

Sets

Note 1

097312afe75d4a3d9eaa0c1f4c63748a

Intuitively speaking, $\{\{c2::a \text{ set}\}\}$ is $\{\{c1::a \text{ collection of objects.}\}\}$

Note 2

85e21cf985524b80a8c00eb4608f34be

Intuitively speaking, a set is a collection of objects. $\{\{c2::\text{Those objects}\}\}$ are referred to as $\{\{c1::\text{the elements of the set.}\}\}$

Note 3

12b96daebbc04070b74e2a6f74e5b268

Given a set A , we write $\{\{c2::x \in A\}\}$ if $\{\{c1::x \text{ is an element of } A.\}\}$

Note 4

b25d749749a64c5b90880253d9839da8

Given a set A , we write $\{\{c2::x \notin A\}\}$ if $\{\{c1::x \text{ is not an element of } A.\}\}$

Note 5

39565306ec4e40e18136e7eb88fc817a

Given two sets A and B , $\{\{c1::\text{the union}\}\}$ is written $\{\{c2::A \cup B.\}\}$

Note 6

73bf0eb1d16c4c5da368e326b4739d5b

Given two sets A , and B , $\{\{c2::\text{the union}\}\}$ is $\{\{c3::\text{defined}\}\}$ by the rule

$$\{\{c1::x \in A \cup B \text{ provided that } x \in A \text{ or } x \in B.\}\}$$

Note 7

8ce7db157931494bbfb6eee706e15efc

Given two sets A and B , $\{\{c1::\text{the intersection}\}\}$ is written $\{\{c2::A \cap B.\}\}$

Note 8

6a277df52de2409a98e48429d69b6d05

Given two sets A and B , $\{\{c2::\text{the intersection}\}\}$ is $\{\{c3::\text{defined}\}\}$ by the rule

$$\{\{c1::x \in A \cap B \text{ provided that } x \in A \text{ and } x \in B.\}\}$$

Note 9

684951afc378458aa7bd27e67cdc499b

The set of natural numbers is denoted \mathbf{N} .

Note 10

49d36a026d4b4678ab86fb6103571cce

$$\mathbf{N} \stackrel{\text{def}}{=} \{1, 2, 3, \dots\}.$$

Note 11

797c81e5adb543e1a5d4cc67e64c5e09

The set of integers is denoted \mathbf{Z} .

Note 12

d3c61bf891744c58b73cef543c6e100d

$$\mathbf{Z} \stackrel{\text{def}}{=} \{\dots, -2, -1, 0, 1, 2, \dots\}.$$

Note 13

57f085776972449f8bc14daf5cff6603

The set of rational numbers is denoted \mathbf{Q} .

Note 14

f7e3370650134607853b41b2b1ecf54b

$$\mathbf{Q} \stackrel{\text{def}}{=} \left\{ \text{all fractions } \frac{p}{q} \text{ where } p, q \in \mathbf{Z} \text{ and } q \neq 0 \right\}.$$

Note 15

faeac83cb5b740b6964551c85ad3e35b

The set of real numbers is denoted \mathbf{R} .

Note 16

6e5da98964d645d09ad6989e85679c74

The empty set is the set that contains no elements.

Note 17

206db0a0f3d042e49a9ca532e222201f

The empty set is denoted \emptyset .

Note 18

2f0448d226db4b71b150acaed349a73b

Two sets A and B are said to be disjoint if $A \cap B = \emptyset$.

Note 19

e5d9d365e86640319ca5460ef8c4f05c

Given two sets A and B , we say $\{\{c2::A \text{ is a subset of } B,\}$ or $\{\{c2::B \text{ contains } A,\}$ if $\{\{c1::\text{every element of } A \text{ is also an element of } B.\}$

Note 20

c2bd27f1fc0d40e296dceef9c9789556

Given two sets A and B , the $\{\{c3::\text{inclusion}\}$ relationship $\{\{c2::A \subseteq B \text{ or } B \supseteq A,\}$ is used to indicate that $\{\{c1::A \text{ is a subset of } B.\}$

Note 21

333e7c6716af48b7b9962ad803f0732f

Given two sets A and B , $\{\{c2::A = B,\}$ means that $\{\{c1::A \subseteq B \text{ and } B \subseteq A.\}$

Note 22

74e93b42d46746dc9ec2b54f8366c435

Let A_1, A_2, A_3, \dots be an infinite collection of sets. Notationally,

$$\bigcup_{n=1}^{\infty} A_n, \quad \bigcup_{n \in \mathbf{N}} A_n, \quad \text{or} \quad A_1 \cup A_2 \cup A_3 \cup \dots$$

are all equivalent ways to indicate $\{\{c1::\text{the set whose elements consist of any element that appears in at least on particular } A_n.\}$

Note 23

69e4627a3e7149ef8be05479a2587b41

Let A_1, A_2, A_3, \dots be an infinite collection of sets. Notationally,

$$\bigcap_{n=1}^{\infty} A_n, \quad \bigcap_{n \in \mathbf{N}} A_n, \quad \text{or} \quad A_1 \cap A_2 \cap A_3 \cap \dots$$

are all equivalent ways to indicate $\{\{c1::\text{the set whose elements consist of any element that appears in every } A_n.\}$

Note 24

11a987e10fce4ceea69672f366597729

Given $A \subseteq \mathbf{R}$, $\{\{c2::\text{the complement of } A,\}$ refers to $\{\{c1::\text{the set of all elements of } \mathbf{R} \text{ not in } A.\}$

Note 25

8b379552450b4672af82c17476c0ff13

Given $A \subseteq \mathbf{R}$, $\{\{c2::\text{the complement of } A,\}$ is written $\{\{c1::A^c.\}$

Note 26

a3459afa53264a7c82d9abd760a0c93e

Given $A, B \subseteq \mathbf{R}$,

$$\{\{c2: (A \cap B)^c\}\} = \{\{c1: A^c \cup B^c.\}\}$$

«{\{c3:De Morgan's Law\}}»

Note 27

c983927aa0304e51949e2f90a2ec2614

Given $A, B \subseteq \mathbf{R}$,

$$\{\{c2: (A \cup B)^c\}\} = \{\{c1: A^c \cap B^c.\}\}$$

«{\{c3:De Morgan's Law\}}»

Functions

Note 1

18930cfe4e4445779bcec8a2fb53f23c

Given $\{\{c3::\text{two sets } A \text{ and } B,\}\}$ $\{\{c2::\text{a function from } A \text{ to } B\}\}$ is $\{\{c1::\text{a rule or mapping that takes each element } x \in A \text{ and associates with it a single element of } B.\}\}$

Note 2

dfa898ef047e418fa8dfe9e9582fd71

$\{\{c1::\text{If } f \text{ is a function from } A \text{ to } B,\}\}$ we write $\{\{c2::f : A \rightarrow B.\}\}$

Note 3

c2730dafa0fe4bf4bede66b7199b48b9

Let $f : A \rightarrow B$. Given $\{\{c3::x \in A,\}\}$ the expression $\{\{c2::f(x)\}\}$ is used to represent $\{\{c1::\text{the element of } B \text{ associated with } x \text{ by } f.\}\}$

Note 4

65568f366ca949888310668475dbe570

Let $f : A \rightarrow B$. $\{\{c2::\text{The set } A\}\}$ is called $\{\{c1::\text{the domain of } f.\}\}$

Note 5

7870a310786142fa938bcc843ca8e1ae

Let $f : A \rightarrow B$. $\{\{c2::\text{The set } \{f(x) \mid x \in A\}\}\}$ is called $\{\{c1::\text{the range of } f.\}\}$

Note 6

f951f5a5136248dcb413f59b3271d389

Given $x \in \mathbf{R}$, $\{\{c2::\text{the absolute value of } x\}\}$ is denoted $\{\{c1::|x|.\}\}$

Note 7

624dda908fd64a1cadae2b61c1277c59

Given $x \in \mathbf{R}$,

$$|x| \stackrel{\text{def}}{=} \begin{cases} \{\{c1::x,\}\} & \text{if } \{\{c2::x \geq 0\}\}, \\ \{\{c1:: -x,\}\} & \text{if } \{\{c2::x < 0\}\}. \end{cases}$$

Note 8

0ab23d0afe1448e397cad330aea55883

Given $a, b \in \mathbf{R}$, $|ab| = \{\{c1::|a| \cdot |b|\}\}$.

Note 9

2b51f36fba524365b72001d318791436

Given $a, b \in \mathbf{R}$, $\|a + b\| \leq \|a\| + \|b\|$.

«Triangle inequality»

Logic and Proofs

Note 1

93f759e32dbf497cb30754e24c5b09f1

When in a proof by contradiction the contradiction is with the theorem's hypothesis, the proof is said to be contrapositive.

Note 2

1f45350926704df98b0abdf205f4319c

Two real number a and b are equal if and only if for every real number $\epsilon > 0$ it follows that $|a - b| < \epsilon$.

Note 3

3ef90c9123e64df39ae9cd34271a7dcd

Two real number a and b are equal \iff for every real number $\epsilon > 0$ it follows that $|a - b| < \epsilon$. What is the key idea in the proof?

By contradiction.

Note 4

aab4bb967d814e87bd85608277093755

Let $S \subseteq \mathbb{N}$. If S contains 1 and whenever S contains n , it also contains $n + 1$, then $S = \mathbb{N}$.

Note 5

3dd92625856f408b9dc93fd36d82588d

Let $S \subseteq \mathbb{N}$. If S contains 1 and whenever S contains n , it also contains $n + 1$, then $S = \mathbb{N}$. This proposition is the fundamental principle behind induction.