

# Definition and Examples

## Note 1

9080791fc8754b0bb88c381c10acbdffc

Let  $G$  be a group. If  $H$  is a subgroup of  $G$  we shall write

$$H \leq G.$$

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## Note 2

66e7f23728af4c9d8839d172e59d716a

Let  $G$  be a group and  $H \leq G$ . We shall denote the operation for  $H$  by the same symbol as the operation for  $G$ .

## Note 3

e76ada2ee6da4b5fb71966e9f7ce3ded

Let  $G$  be a group. If  $H \leq G$  and  $H \neq G$  we shall write  $H < G$ .

## Note 4

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If  $H$  is a subgroup of  $G$  then any equation in the subgroup  $H$  may also be viewed as an equation in the group  $G$ .

## Note 5

8f5b765961884460823141645b5ea08b

Let  $G$  be a group and  $H \leq G$ . What is the identity of  $H$ ?

■ The identity of  $G$ .

## Note 6

7c122a5400f64eba9a76438c1ff296ee

Let  $G$  be a group and  $H \leq G$ . The identity of  $H$  is the identity of  $G$ . What is the key idea in the proof?

■ The identity is unique and it is the identity of  $G$ .

## Note 7

83cba804764b43e2baf282ffec513694

Let  $G$  be a group. What is the minimal subgroup of  $G$ ?

■ The singleton  $\{1\}$ .

### Note 8

d40df9f46a6b43ecb3fea9b5b37e5b1c

Let  $G$  be a group. What is the element that any subgroup of  $G$  must contain?

■ The identity of  $G$ .

### Note 9

bc95ad6358814213a6fff2eb0cfd544b

Let  $G$  be a group and  $H \leq G$ . What is the inverse of an element  $x$  in  $H$ ?

■ The inverse of  $x$  in  $G$ .

### Note 10

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Let  $G$  be a group and  $H \leq G$ . Why is the notation  $x^{-1}$  unambiguous?

■ The inverse in  $H$  is the same as the inverse in  $G$ .

### Note 11

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Let  $G$  be a group. The subgroup  $\{1\}$  of  $G$  is called the trivial subgroup.

### Note 12

eb859714e1f34f4db3dc35755f562945

Let  $G$  be a group. The trivial subgroup is denoted by  $\{1\}$ .

### Note 13

74ab3ceb9a36420fb53dc2b3a22eb5f2

Which subgroups does any group have?

■ The trivial subgroup and the group itself.

### Note 14

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If  $H$  is a subgroup of  $G$  and  $K$  is a subgroup of  $H$ , then  $K$  is a subgroup of  $G$ .

### Note 15

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Which object is considered in the Subgroup Criterion?

- Any subset of a group.

### Note 16

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What are the conditions of the Subgroup Criterion?

- The subset is nonempty and closed under  $(x, y) \mapsto x \cdot y^{-1}$ .

### Note 17

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What is the special case considered in the Subgroup Criterion?

- The subset is finite.

### Note 18

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What are the conditions of the Subgroup Criterion for a finite subset?

- The subset is nonempty and closed under the operation.

### Note 19

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What is the key idea in the proof of the Subgroup Criterion for a finite subset?

- Any element's inverse is its  $n$ -th power.