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An Erosion Model Based on Velocity Fields for the Visual Simulation of Mountain Scenery

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SUMMARY

Visual simulation of natural scenery using computer graphics is an interesting research field with wide applications such as flight simulation and special effects in movies.

There have been many studies of fractal techniques that use 1/*f* noise generated by FFT or the midpoint displacement method as modelling methods for imaginary mountain scenery. These methods are suitable for creating impressive mountain scenes, but they cannot create clear ridge and valley lines, which are notable features of mountains produced by erosion processes. Although a few reports have presented modelling methods that take erosion processes into account, the results have not been satisfactory.

In this paper we present a simple 'quasi-physically based' method for simulating the topography of eroded mountains based on velocity fields of water flow. © 1998 John Wiley & Sons, Ltd.

KEY WORDS: computer graphics; visual simulation; natural phenomena; terrain; erosion processes

1. INTRODUCTION

Visual simulation of natural scenery using computer graphics (CG) is an interesting research field with wide applications such as flight simulation and special effects in movies. In this paper we present a method for simulating the eroded topography of natural mountains, which are the most basic element in natural scenery, based on velocity fields of water flow.

Various methods have been proposed for simulating mountain scenery:

1. methods based on fractal techniques (e.g. References 1 and 2)
2. method based on the creation of a water system[3]
3. method based on the creation of ridge lines[4]
4. method based on fractal techniques and simulation of erosion and weathering[5]

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5. method based on erosion simulation that takes into account geological differences in the topography.[6]

Method (1) is very simple and various modelling procedures have been proposed. This method is suitable for creating impressive mountain scenes, but in most cases the created mountains are isotropic and the creation of ridge and valley lines is difficult. Therefore, in order to create a desirable image, the outline must be created by other modelling procedures considering $1/f$ noise as texture.

Methods (2)–(5), like the method proposed in this paper, aim to create an eroded terrain model including ridge and valley lines.

In methods (2) and (3), eroded mountain topography is simulated by creating a water system or by creating ridge lines. The topography created by the algorithm shows clear ridge and valley lines, which are characteristic of eroded mountains. However, as the mountain slopes are calculated by simple interpolation from the water system or ridge lines, it is difficult to obtain the natural configuration of slopes.

In methods (4) and (5) an initial topographical geometric model is first created (e.g. by a fractal procedure) and then a more natural topographical model is created by a procedure to simulate erosion processes. The algorithms used in these methods are intuitive and easy to understand. However, since they are based on the simple cellular automaton model not taking into account the inertia of flowing water, it is not easy to produce ridge and valley lines of large scale reflecting natural water systems. In the simple cellular automaton model the water flow goes just to one of the neighbour cells having lowest elevation for each time step, i.e. the flow is simulated by checking only local topographical geometry.

In the method proposed in this paper, the velocity fields of water flowing down the face of a mountain are calculated by simulation of the motion of 'water particles', and these velocity fields are used for simulating erosion. This method is classified as an effective 'quasi-physically based' simulation method and is also intuitive and easy to understand.

2. SIMULATION USING VELOCITY FIELDS

A few erosion models based on water flow have been proposed in previous reports.[5,6] However, these models are cellular automaton models, in which the water flow is estimated only at one point of the slope, and the inertia of flowing water is not taken into account. Thus it is not easy to produce ridge and valley lines of large scale reflecting the true flow of water.

In our proposed model we assume that the flow of water can be calculated by integrating the motion of independent water particles. The velocity field of water flow is obtained by simulation of the motion of water particles. Thus, unlike that in past models, the water flow in our model reflects the entire topography and so a more natural erosion process can be expected. Moreover, as the 'motion' rather than just the 'movement' of water particles is calculated, the simulation of erosion in our model takes into account the collision of each water particle with the ground surface. The following is an outline of the algorithm used in our model. Also in our model, the topographical geometry is represented by the set of elevation values sampled at two-dimensional regular grid points. Such a geometric model is usually called a *digital terrain model* or *digital elevation model*.

Outline of erosion algorithm

Step 0. Prepare the following two-dimensional arrays (we define here the 'velocity field' precisely as the set of arrays **W**, **V** and **E**: **T**, elevation; **W**, quantity of water; **V**, velocity vector; **E**, collision energy.

Step 1. Input the initial topography in array **T**.

Step 2. Repeat the following steps as many times as needed.
(a) Obtain the velocity field **W**, **V**, **E** for the current terrain **T**.
(b) Perform erosion processes on the current topography **T** by the velocity field **W**, **V**, **E**.

2.1. Method for obtaining velocity field of water flow

The velocity field is obtained by arranging water particles on each grid point of the digital terrain model and examining the flow of each water particle. The flow of a water particle can be approximated by the motion equation

$$\mathbf{x}(t+\Delta t) = \mathbf{x}(t) + \mathbf{v}(t)\,\Delta t$$

where $\mathbf{x}(t)$, $\mathbf{v}(t)$ and $\mathbf{a}(t)$ are the position, velocity and acceleration of the water particle at time t respectively and Δt is the time step.

Calculation of $\mathbf{a}(t)$

In our model the acceleration of water flow is assumed to occur in the direction with the maximum angle of inclination. The magnitude of the acceleration vector is obtained from the equation

$$|\mathbf{a}| = \frac{\mathrm{d}h}{\sqrt{len^2+\mathrm{d}h^2}}\,G$$

where len is the distance from the current position to the neighbour giving the maximum angle of inclination, dh is the difference in height between the current position and the neighbour and G is gravity.

Modification of $\mathbf{v}(t+\Delta t)$

In the case where the velocity vector is not tangential to the ground surface (see Figure 1), modification is carried out via the equation

$$\mathbf{v}' = (\mathbf{v}_{\mathrm{unit}}\cdot\mathbf{v})\,\mathbf{v}_{\mathrm{unit}}$$

where $\mathbf{v}_{\mathrm{unit}}$ is the unit vector tangential to the ground surface. If the vector $\mathbf{v}(t+\Delta t) = \mathbf{v}'$ goes upward of $\mathbf{v}(t)$ (see Figure 1(c)), it is assumed that water particles

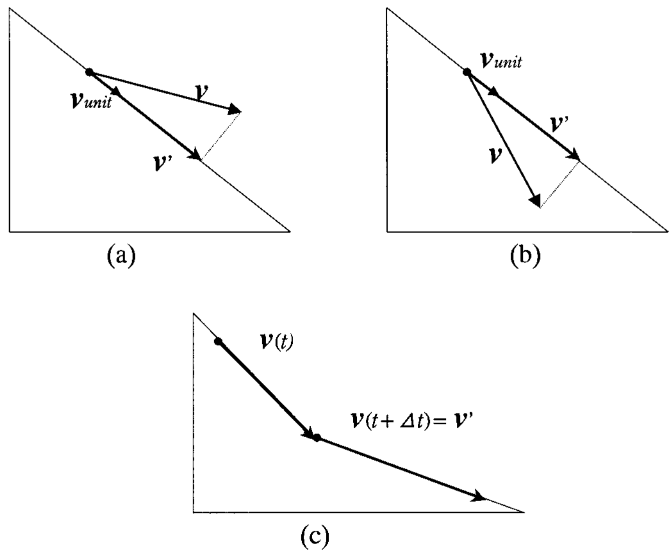

Figure 1. Modification of velocity vector

collide with the ground surface. In this case the energy is calculated by the following equation, in which Δv is the decrease in velocity:

$$e = m\ \Delta v^2/2, \quad m \propto w$$

Details of Step 2(a)

1. Arrange water particles on each grid point of **T** and apply the following step to every water particle.
2. Repeat the following steps until the water particle goes out of range of **T** or stops.
   (a) Determine the velocity $\mathbf{v}$ and the position $\mathbf{x}(t + \Delta t)$ according to the above description.
   (b) Find the digital line segment $\mathbf{x}(t) - \mathbf{x}(t + \Delta t)$ aand add w, $\mathbf{v}$ and e to the elements of the arrays **W**, **V** and **E** respectively corresponding to the digital line segment.
3. Let $\mathbf{v}_{ij} = \mathbf{v}_{ij}/n$ for every (i, j), where $\mathbf{v}_{ij} = \mathbf{V}(i, j)$ and n denotes the total number of passing water particles.

2.2. Erosion, transportation and sedimentation processes

The main physical effects of the flow of water are erosion, transportation and sedimentation. We define these effects based on the velocity field. In our model these effects are defined by the flow quantity Q_{ij} and the collision energy e_{ij} $(= \mathbf{E}(i, j))$.

Here the flow quantity is the quantity of water that passes through a unit section of the river in unit time. The flow quantity is defined by the following equation, in which w_{ij} $(= \mathbf{W}(i, j))$ and $\mathbf{v}_{ij}$ have already been calculated by the computation of the

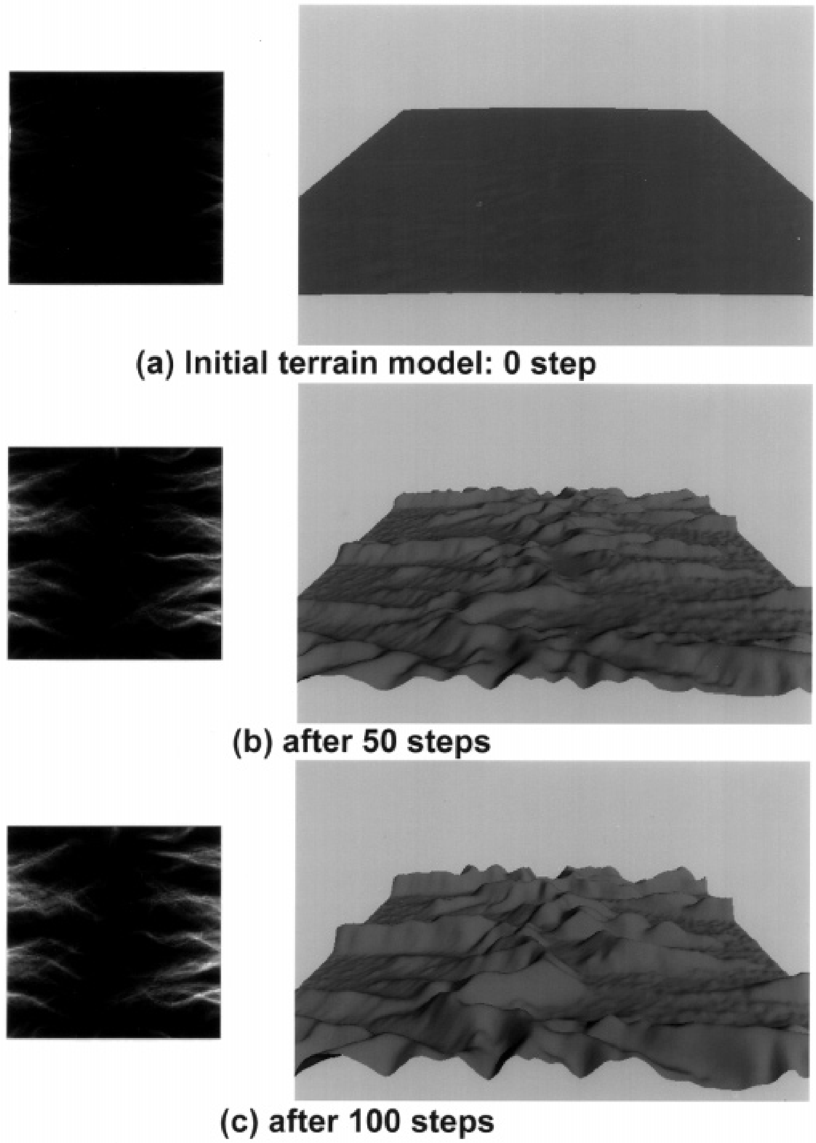

Figure 2. Erosion process with small K_q and small K_e Left images show corresponding velocity fields

velocity field and the constant C is the interval of grid points corresponding to the digital terrain model **T**:

$$Q_{ij} = C\, w_{ij}\, |\mathbf{v}_{ij}|$$

Here $\mathbf{v}_{ij}$, w_{ij} and e_{ij} are the values at the point (i, j) stored in the arrays **V**, **W** and **E** respectively.

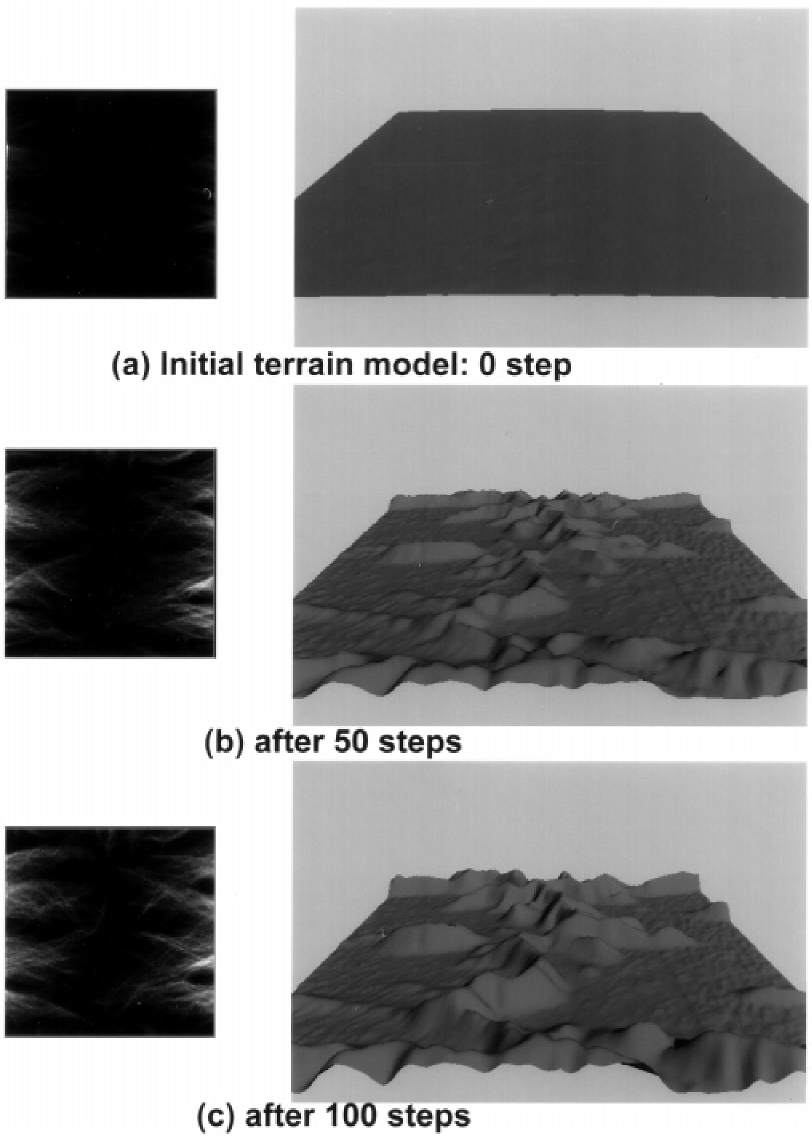

Figure 3. Erosion process with large K_q and small K_e. Left images show corresponding velocity fields

Erosion processes

There are several kinds of complex erosion processes changing the appearance of terrain.[7] Our model considers only one type of erosion of the ground surface, namely *surface run-off* caused by overland flow. We assume that the amount of eroded ground depends on the flow quantity Q and the energy e of water flow decreased by the collision of water with the ground surface. The amounts S_q and S_e of eroded ground caused by these two types of physical quantities are defined by the following equations, in which the coefficients K_q and K_e correspond to the geological features of the ground:

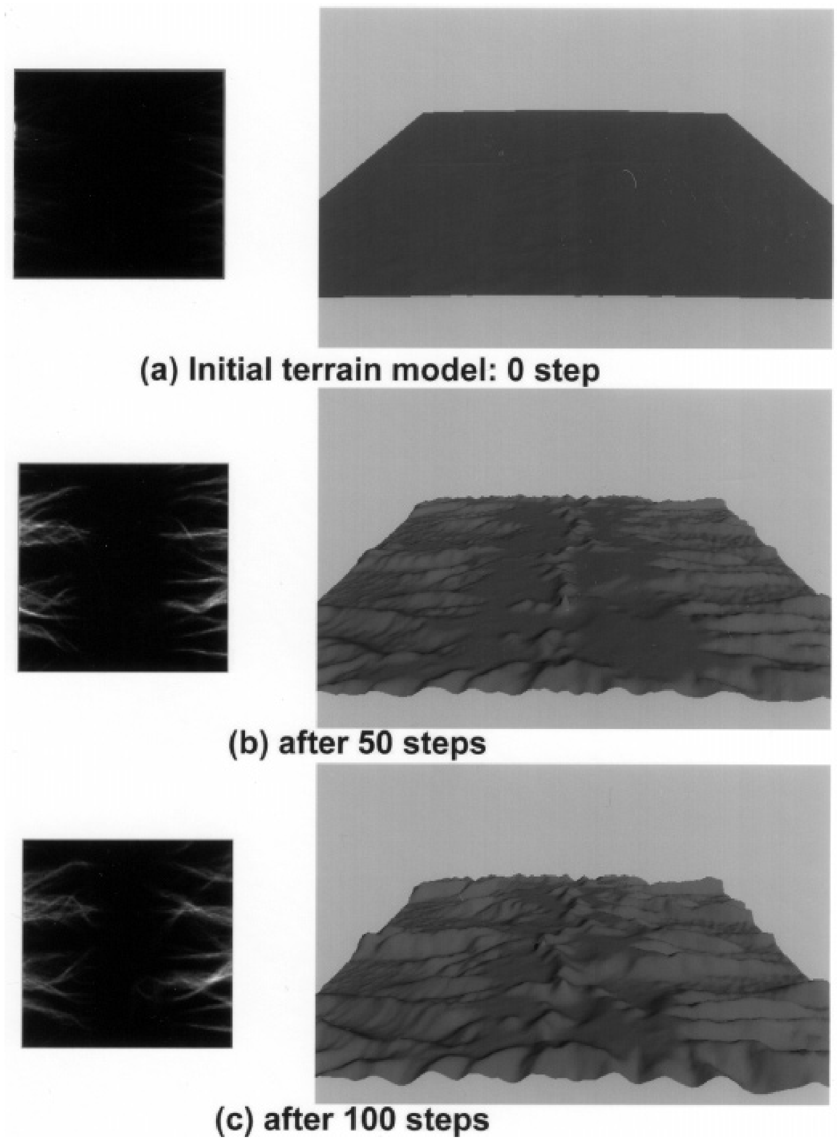

Figure 4. Erosion process with small K_q and large K_e. Left images show corresponding velocity fields

$$S_q = K_q Q$$
$$S_e = K_e e$$

Processes of transportation and sedimentation

We assume that the relationship between the quantity S of suspended earth and sand particles transported by the river and the quantity Q of water flow is given by

$$S = \gamma Q^f$$

Table I Simulation parameters and computation statistics

	Parameters			Size of array	Computation time (s) (SGI O_2 (R10000SC 175 MHz))
	$K>_q$	K_e	K_c		
Figure 2	$1{\cdot}0 \times 10^{-5}$	$1{\cdot}0 \times 10^{-5}$	1·0	128×128	7062
Figure 3	$1{\cdot}0 \times 10^{-3}$	$1{\cdot}0 \times 10^{-5}$	1·0	128×128	5001
Figure 4	$1{\cdot}0 \times 10^{-5}$	$1{\cdot}0 \times 10^{-3}$	1·0	128×128	6189
Plate 1	$1{\cdot}0 \times 10^{-5}$	$1{\cdot}0 \times 10^{-5}$	1·0	128×128	7062
Plate 2	$1{\cdot}0 \times 10^{-3}$	$1{\cdot}0 \times 10^{-3}$	1·0	128×128	5697
(layers from	$1{\cdot}0 \times 10^{-6}$	$1{\cdot}0 \times 10^{-6}$	1·0		
top)	$1{\cdot}0 \times 10^{-2}$	$1{\cdot}0 \times 10^{-2}$	1·0		
	$1{\cdot}0 \times 10^{-4}$	$1{\cdot}0 \times 10^{-4}$	1·0		
	$1{\cdot}0 \times 10^{-3}$	$1{\cdot}0 \times 10^{-3}$	1·0		

where γ is a coefficient and f is a constant.[7,8] Here the quantity of suspended earth and sand particles is the quantity of earth and sand that passes through a unit section of the river in unit time. The maximum transportable quantity of eroded earth and sand is defined as

$$S_{\max} = K_c Q^f$$

where K_c is a coefficient.

In the following equation the quantity of eroded earth and sand is expressed as $S_q + S_e$, and S_k is the quantity of earth and sand transported from another point k. The total quantity S_t of earth and sand is then calculated as

$$S_t = S_q + S_e + \Sigma S_k$$

where the sum is taken over all possible k.

Details of Step 2(b)

(Note that S_k is initially zero for every k). Apply the following steps to each grid point as many times as needed.

1. Determine S_t and $S_{\max}$ as defined above and decrease the elevation in **T** by $\delta(S_q + S_e)$, i.e. erosion, where δ is a coefficient to transfer the quantity of earth and sand to the elevation.
2. If $S_t > S_{\max}$, then add $\delta(S_t - S_{\max})$ to the elevation, i.e. sedimentation, and transport earth and sand of quantity $S_{\max}$ to the neighbour in the direction of the velocity vector in **V**; otherwise transport earth and sand of quantity S_t in the same manner.

3. SIMULATION EXAMPLE

Figure 2 shows the results of simulation at 50-step intervals using an initial terrain model that is a part of a cylinder covered with white noise. The figure shows the gradual formation of ridge and valley lines with the progression of simulation steps.

Although past erosion models only take into account the erosion due to the transfer of water, our model also takes into account the erosion caused by the collision of water with the ground surface. In order to clarify the effects of this erosion, Figures 3 and 4 show the results of simulation with different values of the coefficients K_q and K_e. As in Figure 2, the simulation patterns are shown at 50-step intervals.

In Figure 2, K_q is small and the erosion due to the transfer of water is suppressed, while in Figure 3, K_q is large and there is a great deal of erosion. This difference in K_q results in different topographical patterns. In Figure 2, clear ridge lines extending to the foot of the mountain can be seen, whereas in Figure 3 the ridges near the bottom of the mountain have been eroded, leaving only short ridges.

In Figure 4 the parameter of erosion due to collision, K_e, was increased. Compared with Figure 2, low-elevation regions have been heavily eroded, while the top of the mountain and ridges, regions in which collision erosion cannot easily occur, have retained their initial shape.

Plate 1 shows a created image of forest scenery in winter using the proposed simulation method. The ray-tracing method and volume-rendering method were applied to the three-dimensional texture method to create this image.[9] Although both the mountains and trees are imaginary, the image gives the impression of real natural scenery.

Plate 2 shows an image of terrain generated by eroding ground consisting of geologic strata. There are various types of layers having different hardnesses and thicknesses.

Table 1 gives the parameters and computation statistics used for the simulation.

4. CONCLUSIONS

We proposed a new method of simulating topographical formation using velocity fields of water flow as a modelling method for creating imaginary mountain scenery. Examples of simulation by this new method were presented. With further improvement it is thought that this model may become a sophisticated method for simulating various types of eroded topography.

Topics for future study include (i) improvements to the model to make it possible to create hierarchical structures of ridge lines of various scales and (ii) improvements to the model to be able to control the characteristic size of ridge lines in a wide range.

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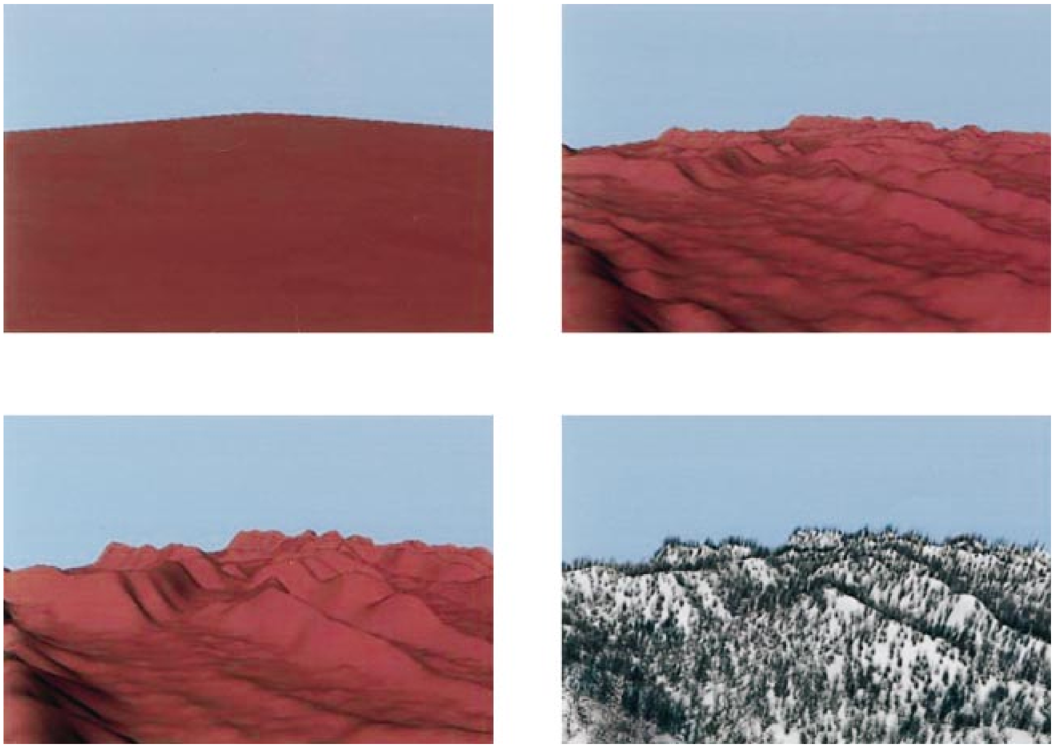

Plate 1 (Chiba et al.). Simulation of forest scenery in winter.

Plate 2 (Chiba et al.). Simulation with geologic strata.