Properties of Infinite Series

Note 1

51836e3c068e46888891ad60f449bd6

Let $\sum_{k=1}^{\infty} a_k = A$ and $c \in \mathbf{R}$. Under which condition does

$$\sum_{k=1}^{\infty} c a_k$$

converge?

> Always.

Note 2

548101004aba462b8e81b2c4f7cbd1b9

If $\sum_{k=1}^{\infty} a_k = A$ and $c \in \mathbf{R}$, then $\sum_{k=1}^{\infty} c a_k =$ {{c1::cA}}.

Note 3

30607fca749d4ea9814ec7460a102865

Let $\sum_{k=1}^{\infty} a_k = A$ and $\sum_{k=1}^{\infty} b_k = B$. Under which condition does

$$\sum_{k=1}^{\infty} a_k + b_k$$

converge?

> Always.

Note 4

4f1064d2b18d4e889fa4e80010f532b1

If $\sum_{k=1}^{\infty} a_k = A$ and $\sum_{k=1}^{\infty} b_k = B$, then

$$\sum_{k=1}^{\infty} a_k + b_k = \text{\{\{c1::}A + B.\text{\}\}}$$

Note 5

6795efea2a204bfb90bf19f3ac01f60a

The series $\sum_{k=1}^{\infty} a_k$ {{c5::converges}} {{c4::if and only if,}} given {{c3::$\epsilon > 0$,}} there exists {{c2::an $N \in \mathbf{N}$}} such that whenever {{c2::$n > m \geq N$}} it follows that {{c1::

$$|a_{m+1} + \cdots + a_n| < \epsilon.$$

}}

Note 6

f83e35fa266b4b71ae674a5ae53196aa

The series $\sum_{k=1}^{\infty} a_k$ converges if and only if, given $\epsilon > 0$, there exists an $N \in \mathbf{N}$ such that whenever $n > m \geq N$ it follows that

$$|a_{m+1} + \cdots + a_n| < \epsilon.$$

«{{c1::Cauchy Criterion}}»

Note 7

255fd1a8d1ca40ddbe4706f396dcaad5

What is the key idea in the proof of the Cauchy Criterion for Series?

> Cauchy Criterion for the sequence of partial sums.

Note 8

2ccccd666d0d4025a48baaa6ac297e88

If the series $\sum_{k=1}^{\infty} a_k$ {{c2::converges,}} then {{c1::$(a_k) \to 0$.}}

Note 9

e553a27c1b0240b4a08a2d2e1291a1c5

If the series $\sum_{k=1}^{\infty} a_k$ converges, then $(a_k) \to 0$. What is the key idea in the proof?

> Apply the Cauchy Criterion with $n = m + 1$.

Note 10

0314d6d2761e4bd1b24b1b858e9c5086

Assume (a_k) and (b_k) are sequences satisfying {{c3::$0 \leq a_k \leq b_k$ for all $k \in \mathbf{N}$.}} If $\sum_{k=1}^{\infty}$ {{c1::b_k}} {{c2::converges,}} then $\sum_{k=1}^{\infty}$ {{c1::a_k}} {{c2::converges.}}

Note 11

03fddbcdb39340e0a421d24fe7298f2e

Assume (a_k) and (b_k) are sequences satisfying {{c3::$0 \leq a_k \leq b_k$ for all $k \in \mathbf{N}$.}} If $\sum_{k=1}^{\infty}$ {{c2::a_k}} {{c2::diverges,}} then $\sum_{k=1}^{\infty}$ {{c1::b_k}} {{c2::diverges.}}

Note 12

d6553b70220c4348a7c7692a58f91271

Assume (a_k) and (b_k) are sequences satisfying $0 \leq a_k \leq b_k$ for all $k \in \mathbf{N}$. If $\sum_{k=1}^{\infty} b_k$ converges, then $\sum_{k=1}^{\infty} a_k$ converges.

«{{c1::Comparison Test}}»

Note 13

7f40a1b03ff44e75af1465ca5e329e3e

What is the key idea in the proof of the Comparison Test for Series?

> Use the Cauchy Criterion explicitly.

Note 14

f49c77a313a747e9b024dd5189511f35

$$\sum_{k=1}^{\infty} \frac{1}{k} = \text{\{\{c1::}\infty.\text{\}\}}$$

Note 15

184fe5e5e62b4c3f8a49c4ea6d26c240

$$\sum_{k=1}^{\infty} \frac{1}{k} = \text{\{\{c1::}\infty.\text{\}\}}$$

What is the key idea in the proof?

> Observe $\frac{1}{k} \geqslant \frac{1}{2^i}$ for every next 2^{i-1} terms.