\sum_{k=1}^{\infty} \frac{1}{k} = \infty.$$

What is the key idea in the proof?

■ Observe  $\frac{1}{k} \geq \frac{1}{2^i}$  for every next  $2^{i-1}$  terms.

## Note 16

1f9364c8930f4fedbf3501d9a92ce2e

Statements about  $\{[c2::\text{convergence}]\}$  of sequences and series are immune to  $\{[c1::\text{changes in some finite number of initial terms.}]\}$

## Note 17

89c3e03f687b4c4aa41185f6c668d327

A series is called  $\{[c2::\text{geometric}]\}$  if it is of the form  $\{[c1::$

$$\sum_{k=1}^{\infty} ar^k.$$

}}

## Note 18

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The series  $\sum_{k=1}^{\infty} ar^k$  converges if and only if  $|r| < 1$ .

## Note 19

f7ab1e58f37b4580a558de06c51dc6f7

Given  $|r| < 1$ ,

$$\sum_{k=1}^{\infty} ar^k = \frac{a}{1-r}.$$

## Note 20

c409ec230f6741b796ea4ef3e8813d9c

Given  $|r| < 1$ ,  $\sum_{k=1}^{\infty} ar^k = \frac{a}{1-r}$ . What is the key idea in the proof?

■ Rewrite partial sums.

## Note 21

28dc84fd3d384adea7a15102e07c644a

If the series  $\sum_{k=1}^{\infty} a_k$  converges, then  $\sum_{k=1}^{\infty} a_k$  converges.

«Absolute Convergence Test»

## Note 22

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What is the key idea in the proof of the Absolute Convergence Test?

■ The Cauchy Criterion and the Triangle Inequality.

## Note 23

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Let  $(a_k)$  be a decreasing sequence and  $(a_k) \rightarrow 0$ . Then

$$\sum_{k=1}^{\infty} (-1)^{n+1} a_k$$

converges.

### Note 24

cb8249219a644a12b50a90701e47c548

We say  $\sum_{k=1}^{\infty} a_k$  converges absolutely, if  $\sum_{k=1}^{\infty} |a_k|$  converges.

### Note 25

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We say  $\sum_{k=1}^{\infty} a_k$  converges conditionally, if it converges and does not converge absolutely.

### Note 26

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A series  $\sum_{k=1}^{\infty} a_k$  is said to be positive, if  $a_k \geq 0$  for all  $k \in \mathbb{N}$ .

### Note 27

c5acade4dde342f8b7ac4acec2278ac6

Any positive series must converge absolutely.

### Note 28

e85b9eb09cfa4056b868f983703a571c

May a positive series diverge?

Only to  $+\infty$ .

### Note 29

b65eba46e51c438e933833ad313a4cf8

A positive series converges if and only if the sequence of partial sums  $(s_n)$  is bounded.

### Note 30

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Let  $\sum_{k=1}^{\infty} a_k$  be a series and  $f : \mathbb{N} \rightarrow \mathbb{N}$  be 1-1 and onto. The series  $\sum_{k=1}^{\infty} a_{f(k)}$  is called a rearrangement of  $\sum_{k=1}^{\infty} a_k$ .

### Note 31

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If a series converges absolutely, then any rearrangement of this series converges to the same limit.

### Note 32

057430cb21934da7ac9bc037ba169eb5

If a series converges absolutely, then any rearrangement of this series converges to the same limit. What is the key idea in the proof?

Substitute the original series' initial terms for the rearrangement's partial sum.

### Note 33

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If a series converges absolutely, then any rearrangement of this series converges to the same limit. In the proof, how many of the original series' initial terms are substituted from the rearrangement's partial sum?

So as to use the definition of convergence and the Cauchy Criterion for absolute convergence.

### Note 34

574ee484bcf94971932baee731b90e95

If a series converges absolutely, then any rearrangement of this series converges to the same limit. In the proof, how many of the rearrangement's terms are taken for the partial sum?

So as to contain the initial terms of the sequence.

### Note 35

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If a series converges absolutely, then any rearrangement of this series converges to the same limit. In the proof we denote  $\{s_n\}$  to be the original series' partial sum.

### Note 36

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If a series converges absolutely, then any rearrangement of this series converges to the same limit. In the proof we denote  $\{t_n\}$  to be the rearrangement' partial sum.

### Note 37

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If a series converges absolutely, then any rearrangement of this series converges to the same limit. In the proof, what do we show about  $t_m - s_N$ ?

■  $|t_m - s_N| < \varepsilon$  due to the Cauchy Criterion.

### Note 38

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If a series converges absolutely, then any rearrangement of this series converges to the same limit. In the proof, how do you show  $|t_m - A| < \varepsilon$ ?

■  $|t_m - s_N + s_N - A|$ .