

Prerequisites

Note 1

621caffff9ce421bb4309fc0c1cf144c

A function is said to be $\{\{c2: \text{multilinear}\}\}$ if and only if it is $\{\{c1: \text{linear}\}$ separately in each variable. $\}$

Note 2

1a514ffb24744a278834d0048496a850

A function is said to be $\{\{c2: \text{bilinear}\}\}$ if and only if $\{\{c1: \text{it is a multilinear function of two argument.}\}$

Note 3

6712178af383453faa8c5bad8aeabc89

$\{\{c2: \text{An endomorphism}\}\}$ of a vector space is $\{\{c1: \text{a linear map from this space to itself.}\}$

Note 4

d6b4c6b47276475dbc8548d1c5240801

$\{\{c2: \text{The characteristic}\}\}$ of a ring R is $\{\{c1: \text{the smallest positive number } n \text{ such that}$

$$\underbrace{1 + \cdots + 1}_n = 0,$$

or 0, if no such n exists. $\}$

Note 5

7ea5080a9df1419e88226f7df77af8db

$\{\{c1: \text{The characteristic}\}\}$ of a ring R is denoted $\{\{c2: \text{char } R.\}\}$

Note 6

e270ce3bf39a4ad4b08726ec08e3353a

Let V be a vector space over a field K . $\{\{c1: \text{A linear map}$

$$V \rightarrow K$$

$\}\}$ is called $\{\{c2: \text{a linear form on the vector space } V.\}\}$

Note 7

49d3a5be7380467e9f537fc3ce7b1197

Let V be a vector space over a field K . $\{\{c1: \text{The set of all the linear forms } V \rightarrow K\}\}$ is called $\{\{c2: \text{the dual space of } V.\}\}$

Note 8

584e4213ae234a62a4eba3b726f581c0

The dual space of a vector space V is denoted V^* .

Note 9

c3c550f6f7284b649efd64a23b3fba07

Let V be a vector space over a field K . A bilinear map

$$V \times V \rightarrow K$$

is called a bilinear form on the vector space V .

Note 10

b578c7a8572d40ffbe4a3f132fda2107

A bilinear form $f : V \times V \rightarrow K$ is said to be nondegenerate if each of its corresponding linear maps $V \rightarrow V^*$ is nondegenerate.

Note 11

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Let $f : V \times V \rightarrow K$ be a bilinear form with a matrix A . Then, f is nondegenerate if and only if $\det A \neq 0$.

Note 12

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Let V be a vector space over a field K and e_1, \dots, e_n be a basis in V . The matrix

$$A = \left(f(e_i, e_j) \right) \sim n \times n$$

is called the matrix of the bilinear form f on the basis e_1, \dots, e_n .

1.1. The notion of Lie algebra

Note 1

686bbc96abfb46e883a4acb108450cc1

At the first place a Lie algebra is a vector space L over a field F .

Note 2

7a252531934f4c00829418ab1f3a1d01

What is the signature of the new operation in the definition of a Lie algebra?

$$L \times L \rightarrow L.$$

Note 3

a1cc6426fa49471dad192df5295fb310

The operation $L \times L \rightarrow L$ from the definition of a Lie algebra is denoted $(x, y) \mapsto [xy]$.

Note 4

8bb3c76247ab416a97f8f6e247a6c2a2

The operation $(x, y) \mapsto [xy]$ from the definition of a Lie algebra is called the bracket or commutator of x and y .

Note 5

6c529b4b819a45c3b91755b1280be2a2

How many axioms are there in the definition of a Lie algebra?

(L1), (L2), (L3).

Note 6

f8d0434e7d3c404b8319bf527f96627c

What is the axiom (L1) from the definition of a lie algebra?

The bracket operation is bilinear.

Note 7

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What is the axiom (L2) from the definition of a lie algebra?

■ $[xx] = 0$ for all $x \in L$.

Note 8

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What is the axiom (L3) from the definition of a Lie algebra?

■ $[x[yz]] + [y[zx]] + [z[xy]] = 0$ for all $x, y, z \in L$.

Note 9

db6289e2261549bcb58877ac4d6f36f7

The axiom (L3) from the definition of a Lie algebra is called the Jacobi identity.

Note 10

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Let L, L' be two Lie algebras over F . A vector space isomorphism $\phi : L \rightarrow L'$ satisfying

$$\phi([xy]) = [\phi(x)\phi(y)] \quad \forall x, y \in L$$

is called an isomorphism of Lie algebras.

Note 11

cbd7fbc2d06c41e29e72d8075fc10e5f

We say that two Lie algebras L, L' over F are isomorphic if there exists a Lie algebra isomorphism $\phi : L \rightarrow L'$.

Note 12

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Let L be a Lie algebra over F . A subspace K of L satisfying

$$[xy] \in K \quad \forall x, y \in K.$$

is called a subalgebra of L .