

# Sets

## Note 1

097312afe75d4a3d9eaa0c1f4c63748a

Intuitively speaking,  $\{\{c2::a \text{ set}\}\}$  is  $\{\{c1::a \text{ collection of objects.}\}\}$

## Note 2

85e21cf985524b80a8c00eb4608f34be

Intuitively speaking, a set is a collection of objects.  $\{\{c2::\text{Those objects}\}\}$  are referred to as  $\{\{c1::\text{the elements of the set.}\}\}$

## Note 3

12b96daebbc04070b74e2a6f74e5b268

Given a set  $A$ , we write  $\{\{c2::x \in A\}\}$  if  $\{\{c1::x \text{ is an element of } A.\}\}$

## Note 4

b25d749749a64c5b90880253d9839da8

Given a set  $A$ , we write  $\{\{c2::x \notin A\}\}$  if  $\{\{c1::x \text{ is not an element of } A.\}\}$

## Note 5

39565306ec4e40e18136e7eb88fc817a

Given two sets  $A$  and  $B$ ,  $\{\{c1::\text{the union}\}\}$  is written  $\{\{c2::A \cup B.\}\}$

## Note 6

73bf0eb1d16c4c5da368e326b4739d5b

Given two sets  $A$ , and  $B$ ,  $\{\{c2::\text{the union}\}\}$  is  $\{\{c3::\text{defined}\}\}$  by the rule

$$\{\{c1::x \in A \cup B \text{ provided that } x \in A \text{ or } x \in B.\}\}$$

## Note 7

8ce7db157931494bbfb6eee706e15efc

Given two sets  $A$  and  $B$ ,  $\{\{c2::\text{the intersection}\}\}$  is written  $\{\{c2::A \cap B.\}\}$

## Note 8

6a277df52de2409a98e48429d69b6d05

Given two sets  $A$  and  $B$ ,  $\{\{c2::\text{the intersection}\}\}$  is  $\{\{c3::\text{defined}\}\}$  by the rule

$$\{\{c1::x \in A \cap B \text{ provided that } x \in A \text{ and } x \in B.\}\}$$

## Note 9

684951afc378458aa7bd27e67cdc499b

The set of natural numbers is denoted  $\mathbf{N}$ .

## Note 10

49d36a026d4b4678ab86fb6103571cce

$$\mathbf{N} \stackrel{\text{def}}{=} \{1, 2, 3, \dots\}.$$

## Note 11

797c81e5adb543e1a5d4cc67e64c5e09

The set of integers is denoted  $\mathbf{Z}$ .

## Note 12

d3c61bf891744c58b73cef543c6e100d

$$\mathbf{Z} \stackrel{\text{def}}{=} \{\dots, -2, -1, 0, 1, 2, \dots\}.$$

## Note 13

57f085776972449f8bc14daf5cff6603

The set of rational numbers is denoted  $\mathbf{Q}$ .

## Note 14

f7e3370650134607853b41b2b1ecf54b

$$\mathbf{Q} \stackrel{\text{def}}{=} \left\{ \text{all fractions } \frac{p}{q} \text{ where } p, q \in \mathbf{Z} \text{ and } q \neq 0 \right\}.$$

## Note 15

faeac83cb5b740b6964551c85ad3e35b

The set of real numbers is denoted  $\mathbf{R}$ .

## Note 16

6e5da98964d645d09ad6989e85679c74

The empty set is the set that contains no elements.

## Note 17

206db0a0f3d042e49a9ca532e222201f

The empty set is denoted  $\emptyset$ .

## Note 18

2f0448d226db4b71b150acaed349a73b

Two sets  $A$  and  $B$  are said to be disjoint if  $A \cap B = \emptyset$ .

### Note 19

e5d9d365e86640319ca5460ef8c4f05c

Given two sets  $A$  and  $B$ , we say  $\{\{c2::A \text{ is a subset of } B\}\}$  or  $\{\{c2::B \text{ contains } A\}\}$  if  $\{\{c1::\text{every element of } A \text{ is also an element of } B\}\}$

### Note 20

c2bd27f1fc0d40e296dceef9c9789556

Given two sets  $A$  and  $B$ , the  $\{\{c3::\text{inclusion}\}\}$  relationship  $\{\{c2::A \subseteq B \text{ or } B \supseteq A\}\}$  is used to indicate that  $\{\{c1::A \text{ is a subset of } B\}\}$

### Note 21

333e7c6716af48b7b9962ad803f0732f

Given two sets  $A$  and  $B$ ,  $\{\{c2::A = B\}\}$  means that  $\{\{c1::A \subseteq B \text{ and } B \subseteq A\}\}$

### Note 22

74e93b42d46746dc9ec2b54f8366c435

Let  $A_1, A_2, A_3, \dots$  be an infinite collection of sets. Notationally,

$$\bigcup_{n=1}^{\infty} A_n, \quad \bigcup_{n \in \mathbf{N}} A_n, \quad \text{or} \quad A_1 \cup A_2 \cup A_3 \cup \dots$$

are all equivalent ways to indicate  $\{\{c1::\text{the set whose elements consist of any element that appears in at least on particular } A_n\}\}$

### Note 23

69e4627a3e7149ef8be05479a2587b41

Let  $A_1, A_2, A_3, \dots$  be an infinite collection of sets. Notationally,

$$\bigcap_{n=1}^{\infty} A_n, \quad \bigcap_{n \in \mathbf{N}} A_n, \quad \text{or} \quad A_1 \cap A_2 \cap A_3 \cap \dots$$

are all equivalent ways to indicate  $\{\{c1::\text{the set whose elements consist of any element that appears in every } A_n\}\}$

### Note 24

11a987e10fce4ceea69672f366597729

Given  $A \subseteq \mathbf{R}$ ,  $\{\{c2::\text{the complement of } A\}\}$  refers to  $\{\{c1::\text{the set of all elements of } \mathbf{R} \text{ not in } A\}\}$

### Note 25

8b379552450b4672af82c17476c0ff13

Given  $A \subseteq \mathbf{R}$ ,  $\{\{c2::\text{the complement of } A\}\}$  is written  $\{\{c1::A^c\}\}$

## Note 26

a3459afa53264a7c82d9abd760a0c93e

Given  $A, B \subseteq \mathbf{R}$ ,

$$\{\{c2:(A \cap B)^c\}\} = \{\{c1:A^c \cup B^c.\}\}$$

«{\{c3:De Morgan's Law\}}»

## Note 27

c983927aa0304e51949e2f90a2ec2614

Given  $A, B \subseteq \mathbf{R}$ ,

$$\{\{c2:(A \cup B)^c\}\} = \{\{c1:A^c \cap B^c.\}\}$$

«{\{c3:De Morgan's Law\}}»